Explain IB Math AA HL Like I'm 5

First Last

Compiled: December 24, 2021

Contents

Preface

Hello and prepare for the hardest two year (or if you're a sadist, one year) **course for a long while**! It's IB Math AA HL, the best course known to at least some portion of the human race!
This course is being written in LaTeX. So please excuse the occasional mishap that occurs throughout the document.
Here is every formula given to you in the booklet. Don't throw this book into the rubbish bin out of sheer agony just yet, as I will go over *every* formula in this booklet as we progress through each topic. (except for the prerequisites, you should already know them from previous maths classes) By the end of this (hopefully) small book, you should be able to understand a large portion of IB Math AA HL.[1]
However, it is up to *you* to get a 7 on the exam. You need to study very hard in order to get a high achieving score.

You can find the AA HL Formula Booklet here.

I'm currently a Math AA SL student, and this seems pretty cool. How can I join AA HL?

Well, first off I would suggest meeting with your coordinator to discuss about possibly going up to HL. There is about 90 extra teaching hours of content to go through for HL. Depending on how your IB School structures their maths courses, it may be possible to go from one level to another as AA SL is a subset of AA HL. (If you don't know what a *subset* is, we'll discuss it in Topic 4.)

What's the point of this PDF?

I made this document to assist people who are going to take the IB Math AA HL exam, which began examination in May 2021 and ends in May 2028[2], but do not have a proper HL course.
e.g: Combined SL and HL classes or an HL class doesn't exist, but is available for examination.

[1] This does NOT mean you can skip buying a textbook! Try getting the books that are in the "Further Reading" section.
[2] It's just a somewhat random guess, I think they change it every 7 or so years.

Alright, assuming you have chosen this course and accepted the consequences of life, you may be asking yourself,

How is this "class" even structured?

Based off of what IB recommends, SL AA should have about 150 hours of teaching. However, HL AA has an extra 90 additional hours of content. Although it is not necessary for SL students to read through this HL document, it may be worth reading for enrichment.

This document, which only goes over the **HL** content, should be completed in around 6 months for those that recently finished the SL content or **10** months for those who are reading this document along with an SL class.

Topic 1: Algebra

Brief Overview

By the end of this topic, you should be able to: This topic achieves the following from the IB Analysis and Approaches Guide:

- AHL 1.10: Counting Principles (Including Permutations and Combinations)
- AHL 1.11: Partial Fractions
- AHL 1.12: Complex Numbers and the Complex plane
- AHL 1.13: Modulus-Arugment and Euler's Form of Complex Numbers
- AHL 1.14: Complex Conjugate Roots and DeMoivre's Theorem
- AHL 1.15: Additional Mehtods of Proofs
- AHL 1.16: Solutions of linear equations

Which means in the first section, we will go over combinations and permutations (which will help immensely in *Topic 4*. From there, we will discuss what happens if we cannot simplify a linear fraction and how to rectify those fractions. Then, we discuss about how we can extend our knowledge about numbers by expanding outside of the real numbers and into the complex plane. There, we have three separate ways of writing out complex numbers: $z = a + bi$, $z = r(\text{cis}(\theta))$, and $re^{i\theta}$. Finally, the topic ends with solving for the roots of complex numbers, which is generalised by using De Moivre's theorem.

Formulae for this Topic

Name	Formula	Meaning of Each Symbol
The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$	u_1 = first term in sequence d = difference between iterations
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d)$ $S_n = \frac{n}{2}(u_1 + u_n)$	u_1 = first term in sequence u_n = nth term of sequence d = difference between iterations
The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$	u_1 = first term in sequence u_n = nth term of sequence r = *ratio* between iterations
Sum of n terms of a **finite** geometric sequence	$S_n = \frac{u_1(r^n-1)}{r-1}$ $S_n = \frac{u_1(1-r^n)}{r-1}$	u_1 = first term in sequence r = *ratio* between iterations $r \neq 1$
Compound interest	$FV = PV(1 + \frac{r}{100k})^{kn}$	FV = Future Value PV = Present Value k = compounds in a year r% nominal annual rate of interest
The sum of an **infinite** geometric sequence	$S_\infty = \frac{u_1}{1-r}$	S_∞ = Sum at "∞'th" iteration u_1 = first term of sequence r = *ratio* between iterations
Binomial Theorem ($x \in \mathbb{N}$)	$(a+b)^n =$ $a^n + \binom{n}{1}a^{n-1}b^1 + ...$ $+\binom{n}{r}a^{n-r}b^r + ... + b^n$	$\binom{n}{r}$ = Combination of n and r
Binomial coefficient	$\binom{n}{r} = {}^nC_r = \frac{n!}{r!-(n-r)!}$	$\binom{n}{r}$ = Combination of n and r

Additional Formulae for AA HL

Name	Formula	Meaning of Each Symbol
Combinations	${}^nC_r = \frac{n!}{r!(n-r)!}$	n = total r = total to choose from
Permutations	${}^nP_r = \frac{n!}{(n-r)!}$	n = total r = total to choose from
Extension of Binomial Theorem $n \in \mathbb{Q}$	$(a+b)^n = a^n(1 + n\frac{b}{a} + \frac{n(n-1)}{2!}\frac{b}{a}^2 + ...)$	a = 1st term b = 2nd term n = Power of binomial
Complex Numbers	$z = a + bi$	a = Real portion of z b = Imaginary portion of z
Modulus-argument Form (Euler's Form)	$z = r(\cos\theta + i\sin\theta) = r(\operatorname{cis}\theta)$	θ can be in radians or degrees $\operatorname{cis}\theta = \cos\theta + i\sin\theta$
De Moivre's Theorem	$z^n = [r(\cos(\theta) + i\sin(\theta))]^n$ $= r^n(\cos(n\theta) + i\sin(n\theta)$ $= r^n e^{in\theta} = r^n \operatorname{cis}(n\theta)$	r = modulus of z θ = argument of z n = nth power of z

