Probability and Statistics

Unit II: Random Variables & Probability Distribution

Random Variables: Sachin Verma

Mukesh Patel School of Technology Management & Engineering,Mumbai, India

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- Probability mass function
- Cumulative distribution function,
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Definition

A random variable (abbreviatively RV) is a function that assigns a real number $X(s)$ to every element $s \in S$, where S is the sample space corresponding to a random experiment E.)

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If X is a random variable (RV) which can take a finite number or countably infinite number of values, X is called a discrete RV. When the RV is discrete, the possible values of X may be assumed as x_1 ,x_2 , x_3, x_n ,...

Definition

If X is a discrete RV which can take the values x_1 ,x_2 , x_3,... such that $P(X = x_i) = p_i$, then p_i is called the probability function or probability mass function or point probability function, provided p_i (i = 1, 2, 3, ...) satisfy the following conditions:

1. $p_i \geq 0$
2. $\sum_i p_i = 1$

Examples

- Bernoulli
- Binomial
- Poisson

Question (Random Variable Q.3.)

PDF of random variable X is

X	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find k, $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

Solution: We know

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$$\sum_{i=1}^{7} P(X=x) = 1$$

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$$k = \frac{1}{49}$$

Question (Random Variable Q.3.)

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$$k=\frac{1}{49}$$

X	0	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

$$P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

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$$=$$

$$P(X \geq 5) = P(X = 5) + P(X = 6)$$

$$P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$=$$

$$P(X \geq 5) = P(X = 5) + P(X = 6) \qquad\qquad =$$

$$P(3 < X \leq 6) = P(X = 4) + P(X = 5) + P(X = 6)$$

Question (Random Variable Q.3.)

A random variable X has following probability function:

X	0	1	2	3
$P(X = x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

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Determine the distributive function of X.

Solution:

$$F(X = x) = P(X \leq x)$$

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X	0	1	2	3
$P(X=x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

$$F(X=x) = P(X \leq x)$$

$$F(X=0) = P(X \leq 0) = P(X=0) = 0$$

Question (Random Variable Q.3.)

A random variable X has following probability function:

X	0	1	2	3
$P(X=x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

$$F(X=x)=P(X\leq x)$$

$$F(X=0)=P(X\leq 0)=P(X=0)=0$$

$$F(X=1)=P(X\leq 1)$$

Question (Random Variable Q.3.)

A random variable X has following probability function:

X	0	1	2	3
$P(X=x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

$$F(X=x) = P(X \leq x)$$

$$F(X=0) = P(X \leq 0) = P(X=0) = 0$$

$$F(X=1) = P(X \leq 1) = P(X=0) + P(X=1) = \frac{1}{5}$$

Question (Random Variable Q.3.)

A random variable X has following probability function:

X	0	1	2	3
$P(X=x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

$$F(X=x) = P(X \leq x)$$

$$F(X=0) = P(X \leq 0) = P(X=0) = 0$$

$F(X=1) = P(X \leq 1) = P(X=0) + P(X=1) = \frac{1}{5}$

$F(X=2) = P(X \leq 2)$

Question (Random Variable Q.3.)

A random variable X has following probability function:

X	0	1	2	3
$P(X=x)$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Determine the distributive function of X.

Solution:

$$F(X=x) = P(X \leq x)$$

$$F(X=0) = P(X \leq 0) = P(X=0) = 0$$

$$F(X=1) = P(X \leq 1) = P(X=0) + P(X=1) = \frac{1}{5}$$

$$F(X=2) = P(X \leq 2) = P(0) + P(1) + P(2) = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

$$F(X = 3) = P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$F(X=3)=P(X\leq 3)=P(0)+P(1)+P(2)+P(3)$$

$$F(X=3)=\frac{1}{5}+\frac{2}{5}+\frac{2}{5}=\frac{5}{5}$$

The distributive function is

x	$P(X=x)$	$F(X=x)$
0	0	0
1	$\frac{1}{5}$	$0+\frac{1}{5}=\frac{1}{5}$
2	$\frac{2}{5}$	$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$
3	$\frac{2}{5}$	$\frac{3}{5}+\frac{2}{5}=\frac{5}{5}=1$

Question (Random Variable Q.1.)

