

# ENM 5310: Data-driven Modeling and Probabilistic Scientific Computing

## *Lecture #5: Linear regression*

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$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

# Supervised learning

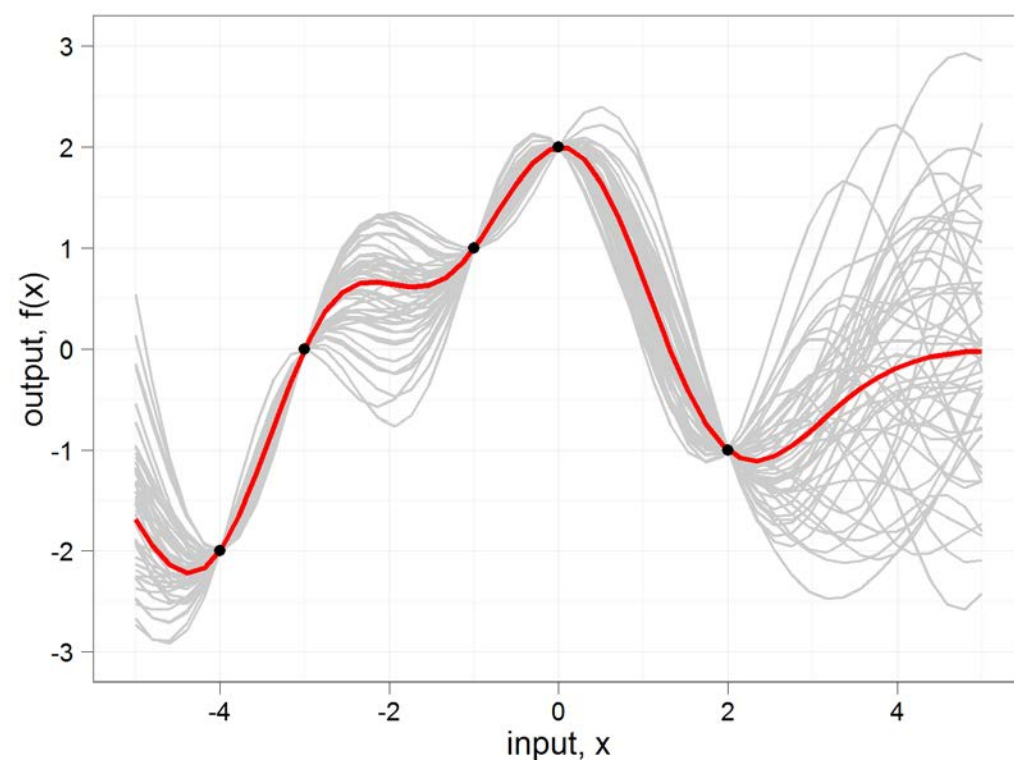
$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

