FINAL EXAM PREP, MATH 425A (F2022)

Exam Format: (Scored out of 125 points; 133 possible)

I reserve the right to alter the format and point distribution slightly. I will notify you if there are any major changes.

1. (20 points; recommended time: 15 minutes or less) Multiple choice and True/False
2. (25 points; recommended time: 20 minutes) Provide examples of mathematical objects satisfying certain properties. No proofs necessary
3. (12 points; recommended time: 10 minutes) Sets, Topology of metric spaces (3 short parts). Possible topics: Open/Closed sets, Interior/Closure, Compactness, Connectedness, Perfect Sets, etc.
4. (20 points; recommended time: 20 minutes) Series (2 parts). Possible topics: Convergence, Absolute Convergence, Power series, radius of convergence (including $\limsup$/$\liminf$), Ratio/Root Test.
5. (20 points; recommended time: 20 minutes) Uniform convergence, uniform continuity, etc. (2 parts). Have your ϵ's, δ's, and N's at the ready, and remember how to prove that uniform convergence does (or does not) hold.
6. (18 points; recommended time: 15 minutes) Riemann Integration, Fundamental Theorem of Calculus, etc. (2 parts. One will be like a quiz problem.)
7. (18 points; recommended time: 15 minutes) Prove a Theorem from class. Like in Exam 2, you'll be given a choice between two statements. The four possibilities are:
   - The Uniform Limit Theorem (the simpler version, which we proved in class).
   - The Weierstrass M-Test
   - The Fundamental Theorem of Calculus, version 1 (both statements)
   - The Integrable Limit Theorem

Other comments:

- The exam is slightly tougher than the midterms, and time will be tighter. (In Exam 2, you might have had to 'walk briskly.' The final exam is slightly faster-paced: perhaps a 'jog.')
- You'll notice that the total 'recommended time' adds up to 115 minutes, just shy of the 120 minute time limit. You can ensure that time is a little less tight by making sure you're well-prepared for problem 7, and working quickly through problem 1.
- Definitely work through the problems on the following page. They're intended to make you think about a few issues that you'll see on the exam, which were not emphasized in class.

(JUST A FEW) PRACTICE PROBLEMS

1. Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers such that the radius of convergence R of the series $\sum_{n=0}^{\infty} a_n z^n$ is 1.5. Prove that $a_n \to 0$ as $n \to \infty$. For which $R \in [0, \infty)$ does this conclusion hold?

2. Define $f : [-10, 10] \to \mathbb{R}$ by $f(x) = 1$ if $x \in [5, 6]$ or $x = -5$, and $f(x) = 0$ otherwise. Define $F : [-10, 10] \to \mathbb{R}$ by $F(x) = \int_{-10}^{x} f(t)\,\mathrm{d}t$. For which $x \in (-10, 10)$ is F differentiable at x? For which x is $F'(x) = f(x)$?

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, and define $f_n(x) = f(x/n)$ for each $n \in \mathbb{N}$.
   (a) Prove that $f_n(x) \to f(0)$ pointwise on $\mathbb{R}$.
   (b) Prove that $f_n \to f(0)$ uniformly on any bounded subset of $\mathbb{R}$.
   (c) Does $f_n \to f(0)$ uniformly on all of $\mathbb{R}$? If so, prove it; if not, give a counterexample.