Signals and Systems

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Chapter 1

Signal Analysis

B.Tech ECE: Signals and Systems **II-I Semester**

Chapter 1: Signal Analysis

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Signals can be used to describe a wide range of natural phenomena. A signal is generally imagined as a pattern of variations of some quantity with respect to another independent quantity. In the following section we shall look into the definition of a signal as well as a system along with some examples

1.1 Introduction

Signal It is defined as a function of any independent variable. Generally speaking, a signal is a function of time which conveys some sort of information.
E.g;

- Speech or Voice signals
- Image signals
- etc

System It is a collection of objects which work together to perform a particular task.
From a communications standpoint, systems are used to process signals.

1.2 Classification of Signals

If a signal is defined in terms of only one independent variable, it is called a one dimensional signal otherwise it is called a multi-dimensional signal.
in addition to dimensions,signals can be classified on the basis of various parameters:

On the basis of time t

Continuous Time Signals
A signal which is defined continuously for all values of time t is called a continuous time

signal. It is represented by $x(t)$ (in parentheses) and are also called analog signals.

Discrete Time Signals
A signal that is defined for only specific instances of time or at discrete values of time.It is represented by $x[n]$ (in brackets).It is generally obtained by sampling an analog signal.

These signals can be further classified as follows:

On the basis of periodicity

Periodic Signals : Continuous Time Periodic Signals
a signal is said to be CT-periodic if it repeats after a certain time interval T_0
Mathematically, it is defined as the signal which satisfies :

$$x(t) = x(t + T_0) \forall t \in R$$

Discrete Time Periodic Signals
a signal is said to be DT-periodic if it repeats after a certain time interval N_0
Mathematically, it is defined as the signal which satisfies :

$$x[n] = x[n + N_0] \forall n \in Z$$

Aperiodic Signals :
A signal that does not satisfy the above conditions is called aperiodic signal. It may be viewed as a limiting case of a periodic signal in which period tends to Infinity.

Even and Odd signals

a signal $x(t)$ is said to be even if it satisfies :

$$\begin{aligned} x(t) &= x(-t) \quad (CT) \\ x[n] &= x[-n] \quad (DT) \end{aligned}$$

a signal x(t) is said to be odd if it satisfies :

$$\begin{aligned} x(t) &= -x(-t) \quad (CT) \\ x[n] &= -x[-n] \quad (DT) \end{aligned}$$

if it satisfies neither it is said to be neither even nor odd.

Deterministic and Random Signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.

A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} |x\,(t)|^2 dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

$$\text{Power of energy signal} = 0$$
$$\text{Energy of power signal} =\infty$$

1.3 Standard Signals

There are some standard signals which are used repeatedly in signals and systems. Let us take a look on some of them.

Standard signals list:

1. Unit step signal
2. Unit Ramp signal
3. Unit parabolic
4. Signum
5. Real And Complex Exponential Signals
6. Sinusoidal Signals
7. Rectangular Signals
8. Triangular signal
9. Sampling Function
10. Sinc Function
11. Gaussian Pulse
12. Unit Impulse

Unit step signal

for continuous time :

$$x(t) = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

at t=0 there is a discontinuity. but generally it can be said that the value converges to 0.5.
for discrete time :

$$x[n] = \begin{cases} 1 & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

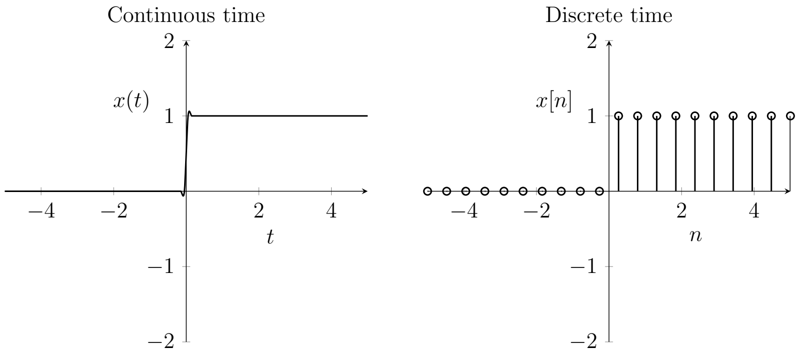

Figure 1.1: the unit step signal

Ramp signal

for continuous time :

$$x(t) = \begin{cases} t & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} n & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

note: The ramp signal can be related to the unit step by the following relation :

