ACTSC 372 Project: Markowitz Portfolio Optimization

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a. Data Collection

In this project, we aim to optimize the following portfolio of stocks that are publicly traded in the Toronto Stock Exchange (TSE):

Ticker (TSE)	Full Name	Industry	Sub-Industry
RY	Royal Bank of Canada	Financial Services	Bank
MFC	Manulife Corporation	Financial Services	Insurance
SHOP	Shopify Inc.	Information Technology	E-Commerce (Service)
CSU	Constellation Software Inc.	Information Technology	Software (Holding)
DOL	Dollarama Inc.	Consumer Staples	Retail
CTC	Canadian Tire Corporation	Consumer Discretionary	Equipments Retail
CNR	Canadian National Railway	Industrials	Transportation
T	TELUS Corporation	TMT	Telecommunication
ENB	Enbridge Inc	Energy	Oil and Gas
BEP.UN	Brookfield Renewable Partners LP	Utilities	Renewable Energy

We will be using Quantmod library connected with Yahoo Finance to get the adjusted stock price from November 8, 2018 to November 6, 2020 (24 months). We will use the data from November 8, 2018 to July 7, 2020 (20 months) as our training set, and the data from July 8, 2020 to November 6, 2020 (4 months) as our testing set.

Here are the first 6 (from training set 1:6) and last 6 (from testing set 1:6) adjusted prices

```
##         Date    RY.TO   MFC.TO SHOP.TO   CSU.TO   DOL.TO   CTC.TO   CNR.TO
## 1 2018-11-08 88.78171 20.28227  197.02 886.6312 37.25510 223.2063 109.1542
## 2 2018-11-09 88.19345 20.28227  186.23 871.3427 36.68958 232.1345 109.1638
## 3 2018-11-12 87.62358 20.13695  179.57 867.2573 36.11414 232.1345 107.6687
## 4 2018-11-13 88.00962 20.28227  183.93 869.8802 35.20136 226.3263 108.4307
## 5 2018-11-14 87.59598 20.10062  189.00 877.8950 35.09222 226.3263 108.4404
## 6 2018-11-15 87.91769 19.98254  197.87 877.5245 34.14968 226.3263 108.3535
##       T.TO   ENB.TO BEP-UN.TO
## 1 20.92064 37.34558  27.07776
## 2 20.89787 37.49215  27.17885
## 3 21.02085 37.51801  27.26550
## 4 21.04362 37.28524  27.02721
## 5 21.12105 37.31239  26.57953
## 6 21.06640 37.25984  26.22571
```

```
##          Date RY.TO MFC.TO SHOP.TO  CSU.TO DOL.TO CTC.TO CNR.TO  T.TO   ENB.TO
## 80 2020-10-30 93.16  18.06 1228.23 1398.59  45.88 202.65 132.35 22.78 35.96345
## 81 2020-11-02 93.29  18.19 1210.74 1380.00  46.05 202.65 135.59 22.54 35.66955
## 82 2020-11-03 94.94  19.04 1237.01 1447.87  46.51 202.75 138.02 22.64 35.71853
## 83 2020-11-04 97.23  18.53 1303.72 1516.97  47.27 202.94 134.13 23.01 35.80670
## 84 2020-11-05 96.27  18.87 1352.84 1531.72  49.89 210.00 136.51 23.03 36.36511
## 85 2020-11-06 96.74  18.83 1361.45 1572.59  49.02 204.67 137.12 23.32 35.09155
##    BEP-UN.TO
## 80     72.20
## 81     75.44
## 82     76.08
## 83     73.61
## 84     75.07
## 85     77.05
```

