Signals and Systems

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August 2019

Contents

Chapter 1

Signal Analysis

B.Tech ECE: Signals and Systems **II-I Semester**

Chapter 1: Signal Analysis

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Signals can be used to describe a wide range of natural phenomena. A signal is generally imagined as a pattern of variations of some quantity with respect to another independent quantity. In the following section we shall look into the definition of a signal as well as a system along with some examples

1.1 Introduction

Signal It is defined as a function of any independent variable. Generally speaking, a signal is a function of time which conveys some sort of information.
E.g;

- Speech or Voice signals
- Image signals
- etc

System It is a collection of objects which work together to perform a particular task.
From a communications standpoint, systems are used to process signals.

1.2 Classification of Signals

If a signal is defined in terms of only one independent variable, it is called a one dimensional signal otherwise it is called a multi-dimensional signal.
in addition to dimensions,signals can be classified on the basis of various parameters:

On the basis of time t

- **Discrete time signal:** X-axis (time) is discrete and Y-axis (amplitude) may be continuous or discrete.It is represented by $x[n]$ (in brackets)

- **Continuous time signal:** X-axis (time) is continuous and Y-axis (amplitude) may be continuous or discrete.It is represented by $x(t)$ (in parentheses)
- **Digital signal:** Y-axis (amplitude) is discrete and X-axis (time) may be continuous or discrete.
- **Analog signal:** Y-axis (amplitude) is continuous and X-axis (time) may be continuous or discrete.

These signals can be further classified as follows:

On the basis of periodicity

Periodic Signals : Continuous Time Periodic Signals
a signal is said to be CT-periodic if it repeats after a certain time interval T_0
Mathematically, it is defined as the signal which satisfies :

$$x(t) = x(t + T_0) \forall t \in R$$

Discrete Time Periodic Signals
a signal is said to be DT-periodic if it repeats after a certain time interval N_0
Mathematically, it is defined as the signal which satisfies :

$$x[n] = x[n + N_0] \forall n \in Z$$

Aperiodic Signals :
A signal that does not satisfy the above conditions is called aperiodic signal. It may be viewed as a limiting case of a periodic signal in which period tends to Infinity.

Even and Odd signals

a signal $x(t)$ is said to be even if it satisfies :

$$x(t) = x(-t) \quad (CT)$$
$$x[n] = x[-n] \quad (DT)$$

a signal x(t) is said to be odd if it satisfies :

$$x(t) = -x(-t) \quad (CT)$$
$$x[n] = -x[-n] \quad (DT)$$

if it satisfies neither it is said to be neither even nor odd.

Deterministic and Random Signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.

A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} |x\left(t\right)|^2 dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

$$\text{Power of energy signal} = 0$$
$$\text{Energy of power signal} = \infty$$

1.3 Standard Signals

There are some standard signals which are used repeatedly in signals and systems. Let us take a look on some of them.

Standard signals list:

1. Unit step signal
2. Unit Ramp signal
3. Unit parabolic
4. Signum
5. Real And Complex Exponential Signals
6. Sinusoidal Signals
7. Rectangular Signals
8. Triangular signal
9. Sampling Function
10. Sinc Function
11. Gaussian Pulse
12. Unit Impulse

Unit step signal

for continuous time :

$$x(t) = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

at t=0 there is a discontinuity. but generally it can be said that the value converges to 0.5.
for discrete time :

$$x[n] = \begin{cases} 1 & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

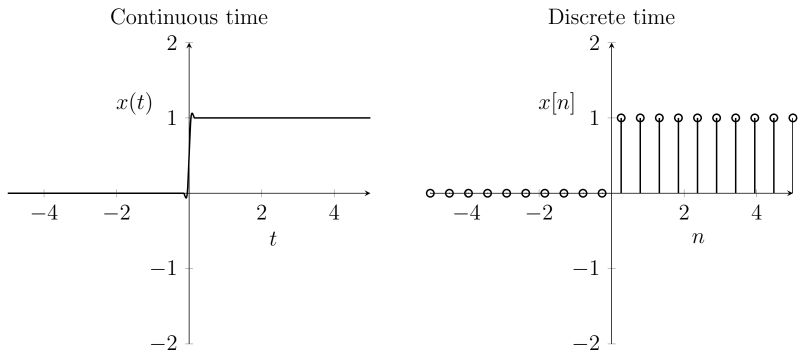


Figure 1.1: the unit step signal

Ramp signal

for continuous time :

