



# The use of the photogrammetric method for measurement of the repose angle of granular materials

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## ABSTRACT

The paper addresses a vitally important issue of precise determination of the angle of repose of granular materials. For calculation of the said angle, a photogrammetric 3D coordinate measurement method has been proposed. With the view of method verification, 600 independent measurement results were obtained, based on which the angle of repose of examined plant granular materials (triticale) was determined with a statement of the associated measurement uncertainty. The conducted analysis has shown that the proposed method is useful and capable of low (as for biological materials) measurement uncertainty congruent with the requirements of automated systems, and as such may be helpful in laying down the standard specification for measuring of the angle of repose, using coordinate metrology.

## 1. Introduction

Granular materials can be viewed as raw materials consisting of solid particles exhibiting specific granulometric composition. They are in widespread use and find application in any human economic activity, i.e., food, agriculture, construction, energy, and pharmaceuticals. Regardless of the application of a given granular material, it can be generally stated that during industrial processing they are subjected to the same technological processes and single operations. Table 1 shows the examples of particulate solids and the areas of their application.

The prevalence of these materials encourages many researchers to explore and catalog their physical properties in order to understand their behavior during the mentioned technological operations [1,2]. This knowledge is crucial for the design of appropriate machines and equipment, especially automated systems. The properties describing a given granular material comprise granulometric composition, shape, bulk density, porosity, mechanical features, external and internal friction, and the angle of repose.

One of the basic parameters characterizing granular materials, the key one – in author's opinion – in the design of automated discharge and transport systems, is the angle of repose. Broadly speaking, when particles are poured onto a flat surface, a conical pile is formed. The angle between the slant height and the base of a cone is known as the

angle of repose. The value of the angle of repose affects the behavior of a given granular material during transport and discharge, owing to which it is perceived as a decisive factor in the design of conveyors and silos. Its value depends on friction between moving particles, which – to a large extent – is dependent on moisture, granulometric composition, shape, size, and surface texture of the particles.

The angle of repose becomes the subject of interest of numerous researchers either when a new kind of particulate matter is to be investigated or once the behavior of a given granular material has to be verified under altered conditions. Therefore, there is no universal method by which the angle of repose can be measured. The process of repose angle determination can be divided into two stages: (1) the study of how a piled cone is formed; and (2) the study of how the acquired repose angle is measured.

The basic methods of cone formation include “emptying” (Fig. 1a), “piling” (Fig. 1b), “submerging” (Fig. 1c), “pouring” (Fig. 1d) [3–12]. There are also “rotating” and “aerating” methods [13–16]. Considering that the methods slightly differ from one another, they are accompanied by various physical conditions (e.g., molecular kinetic energy), thereby producing somewhat different results.

The basic issue, therefore, is to measure the angle of repose, i.e., the one formed between the slant height and the base of a cone (Fig. 1e), and – what is hugely important – to perform measurement with utmost

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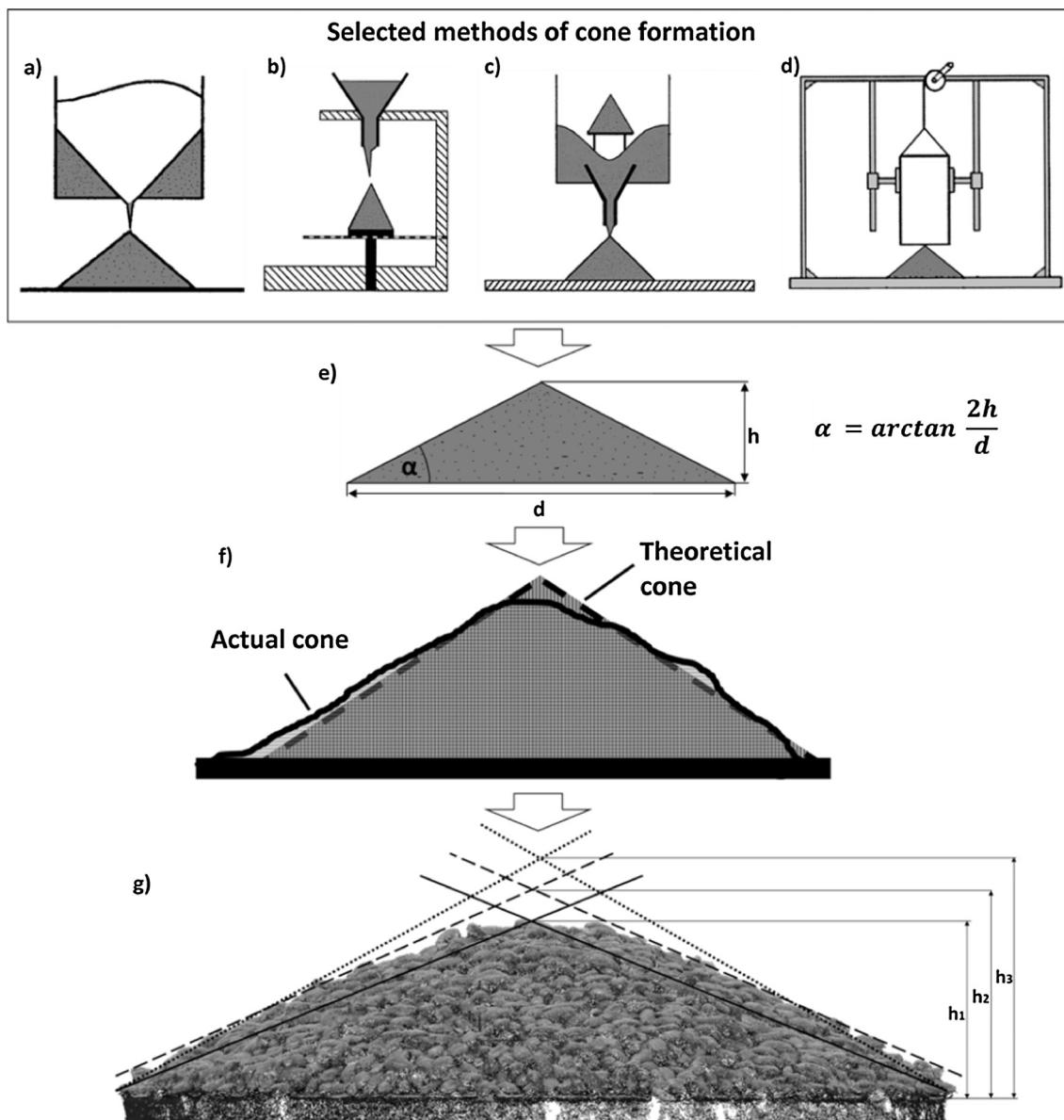
E-mail addresses: [artur.wojcik@urk.edu.pl](mailto:artur.wojcik@urk.edu.pl) (A. Wójcik), [przemyslaw.klapa@wp.pl](mailto:przemyslaw.klapa@wp.pl) (P. Klapa), [bartosz.mitka@ur.krakow.pl](mailto:bartosz.mitka@ur.krakow.pl) (B. Mitka), [sladek@mech.pk.edu.pl](mailto:sladek@mech.pk.edu.pl) (J. Śladek).