Write down the probability distribution of the sum of numbers appearing on the toss of two unbiased dice.

Solution: Two unbiased dice are rolled. Sample Space

$$S=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),\\ (2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),\\ (5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

X: Sum of numbers appearing on dice.

$x = 2, 3, 4, 5, \ldots, 12.$

x	elements	$P(X = x)$
2	$\{(1,1)\}$	$\frac{1}{36}$
3	$\{(1,2),(2,1)\}$	$\frac{2}{36}$
4	$\{(1,3),(3,1),(2,2)\}$	$\frac{3}{36}$
5	$\{(1,4),(4,1),(2,3),(3,2)\}$	$\frac{4}{36}$
6	$\{(2,4),(4,2),(1,5),(5,1),(3,3)\}$	$\frac{5}{36}$
7	$\{(2,5),(5,2),(1,6),(6,1),(3,4),(4,3)\}$	$\frac{6}{36}$
8	$\{(3,5),(5,3),(2,6),(6,2),(4,4)\}$	$\frac{5}{36}$
9	$\{(5,4),(4,5),(6,3),(3,6)\}$	$\frac{4}{36}$
10	$\{(4,6),(6,4),(5,5)\}$	$\frac{3}{36}$
11	$\{(5,6),(6,5)\}$	$\frac{2}{36}$
12	$\{(6,6)\}$	$\frac{1}{36}$

Question (Random Variable Q.2.)

For the above distribution,

1. *Find the probability that X is an odd number.*
2. *Find the probability that X lies between 3 and 9.*

Solution:

$P(\text{odd number})$

$$= P(X = 3) + P(X = 5) + P(X = 7) + P(X = 9) + P(X = 11)$$

$$= \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \frac{18}{36} = \frac{1}{2}$$

$$P(3 \leq X \leq 9) = P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9)$$

$$= \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} = \frac{29}{36}$$

Question (Random Variable Q.5.)

If X is a RV taking 0,1,2,3,4 with probabilities 0.20,0.30,0.25,0.15,0.10, find the probability distribution of $Y = 2X^2 + 3$ *and find the probability that* $Y \geq 20$

Solution:

X	$P(X = x)$	$Y = 2X^2 + 3$
0	0.20	$2(0)^2 + 3 = 3$
1	0.30	$2(1)^2 + 3 = 5$
2	0.25	$2(2)^2 + 3 = 11$
3	0.15	$2(3)^2 + 3 = 21$
4	0.10	$2(4)^2 + 3 = 35$

$$\begin{aligned} P(Y \geq 20) &= P(Y = 21) + P(Y = 35) \\ &= P(X = 3) + P(X = 4) \\ &= 0.15 + 0.10 = .25 \end{aligned}$$

Continuous Random Variable

If X is an RV which can take all values (i.e., infinite number of values) in an interval, then X is called a continuous RV.
For example, the length of time during which a vacuum tube installed in a circuit functions is a continuous RV.

Probability Density Function

If X is a continuous RV such that

$$P\{x - \frac{1}{2}dx \leq X \leq x + \frac{1}{2}dx\} = f(x)dx$$

then f (x) is called the probability density function (shortly denoted as pdf) of X, provided f (x) satisfies the following conditions:

1. $f(x) \geq 0, \ \ for \ all \ x \in R_x$
2. $\int_{R_x} f(x)dx = 1$

Examples

- Uniform
- normal
- Rayleig

Cumulative Distribution Function

If X is an RV, discrete or continuous, then $P(X \leq x)$ is called the cumulative distribution function of X or distribution function of X and denoted as F(x). If X is continuous,

$$F(x) = P(-\infty \leq X \leq x) = \int_{-\infty}^{x} f(x)dx$$

Properties of cdf

Properties of the *cdf* F(x)

1. $F(x)$ is a non-decreasing function of x, i.e., if $x_1 < x_2$, then $F(x_1) \le F(x_2)$.
2. $F(-\infty) = 0$ and $F(\infty) = 1$.
3. If X is a discrete RV taking values $x_1, x_2, \ldots$, where $x_1 < x_2 < x_3 < \ldots < x_{i-1} < x_i < \ldots$, then $P(X = x_i) = F(x_i) - F(x_{i-1})$.
4. If X is a continuous RV, then $\frac{d}{dx} F(x) = f(x)$, at all points where $F(x)$ is differentiable.