$$x(t) = \begin{cases} t & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} n & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

note: The ramp signal can be related to the unit step by the following relation :

$$\frac{d}{dt} r(t) = u(t)$$

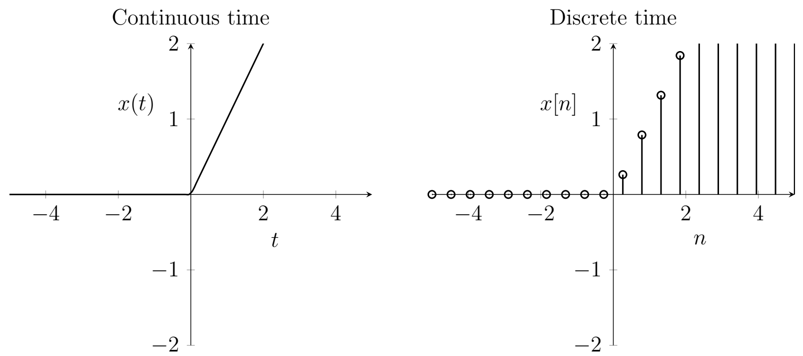


Figure 1.2: the ramp signal

Parabolic signal

for continuous time :

$$x(t) = \begin{cases} \frac{t^2}{2} & \text{if } t > 0; \\ 0 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} \frac{n^2}{2} & \text{if } n \geq 0; \\ 0 & \text{if } n < 0; \end{cases}$$

note: The parabolic signal can be related to the ramp signal by the following relation :

$$\frac{d}{dt}p(t) = r(t)$$

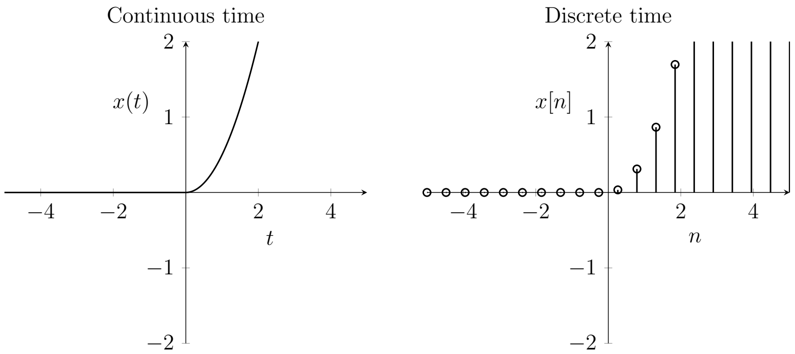


Figure 1.3: the parabolic signal

signum function

for continuous time :

$$x(t) = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t = 0; \\ -1 & \text{if } t < 0; \end{cases}$$

for discrete time :

$$x[n] = \begin{cases} 1 & \text{if } t > 0; \\ 0 & \text{if } t = 0; \\ -1 & \text{if } t < 0; \end{cases}$$

note: signum can be related to the unit step signal by the following relation :