**Table 1**  
Areas of application of granular materials.

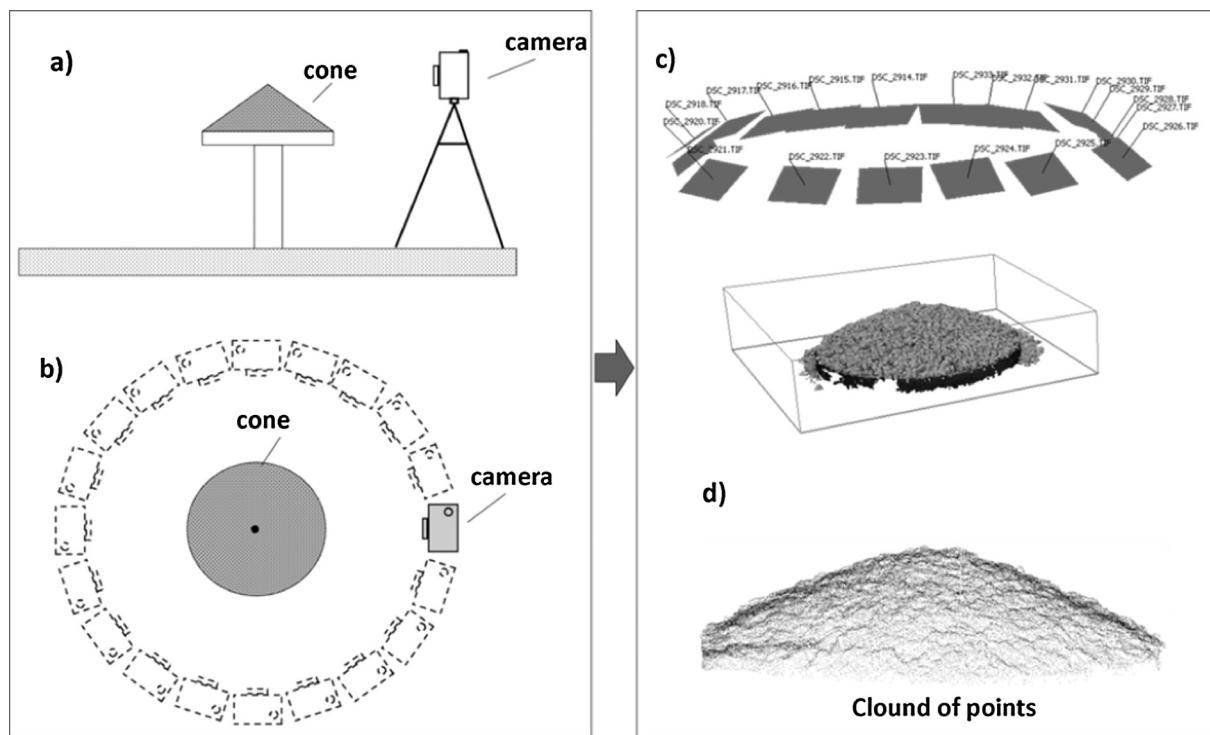
| Areas of application    | Granular materials   | Technological process     |
|-------------------------|--|---------------------------|
| Pharmaceutics cosmetics | Powders  | Crushing                  |
| Food                    | Reinforcing materials<br>Comminuted fruit<br>Vegetables<br>Flour<br>Cereal | Sorting<br>Mixing         |
| Agriculture             | Grain seeds<br>Mineral fertilizers   | Agglomerating             |
| Construction            | Animal feed<br>Aggregates<br>Sand<br>Cement                                | Storing<br>Precise dosing |
| Energy                  | Coal<br>Slag<br>Comminuted biomass   | Transporting              |