Note *Although we may talk of probability distribution of a continuous RV, it cannot be represented by a table as in the case of a discrete RV. The probability distribution of a continuous RV is said to be known, if either its pdf or cdf is given.*

Question (Q.1.)

If $f(x)$ is a density function

$$f(x) = \begin{cases} cx^2, & 0 < x < 3 \\ 0, & elsewhere \end{cases}$$

Then find (i) c, (ii) $P(1 < X < 2)$ (iii) cdf

Solution: We know,

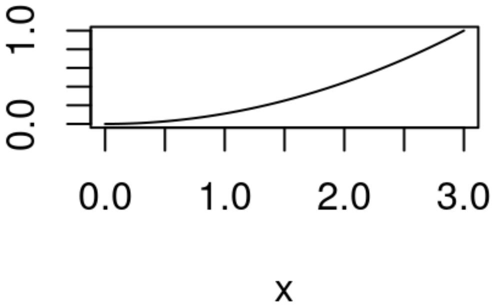
$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{0}^{3} cx^2 dx = 1$$

$$c\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$c = \frac{1}{9}$$

$$f(x) = \begin{cases} \frac{x^2}{9}, & 0 < x < 3 \end{cases}$$


1.0
0.0
0.0
1.0
2.0
3.0
x

$$P(1 < X < 2) = \int_1^2 \frac{x^2}{9} dx = \frac{7}{27}$$

cdf

$$F(x) = P(-\infty \leq X \leq x) = \int_{-\infty}^{x} f(x) dx$$

$$F(x) = \int_0^x \frac{x^2}{9} dx = \frac{x^3}{27}$$

$$\boxed{F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^3}{27}, & 0 < x < 3 \\ 1, & x \geq 3 \end{cases}}$$

Question (Q.2.)

$$f(x) = \begin{cases} xe^{\frac{-x^2}{2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

1. *Show that $f(x)$ is a pdf of a continuous random variable.*
2. *Find its distribution function*

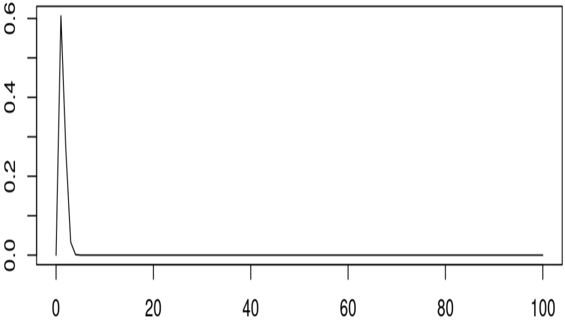
Solution: Consider

$$I = \int_0^\infty xe^{\frac{-x^2}{2}} dx$$

Put $\frac{x^2}{2} = t$, $xdx = dt$

$$I = \int_0^\infty e^{-t} dt = \left[-e^{-t} \right]_0^\infty$$

$$= -e^{-\infty} + e^0 = 0 + 1 = 1$$


0.0
0.2
0.4
0.6
0
20
40
60
80
100

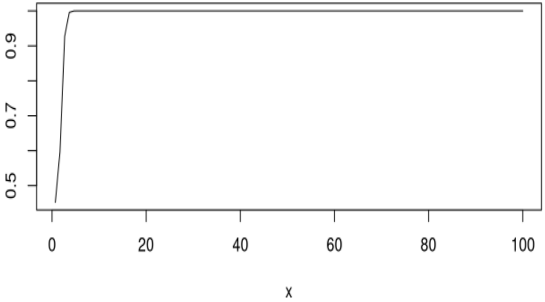
Distribution function is given by

$$F(x) = P(-\infty \leq X \leq x) = \int_{-\infty}^{x} f(x)dx$$

$$F(x) = \int_{0}^{x} xe^{\frac{-x^2}{2}} dx = 1 - e^{\frac{-x^2}{2}}$$

$$F(x) = \begin{cases} 1 - e^{\frac{-x^2}{2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

cdf



Question (Q.3.)