$$\mathcal{D} = \{x, y\}, \quad x \in \mathcal{X}, \quad y \in \mathcal{Y}$$

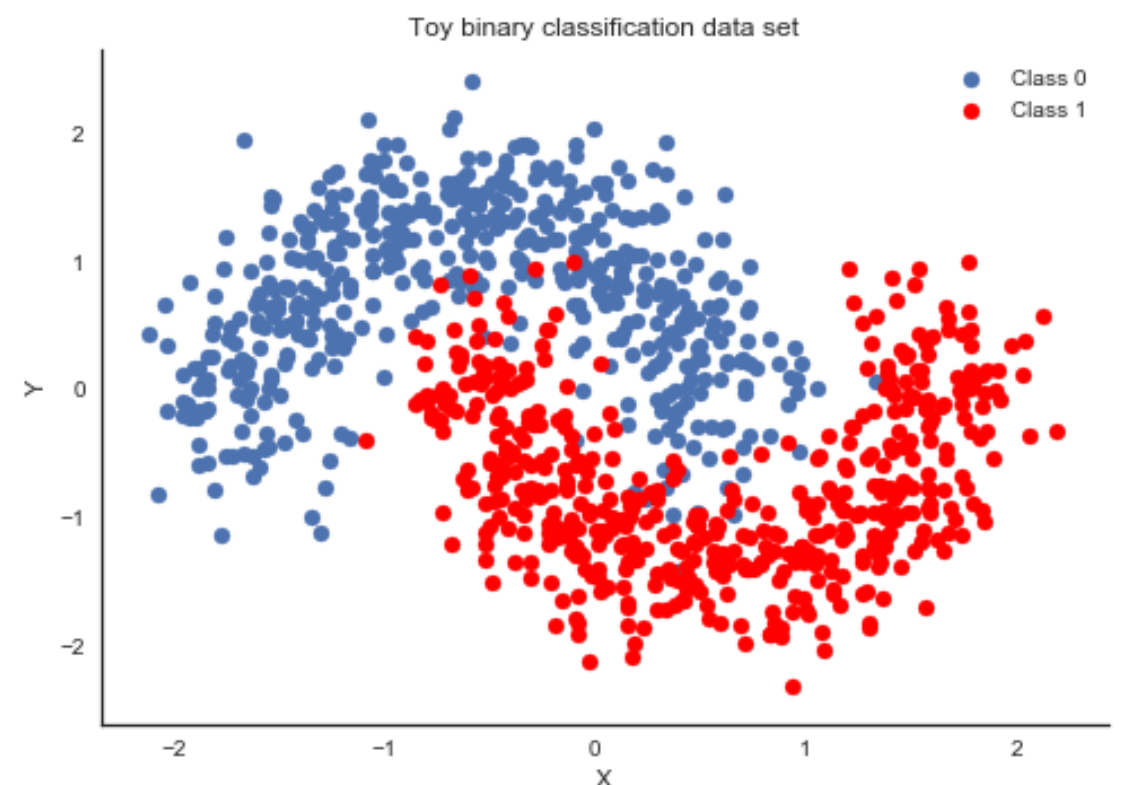
$$y = f(x) + \epsilon$$

$$p(f(x^*)|x^*, \mathcal{D})$$

Regression



Classification



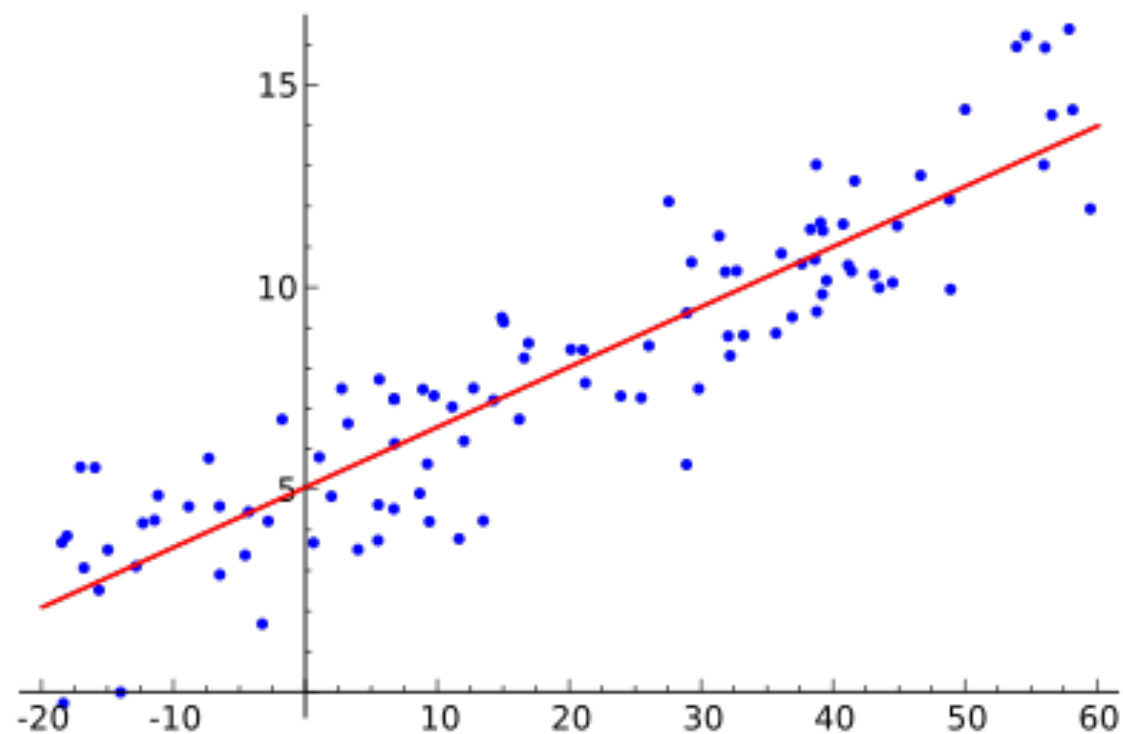