$$u(t) = \frac{1}{2}(sgn(t) + 1)$$

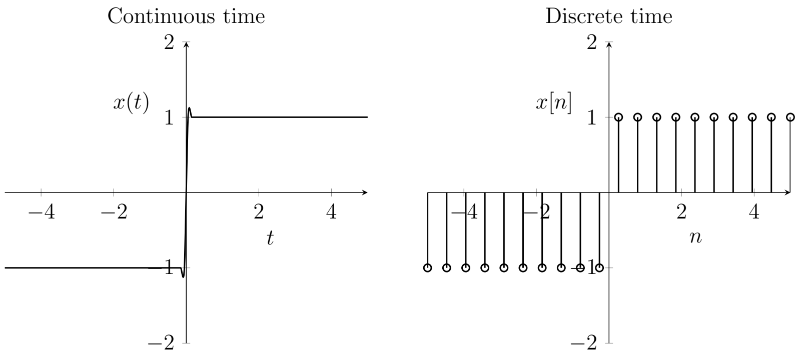


Figure 1.4: Signum Function

Real Exponential signal

for continuous time :
$x(t) = Ca^t \quad$ for $a > 0$

for discrete time :
$x[n] = Ca^n \quad$ for $a > 0$

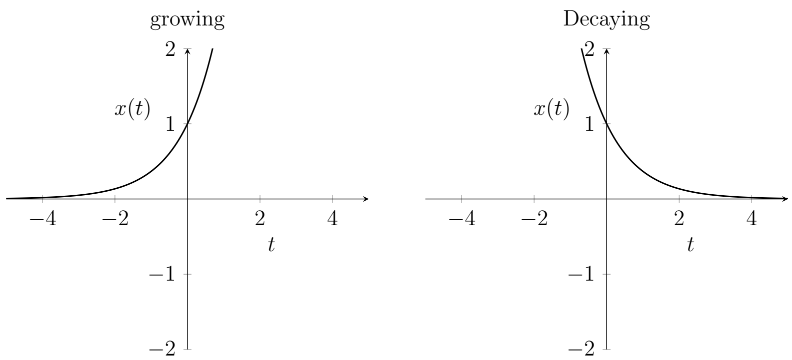


Figure 1.5: real exponential signal

Complex Exponential signal

for continuous time :
$x(t) = Ca^t \qquad \text{for } a < 0$

for discrete time :
$x[n] = Ca^n \qquad \text{for } a < 0$

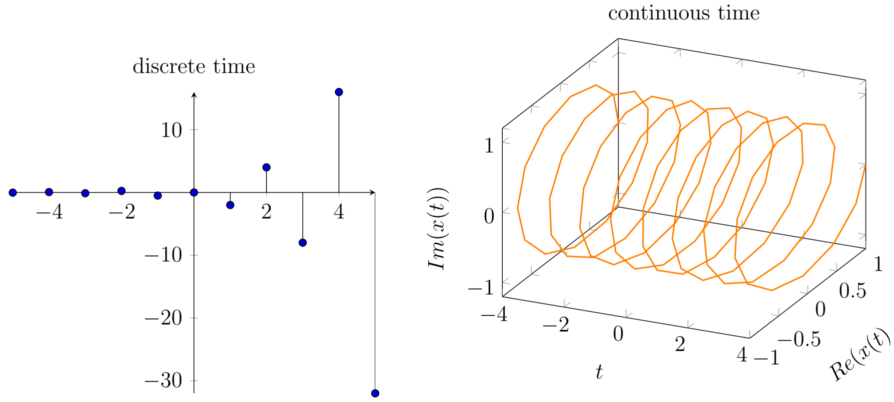


Figure 1.6: complex exponential signal

Sinusoidal signals

for continuous time :

$$x(t) = sin(\omega_0 t + \theta)$$

where, $\omega_0 = \frac{2\pi}{T_0}$ is the fundamental frequency and θ is the phase.

Rectangular signal a.k.a Gating pulse

It is denoted by $rect(t/T)$ where T is the period or $\Pi(t)$
for continuous time :

$$\Pi(t) = \begin{cases} A & \text{if } \frac{-T}{2} < t < \frac{T}{2}; \\ 0 & \text{otherwise} \end{cases}$$

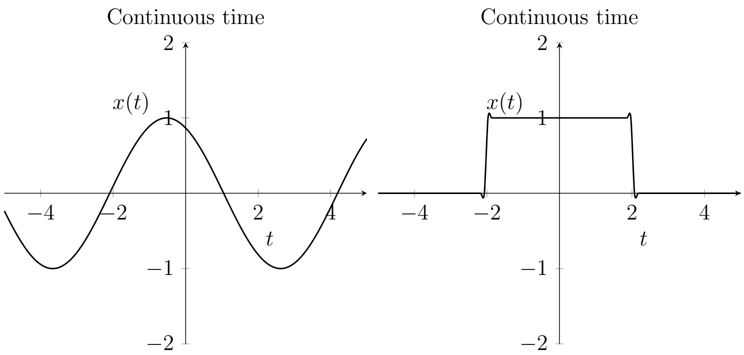


Figure 1.7: a sinusoid and rectangular signal

Triangular pulse

for continuous time :

$$Tri(t) = \begin{cases} \frac{A}{T}t + A & \text{if } T \leq t \leq 0; \\ \frac{-A}{T}t + A & \text{if } 0 \leq t \leq T; \end{cases}$$

