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A five-factor asset pricing model☆

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ABSTRACT

A five-factor model directed at capturing the size, value, profitability, and investment patterns in average stock returns performs better than the three-factor model of Fama and French (FF, 1993). The five-factor model's main problem is its failure to capture the low average returns on small stocks whose returns behave like those of firms that invest a lot despite low profitability. The model's performance is not sensitive to the way its factors are defined. With the addition of profitability and investment factors, the value factor of the FF three-factor model becomes redundant for describing average returns in the sample we examine.

1. Introduction

There is much evidence that average stock returns are related to the book-to-market equity ratio, B/M. There is also evidence that profitability and investment add to the description of average returns provided by B/M. We can use the dividend discount model to explain why these variables are related to average returns. The model says the market value of a share of stock is the discounted value of expected dividends per share,

$$m_t = \sum_{\tau=1}^{\infty} \mathrm{E}(d_{t+\tau})/(1+r)^{\tau}. \qquad (1)$$

In this equation, m_t is the share price at time t, $\mathrm{E}(d_{t+\tau})$ is the expected dividend per share for period $t+\tau$, and r is (approximately) the long-term average expected stock return or, more precisely, the internal rate of return on expected dividends.

Eq. (1) says that if at time t the stocks of two firms have the same expected dividends but different prices, the stock with a lower price has a higher (long-term average) expected return. If pricing is rational, the future dividends of the stock with the lower price must have higher risk. The predictions drawn from (1), here and below, are, however, the same whether the price is rational or irrational.

With a bit of manipulation, we can extract the implications of Eq. (1) for the relations between expected return and expected profitability, expected investment, and B/M. Miller and Modigliani (1961) show that the time t total market value of the firm's stock implied by (1) is,

$$M_t = \sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}. \qquad (2)$$

In this equation, $Y_{t+\tau}$, is total equity earnings for period $t+\tau$ and $dB_{t+\tau}=B_{t+\tau}-B_{t+\tau-1}$ is the change in total book

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equity. Dividing by time t book equity gives,

$$\frac{M_t}{B_t} = \frac{\Sigma_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}. \tag{3}$$

Eq. (3) makes three statements about expected stock returns. First, fix everything in (3) except the current value of the stock, M_t, and the expected stock return, r. Then a lower value of M_t, or equivalently a higher book-to-market equity ratio, B_t/M_t, implies a higher expected return. Next, fix M_t and the values of everything in (3) except expected future earnings and the expected stock return. The equation then tells us that higher expected earnings imply a higher expected return. Finally, for fixed values of B_t, M_t, and expected earnings, higher expected growth in book equity – investment – implies a lower expected return. Stated in perhaps more familiar terms, (3) says that B_t/M_t is a noisy proxy for expected return because the market cap M_t also responds to forecasts of earnings and investment.

The research challenge posed by (3) has been to identify proxies for expected earnings and investments. Novy-Marx (2013) identifies a proxy for expected profitability that is strongly related to average return. Aharoni, Grundy, and Zeng (2013) document a weaker but statistically reliable relation between investment and average return. (See also Haugen and Baker, 1996; Cohen, Gompers, and Vuolteenaho, 2002; Fairfield, Whisenant, and Yohn, 2003; Titman, Wei, and Xie, 2004; and Fama and French, 2006, 2008.) Available evidence also suggests that much of the variation in average returns related to profitability and investment is left unexplained by the three-factor model of Fama and French (FF, 1993). This leads us to examine a model that adds profitability and investment factors to the market, size, and B/M factors of the FF three-factor model.

Many "anomaly" variables are known to cause problems for the three-factor model, so it is reasonable to ask why we choose profitability and investment factors to augment the model. Our answer is that they are the natural choices implied by Eqs. (1) and (3). Campbell and Shiller (1988) emphasize that (1) is a tautology that defines the internal rate of return, r. Given the stock price and estimates of expected dividends, there is a discount rate r that solves Eq. (1). With clean surplus accounting, Eq. (3) follows directly from (1), so it is also a tautology. Most asset pricing research focuses on short-horizon returns – we use a one-month horizon in our tests. If each stock's short-horizon expected return is positively related to its internal rate of return in (1) – if, for example, the expected return is the same for all horizons – the valuation equation implies that the cross-section of expected returns is determined by the combination of current prices and expectations of future dividends. The decomposition of cashflows in (3) then implies that each stock's relevant expected return is determined by its price-to-book ratio and expectations of its future profitability and investment. If variables not explicitly linked to this decomposition, such as *Size* and momentum, help forecast returns, they must do so by implicitly improving forecasts of profitability and investment or by capturing horizon effects in the term structure of expected returns.

We test the performance of the five-factor model in two steps. Here we apply the model to portfolios formed on size, B/M, profitability, and investment. As in FF (1993), the portfolio returns to be explained are from finer versions of the sorts that produce the factors. We move to more hostile territory in Fama and French (FF, 2014), where we study whether the five-factor model performs better than the three-factor model when used to explain average returns related to prominent anomalies not targeted by the model. We also examine whether model failures are related to shared characteristics of problem portfolios identified in many of the sorts examined here – in other words, whether the asset pricing problems posed by different anomalies are in part the same phenomenon.

We begin (Section 2) with a discussion of the five-factor model. Section 3 examines the patterns in average returns the model is designed to explain. Definitions and summary statistics for different versions of the factors are in

Sections 4 and 5. Section 6 presents summary asset pricing tests. One Section 6 result is that for portfolios formed on size, B/M, profitability, and investment, the five-factor model provides better descriptions of average returns than the FF three-factor model. Another result is that inferences about the asset pricing models we examine do not seem to be sensitive to the way factors are defined, at least for the definitions considered here. One result in Section 6 is so striking we caution the reader that it may be specific to this sample: When profitability and investment factors are added to the FF three-factor model, the value factor, *HML*, seems to become redundant for describing average returns. Section 7 confirms that the large average *HML* return is absorbed by the exposures of *HML* to the other four factors, especially the profitability and investment factors. Section 8 provides asset pricing details, specifically, intercepts and pertinent regression slopes. An interesting Section 8 result is that the portfolios that cause major problems in different sorts seem to be cast in the same mold, specifically, small stocks whose returns behave like those of firms that invest a lot despite low profitability. Finally, the paper closest to ours is Hou, Xue, and Zhang (2012). We discuss their work in the concluding Section 9, where contrasts with our work are easily described.

2. The five-factor model

The FF (1993) three-factor model is designed to capture the relation between average return and *Size* (market capitalization, price times shares outstanding) and the relation between average return and price ratios like B/M. At the time of our 1993 paper, these were the two well-known patterns in average returns left unexplained by the CAPM of Sharpe (1964) and Lintner (1965).

Tests of the three-factor model center on the time-series regression,

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + e_{it}. \tag{4}$$

In this equation R_{it} is the return on security or portfolio i for period t, R_{Ft} is the riskfree return, R_{Mt} is the return on the value-weight (VW) market portfolio, SMB_t is the return on a diversified portfolio of small stocks minus the return

Table 1
Average monthly percent excess returns for portfolios formed on *Size* and *B/M*, *Size* and *OP*, *Size* and *Inv*; July 1963–December 2013, 606 months.

At the end of each June, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Low to High), again using NYSE breakpoints. The intersections of the two sorts produce 25 value-weight *Size-B/M* portfolios. In the sort for June of year t, B is book equity at the end of the fiscal year ending in year $t-1$ and M is market cap at the end of December of year $t-1$, adjusted for changes in shares outstanding between the measurement of B and the end of December. The *Size-OP* and *Size-Inv* portfolios are formed in the same way, except that the second sort variable is operating profitability or investment. Operating profitability, *OP*, in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, *Inv*, is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets. The table shows averages of monthly returns in excess of the one-month Treasury bill rate.

	Low	2	3	4	High
Panel A: Size-B/M portfolios					
Small	0.26	0.81	0.85	1.01	1.15
2	0.48	0.72	0.94	0.94	1.02
3	0.50	0.78	0.79	0.88	1.07
4	0.60	0.57	0.71	0.85	0.86
Big	0.46	0.51	0.48	0.56	0.62
Panel B: Size-OP portfolios					
Small	0.56	0.94	0.90	0.95	0.88
2	0.59	0.78	0.84	0.81	0.98
3	0.53	0.77	0.72	0.78	0.94
4	0.57	0.65	0.63	0.70	0.82
Big	0.39	0.33	0.43	0.47	0.57
Panel C: Size-Inv portfolios					
Small	1.01	0.98	0.99	0.89	0.35
2	0.92	0.91	0.92	0.90	0.48
3	0.90	0.93	0.81	0.82	0.50
4	0.79	0.72	0.71	0.75	0.54
Big	0.71	0.52	0.49	0.48	0.42

on a diversified portfolio of big stocks, HML_t is the difference between the returns on diversified portfolios of high and low B/M stocks, and e_{it} is a zero-mean residual. Treating the parameters in (4) as true values rather than estimates, if the factor exposures b_i, s_i, and h_i capture all variation in expected returns, the intercept a_i is zero for all securities and portfolios i.

The evidence of Novy-Marx (2013), Titman, Wei, and Xie (2004), and others says that (4) is an incomplete model for expected returns because its three factors miss much of the variation in average returns related to profitability and investment. Motivated by this evidence and the valuation Eq. (3), we add profitability and investment factors to the three-factor model,

$$R_{it}-R_{Ft} = a_i + b_i(R_{Mt}-R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}. \quad (5)$$

In this equation RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA_t is the difference between the returns on diversified portfolios of the stocks of low and high investment firms, which we call conservative and aggressive. If the exposures to the five factors, b_i, s_i, h_i, r_i, and c_i, capture all variation in expected returns, the intercept a_i in (5) is zero for all securities and portfolios i.

We suggest two interpretations of the zero-intercept hypothesis. Leaning on Huberman and Kandel (1987), the first proposes that the mean-variance-efficient tangency portfolio, which prices all assets, combines the riskfree asset, the market portfolio, *SMB*, *HML*, *RMW*, and *CMA*. The more ambitious interpretation proposes (5) as the regression equation for a version of Merton's (1973) model in which up to four unspecified state variables lead to risk premiums that are not captured by the market factor. In this view, *Size*, *B/M*, *OP*, and *Inv* are not themselves state variables, and *SMB*, *HML*, *RMW*, and *CMA* are not state variable mimicking portfolios. Instead, in the spirit of Fama (1996), the factors are just diversified portfolios that provide different combinations of exposures to the unknown state variables. Along with the market portfolio and the riskfree asset, the factor portfolios span the relevant multifactor efficient set. In this scenario, the role of the valuation Eq. (3) is to suggest factors that allow us to capture the expected return effects of state variables without identifying them.

3. The playing field

Our empirical tests examine whether the five-factor model and models that include subsets of its factors explain average returns on portfolios formed to produce large spreads in *Size*, *B/M*, profitability, and investment. The first step is to examine the *Size*, *B/M*, profitability, and investment patterns in average returns we seek to explain.

Panel A of Table 1 shows average monthly excess returns (returns in excess of the one-month U.S. Treasury bill rate) for 25 value-weight (VW) portfolios from independent sorts of stocks into five *Size* groups and five *B/M* groups. (We call them 5 × 5 *Size-B/M* sorts, and for a bit of color we typically refer to the smallest and biggest *Size* quintiles as microcaps and megacaps.) The *Size* and *B/M* quintile breakpoints use only NYSE stocks, but the sample is all NYSE, AMEX, and NASDAQ stocks on both CRSP and Compustat with share codes 10 or 11 and data for *Size* and *B/M*. The period is July 1963–December 2013. Fama and French (1993) use these portfolios to evaluate the three-factor model, and the patterns in average returns in Table 1 are like those in the earlier paper, with 21 years of new data.

In each *B/M* column of Panel A of Table 1, average return typically falls from small stocks to big stocks – the size effect. The first column (low *B/M* extreme growth stocks) is the only exception, and the glaring outlier is the low average return of the smallest (microcap) portfolio. For the other four portfolios in the lowest *B/M* column, there is no obvious relation between *Size* and average return.

The relation between average return and *B/M*, called the value effect, shows up more consistently in Table 1. In every *Size* row, average return increases with *B/M*. As is well-known, the value effect is stronger among small stocks. For example, for the microcap portfolios in the first row, average excess return rises from 0.26% per month for the lowest *B/M* portfolio (extreme growth stocks) to 1.15%

per month for the highest B/M portfolio (extreme value stocks). In contrast, for the biggest stocks (megacaps) average excess return rises only from 0.46% per month to 0.62%.

Panel B of Table 1 shows average excess returns for 25 VW portfolios from independent sorts of stocks into *Size* and profitability quintiles. The details of these 5×5 sorts are the same as in Panel A, but the second sort is on profitability rather than B/M. For portfolios formed in June of year t, profitability (measured with accounting data for the fiscal year ending in $t-1$) is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity at the end of fiscal year $t-1$. We call this variable operating profitability, *OP*, but it is operating profitability minus interest expense. As in all our sorts, the *OP* breakpoints use only NYSE firms.

The patterns in the average returns of the 25 *Size-OP* portfolios in Table 1 are like those observed for the *Size-B/M* portfolios. Holding operating profitability roughly constant, average return typically falls as *Size* increases. The decline in average return with increasing *Size* is monotonic in the three middle quintiles of *OP*, but for the extreme low and high *OP* quintiles, the action with respect to *Size* is almost entirely due to lower average returns for megacaps.

The profitability effect identified by Novy-Marx (2013) and others is evident in Panel B of Table 1. For every *Size* quintile, extreme high operating profitability is associated with higher average return than extreme low *OP*. In each of the first four *Size* quintiles, the middle three portfolios have similar average returns, and the profitability effect is a low average return for the lowest *OP* quintile and a high average return for the highest *OP* quintile. In the largest *Size* quintile (megacaps), average return increases more smoothly from the lowest to the highest *OP* quintile.

Panel C of Table 1 shows average excess returns for 25 *Size-Inv* portfolios again formed in the same way as the 25 *Size-B/M* portfolios, but the second variable is now investment (*Inv*). For portfolios formed in June of year t, *Inv* is the growth of total assets for the fiscal year ending in $t-1$ divided by total assets at the end of $t-2$. In the valuation Eq. (3), the investment variable is the expected growth of book equity, not assets. We have replicated all tests using the growth of book equity, with results similar to those obtained with the growth of assets. The main difference is that sorts on asset growth produce slightly larger spreads in average returns. (See also Aharoni, Grundy, and Zeng, 2013.) Perhaps the lagged growth of assets is a better proxy for the infinite sum of expected future growth in book equity in (3) than the lagged growth in book equity. The choice is in any case innocuous for all that follows.