Suppose that in a certain region the daily rainfall (in inches) is a continuous RV X with p.d.f $f(x)$ given by

$$f(x) = \begin{cases} \frac{3}{4}(2x - x^3), & 0 < x < 2 \\ 0, & elsewhere \end{cases}$$

Find the probability that on a given day in this region the rainfall is

1. *not more than 1 inch*
2. *greater than 1.5 inches*
3. *between 0.5 and 1.5 inches*

Hint: X: daily rainfall in inches

1. $$P(\text{not more than 1 inch}) = P(X \leq 1) = \int_0^1 \frac{3}{4}(2x - x^3)dx$$

2. $$P(\text{greater than 1.5 inches}) = P(X \geq 1.5) = \int_{1.5}^2 \frac{3}{4}(2x - x^3)dx$$

3. $$P(\text{between 0.5 and 1.5 inches}) = P(0.5 \leq X \leq 1.5) = \int_{0.5}^{1.5} \frac{3}{4}(2x - x$$

Expectation

X is a discrete random variable.

$$E(X) = \sum x.P(X)$$

$$E(X^2) = \sum x^2.P(X = x)$$

$$Var(X) = E(X^2) - [E(X)]^2$$

Question

For the continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} cx(2-x) & \textit{if } 0 \leq x \leq 2 \\ 0 & \textit{otherwise} \end{cases}$$

Find c, mean and variance.

Question (Expectation, Mean and Variance Q.4.)

A random variate X has the probability distribution;

x	-3	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$, $E(2X+1)^2$

Solution:

$$E(X) = \sum xP(X=x) = \left(-3 \times \tfrac{1}{6}\right) + \left(6 \times \tfrac{1}{2}\right) + \left(9 \times \tfrac{1}{3}\right) =$$

$$E(X^2) = \sum x^2 P(X=x) = \left((-3)^2 \times \tfrac{1}{6}\right) + \left(6^2 \times \tfrac{1}{2}\right) + \left(9^2 \times \tfrac{1}{3}\right) =$$

$$\begin{aligned} E(2X+1)^2 &= E(4X^2+4X+1) \\ ` &= E(4X^2) + E(4X) + 1 \\ &= 4E(X^2) + 4E(X) + 1 \end{aligned}$$

Question (Expectation, Mean and Variance Q.1.)

A fair coin is tossed 3 times. A person receives Rs. X^2 , if he gets X heads. Find his expectation.

Solution: A fair coin is tossed 3 times.

S=$\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$, $\quad n(S) = 8$

X: number of heads.

$x = 0, 1, 2, 3$

X	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
X^2	0	1	4	9

$$E(X^2) = \sum x^2 P(X = x)$$

$$= (0 \times \tfrac{1}{8}) + (1^2 \times \tfrac{3}{8}) + (2^2 \times \tfrac{3}{8}) + (3^2 \times \tfrac{1}{8})$$

Question (Homework problem)

A discrete variable has the pdf given below:

x	-2	-1	0	1	2	3
$P(X = x)$	.2	k	0.1	$2k$	0.1	$2k$

Find k, mean and variance.

Expectation and Variance

If X is a continuous random variable, then the expectation (or expected value or mean) of X, denoted by E[X], is defined by

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)dx$$

Variance

$$V(X) = E[X^2] - [E(X)]^2$$

Standard deviation

$$SD = \sqrt{V(X)}$$

Gamma Function

$$\boxed{\int_0^{\infty} e^{-x} x^{n-1} dx = \Gamma(n) = (n-1)!}$$

Question

A continuous RV X has a pdf

$$f(x) = kx^2e^{-x}; x \geq 0.$$

Find k, mean and variance.