b. Estimation

Here, we are going to estimate the mean and covariance matrix of the stock daily returns. SHown benlow is the first 6 daily returns from the training and testing set, respectively.

```
##         Date        RY.TO        MFC.TO      SHOP.TO        CSU.TO      DOL.TO
## 1 2018-11-09 -0.006625881  0.0000000000 -0.054766053 -0.0172433066 -0.01517969
## 2 2018-11-12 -0.006461591 -0.0071652715 -0.035762171 -0.0046886121 -0.01568410
## 3 2018-11-13  0.004405663  0.0072169832  0.024280146  0.0030243204 -0.02527478
## 4 2018-11-14 -0.004699895 -0.0089565401  0.027564874  0.0092137194 -0.00310042
## 5 2018-11-15  0.003672645 -0.0058742979  0.046931190 -0.0004220835 -0.02685903
## 6 2018-11-16 -0.002927306  0.0004543467  0.004700096  0.0149999714  0.02149935
##        CTC.TO        CNR.TO         T.TO        ENB.TO    BEP-UN.TO
## 1  0.03999988  8.835209e-05 -0.001088590  0.0039244532  0.003733211
## 2  0.00000000 -1.369622e-02  0.005885002  0.0006898245  0.003188252
## 3 -0.02502063  7.077554e-03  0.001083305 -0.0062041938 -0.008739506
## 4  0.00000000  8.894158e-05  0.003679452  0.0007281434 -0.016564158
## 5  0.00000000 -8.006520e-04 -0.002587702 -0.0014082990 -0.013311635
## 6 -0.02328748  4.184164e-03  0.009297270  0.0030553538  0.004680559
```

```
##         Date        RY.TO       MFC.TO      SHOP.TO       CSU.TO       DOL.TO
## 1 2020-07-08 -0.005299053 -0.005991315 -0.002622602  0.035810562  0.019850049
## 2 2020-07-09 -0.007719175 -0.017534207  0.029070300  0.002676890 -0.005325179
## 3 2020-07-10  0.019831330  0.022866657 -0.007347167 -0.010648453  0.014145729
## 4 2020-07-13  0.003867755  0.019083984 -0.062228604 -0.004336224  0.001502159
## 5 2020-07-14  0.014875859  0.003745278  0.007618975 -0.006311563  0.016713181
## 6 2020-07-15  0.010017787  0.011727029 -0.022533116  0.005411146  0.024235930
##         CTC.TO       CNR.TO         T.TO       ENB.TO    BEP-UN.TO
## 1  0.009345835  0.009990815 -0.001336315 -0.001961767  0.009036191
## 2  0.000000000 -0.004286478 -0.004014310 -0.012284926  0.008805917
## 3  0.000000000  0.009023880  0.016121867 -0.001243775 -0.017014365
## 4 -0.030277711  0.001148668  0.002203560  0.005977595  0.017459324
## 5  0.000000000  0.024913972  0.018469694  0.033176479  0.040828401
## 6  0.002864383  0.009755339  0.016839314  0.003115279  0.026861879
```

Here is the expected return $\hat{\mu} = [\mu_1, ..., \mu_{10}]$ for the training set

```
##  [1] 2.135890e-04 6.619368e-05 5.341722e-03 1.557148e-03 6.791974e-04
##  [6] 1.495341e-04 3.698461e-04 2.826355e-04 3.995890e-04 1.875938e-03
```