In every *Size* quintile the average return on the portfolio in the lowest investment quintile is much higher than the return on the portfolio in the highest *Inv* quintile, but in the smallest four *Size* quintiles this is mostly due to low average returns on the portfolios in the highest *Inv* quintile. There is a size effect in the lowest four quintiles of *Inv*; that is, portfolios of small stocks have higher average returns than big stocks. In the highest *Inv* quintile, however, there is no size effect, and the microcap portfolio in the highest *Inv* group has the lowest average excess return in the matrix, 0.35% per month. The five-factor regressions will show that the stocks in this portfolio are like the microcaps in the lowest B/M quintile of Panel A of Table 1; specifically, strong negative five-factor *RMW* and *CMA* slopes say that their stock returns behave like those of firms that invest a lot despite low profitability. The low average returns of these portfolios are lethal for the five-factor model.

Eq. (3) predicts that controlling for profitability and investment, B/M is positively related to average return, and there are similar conditional predictions for the relations between average return and profitability or investment. The valuation equation does not predict that B/M, *OP*, and *Inv* effects show up in average returns without the appropriate controls. Moreover, Fama and French (1995) show that the three variables are correlated. High B/M value stocks tend to have low profitability and investment, and low B/M growth stocks – especially large low B/M stocks – tend to be profitable and invest aggressively. Because the characteristics are correlated, the *Size-B/M*, *Size-OP*, and *Size-Inv* portfolios in Table 1 do not isolate value, profitability, and investment effects in average returns.

To disentangle the dimensions of average returns, we would like to sort jointly on *Size*, B/M, *OP*, and *Inv*. Even $3 \times 3 \times 3 \times 3$ sorts, however, produce 81 poorly diversified portfolios that have low power in tests of asset pricing models. We compromise with sorts on *Size* and pairs of the other three variables. We form two *Size* groups (small and big), using the median market cap for NYSE stocks as the breakpoint, and we use NYSE quartiles to form four groups for each of the other two sort variables. For each combination of variables we have $2 \times 4 \times 4 = 32$ portfolios, but correlations between characteristics cause an uneven allocation of stocks. For example, B/M and *OP* are negatively correlated, especially among big stocks, so portfolios of stocks with high B/M and high *OP* can be poorly diversified. In fact, when we sort stocks independently on *Size*, B/M, and *OP*, the portfolio of big stocks in the highest quartiles of B/M and *OP* is often empty before July 1974. To spread stocks more evenly in the $2 \times 4 \times 4$ sorts, we use separate NYSE breakpoints for small and big stocks in the sorts on B/M, *OP*, and *Inv*.

Table 2 shows average excess returns for the 32 *Size-B/M-OP* portfolios, the 32 *Size-B/M-Inv* portfolios, and the 32 *Size-OP-Inv* portfolios. For small stocks, there are strong value, profitability, and investment effects in average returns. Controlling for *OP* or *Inv*, average returns of small stock portfolios increase with B/M; controlling for B/M or *Inv*, average returns also increase with *OP*; and controlling for B/M or *OP*, higher *Inv* is associated with lower average returns. Though weaker, the patterns in average returns are similar for big stocks.

In the tests of the five-factor model presented later, two portfolios in Table 2 display the lethal combination of *RMW* and *CMA* slopes noted in the discussion of the *Size-B/M* and *Size-Inv* portfolios of Table 1. In the *Size-B/M-OP* sorts, the portfolio of small stocks in the lowest B/M and *OP* quartiles has an extremely low average excess return, 0.03% per month. In the Appendix we document that this portfolio has negative five-factor exposures to *RMW* and *CMA* (typical of firms that

Table 2

Averages of monthly percent excess returns for value-weight (VW) portfolios formed on (i) *Size*, *B/M*, and *OP*, (ii) *Size*, *B/M*, and *Inv*, and (iii) *Size*, *OP*, and *Inv*; July 1963–December 2013, 606 months.

At the end of June each year *t*, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median market cap as breakpoint. Stocks in each *Size* group are allocated independently to four *B/M* groups (Low *B/M* to High *B/M* for fiscal year $t-1$), four *OP* groups (Low *OP* to High *OP* for fiscal year $t-1$), and four *Inv* groups (Low *Inv* to High *Inv* for fiscal year $t-1$) using NYSE breakpoints specific to the *Size* group. The table shows averages of monthly returns in excess of the one-month Treasury bill rate on the 32 portfolios formed from each of the three sorts.

	Small				Big			
Panel A: Portfolios formed on Size, B/M, and OP								
B/M→	Low	2	3	High	Low	2	3	High
Low *OP*	0.03	0.72	0.84	0.93	0.24	0.23	0.37	0.60
2	0.67	0.76	0.88	1.08	0.41	0.50	0.47	0.69
3	0.66	0.88	1.07	1.30	0.40	0.59	0.68	0.91
High *OP*	0.81	1.13	1.22	1.63	0.53	0.64	0.79	0.71
Panel B: Portfolios formed on Size, B/M and Inv								
B/M →	Low	2	3	High	Low	2	3	High
Low *Inv*	0.69	0.99	1.18	1.23	0.58	0.70	0.62	0.77
2	0.87	0.92	0.93	1.08	0.49	0.54	0.54	0.60
3	0.84	0.95	1.01	0.97	0.49	0.54	0.56	0.72
High *Inv*	0.39	0.75	0.87	1.01	0.49	0.44	0.39	0.64
Panel C: Portfolios formed on Size, OP, and Inv								
OP →	Low	2	3	High	Low	2	3	High
Low *Inv*	0.85	1.01	1.19	1.27	0.63	0.66	0.79	0.70
2	0.94	0.90	0.92	1.04	0.32	0.43	0.64	0.64
3	0.61	0.93	0.94	1.06	0.52	0.57	0.48	0.53
High *Inv*	−0.09	0.58	0.76	0.76	0.29	0.25	0.38	0.65

invest a lot despite low profitability) that, at least for small stocks, are associated with low average returns left unexplained by the five-factor model. In the *Size-OP-Inv* sorts, the portfolio of small stocks in the lowest *OP* and highest *Inv* quartiles has an even lower average excess return, −0.09% per month. In this case, the five-factor slopes simply confirm that the small stocks in this portfolio invest a lot despite low profitability.

The portfolios in Tables 1 and 2 do not cleanly disentangle the value, profitability, and investment effects in average returns predicted by the valuation Eq. (3), but we shall see that they expose variation in average returns sufficient to provide strong challenges in asset pricing tests.

4. Factor definitions

To examine whether the specifics of factor construction are important in tests of asset pricing models, we use three sets of factors to capture the patterns in average returns in Tables 1 and 2. The three approaches are described formally and in detail in Table 3. Here we provide a brief summary.

The first approach augments the three factors of Fama and French (1993) with profitability and investment factors defined like the value factor of that model. The *Size* and value factors use independent sorts of stocks into two *Size* groups and three *B/M* groups (independent 2×3 sorts). The *Size* breakpoint is the NYSE median market cap, and the *B/M* breakpoints are the 30th and 70th percentiles of *B/M* for NYSE stocks. The intersections of the sorts produce six VW portfolios. The *Size* factor, SMB_{BM}, is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns. The value factor *HML* is the average of the two high *B/M* portfolio returns minus the average of the two low *B/M* portfolio returns. Equivalently, it is the average of small and big value factors constructed with portfolios of only small stocks and portfolios of only big stocks.

The profitability and investment factors of the 2×3 sorts, *RMW* and *CMA*, are constructed in the same way as *HML* except the second sort is either on operating profitability (robust minus weak) or investment (conservative minus aggressive). Like *HML*, *RMW* and *CMA* can be interpreted as averages of profitability and investment factors for small and big stocks.

The 2×3 sorts used to construct *RMW* and *CMA* produce two additional *Size* factors, SMB_{OP} and SMB_{Inv}. The *Size* factor *SMB* from the three 2×3 sorts is defined as the average of $SMB_{B/M}$, SMB_{OP}, and SMB_{Inv}. Equivalently, *SMB* is the average of the returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the returns on the nine big stock portfolios.

When we developed the three-factor model, we did not consider alternative definitions of *SMB* and *HML*. The choice of a 2×3 sort on *Size* and *B/M* is, however, arbitrary. To test the sensitivity of asset pricing results to this choice, we construct versions of *SMB*, *HML*, *RMW*, and *CMA* in the same way as in the 2×3 sorts, but with 2×2 sorts on *Size* and *B/M*, *OP*, and *Inv*, using NYSE medians as breakpoints for all variables (details in Table 3).

Since *HML*, *RMW*, and *CMA* from the 2×3 (or 2×2) sorts weight small and big stock portfolio returns equally, they are roughly neutral with respect to size. Since *HML* is constructed without controls for *OP* and *Inv*, however, it is not neutral with respect to profitability and investment. This likely means that the average *HML* return is a mix of premiums related to *B/M*, profitability, and investment. Similar comments apply to *RMW* and *CMA*.

To better isolate the premiums in average returns related to *Size*, *B/M*, *OP*, and *Inv*, the final candidate factors

Table 3
Construction of *Size*, *B/M*, profitability, and investment factors.

We use independent sorts to assign stocks to two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label these portfolios with two or four letters. The first always describes the *Size* group, small (*S*) or big (*B*). In the 2 × 3 sorts and 2 × 2 sorts, the second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (*C*), neutral (*N*), or aggressive (*A*). In the 2 × 2 × 2 × 2 sorts, the second character is *B/M* group, the third is *OP* group, and the fourth is *Inv* group. The factors are *SMB* (small minus big), *HML* (high minus low *B/M*), *RMW* (robust minus weak *OP*), and *CMA* (conservative minus aggressive *Inv*).

Sort	Breakpoints	Factors and their components
2 × 3 sorts on *Size* and *B/M*, or *Size* and *OP*, or *Size* and *Inv*	*Size*: NYSE median	$SMB_{B/M}=(SH+SN+SL)/3-(BH+BN+BL)/3$
		$SMB_{OP}=(SR+SN+SW)/3-(BR+BN+BW)/3$
		$SMB_{Inv}=(SC+SN+SA)/3-(BC+BN+BA)/3$
		$SMB=(SMB_{B/M}+SMB_{OP}+SMB_{Inv})/3$
	B/M: 30th and 70th NYSE percentiles	$HML=(SH+BH)/2-(SL+BL)/2=[(SH-SL)+(BH-BL)]/2$
	OP: 30th and 70th NYSE percentiles	$RMW=(SR+BR)/2-(SW+BW)/2=[(SR-SW)+(BR-BW)]/2$
	Inv: 30th and 70th NYSE percentiles	$CMA=(SC+BC)/2-(SA+BA)/2=[(SC-SA)+(BC-BA)]/2$
2 × 2 sorts on *Size* and *B/M*, or *Size* and *OP*, or *Size* and *Inv*	*Size*: NYSE median	$SMB=(SH+SL+SR+SW+SC+SA)/6-(BH+BL+BR+BW+BC+BA)/6$
	B/M: NYSE median	$HML=(SH+BH)/2-(SL+BL)/2=[(SH-SL)+(BH-BL)]/2$
	OP: NYSE median	$RMW=(SR+BR)/2-(SW+BW)/2=[(SR-SW)+(BR-BW)]/2$
	Inv: NYSE median	$CMA=(SC+BC)/2-(SA+BA)/2=[(SC-SA)+(BC-BA)]/2$
2 × 2 × 2 × 2 sorts on *Size*, *B/M*, *OP*, and *Inv*	*Size*: NYSE median	$SMB=(SHRC+SHRA+SHWC+SHWA+SLRC+SLRA+SLWC+SLWA)/8$ $-(BHRC+BHRA+BHWC+BHWA+BLRC+BLRA+BLWC+BLWA)/8$
	B/M: NYSE median	$HML=(SHRC+SHRA+SHWC+SHWA+BHRC+BHRA+BHWC+BHWA)/8$ $-(SLRC+SLRA+SLWC+SLWA+BLRC+BLRA+BLWC+BLWA)/8$
	OP: NYSE median	$RMW=(SHRC+SHRA+SLRC+SLRA+BHRC+BHRA+BLRC+BLRA)/8$ $-(SHWC+SHWA+SLWC+SLWA+BHWC+BHWA+BLWC+BLWA)/8$
	Inv: NYSE median	$CMA=(SHRC+SHWC+SLRC+SLWC+BHRC+BHWC+BLRC+BLWC)/8$ $-(SHRA+SHWA+SLRA+SLWA+BHRA+BHWA+BLRA+BLWA)/8$

use four sorts to control jointly for the four variables. We sort stocks independently into two *Size* groups, two *B/M* groups, two *OP* groups, and two *Inv* groups using NYSE medians as breakpoints. The intersections of the groups are 16 VW portfolios. The *Size* factor *SMB* is the average of the returns on the eight small stock portfolios minus the average of the returns on the eight big stock portfolios. The value factor *HML* is the average return on the eight high *B/M* portfolios minus the average return on the eight low *B/M* portfolios. The profitability factor, *RMW*, and the investment factor, *CMA*, are also differences between average returns on eight portfolios (robust minus weak *OP* or conservative minus aggressive *Inv*). Though not detailed in Table 3, we can, as usual, also interpret the value, profitability, and investment factors as averages of small and big stock factors.

In the 2 × 2 × 2 × 2 sorts, *SMB* equal weights high and low *B/M*, robust and weak *OP*, and conservative and aggressive *Inv* portfolio returns. Thus, the *Size* factor is roughly neutral with respect to value, profitability, and investment, and this is what we mean when we say the *Size* factor controls for the other three variables. Likewise, *HML* is roughly neutral with respect to *Size*, *OP*, and *Inv*, and similar comments apply to *RMW* and *CMA*. We shall see, however, that neutrality with respect to characteristics does not imply low correlation between factor returns.

Joint controls likely mean that the factors from the 2 × 2 × 2 × 2 sorts better isolate the premiums in average returns related to *B/M*, *OP*, and *Inv*. But factor exposures are more important in our eventual inferences. Since multivariate regression slopes measure marginal effects, the five-factor slopes for *HML*, *RMW*, and *CMA* produced by the factors from the 2 × 3 or 2 × 2 sorts may isolate exposures to value, profitability, and investment effects in returns as effectively as the factors from the 2 × 2 × 2 × 2 sorts.

5. Summary statistics for factor returns

Table 4 shows summary statistics for factor returns. Summary statistics for returns on the portfolios used to construct the factors are in Appendix Table A1.

Average *SMB* returns are 0.29% to 0.30% per month for the three versions of the factors (Panel A of Table 4). The standard deviations of *SMB* are similar, 2.87–3.13%, and the correlations of the different versions of *SMB* are 0.98 and 1.00 (Panel B of Table 4). The high correlations and the similar means and standard deviations are not surprising since the *Size* breakpoint for *SMB* is always the NYSE median market cap, and the three versions of *SMB* use all stocks. The average *SMB* returns are more than 2.3 standard errors from zero.

The summary statistics for *HML*, *RMW*, and *CMA* depend more on how they are constructed. The results from the 2 × 3 and 2 × 2 sorts are easiest to compare. The standard deviations of the three factors are lower when only two *B/M*, *OP*, or *Inv* groups are used, due to better diversification. In the 2 × 2 sorts, *HML*, *RMW*, and *CMA* include all stocks, but in the 2 × 3 sorts, the factors do not use the stocks in the middle 40% of *B/M*, *OP*, and *Inv*. The 2 × 3 sorts focus more on the extremes of the three variables, and so produce larger average *HML*, *RMW*, and *CMA* returns. For example, the average *HML* return is 0.37% per month in the 2 × 3 *Size-B/M* sorts, versus 0.28% in the 2 × 2 sorts. Similar

Table 4

Summary statistics for monthly factor percent returns; July 1963–December 2013, 606 months.

