Unit I

Unit I: Basic Probability

Probability spaces, conditional probability, independence; Bayes theorem

Probability spaces

Consider an experiment whose outcome is not predictable with certainty. However, although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known. This set of all possible outcomes of an experiment is known as the **SAMPLE SPACE** of the experiment and it is denoted by S.

Example: If the experiment consists of flipping two coins, then the sample space consists of the following four points S = {HH, HT, TH, TT }

Each outcome in a sample space is called a Sample Point Number of sample points in a sample space S is $\mathbf{n(S) = n^k}$ Where n = number of outcomes and k = number of objects

Probability:

If an experiment results in 'n' exhaustive, mutually exclusive and equally likely cases and 'm' of them are favorable to the happening of an event 'A' then Probability of happening of A is

$$P(A) = \frac{m}{n}$$

Since the number of cases in which the event A will not happen is 'n – m', the probability that event A will not happen is:

$$P(\overline{A}) = \frac{n-m}{n}$$

Therefore: $P(A) + P(\overline{A}) = 1$ This formula

Axioms of probability:

Consider an experiment whose sample space is S. For each event E of the sample space S, then

This theory

1. $0 \le P(E) \le 1$
2. $P(S) = 1$

Laws of probability: This theory

1. Addition theorem:
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   If A and B are exclusive events i.e. disjoint sets, then: $P(A \cup B) = P(A) + P(B)$
2. Addition theorem (for three events):If A, B and C are pairwise exclusive events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Complementary Event: $P(A') = 1 - P(A)$

Conditional Probability and Independence

If A and B are two events in a sample space S, then the probability of the event A when the event B has already occurred is called the conditional probability of A and is denoted by P(A | B) and defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ This theory

The probability P(A | B) is an updating of P(A) based on the knowledge that event B has already occurred.

Multiplication law of probability: $P(A \cap B) = P(B|A).P(A) = P(A|B).P(B)$ This formula

Independent events:

A set of events is said to be independent if the occurrence of any one of them does not depend on the occurrence or non-occurrence of the others. If two events A and B are independent then:

$P(A \cap B) = P(A).P(B)$ This formula

Mutually exclusive is the same as disjoint, i.e. when talking of events, they can't happen at the same time.

Exhaustive means that the events together make up everything that can possibly happen.

Theorem of total probability: This theory

If B_1, B_2,……B_n be a set of exhaustive and mutually exclusive events and A is another event associated with B_i, then

$$P(A) = \sum_{i=1}^{n} P(B_i) P\left(\frac{A}{B_i}\right)$$

Baye's theorem: This theory

If E1, E2, E3, . . . En are mutually exclusive and exhaustive events with P(Ei) $\neq 0$ for i = 1 to n of a RANDOM experiment then for any arbitrary event 'A' of the sample spaces of the above experiment with P(A) > 0 , we have i=1

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i).P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$$

References of the entire unit

Solution to all questions below with all steps

SVKM's Narsee Monjee Institute of Management Studies
Mukesh Patel School of Technology Management & Engineering

1. *Probability, Statistics and Random Processes, T. Veerarajan, Tata McGraw Hill, 3rd edition.*
2. *Fundamentals of Mathematical Statictics, S.C. Gupta & V.K. Kapoor, Sultan Chand & Sons*

Session 01(Basic Probability)

Q.01 If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

1/6

Q.02 A fair coin is tossed 4 times. Define the sample space corresponding to this random experiment. Also give the subsets corresponding to the following events and find the respective probabilities.

b. More heads than tails are obtained.
c. Tails occur on the even numbered tosses.

a. 5/16, b. 1/4

Q.03 If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and the other two black?

4/11

Q.04 A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

240/1001

Session 02(Conditional Probability & Bayes Theorem)

Q.01 A box contains 4 bad and 6 good tubes. Two are drawn out from the box one after other. One of them is tested and found to be good. What is the probability that the other one is also good?

5/9

Q.02 Two fair dice are thrown independently. Three events A, B, and C are defined as follows:

(i) Odd face with the first die.
(ii) Odd face with second die.
(iii) Sum of the numbers in the 2 dice is odd. Are the events A, B and C mutually independent?

No.

Q.03 From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is positive?

505/1001

Q04 A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random (without replacement). Find the probability that

(i) Both are good.
(ii) Both have major defects.
(iii) At least 1 is good.
(iv) At most 1 is good.
(v) Exactly I is good
(vi) Neither has major defects
(vii) Neither is good

3/8,1/120,7/8,5/8,1/2,91/120,1/8

Q.05 There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used?

16/19

Q.06 A bag contains 5 balls and it is known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white

1/2