# Kružnica

$S(p, q)$, $T(x, y)$

$|ST| = r$

$\sqrt{(x-p)^2+(y-q)^2} = r \quad /^2$

$k \ldots (x-p)^2+(y-q)^2 = r^2$, $S(p, q)$, $r$

$x^2-2xp+p^2+y^2-2yq+q^2-r^2=0$

$x^2+y^2\underbrace{-2p}_{a}x\underbrace{-2q}_{b}y\underbrace{+p^2+q^2-r^2}_{c}=0$

$a=-2p$

$b=-2q$

$c=p^2+q^2-r^2$

Pr. ① iz jednadžbe kružnice odredite koordinate središta i duljinu polumjera

$k \ldots x^2+y^2-4x+6y-2=0$

I. $-4=-2p \;/:(-4)$ $\quad$ $6=-2q \;/:(-2)$ $\quad \Rightarrow S(2,-3)$

$\underline{p=2}$ $\quad$ $\underline{q=-3}$

$-2=p^2+q^2-r^2$

$r^2=4+9+2 \Rightarrow \boxed{r=\sqrt{15}}$

II. $x^2 + y^2 - 4x + 6y - 2 = 0$

$x^2 - 4x + y^2 + 6y = 2$

$\underbrace{x^2 - 2x \cdot 2 + 2^2} + \underbrace{y^2 + 2y \cdot 3 + 3^2} = 2 + 2^2 + 3^2$

$(x-2)^2 + (y+3)^2 = 15 \Rightarrow S(2,-3),\ r = \sqrt{15}$

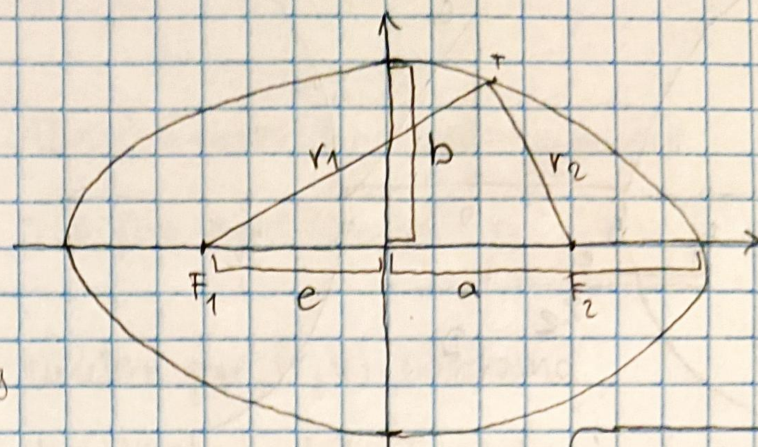
# elipsa i hiperbola

<u>elipsa</u> - skup točaka za koje vrijedi $|TF_1| + |TF_2| = \text{konst.}$

a - velika poluos

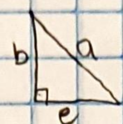
b - mala poluos

e - ekscentricitet linearni (žarište)




$$r_1 + r_2 = 2a$$

$e = \sqrt{a^2 - b^2}$ → primjena pitagorinog poučka




ELIPSA SA SREDIŠTEM U S(0,0)

→ kanonska jednadžba:

$$\boxed{b^2x^2 + a^2y^2 = a^2b^2} \quad / : a^2b^2$$

→ segmentni oblik

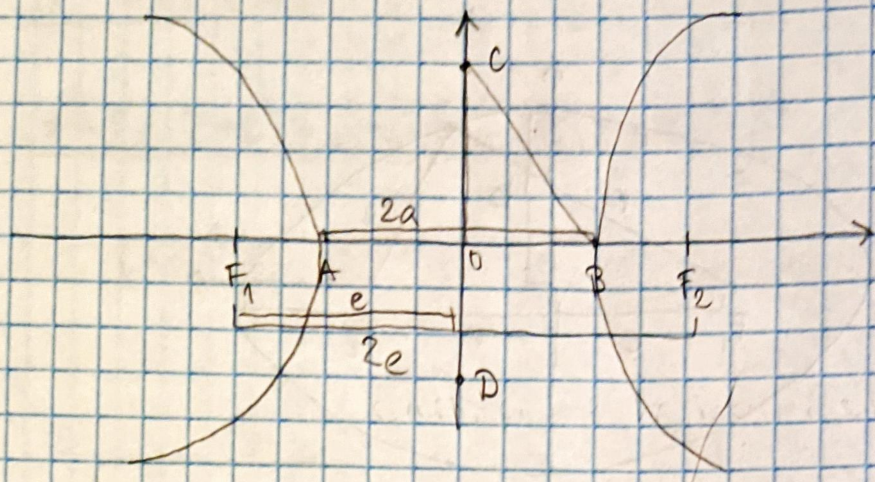
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

