Feedback — Homework 1

Help Center

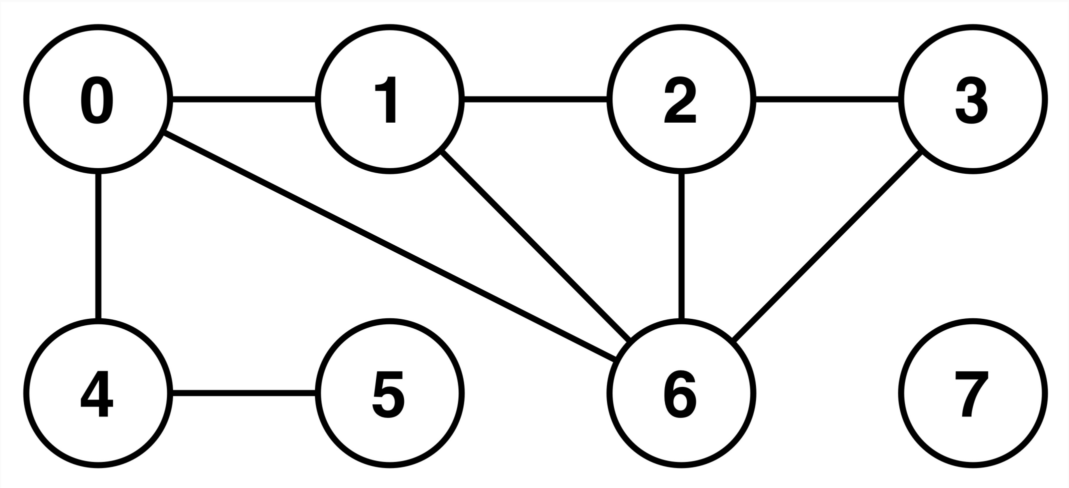
You submitted this homework on **Thu 17 Sep 2015 9:00 PM BST**. You got a score of **85.00** out of **100.00**.

This Homework covers the basics of graphs and guides you through the mathematical analysis of the properties of a simple class of random graphs. We'll consider this class of graphs further in the Application portion of this module.

Please review the notes on graph basics as needed.

Question 1

For Questions 1-3, please refer to the following undirected graph:



If you prefer, you can open this figure in a separate tab.

What is the degree of node 7? Enter your answer below as a single integer. For example, if the answer is 25, just enter 25 in the box, and nothing else.

You entered:

0

Your Answer		Score	Explanation
0	✔	5.00	Correct. Node 7 has no neighbors.
Total		5.00 / 5.00	

Question Explanation

Review the definition of the degree of a node in graph basics if you missed this problem.

Question 2

If A is the adjacency matrix representation for the undirected graph in Question 1, what is the value of $A[2, 6]$?

Enter your answer below as a single integer.

You entered:

1

Your Answer		Score	Explanation
1	✔	5.00	Correct. The entry $A[2, 6]$ has value 1 if there is an edge between nodes 2 and 6.
Total		5.00 / 5.00	

Question Explanation

Review the definition of an adjacency matrix in graph basics if you missed this problem.

Question 3

Which one of the following is a (Python) dictionary representation of the degree distribution of the graph in Question 1?

Remember that the sum of the values in the dictionary should be normalized to sum to one.

Your Answer	Score	Explanation
◯ `{0 : set([1, 4, 6]), 1 : set([0, 2, 6]), 2 : set([1, 3, 6]), 3 : set([2, 6]), 4 : set([0, 5]), 5 :`		

`set([4]), 6 : set([0, 1, 2, 3]), 7 : set([])}`

- ( ) `{0 : 3, 1 : 3, 2 : 3, 3 : 2, 4 : 2, 5 : 1, 6 : 4, 7 : 0}`
- ( ) `{0 : 1, 1 : 1, 2 : 2, 3 : 3, 4 : 1}`
- (•) `{0 : 0.125, 1 : 0.125, 2 : 0.25, 3 : 0.375, 4 : 0.125}` ✔ 5.00 Yes. Each key in the dictionary corresponds to a degree, and its corresponding value is the fraction of nodes that have that degree.