$$\frac{d}{dt} r(t) = u(t)$$

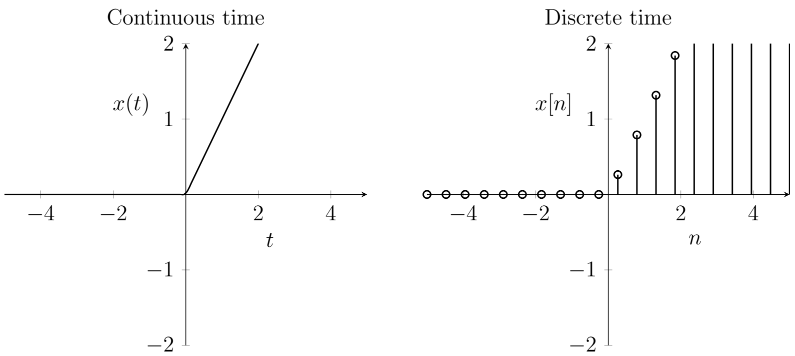

Figure 1.2: the ramp signal

Parabolic signal

for continuous time :

$$x(t) = \begin{cases} \frac{t^2}{2} & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} \frac{n^2}{2} & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

note: The parabolic signal can be related to the ramp signal by the following relation :

$$\frac{d}{dt}p(t) = r(t)$$

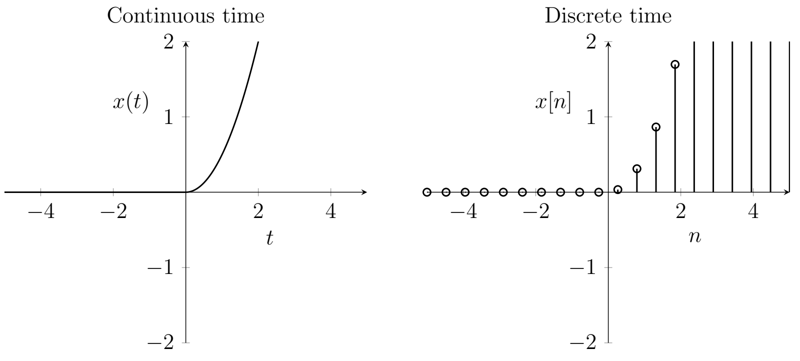

Figure 1.3: the parabolic signal

signum function

for continuous time :

$$x(t) = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t = 0; \\ -1 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t = 0; \\ -1 & \text{if } t < 0; \end{cases}$$

note: signum can be related to the unit step signal by the following relation :

$$u(t) = \frac{1}{2}(sgn(t) - 1)$$

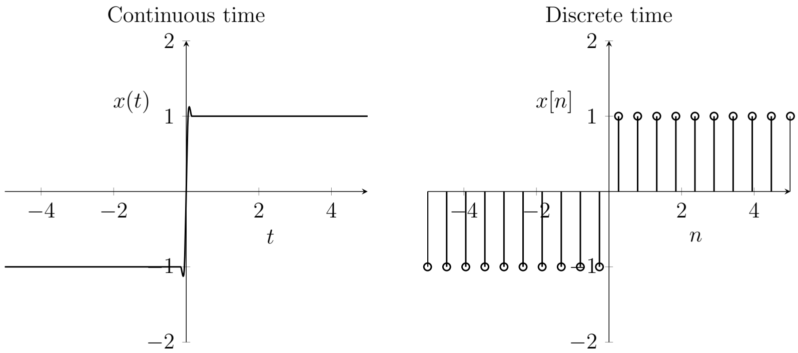

Figure 1.4: Signum Function

Real Exponential signal

for continuous time :

$x(t) = Ca^t$ for $a > 0$

for discrete time :

$x[n] = Ca^n$ for $a > 0$

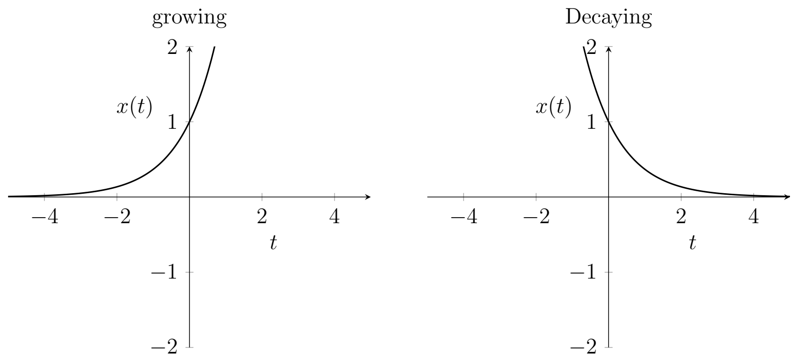

Figure 1.5: real exponential signal

Complex Exponential signal

for continuous time :

$x(t) = Ca^t \quad$ for $a < 0$

for discrete time :

