Unit II & IV

Unit II: Random variables and Probability Distributions

Discrete random variables, probability mass function , cumulative distribution function, Independent random variables, Continuous random variables, distribution functions and densities, expectation, variance, raw and central moments of random variables, The multinomial distribution, Poisson approximation to the binomial distribution, Infinite sequences of Bernoulli trials, Normal distribution, exponential and gamma densities.

Unit-IV: Basic Statistics

Measures of Central tendency; Moments, skewness, Kurtosis. Mean and variance of Binomial distribution & Poisson distribution. Moments, skewness & kurtosis for Normal distribution.

Random Variable M1

A random variable is a function that assigns a real number to every element of sample space. Let S be the sample space of an experiment. Here we assign a specific number to each outcome of the sample space. A random variable X is a function from S to the set of real numbers R i.e. X: S →R

Ex. Suppose a coin is tossed twice S = {HH, HT, TH, TT}.Let **X:** represents number of heads on top face. So to each sample point we can associate a number X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0.Thus X is a random variable with range space RX = { 0, 1, 2}

Types of random Variable:

Discrete Random Variable: A random variable which takes finite number of values or countable infinite number of values is called discrete random variable.

Example: Number of alpha particles emitted by a radioactive source.

Continuous Random Variable: A random variable which takes non-countable infinite number of values is called discrete random variable. Example: length of time during which a vacuum tube is installed in a circuit functions is a continuous RV.

Discrete Probability Distribution

Suppose a discrete variate X is the outcome of some experiment. If the probability that X takes the value x_i is p_i, then

$P(X = x_i) = p_i \ or \ p(x_i)$ for $i = 1, 2....n$

Where

a. $p(x_i) \geq 0$

b. $\sum p(x_i) = 1$

The set of values x_i with their probabilities p_i i.e. (x_i,p_i) constitute a discrete probability distribution of the discrete variate X. The function p is called probability mass function (pmf) or probability density function (pdf).

Cumulative Distribution Function (CDF) or Distribution Function of discrete random variable X is defined by $F(x) = p(X \le x)$ where x is a real number $(-\infty < x < \infty)$

$$F(x=u)=\sum_{x\le u} p(x)$$

Expectation M I

If an experiment is conducted repeatedly for large number of times under essentially homogeneous conditions, then the average of actual outcomes i.e. the mean value of the probability distribution of random variable is the expected value. Let X be a discrete random variable with PMF p(x) or PDF f(x) then its mathematical expectation is denoted by E(x) and is defined as

$$\mu = E(X)=\sum xp(x)$$
$$E(X^2)=\sum x^2 p(x)$$

Properties:

$1. E(a)=a.$

$2. E(ax+b)=aE(x)+b$

$3. E(x+y)=E(x)+E(y)$

$4. E(xy)=E(x).E(y),$ if x and v are independent R.V.

Variance: Variance of r.v. X is defined as

$$\sigma^2=\sum_i (x_i-\mu)^2 . f(x_i) = \sum_i (x-x_i)^2 . p(x_i)$$

Also: $Var(X)=E(X^2)-[E(X)]^2$

Properties:

$1. Var(a)=0$

$2. Var(aX+b)=a^2 Var(X)$

$3. Var(X)\ge 0$

Standard Deviation: $S.D.=\sigma=\sqrt{Var(X)}$

Moments

1. **The r^{th} moment of a r.v. X about any point (X=A) is given by** $\mu_r(about X = A) = E\left[(X-A)^r\right]$

2. **Ordinary moments /Raw moments (moments about origin):**

The r^{th} raw moment of a r.v. X (i.e. about A= 0) is given by

$$\mu'_r(\text{about origin}) = E\left[X^r\right]$$
$$r = 0 \Rightarrow \mu'_0(\text{about origin}) = 1$$
$$r = 1 \Rightarrow \mu'_1(\text{about origin}) = E\left[X\right] = Mean$$
$$r = 2 \Rightarrow \mu'_2(\text{about origin}) = E\left[X^2\right]$$