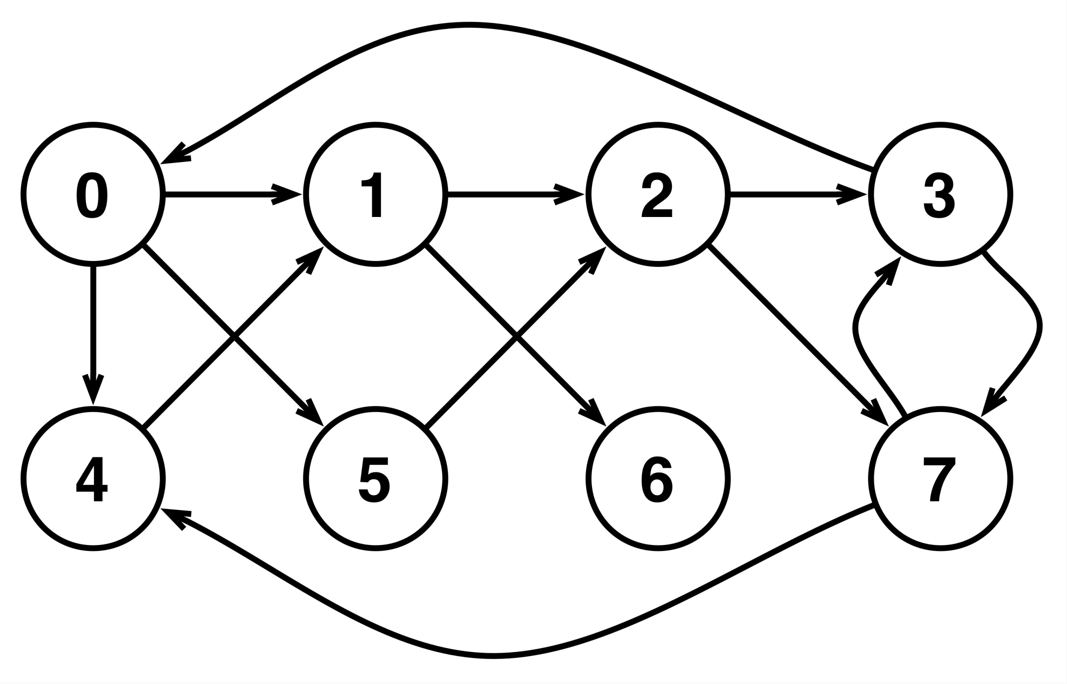
Total 5.00 / 5.00

Question Explanation

Review the definition of a degree distribution in graph basics if you missed this problem.

Question 4

For Questions 4-6, please refer to the following directed graph:



If you find it easier to refer to, you can open this figure in another tab.

How many nodes in the graph have in-degree 2? Enter your answer below as a single integer.

You entered:

5

Your Answer		Score	Explanation
5	✔	5.00	Correct. The in-degree corresponds to the number of edges whose head matches a node.
Total		5.00 / 5.00	

Question Explanation

Review the definition of out-degree in graph basics if you missed this problem.

Question 5

If A is the adjacency matrix of the graph in Question 4, what is the value of entry $A[2, 1]$? Enter your answer below as a single integer.

You entered:

0

Your Answer		Score	Explanation
0	✔	5.00	Correct. There is an edge (1, 2) from node 1 to node 2. However, there is not an edge from node 2 to node 1.
Total		5.00 / 5.00	

Question Explanation

Review the definition of an adjacency matrix in graph basics if you missed this problem.

Question 6

How many nodes in the graph of Question 4 have out-degree 2? Enter your answer below as a single integer.

You entered:

4

Your Answer		Score	Explanation
4	✔	5.00	Correct. Nodes 1, 2, 3, and 7 have out-degree 2.
Total		5.00 / 5.00	

Question Explanation

Review the definition of out-degree in graph basics if you missed this problem.

Question 7

Consider an undirected graph $g = (V, E)$ where $|V| = n$. What is the maximum possible degree of a node in g?

Enter your answer below as a math expression in n. To format your expression correctly, please consult this page. Remember to use the Preview button (as well as the Help link) to make sure that your expression is formatted correctly.

You entered:

n-1

Preview Help

Your Answer		Score	Explanation
n-1	✔	5.00	Correct. There are $n - 1$ remaining nodes that can be connected to a given node.
Total		5.00 / 5.00	

Question 8

Consider an undirected graph $g = (V, E)$ where $|V| = n$. What is the maximum possible number of edges in g? That is, what is the largest size that E can be?

Enter your answer below as a math expression in n. Remember to use the Preview button (as well as the Help link) to make sure that your expression is formatted correctly.

You entered:

(n * (n-1))/2

Preview Help

Your Answer		Score	Explanation
(n * (n-1))/2	✔	5.00	Correct.
Total		5.00 / 5.00	

Question Explanation

Consider adding one node at a time to the graph. How many new edges are possible each time a new node is added? Use the answer to this question to build an arithmetic sum that models this process.

Question 9

In an undirected graph $g = (V, E)$ with n nodes, in how many ways can we connect a specific node u to other nodes in g so that u has degree k (where $0 \leq k < n$)?

Remember that the notation $\binom{n}{k}$ denotes the number of combinations of k elements from a set of n elements; that is, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. See this page for a quick review of permutations and combinations.

Your Answer		Score	Explanation
◉ $\binom{n-1}{k}$	✔	5.00	Correct. We can choose k edges out of $n-1$ possible edges.
○ $\binom{n}{k}$			
○ $(n-1)k$			
○ $(n-1)^k$			

k^{n-1}

Total	5.00 / 5.00

Question 10

For Questions 10-16, we will be referring to the following algorithm for generating random undirected graphs:

Algorithm 1: ER.

Input: Number of nodes n; probability p.
Output: A graph $g = (V, E)$ where $g \in G(n, p)$.