$x[n] = Ca^n \quad$ for $a < 0$

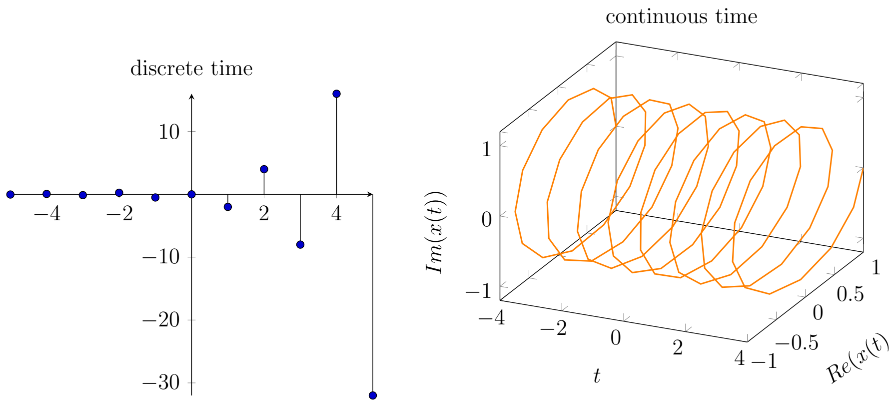

Figure 1.6: complex exponential signal

Sinusoidal signals

for continuous time :

$$x(t) = sin(\omega_0 t + \theta)$$

where, $\omega_0 = \frac{2\pi}{T_0}$ is the fundamental frequency and θ is the phase.

Rectangular signal a.k.a Gating pulse

for continuous time :

$$x(t) = \begin{cases} A & \text{if } \frac{-T}{2} < t < \frac{T}{2}; \\ 0 & \text{otherwise} \end{cases}$$

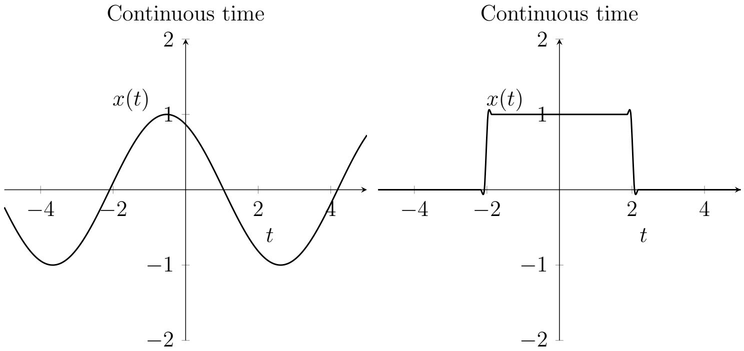

Figure 1.7: a sinusoid and rectangular signal

Triangular pulse

for continuous time :

$$x(t) = \begin{cases} \frac{A}{T}t + A & \text{if } T \leq t \leq 0; \\ \frac{-A}{T}t + A & \text{if } 0 \leq t \leq T; \end{cases}$$

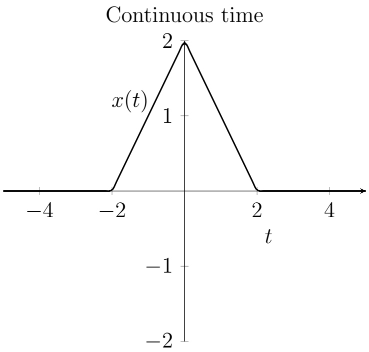

Figure 1.8: Triangular pulse

Sampling function $Sa(t)$

for continuous time :

$$Sa(t) = \frac{sin(t)}{t} \quad \forall t \in R$$

Sinc function $Sinc(t)$

The sinc function is a time scaled version of the sampling function. specifically , the time variable t is scaled as

$$t \longrightarrow \pi t$$

The full name of the function is "sine cardinal," but it is commonly referred to by its abbreviation, "sinc."
It is a function that arises frequently in signal processing and the theory of Fourier transforms.
for continuous time :

$$Sinc(t) = \frac{sin(\pi t)}{\pi t} \quad \forall t \in R$$

it may also be defined as:

$$sinc(t) = Sa(\pi t)$$

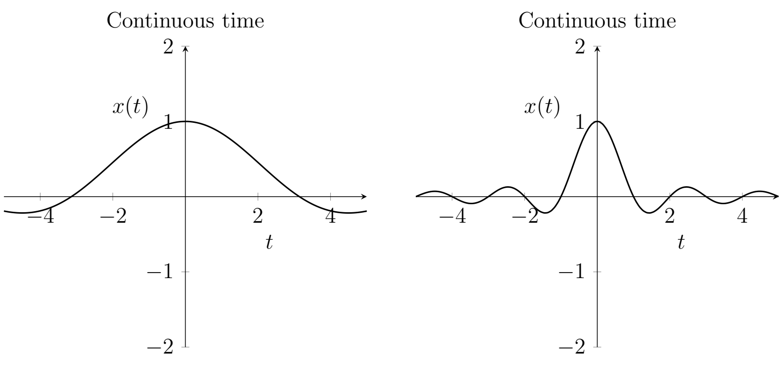

Figure 1.9: Sampling and Sinc functions

Gaussian Pulse

for continuous time :

