



AMRITA
VISHWA VIDYAPEETHAM

DESIGN AND ANALYSIS OF
ALGORITHMS
LAB WORKBOOK
WEEK - 5

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ROLL NUMBER : CH.SC.U4CSE24140

CLASS : CSE-B

Question 1:

Construct an AVL tree with these numbers:

157, 110, 147, 122, 149, 151, 111, 141, 112, 123, 133, 117

- (i) Print the tree showing each level.
- (ii) Check that all nodes are balanced.

CODE:

```
1 //CH.SC.U4CSE24140 - Rohith S
2 #include<stdio.h>
3 #include<stdlib.h>
4
5 struct Node{
6     int key;
7     struct Node*left;
8     struct Node*right;
9     int height;
10 };
11
12 int maximum(int a,int b){
13     return(a>b)?a:b;
14 }
15
16 int height(struct Node*n){
17     if(n==NULL)
18         return 0;
19     return n->height;
20 }
21
22 struct Node*newNode(int key){
23     struct Node*node=(struct Node*)malloc(sizeof(struct Node));
24     node->key=key;
25     node->left=NULL;
26     node->right=NULL;
27     node->height=1;
28     return node;
29 }
30
31 struct Node*rightRotate(struct Node*y){
32     struct Node*x=y->left;
33     struct Node*T2=x->right;
34     x->right=y;
35     y->left=T2;
36     y->height=maximum(height(y->left),height(y->right))+1;
37     x->height=maximum(height(x->left),height(x->right))+1;
```

```
38     |     return x;
39 }
40
41 struct Node*leftRotate(struct Node*x){
42     struct Node*y=x->right;
43     struct Node*T2=y->left;
44     y->left=x;
45     x->right=T2;
46     x->height=maximum(height(x->left),height(x->right))+1;
47     y->height=maximum(height(y->left),height(y->right))+1;
48     return y;
49 }
50
51 int getBalance(struct Node*n){
52     if(n==NULL)
53         return 0;
54     return height(n->left)-height(n->right);
55 }
56
57 struct Node*insert(struct Node*node,int key){
58     if(node==NULL)
59         return newNode(key);
60
61     if(key<node->key)
62         node->left=insert(node->left,key);
63     else if(key>node->key)
64         node->right=insert(node->right,key);
65     else
66         return node;
67
68     node->height=1+maximum(height(node->left),height(node->right));
69
70     int balance=getBalance(node);
71
72     if(balance>1&&key<node->left->key)
73         return rightRotate(node);
```

```
75     if(balance<-1&&key>node->right->key)
76         return leftRotate(node);
77
78     if(balance>1&&key>node->left->key){
79         node->left=leftRotate(node->left);
80         return rightRotate(node);
81     }
82
83     if(balance<-1&&key<node->right->key){
84         node->right=rightRotate(node->right);
85         return leftRotate(node);
86     }
87
88     return node;
89 }
90
91 int isBalanced(struct Node*root){
92     if(root==NULL)
93         return 1;
94
95     int bf=getBalance(root);
96
97     if(bf>1||bf<-1)
98         return 0;
99
100    return isBalanced(root->left)&&isBalanced(root->right);
101 }
102
103 int treeHeight(struct Node*root){
104     if(root==NULL)
105         return 0;
106     return 1+maximum(treeHeight(root->left),treeHeight(root->right));
107 }
```

```

109 void printLevel(struct Node*root,int level){
110     if(root==NULL)
111         return;
112     if(level==1)
113         printf("%d ",root->key);
114     else{
115         printLevel(root->left,level-1);
116         printLevel(root->right,level-1);
117     }
118 }
119 }
120
121 void printTreeLevels(struct Node*root){
122     int h=treeHeight(root);
123     for(int i=1;i<=h;i++){
124         printf("Level %d: ",i-1);
125         printLevel(root,i);
126         printf("\n");
127     }
128 }
129
130 int main(){
131     struct Node*root=NULL;
132
133     int keys[]={157,110,147,122,149,151,111,141,112,123,133,117};
134     int n=sizeof(keys)/sizeof(keys[0]);
135
136     for(int i=0;i<n;i++)
137         root=insert(root,keys[i]);
138
139     printf("AVL Tree Level-wise:\n");
140     printTreeLevels(root);
141
142     if(isBalanced(root))
143         printf("\nAll nodes are balanced. AVL Tree is valid.\n");
144     else
145         printf("\nTree is NOT balanced.\n");
146
147     return 0;
148 }
149

```

OUTPUT:

```

AVL Tree Level-wise:
Level 0: 122
Level 1: 111 147
Level 2: 110 112 133 151
Level 3: 117 123 141 149 157

```

All nodes are balanced. AVL Tree is valid.