$R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate. At the end of each June, stocks are assigned to two *Size* groups using the NYSE median market cap as the breakpoint. Stocks are also assigned independently to two or three book-to-market equity (*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using NYSE medians of *B/M*, *OP*, and *Inv* or the 30th and 70th NYSE percentiles. In the first two blocks of Panel A, the *B/M* factor, *HML*, uses the VW portfolios formed from the intersection of the *Size* and *B/M* sorts ($2 \times 2 = 4$ or $2 \times 3 = 6$ portfolios), and the profitability and investment factors, *RMW* and *CMA*, use four or six VW portfolios from the intersection of the *Size* and *OP* or *Inv* sorts. In the third block, *HML*, *RMW*, and *CMA* use the intersections of the *Size*, *B/M*, *OP*, and *Inv* sorts ($2 \times 2 \times 2 \times 2 = 16$ portfolios). HML_B is the average return on the portfolio(s) of big high *B/M* stocks minus the average return on the portfolio(s) of big low *B/M* stocks, HML_S is the same but for portfolios of small stocks, *HML* is the average of HML_S and HML_B, and $HML_{S\text{-}B}$ is the difference between them. RMW_S, RMW_B, *RMW*, and $RMW_{S\text{-}B}$ and CMA_S, CMA_B, *CMA*, and $CMA_{S\text{-}B}$ are defined in the same way, but using high and low *OP* or *Inv* instead of *B/M*. In the $2 \times 2 \times 2 \times 2$ sorts, *SMB* is the average return on the eight portfolios of small stocks minus the average return on the eight portfolios of big stocks. In the separate 2×3 *Size-B/M*, Size-*OP*, and *Size-Inv* sorts, there are three versions of *SMB*, one for each 2×3 sort, and *SMB* is the average of the three. *SMB* in the separate 2×2 sorts is defined similarly. Panel A of the table shows average monthly returns (Mean), the standard deviations of monthly returns (Std dev.) and the *t*-statistics for the average returns. Panel B shows the correlations of the same factor from different sorts and Panel C shows the correlations for each set of factors.

Panel A: Averages, standard deviations, and t-statistics for monthly returns

	2×3 Factors					2×2 Factors					$2 \times 2 \times 2 \times 2$ Factors				
	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.50	0.29	0.37	0.25	0.33	0.50	0.30	0.28	0.17	0.22	0.50	0.30	0.30	0.25	0.14
Std dev.	4.49	3.07	2.88	2.14	2.01	4.49	3.13	2.16	1.52	1.48	4.49	2.87	2.13	1.49	1.29
t-Statistic	2.74	2.31	3.20	2.92	4.07	2.74	2.33	3.22	2.79	3.72	2.74	2.60	3.43	4.09	2.71

	HML_S	HML_B	$HML_{S\text{-}B}$	RMW_S	RMW_B	$RMW_{S\text{-}B}$	CMA_S	CMA_B	$CMA_{S\text{-}B}$
2×3 factors									
Mean	0.53	0.21	0.32	0.33	0.17	0.16	0.45	0.22	0.23
Std dev.	3.24	3.11	2.69	2.69	2.35	2.68	2.00	2.66	2.47
t-Statistic	4.05	1.69	2.94	3.06	1.81	1.48	5.49	2.00	2.29
2×2 Factors									
Mean	0.40	0.16	0.24	0.22	0.13	0.09	0.33	0.11	0.22
Std dev	2.39	2.36	1.97	1.93	1.69	2.00	1.53	1.87	1.70
t-Statistic	4.16	1.68	3.05	2.76	1.86	1.09	5.37	1.50	3.17
$2 \times 2 \times 2 \times 2$ Factors									
Mean	0.37	0.22	0.16	0.30	0.21	0.09	0.23	0.07	0.17
Std dev.	2.40	2.36	2.01	2.18	1.53	2.22	1.23	1.58	1.59
t-Statistic	3.83	2.28	1.91	3.41	3.38	1.02	4.64	1.03	2.56

Panel B: Correlations between different versions of the same factor

	SMB			*HML*			*RMW*			*CMA*		
	2×3	2×2	$2 \times 2 \times 2 \times 2$	2×3	2×2	$2 \times 2 \times 2 \times 2$	2×3	2×2	$2 \times 2 \times 2 \times 2$	2×3	2×2	$2 \times 2 \times 2 \times 2$
2×3	1.00	1.00	0.98	1.00	0.97	0.94	1.00	0.96	0.80	1.00	0.95	0.83
2×2	1.00	1.00	0.98	0.97	1.00	0.96	0.96	1.00	0.83	0.95	1.00	0.87
$2 \times 2 \times 2 \times 2$	0.98	0.98	1.00	0.94	0.96	1.00	0.80	0.83	1.00	0.83	0.87	1.00

Panel C: Correlations between different factors

	2×3 Factors					2×2 Factors					$2 \times 2 \times 2 \times 2$ Factors				
	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*
R_M-R_F	1.00	0.28	−0.30	−0.21	−0.39	1.00	0.30	−0.34	−0.13	−0.43	1.00	0.25	−0.33	−0.27	−0.42
SMB	0.28	1.00	−0.11	−0.36	−0.11	0.30	1.00	−0.16	−0.32	−0.13	0.25	1.00	−0.21	−0.33	−0.21
HML	−0.30	−0.11	1.00	0.08	0.70	−0.34	−0.16	1.00	0.04	0.71	−0.33	−0.21	1.00	0.63	0.37
RMW	−0.21	−0.36	0.08	1.00	−0.11	−0.13	−0.32	0.04	1.00	−0.19	−0.27	−0.33	0.63	1.00	0.15
CMA	−0.39	−0.11	0.70	−0.11	1.00	−0.43	−0.13	0.71	−0.19	1.00	−0.42	−0.21	0.37	0.15	1.00

differences are observed in average *RMW* and *CMA* returns. The *t*-statistics (and thus the Sharpe ratios) for average *HML*, *RMW*, and *CMA* returns are, however, similar for the 2×3 and 2×2 sorts. The correlations between the factors of the two sorts (Panel B of Table 4) are also high, 0.97 (*HML*), 0.96 (*RMW*), and 0.95 (*CMA*).

Each factor from the 2×2 and 2×3 sorts controls for *Size* and one other variable. The factors from the $2\times2\times2\times2$ sorts control for all four variables. Joint controls have little effect on *HML*. The correlations of the $2\times2\times2\times2$ version of *HML* with the 2×2 and 2×3 versions are high, 0.94 and 0.96. The 2×2 and $2\times2\times2\times2$ versions of *HML*, which split stocks on the NYSE median *B*/*M*, have almost identical means and standard deviations, and both means are more than 3.2 standard errors from zero (Panel A of Table 4).

The correlations of *RMW* and *CMA* from the $2\times2\times2\times2$ sorts with the corresponding 2×3 and 2×2 factors are lower, 0.80 to 0.87, and joint controls produce an interesting result − a boost to the profitability premium at the expense of the investment premium. The $2\times2\times2\times2$ and 2×2 versions of *RMW* have similar standard deviations, 1.49% and 1.52% per month, but the $2\times2\times2\times2$ *RMW* has a larger mean, 0.25% (t=4.09) versus 0.17% (t=2.79). The standard deviation of *CMA* drops from 1.48 for the 2×2 version to 1.29 with four-variable controls, and the mean falls from 0.22% (t=3.72) to 0.14% (t=2.71). Thus, with joint controls, there is reliable evidence of an investment premium, but its average value is much lower than those of the other $2\times2\times2\times2$ factor premiums.

The value, profitability, and investment factors are averages of small and big stock factors. Here again, joint controls produce interesting changes in the premiums for small and big stocks (Panel A of Table 4). The factors from the 2×3 and 2×2 sorts confirm earlier evidence that the value premium is larger for small stocks (e.g., Fama and French, 1993, 2012; Loughran, 1997). For example, in the 2×3 *Size-B*/*M* sorts the average HML_S return is 0.53% per month (t=4.05), versus 0.21% (t=1.69) for HML_B. The evidence of a value premium in big stock returns is stronger if we control for profitability and investment. The average value of HML_B in the 2×2 and 2×3 sorts is less than 1.7 standard errors from zero, but more than 2.2 standard errors from zero in the $2\times2\times2\times2$ sorts. Controls for profitability and investment also reduce the spread between the value premiums for small and big stocks. The average difference between HML_S and HML_B falls from 0.24 (t=3.05) in the 2×2 sorts to 0.16 (t=1.91) in the $2\times2\times2\times2$ sorts.

For all methods of factor construction, there seem to be expected profitability and investment premiums for small stocks; the average values of RMW_S and CMA_S are at least 2.76 standard errors from zero. The average profitability premium is larger for small stocks than for big stocks, but the evidence that the expected premium is larger is weak. In the 2×3 sorts the average difference between RMW_S and RMW_B is 1.48 standard errors from zero. In the 2×2 and $2\times2\times2\times2$ sorts the average difference between RMW_S *and* RMW_B is less than 1.1 standard errors from zero.

In contrast, there is strong evidence that the expected investment premium is larger for small stocks. The average value of CMA_S is 4.64–5.49 standard errors from zero, but the average value of CMA_B is only 1.03–2.00 standard errors from zero, and it is more than 2.2 standard errors below the average value of CMA_S. In the $2\times2\times2\times2$ sorts the average value of CMA_B is 0.07% per month (t=1.03), and almost all the average value of *CMA* is from small stocks.

Panel C of Table 4 shows the correlation matrix for each set of factors. With 606 monthly observations, the standard error of the correlations is only 0.04, and most of the estimates are more than three standard errors from zero. The value, profitability, and investment factors are negatively correlated with both the market and the size factor. Since small stocks tend to have higher market betas than big stocks, it makes sense that *SMB* is positively correlated with the excess market return. Given the positive correlation between profitability and investment, it is perhaps surprising that the correlation between the profitability and investment factors is low, −0.19 to 0.15.

The correlations of the value factor with the profitability and investment factors merit comment. When *HML* and *CMA* are from separate 2×2 or 2×3 sorts, the correlation between the factors is about 0.70. This is perhaps not surprising given that high *B*/*M* value firms tend to be low investment firms. In the $2\times2\times2\times2$ sorts the correlation falls about in half, to 0.37, which also is not surprising since the factors from these sorts attempt to neutralize the effects of other factors.

The correlations between *HML* and *RMW* are surprising. When the two factors are from separate *Size-B*/*M* and *Size-OP* sorts, the correlation is close to zero, 0.04 in the 2×2 sorts and 0.08 in the 2×3 sorts. When the sorts jointly control for *Size*, *B*/*M*, *OP*, and *Inv*, the correlation between *HML* and *RMW* jumps to 0.63. There is a simple explanation. Among the 16 portfolios used to construct the $2\times2\times2\times2$ factors, the two with by far the highest return variances (small stocks with low *B*/*M*, weak *OP*, and low or high *Inv*) are held short in *HML* and *RMW*. Similarly, the portfolio of big stocks with the highest return variance is held long in the two factors, and the big stock portfolio with the second highest return variance is in the short end of both factors. The high correlation between *HML* and *RMW* is thus somewhat artificial, and it is a negative feature of the factors constructed with joint controls.

Finally, initiated by Carhart (1997), the FF three-factor model is often augmented with a momentum factor. The liquidity factor of Pástor and Stambaugh (2003) is another common addition. We do not show results for models that include these factors since for the left-hand-side (LHS) portfolios examined here, the two factors have regression slopes close to zero and so produce trivial changes in model performance. The same is true for the LHS anomaly portfolios in FF (2014), except when the LHS portfolios are formed on momentum, in which case including a momentum factor is crucial.

6. Model performance summary

We turn now to our primary task, testing how well the three sets of factors explain average excess returns on the portfolios of Tables 1 and 2. We consider seven asset

Table 5

Summary statistics for tests of three-, four-, and five-factor models; July 1963–December 2013, 606 months.

The table tests the ability of three-, four-, and five-factor models to explain monthly excess returns on 25 *Size-B/M* portfolios (Panel A), 25 *Size-OP* portfolios (Panel B), 25 *Size-Inv* portfolios (Panel C), 32 *Size-B/M-OP* portfolios (Panel D), 32 *Size-B/M-Inv* portfolios (Panel E), and 32 *Size-OP-Inv* portfolios (Panel F). For each set of 25 or 32 regressions, the table shows the factors that augment $R_M - R_F$ and *SMB* in the regression model, the *GRS* statistic testing whether the expected values of all 25 or 32 intercept estimates are zero, the average absolute value of the intercepts, $A|a_i|$, $A|a_i|/A|\bar{r}_i|$, the average absolute value of the intercept a_i over the average absolute value of $\bar{r}_i$, which is the average return on portfolio i minus the average of the portfolio returns, and $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$, which is $A(a_i^2)/A(\bar{r}_i^2)$, the average squared intercept over the average squared value of $\bar{r}_i$, corrected for sampling error in the numerator and denominator.