accuracy. In the majority of the quoted works, this issue seems to have been overlooked or at least insufficiently discussed. However, it belongs to key factors which need to be incorporated into the methodology in order to avoid misinterpretation of the value of the repose angle. In most cases, the quoted authors invoke optical methods inherent in image analysis carried out on the basis of a single or a set of pictures [10,13–15,17,18]. The use of a laser device for the assessment of the repose angle of landslide debris deposits may serve as another example [19]. The full methodology and algorithm for the determination of the angle of repose are provided by [3]; the method involves precise positioning of the camera relative to the pile, measured using a laser device as well as being based on the analysis of the inclined plane (slant height) of the pile in order to calculate a regression line; the proposed method, however, lacks the analysis of measurement uncertainty. Still, these are all considerations relating to a two-dimensional (2D) plane.

In the authors' view, indicative determination of the angle of repose, in many studies, e.g., the ones bordering on agronomy and physics, appears to be sufficient, however, from the metrological point of view, the issue requires more attention. Moreover, the analysis of the issue



**Fig. 1.** Selected methods of cone formation. (a) “emptying”, (b) “piling”, (c) “submerging”, (d) “pouring”, (e) analytical determination of the angle of repose, (f) actual cone vs. theoretical cone, (g) the interpretation problems of the right edge and the height of a pile.



**Fig. 2.** Data acquisition method: (a) testing facility - side view, (b) testing facility – top view, (c) distribution of taken pictures, (d) obtained point cloud.

with respect to 2D planes suffers from methodological bias, entailing errors resulting from the measuring characteristics of an image as well as its geometrical features captured by the lens of a camera. First of all, ideal imaging can be regarded as a perspective projection. From the features of perspective projection, it follows that homogeneous image coordinates can only be obtained when the photograph of the flat surface orthogonal to the axis of the camera is taken. In other cases, perspective or radial distortions of an image, caused by its spatial form, may occur.

A radial displacement of image points is expressed by the following formula (1):

$$dr = \frac{r}{M_z C_k} \Delta h \quad (1)$$

where:

$d_r$  – radial displacement of image points caused by elevation differences across an object.

$r$  – radial distance.

$M_z$  – photo scale denominator.

$C_k$  – calibrated focal length.

$\Delta h$  – object elevation differences (deviation from a reference plane).

From the above equation, it may be inferred that the geometrical form of an image varies from the minimum at its principal point to the maximum at its extremities.

From the geometrical properties of an oblique images, it follows that for the entire image surface, the measurement of angles involves errors resulting from the angle of inclination and elevation differences across the photographed object. For such a picture, there are two characteristic points lying along the principal line: the nadir point at which angle measurement does not entail errors arising from elevation differences; isocenter point at which angle measurement is not affected by image inclination. Between the nadir and the isocenter, there is a point which is least influenced by both distortions [20].

In addition, image geometry is subjected to radial distortion and tangential distortion of a camera lens used to make images.

The above-mentioned arguments completely disqualify a single picture from being enough for analysis of spatial objects including granular piles. Even using a set of single photographs, it is utterly impossible to obtain reliable and accurate geometric and metrological information on the shape and the size of the examined object.

The application of photogrammetric tools using stereograms or pairs of photographic images allows the incorporation of all the mentioned factors into processing and the acquisition of reliable measurement data. They are obtained either by having produced a relevant stereogram or by having converted a properly generated pair of images into the 3D display of an object both in the form of a point cloud and a 3D geometric model.

In addition, Fig. 1f and g show one of many problems concerning the interpretation of the right edge and the height of a pile. It has to be mentioned, however, that rarely does a granular pile assume the shape of a perfect cone; conversely, it is often truncated at the top, while at the base the material piles up. As a matter of fact, a pile frequently lacks symmetry.