$$x(t) = e^{-at^2} \quad \forall t \in R$$

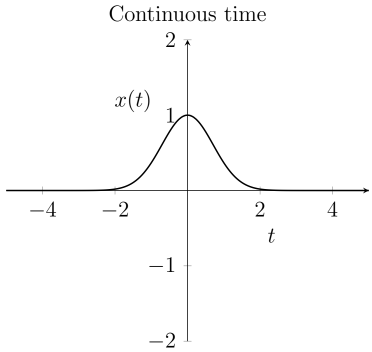

Figure 1.10: Gaussian Pulse

Impulse signal

The Impulse signal in continuous time is also called the 'Dirac Delta'. It is represented by $\delta(t)$.
formally, the defintion for this function is :

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

for continuous time :

$$x(t) = \begin{cases} \infty & \text{for } t = 0; \\ 0 & \text{otherwise;} \end{cases}$$

In discrete time, it is called the Kronecker delta or 'Unit impulse'. **for discrete time :**

$$x[n] = \begin{cases} 1 & \text{if } n = 0; \\ 0 & \text{otherwise} \end{cases}$$

Note:

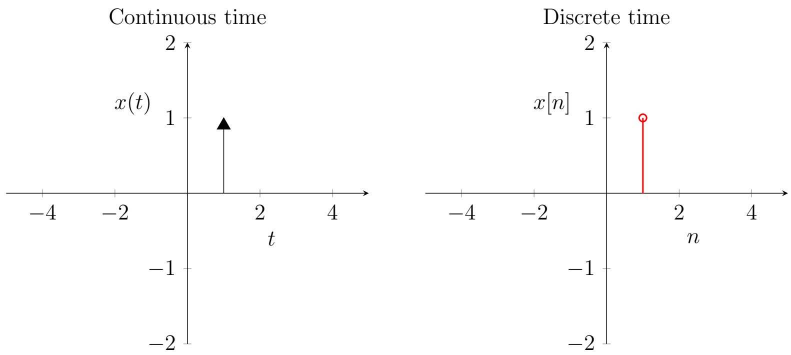

Figure 1.11: $\delta(t-1)$

- the unit impulse is the first difference of the unit step sequence.

$$\delta[n] = u[n] - u[n-1]$$

- alternatively:

$$u[n] = \sum_{n=-\infty}^{+\infty} \delta[n]$$

- the dirac delta is the first derivative of the unit step signal.

$$\delta(t) = \frac{d}{dt}u(t)$$

- alternatively:

$$u(t) = \int_{-\infty}^{+\infty} \delta(t)$$

Properties of impulse signal:

- The integral of the impulse over R is unity.

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Sifting

$$\int_a^b x(t)\delta(t-T)dt = \begin{cases} x(T) fora < T < b; \\ 0 \quad otherwise; \end{cases}$$

  thus it can be imagined that the impulse function $\delta(t-T)$ sifts through the function f(t) and pulls out the value f(T).

- Convolution and Sifting
  Convolution of a function with a shifted impulse yields a shifted version of that function. whereas convolution of a function with an impulse at t=0 yields the function itself.

$$x(t) * \delta(t-T) = x(t-T)$$

- Time scaling
  The unit impulse time-scaled by a factor of a is the unit impulse amplitude-scaled by a factor of 1/—a—

$$\delta(at) = \frac{1}{a}\delta(t)$$

- Similarly sifting in discrete time:

$$x[n_0] = \sum_{n=-\infty}^{\infty} \delta[n-n_0]x[n]$$

- and convolution in discrete time:

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

1.4 Next topic

Here is how to define things in the proper mathematical style. Let f_k be the $AND - OR$ function, defined by

$$f_k(x_1, x_2, \ldots, x_{2^k}) = \begin{cases} x_1 & \text{if } k = 0; \\ AND(f_{k-1}(x_1, \ldots, x_{2^{k-1}}), f_{k-1}(x_{2^{k-1}+1}, \ldots, x_{2^k})) & \text{if } k \text{ is even;} \\ OR(f_{k-1}(x_1, \ldots, x_{2^{k-1}}), f_{k-1}(x_{2^{k-1}+1}, \ldots, x_{2^k})) & \text{otherwise.} \end{cases}$$

Here is a citation, just for fun [CW87].

References

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