# Linear regression

$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

$$\mathcal{D} = \{x, y\}, \quad x \in \mathcal{X}, \quad y \in \mathcal{Y}$$

$$y = f(x) + \epsilon$$

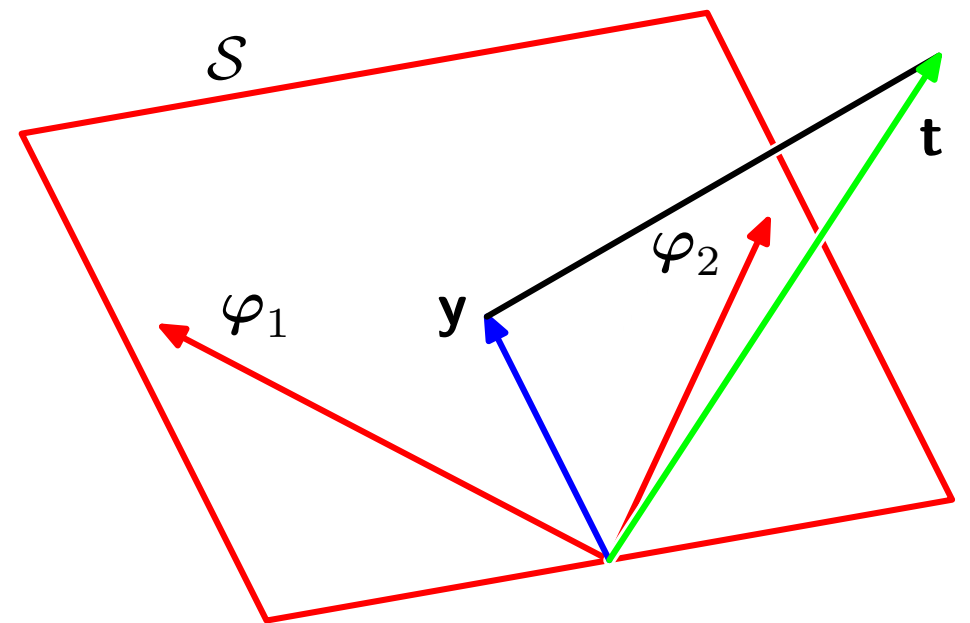
$$f(x) = w^T x$$



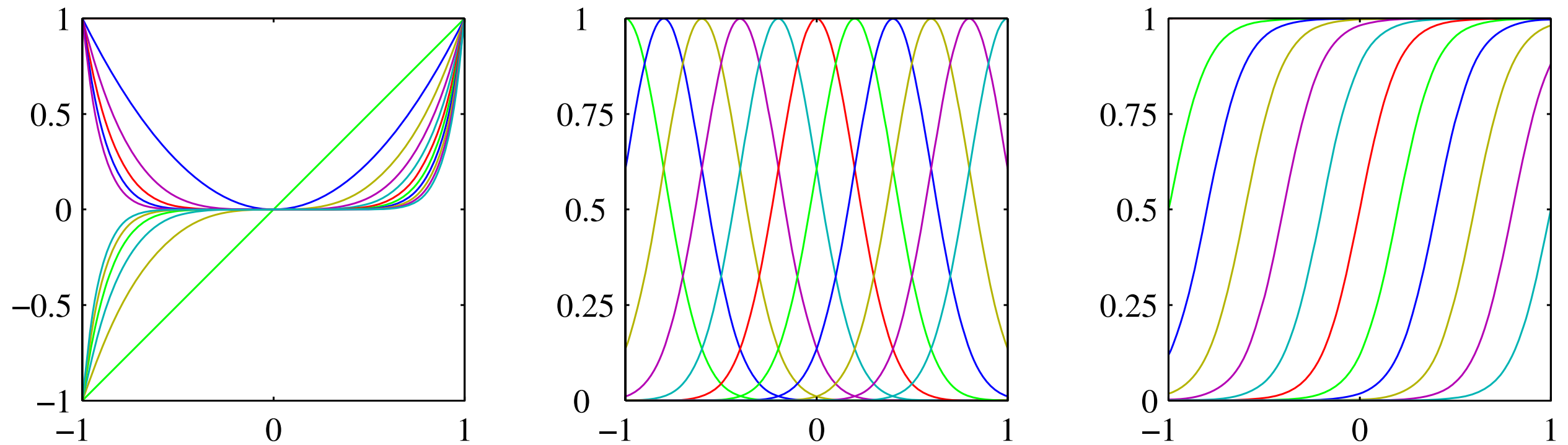
*“It’s not just about lines and planes!”*