Here is the estimated covariance matrix $\hat{\Sigma}$ for the training set

```
##                  RY.TO       MFC.TO      SHOP.TO       CSU.TO       DOL.TO
## RY.TO     0.0003431246 3.940049e-04 2.070506e-04 1.564132e-04 1.788827e-04
## MFC.TO    0.0003940049 6.728955e-04 3.164302e-04 2.457946e-04 2.576744e-04
## SHOP.TO   0.0002070506 3.164302e-04 1.316389e-03 3.029711e-04 1.514517e-04
## CSU.TO    0.0001564132 2.457946e-04 3.029711e-04 3.778649e-04 1.467959e-04
## DOL.TO    0.0001788827 2.576744e-04 1.514517e-04 1.467959e-04 4.229221e-04
## CTC.TO    0.0001070475 9.634327e-05 3.682125e-05 2.337044e-05 6.053916e-05
## CNR.TO    0.0002377664 3.279252e-04 2.614025e-04 1.785253e-04 1.526207e-04
## T.TO      0.0002302833 2.885144e-04 1.703796e-04 1.257884e-04 1.439302e-04
## ENB.TO    0.0003447157 4.486762e-04 3.135094e-04 2.029303e-04 2.065480e-04
```

```
## BEP-UN.TO 0.0003085459 3.868829e-04 2.818222e-04 1.676073e-04 1.886133e-04
##                 CTC.TO       CNR.TO         T.TO       ENB.TO    BEP-UN.TO
## RY.TO     1.070475e-04 2.377664e-04 0.0002302833 3.447157e-04 0.0003085459
## MFC.TO    9.634327e-05 3.279252e-04 0.0002885144 4.486762e-04 0.0003868829
## SHOP.TO   3.682125e-05 2.614025e-04 0.0001703796 3.135094e-04 0.0002818222
## CSU.TO    2.337044e-05 1.785253e-04 0.0001257884 2.029303e-04 0.0001676073
## DOL.TO    6.053916e-05 1.526207e-04 0.0001439302 2.065480e-04 0.0001886133
## CTC.TO    5.568276e-04 6.012697e-05 0.0000781485 9.511308e-05 0.0001325766
## CNR.TO    6.012697e-05 2.996145e-04 0.0001838936 2.786516e-04 0.0002137043
## T.TO      7.814850e-05 1.838936e-04 0.0002818179 2.761405e-04 0.0002445297
## ENB.TO    9.511308e-05 2.786516e-04 0.0002761405 5.570700e-04 0.0003620844
## BEP-UN.TO 1.325766e-04 2.137043e-04 0.0002445297 3.620844e-04 0.0005482562
```

c. Construction of Portfolios

According to the Markowitz Portfolio Optimization Model, to optimize the objective function

$$\max \quad \tau w^T \mu - \frac{1}{2} w^T \Sigma w \quad \text{s.t.} \quad e^T w = 1$$

, we get that

$$w_{opt} = \tau w_z + w_m$$

where w_{opt} is the optimum portfolio weight, w_z is the zero-covariance portfolio, and w_m is the minimum risk portfolio that is given as follows:

$$w_z = \Sigma^{-1}\mu - \frac{e^T \Sigma^{-1} \mu}{e^T \Sigma^{-1} e} \Sigma^{-1} e$$

and

$$w_m = \frac{\Sigma^{-1} e}{e^T \Sigma^{-1} e}$$

Here is the minimum risk portfolio w_m

```
##                   [,1]
## RY.TO       0.178699344
## MFC.TO     -0.261506146
## SHOP.TO    -0.001533679
## CSU.TO      0.228889445
## DOL.TO      0.167666353
## CTC.TO      0.189858220
## CNR.TO      0.281507101
## T.TO        0.340490457
## ENB.TO     -0.121681416
## BEP-UN.TO -0.002389678
```

Here is the zero-covariance portfolio w_z

```
##                  [,1]
## RY.TO      -2.0628537
## MFC.TO     -2.0172022
## SHOP.TO     3.9889595
## CSU.TO      1.6897759
## DOL.TO      0.2968893
## CTC.TO     -0.9694562
## CNR.TO     -2.7786866
## T.TO       -2.0223412
## ENB.TO     -1.4141692
## BEP-UN.TO   5.2890845
```

Combining w_z and w_m, here are some optimal portfolios that is constructed from the several tau as selected below

```
##                       0        0.01        0.02        0.05          0.1
## RY.TO      0.178699344  0.15807081  0.13744227  0.07555666 -0.027586029
## MFC.TO    -0.261506146 -0.28167817 -0.30185019 -0.36236625 -0.463226364
## SHOP.TO   -0.001533679  0.03835592  0.07824551  0.19791430  0.397362273
## CSU.TO     0.228889445  0.24578720  0.26268496  0.31337824  0.397867030
## DOL.TO     0.167666353  0.17063525  0.17360414  0.18251082  0.197355284
## CTC.TO     0.189858220  0.18016366  0.17046909  0.14138541  0.092912596
## CNR.TO     0.281507101  0.25372023  0.22593337  0.14257277  0.003638436
## T.TO       0.340490457  0.32026705  0.30004363  0.23937340  0.138256337
## ENB.TO    -0.121681416 -0.13582311 -0.14996480 -0.19238988 -0.263098336
## BEP-UN.TO -0.002389678  0.05050117  0.10339201  0.26206455  0.526518773
##                     0.2        0.5
## RY.TO     -0.233871401 -0.8527275
## MFC.TO    -0.664946581 -1.2701072
## SHOP.TO    0.796258226  1.9929461
## CSU.TO     0.566844615  1.0737774
## DOL.TO     0.227044214  0.3161110
## CTC.TO    -0.004033029 -0.2948699
## CNR.TO    -0.274230228 -1.1078362
## T.TO      -0.063977784 -0.6706801
## ENB.TO    -0.404515255 -0.8287660
## BEP-UN.TO  1.055427224  2.6421526
```