	2 × 3 Factors				2 × 2 Factors				2 × 2 × 2 × 2 Factors			
	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$
Panel A: 25 Size-B/M portfolios												
HML	3.62	0.102	0.54	0.38	3.54	0.101	0.53	0.36	3.40	0.096	0.51	0.36
HML RMW	3.13	0.095	0.50	0.24	3.11	0.096	0.51	0.26	3.29	0.089	0.47	0.24
HML CMA	3.52	0.101	0.53	0.39	3.46	0.100	0.53	0.37	3.18	0.096	0.51	0.35
RMW CMA	2.84	0.100	0.53	0.22	2.78	0.093	0.49	0.19	2.78	0.087	0.46	0.13
HML RMW CMA	2.84	0.094	0.50	0.23	2.80	0.093	0.49	0.23	2.82	0.088	0.46	0.18
Panel B: 25 Size-OP portfolios												
HML	2.31	0.108	0.68	0.51	2.31	0.109	0.68	0.51	1.91	0.089	0.56	0.37
RMW	1.71	0.067	0.42	0.12	1.82	0.078	0.49	0.16	1.73	0.059	0.37	0.05
HML RMW	1.64	0.062	0.39	0.16	1.74	0.058	0.36	0.03	1.62	0.064	0.40	0.06
HML CMA	3.02	0.137	0.86	0.90	2.85	0.135	0.85	0.86	2.06	0.102	0.64	0.49
RMW CMA	1.87	0.075	0.47	0.12	1.67	0.066	0.42	0.05	1.61	0.068	0.43	0.05
HML RMW CMA	1.87	0.073	0.46	0.12	1.73	0.066	0.42	0.06	1.60	0.069	0.43	0.07
Panel C: 25 Size-Inv portfolios												
HML	4.56	0.112	0.64	0.57	4.40	0.107	0.61	0.53	4.32	0.100	0.57	0.56
CMA	4.03	0.105	0.60	0.47	4.05	0.106	0.61	0.47	4.23	0.123	0.70	0.62
HML RMW	4.40	0.106	0.61	0.57	4.26	0.103	0.59	0.52	4.45	0.116	0.66	0.66
HML CMA	4.00	0.099	0.57	0.43	3.97	0.098	0.56	0.41	3.70	0.084	0.48	0.35
RMW CMA	3.33	0.085	0.49	0.29	3.28	0.082	0.47	0.26	3.50	0.082	0.47	0.27
HML RMW CMA	3.32	0.085	0.49	0.29	3.27	0.082	0.47	0.27	3.59	0.082	0.47	0.28
Panel D: 32 Size-B/M-OP portfolios												
HML	2.50	0.152	0.61	0.35	2.57	0.151	0.60	0.34	2.31	0.134	0.53	0.26
HML RMW	1.96	0.110	0.44	0.13	2.30	0.112	0.45	0.14	1.90	0.096	0.38	0.12
HML CMA	3.00	0.169	0.67	0.45	2.99	0.165	0.66	0.42	2.29	0.145	0.58	0.26
RMW CMA	2.02	0.137	0.55	0.16	2.06	0.129	0.51	0.13	1.73	0.108	0.43	0.07
HML RMW CMA	2.02	0.134	0.54	0.17	2.21	0.129	0.51	0.15	1.74	0.111	0.44	0.10
Panel E: 32 Size-B/M-Inv portfolios												
HML	2.72	0.129	0.64	0.38	2.80	0.134	0.66	0.40	2.82	0.131	0.65	0.40
HML RMW	2.32	0.120	0.60	0.38	2.49	0.128	0.64	0.42	2.49	0.122	0.61	0.37
HML CMA	2.43	0.102	0.51	0.25	2.52	0.108	0.54	0.26	2.36	0.114	0.57	0.27
RMW CMA	1.70	0.097	0.48	0.18	1.70	0.092	0.46	0.14	1.82	0.080	0.40	0.07
HML RMW CMA	1.73	0.091	0.45	0.18	1.87	0.092	0.46	0.18	1.86	0.084	0.42	0.13
Panel F: 32 Size-OP-Inv portfolios												
HML	4.38	0.182	0.79	0.69	4.17	0.179	0.78	0.67	4.01	0.170	0.74	0.61
HML RMW	3.80	0.140	0.61	0.37	3.82	0.140	0.61	0.37	3.55	0.151	0.66	0.43
HML CMA	3.91	0.177	0.77	0.68	3.82	0.177	0.77	0.67	3.66	0.142	0.62	0.48
RMW CMA	2.92	0.103	0.45	0.20	3.04	0.098	0.42	0.20	2.99	0.102	0.44	0.19
HML RMW CMA	2.92	0.103	0.45	0.21	3.04	0.097	0.42	0.20	3.03	0.101	0.44	0.19

pricing models: (i) three three-factor models that combine $R_M - R_F$ and *SMB* with *HML*, *RMW*, or *CMA*; (ii) three four-factor models that combine $R_M - R_F$, *SMB*, and pairs of *HML*, *RMW*, and *CMA*; and (iii) the five-factor model.

With seven models, six sets of left-hand-side portfolios, and three sets of right-hand-side (RHS) factors, it makes sense to restrict attention to models that fare relatively well in the tests. To judge the improvements provided by the profitability and investment factors, we show summary statistics for the original FF (1993) three-factor model, the five-factor model, and the three four-factor models for all sets of LHS portfolios and RHS factors. But we show results for alternative three-factor models only for the 5 × 5 sorts on *Size* and *OP* or *Inv* and only for the model in which the third factor – *RMW* or *CMA* – is aimed at the second LHS sort variable.

If an asset pricing model completely captures expected returns, the intercept is indistinguishable from zero in a regression of an asset's excess returns on the model's factor returns. Table 5 shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989) that tests this hypothesis for combinations of LHS portfolios and factors. The *GRS* test easily rejects all models considered for all LHS portfolios and RHS factors. To save space, the probability, or *p*-value, of getting a *GRS* statistic larger than the one observed if the true intercepts are all zero, is not shown. We can report that for four of the six sets of LHS returns, the p-values for all models round to zero to at least three decimals. The models fare best in the tests on the 25 *Size-OP* portfolios, but the p-values are still less than 0.04. In short, the *GRS* test says all our models are incomplete descriptions of expected returns.

Asset pricing models are simplified propositions about expected returns that are rejected in tests with power. We are less interested in whether competing models are rejected than in their relative performance, which we judge using *GRS* and other statistics. We want to identify the model that is the best (but imperfect) story for average returns on portfolios formed in different ways.

We are interested in the improvements in descriptions of average returns provided by adding profitability and investment factors to the FF three-factor model. For all six sets of LHS portfolios, the five-factor model produces lower *GRS* statistics than the original three-factor model. Table 5 shows that the average absolute intercepts, $A|a_i|$, are also smaller for the five-factor model. For the 25 *Size-B/M* portfolios, the five-factor model produces minor improvements, less than a basis point, in the average absolute intercept. The improvements are larger for the 25 *Size-OP* portfolios (2.0–4.3 basis points), the 25 *Size-Inv* portfolios (1.8–2.7 basis points), the 32 *Size-B/M-OP* portfolios (1.8–2.3 basis points), and the 32 *Size-B/M-Inv* portfolios (3.8–4.7 basis points).

Relative to the FF three-factor model, the biggest improvements in the average absolute intercept (6.9–8.2 basis points per month) are produced by the five-factor model when applied to the 32 *Size-OP-Inv* portfolios. This is not surprising since these portfolios are formed on two variables (profitability and investment) not directly targeted by the three-factor model. The results suggest that the FF three-factor model is likely to fare poorly when applied to portfolios with strong profitability and investment tilts.

Table 5 also shows two ratios that estimate the proportion of the cross-section of expected returns left unexplained by competing models. The numerator of each is a measure of the dispersion of the intercepts produced by a given model for a set of LHS portfolios; the denominator measures the dispersion of LHS expected returns. Define $\overline{R}_i$ as the time-series average excess return on portfolio i, define $\overline{R}$ as the cross-section average of $\overline{R}_i$, and define $\overline{r}_i$ as portfolio i's deviation from the cross-section average, $\overline{r}_i = \overline{R}_i - \overline{R}$. The first estimate is $A|a_i|/A|\overline{r}_i|$, the average absolute intercept over the average absolute value of $\overline{r}_i$.

The results for $A|a_i|/A|\overline{r}_i|$ in Table 5 tell us that for different sets of LHS portfolios and factor definitions, the five-factor model's average absolute intercept, $A|a_i|$, ranges from 42% to 54% of $A|\overline{r}_i|$. Thus, measured in units of return, the five-factor model leaves 42–54% of the dispersion of average excess returns unexplained. The dispersion of average excess returns left unexplained by the three-factor model is higher, 54–68%. Though not shown in Table 5, we can report that when the CAPM is estimated on the six sets of LHS portfolios, $A|a_i|/A|\overline{r}_i|$ ranges from 1.26 to 1.55. Thus, CAPM intercepts are more disperse than average returns, a result that persists no matter how we measure dispersion.

Measurement error inflates both the average absolute intercept $A|a_i|$ and the average absolute deviation $A|\overline{r}_i|$. The estimated intercept, a_i, is the true intercept, α_i, plus estimation error, $a_i = \alpha_i + e_i$. Similarly, $\overline{r}_i$ is μ_i, portfolio i's expected deviation from the grand mean, plus estimation error, $\overline{r}_i = \mu_i + \varepsilon_i$. We can adjust for measurement error if we focus on squared intercepts and squared deviations.

The cross-section average of μ_i is zero, so $A(\mu_i^2)$ is the cross-section variance of expected portfolio returns, and $A(\alpha_i^2)/A(\mu_i^2)$ is the proportion of $A(\mu_i^2)$ left unexplained by a model. Since α_i is a constant, the expected value of the square of an estimated intercept is the squared value of the true intercept plus the sampling variance of the estimate, $E(a_i^2) = \alpha_i^2 + E(e_i^2)$. Our estimate, $\hat{\alpha}_i^2$, of the square of the true intercept, α_i^2, is the difference between the squared estimates of the regression intercept and its standard error. Similarly, our estimate of μ_i^2, $\hat{\mu}_i^2$, is the difference between the square of the realized deviation, $\overline{r}_i^2$, and the square of its standard error. The ratio of averages, $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$, then estimates the proportion of the variance of LHS expected returns left unexplained. (As such, it is akin to $1-R^2$ in the regression of LHS expected returns on the expected returns from a model.)

In part because it is in units of return squared and in part because of the corrections for sampling error, $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$ provides a more positive picture of the five-factor model than $A|a_i|/A|\overline{r}_i|$. In the 5 × 5 sorts, the *Size-Inv* portfolios present the biggest challenge, but the estimates suggest that the five-factor model leaves only around 28% of the cross-section variance of expected returns unexplained. The estimate drops to less than 25% for the 25 *Size-B/M* portfolios and 6–12% for the 25 *Size-OP* portfolios. These are far less than the variance ratios produced by the FF three-factor model, which are mostly greater than 50% for the *Size-Inv* and *Size-OP* portfolios and about 37% for the *Size-B/M* portfolios. For the 25 *Size-OP* portfolios, however, the five-factor model is not systematically better on any metric than the three-factor model that substitutes *RMW* for *HML*.

The estimates of the cross-section variance of expected returns left unexplained by the five-factor model are lower for the LHS portfolios from the 2 × 4 × 4 sorts. For the 32 *Size-OP-Inv* portfolios, $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$ suggests that only about 20% of the cross-section variance of expected returns is left unexplained, versus 61–69% for the original three-factor model. The five-factor estimates drop to 13–18% for the 32 *Size-B/M-Inv* portfolios and 10–17% for the *Size-B/M-OP* portfolios, and most are less than half the estimates for the three-factor model.

Two important general results show up in the tests for each of the six sets of LHS portfolios. First, the factors from the 2 × 3, 2 × 2, and 2 × 2 × 2 × 2 sorts produce much the same results in the tests of a given model. Second, and more interesting, the five-factor model outperforms the FF three-factor model on all metrics and it generally outperforms other models, with one major exception. Specifically, the five-factor model and the four-factor model that excludes *HML* are similar on all measures of performance, including the *GRS* statistic. We explore this result in Section 7.

Finally, we do not show average values of R^2 in Table 5, but we can report that on average our models absorb a smaller fraction of return variance for the LHS portfolios

Table 6
Using four factors in regressions to explain average returns on the fifth: July 1963–December 2013, 606 months.

$R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate; *SMB* (small minus big) is the size factor; *HML* (high minus low *B/M*) is the value factor; *RMW* (robust minus weak *OP*) is the profitability factor; and *CMA* (conservative minus aggressive *Inv*) is the investment factor. The 2×3 factors are constructed using separate sorts of stocks into two *Size* groups and three *B/M* groups (*HML*), three *OP* groups (*RMW*), or three *Inv* groups (*CMA*). The 2×2 factors use the same approach except the second sort for each factor produces two rather than three portfolios. Each factor from the 2×3 and 2×2 sorts uses $2\times3=6$ or $2\times2=4$ portfolios to control for *Size* and one other variable (*B/M*, *OP*, or *Inv*). The $2\times2\times2\times2$ factors use the $2\times2\times2\times2=16$ portfolios to jointly control for *Size*, *B/M*, *OP*, and *Inv*. *Int* is the regression intercept.

	Int	$R_M - R_F$	*SMB*	*HML*	*RMW*	*CMA*	R^2
2×3 Factors							
$R_M - R_F$							
Coef	0.82		0.25	0.03	−0.40	−0.91	0.24
t-Statistic	4.94		4.44	0.38	−4.84	−7.83	
SMB							
Coef	0.39	0.13		0.05	−0.48	−0.17	0.17
t-Statistic	3.23	4.44		0.81	−8.43	−1.92	
HML							
Coef	−0.04	0.01	0.02		0.23	1.04	0.51
t-Statistic	−0.47	0.38	0.81		5.36	23.03	
RMW							
Coef	0.43	−0.09	−0.22	0.20		−0.44	0.21
t-Statistic	5.45	−4.84	−8.43	5.36		−7.84	
CMA							
Coef	0.28	−0.10	−0.04	0.45	−0.21		0.57
t-Statistic	5.03	−7.83	−1.92	23.03	−7.84		
2×2 Factors							
$R_M - R_F$							
Coef	0.78		0.28	−0.00	−0.43	−1.30	0.25
t-Statistic	4.80		5.09	−0.02	−3.71	−8.12	
SMB							
Coef	0.38	0.15		−0.03	−0.63	−0.18	0.17
t-Statistic	3.10	5.09		−0.36	−7.60	−1.42	
HML							
Coef	0.00	−0.00	−0.01		0.25	1.08	0.53
t-Statistic	0.01	−0.02	−0.36		5.66	23.13	
RMW							
Coef	0.30	−0.05	−0.14	0.21		−0.51	0.21
t-Statistic	5.22	−3.71	−7.60	5.66		−9.29	
CMA							
Coef	0.19	−0.08	−0.02	0.43	−0.25		0.60
t-Statistic	4.72	−8.12	−1.42	23.13	−9.29		
$2\times2\times2\times2$ Factors							
$R_M - R_F$							
Coef	0.79		0.19	−0.23	−0.33	−1.29	0.24
t-Statistic	4.77		3.23	−2.26	−2.30	−8.63	
SMB							
Coef	0.42	0.09		0.13	−0.64	−0.33	0.15
t-Statistic	3.73	3.23		1.82	−6.78	−3.04	
HML							
Coef	0.02	−0.04	0.04		0.84	0.48	0.48
t-Statistic	0.23	−2.26	1.82		18.61	8.05	
RMW							
Coef	0.20	−0.03	−0.11	0.43		−0.20	0.46
t-Statistic	4.28	−2.30	−6.78	18.61		−4.50	
CMA							
Coef	0.19	−0.09	−0.05	0.20	−0.16		0.26
t-Statistic	4.39	−8.63	−3.04	8.05	−4.50		

from the $2 \times 4 \times 4$ sorts than for the portfolios from the 5×5 sorts. For example, average R^2 in the five-factor regressions is 0.91–0.93 for the 5×5 sorts, versus 0.85–0.89 for the $2 \times 4 \times 4$ sorts. Average R^2 is lower because the LHS portfolios with three sort variables are less diversified. First, the $2 \times 4 \times 4$ sorts produce 32 portfolios and the 5×5 sorts produce only 25. Second, correlation between variables limits the diversification of some LHS portfolios. For example, the negative correlation between *OP* and *B/M* means there are often few big stocks in the top quartiles of *OP* and *B/M* (highly profitable extreme value stocks).

7. *HML*: a redundant factor

The five-factor model never improves the description of average returns from the four-factor model that drops *HML* (Table 5). The explanation is interesting. The average *HML* return is captured by the exposures of *HML* to other factors. Thus, in the five-factor model, *HML* is redundant for describing average returns, at least in U.S. data for 1963–2013.