EKSCENTRICITET

linearni → $e = \sqrt{a^2 - b^2}$

numerički → $\varepsilon = \frac{e}{a}$

hiperbola - skup svih točaka $|F_1T| - |F_2T| = 2a$




O - središte (centar)

A, B, C, D - tjemena

pravci AB i CD - simetrije hiperbole

$\overline{AB}$ - realna os hiperbole

$e^2 = a^2 + b^2$

$e = \sqrt{a^2 + b^2}$ → linearni ekscentricitet

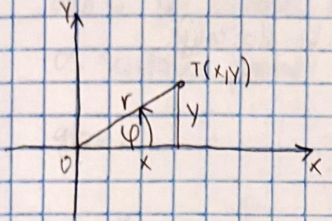
$\varepsilon = \frac{e}{a}$, $e > a \Rightarrow \varepsilon > 1$ → numerički ekscentricitet

$$\boxed{b^2x^2 - a^2y^2 = a^2b^2 \quad , \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

# POLARNE KOORDINATE

- polarni koordinatni sustav

Polarni sustav




$T(x,y) \Rightarrow$ pravokutni koordinatni sustav

$T(r,\varphi) \Rightarrow$ POLARNI koordinatni sustav

uređeni par $(r,\varphi)$ nazivamo koordinatama točke T u polarnom sustavu

$r \geq 0$

prijelaz iz pravokutnog u polarni koordinatni sustav

$\cos\varphi = \frac{x}{r} \Rightarrow \boxed{x = r\cos\varphi}$

$\sin\varphi = \frac{y}{r} \Rightarrow \boxed{y = r\sin\varphi}$

$r = \sqrt{x^2 + y^2}$

$\boxed{tg\,\varphi = \frac{y}{x}}$

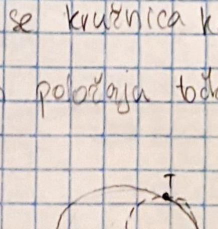
polarna jednadžba pravca

$y = kx + l \Rightarrow \boxed{r = \frac{l}{\sin\varphi - k\cos\varphi}}$

KARDIOIDA

→ nastaje tako da se kružnica $k_1$ kotrlja po kružnici $k_2$ istog polumjera

- u početnom položaju točka T se nalazi u točki A fiksne kružnice




A - pol kardioide

$(\rho - 2a\cos\varphi)^2 = 4a^2 \Rightarrow$ polarni

$(x^2+y^2-2ax)^2 - 4a^2(x^2+y^2) = 0 \Rightarrow$ pravokutni

ARHIMEDOVA SPIRALA

$r = a \cdot \varphi, \quad a > 0$

$$\left.\begin{aligned} r &= \sqrt{x^2+y^2} \\ \operatorname{tg}\varphi &= \frac{y}{x} \\ \varphi &= \operatorname{arctg}\frac{y}{x} \end{aligned}\right\}$$

jednadžba arhimedove spirale u pravokutnom koordinatnom sustavu izgledala bi složenije