d. In-Sample Performance Evaluation

In this section, we want to evaluate the performance of our optimal weighting by backtesting it to our training set/prices.

We have the S&P 500 as our market portfolio. The first few daily returns of the S&P 500, as well as their mean daily returns and standard deviations during training is as follows:

```
##         Date      S&P 500
## 1 2018-11-09 -0.009199014
## 2 2018-11-12 -0.019701489
## 3 2018-11-13 -0.001481920
## 4 2018-11-14 -0.007567411
## 5 2018-11-15  0.010593753
## 6 2018-11-16  0.002223306
```

```
## [1] "The mean return of the S&P 500 is 0.000432"
```

```
## [1] "The standard deviation of the S&P 500 strategy is 0.017699"
```

We also have our 1/N strategy (each stock given a weighting of 0.1) for the performance comparison. Below are the mean daily returns and daily standard deviations.

```
## [1] "The mean return of the 1/N strategy is 0.001094"
```

```
## [1] "The standard deviation of the 1/N strategy is 0.015660"
```

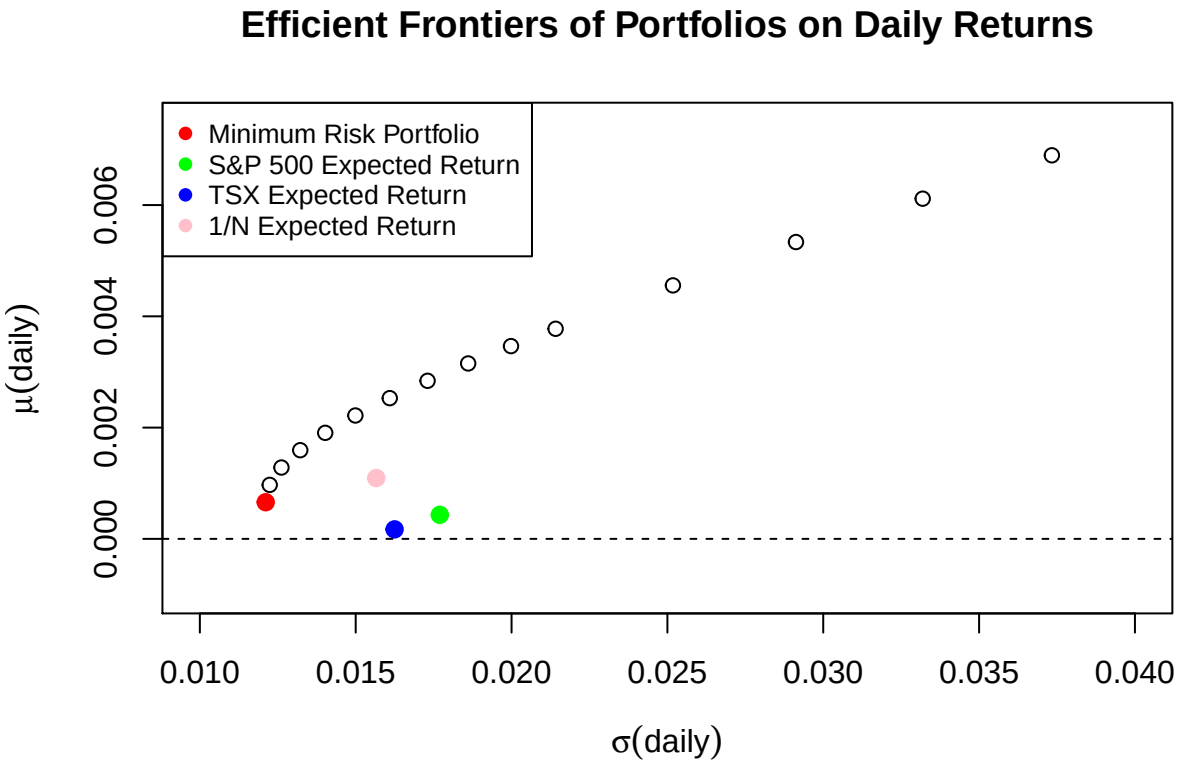
Here is the mean and standard deviation of daily interest rate yield of the US 10-Year Treasury Bond, assuming that there are 250 trading days (for compounded daily rates)

```
##         Date    Risk-Free
## 1 2018-11-09 0.0001293908
## 2 2018-11-12 0.0001276182
## 3 2018-11-13 0.0001275000
## 4 2018-11-14 0.0001258843
## 5 2018-11-15 0.0001248987
## 6 2018-11-16 0.0001248199
```

```
## [1] "The mean return of the 10-Year US Treasury is 0.000075"
```

```
## [1] "The standard deviation of the 10-Year US Treasury is 0.000029"
```