Solution: A continuous RV X has a pdf

$$f(x) = kx^2e^{-x}; x \geq 0.$$

$$\int_0^\infty kx^2e^{-x}dx = 1 \qquad \boxed{\int_{-\infty}^{\infty} f(x)dx = 1}$$

$$k = \frac{1}{2}$$

$$\boxed{f(x) = \frac{1}{2}x^2e^{-x}; x \geq 0.}$$

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx, \qquad \text{X is continuous}$$

$$E[X] = \int_{0}^{\infty} x.\frac{1}{2}.x^2 e^{-x} dx$$

$$= \frac{1}{2}\int_{0}^{\infty} x^3 e^{-x} dx = \frac{1}{2} \times 3!$$

$$\boxed{E(X) = 3}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$E[X^2] = \frac{1}{2}\int_{0}^{\infty} x^4 e^{-x} dx = 12$$

$$V(X) = E[X^2] - [E(X)]^2 = 12 - 9 = 3$$

Bernoulli trial

Suppose that a trial, or an experiment, whose outcome can be classified as either a success or a failure is performed.
If we let $X = 1$ when the outcome is a success and $X = 0$ when it is a failure, then the probability mass function of X is given by
$p(0) = P(X = 0) = 1 - p$
$p(1) = P(X = 1) = p$

Sequence of Bernoulli trials

A sequence of n trials is said to be a sequence of n Bernoulli trials if The trials are independent Each trial results in exactly one of the 2 outcomes – success and failure. The probability of success in each trial is p

Binomial distribution

Suppose X denotes the number of successes in a sequence of n Bernoulli trials and let the probability of success in each trial be p. Then X is said to follow a Binomial distribution with parameters n and p if the probability distribution of X is given by

$$B(n,p) = P(X = x) = {}^{n}C_{x}\; p^{x} q^{n-x}, x = 0, 1, 2, 3, ..., n$$

Mean and variance of the Binomial distribution

$$E(X) = np$$

$$V(X) = E(X^2) - (E(X))^2 = npq$$

Question

Find the binomial distribution if mean is 2 and variance is $\frac{4}{3}$

Question

Two dice are thrown 120 times. Find the average number of times in which the number on the first die exceeds the number on the second die.

Solution The number on the first die exceeds that on the second die, in the following combinations:
(2, 1); (3, 1), (3, 2); (4, 1), (4, 2), (4, 3); (5, 1), (5, 2), (5, 3); (5, 4); (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),
P(success) = P(no. in the first dice exceeds the no. in the second dice)$=\frac{15}{36}$
This probability remains the same in all the throws that are independent.
If X is the no. of successes, then X follows a binomial distribution with n=120, $p = \frac{15}{36}$

$$E(X) = np = 120 \times \frac{15}{36}$$

Question

An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets?

Solution: Let the probability of getting an even number with the unfair dice be p.
Let X denote the number of even numbers obtained in 5 trials .

$$P(X = 3) = 2P(X = 2) = ^5C_3\ p^3q^2 = 2 \times ^5C_2\ p^2q^3$$

$p = \frac{2}{3}$
Now P(getting no even number)$= P(X = 0) = ^5C_0\ p^0q^5 = \frac{1}{243}$
Number of sets having no success (even number) out of N sets
$=$ N $\times$ P(X $=$ 0)
Required number of sets $=$ 2500 $\times \frac{1}{243}$

Question

Out of 800 families with 4 children each, how many families would be expected to have

1. *2 boys and 2 girls*
2. *at least 1 boy*
3. *at most 2 girls*
4. *children of both sexes*

Assume equal probabilities for boys and girls.

Solution Considering each child as a trial, n = 4.
p=$\frac{1}{2}$, q=$\frac{1}{2}$
Let X denote the number of boys.