# Geometrical interpretation

**Figure 3.2** Geometrical interpretation of the least-squares solution, in an  $N$ -dimensional space whose axes are the values of  $t_1, \dots, t_N$ . The least-squares regression function is obtained by finding the orthogonal projection of the data vector  $\mathbf{t}$  onto the subspace spanned by the basis functions  $\phi_j(\mathbf{x})$  in which each basis function is viewed as a vector  $\varphi_j$  of length  $N$  with elements  $\phi_j(\mathbf{x}_n)$ .

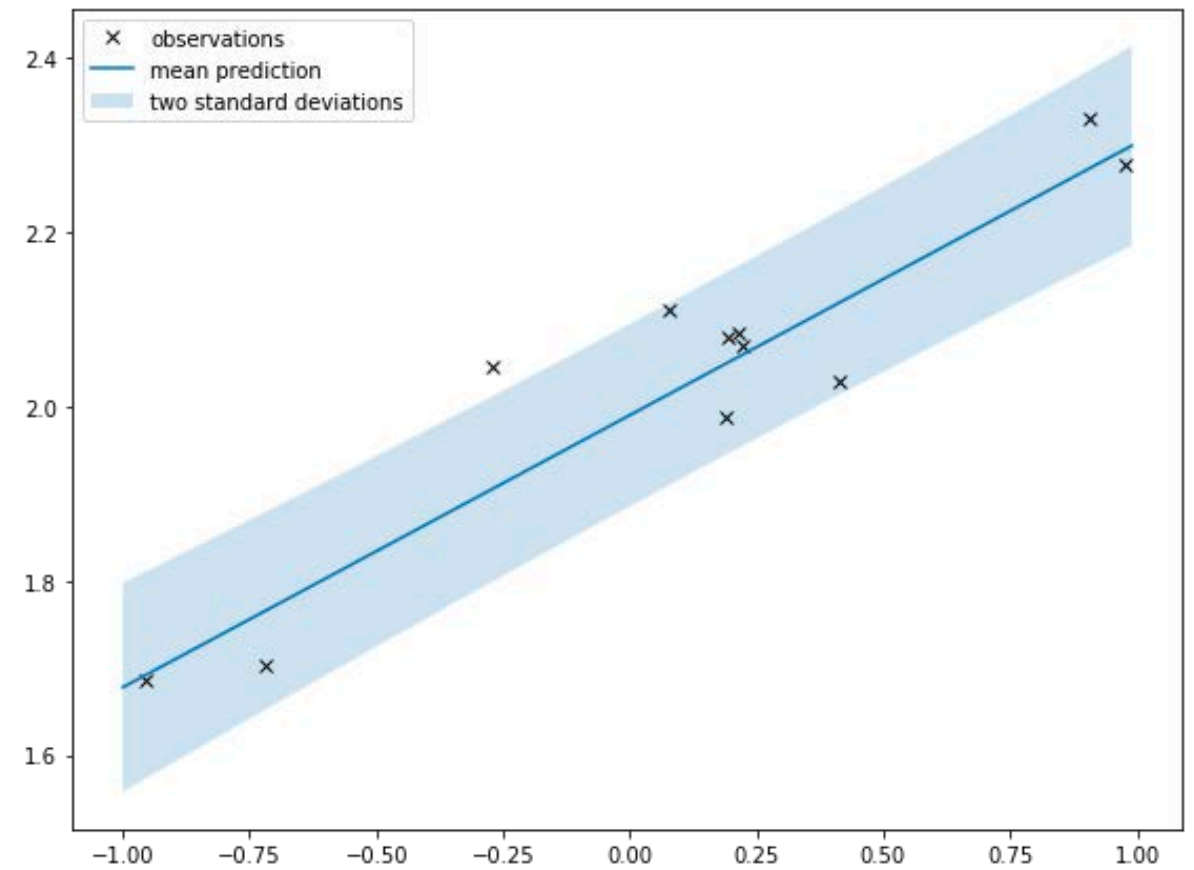
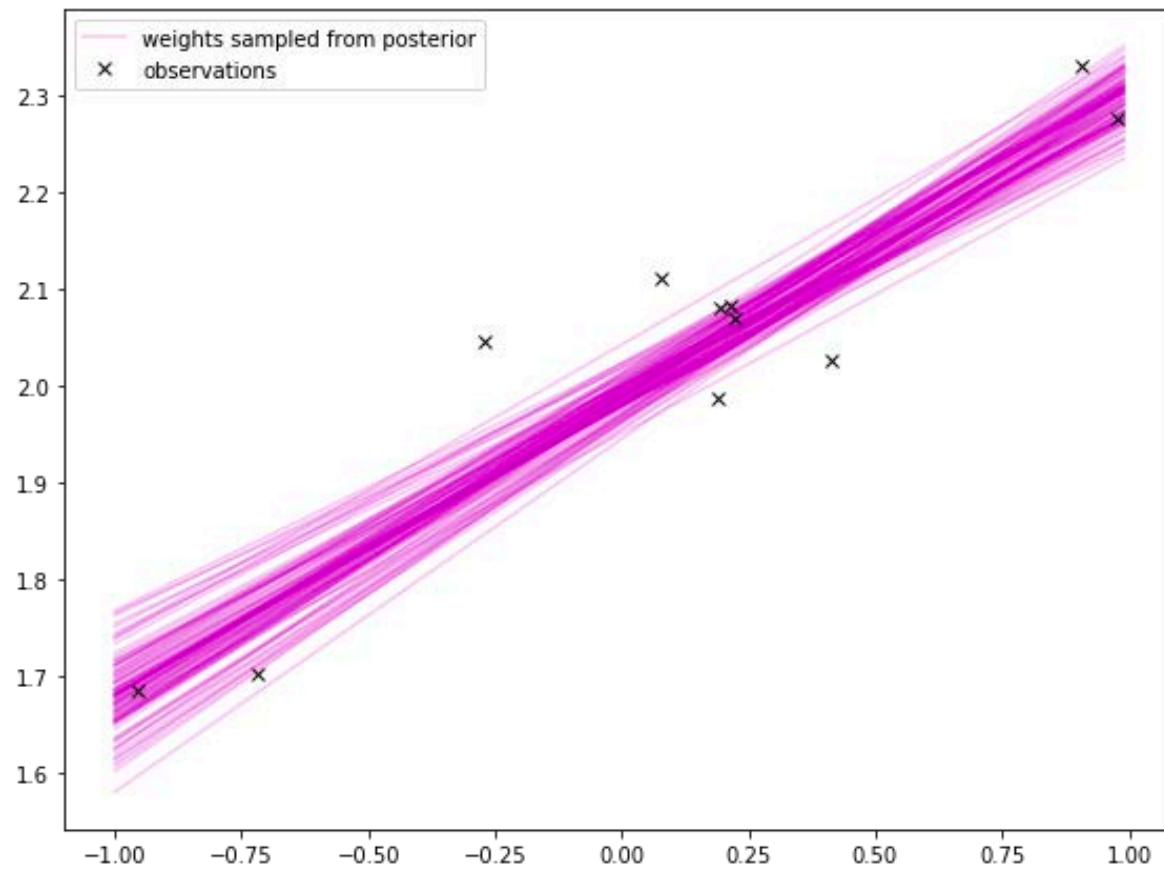