$V \leftarrow \{0, 1, \ldots, n-1\}$;
$E \leftarrow \emptyset$;
foreach $\{i, j\} \subseteq V$, *where* $i \neq j$ **do**
  $a \leftarrow random(0, 1)$;   // a is a random real number in $[0, 1)$
  if $a < p$ **then**
    $E \leftarrow E \cup \{\{i, j\}\}$;
return $g = (V, E)$;

If you find it more convenient, you can open this figure in another tab.

Suppose that the graph g was produced by running Algorithm **ER**(n, p). What are lower and upper bounds on the degrees for nodes in g?

Your Answer		Score	Explanation
Lower = 0 and upper = $n-1$	✔	5.00	Correct.
Lower = 0 and upper = n			
Lower = 0 and the upper bound is unbounded (that is, the largest degree can be any positive integer, regardless of the values of n and p)			
Lower = $n/2$ and upper = n			
Lower = $n \times p$ and upper = $n-1$			
Total		5.00 / 5.00	

Question 11

How many edges are there in a graph g that is obtained by running Algorithm **ER**$(10, 0)$? Enter your answer below as a single integer.

You entered:

0

Your Answer		Score	Explanation
0	✔	5.00	Correct. The probability of generating any edge is zero.
Total		5.00 / 5.00	

Question Explanation

Remember that the second parameter p is the probability of creating an edge between two nodes.

Question 12

How many edges are there in a graph g that is obtained by running Algorithm **ER**$(10, 1)$? Enter your answer below as a single integer.

You entered:

45

Your Answer		Score	Explanation
45	✔	5.00	Correct. Since the probability is one, an edge is always added.
Total		5.00 / 5.00	

Question 13

What is the expected value of the degree of a node in a graph g that is obtained by running **ER**(n, p)?

Review the notes on basic probability and expected value if necessary.

Your Answer		Score	Explanation
$n/2$			
$(n-1)/2$			
$(n-1)p$	✔	5.00	Correct. There are $n-1$ possible edges and each edge has probability p of being added.
$p\frac{n(n-1)}{2}$			
Total		5.00 / 5.00	

Question Explanation

Remember that the probability that any particular edge is added is p.

Question 14

What is the probability that a particular node in a graph g produced by running **ER**(n, p) has degree k where $0 \leq k < n$?

Hint: The **ER** procedure can be viewed as conducting a trial on each possible edge in the graph. Each trial has one of two outcomes: generate an edge or don't generate an edge. The probabilities associated with these outcomes are p and $1-p$.

To determine the appropriate formula, we suggest that you review the Wikipedia page on binomial distributions.

Your Answer	Score	Explanation

- $\frac{1}{2^n}\binom{n}{k}$
- $p^k(1-p)^{n-1-k}$

Your Answer		Score	Explanation
$\binom{n-1}{k}p^k(1-p)^{n-1-k}$	✔	10.00	Correct. There are $n-1$ possible edges that connect to a particular node.
$\frac{1}{n-1}k$			
$\binom{n}{k}p^k(1-p)^{n-k}$			
Total		10.00 / 10.00	

Question 15

Consider a graph g that is obtained by running **ER**(n, p). If g is represented by its adjacency matrix, which of the following expressions in n and p grows at the same rate as the running time of **ER**(n, p)?

Your Answer		Score	Explanation
n^2	✔	5.00	Correct. The method must compute the value for each entry of the adjacency matrix, which has size n^2.
$n(n-1)p$			
$(n-1)p$			
$n \times p$			
n^2p			
Total		5.00 / 5.00	

Question Explanation

Think about the question: Does the value of p affect the number of entries in the adjacency matrix of g that will be visited by the algorithm?

Question 16

Consider a graph g that is obtained by running **ER**(n, p) where $p = 0$. If g is represented by its adjacency matrix, which of the following expressions in n and p grows at the same rate as the running time of **ER**(n, p)?

Your Answer		Score	Explanation
◉ n^2	✔	5.00	Correct. Even though the graph contains no edges, setting the entries of the adjacency matrix to all be zero requires n^2 operations.
○ Some constant $c > 0$			
○ n			
○ 0			
Total		5.00 / 5.00	

Question Explanation

Does the algorithm check the value of p before it computes the entries of the adjacency matrix for g? Further, even if a graph has no edges, how long does it take to fill out all zeros in the entries?

Question 17

Let g be an undirected graph that has n nodes and m edges. If g is given by its adjacency matrix, which of the following expressions grows at the same rate as the time that it takes to compute the degree distribution of g?

Your Answer	Score	Explanation
○ n^2		
○ 2^m		
○ m^2		
○ m		

2^n	✖	0.00	Incorrect.
Total		0.00 / 5.00	

Question Explanation

Think about the question: Does the algorithm have to inspect every entry in the matrix or not?

Question 18

Let g be an undirected graph that has n nodes and m edges. If g is given by its adjacency list, which of the following expressions grows at the same rate as the time that it takes to compute the degree distribution of g?

For this problem, you should assume that it takes time proportional to the length of a list to compute its length.

Your Answer		Score	Explanation
n^2			
m			
2^m			
$m + n$			
2^n	✖	0.00	Incorrect.
Total		0.00 / 10.00	

Question Explanation

Think about the cases where, for example, the graph has no edges (or a very small number of edges).