# Polarna jednadžba kružnice

Pr. 1 Odredi polarnu jednadžbu kružnice

a) $k \ldots x^2+y^2=1$

$(r\cos\varphi)^2+(r\sin\varphi)^2=1$

$r^2\cos^2\varphi+r^2\sin^2\varphi=1$

$r^2\underbrace{(\cos^2\varphi+\sin^2\varphi)}_{=1}=1$

$r^2=1 \quad |\sqrt{\ }$

$\boxed{r=1}$

b) $x^2+y^2-x=0$

$(r\cos\varphi)^2+(r\sin\varphi)^2-r\cos\varphi=0$

$r^2\cos^2\varphi+r^2\sin^2\varphi-r\cos\varphi=0$

$r^2(\cos^2\varphi+\sin^2\varphi)-r\cos\varphi=0$

$r^2\cdot 1-r\cos\varphi=0$

$r(r-\cos\varphi)=0$

$\cancel{r=0}$ ne postoji kružnica $r=0 \to$ točka

$r-\cos\varphi=0$

$\boxed{r=\cos\varphi}$

c) $x^2+y^2=x+y$

$x^2+y^2-x-y=0$

$(r\cos\varphi)^2+(r\sin\varphi)^2-r\cos\varphi-r\sin\varphi=0$

$r^2\cos^2\varphi+r^2\sin^2\varphi-r\cos\varphi-r\sin\varphi=0$

$r^2(\cos^2\varphi+\sin^2\varphi)-r\cos\varphi-r\sin\varphi=0$

$r^2-r\cos\varphi-r\sin\varphi=0$

$r^2-(\cos\varphi-\sin\varphi)=0$

$r(r-\cos\varphi-\sin\varphi)=0$

$\cancel{r=0}$ $\quad r-\cos\varphi-\sin\varphi=0$

$r=\cos\varphi+\sin\varphi$

# jednadžbe krivulja drugog reda u polarnim koordinatama

ELIPSA, PARABOLA, HIPERBOLA:

ako se pol sustava nalazi u žarištu krivulje drugog reda, a os sustava prolazi kroz os krivulje, tada njezina jednadžba glasi:

$$r = \frac{p}{1 - \varepsilon \cos\varphi}$$

p - poluparametar krivulje

$\varepsilon$ - numerički ekscentricitet

$p = \frac{b^2}{a}$ → omjer kvadrata duljine male poluosi i velike krivulje

$\varepsilon = \frac{e}{a}$ → omjer linearnog ekscentriciteta i velike poluosi krivulje

→ Karakter krivulje ovisi o vrijednosti numeričkog ekscentriciteta $\varepsilon$:

ELIPSA: $\varepsilon < 1$

HIPERBOLA: $\varepsilon > 1$

PARABOLA: $\varepsilon = 1$

Pr. ① Koje su krivulje zadane sljedećim jednadžbama:

a) $r = \frac{10}{2 - \cos\varphi} = \frac{\cancel{10}\ 5}{\cancel{2}(1 - \frac{1}{2}\cos\varphi)} = \frac{5}{1 - \frac{1}{2}\cos\varphi}$

$p = 5$, $\boxed{\varepsilon = \frac{1}{2}} < 1$ ⇒ elipsa

b) $r = \frac{1}{3 - 3\cos\varphi} = \frac{1}{3(1 - \cos\varphi)} = \frac{\frac{1}{3}}{1 - \cos\varphi}$

$p = \frac{1}{3}$, $\boxed{\varepsilon = 1} = 1$ ⇒ parabola

c) $r = \frac{20}{2 - 3\cos\varphi} = \frac{\cancel{20}\ 10}{2(1 - \frac{3}{2}\cos\varphi)} = \frac{10}{1 - \frac{3}{2}\cos\varphi}$

$\boxed{\varepsilon = \frac{3}{2}} = 1{,}5 > 1$ ⇒ hiperbola

Pr. 3 odredite polarnu jednadžbu elipse

$E \ldots 3x^2 + 4y^2 = 48$

segmentni oblik $E \ldots \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$3x^2 + 4y^2 = 48 \,/ : 48$

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

$a^2 = 16 \Rightarrow a = 4$

$b^2 = 12 \Rightarrow b = 2\sqrt{3}$

$r = \frac{p}{1 - \varepsilon \cos\varphi}$

$p = \frac{b^2}{a} = \frac{12}{4} = 3$

$\varepsilon = \frac{e}{a} = \frac{\sqrt{a^2 - b^2}}{a}$

$= \frac{\sqrt{16 - 12}}{4} = \frac{2}{4} = \frac{1}{2} > 1 \Rightarrow$ elipsa

$$\boxed{r = \frac{3}{1 - \frac{1}{2}\cos\varphi}}$$

$H \ldots \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$e^2 = a^2 + b^2$

Pr. 4 polarna jednadžba hiperbole:

$H \ldots \frac{x^2}{144} - \frac{y^2}{25} = 1$

$a^2 = 144 \Rightarrow a = 12$

$b^2 = 25 \Rightarrow b = 5$

$r = \frac{p}{1 - \varepsilon \cos\varphi}$

$p = \frac{25}{12}$

$\varepsilon = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{144 + 25}}{12}$

$\varepsilon = \frac{13}{12}$

$r = \frac{\frac{25}{12}}{1 - \frac{13}{12}\cos\varphi}$