3. **Central moments (moments about mean):**

The r^{th} central moment of a r.v. X (about mean) is given by

$$\mu_r(about\ \text{mean}) = E\left[\left(X-\bar{x}\right)^r\right]$$
$$r = 0 \Rightarrow \mu_0 = \text{E}(1) = 1.$$
$$r = 1 \Rightarrow \mu_1 = E\left[\left(X-\bar{x}\right)\right] = E(X) - E\left(\bar{x}\right) = \bar{x} - \bar{x} = 0$$
$$\begin{aligned} r = 2 \Rightarrow \mu_2 &= E\left[\left(X-\bar{x}\right)^2\right] \\ &= E\left[X^2 - 2X\bar{x} - \bar{x}^2\right] \\ &= E\left[X^2\right] - 2\bar{x}E\left[X\right] - \bar{x}^2 \\ &= E\left[X^2\right] - 2\bar{x}.\bar{x} - \bar{x}^2 \\ &= E\left[X^2\right] - E\left[X\right]^2 \end{aligned}$$

4. **Central Moments in terms of Raw moments**

$$\mu_0 = 1, \mu_1 = 0$$
$$Var(X) = \mu_2 = \mu'_2 - \mu'^2_1$$
$$\mu_3 = \mu'_3 - 3\mu'_1\ \mu'_2 + 2\mu'^3_1$$
$$\mu_4 = \mu'_4 - 4\mu'_1\ \mu'_3 + 6\mu'^2_1\ \mu'_2 - 3\mu'^4_1$$

Moment Generating Function:

Suppose X is a discrete random variable, discrete or continuous. The Moment generating function (mgf or MGF) is defined and denoted by :

$$M_X(t) = E\left[e^{tx}\right] = \sum_{i=1}^{n} e^{tx_i}\, \mathrm{P}(x_i)$$ (for discrete variable)

$$M_X(t) = E\left[e^{tx}\right] = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$ (for continuous variable)

Also by Taylor series $M_X(t) = 1 + \mu'_1 t + \mu'_2 \frac{t^2}{2!} + \ldots.\mu'_r \frac{t^r}{\mathrm{r}!} + \ldots$

Remark: If the mgf exists for a random variable X, we will be able to obtain all the moments of X. It is very plainly put, one function that generates all the moments of X.

Result: Suppose X is a random variable (discrete or continuous) with moment generating function then the rth raw moment is given by

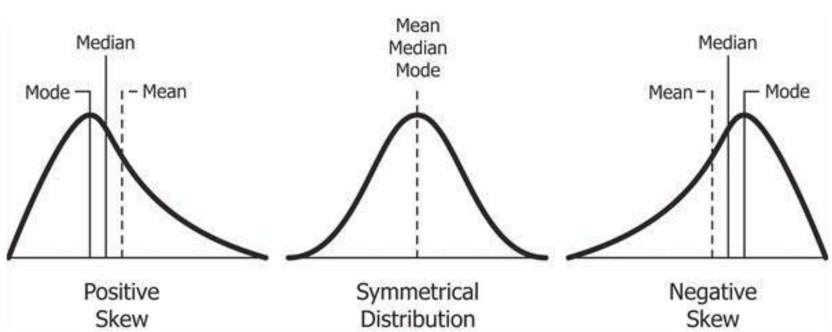
$$\mu'_r = \begin{cases} \text{coefficient of } \dfrac{t^r}{r!} \text{ in the expansion of } M_X(t) \\ \dfrac{d^r}{dt^r}\left[M_X(t)\right]_{t=0} \end{cases}, \quad r = 1, 2, \ldots$$

Skewness

Skewness, which means lack of symmetry, is the property of a random variable or its distribution by which we get an idea about the shape of the probability curve of the distribution. If the probability curve is not symmetrical but has a longer tail on one side than on the other, the distribution is said to be skewed. If a distribution is skewed, then the averages mean, median and mode will take different values and the quartiles will not be equidistant from the median.