The evidence is in Table 6, which shows regressions of each of the five factors on the other four. In the $R_M - R_F$ regressions, the intercepts (average returns unexplained by exposures to *SMB*, *HML*, *RMW*, and *CMA*) are around 0.80% per month, with *t*-statistics greater than 4.7. In the regressions to explain *SMB*, *RMW*, and *CMA*, the intercepts are more than three standard errors from zero. In the *HML* regressions, however, the intercepts are −0.04% ($t = -0.47$) for the 2×3 factors, 0.00% ($t = 0.01$) for the 2×2 factors, and 0.02% ($t = 0.23$) for the $2 \times 2 \times 2 \times 2$ factors.

In the spirit of Huberman and Kandel (1987), the evidence suggests that in U.S. data for 1963–2013, adding *HML* does not improve the mean-variance-efficient tangency portfolio produced by combining the riskfree asset, the market portfolio, *SMB*, *RMW*, and *CMA*. It will be interesting to examine whether this result shows up in U.S. data for the pre-1963 period or in international data.

The slopes in the Table 6 regressions often seem counterintuitive. For example, in the *HML* regressions, the large average *HML* return is mostly absorbed by the slopes for *RMW* and *CMA*. The *CMA* slopes are strongly positive, which is in line with the fact that high *B/M* value firms tend to do little investment. But the *RMW* slopes are also strongly positive, which says that controlling for other factors, value stocks behave like stocks with robust profitability, even though unconditionally, value stocks tend to be less profitable. The next section provides more examples of multivariate regression slopes that do not line up with univariate characteristics.

8. Regression details

For insights into model performance we next examine regression details, specifically, intercepts and pertinent slopes. To simplify the task, we could drop the five-factor model, given that *HML* is redundant for describing average returns. Though captured by exposures to other factors, however, there is a large value premium in average returns that is often targeted by money managers. Thus, in evaluating investment performance, we probably want to know the exposures of LHS portfolios to the *Size*, *B/M*, *OP*, and *Inv* factors. But we also want other factors to have slopes that reflect the fact that, at least in this sample, the four-factor model that drops *HML* captures average returns as well as the five-factor model.

A twist on the five-factor model (suggested by the referee) meets these goals. Define *HMLO* (orthogonal *HML*) as the sum of the intercept and residual from the regression of *HML* on $R_M - R_F$, *SMB*, *RMW*, and *CMA*. Substituting *HMLO* for *HML* in (5) produces an alternative five-factor model,

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HMLO_t + r_i RMW_t + c_i CMA_t + e_{it}. \tag{6}$$

The intercept and residual in (6) are the same as in the five-factor regression (5), so the two regressions are equivalent for judging model performance. (The results in Table 5, for example, do not change if we use Eq. (6) rather than (5).) The *HMLO* slope in (6) is also the same as the *HML* slope in (5), so (6) produces the same estimate of the value tilt of the LHS portfolio. But the estimated mean of *HMLO* (the intercept in the *HML* regressions in Table 6) is near zero, so its slope adds little to the estimate of the expected LHS return from (6). (Table 6 also says that the variance of *HMLO* is about half that of *HML*.) The slopes on the other factors in (6) are the same as in the four-factor model that drops *HML*, so the other factors have slopes that reflect the fact that they capture the information in *HML* about average returns.

The slopes in (6) for different versions of the factors are estimates of the same marginal effects, and we can report that the stories told by the slopes are similar for different versions of the factors. The three versions of the factors also produce much the same descriptions of average returns (Table 5). Thus, we keep the presentation of regression details manageable by focusing on one set of factors. Driven by precedent, we choose the factors from the 2×3 sorts – the FF (1993) approach.

We show regression intercepts and pertinent slopes from (6) for the 25 *Size-B/M*, the 25 *Size-OP*, the 25 *Size-Inv*, and the 32 *Size-OP-Inv* portfolios. Results for the 32 portfolios formed on *Size*, *B/M*, and either *OP* or *Inv* are relegated to Appendix A since they just reinforce the results for other LHS portfolios. For perspective on the five-factor results, we usually show the regression intercepts from the FF three-factor model, using *HML* rather that *HMLO* as the value factor.

The discussion of regression details is long, and a summary is helpful. Despite rejection on the *GRS* test, the five-factor model performs well: unexplained average returns for individual portfolios are almost all close to zero. The major exception is a portfolio that shows up in many sorts. The stocks in the offending portfolio are small and have negative exposures to *RMW* and *CMA*; that is, their returns behave like those of firms that invest a lot despite low profitability. In each sort that produces such a portfolio, its five-factor intercept is so negative that, using Bonferroni's inequality, we can easily reject the model for the entire set of 25 or 32 LHS portfolios.

8.1. 25 Size-B/M portfolios

Panel A of Table 7 shows intercepts from the FF three-factor regressions for the 25 *Size-B/M* portfolios. As

Table 7

Regressions for 25 value-weight *Size-B/M* portfolios; July 1963 to December 2013, 606 months.

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Low *B/M* to High *B/M*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-B/M* portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-B/M* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML* or its orthogonal version, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on *Size* and each of *B/M*, *OP*, and *Inv*. Panel A of the table shows three-factor intercepts produced by the *Mkt*, *SMB*, and *HML*. Panel B shows five-factor intercepts, slopes for *HMLO*, *RMW*, and *CMA*, and *t*-statistics for these coefficients.. The five-factor regression equation is,

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO(t)+rRMW(t)+cCMA(t)+e(t).$$

B/M →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: R_M-R_F, SMB, and HML										
			a					*t*(*a*)		
Small	−0.49	0.00	0.02	0.16	0.14	−5.18	0.07	0.40	2.88	2.37
2	−0.17	−0.04	0.12	0.07	−0.02	−2.75	−0.80	2.24	1.40	−0.38
3	−0.06	0.06	0.02	0.06	0.12	−0.98	0.92	0.33	0.96	1.66
4	0.14	−0.10	−0.04	0.07	−0.08	2.24	−1.46	−0.55	1.05	−0.94
Big	0.17	0.02	−0.07	−0.11	−0.18	3.53	0.40	−0.95	−1.86	−1.92
Panel B: Five-factor coefficients: R_M-R_F, SMB, HMLO, RMW, and CMA										
			a					*t*(a)		
Small	−0.29	0.11	0.01	0.12	0.12	−3.31	1.61	0.17	2.12	1.99
2	−0.11	−0.10	0.05	−0.00	−0.04	−1.73	−1.88	0.95	−0.04	−0.64
3	0.02	−0.01	−0.07	−0.02	0.05	0.40	−0.10	−1.06	−0.25	0.60
4	0.18	−0.23	−0.13	0.05	−0.09	2.73	−3.29	−1.81	0.73	−1.09
Big	0.12	−0.11	−0.10	−0.15	−0.09	2.50	−1.82	−1.39	−2.33	−0.93
			h					*t*(*h*)		
Small	−0.43	−0.14	0.10	0.27	0.52	−10.11	−4.38	3.90	10.12	17.55
2	−0.46	−0.01	0.29	0.43	0.69	−15.22	−0.45	11.77	16.78	24.44
3	−0.43	0.12	0.37	0.52	0.67	−14.70	3.71	12.28	17.07	18.75
4	−0.46	0.09	0.38	0.52	0.80	−15.18	2.76	11.03	15.88	20.26
Big	−0.31	0.03	0.26	0.62	0.85	−14.12	1.09	7.54	21.05	18.74
			r					*t*(*r*)		
Small	−0.58	−0.34	0.01	0.11	0.12	−13.26	−10.56	0.31	3.89	3.95
2	−0.21	0.13	0.27	0.26	0.21	−6.75	4.89	10.35	9.86	7.04
3	−0.21	0.22	0.33	0.28	0.33	−6.99	6.77	10.36	8.98	8.88
4	−0.19	0.27	0.28	0.14	0.25	−6.06	7.75	7.99	4.16	6.14
Big	0.13	0.25	0.07	0.23	0.02	5.64	8.79	2.07	7.62	0.49
			c					*t*(c)		
Small	−0.57	−0.12	0.19	0.39	0.62	−12.27	−3.46	6.59	13.15	19.10
2	−0.59	0.06	0.31	0.55	0.72	−17.76	1.94	11.27	19.39	22.92
3	−0.67	0.13	0.42	0.64	0.78	−20.59	3.64	12.52	18.97	19.62
4	−0.51	0.31	0.51	0.60	0.79	−15.11	8.33	13.35	16.41	18.03
Big	−0.39	0.26	0.41	0.66	0.73	−16.08	8.38	10.80	19.88	14.54

in Fama and French (1993, 2012), extreme growth stocks (left column of the intercept matrix) are a problem for the three-factor model. The portfolios of small extreme growth stocks produce negative three-factor intercepts and the portfolios of large extreme growth stocks produce positive intercepts. Microcap extreme growth stocks (upper left corner of the intercept matrix) are a huge problem. By itself, the three-factor intercept for this portfolio, −0.49% per month ($t=-5.18$), is sufficient to reject the three-factor model as a description of expected returns on the 25 *Size-B/M* portfolios.

The five-factor regression (6) reduces these problems. The intercept for the microcap extreme growth portfolio rises 20 basis points to −0.29 ($t=-3.31$), and the intercepts for three of the other four extreme growth portfolios shrink toward zero (Panel B of Table 7). But the pattern in the extreme growth intercepts – negative for small stocks and positive for large – survives in the five-factor model.

Panel B of Table 7 shows the five-factor slopes for *HMLO*, *RMW*, and *CMA*. The market and *SMB* slopes are not shown. The market slopes are always close to 1.0, and the *SMB* slopes are strongly positive for small stocks and slightly negative for big stocks. The market and *SMB* slopes are similar for different models, so they cannot account for changes in the intercepts observed when factors are added. To save space, here and later, we concentrate on *HMLO*, *RMW*, and *CMA* slopes.

The five-factor slopes provide information about stocks in the troublesome microcap portfolio in the lowest *B/M* quintile. The portfolio's *HMLO* slope (−0.43, $t=-10.11$), and its *CMA* slope (−0.57, $t=-12.27$) are similar to those of other extreme growth portfolios. But the portfolio has the most negative *RMW* slope, −0.58 ($t=-13.26$). The *RMW* and *CMA* slopes

Table 8

Time-series averages of book-to-market ratios (*B*/*M*), profitability (*OP*), and investment (*Inv*) for portfolios formed on (i) *Size* and *B*/*M*, (ii) *Size* and *OP*, (iii) *Size* and *Inv*, and (iv) *Size*, *OP*, and *Inv*.

In the sort for June of year t, *B* is book equity at the end of the fiscal year ending in year $t-1$ and *M* is market cap at the end of December of year t-1, adjusted for changes in shares outstanding between the measurement of *B* and the end of December. Operating profitability, *OP*, in the sort for June of year t is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, *Inv*, is the rate of growth of total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$. Each of the ratios for a portfolio for a given year is the value-weight average (market cap weights) of the ratios for the firms in the portfolio. The table shows the time-series averages of the ratios for the 51 portfolio formation years 1963–2013.

	B/*M*					*OP*					*Inv*				
25 *Size*-*B*/*M* portfolios															
B/*M* →	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Small	0.25	0.54	0.77	1.05	1.95	0.28	0.22	0.22	0.19	0.13	0.29	0.25	0.15	0.10	0.04
2	0.26	0.54	0.77	1.04	1.81	0.41	0.28	0.24	0.22	0.16	0.35	0.21	0.13	0.11	0.07
3	0.27	0.54	0.77	1.04	1.75	0.37	0.29	0.25	0.22	0.16	0.33	0.17	0.13	0.10	0.07
4	0.27	0.54	0.77	1.04	1.72	0.43	0.30	0.25	0.21	0.16	0.25	0.15	0.11	0.09	0.07
Big	0.26	0.53	0.76	1.04	1.61	0.50	0.32	0.27	0.23	0.19	0.16	0.12	0.11	0.10	0.11
25 *Size*-*OP* portfolios															
OP →	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Small	1.11	1.06	0.92	0.77	0.54	−0.37	0.19	0.25	0.32	1.63	0.07	0.18	0.20	0.24	0.31
2	1.02	0.94	0.80	0.66	0.46	−0.10	0.19	0.25	0.32	0.95	0.13	0.18	0.19	0.21	0.27
3	1.04	0.94	0.75	0.61	0.42	−0.09	0.19	0.25	0.32	0.67	0.16	0.15	0.15	0.19	0.24
4	1.09	0.92	0.71	0.56	0.40	0.03	0.19	0.25	0.32	0.61	0.14	0.13	0.14	0.16	0.19
Big	1.01	0.84	0.69	0.51	0.35	0.08	0.19	0.25	0.33	0.59	0.19	0.12	0.12	0.13	0.13
25 *Size*-*Inv* portfolios															
Inv →	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High
Small	1.14	1.12	1.00	0.87	0.65	−0.12	0.19	0.25	0.25	0.29	−0.14	0.02	0.08	0.15	0.71
2	0.99	0.96	0.87	0.74	0.55	0.14	0.45	0.26	0.28	0.28	−0.10	0.02	0.08	0.15	0.64
3	0.95	0.90	0.80	0.68	0.51	0.18	0.26	0.30	0.33	0.29	−0.08	0.03	0.08	0.15	0.58
4	0.90	0.87	0.75	0.62	0.49	0.26	0.29	0.30	0.33	0.32	−0.08	0.03	0.08	0.15	0.51
Big	0.75	0.71	0.61	0.49	0.42	0.35	0.33	0.34	0.37	0.48	−0.07	0.03	0.08	0.14	0.43

32 *Size*-*OP*-*Inv* portfolios	*B*/*M*				*OP*				*Inv*			
OP →	Low	2	3	High	Low	2	3	High	Low	2	3	High
						Small						
Low *Inv*	1.20	1.16	0.96	0.66	−0.50	0.18	0.26	1.71	−0.14	−0.08	−0.08	−0.10
2	1.30	1.13	0.90	0.66	−0.04	0.18	0.26	0.61	0.02	0.03	0.03	0.03
3	1.11	0.99	0.80	0.59	−0.03	0.19	0.27	0.57	0.11	0.11	0.11	0.11
High *Inv*	0.74	0.76	0.64	0.45	−0.30	0.18	0.27	0.66	0.93	0.51	0.42	0.45
						Big						
Low *Inv*	1.12	0.83	0.65	0.48	0.11	0.24	0.32	0.59	0.04	−0.02	−0.02	−0.03
2	1.01	0.78	0.59	0.41	0.14	0.24	0.32	0.54	0.06	0.06	0.06	0.06
3	0.90	0.69	0.51	0.34	0.14	0.24	0.32	0.54	0.12	0.12	0.12	0.12
High *Inv*	0.75	0.55	0.45	0.31	0.11	0.24	0.32	0.66	0.57	0.37	0.35	0.34

say the portfolio is dominated by microcaps whose returns behave like those of unprofitable firms that grow rapidly. The portfolio's negative five-factor loadings on *RMW* and *CMA* absorb about 40% of its three-factor intercept (-0.49, $t=-5.18$), but the five-factor model still leaves a large unexplained average return (-0.29, $t=-3.31$). There is a similar negative intercept in the results to come whenever the LHS assets include a portfolio of small stocks with strong negative *RMW* and *CMA* slopes.