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Time Complexity for

- (i) Search: O(log N)** : Tree height is $O(\log n)$, search follows one path from root to leaf.
- (ii) Insertion: O(log N)** : BST insertion $O(\log n)$ + at most 2 rotations $O(1) = O(\log n)$.
- (iii) Deletion: O (log N)** : BST deletion $O(\log n)$ + rebalancing up to $O(\log n)$ rotations= $O(\log n)$
- (iv) Traversal: O(N)** : Must visit all n nodes exactly once.
- (v) Rotation: O(1)** : Only changes a constant number of pointers.

Space Complexity: O(N) : Stores all the N nodes along with the data, pointer and height.

Question 2: Construct a Red-Black tree with the numbers:

157, 110, 147, 122, 149, 151, 111, 141, 112, 123, 133, 117

Print the tree showing R (red) or B (black) for each node. Check that:

- (i) Root is black**
- (ii) No red node has red children**

CODE:

```
1 //CH.SC.U4CSE24140 - Rohith S
2 #include<stdio.h>
3 #include<stdlib.h>
4
5 #define RED 1
6 #define BLACK 0
7
8 struct Node{
9     int data;
10    int color;
11    struct Node*left;
12    struct Node*right;
13    struct Node*parent;
14 };
15
16 struct Node*root=NULL;
17
18 struct Node*newNode(int data){
19     struct Node*node=(struct Node*)malloc(sizeof(struct Node));
20     node->data=data;
21     node->color=RED;
22     node->left=NULL;
23     node->right=NULL;
24     node->parent=NULL;
25     return node;
26 }
27
28 void leftRotate(struct Node*x){
29     struct Node*y=x->right;
30     x->right=y->left;
31     if(y->left!=NULL)
32         y->left->parent=x;
33     y->parent=x->parent;
34     if(x->parent==NULL)
35         root=y;
36     else if(x==x->parent->left)
37         x->parent->left=y;
```

```
38     else
39         |   x->parent->right=y;
40     y->left=x;
41     x->parent=y;
42 }
43
44 void rightRotate(struct Node*y){
45     struct Node*x=y->left;
46     y->left=x->right;
47     if(x->right!=NULL)
48         |   x->right->parent=y;
49     x->parent=y->parent;
50     if(y->parent==NULL)
51         |   root=x;
52     else if(y==y->parent->left)
53         |   y->parent->left=x;
54     else
55         |   y->parent->right=x;
56     x->right=y;
57     y->parent=x;
58 }
59
60 void fixInsert(struct Node*z){
61     while(z!=root && z->parent->color==RED){
62         if(z->parent==z->parent->parent->left){
63             struct Node*y=z->parent->parent->right;
64             if(y!=NULL && y->color==RED){
65                 z->parent->color=BLACK;
66                 y->color=BLACK;
67                 z->parent->parent->color=RED;
68                 z=z->parent->parent;
69             }else{
70                 if(z==z->parent->right){
71                     z=z->parent;
72                     leftRotate(z);
73                 }
74             }
75         }
76     }
77 }
```

```
74         z->parent->color=BLACK;
75         z->parent->parent->color=RED;
76         rightRotate(z->parent->parent);
77     }
78 }else{
79     struct Node*y=z->parent->parent->left;
80     if(y!=NULL && y->color==RED){
81         z->parent->color=BLACK;
82         y->color=BLACK;
83         z->parent->parent->color=RED;
84         z=z->parent->parent;
85     }else{
86         if(z==z->parent->left){
87             z=z->parent;
88             rightRotate(z);
89         }
90         z->parent->color=BLACK;
91         z->parent->parent->color=RED;
92         leftRotate(z->parent->parent);
93     }
94 }
95 }
96 root->color=BLACK;
97 }

98 void insert(int data){
99     struct Node*z=newNode(data);
100    struct Node*y=NULL;