The measure of skewness used in common is the third order central moment (μ_3).

The moment coefficient of skewness is defined as $v_1 = \dfrac{\mu_3}{\mu_2^{3/2}}$.



Kurtosis

Even if we know the measures of central tendency, dispersion and skewness of a random variable (or its distribution). we cannot get a complete idea about the distribution. In order to analyze the distribution completely, another characteristic kurtosis is also required. Kurtosis means the convexity of the probability curve of the distribution. Using the measure of coefficient of kurtosis, we can have an idea about the flatness or peakedness of the probability curve near its top.

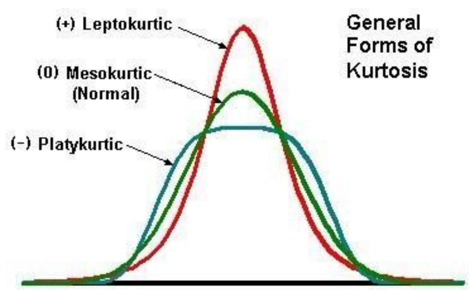
The only measure of kurtosis used is the fourth order central moment (μ_4).

The coefficient of kurtosis is defined as $\beta_2 = \frac{\mu_4}{\mu_2^2}$.

Note: 1. Curve which is neither flat nor peaked is called a mesokurtic curve, for which $\beta_2 = 3$

2. Curve which is flatter than the curve 1 is called platykurtic curve, for which $\beta_2 < 3$

3. Curve which is more peaked than the curve 1 is called leptokurtic curve, for which $\beta_2 > 3$



Bernoulli Experiment

A Bernoulli Experiment is a random experiment, the outcome of which can be classified in exactly one of two mutually exclusive and exhaustive ways, say, success or failure (e.g. female or male, non-defective or defective)

Bernoulli Random Variable:

Suppose that a trial, or an experiment, whose outcome can be classified as either a success or a failure is performed. If we let X = 1 when the outcome is a success and X = 0 when it is a failure, then the probability mass function of X is given by

p(0) = P{X = 0} = 1 − p

p(1) = P{X = 1} = p

where p, $0 \le p \le 1$, is the probability that the trial is a success.

A random variable X is said to be a Bernoulli random variable (after the Swiss mathematician James Bernoulli) if its probability mass function is given by above equations for some p ∈(0, 1).

Sequence of Bernoulli trials

A sequence of n trials is said to be a sequence of n Bernoulli trials if The trials are independent

Each trial results in exactly one of the 2 outcomes – **success** and **failure.** The probability of success in each trial is p, $0 \le p \le 1$

Binomial Distribution

Suppose X denotes the number of successes in a sequence of n Bernoulli trials and let the probability of success in each trial be p. Then X is said to follow a **Binomial distribution** with parameters n and p if the probability distribution of X is given by

$$B(n,p) = P(X = x) = p_x = {}^nC_x p^x q^{n-x}, x = 0,1,2......n$$

Example:

Suppose there are 2000 computer chips in a batch and there is a 2% probability that any one chip is faulty. Then the **number of faulty computer chips** in the batch follows a Binomial distribution with parameters n=2000 and p=2%.

Mean and variance of the Binomial distribution:

$$E(X) = np$$
$$E(X^2) = n(n-1)p^2 + np$$
$$Var(X) = E(X^2) - \{E(X)\}^2$$
$$= n(n-1)p^2 + np - n^2p^2$$
$$= npq$$

Recurrence relation for central moments of Binomial Distribution: $\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$

Moment generating function of the Binomial Distribution: $M_x(t) = (q + pe^t)^n$

Measure of Skewness = $npq(q-p)$

Measure of Kurtosis = $npq\left[1 + 3pq(n-2)\right]$

Poisson Distribution

A random variable X that takes on one of the values 0, 1, 2, . . . is said to be a Poisson random variable with parameter λ if, for some λ>0

$$P(X = x) = \frac{e^{-\lambda}.\lambda^{x}}{x!}, x = 0,1,2..........$$

Poisson distribution as a limiting case of the Binomial distribution:

The Poisson random variable has a tremendous range of applications in diverse areas because it may be used as an approximation for a binomial random variable with parameters (n, p) when n is large and p is small enough so that np is of moderate size.