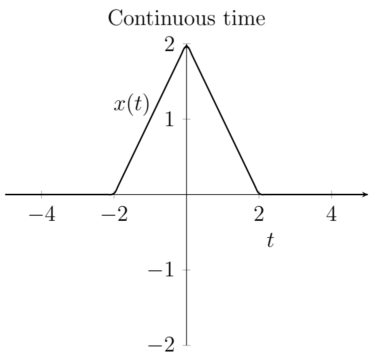


Figure 1.8: Triangular pulse

Sampling function $Sa(t)$

for continuous time :

$$Sa(t) = \frac{sin(t)}{t} \quad \forall t \in R$$

Sinc function $Sinc(t)$

The sinc function is a time scaled version of the sampling function. specifically , the time variable t is scaled as

$$t \longrightarrow \pi t$$

The full name of the function is "sine cardinal," but it is commonly referred to by its abbreviation, "sinc."
It is a function that arises frequently in signal processing and the theory of Fourier transforms.
for continuous time :

$$Sinc(t) = \frac{sin(\pi t)}{\pi t} \quad \forall t \in R$$

it may also be defined as:

$$sinc(t) = Sa(\pi t)$$

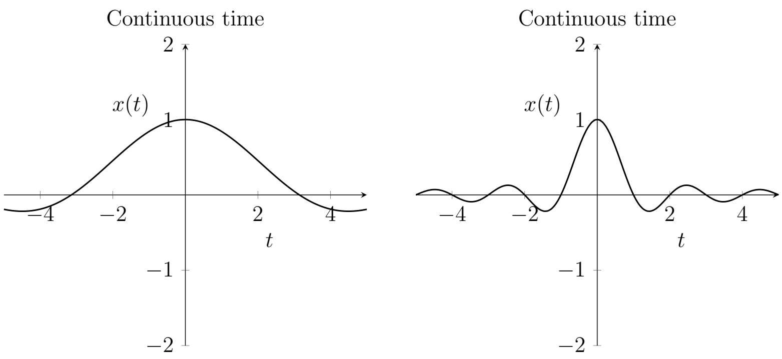


Figure 1.9: Sampling and Sinc functions

Gaussian Pulse

for continuous time :

$$x(t) = e^{-at^2} \quad \forall t \in R$$

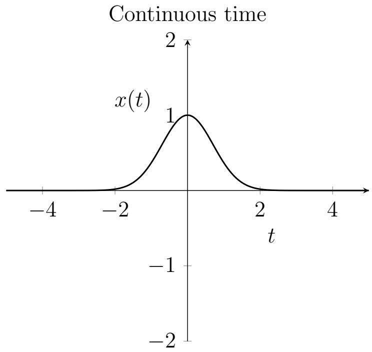


Figure 1.10: Gaussian Pulse

Impulse signal

The Impulse signal in continuous time is also called the 'Dirac Delta'. It is represented by $\delta(t)$.
formally, the defintion for this function is :

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

for continuous time :

$$x(t) = \begin{cases} \infty & \text{for } t = 0; \\ 0 & \text{otherwise;} \end{cases}$$

In discrete time, it is called the Kronecker delta or 'Unit impulse'. **for discrete time :**

$$x[n] = \begin{cases} 1 & \text{if } n = 0; \\ 0 & \text{otherwise} \end{cases}$$

Note:

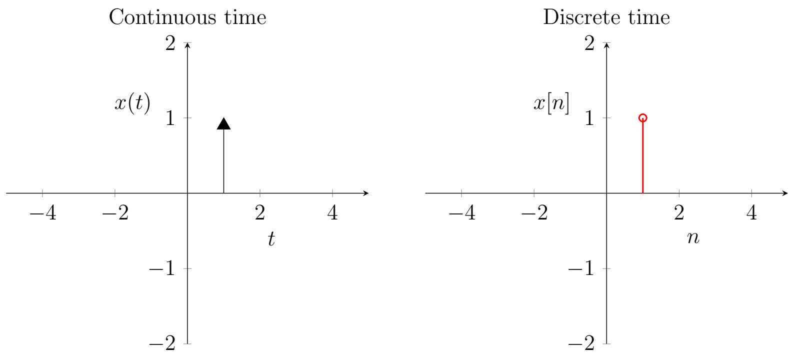


Figure 1.11: $\delta(t-1)$

- the unit impulse is the first difference of the unit step sequence.

$$\delta[n] = u[n] - u[n-1]$$

- alternatively:

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

- the dirac delta is the first derivative of the unit step signal.

$$\delta(t) = \frac{d}{dt}u(t)$$

- alternatively:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

Properties of impulse signal:

- The integral of the impulse over R is unity.