Since lots of what is common in the story for average returns for different sets of LHS portfolios centers on the slopes for *RMW*, *CMA*, and in some cases *HMLO*, an interesting question is whether the factor slopes line up with the profitability (*OP*), investment (*Inv*), and *B*/*M* characteristics. Summary statistics for the portfolio characteristics, in Table 8, say the answer is often, but not always, yes. The regression slopes always line up with the characteristics used to form a set of LHS portfolios, but not always with other characteristics. For example, the *HMLO* slopes for the 25 *Size*-*B*/*M* portfolios in Panel B of Table 7 have a familiar pattern – strongly negative for low *B*/*M* growth stocks and strongly positive for high *B*/*M* value stocks. The *Size*-*B*/*M* portfolios are not formed on investment, but strong negative *CMA* slopes for low *B*/*M* growth stocks and strong positive *CMA* slopes for high *B*/*M* value stocks line up with the evidence in Table 8 that low *B*/*M* stocks invest aggressively and high *B*/*M* stocks invest conservatively. On the other hand, profitability is higher for low *B*/*M* growth portfolios than for high *B*/*M* value portfolios (Table 8), but (megacaps aside) this is the reverse of the pattern in the *RMW* slopes (Table 7).

There is, however, no reason to expect that univariate characteristics line up with multivariate regression slopes, which estimate marginal effects holding constant other explanatory variables. Moreover, the characteristics are measured with lags relative to returns. Since pricing should be forward looking, an interesting question for future research is whether *RMW*, *CMA*, and *HMLO* slopes line up better with future values of the corresponding characteristics than with past values.

Since characteristics do not always line up with regression slopes, we are careful when describing the slopes. For example, for the microcap portfolio in the lowest *B/M* quintile, we say that strong negative *RMW* and *CMA* slopes imply that the portfolio contains stocks whose returns “behave like” those of unprofitable firms that grow rapidly. Table 8 says that these firms have grown rapidly, and they are less profitable than extreme growth (low *B/M*) portfolios in larger size quintiles, but they are more profitable than other microcap portfolios.

8.2. 25 Size-OP portfolios

The *GRS* test and other statistics in Table 5 say that the five-factor model and the three-factor model that includes *RMW* provide similar descriptions of average returns on the 25 portfolios formed on *Size* and profitability. The five-factor intercepts for the portfolios (Panel B of Table 9) show no patterns and most are close to zero. This is in line with the evidence in Table 5 that average absolute intercepts are smaller for the *Size-OP* portfolios than for other LHS portfolios. The highest profitability microcap portfolio produces the most extreme five-factor intercept, -0.15 ($t=-2.05$), but it is modest relative to the most extreme intercept in other sorts.

The tests on the 25 *Size-OP* portfolios tell us that for small and big stocks, low profitability per se is not a five-factor asset pricing problem. For example, the five-factor intercept for the microcap portfolio in the lowest profitability quintile is -0.10% per month ($t=-1.28$). This portfolio has strong negative exposure to *RMW* (-0.67, $t=-17.70$) but modest exposure to *CMA* (-0.06, $t=-1.42$). This is in contrast to the *Size-B/M* sorts, in which the big problem is microcaps with extreme negative exposures to *RMW* and *CMA*. In short, portfolios formed on *Size* and *OP* are less of a challenge for the five-factor model than portfolios formed on *Size* and *B/M* in large part because the *Size-OP* portfolios do not isolate small stocks whose returns behave like those of firms that invest a lot despite low profitability.

The *Size-OP* portfolios are a problem for the FF three-factor model. Panel A of Table 9 shows that the model produces negative intercepts far from zero for the three small stock portfolios in the lowest *OP* quintile. The estimate for the low *OP* microcap portfolio, for example, is -0.30% per month ($t=-3.25$). Four of the five portfolios in the highest *OP* quintile produce positive three-factor intercepts, all more than two standard errors from zero. The results suggest that the three-factor model is likely to have problems in applications when portfolios have strong tilts toward high or low profitability.

8.3. 25 Size-Inv portfolios

Table 5 says that the five-factor model improves the description of average returns on the 25 *Size-Inv* portfolios provided by the FF three-factor model. Panel A of Table 10 shows that the big problems of the three-factor model are strong negative intercepts for the portfolios in the three smallest *Size* quintiles and the highest *Inv* quintile. Switching to the five-factor model moves these intercepts toward zero. The improvements trace to negative slopes for the investment and profitability factors, which lower five-factor estimates of expected returns. For example, the microcap portfolio in the highest *Inv* quintile produces the most extreme three-factor intercept, -0.48% ($t=-7.19$), but the portfolio's negative *RMW* and *CMA* slopes (-0.19, $t=-5.93$, and -0.31, $t=-8.78$) lead to a less extreme five-factor intercept, -0.35% ($t=-5.30$). This intercept is still sufficient (on Bonferroni's inequality) for a strong rejection of the five-factor model as a description of expected returns on the 25 *Size-Inv* portfolios.

The problem for the five-factor model posed by the microcap portfolio in the highest *Inv* quintile is similar to that posed by the microcap portfolio in the lowest *B/M* quintile in Table 7. Both show negative exposures to *RMW* and *CMA*, like those of firms that invest a lot despite low profitability, but their *RMW* and *CMA* slopes do not suffice to explain their low average returns (Table 1).

Given that the second-pass sort variable is investment, the *CMA* slopes for the *Size-Inv* portfolios show the expected pattern — positive for low investment portfolios and negative for high investment portfolios. There is less correspondence between the *HMLO* and *RMW* slopes and the *B/M* and *OP* characteristics. Table 8 says low investment is associated with value (high *B/M*) and high investment is associated with growth (low *B/M*). Confirming one end of this pattern, the *HMLO* slopes in the highest *Inv* quintile in Table 10 are zero to slightly negative, which is typical of growth stocks. But the portfolios in the lowest *Inv* quintile have rather low *HMLO* slopes (two are negative), which does not line up with their rather high average *B/M* in Table 8. Low investment firms are typically less profitable, and high investment firms are more profitable (Table 8). *RMW* slopes that are negative or close to zero for low investment portfolios in Table 10 indeed suggest low profitability, but the *RMW* slopes for the portfolios in the highest *Inv* quintile are also negative, and profitability is not low for these portfolios (Table 8).

Again, there is no reason to expect that multivariate regression slopes relate directly to univariate characteristics. Still, if one interprets the results for the 25 *Size-Inv* portfolios in terms of characteristics rather than factor exposures, the evidence suggests that high investment per se is a five-factor asset pricing problem, in particular, negative five-factor intercepts for high investment portfolios of small stocks and positive intercepts for high investment portfolios of big stocks. The *Size-B/M* portfolios of Table 7 also suggest this conclusion.

Adding fuel to the fire, Table 8 shows that average annual rates of investment in the highest *Inv* quintile are impressive, rising from 43% of assets for megacaps to 71% for microcaps. It seems likely that lots of these firms issue new stock and do mergers financed with stock — actions known to be associated with low subsequent stock returns (Ikenberry, Lakonishok, and Vermaelen, 1995; Loughran and Ritter, 1995; Loughran and Vijh, 1997). The overlap among new issues, mergers financed with stock, and high investment is an interesting topic for future research. For example, are the three patterns in unexplained returns somewhat independent or are they all subsumed by investment?

8.4. Size-OP-Inv portfolios

Table 11 shows three-factor and five-factor regression intercepts and five-factor *RMW* and *CMA* slopes for the 32 portfolios from $2\times4\times4$ sorts on *Size*, *OP*, and *Inv*. (To save

Table 9

Regressions for 25 value-weight *Size-OP* portfolios; July 1963–December 2013, 606 months.

At the end of each June, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *OP* (profitability) groups (Low *OP* to High *OP*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-OP* portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-OP* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML* or its orthogonal version, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on *Size* and each of *B/M*, *OP*, and *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts, slopes for *HMLO*, *RMW*, and *CMA*, and *t*-statistics for these coefficients.

$$R(t) - R_F(t) = a + b[R_M(t) - R_F(t)] + sSMB(t) + hHMLO(t) + rRMW(t) + cCMA(t) + e(t)$$

OP→	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: $R_M - R_F$, SMB, and HML										
			a					*t*(*a*)		
Small	−0.30	0.10	0.05	0.09	−0.02	−3.25	1.54	0.85	1.30	−0.30
2	−0.24	−0.03	0.05	0.04	0.16	−3.16	−0.55	0.94	0.58	2.08
3	−0.21	0.07	0.01	0.05	0.20	−2.27	1.04	0.14	0.79	2.51
4	−0.11	−0.02	−0.05	0.06	0.18	−1.15	−0.24	−0.73	0.96	2.43
Big	−0.17	−0.20	−0.03	0.05	0.22	−1.90	−2.94	−0.58	1.20	4.03
Panel B: Five-factor coefficients: $R_M - R_F$, SMB, HMLO, RMW, and CMA										
			a					*t*(a)		
Small	−0.10	0.04	−0.05	−0.05	−0.15	−1.28	0.64	−0.80	−0.80	−2.05
2	−0.05	−0.11	−0.03	−0.11	0.00	−0.83	−1.86	−0.64	−1.92	0.02
3	0.08	0.04	−0.06	−0.07	0.03	1.15	0.67	−1.05	−1.23	0.43
4	0.16	0.02	−0.12	−0.09	0.05	1.91	0.26	−1.97	−1.52	0.76
Big	0.14	−0.11	−0.03	0.02	0.08	2.08	−1.67	−0.57	0.42	1.85
			h					*t*(*h*)		
Small	−0.14	0.24	0.26	0.28	0.21	−3.82	8.05	9.32	9.31	6.17
2	−0.12	0.17	0.23	0.18	0.15	−3.96	5.84	9.51	6.38	5.08
3	0.00	0.14	0.21	0.19	0.09	0.11	4.36	7.68	6.74	2.93
4	0.03	0.15	0.21	0.10	0.02	0.72	4.80	7.19	3.60	0.69
Big	0.22	0.16	0.04	−0.00	−0.13	6.70	5.33	1.42	−0.19	−6.13
			r					*t*(*r*)		
Small	−0.67	0.21	0.30	0.47	0.45	−17.70	6.98	10.59	15.08	12.95
2	−0.60	0.21	0.29	0.45	0.55	−19.94	6.90	11.32	15.76	17.91
3	−0.76	0.03	0.24	0.38	0.57	−21.06	0.93	8.33	13.12	17.19
4	−0.75	−0.15	0.23	0.39	0.37	−18.94	−4.54	7.49	12.95	11.09
Big	−0.71	−0.26	−0.08	0.12	0.35	−21.05	−8.41	−2.82	5.66	15.54
			c					*t*(c)		
Small	−0.06	0.25	0.34	0.31	0.14	−1.42	7.58	10.89	9.08	3.76
2	−0.09	0.29	0.26	0.23	0.05	−2.65	8.94	9.52	7.44	1.56
3	−0.17	0.26	0.24	0.23	0.02	−4.41	7.31	7.89	7.49	0.65
4	−0.02	0.30	0.30	0.26	0.02	−0.41	8.56	9.08	8.12	0.48
Big	−0.03	0.23	0.19	−0.04	−0.12	−0.83	6.82	6.16	−1.82	−5.22

space the five-factor *HMLO* slopes are not shown.) These sorts are interesting because the profitability and investment characteristics of the stocks in the portfolios line up with their *RMW* and *CMA* slopes. For small and big stocks, *RMW* slopes are positive for high profitability quartiles and negative for low *OP* quartiles, and *CMA* slopes are positive for low investment quartiles and negative for high *Inv* quartiles. The correspondence between characteristics and regression slopes facilitates inferences about the nature of the stocks in troublesome portfolios.

The biggest problem for the five-factor model in Table 11 is the portfolio of small stocks in the lowest profitability and highest investment quartiles. Its intercept, −0.47% per month ($t = -5.89$) easily rejects the model as a description of expected returns on the 32 *Size-OP-Inv* portfolios. Low profitability per se is not a problem for the five-factor model in the results for small stocks. Two of the other three portfolios in the lowest *OP* quartile produce positive intercepts and one is 2.59 standard errors from zero. There is again suggestive evidence that for small stocks, high investment alone is associated with five-factor problems. The other three small stock portfolios in the highest *Inv* quartile also produce negative five-factor intercepts and two are more than two standard errors below zero.

If one looks to big stocks for confirmation of the five-factor problems observed for small stocks, none is found. The portfolio of big stocks in the lowest *OP* and highest *Inv* quartiles (the lethal combination for small stocks) produces a small positive five-factor intercept, 0.12% per month ($t = 1.37$). Moreover, the intercepts for the four big stock portfolios in the highest *Inv* quartile split evenly

Table 10
Regressions for 25 value-weight *Size-Inv* portfolios; July 1963–December 2013, 606 months.

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *Inv* (investment) groups (Low *Inv* to High *Inv*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-Inv* portfolios. The LHS variables are the monthly excess returns on the 25 *Size-Inv* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML* or its orthogonal version, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using independent 2×3 sorts on *Size* and each of *B/M*, *OP*, and *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts, slopes for *HMLO*, *RMW*, and *CMA*, and *t*-statistics for these coefficients.