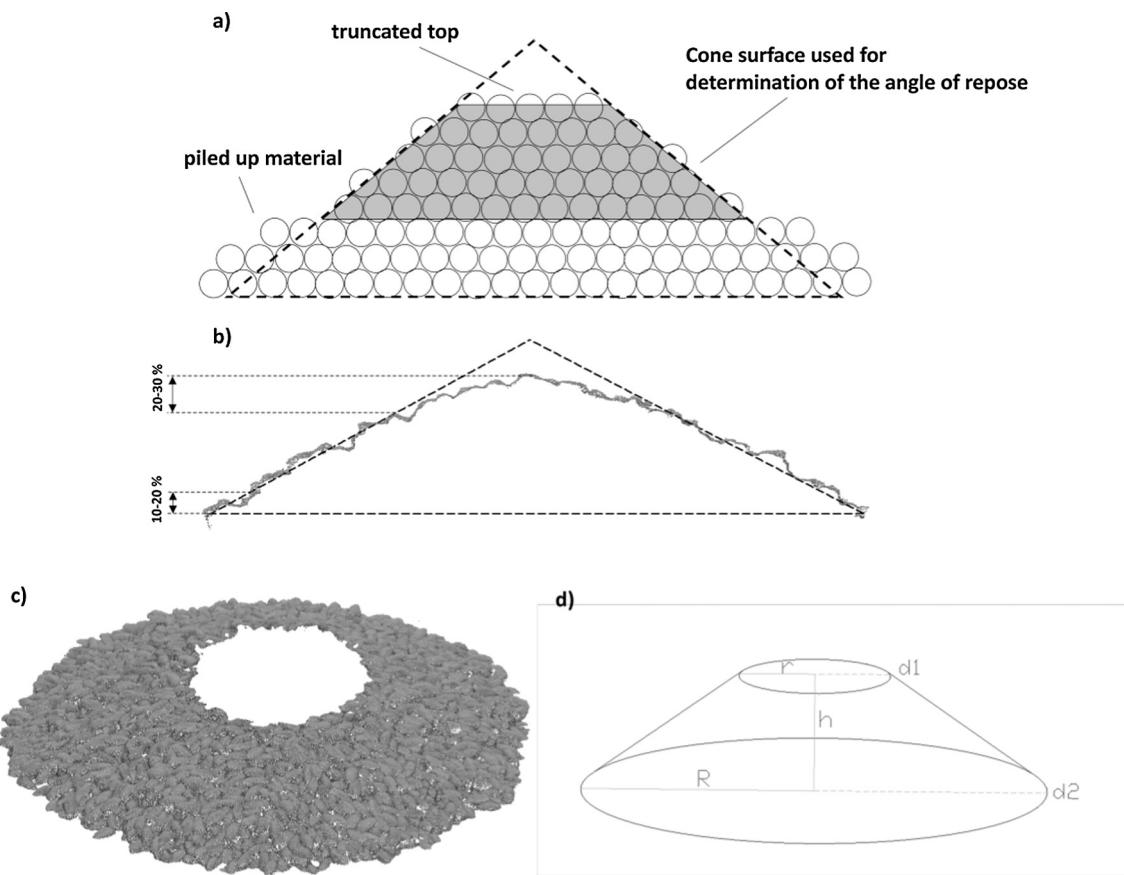
Taking into account the above analysis, the authors decided to propose a new method of precise measurement of the angle of repose, employing close-range photogrammetric technology.

Digital image photogrammetry has become the dominant and commonly used method of 3D coordinate measurement due to the capability to automatically process images while maintaining sub-pixel accuracy [21]. The measured object of which a set of digital photographs has been made can be viewed as a 3D model and subsequently subjected to analysis characteristic of coordinate metrology.

## 2. Material and method

Photographs were taken with the use of AF-S NIKKOR 35 mm fixed focal length lens mounted on Nikon D800 and set at ISO 100 – f/16 ensuring a low level of noise and an appropriate depth of field, respectively. The background was formed by graph paper attached to a rigid substrate, used for plotting out coordinates allowing orientation of single photos and metrological control of a produced image.

In the course of tests, the entire measuring procedure was repeated



**Fig. 3.** Cone analysis. (A) cone model, (b) cone area under investigation, (c) actual cone surface used for determination of the angle of repose, (d) cone model after fitting to a point cloud.

20 times. Photographs were taken under fixed lighting conditions. The camera was held at fixed distance from the measured piled cone and rotated by the angle of 18° (20 positions). The captured images were imported into Agisoft PhotoScan Professional in which the mutual orientation of the images was estimated; the generated pairs of photos were georeferenced by placing markers on single pictures, owing to which a coordinate system was set up; the parameters of the internal orientation of the photographs and the lens distortion parameters were determined. Then, on the basis of the captured images, a dense point cloud representing the examined object was built. Fig. 2 shows the diagram of the measuring procedure.

For cone formation, the “pouring” method was adopted (Fig. 1d). The granular material used for the study purposes was triticale.

To determine the geometric parameters of acquired 3D point clouds, Leica Cyclone software was used, equipped with the fit-to-cloud function allowing the best fit of the geometry of an object to a collection of data points. This command configures the geometric parameters of a given solid (in this case, the cone) so that its geometry best fits a tested point cloud. The adjustment was made by extracting the geometry of a solid from the points, the position of which was determined automatically according to the least squares method. Statistically, this technique most accurately reflects the actual shape and size of a given object. This is the most precise method for adjusting lines, planes, and whole objects since it minimizes the sum of squared deviations of each point from the surface of the generated model of a pile.

For research purposes, 20 piled cones (measurement series) expressed by  $n = \{1, \dots, 20\}$  were formed, along with generated point clouds; however, for each cone, 3D point cloud processing and the best-fit algorithm was performed 30 times, according to  $m = \{1, \dots, 30\}$  repetition scheme, which produced 600 output quantities of the repose angle as per the  $n$ -by- $m$  matrix pictured below:

$$\alpha_{n,m} = \begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,30} \\ \vdots & \dots & \vdots \\ \alpha_{20,1} & \dots & \alpha_{20,30} \end{bmatrix}$$

In the following step, the obtained models were subjected to further statistical analysis.