1. $P(2\ boys\ and\ 2\ girls) = P(X = 2)$
$=^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} =$

No. of families having 2 boys and 2 girls
= N (P(X = 2) (where N is the total no. of families)
$800 \times \frac{3}{}$

2 $P(at\ \ least\ \ 1\ \ boy) = P(X \geq 1)$
$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

3 $P(at\ \ most\ \ 2\ \ girls) = P(exactly\ 0\ girl,\ 1\ girl\ or\ 2\ girls)$

4 $P(children\ \ of\ \ both\ \ sexes)$
$= 1 – P(children\ \ of\ \ the\ \ same\ \ sex)$
$= 1 – \{P(all\ \ are\ \ boys) + P(all\ \ are\ \ girls)\}$
$= 1 – \{P(X = 4) + P(X = 0)\}$

Question

Fit a binomial distribution for the following data:

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

Question

With usual notation find p of Binomial distribution if $n = 6$,

$$9(X = 4) = P(X = 2)$$

Also find mean, variance

Applications

The Poisson distribution has many applications in science and engineering.

- The number of telephone calls arriving at a switchboard during various intervals of time.
- Number of customers arriving at a bank during various intervals of time.

are usually modeled by Poisson random variables.

Poisson distribution

A discrete random variable X is called a Poisson random variable with parameter λ, where $\lambda > 0$, if its PMF is given by

$$p_X(x) = \frac{\lambda^x}{x!} e^{-\lambda} \qquad x = 0, 1, 2, \dots$$

The CDF of X is given by

$$F_X(x) = P[X \leq x] = \sum_{r=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda}$$

Expectation and Variance

The expected value of X is given by

$$E[X] = \lambda$$

The second moment of is given by

$$E[X^2] = \lambda^2 + \lambda$$

The variance of X is given by

$$\sigma_X^2 = E[X^2] - (E[X])^2 = \lambda$$

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If X is a poisson variate such that $E(X^2) = 6$, find E(X).

If X is a Poisson variate such that

$$2P(X=0)+P(X=2)=2P(X=1)$$

, find $E(X)$.

A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day follows a Poisson distribution with mean 1.5. Calculate the proportion of days on which

1. neither car is used and
2. some demand is not fulfilled.

Problems

Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

1. Exactly two messages arrive within one hour.
2. No message arrives within one hour.
3. At least three messages arrive within one hour.

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Exactly two messages arrive within one hour.

No message arrives within one hour.

At least three messages arrive within one hour.

Normal distribution

A continuous random variable X is defined to be a normal random variable with parameters μ_X and σ_X^2 if its PDF is given by

$$f_X(x) = \frac{1}{\sigma_X\sqrt{2\pi}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} \qquad -\infty < x < \infty$$

The PDF is a bell-shaped curve that is symmetric about μ_X , which is the mean of X

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The CDF of X is given by

$$F_X(x) = P[X \leq x] = \frac{1}{\sigma_X\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(u-\mu_X)^2}{2\sigma_X^2}} du$$

SNV

pdf of SNV is

$$\Phi_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy$$

The CDF of X is given by

$$F_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{u-\mu_X}{\sigma_X}} e^{-\frac{y^2}{2}} dy$$

Question (ND Q.3.)

If X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find

1. $P(2 < X < 5)$;
2. $P(X > 0)$;

Solution

Question (ND Q.7.)

For a normal variate X with mean 25 and standard deviation 10, find the area between

1. $X = 25, X = 35$
2. $X = 15, X = 35$
3. $X \geq 15$
4. $X \geq 35$

Solution

Question (ND Q.8.)

If the height of 500 students is normally distributed with mean 68 inches and SD 4 inches, estimate the number of students having heights

1. *greater than 72 inches*
2. *less than 62 inches*
3. *between 65 and 71 inches.*

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Question (ND Q.10.)

For a normally distributed variate X with mean 1 and s.d 3, find

1. $P(3.43 \leq X \leq 6.19)$
2. $P(-1.43 \leq X \leq 2.3)$