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO(t)+rRMW(t)+cCMA(t)+e(t)$$

Inv →	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: R_M-R_F, SMB, and HML										
			a					*t*(*a*)		
Small	0.09	0.15	0.17	0.06	−0.48	1.01	2.74	2.76	1.00	−7.19
2	0.01	0.10	0.15	0.08	−0.26	0.14	1.72	2.74	1.45	−4.71
3	0.09	0.19	0.10	0.11	−0.17	1.11	3.15	1.80	1.73	−2.50
4	0.02	0.01	0.04	0.14	−0.03	0.24	0.19	0.66	2.09	−0.38
Big	0.15	0.07	0.02	0.07	0.05	1.86	1.18	0.39	1.43	0.75
Panel B: Five-factor coefficients: R_M-R_F, SMB, HMLO, RMW, and CMA										
			a					*t*(*a*)		
Small	0.21	0.11	0.09	0.02	−0.35	2.66	1.93	1.47	0.32	−5.30
2	−0.01	−0.01	0.06	0.02	−0.14	−0.14	−0.21	1.12	0.30	−2.59
3	0.03	0.10	−0.01	0.09	−0.02	0.40	1.74	−0.21	1.37	−0.33
4	−0.09	−0.09	−0.04	0.08	0.15	−1.20	−1.42	−0.73	1.22	2.05
Big	−0.04	−0.07	−0.06	0.04	0.20	−0.49	−1.42	−1.31	0.90	3.33
			h					*t*(*h*)		
Small	−0.10	0.17	0.16	0.12	0.00	−2.67	6.53	5.50	4.35	0.14
2	0.06	0.26	0.14	0.25	−0.11	2.33	9.11	5.26	10.24	−4.36
3	0.13	0.21	0.21	0.18	−0.04	3.53	7.26	7.99	6.12	−1.40
4	0.15	0.29	0.25	0.08	−0.19	4.34	9.41	8.63	2.50	−5.57
Big	−0.10	−0.04	0.10	−0.00	−0.06	−2.94	−1.86	4.47	−0.18	−2.04
			r					*t*(*r*)		
Small	−0.55	0.04	0.15	0.11	−0.19	−14.42	1.52	5.13	3.79	−5.93
2	−0.18	0.27	0.17	0.30	−0.15	−6.54	9.37	6.02	11.72	−5.86
3	−0.01	0.11	0.29	0.18	−0.13	−0.36	3.71	10.65	5.99	−4.20
4	0.05	0.21	0.21	0.16	−0.31	1.51	6.68	7.29	5.02	−8.77
Big	0.05	0.07	0.17	0.15	−0.02	1.50	2.74	7.20	6.05	−0.71
			c					*t*(*c*)		
Small	0.22	0.38	0.34	0.18	−0.31	5.27	13.11	10.50	5.69	−8.78
2	0.47	0.47	0.36	0.17	−0.51	15.85	15.12	12.21	6.28	−18.17
3	0.47	0.53	0.37	0.06	−0.56	11.59	16.71	12.66	1.83	−16.72
4	0.64	0.56	0.39	0.11	−0.60	16.64	16.55	12.46	3.10	−16.03
Big	0.69	0.48	0.25	−0.12	−0.76	18.03	18.80	10.27	−4.59	−24.15

between positive and negative, and the troublesome one is positive (0.36% per month, $t=4.36$, for the big stock portfolio in the highest *OP* and *Inv* quartiles). Thus, if the market overprices small stocks that invest a lot, the problem does not carry over to big stocks. Indeed, the asset pricing problem for big stocks is the high average return of highly profitable firms that invest a lot.

The FF three-factor model's problems in the tests on the 32 *Size-OP-Inv* portfolios are more severe. For example, portfolios of small or big stocks that combine high *OP* and low *Inv* produce strong positive intercepts in the three-factor model, but in the five-factor model the high average returns of these portfolios are absorbed by strong positive *RMW* and *CMA* slopes. The lethal combination that dooms the five-factor model is even more deadly in the three-factor model. The three-factor intercept for the portfolio of small stocks in the lowest *OP* and highest *Inv* quartiles is −0.87% per month ($t=-8.45$), but negative *RMW* and *CMA* slopes shrink the intercept to −0.47% ($t=-5.89$) in the five-factor model. The *Size-OP-Inv* sorts provide the most direct evidence that strong profitability and investment tilts are problems for the three-factor model.

9. Conclusions

There are patterns in average returns related to *Size*, *B/M*, profitability, and investment. The *GRS* test easily rejects a five-factor model directed at capturing these patterns, but we estimate that the model explains between 71% and 94%

Table 11
Regressions for 32 value-weight *Size-OP-Inv* portfolios; July 1963–December 2013, 606 months.

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *OP* groups (Low *OP* to High *OP*) and four *Inv* groups (Low *Inv* to High *Inv*), using NYSE *OP* and *Inv* breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-OP-Inv* portfolios. The LHS variables in the 32 regressions are the excess returns on the 32 *Size-OP-Inv* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the *B/M* factor, *HML* or its orthogonal version *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2×3 sorts on *Size* and *B/M*, *OP*, or *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts, slopes for *RMW* and *CMA*, and their *t*-statistics.

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO(t)+rRMW(t)+cCMA(t)+e(t)$$

	Small								Big							
OP →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Panel A: Three-factor intercepts: $R_M - R_F$, SMB, and HML																
	a				*t*(*a*)				*a*				*t*(*a*)			
Low *Inv*	−0.09	0.11	0.32	0.34	−0.92	1.45	3.71	3.74	−0.01	0.10	0.21	0.17	−0.07	1.12	2.39	1.97
2	0.11	0.09	0.15	0.21	1.37	1.51	2.72	2.87	−0.25	−0.11	0.16	0.20	−2.88	−1.40	2.22	2.53
3	−0.24	0.18	0.17	0.28	−2.72	2.99	3.01	4.21	−0.11	0.01	0.03	0.15	−1.25	0.19	0.40	1.86
High *Inv*	−0.87	−0.23	−0.02	−0.05	−8.45	−2.80	−0.40	−0.66	−0.23	−0.27	−0.06	0.29	−2.34	−3.04	−0.71	3.24
Panel B: Five-factor coefficients: $R_M - R_F$, SMB, HMLO, RMW, and CMA																
	a				*t*(*a*)				*a*				*t*(*a*)			
Low *Inv*	0.05	0.00	0.11	0.11	0.66	0.04	1.29	1.30	0.05	−0.11	−0.03	−0.10	0.63	−1.36	−0.43	−1.27
2	0.18	−0.03	0.03	−0.01	2.59	−0.45	0.60	−0.21	−0.18	−0.10	0.05	0.03	−2.11	−1.29	0.75	0.33
3	−0.14	0.15	0.06	0.09	−1.67	2.33	1.21	1.72	0.07	−0.01	−0.05	−0.01	0.80	−0.11	−0.73	−0.09
High *Inv*	−0.47	−0.23	−0.05	−0.13	−5.89	−2.91	−0.96	−2.44	0.12	−0.17	−0.01	0.36	1.37	−1.88	−0.11	4.36
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.65	0.15	0.38	0.51	−18.43	3.88	9.37	11.88	−0.37	0.15	0.39	0.38	−9.86	3.68	10.03	9.65
2	−0.43	0.25	0.33	0.57	−12.40	8.92	12.53	18.64	−0.23	−0.11	0.17	0.37	−5.32	−2.75	4.97	9.71
3	−0.36	0.07	0.32	0.58	−8.74	2.35	12.27	22.30	−0.36	0.10	0.22	0.43	−8.62	2.47	6.48	11.62
High *Inv*	−0.89	0.18	0.22	0.48	−22.84	4.65	8.10	19.02	−0.62	−0.06	0.12	0.15	−14.89	−1.43	2.88	3.77
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	0.23	0.65	0.69	0.66	6.02	15.86	15.95	14.34	0.48	0.74	0.67	0.59	11.95	17.31	15.78	13.79
2	0.26	0.59	0.50	0.52	7.04	19.27	17.88	15.73	0.33	0.38	0.27	0.25	7.29	8.85	7.28	6.14
3	0.11	0.24	0.34	0.27	2.58	7.21	12.39	9.67	0.23	0.23	0.05	−0.07	5.21	5.41	1.46	−1.62
High *Inv*	−0.67	−0.12	−0.11	−0.21	−15.95	−2.78	−3.61	−7.57	−0.49	−0.34	−0.49	−0.77	−11.04	−7.10	−10.97	−18.03

of the cross-section variance of expected returns for the *Size*, *B/M*, *OP*, and *Inv* portfolios we examine.

Judged on regression intercepts, the three sets of factors we use — (i) separate 2×3 sorts on *Size* and *B/M*, *OP*, or *Inv*, (ii) separate 2×2 sorts, and (iii) $2 \times 2 \times 2 \times 2$ sorts that jointly control for *Size*, *B/M*, *OP*, and *Inv* – provide similar descriptions of average returns on the LHS portfolios examined.

Armed with the evidence presented here, which version of the factors would we choose if starting fresh? We might prefer the factors from the 2×2 *Size-B/M*, *Size-OP*, and *Size-Inv* sorts over those from the 2×3 sorts (the original approach). Since the 2×2 versions of *HML*, *RMW*, and *CMA* use all stocks and the 2×3 versions exclude 40%, the 2×2 factors are better diversified. In the tests of the five-factor model here and in Fama and French (2014), however, the performance of the two sets of factors is similar for the LHS portfolios we examine, so the choice between them seems inconsequential.

The joint controls of the $2 \times 2 \times 2 \times 2$ sorts are attractive for isolating estimates of factor premiums. But given that multivariate regression slopes measure marginal effects, it's not clear that the factors from the $2 \times 2 \times 2 \times 2$ sorts better isolate exposures to variation in returns related to *Size*, *B/M*, profitability, and investment. And inevitable uncertainty about the eventual list of factors lessens the attraction of the $2 \times 2 \times 2 \times 2$ factors. Controlling for more factors is problematic. If we add momentum, for example, correlations among the five variables are likely to result in poor diversification of some of the portfolios used to construct factors. If one shortens the list of factors (for example, dropping *HML*), one should reconstruct the factors since controlling for unused characteristics is potentially harmful (though apparently not an issue in the Table 5 tests of three-factor and four-factor models).

In the end, precedent, flexibility in accommodating more or fewer factors, and the fact that they perform as well as the 2×2 and $2 \times 2 \times 2 \times 2$ factors in our tests of asset pricing models lead us back to the factors from the 2×3 sorts.

If parsimony is an issue, our results suggest that *HML* is a redundant factor in the sense that its high average return is fully captured by its exposures to $R_M - R_F$, *SMB*, and especially *RMW* and *CMA*. Thus, in applications where the sole interest is abnormal returns (measured by regression intercepts), our tests suggest that a four-factor model that drops *HML* performs as well as the five-factor model. But if one is also interested in portfolio tilts toward *Size*, value, profitability, and investment premiums, the five-factor model is the choice. As a concession to the evidence that suggests *HML* is redundant, however, one might substitute *HMLO* for *HML* in the five-factor model.

One of our more interesting results is that portfolios of small stocks with negative exposures to *RMW* and *CMA* are the biggest asset pricing problem in four of the six sets of LHS portfolios examined here. The negative *CMA* exposures of the troublesome portfolios always line up with evidence that the firms in these portfolios invest a lot, but negative exposures to *RMW* in the 5×5 *Size-B/M* and *Size-Inv* sorts (Tables 7 and 10) do not correspond to particularly low profitability. For these portfolios, we say their returns behave like those of the stocks of firms that invest a lot despite low profitability, but there are hints that for small stocks, high investment alone might be the prime problem. For the $2 \times 4 \times 4$ *Size-OP-Inv* portfolios in Table 11, there is less ambiguity. In this sort, negative *RMW* and *CMA* slopes line up nicely with low *OP* and high *Inv*, and we conclude that the lethal portfolios contain small stocks of firms that invest a lot despite low profitability. As a lure for potential readers of FF (2014) we can report that small stock portfolios with similar properties play a big role in our tests of the five-factor model on prominent anomaly variables, specifically, accruals, net share issues, and volatility.

Behavioral stories for the low average returns of small stocks that invest a lot despite low profitability face a serious challenge: The unexplained average returns of big stocks that invest a lot despite low profitability are positive.

Finally, the paper closest to ours is Hou, Xue, and Zhang (2012). They examine a four-factor model that, in addition to $R_M - R_F$, includes factors much like *SMB*, *RMW*, and *CMA* that are constructed from $2 \times 3 \times 3$ sorts that jointly control for *Size*, profitability, and investment. They do not comment on why *HML* is not in the model, and they only compare the performance of their four-factor model to that of the CAPM, the FF three-factor model, and Carhart's (1997) four-factor model, which adds a momentum factor. Their investigation of models is more restricted than ours, and they do not consider alternative factor definitions. More important, they are primarily concerned with explaining the returns associated with anomaly variables not used to construct their factors, and they focus on VW portfolios from univariate sorts on each variable. Value-weight portfolios from univariate sorts on variables other than *Size* are typically dominated by big stocks, and one of the main messages here and in Fama and French (1993, 2012, 2014) is that the most serious problems of asset pricing models are in small stocks.

Appendix A

A.1. Summary statistics for the components of the factors

Table A1 shows the means, standard deviations, and *t*-statistics for the means for returns on the portfolios used to construct *SMB*, *HML*, *RMW*, and *CMA*.

A.2. Five-factor regressions to explain the returns for Size-B/M-OP and Size-B/M-Inv Portfolios

Table A2 shows intercepts and *HMLO*, *RMW*, and *CMA* slopes from five-factor regression (6) for monthly excess returns for the 32 portfolios from $2 \times 4 \times 4$ sorts on *Size*, *B/M*, and operating profitability *OP*. The portfolios of small and big stocks with the highest *B/M* and *OP* (highly profitable extreme value stocks) produce rather extreme intercepts, negative for big stocks (−0.20% per month), and positive for small stocks (0.35%), but they are only −0.97 and 1.80 standard errors from zero, suggestive of chance results. The imprecision of these intercepts is due to poor diversification: highly profitable extreme value stocks are rare, especially among big stocks. The regression R^2 for these portfolios (not shown in Table A2) are low, 0.57 for big stocks and 0.67 for small stocks.

For small and big stocks, the *HMLO* slopes for the 32 *Size-B/M-OP* portfolios increase monotonically from the low *B/M*

Table A1

Average percent returns, standard deviations (Std dev.), and *t*-statistics for the average return for the portfolios used to construct *SMB*, *HML*, *RMW*, and *CMA*; July 1963–December 2013, 606 months.

We use independent sorts to form two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label the portfolios with two or four letters. The first is small (*S*) or big (*B*). In the 2×3 and 2×2 sorts, the second is the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (*C*), neutral (*N*), or aggressive (*A*). In the $2 \times 2 \times 2 \times 2$ sorts, the second character is the *B/M* group, the third is the *OP* group, and the fourth is the *Inv* group.

	2×3 Sorts						2×2 Sorts			
Size-B/M	*SL*	*SN*	*SH*	*BL*	*BN*	*BH*	*SL*	*SH*	*BL*	*BH*
Mean	0.93	1.31	1.46	0.89	0.94	1.10	1.03	1.43	0.88	1.04
Std dev.	6.87	5.44	5.59	4.65	4.34	4.68	6.41	5.42	4.50	4.38
t-Statistic	3.32	5.93	6.44	4.69	5.36	5.78	3.95	6.51	4.82	5.86
Size-OP	*SW*	*SN*	*SR*	*BW*	*BN*	*BR*	*SW*	*SR*	*BW*	*BR*
Mean	1.02	1.27	1.35	0.81	0.87	0.98	1.10	1.32	0.82	0.95
Std dev	6.66	5.32	5.96	4.98	4.38	4.39	6.16	5.69	4.53	4.39
t-Statistic	3.77	5.87	5.60	4.00	4.91	5.50	4.41	5.71	4.47	5.33
Size-Inv	*SC*	*SN*	*SA*	*BC*	*BN*	*BA*	*SC*	*SA*	*BC*	*BA*
Mean	1.41	1.34	0.96	1.07	0.94	0.85	1.40	1.06	0.99	0.88
Std dev	6.12	5.22	6.59	4.38	4.08	5.18	5.73	6.17	4.09	4.69
t-Statistic	5.66	6.35	3.59	5.99	5.69	4.03	6.01	4.25	5.98	4.62

	$2 \times 2 \times 2 \times 2$ *Size-B/M-OP-Inv* Sorts							
	SLWC	*SLWA*	*SLRC*	*SLRA*	*SHWC*	*SHWA*	*SHRC*	*SHRA*
Mean	1.13	0.70	1.36	1.16	1.43	1.24	1.64	1.54
Std dev	7.18	7.36	5.38	6.15	5.55	5.62	5.23	5.52
t-Statistic	3.89	2.34	6.24	4.64	6.34	5.42	7.72	6.88
	BLWC	*BLWA*	*BLRC*	*BLRA*	*BHWC*	*BHWA*	*BHRC*	*BHRA*
Mean	0.77	0.78	1.02	0.91	1.02	0.93	1.24	1.17
Std dev	5.16	5.47	4.16	4.74	4.36	4.69	4.79	5.51
t-Statistic	3.69	3.51	6.04	4.75	5.78	4.87	6.38	5.23

Table A2

Five-factor regression results for 32 value-weight *Size-B/M-OP* portfolios; July 1963–December 2013, 606 months.