The Poisson distribution is a limiting case of the binomial distribution under the following conditions:

n, the no of trails is indefinitely large, i.e. $n \to \infty$.

p ,the constant probability for the success of each trail is indefinitely small, i.e. $p \to 0$.

$np = \lambda$ is finite.

Moment Generating Function of the Poisson distribution= $M_X(t) = e^{-\lambda} e^{\lambda e^t}$

Mean of the Poisson distribution= $Mean = E(X) = \lambda$

Variance of the Poisson distribution= $Var(X) = \lambda$

In general, any $(k+1)^{th}$ **order central moment for Poisson distribution=** $\mu_{k+1} = \lambda\left[\frac{d\mu_k}{d\lambda} + k\mu_{k-1}\right]$

Measure of Skewness = λ

Measure of Kurtosis = $\lambda(3\lambda + 1)$

(Random Variable) M 1

Q.01 Write down the probability distribution of the sum of numbers appearing on the toss of two unbiased dice.

Q.02 For the above distribution,

i. find the probability that X is an odd number
ii. Find the probability that X lies between 3 and 9.

½,29/36

Q.03 A random variable X has following probability function:

X	0	1	2	3

p(X=x)	0	1/5	2/5	2/5

Determine the distributive function of x.

0,1/5,3/5,1

Q.04 PDF of random variable X is

X	0	1	2	3	4	5	6
p(X=x)	k	3k	5k	7k	9k	11k	13k

Find P(X < 4), P(X≥5) and P(3 < X ≤ 6).What will be the minimum value of k so that P(X≤2)>3.

16/49,24/49,33/49,1/3

Q.05 If X is a RV taking 0,1,2,3,4 with probabilities 0.20,0.30,0.25,0.15,0.10, find the probability distribution of $Y = 2X^2 + 3$ and find the probability that $Y \geq 20$.

Y	3	5	11	21	35
p_i	0.20	0.0	0.25	0.15	0.10

0.25

Q.06 The pmf of a RV X is zero except at the points X=0, 1, 2. At these points $P(0) = 3c^3, P(1) = 4c - 10c^2, P(2) = 5c - 1$. Determine

i. c

ii. Find $P(X < 1), P(1 < X \leq 2), P(0 < X \leq 2)$

1/3,1/9,2/3,8/9

Q.07 A random variable X has the following pdf

X	0	1	2	3	4	5	6	7
p(X=x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2$+k

Find i) k ii) P(0 < X < 5), P(X < 6), P(X ≥ 6).

1/10,4/5,81/100,19/100

Q.08 Let X be the RV taking values 1, 2, ……n and let $P(X = n) = \left(\frac{1}{2}\right)^n$.Let Y=1 when X is even and Y=-1 when X is odd. Find pdf of Y.

1/3,2/3

(Expectation, Mean Variance) M 1

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Q.01 A fair coin is tossed 3 times. A person receives Rs. X^2, if he gets X heads. Find his expectation.

3 Rs.

Q.02 Three urns contain resp. 3 green and 2 white balls, 5 green and 6 white balls and 2 green and 4 white balls. One ball is drawn from each urn. Find the expected number of white ball drawn.