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Sifting

$$\int_{a}^{b} x(t)\delta(t-T)dt = \begin{cases} x(T)\ for\ a < T < b; \\ 0 \quad otherwise; \end{cases}$$

thus it can be imagined that the impulse function $\delta(t-T)$ sifts through the function f(t) and pulls out the value f(T).

- Convolution and Sifting
  Convolution of a function with a shifted impulse yields a shifted version of that function. whereas convolution of a function with an impulse at t=0 yields the function itself.

$$x(t) * \delta(t-T) = x(t-T)$$

- Time scaling
  The unit impulse time-scaled by a factor of a is the unit impulse amplitude-scaled by a factor of 1/—a—

$$\delta(at) = \frac{1}{a}\delta(t)$$

- Similarly sifting in discrete time:

$$x[n_0] = \sum_{n=-\infty}^{\infty} \delta[n-n_0]x[n]$$

- and convolution in discrete time:

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

1.4 Operations on Signals

A variety of operations can be carried out on signals to obtain new signals. These operations can be classified into two categories - operations that are performed on the dependent variable and operations that can be performed on the independent variable. Recall that for a signal $x(t)$, $t$is the independent variable and $x(t)$is the dependent variable.

1.4.1 Operations performed on the dependent variable:

Amplitude Scaling:
Signals are often amplified or attenuated, i.e., the amplitude of the signal is scaled. Mathematically, this corresponds to multiplying the signal $x(t)$ by a (real or complex constant) c to obtain a new signal, say $y(t)$ given by :

$$\begin{aligned} &\text{For CT signals :} y(t) = c\ x(t) \\ &\text{For DT signals :} y[n] = c\ x[n] \end{aligned} \tag{1.1}$$

Addition, Subtraction, Multiplication, and Division
Take two signals $x_1(t)$ and $x_2(t)$

. We can add, subtract, multiply and divide these signals. These operations are performed for every value of t . This is similar to performing operations on functions of t . Thus we can obtain a new signal $y(t)$ from two signals $x_1(t)$ and $x_2(t)$ according to,

$$\text{Addition:} y(t) = x_1(t) + x_2(t) \tag{1.2}$$

$$\text{Subtraction:} y(t) = x_1(t) - x_2(t) \tag{1.3}$$

$$\text{Multiplication:} y(t) = x_1(t).x_2(t) \tag{1.4}$$

$$\text{Division:} y(t) = x_1(t)/x_2(t) \tag{1.5}$$

Derivatives and Integrals of CT signals
For continuous-time signals (CT) signals, we define the derivative and integral of the signal as follows. It is important to note the notation used in defining the integral. Notice that the variable of integration is τ and t appears in the limit of the integral making the result of the integral a function of t.

$$\text{Derivative:} y(t) = \frac{d}{dt}x(t) \tag{1.6}$$

$$\text{Integral:} y(t) = \int_{-\infty}^{t} x(\tau)d\tau. \tag{1.7}$$

Difference and Partial sums of DT signals
For discrete-time signals, the operation of derivative and integrals are not well defined. The equivalent notion of derivative and integral for DT signals are given by backward difference and partial sum as given below:

$$\text{Backward difference:} y[n] = x[n] - x[n-1] \tag{1.8}$$

$$\text{Partial sum:} y[n] = \sum_{m=-\infty}^{n} x[m]. \tag{1.9}$$

1.4.2 Operations performed on the independent variable:

The various operations are:

1. Time Scaling:
   Given a signal $x(t)$,Time scaling is defined as-

$$y(t) = x(at) \tag{1.10}$$

2. Time Reversal or Reflection:

$$y(t) = x(-t) \tag{1.11}$$

3. Time Shifting
   Given a signal $x(t)$,Time shifting is defined as-

$$y(t) = x(t - t_0) \tag{1.12}$$

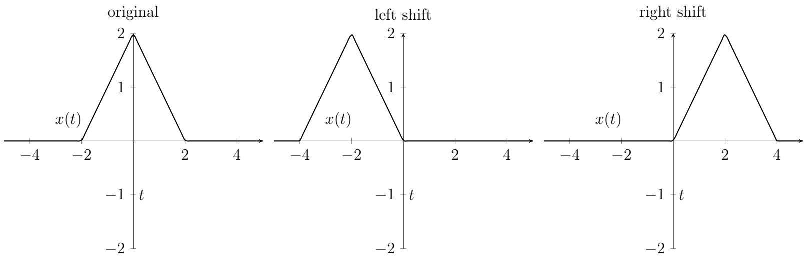


Figure 1.12: Triangular pulse time shifted to left and right by 2 sec respectively

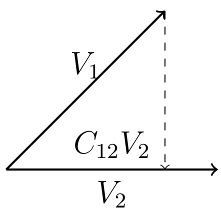
1.5 Signal Analysis

1.5.1 Analogy Between Vectors and Signals

While studying a new topic it is often very helpful to relate it to another topic which we are quite familiar with. Such an analogy can be developed between Vectors and signals.

Vector

A vector may be imagined as an arrow with some length in a vector plane. the length represents magnitude and arrow points to the direction.
let V is a vector with magnitude V. Consider two vectors V_1 and V_2 as shown in the following diagram. Let the component of V_1 along with V_2 is given by $C_{12}V_2$.
The component of a vector V_1 along with the vector V_2 can obtained by taking a perpendicular from the end of V_1 to the vector V_2 as shown in diagram:



The vector V1 can be expressed in terms of vector V2

$$V_1 = C_{12}V_2 + V_e$$

Where V_e is the error vector.
The error signal is minimum for large component value. If C_{12}=0, then two signals are said to be orthogonal. Dot Product of Two Vectors

$$V_1.V_2 = V_1.V_2 cos\theta$$

$\theta =$ Angle between V1 and V2

$$V_1.V_2 = V_2.V_1$$

The components of V_1 along V_2 =

$$V_1 cos\theta = \frac{V_1.V_2}{V_2}$$

therefore,

$$C_{12} = \frac{V_1.V_2}{V_2}$$

Signals:
The concept of orthogonality can be applied to signals. Let us consider two signals $x_1(t)$ and $x_2(t)$. Similar to vectors, you can approximate $x_1(t)$ in terms of $x_2(t)$ as

$$x_1(t) = C_{12}x_2(t) + x_e(t)$$

over the interval

$$t \in [t_1, t_2]$$

in order to evaluate the error, it is not a good choice to take the integral of the error function $x_e(t)$ over t1 to t2, rather error can be estimated more efficiently by taking the mean square value of that function over t1 to t2.

$$let, \varepsilon = \frac{1}{t_2 - t_1}\int_{-\infty}^{\infty} |x_e(t)|^2 dt \tag{1.13}$$

so,

$$\varepsilon = \frac{1}{t_2 - t_1}\int_{-\infty}^{\infty} |x_1(t) - C_{12}x_2(t)|^2 dt$$

Where ε is the mean square value of error signal. The value of C12 which minimizes the error, we need to calculate

$$\frac{d}{dC_{12}}\varepsilon = 0$$

$$\Rightarrow \frac{d}{dC_{12}}[\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [x_1(t) - C_{12}x_2(t)]^2 dt] = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [\frac{d}{dC_{12}}x_1^2(t) - \frac{d}{dC_{12}}2x_1(t)C_{12}x_2(t) + \frac{d}{dC_{12}}x_2^2(t)C_{12}^2]dt = 0$$

Derivative of the terms which do not have C12 term are zero.