## 2.1. The methodology for the determination of the angle of repose

Leaving aside agents and phenomena occurring between single particles, which determine cone formation, for many granular materials (e.g., grains), there is a general tendency for the cone to get truncated at the top and have bulk material piled up at its base (Fig. 3). These areas lie at a great distance from the theoretical cone and may, therefore, impair the correct calculation of the angle of repose. For this very reason, in the fit-to-cloud procedure, these two areas were omitted.

The first level (calculated from the bottom) takes up 10–20% of the piled cone, while the third level, i.e. the top of the cone, accounts for 20–30% of the total pile. These values have been established for different types of granular materials on the basis of observations. The remaining 50–70% of the point cloud is the actual representation of the lateral surface of the cone and only this part is considered in further analysis (Fig. 3). The angle of repose denoted by  $\alpha_{n,m}$  was calculated according to the formula (2), (Fig. 3d). The trimmed-off matter impairing the lateral surface of a cone needs to be considered in relation to a given biological material.

$$\alpha_{n,m} = \arctan \frac{h_{n,m}}{R_{n,m} - r_{n,m}} \quad (2)$$

where:

$\alpha_{n,m}$  – the repose angle for an array of cones  $n = \{1, \dots, 20\}$ ,  $m =$

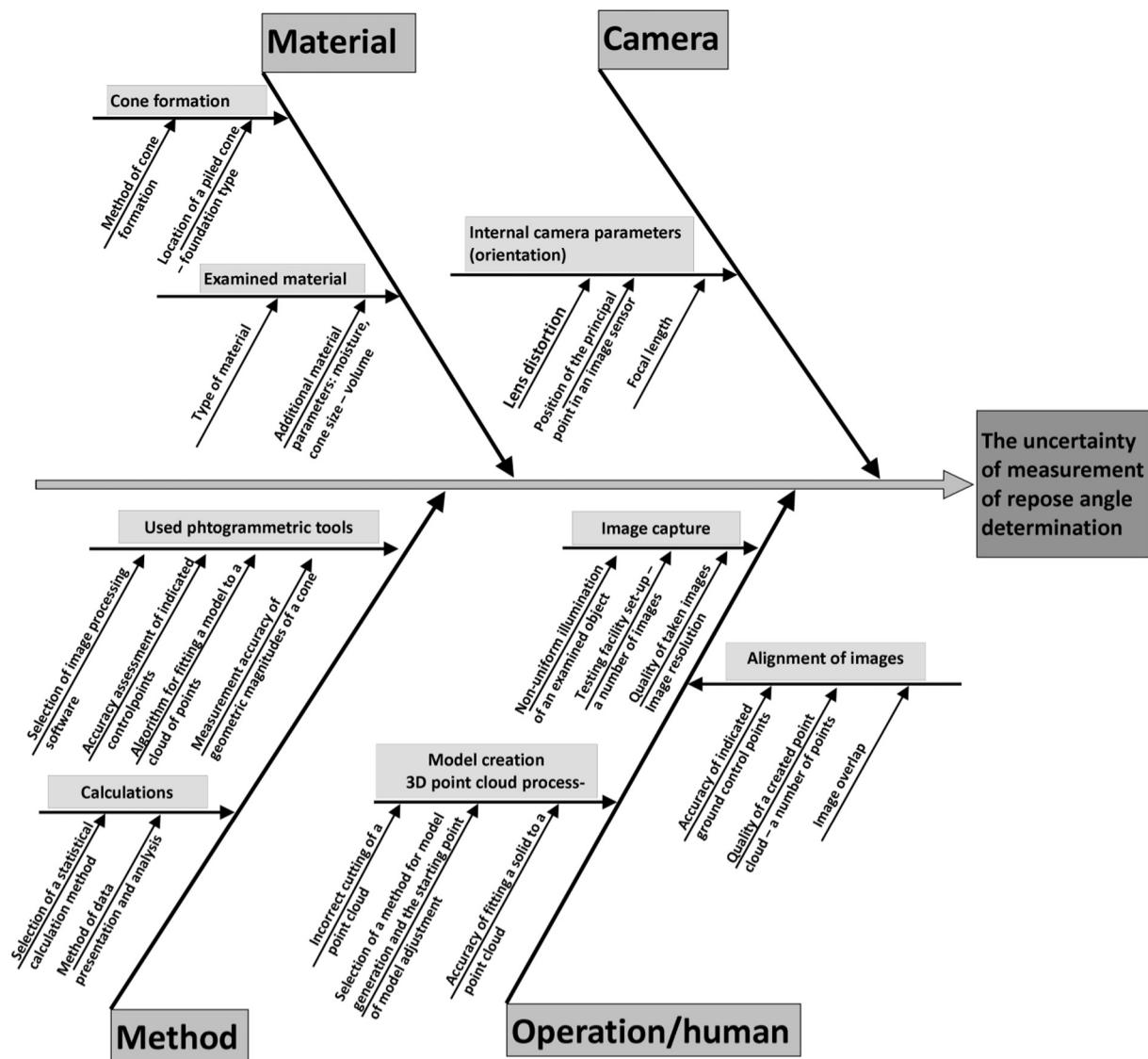


Fig. 4. Factors affecting the measurement uncertainty of angle determination using a photogrammetric method.