Here is the plot of the efficient frontier made from optimal portfolios, as well as the market portfolio (S&P 500 and TSX), 1/N portfolio, and the min-risk portfolio.



As seen from the picture above, the optimum portfolios will form an efficient frontier that signifies the best expected daily return given the daily standard deviation. The market portfolios (S&P 500 and TSX) as well as the 1/N portfolio all lies below the efficient frontier, signifying that their expected daily returns are inefficient (not optimal) given their daily standard deviation (risk). The min-risk portfolio is located at the left-most part of the frontier, and since it has the lowest risk, it has a lower reward as well. However, we can see that even with this minimum risk, this portfolio still have a higher expected return compared to the market portfolio. This becomes a basis for us to explore deeper and test if our set of efficient portfolio can actually outperform the market.

Here is the daily returns of our portfolios with ranging tau and 1/N strategy.

```
##         Date       Tau = 0   Tau = 0.01    Tau = 0.02   Tau = 0.05    Tau = 0.1
## 1 2018-11-09 -0.0008299131 -0.003440534 -0.0060511543 -0.013883016 -0.026936119
## 2 2018-11-12 -0.0048722956 -0.005726360 -0.0065804245 -0.009142618 -0.013412940
## 3 2018-11-13 -0.0062960453 -0.005938427 -0.0055808081 -0.004507952 -0.002719860
## 4 2018-11-14  0.0042779708  0.004838360  0.0053987500  0.007079919  0.009881867
## 5 2018-11-15 -0.0033827815 -0.002164422 -0.0009460627  0.002709015  0.008800812
## 6 2018-11-16  0.0059281427  0.006609969  0.0072917956  0.009337275  0.012746407
##       Tau = 0.2   Tau = 0.5           1/N
## 1 -0.0530423250 -0.13136094  0.0003733211
## 2 -0.0219535853 -0.04757552  0.0003188252
## 3  0.0008563263  0.01158488 -0.0008739506
## 4  0.0154857624  0.03229745 -0.0016564158
## 5  0.0209844059  0.05753519 -0.0013311635
## 6  0.0195646721  0.04001947  0.0004680559
```