1.61

Q.03 A box contains a white balls and b black balls of white balls. C balls are drawn from the box at random. Find the expected value of the number of white balls.

ac/(a+b)

Q.04 A random variate X has the probability distribution;

x	-3	6	9
P(X=x)	1/6	½	1/3

Find E(X), E(X^2) and E$(2X+1)^2$

11/2,93/2,209

Q05 A discrete variable has the pdf given below:

x	-2	-1	0	1	2	3
P(X=x)	0.2	k	0.1	2k	0.1	2k

Find k, mean and variance

3/25,6/25,293/625

(Moments)

Q.01 The random variable X can assume the values 1 and -1 with probability ½ each. Find (a) the moment generating function and (b) the first four moments about origin.

$$M_X(t) = \frac{1}{2}\left(e^t + e^{-t}\right),\ \ \mu = 0, \mu_2^{'} = 1, \mu_3^{'} = 0, \mu_4^{'} = 1$$

Q.02

A random variable X has the density function given by $f(x) = \begin{cases} 2e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

Find (a) the moment generating function and (b) the first four moments about origin.

$$M_X(t) = \frac{2}{2-t}, \text{assuming t} < 2;\ \ \mu = 1/2, \mu_2^{'} = 1/2, \mu_3^{'} = 3/4, \mu_4^{'} = 3/2$$

Q.03 Find the first four moments (a) about the origin, (b) about the mean, for a random variable X having density function $f(x)=\begin{cases}4x(9-x^2)/81 & 0\le x\le 3\\ 0 & otherwise\end{cases}$

$\mu_1^{'}=\mu=8/5, \mu_2^{'}=3, \mu_3^{'}=216/35, \mu_4^{'}=27/2$,

$\mu_1=0, \mu_1=11/25=\sigma^2, \mu_3=-32/875, \mu_4=3693/8750$

Q.04 (a) Find the moment generating function of a random variable X having density function $f(x)=\begin{cases}x/2 & 0\le x\le 2\\ 0 & otherwise\end{cases}$

(b) Use the generating function of (a) to find the first four moments about the origin.

$(a)(1+2te^{2t}-e^{2t})/2t^2 \quad (b)\mu=4/3, \mu_2^{'}=2, \mu_3^{'}=16/5, \mu_4^{'}=16/3$

Q.05 If X denotes the outcome when a fair die is tossed, Find the moment generating function of X and hence find the mean and variance of X.

$\frac{1}{6}\left(e^t+e^{2t}+...+e^{6t}\right), Mean=7/2, Variance=35/12$

Q.06 A r.v. X takes values 0 and 1 with probabilities q and p respectively with q+p=1. Find the mgf of X and show that all the moments about the origin equal p.

$M_X(t)=q+pe^t$

(Skewness and Kurtosis)

Q.01 The first four moments of a distribution about the value 5 of the random variable X are 2, 20, 40 and 50. Compute a measure, each of central tendency, dispersion, skewness and kurtosis. Comment on the skewness and kurtosis of the distribution.

[Ans. mean=7, SD=4, skewness= - 64, Kurtosis= 162]

Q.02 The first three moments of a distribution about the value 2 of the random variable X are 1, 16 and - 40. Find the coefficient of skewness.

-1.480

Q.03 The first four moments of a distribution about X = 4 are 1, 4, 10, 45 respectively. Find the mean, variance, coefficient of skewness and coefficient of Kurtosis.

[Ans. 5; 3; 0; 26/9]

Q.04 The distribution of a random variable X has mean 10, variance 16, coefficient of skewness 1 and coefficient of kurtosis 4. Obtain the first four moments of X about origin.

[Ans. 10, 116, 1544, 23184]

Q.05 Compute coefficient of skewness and coefficient of Kurtosis for the following distribution

$X=x_i$	0	1	2	3	4	5	6	7	8
p_i	0.004	0.036	0.1	0.232	0.280	0.204	0.112	0.028	0004

(Binomial Distribution)

Q.01 Find the Binomial distribution if the mean is 2 and variance is 4/3.

64/729,192/729,240/729,160/729,60/729,12/729,1/729.

Q.02 Out of 800 families with 4 children each, how many families would be expected to have

1. 2 boys and 2 girls
2. At least 1 boy
3. At most 2 girls
4. Children of both sexes.

Assume equal probability for boys and girls.