{1, ..., 30}.

$R_{n,m}$  – the radius of the base of the truncated cone.

$r_{n,m}$  – the upper radius of the truncated cone.

$h_{n,m}$  – the height between the base and the top surface.

Each of 600 models is described in a report comprising information on adjustment accuracy and cone parameters, i.e. the base diameter of a cone, the diameter of a truncated cone, and the distance between them. These data will be used to calculate the angle of repose and to determine measurement uncertainty.

## 2.2. The value of the angle of repose and the analysis of measurement uncertainty

Digital image processing, 3D modeling, and point cloud generation entail certain errors which may affect the quality of images and the uncertainty of measurement results.

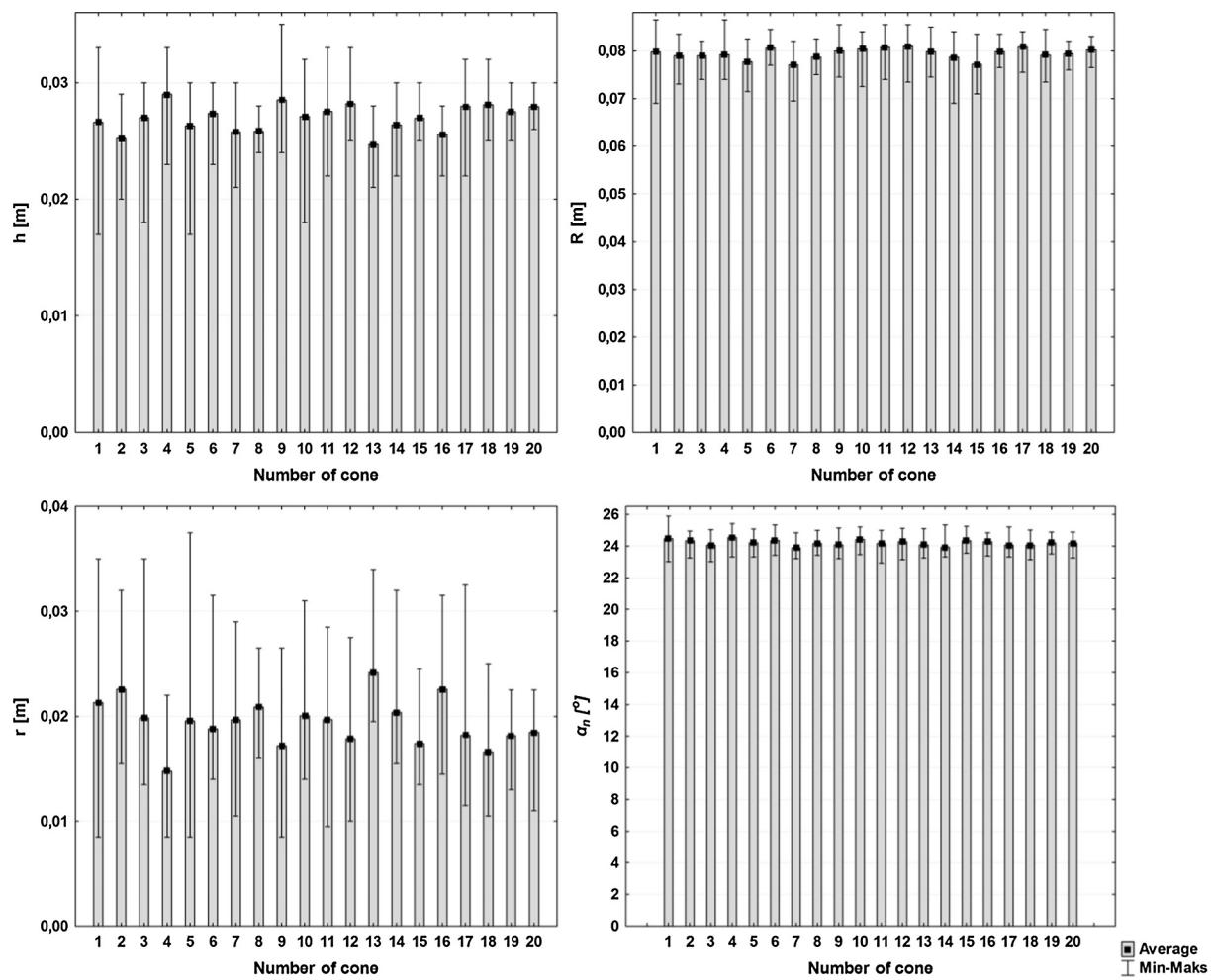
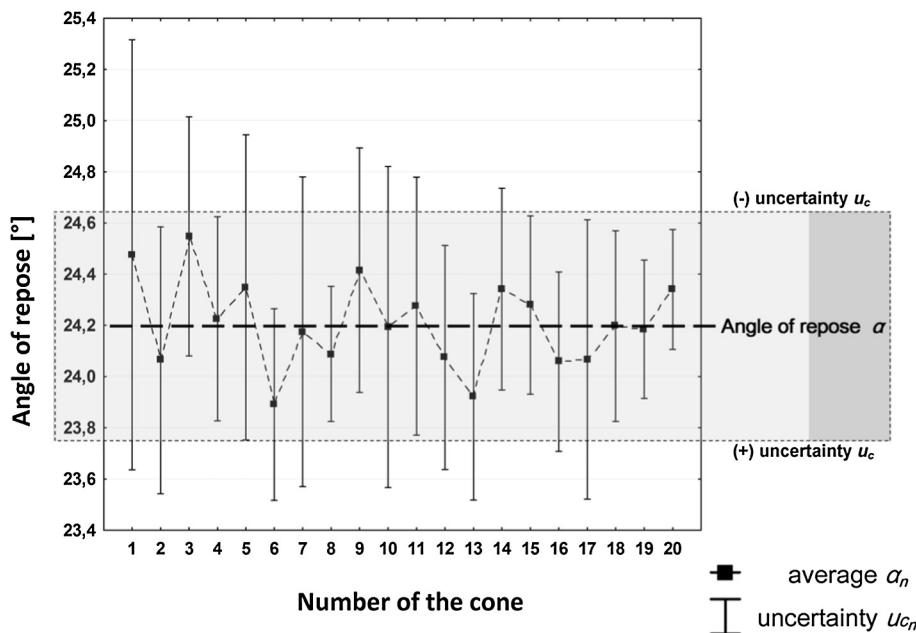
The first thing influencing the measurement uncertainty of the outcome is a digital photo. For the correct determination of perspective projection performed with the camera fitted with the lens, it is necessary to determine intrinsic camera parameters, i.e. focal length and the position of the principal point in an image sensor as well as estimating radial and tangential lens distortion. The measure which is equally