Chapter 1

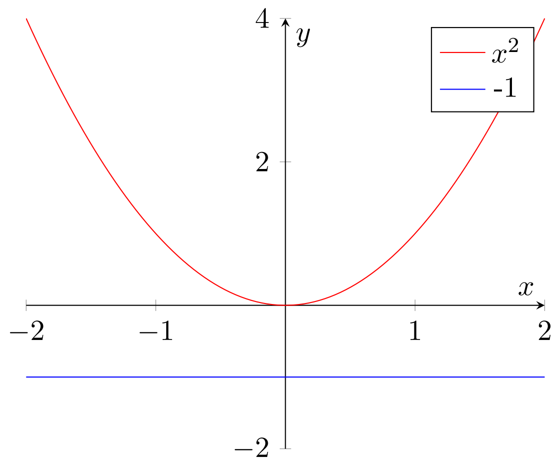
An Introduction to Complex Numbers

1.1 An intuitive way of defining the 'imaginary' numbers.

As you may possibly know from previous maths courses, the largest set that has been discussed so far is the set of the real numbers ($\mathbb{R}$), which contains such elements as:

$$1, \pi, \text{ and } \frac{98763}{4865}.$$

One scenario that was (most likely) discussed during a previous maths course (or AA SL) is to discard discriminants that are less than 0. Now attempt to consider a real number that would have a negative output. This would imply that $x^2 = n$ where n is some number in the negative reals. Now, look at the following graph.



The definition of a complex number.

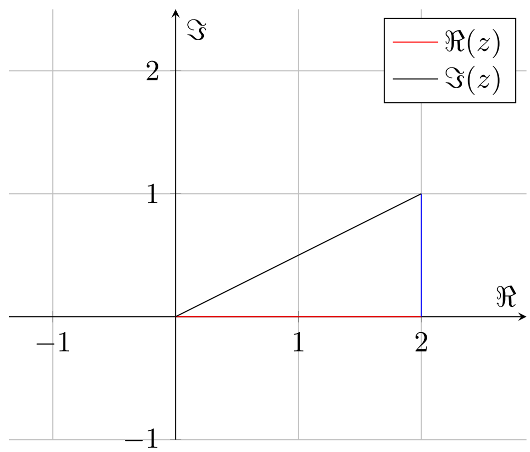
Consider a function, z that has two components: a *real* component, and its *imaginary* component. As place holders, we usually use the real component's variable as a and the imaginary component as b.

Therefore, we can write the equation as $z = a + b\,i$.

As seen here, there is no number in the reals that can intersect with the parabola, so we need to define a number that does so. Therefore we have to create a new type of number, one that makes $x^2 = -1$ possible. We call this number "i". We define these types of numbers as the possible solutions for *negative* square roots. However, we generally call them complex numbers.

1.2 The Complex Plane

Now consider how we would be able to graph those complex numbers. Instead of using an x-y plane, we use a $\Re$ and $\Im$ plane, with the axes for real numbers and imaginary numbers respectively. It looks very similar to a plane you are very used to at this point. **However**, there is a very important distinction to make between the Cartesian Plane that you're used to and the Argand Plane. You have to consider the equation by it's separate parts. Imagine for example, $z = 2 + i$. You could consider the equation as a little point on the plane, which you can add the components of the real and imaginary parts together to get to that point. [1] A similar way to consider imaginary numbers on the plane is to consider it a lot like a coordinate point.



$\Re$ and $\Im$ specify which part of the complex number is used. For example, with $z = 2 + i$, the real portion of the complex number would be 2, and the imaginary portion of the complex number would be 1, as there is 1 imaginary unit in z.
It is also important to consider when a z is purely *imaginary* or purely *real*. Now, before you spend 20 minutes trying to write a fancy $\Re$ or $\Im$ on your exams, it is possible to write "Re(z)" and "Im(z)" as is and still convey the same meaning. As seen with the previous figure, the components of complex numbers were $\Re(z)$ and $\Im(z)$. It is imperative to understand complex numbers as a set of coordinates that can be separated while problem solving.

1.3 Euler's Form

This topic will only make sense after going through the Trigonometry section. As you may recall a unit circle is defined as $x^2 + y^2 = 1$. *

[1] If you have already studied vectors, think of it like a vector.

$\Re$ and $\Im$

Consider a function, z that has two components: a *real* component, and its *imaginary* component. The $\Re$ and $\Im$ give values that correspond with the real or imaginary portion of the function respectively.

$$z = a + b\,i$$
$$\therefore \Re(z) = a;\ \Im(z) = b$$

Topic 2: Functions

Topic 3, Part 1: Trigonometry

Topic 3, Part 2: Geometry and Vectors

Topic 4: Statistics

Topic 5: Calculus

Exam Prep