300,750,11/16,550

Q.03 It is know that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets by using Binomial distribution

184,264,920

Q.04 Fit a binomial distribution for the following data:

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

Also find skewness and kurtosis

X	0	1	2	3	4	5	6

F	4	15	25	22	11	3	0

0.288,5.58

Q.05 Find the mean, variance, skewness and kurtosis of the probability distribution of the number of heads obtained in three flips of a balanced coin.

Q.06 With usual notation find p of Binomial distribution if n=6, $9P\left(X=4\right)=P\left(X=2\right)$ Also find mean, variance, skewness and kurtosis

¼,3/2,9/8,9/16,45/32

Q.07 An irregular 6 faced dice is such that the probability that it gives 3 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets?

10

Q.08 Two dice are thrown 120 times. Find the average number of times in which the number on the first die exceeds the number on the second die.

50

Q.09 Find the mean and standard deviation, skewness and kurtosis of the following probability distribution.

$X=x_i$	0	1	2	3	4	5	6	7	8
p_i	0.004	0.036	0.1	0.232	0.280	0.240	0.112	0.028	0.004

[Ans. mean=3.972, SD=1.410]

Q.10 The sum and product of mean and variance of a Binomial distribution are 24 and 128. Find the distribution

N=32, p=q=1/2

(Poisson Distribution)

Q.01 A variable X follows a Poisson distribution with variance 3. Calculate

i. $P(X=2)$

ii. $P(X\geq 4)$

0.224,0.353

Q.02 The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month

1. Without a breakdown
2. With only one breakdown

3. With at least one breakdown

0.1653,0.2975,0.8347

Q.03 Find the probability that at most 5 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective.

0.7845

Q.04 If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals

i. Exactly 3
ii. More than 2
iii. None
iv. More than 1

Individual will suffer a bad reaction.

0.180,0.323,0.135,0.594

Q.05 A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used, (ii) some demand is refused

Ans. 0.2231, 0.1912

Q.06 Fit a Poisson distribution for the following: Also find mean, variance, skewness and kurtosis.

x	0	1	2	3	4
F	123	59	14	3	1

x	0	1	2	3	4
F	121	61	15	3	1

0.5,0.5,0.5,1.25

Q.07 Fit a Poisson distribution for the following data:

x:	0	1	2	3	4	5	Total
f:	142	156	69	27	5	1	400

Also find mean, variance, skewness and kurtosis

Q.08 If a random variable X follows Poisson distribution such that $P(X=1)=2P(X=2)$, find the mean, variance, skewness and kurtosis of the distribution. Also find P(X=3).

1,1,1,4,0.06143

Q.09 Find out the fallacy of the statement: "If X is a Poisson varaite such that $P(X=2)=9P(X=4)+90P(X=6)$ Then mean of X is 1." Also find mean, variance, coefficient of skewness and kurtosis.

Correct,1,1,1,4

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Continuous random variables and their properties, distribution functions and densities

Q.01 Suppose that in a certain region the daily rainfall (in inches) is a continuous RV X with p.d.f f(x) given by

M1

$$f(x)=\frac{3}{4}(2x-x^2), 0<x<2 \;\; \& \; f(x)=0 \; elsewhere$$

Find the probability that on a given day in this region, the rainfall is :

i. not more than 1 inch
ii. greater than 1.5 inches
iii. between 0.5 and 1.5 inches

Ans: ½,0.1562,0

Q.02 If f(x) is a density function

$$f(x)=\begin{cases} cx^2, & 0<x<3 \\ 0, & \text{otherwise} \end{cases}$$

then find i) c, ii) p(1 < x < 2) iii) C.D.F.