important is ground resolution of an image, defining – by software algorithms – the achievable accuracy of tie points between images from which mutual orientation can be estimated as well as determining the accuracy of indicated ground control points having coordinates in a reference system. The accuracy of determined elements depends on errors embedded in the indication of control points in single images as well as being affected by the mean error concerning the entire geometrical system (stereopairs). Broadly speaking, the uncertainty of measurement results is primarily influenced by inaccurate metrological information, image alignment parameters, and the level of precision in fitting of a geometric shape to a point cloud. Factors that contribute to the uncertainty of measurement results in the proposed method have been shown in Fig. 4.

A single measurement series consisted of 30 independently performed measurements of three cone input quantities  $h_{n,m}$ ,  $R_{n,m}$ ,  $r_{n,m}$ , whereby the angle of repose  $\alpha_{n,m}$  was calculated. For each series, the mean values for  $h_n$ ,  $R_n$ ,  $r_n$  as well as the mean value for  $\alpha_n$  with the standard deviation were determined.

The calculation of the angle of repose  $\alpha_{n,m}$ , made according to the formula (2) was a typical indirect measurement in which the value of the output quantity was found from several input parameters. Fig. 5.

The measurement uncertainty  $u_c$  in indirect measurement for each independent output quantity was estimated by calculating the

Fig. 5. Mean values for parameters  $h_n$ ,  $R_n$ ,  $r_n$ ,  $\alpha_n$  calculated for  $m = 30$  repetitions.Fig. 6. The values of the angle of repose with associated uncertainties for independent  $n$  series of measurements.

**Table 2**

The output quantities of the angle of repose for n = 5 and n = 3 measurement series.

| Number of measurement series, n | Angle of repose, $\alpha$ [ $^{\circ}$ ] | Measurement uncertainty ( $\pm$ ) $u_c$ |
|---------------------------------|--|---|
| 5                               | 24.19                                    | 0.37                                    |
|                                 | 24.28                                    | 0.40                                    |
|                                 | 24.22                                    | 0.46                                    |
| 3                               | 24.20                                    | 0.36                                    |
|                                 | 24.15                                    | 0.41                                    |
|                                 | 24.17                                    | 0.42                                    |

combined standard uncertainty determined in accordance with the error propagation formula (root-mean-square error) [22]:

$$u_c^2 = \sum_{i=1}^m \left( \frac{\partial f}{\partial} \right)^2 (u(x_i))^2 \quad (3)$$

$$u(x)_i = \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^m (x_i - \bar{x})^2} \quad (4)$$

where  $u(x)_i$  denotes the standard uncertainty.