101    struct Node*x=root;
102
103    while(x!=NULL){
104        y=x;
```

```
106     |         if(z->data<x->data)
107     |             x=x->left;
108     |         else
109     |             x=x->right;
110     }
111
112     z->parent=y;
113
114     if(y==NULL)
115         root=z;
116     else if(z->data<y->data)
117         y->left=z;
118     else
119         y->right=z;
120
121     fixInsert(z);
122 }
123
124 int treeHeight(struct Node*node){
125     if(node==NULL)
126         return 0;
127     int lh=treeHeight(node->left);
128     int rh=treeHeight(node->right);
129     return (lh>rh?lh:rh)+1;
130 }
131
132 void printLevel(struct Node*node,int level){
133     if(node==NULL)
134         return;
135     if(level==1)
136         printf("%d(%c) ",node->data,node->color==RED?'R':'B');
137     else{
138         printLevel(node->left,level-1);
139         printLevel(node->right,level-1);
140     }
```

```

141     }
142
143 void printTree(){
144     int h=treeHeight(root);
145     for(int i=1;i<=h;i++){
146         printf("Level %d: ",i-1);
147         printLevel(root,i);
148         printf("\n");
149     }
150 }
151
152 int checkRootBlack(){
153     if(root==NULL)
154         return 1;
155     return root->color==BLACK;
156 }
157
158 int checkNoRedRed(struct Node*node){
159     if(node==NULL)
160         return 1;
161
162     if(node->color==RED){
163         if((node->left!=NULL && node->left->color==RED) ||
164             (node->right!=NULL && node->right->color==RED))
165             return 0;
166     }
167
168     return checkNoRedRed(node->left) && checkNoRedRed(node->right);
169 }
170
171 int main(){
172     int keys[]={157,110,147,122,149,151,111,141,112,123,133,117};
173     int n=sizeof(keys)/sizeof(keys[0]);
174
175     for(int i=0;i<n;i++)
176         insert(keys[i]);
177
178     printf("Red-Black Tree Level-wise:\n");
179     printTree();
180
181     printf("\nCheck Conditions:");
182
183     if(checkRootBlack())
184         printf("(i) Root is Black: YES\n");
185     else
186         printf("(i) Root is Black: NO\n");
187
188     if(checkNoRedRed(root))
189         printf("(ii) No Red node has Red child: YES\n");
190     else
191         printf("(ii) No Red node has Red child: NO\n");
192
193     return 0;
194 }
```

OUTPUT:

```
Red-Black Tree Level-wise:
```

```
Level 0: 122(B)
Level 1: 111(R) 147(R)
Level 2: 110(B) 112(B) 133(B) 151(B)
Level 3: 117(R) 123(R) 141(R) 149(R) 157(R)
```

```
Check Conditions:
```

- (i) Root is Black: YES
- (ii) No Red node has Red child: YES

```
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```

Time Complexity for

- (i) Search:** $O(\log N)$: Red-Black properties guarantee tree height $\leq 2\log_2(n+1)$, so we traverse at most logarithmic levels from root to leaf.
- (ii) Insertion:** $O(\log N)$: BST insertion takes $O(\log n)$ to find the position, and fixing violations requires at most $O(\log n)$ recolouring plus at most 2 rotations ($O(1)$ each).
- (iii) Deletion:** $O(\log N)$: BST deletion takes $O(\log n)$ to find the node, and fixing violations requires at most $O(\log n)$ recolouring plus at most 3 rotations ($O(1)$ each).
- (iv) Traversal:** $O(N)$: We must visit every node exactly once in in order /preorder/ post order traversal, and there are n nodes.
- (v) Rotation:** $O(1)$: It only updates a constant number of pointers (parent, left, right) regardless of tree size.

Space Complexity: $O(N)$: Stores all the N nodes along with the data, pointer and colour information.