$$\Rightarrow \int_{t_1}^{t_2} -2x_1(t)x_2(t)dt + 2C_{12}\int_{t_1}^{t_2} [x_2^2(t)]dt = 0$$

$$Thus, C_{12} = \frac{\int_{t_1}^{t_2} x_1(t)x_2(t)dt}{\int_{t_1}^{t_2} x_2^2(t)dt} \tag{1.14}$$

if $C_{12} = 0$ then the signals are said to be orthogonal
condition for orthogonality

$$\int_{t_1}^{t_2} x_1(t)x_2(t)dt = 0 \tag{1.15}$$

1.5.2 Orthogonal Vector Space

A complete set of orthogonal vectors is referred to as orthogonal vector space.
Consider a vector A at a point (X1, Y1, Z1). Consider three unit vectors (VX, VY, VZ) in the direction of X, Y, Z axis respectively.
Since these unit vectors are mutually orthogonal, it satisfies:

$$V_X.V_Y = V_Y.V_Z = V_Z.V_X = 0$$

and their magnitude is unity (each). hence, they can be written as:

$$V_a.V_b = \begin{cases} 1 & a = b \\ 0 & a \neq b \end{cases}$$

The vector A can be represented in terms of its components and unit vectors as:

$$A = X_1V_X + Y_1V_Y + Z_1V_Z$$

If you consider n dimensional space, then any vector A in that space can be represented as:

$$A = X_1V_X + Y_1V_Y + Z_1V_Z + ... + N_1V_N$$

1.5.3 Orthogonal Signal Space

Let us consider a set of n mutually orthogonal functions x1(t), x2(t)... xn(t) over the interval t1 to t2. As these functions are orthogonal to each other, any two signals xj(t), xk(t) have to satisfy the orthogonality condition. i.e.

$$\int_{t_1}^{t_2} x_j(t)x_k(t)dt = 0 \text{ where } j \neq k$$

Now,

$$\text{Let} \int_{t_1}^{t_2} x_k^2(t)dt = \text{some constant } K_k$$

Let a function f(t), it can be approximated with this orthogonal signal space by adding the components along mutually orthogonal signals i.e.

$$f(t) = C_1x_1(t) + C_2x_2(t) + ... + C_nx_n(t) + f_e(t)$$

$$\text{or } f(t) = \sum_{r=1} nC_rx_r$$

and thus,

$$f_e(t) = f(t) - \sum_{r=1}^{n} C_rx_r(t)$$

as derived before,

$$\varepsilon = \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f[t] - \sum_{r=1}^{n} C_rx_r(t)]^2 dt$$

in order to minimise error along each component, we must evaluate for some parameter k,

$$\frac{d\varepsilon}{dC_k} = 0$$

$$\frac{d}{dC_k}[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f(t) - \Sigma_{r=1}^{n} C_rx_r(t)]^2 dt] = 0$$

All terms that do not contain Ck is zero. i.e. in summation, r=k term remains and all other terms are zero. thus,

$$C_k = \frac{\int_{t_1}^{t_2} f(t)x_k(t)dt}{\int_{t_1}^{t_2} x_k^2(t)dt} \tag{1.16}$$

$$\text{or we can say} \Rightarrow \int_{t_1}^{t_2} f(t)x_k(t)dt = C_kK_k$$

$$\varepsilon = \frac{1}{t_2 - t_1}[\int_{t_1}^{t_2} [f^2(t)]dt + (C_1^2K_1 + C_2^2K_2 + ... + C_n^2K_n)]$$

1.5.4 Closed and Complete Set of Orthogonal Signals

Let us consider a set of n mutually orthogonal functions $x_1(t), x_2(t)...x_n(t)$ over the intervalt_1 to t_2. This is called as closed and complete set when there exist **no function** f(t) satisfying the condition:

$$\int_{t_1}^{t_2} f(t)x_k(t)dt = 0$$

In such a case f(t) can be approximated by the set of orthogonal signals as:

$$f(t) = C_1 x_1(t) + C_2 x_2(t) + ... + C_n x_n(t) + f_e(t)$$

if the series is infinite then it converges to $f(t)$ and thus error becomes zero.

1.5.5 Orthogonality in Complex Functions

If $x_1(t)$ and $x_2(t)$ are two complex valued functions, then $x_1(t)$ can be expressed in terms of $x_2(t)$ as :

$$x_1(t) = C_{12} x_2(t)$$

where $C_{12} = \frac{\int_{t_1}^{t_2} x_1(t) x_2^*(t) dt}{\int_{t_1}^{t_2} |x_2(t)|^2 dt}$ **condition for orthogonality**

$$\Rightarrow \int_{t_1}^{t_2} x_1(t) x_2^*(dt) = 0$$

1.6 Next topic

Here is how to define things in the proper mathematical style. Let f_k be the $AND - OR$ function, defined by

$$f_k(x_1, x_2, \dots, x_{2^k}) = \begin{cases} x_1 & \text{if } k = 0; \\ AND(f_{k-1}(x_1, \dots, x_{2^{k-1}}), f_{k-1}(x_{2^{k-1}+1}, \dots, x_{2^k})) & \text{if } k \text{ is even;} \\ OR(f_{k-1}(x_1, \dots, x_{2^{k-1}}), f_{k-1}(x_{2^{k-1}+1}, \dots, x_{2^k})) & \text{otherwise.} \end{cases}$$

Here is a citation, just for fun [CW87].