Variables h, R, r were determined directly by measuring the model of a cone. The standard uncertainties of the examined variables are estimated by means of the standard deviation of the mean, which in the given case can be expressed by the following equations:

$$\begin{aligned} u(h)_{n=\{1, \dots, 20\}} &= \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^m (h_i - \bar{h})^2} \\ u(R)_{n=\{1, \dots, 20\}} &= \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^m (R_i - R)^2} \\ u(r)_{n=\{1, \dots, 20\}} &= \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^m (r_i - r)^2} \end{aligned} \quad (5)$$

Taking into account the formulas (2) and (3), Eq. (3) will assume the following form, allowing determination of measurement uncertainty for independent datasets, n = 20.

$$u_{c_n} = \sqrt{\left( \frac{\bar{R}_n - \bar{r}_n}{\bar{h}_n + (\bar{R}_n - \bar{r}_n)^2} u(h)_n \right)^2 + \left( \frac{-\bar{h}_n}{\bar{h}_n^2 + (\bar{R}_n - \bar{r}_n)^2} u(R)_n \right)^2 + \left( \frac{\bar{r}_n}{\bar{h}_n^2 + (\bar{R}_n - \bar{r}_n)^2} u(r)_n \right)^2} \quad (6)$$

From the above-mentioned calculations, for a given n series, the final value of the repose angle with the associated uncertainty, equal to  $\bar{\alpha}_n \pm u_{c_n}$ , was determined.

The output quantity of the angle of repose  $\alpha$  with the uncertainty attributed to it was estimated by means of the arithmetic mean weighted for independent measurement results within a given series, the final value of which was equal to  $\alpha \pm u_c$ , as expressed in the formula below:

$$u_c = \frac{\sum_{i=1}^n u_c w_n}{\sum_{i=1}^n w_n} \quad (7)$$

where:

$w_n$  – weighted mean the value of which is evaluated from measurement uncertainties; for a given series, it is expressed by the following formula:

$$w_{n=\{1, \dots, 20\}} = \frac{1}{u_{P_n}^2} \quad (8)$$

where:

$u_{P_n}$  – measurement uncertainty of the measurement average in a given series, n = {1, ..., 20}, which – according to [BIPM 1993] – is characterized by the experimental standard deviation of the mean, defined by the formula below:

$$u_{P_{n=\{1, \dots, 20\}}} = \frac{S_p(\alpha_{nm})}{\sqrt{m}} \quad (9)$$

where:

$S_p(\alpha_{nm})$  - experimental standard deviation.

The weighted arithmetic mean of these values provides the final measurement result along with the associated uncertainty,  $\alpha \pm u_c$ , equal to  $24.20 \pm 0.43$  [ $^{\circ}$ ] for the examined object/ material (Fig. 6).

In order to investigate the possibility of precipitating the measurement procedure, the entire described calculation procedure was performed three times for randomly selected series (n = 5 and n = 3) (from all the datasets collected, n = 20), Table 2 shows the selected measurement results.

The results obtained for datasets of 3 and 5 items are consistent with the results produced for the entire data collection within measurement uncertainty, which allows for a significant reduction in the number of observations, without reducing their reliability.

Further reduction of measurement series (n) and measurement repetitions (m) will certainly reduce reliability of results; yet, it has to be remembered that this approach requires estimation of errors of measurement, based on factors described in (Fig. 4). While a single inaccuracy can be easily predicted, it is hard to say how it may affect the final measurement error. It is worth mentioning that in the case of repose angle determination, there is no reference model a cone can be referred to.

### 3. Summary

In the article, the authors put forward a new approach to determination of the angle of repose, pointing to the limitations of image analysis drawing on single photos. The developed method takes advantage of photogrammetric technology using the concept of measuring geometric characteristics known from coordinate metrology. The authors proposed that the angle of repose might be determined by measuring a selected cone area (Fig. 3). The method was tested by forming n = 20 conical piles, each of which was 30 times subjected to the (3D) point-cloud creation procedure along with determination of geometric parameters allowing calculation of the repose angle ( $\alpha$ ). The appropriate number of observations was necessary to estimate the associated measurement uncertainty. The authors showed that the reduction of repetitions does not influence the overall consistency of a measure. These studies, however, require further investigation.

It has to be underlined that the method for repose angle determination and achieved dimensional accuracy (measurement uncertainty) may directly contribute to more precise design of automated conveying and dosing systems intended for granular materials. Increased measurement reliability of repose angle determination allows detailed specification of allowable variation in mechanical parts and assemblies, thereby enabling more precise control of technological processes which granular materials are subjected to.

If some materials require more accurate determination of the angle of repose, within the principles of the method proposed by the authors, a different device for imaging of a surface cone can be employed, e.g., 3D optical scanner or triangulation based sensor head mounted on a measuring arm or on a coordinate measuring machine. It is worth mentioning, however, that such tools hugely increase the cost of a measuring system. Yet, this is only one of many aspects of improving measurement reliability. Due to the fact that the method does not allow individual interpretation of obtained results, it ensures high reproducibility of a measurement system. But, from the obtained results (e.g. Fig. 6), it can be observed that with every subsequent conical pile measurement uncertainty decreases. This phenomenon occurs whenever the uncertainty of measurement is affected by a deal of factors and depends on manual skills and due diligence of an operator. Thus, it is

advisable to establish a procedure whereby impact of an operator would be reduced to a minimum or at least to perform gauge R & R (repeatability and reproducibility) for an entire measurement system.

All in all, a set of measuring devices proposed by the authors for the measured material (biological material – grain seeds – triticale) seems to be optimal due to the obtained uncertainty and economic efficiency.

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