# Linear regression with basis functions

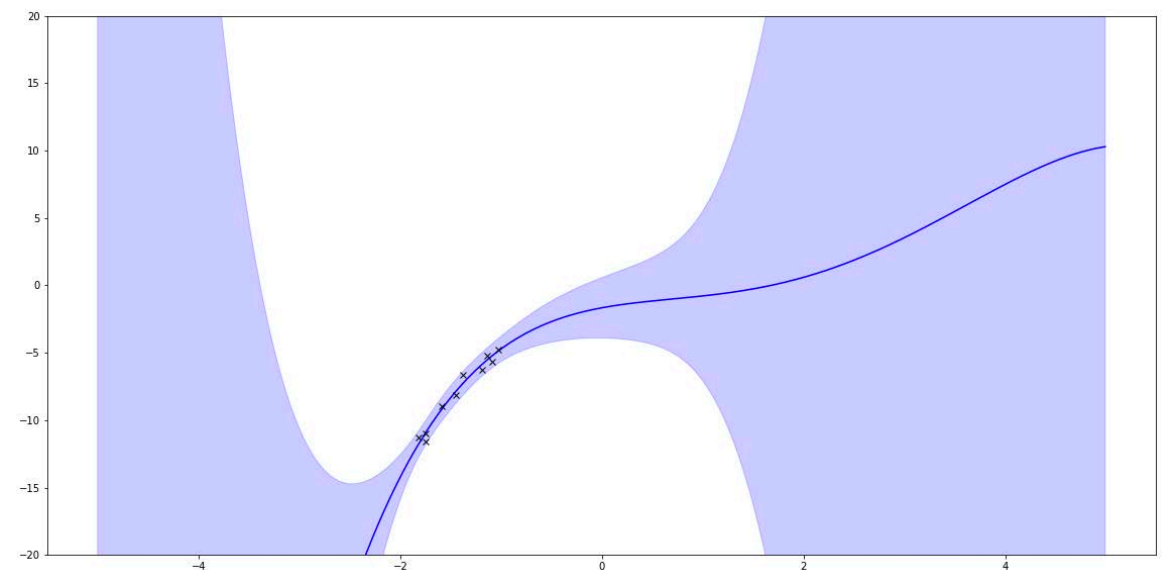
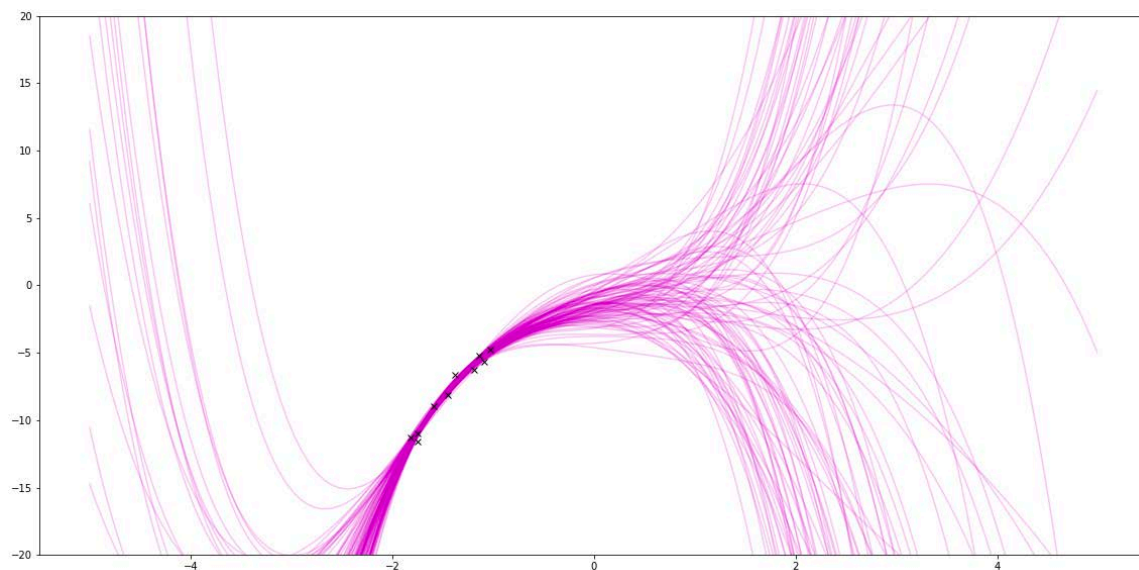


**Figure 3.1** Examples of basis functions, showing polynomials on the left, Gaussians of the form (3.4) in the centre, and sigmoidal of the form (3.5) on the right.

# Bayesian linear regression with basis functions



Nonlinear functions can be approximating using basis functions (or features)



$$\mathbf{y} = \mathbf{w}^T \phi(\mathbf{x}) + \epsilon$$