Ans: 1,7/27,x^3/27

Q.03

If $p(x)=\begin{cases} xe^{-x^2/2}, & x \geq 0 \\ 0, & x<0 \end{cases}$

i. Show that p(x) is a pdf of a continuous RV
ii. Find its distribution function.

$$F(x)=\begin{cases} 1-e^{-x^2/2}, & x \geq 0 \\ 0, & x<0 \end{cases}$$

Ans: 0.7937,0.9830

Continuous random variables and their properties, distribution functions and densities

Q.01 If the density function of a continuous RV is given by:

M1

$$f(x)=\begin{cases} ax, 0 \leq x \leq 1 \\ a, 1 \leq x \leq 2 \\ 3a-ax, 2 \leq x \leq 3 \\ 0, elsewhere \end{cases}$$

i. Find the value of a.

ii. Find the cdf of X

iii. If x_1,x_2 and x_3 are 3 independent observations of X, what is the probability that exactly one of these 3 is greater than 1.5.

Ans: ½,1,1/2

Q.02 A continuous RV X has a pdf $f(x)=kx^2e^{-x}, x\geq 0$. Find k, mean and variance.

Ans: ½,3,3

Q.03 A continuous RV has a pdf $f(x)=kx(1-x),\ 0\leq x\leq 1$. Find k and determine b such that

$P(X\leq b)=P(X>b)$

Ans: 6, 1/2

Q.04 A frequency distribution is defined by:

$$f(x)=\begin{cases} x^3, 0\leq x\leq 1 \\ 3(2-x)^3, 1\leq x\leq 2 \end{cases}$$

Prove that f(x) is a pdf. Also find mean and SD.

Ans: 11/10, $\sqrt{17/300}$

Q.05 *X is a continuous RV with pdf given by:*

$$f(x)=\begin{cases} kx & (0\leq x<2) \\ 2k & (2\leq x<4) \\ -kx+6k & (4\leq x<6) \end{cases}$$

Find k and mean value of X.

Ans: 1/8,3

Normal distribution

Q.01 In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution?

Ans: 50,10

Q.02 In an intelligence test administered to 1000 students, the average was 42 and stan dard deviation was 24. Find the number of students

i. Exceeding the score 50

ii. Between 30 and 54

Ans: 371, 383

Q.03 If X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find

(a) $P\{2 < X < 5\}$; (b) $P\{X > 0\}$; (c) $P\{|X - 3| > 6$

Ans: 0.3779,0.8413, .0456

Q.04 A sample of 100 dry battery cells tested to find the length of life produced the following results: Mean=12 hrs. SD=3 hrs. Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

i. More than 15 hrs.
ii. Less than 6 hrs.
iii. Between 10 & 14 hrs.

Ans: 15.87%,2.28%,49.74%

Q.05 If the actual amount of instant coffee which is filling machine puts into 6 ounce jars is a RV having a normal distribution with SD=0.05 ounce and if only 3% of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars?

Ans: 0.47,6.094

Normal distribution

Q.02 The income of a group of 10000 persons were found to be normally distributed with mean rs 520 and S.D. Rs 60. Find (i) the number of persons having incomes between Rs 400 and Rs 550? (ii) The lowest income of the richest 500.

Ans: 6687, Rs 618.40

Q.03 For a normal variate X with mean 25 and standard deviation 10, find the area between (i) X = 25, X = 35, (ii) X = 15, X = 35 and also the area such that, (iii) $X \geq 15$, (iv) $X \geq 35$.

Ans: 0.3413, 0.6826, 0.8413, 0.1587

Q.04 If the height of 500 students is normally distributed with mean 68 inches and SD 4 inches, estimate the number of students having heights (i) greater than 72 inches, (ii) less than 62 inches, (iii) between 65 and 71 inches.

Ans: 79, 33, 273

Q.05 Assume that the diameters of 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515 inches and SD 0.0020 inches. How many of the plugs are likely to be rejected if the diameter is to be 0.752 ± 0.004 inches?

Ans: 52

Q.06	For a normally distributed variate X with mean 1 and s.d 3, find $P(3.43 \le X \le 6.19)$ & $P(-1.43 \le X \le 2.3)$ Ans: 0.1672, 0.4574