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *B/M* groups (Low *B/M* to High *B/M*) and four *OP* groups (Low *OP* to High *OP*), using NYSE *B/M* and *OP* breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-B/M-OP* portfolios. The LHS variables are the excess returns on the 32 *Size-B/M-OP* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the *B/M* factor, *HML* or its orthogonal version, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2×3 sorts on *Size* and *B/M*, *OP*, or *Inv*. The table shows five-factor regression intercepts, *HMLO*, *RMW*, and *CMA* slopes, and *t*-statistics for the intercepts and slopes.

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO(t)+rRMW(t)+cCMA(t)+e(t)$$

	Small								Big							
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
	a				*t*(*a*)				*a*				*t*(*a*)			
Low *OP*	−0.33	0.03	−0.08	−0.11	−3.49	0.36	−1.09	−1.67	0.34	−0.18	−0.13	−0.11	2.01	−1.72	−1.53	−1.88
2	−0.00	−0.13	−0.05	0.01	−0.02	−1.84	−0.90	0.13	0.20	−0.17	−0.21	−0.12	1.79	−1.97	−2.58	−1.50
3	−0.13	−0.06	0.09	0.25	−2.14	−1.10	1.67	2.04	0.05	−0.06	−0.12	0.18	0.69	−0.93	−1.34	1.42
Low *OP*	−0.14	0.08	0.12	0.35	−2.94	1.34	1.26	1.80	0.07	−0.13	0.07	−0.20	1.25	−1.36	0.53	−0.97
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *OP*	−0.54	−0.14	0.13	0.55	−12.12	−3.27	3.71	16.89	−0.53	−0.08	0.20	0.63	−6.58	−1.60	5.16	22.21
2	−0.34	0.17	0.38	0.79	−7.52	5.12	14.56	23.38	−0.60	−0.04	0.17	0.84	−11.40	−0.91	4.45	22.73
3	−0.16	0.29	0.54	0.79	−5.71	11.66	20.64	13.82	−0.32	0.05	0.44	0.76	−9.91	1.52	10.39	12.57
Low *OP*	0.01	0.52	0.67	0.74	0.40	18.56	14.91	7.96	−0.26	0.06	0.45	1.01	−9.55	1.45	6.97	10.63
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *OP*	−1.05	−0.52	−0.12	0.03	−22.91	−11.60	−3.24	1.04	−1.02	−0.38	−0.21	0.02	−12.23	−7.47	−5.25	0.61
2	−0.12	0.31	0.29	0.33	−2.48	8.86	10.67	9.30	−0.43	0.26	0.13	0.34	−7.77	6.26	3.24	8.95
3	0.18	0.44	0.42	0.33	6.25	16.69	15.62	5.58	0.15	0.30	0.41	0.26	4.55	9.32	9.35	4.09
Low *OP*	0.57	0.58	0.57	0.50	24.51	20.26	12.18	5.17	0.36	0.52	0.33	0.43	12.82	11.44	4.89	4.32
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *OP*	−0.73	−0.06	0.40	0.67	−14.79	−1.17	10.47	18.62	−1.28	−0.08	0.30	0.67	−14.30	−1.39	7.10	21.40
2	−0.41	0.29	0.58	0.80	−8.05	7.71	20.05	21.27	−0.61	0.27	0.58	0.75	−10.33	5.94	13.75	18.42
3	−0.16	0.45	0.61	0.71	−4.94	16.03	21.01	11.16	−0.45	0.26	0.62	0.45	−12.73	7.75	13.25	6.72
Low *OP*	−0.05	0.48	0.68	0.86	−1.95	15.56	13.50	8.33	−0.28	0.36	0.31	0.61	−9.31	7.38	4.27	5.75

Table A3

Five-factor regression results for 32 *Size-B/M-Inv* portfolios; July 1963–December 2013, 606 months.

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *B/M* groups (low *B/M* to High *B/M*) and four *Inv* groups (low *Inv* to High *Inv*) using NYSE breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-B/M-Inv* portfolios. The LHS variables in the 32 regressions are the excess returns on the 32 *Size-B/M-Inv* portfolios. The RHS variables are the excess market return, $R_M - R_F$, the *Size* factor, *SMB*, the orthogonal version of the *B/M* factor, *HMLO*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2×3 sorts on *Size* and *B/M*, *OP*, or *Inv*. The table shows five-factor regression intercepts, *HMLO*, *RMW*, and *CMA* slopes, and *t*-statistics for the intercepts and slopes.

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO\,(t)+rRMW(t)+cCMA(t)+e(t)$$

	Small								Big							
B/M →	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
	a				*t*(*a*)				*a*				*t*(*a*)			
Low *Inv*	−0.05	0.06	0.18	0.04	−0.64	0.81	2.46	0.53	−0.06	−0.08	−0.18	−0.10	−0.64	−0.95	−2.20	−1.37
2	0.10	−0.01	−0.04	0.04	1.37	−0.18	−0.83	0.54	−0.03	−0.09	−0.08	−0.11	−0.32	−1.07	−0.95	−1.39
3	0.08	0.02	0.07	−0.06	1.45	0.30	1.36	−0.68	0.06	−0.08	−0.12	−0.02	0.83	−0.99	−1.39	−0.21
High *Inv*	−0.20	−0.07	−0.03	−0.02	−4.18	−1.27	−0.47	−0.14	0.37	−0.18	−0.22	−0.06	5.39	−2.07	−2.18	−0.60
	h				*t*(*h*)				*h*				*t*(*h*)			
Low *Inv*	−0.45	−0.07	0.25	0.58	−11.39	−1.92	7.43	15.13	−0.44	−0.20	0.10	0.53	−9.85	−5.19	2.65	15.52
2	−0.25	0.23	0.35	0.73	−7.49	8.35	13.91	21.74	−0.28	−0.05	0.27	0.73	−7.02	−1.43	6.74	20.22
3	−0.18	0.27	0.45	0.66	−7.48	10.22	17.34	15.51	−0.22	0.02	0.33	1.00	−6.59	0.63	7.72	23.05
High *Inv*	−0.23	0.32	0.46	0.83	−10.26	11.63	13.54	15.97	−0.42	0.29	0.53	0.73	−12.73	6.94	11.32	14.41
	r				*t*(*r*)				*r*				*t*(*r*)			
Low *Inv*	−0.49	−0.12	0.05	0.14	−12.01	−3.47	1.48	3.55	0.18	0.26	0.18	0.23	3.97	6.61	4.47	6.57
2	−0.07	0.32	0.32	0.22	−2.08	11.02	12.17	6.46	0.27	0.17	0.07	0.17	6.51	4.37	1.69	4.44
3	0.12	0.41	0.34	0.21	4.73	14.86	12.76	4.70	0.28	0.31	0.21	0.16	8.06	8.18	4.75	3.65
High *Inv*	−0.15	0.16	0.15	0.26	−6.48	5.66	4.21	4.79	−0.15	0.30	0.16	0.15	−4.49	7.04	3.35	2.82
	c				*t*(*c*)				*c*				*t*(*c*)			
Low *Inv*	−0.00	0.44	0.64	0.92	−0.06	11.54	16.97	21.72	0.21	0.61	0.76	0.89	4.15	14.60	17.88	23.20
2	0.07	0.52	0.66	0.80	1.89	16.80	23.59	21.44	0.01	0.31	0.52	0.64	0.17	7.32	11.68	16.01
3	−0.06	0.38	0.55	0.70	−2.31	12.85	18.96	14.81	−0.22	0.14	0.49	0.61	−5.88	3.55	10.46	12.57
High *Inv*	−0.65	0.06	0.32	0.41	−25.95	1.91	8.55	7.04	−1.06	−0.02	0.10	0.42	−29.26	−0.44	1.94	7.39

portfolio to the high *B/M* portfolios, and the *RMW* slopes increase with profitability. This is not surprising, given that the LHS sorts are on *Size*, *B/M*, and *OP*. Investment is not a sort variable, but the *CMA* slopes in Table A2 also line up nicely with the evidence in Table A4 that investment is higher for low *B/M* (growth) portfolios and lower for high *B/M* value portfolios. The *CMA* slopes are more negative (Table A2) and investment is stronger (Table A4) for less profitable small stocks in the lowest *B/M* quartile. It is tempting to infer that this result is driven by unprofitable startups, but the same pattern in *CMA* slopes and *Inv* is observed for big stocks in the lowest *B/M* quartile. In any case, the correspondence between *B/M*, *OP*, and *Inv* characteristics and *HMLO*, *RMW*, and *CMA* slopes makes the five-factor regression results easy to interpret.

The big problem for the five-factor model in Table A2 is the negative intercept (−0.33% per month, $t=-3.49$) for the portfolio of small stocks in the lowest *OP* and *B/M* quartiles (small, low profitability growth stocks). This portfolio has negative *HMLO*, *RMW*, and *CMA* slopes, but the reductions to expected return implied by the slopes don't fully explain the low average excess return on the portfolio, 0.03% per month (Table 2). The problem for the five-factor model posed by this portfolio is like the big problem in the tests on the 32 *Size-OP-Inv* portfolios (Table 11). In a nutshell, small stocks that invest a lot despite low profitability fare much worse than predicted by the five-factor model. The $2\times4\times4$ sorts on *Size*, *B/M*, and *OP* add to the puzzle since the portfolio of big stocks in the lowest *B/M* and *OP* quartiles also has strong negative exposures to *HMLO*, *RMW*, and *CMA*, but it has a positive five-factor intercept (0.34% per month, $t=2.01$). Thus, judged by the five-factor model, the average returns of big growth stocks that invest a lot despite low profitability are, if anything, too high.

Table A3 shows five-factor intercepts and *HMLO*, *RMW*, and *CMA* slopes for the 32 portfolios from $2\times4\times4$ sorts on *Size*, *B/M*, and *Inv*. The *HML* and *CMA* slopes behave as expected, given that the LHS sorts are on *B/M* and *Inv*. The *HML* slopes are negative for low *B/M* portfolios and strongly positive for high *B/M* portfolios. The *CMA* slopes fall from strongly positive for low investment portfolios to strongly negative for high *Inv* portfolios. There is less correspondence between the *RMW* slopes in Table A3 and average *OP* in Table A4. Except for the lowest *Inv* quartile of small stocks, *OP* tends to be higher for small and big low *B/M* growth stocks than for high *B/M* value stocks (Table A4). This pattern does not show up in the *RMW* slopes for small stocks and it does not show up consistently for big stocks (Table A3).

The portfolios of small and big stocks in the lowest *B/M* quartile and the highest *Inv* quartile (growth stocks that invest a lot) produce intercepts more than 3.5 standard errors from zero but of opposite sign — negative (−0.20% per month, $t=-4.18$) for small stocks and positive (0.37%, $t=5.39$) for big stocks. Both portfolios have strong negative exposures to *CMA* and weaker negative exposures to *RMW*. The negative intercept for the small stock portfolio in the lowest *B/M* quartile and the highest *Inv* quartile is

Table A4
Time-series averages of book-to-market ratios (*B*/*M*), profitability (*OP*), and investment (*Inv*) for 32 portfolios formed on *Size*, *B*/*M*, and *OP* or *Inv*.

In the sort for June of year *t*, *B* is book equity at the end of the fiscal year ending in year $t-1$ and *M* is market cap at the end of December of year *t*-1, adjusted for changes in shares outstanding between the measurement of *B* and the end of December. Operating profitability, *OP*, in the sort for June of year *t* is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, *Inv*, is the rate of growth of total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$. Each of the ratios for a portfolio for a given year is the value-weight average (market cap weights) of the ratios for the stocks in the portfolio. The table shows the time-series averages of the ratios for the 51 portfolio formation years 1963–2013.

	B/*M*				*OP*				*Inv*			
B/*M*→	Low	2	3	High	Low	2	3	High	Low	2	3	High
32 *Size-B*/*M-OP* portfolios												
						Small						
Low *OP*	0.32	0.77	1.11	2.12	−0.67	−0.01	0.03	0.03	0.37	0.15	0.09	0.05
2	0.41	0.77	1.10	1.81	0.19	0.19	0.18	0.18	0.35	0.14	0.10	0.08
3	0.42	0.76	1.08	1.76	0.27	0.27	0.26	0.26	0.27	0.13	0.10	0.08
High *OP*	0.34	0.74	1.07	1.82	0.88	0.42	0.46	0.46	0.25	0.14	0.12	0.10
						Big						
Low *OP*	0.27	0.54	0.80	1.34	0.00	0.14	0.14	0.13	0.42	0.18	0.14	0.11
2	0.29	0.53	0.78	1.21	0.24	0.24	0.24	0.23	0.23	0.11	0.08	0.08
3	0.29	0.51	0.77	1.20	0.33	0.32	0.32	0.31	0.17	0.13	0.11	0.11
High *OP*	0.23	0.51	0.77	1.24	0.65	0.45	0.46	0.49	0.16	0.11	0.11	0.10
32 *Size-B*/*M-Inv* portfolios												
						Small						
Low *Inv*	0.36	0.76	1.11	2.07	0.06	0.15	0.14	0.08	−0.14	−0.10	−0.09	−0.10
2	0.41	0.77	1.09	2.00	0.35	0.25	0.22	0.15	0.03	0.03	0.03	0.03
3	0.41	0.76	1.09	1.83	0.39	0.26	0.23	0.17	0.11	0.11	0.11	0.11
High *Inv*	0.34	0.75	1.08	1.84	0.30	0.26	0.22	0.16	0.63	0.44	0.42	0.50
						Big						
Low *Inv*	0.28	0.53	0.79	1.36	0.51	0.31	0.25	0.19	−0.04	−0.03	−0.02	−0.03
2	0.27	0.53	0.79	1.25	0.48	0.32	0.26	0.21	0.06	0.06	0.06	0.06
3	0.25	0.51	0.78	1.22	0.46	0.32	0.27	0.21	0.12	0.12	0.12	0.12
High *Inv*	0.23	0.51	0.78	1.24	0.53	0.31	0.26	0.21	0.38	0.37	0.40	0.48

consistent with all the evidence of underperformance for small stocks whose returns behave like those of firms that invest a lot despite low profitability. The strong positive five-factor intercept for the otherwise similar big stock portfolio adds to the puzzle.

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