References

pass

Chapter 2

Fourier Series

B.Tech ECE: Signals and Systems **II-I Semester**

Chapter 2: Fourier Series

Lecturer: Syed Munavvar Hussain *Scribes: Shriram R*

the Fourier Series is used to represent a continuous time or discrete time periodic signal. It can be use to evaluate the frequency spectrum of those signals as well. The Fourier Series is used to analyse only periodic signals.

2.1 Trigonometric Fourier Series

Using the Fourier Series, any periodic signal $x(t)$ can be represented as a linear combination of an infinite set of mutually orthogonal functions.When these orthogonal functions are The Trigonometric functions sine and cosine, the fourier series is known as a Trigonometric fourier series.
Mathematically, a periodic signal $x(t)$ with period T can be represented as :

$$x\left(t\right) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(n\omega_0 t\right) + b_n \sin\left(n\omega_0 t\right)\right) \quad \text{where } \omega_0 = \frac{2\pi}{T} \tag{2.1}$$

Note : finding a fourier series of a function involves evaluation of the Coefficients X_n mainly.

2.1.1 Evaluation of T.F.S Coefficient

$$a_n = \frac{2}{T}\int_T x(t)\cos\left(n\omega_0 t\right) dt b_n = \frac{2}{T}\int_T x_e(t)\sin\left(n\omega_0 t\right) dt \tag{2.2}$$

2.1.2 Even and Odd parts of a function

Any function can be composed of an even and an odd part. Given a function x(t), we can create even and odd functions

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

2.1.3 The Exponential Fourier Series

A more compact representation of the Fourier Series uses complex exponentials. In this case we end up with the following synthesis and analysis equations:

$$x_T(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t} \quad \text{Synthesis Equation} \tag{2.3}$$

$$c_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt \quad \text{Analysis Equation} \tag{2.4}$$

2.2 Equivalence of the Trigonometric and Exponential Series

let,

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right)$$

and also,

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$$

now, since they are equivalent,

$$a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$$

now , let us look at constant terms, by observing, we can say $a_0 = c_0$

also, in general, we can see that

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) = c_{-n}e^{-jn\omega_0 t} + c_n e^{+jn\omega_0 t}$$

$$\Rightarrow = \left(c_{n,r} - jc_{n,i}\right)\left(\cos(n\omega_0 t) - j\sin(n\omega_0 t)\right) + \left(c_{n,r} + jc_{n,i}\right)\left(\cos(n\omega_0 t) + j\sin(n\omega_0 t)\right)$$

$$\Rightarrow = 2c_{n,r}\cos(n\omega_0 t) - 2c_{n,i}\sin(n\omega_0 t) + j\left(\cos(n\omega_0 t)(c_{n,r} - c_{n,r}) + \sin(n\omega_0 t)(c_{n,i} - c_{n,i})\right)$$
$$= 2c_{n,r}\cos(n\omega_0 t) - 2c_{n,i}\sin(n\omega_0 t)$$

Thus, Finally,

$$a_n = 2c_{n,r}$$

$$b_n = -2c_{n,i}$$

$$\text{and conversely,} c_n = \frac{a_n}{2} - j\frac{b_n}{2},\ n \neq 0, \quad with\ c_{-n} = c_n^*$$

2.3 Dirichlet Conditions for Convergence of Fourier Series

1. $x(t)$ must be absolutely integrable over R. i.e;

$$\int_{-\infty}^{\infty} x(t) \ < \ \infty$$

2. $x(t)$ must be single valued over R.
3. $x(t)$ should have a finite number of maxima and minima over the given interval.
4. $x(t)$ must have a finite number of discontinuities over the given inteval.

   These 4 conditions make sure that the fourier series expansion converges to the function $x(t)$