We can compare the performance of these portfolios using some financial metrics as follows:

```
##                                 Tau = 0   Tau = 0.01  Tau = 0.02  Tau = 0.05
## Mean Returns (Daily)       0.0006585929 0.0009703862 0.001282180 0.002217560
## Standard Deviation (Daily) 0.0121136735 0.0122416918 0.012617956 0.014989644
## Cumulative Returns (20m)   0.2630338718 0.5632078396 0.863381807 1.763903711
## Sharpe Ratio               0.8359149729 1.3205133376 1.798365260 3.042111853
## Treynor Ratio              0.3559724482 0.5583782417 0.770386898 1.472479318
## Jensen's Alpha             0.0004068587 0.0007265712 0.001046284 0.002005421
## Beta                       0.4497711428 0.4577472593 0.465723376 0.489651726
##                              Tau = 0.1   Tau = 0.2   Tau = 0.5      S&P 500
## Mean Returns (Daily)       0.003776526 0.006894459  0.01624826 0.0004319342
## Standard Deviation (Daily) 0.021413417 0.037335163  0.08911551 0.0176991982
## Cumulative Returns (20m)   3.264773550 6.266513228 15.27173226 0.1205951128
## Sharpe Ratio               4.569742739 7.712573819 39.18223044 0.3401663239
## Treynor Ratio              2.921832140 7.472412962 65.06107281 0.0951951702
## Jensen's Alpha             0.003603984 0.006801109  0.01639248 0.0000000000
## Beta                       0.529532308 0.609293474  0.84857697 1.0000000000
##                                    1/N
## Mean Returns (Daily)       0.0010935393
## Standard Deviation (Daily) 0.0156601013
## Cumulative Returns (20m)   0.7981309224
## Sharpe Ratio               1.1930206320
## Treynor Ratio              0.3787471256
## Jensen's Alpha             0.0007534857
## Beta                       0.7799435691
```

From the above performance comparison, we can see that the optimal portfolio outperforms the market portfolio during the in-sample (training) period. This can be clearly seen from the mean daily returns and cumulative returns. The min-risk portfolio can generate higher expected return and cumulative return than the market, while still maintaining lower risk (low standard deviation and beta) compared to the market. We can also see an increasing trend in expected return as we increase the tau (trade-off parameter).
One of the optimal portfolios that "wins" in all metrics against the market and 1/N portfolio is the portfolio with $\tau = 0.05$. Aside from having higher returns and lower risk (lower σ and β), it also has a higher Sharpe Ratio and Treynor Ratio. This means that with every increase in σ and β of the portfolio, the portfolio is able to return higher net yield (expected return - risk-free) compared to the market and 1/N portfolios. Additionally, it also has a daily alpha "advantage" of 0.002, suggesting that it outperforms the required rate of return.
Looking back into the portfolio weighting, we have that the min-risk portfolio invests pretty heavily in more "conservative" stocks such as Canadian Rail (TSX: CNR) and TELUS (TSX: T), where these stocks have relatively low beta. This makes sense since the nature of these businesses are more "essential" and unaffected by cyclicalties that might happen in the market. People will still need communication providers like TELUS regardless of how the market performs. Similarly, Canadian Rail has a huge presence in rail transportation, and companies will still need them anyway. They are more prone to swings in the market since they expect revenues to be relatively stable from time to time, and that there are usually less breakthroughs announced compared with growth companies.
On the other hand, the zero-covariance portfolio w_z suggests that we should push for growth stocks such as Brookfield Renewable Partners (TSX:BEP-UN), Shopify (TSX: SHOP), and Constellation Software (TSX: CSU). These companies are in their growing years and are expected to grow their enterprise value by several times before reaching peak. Moreover, with these companies being in the technology and renewable energy sectors, they are competing in the "niche" and futuristic industry, where their presence can be really felt in the years ahead, but requires a massive amount of upfront costs/capital before breaking even in the future. In fact, after the March 2020 crash, these sectors are among the fastest to rebound due to sudden fluctuations in daily technology usage that is not normally as high. In contrast, this portfolio also instructed us to short

the “laggards” such as Enbridge (TSX:ENB) and Manulife (TSX:MFC). The recent oil price slump in late April 2020 and COVID crash during March 2020 have wiped the market values of Canadian companies in the energy and financial sector.

Aside from the cyclicalities shown in different sectors in response to the COVID crash and rebound, it is worth noting that from April to July and beyond, the interest rate (US 10 Year Yields) has fallen to record low levels in decades. This is due to efforts by the Feds to lower interest rates in order to re-ignite the economy, as well as more investors trying to find save investments during the current volatile times. As the result, the Sharpe Ratio and Treynor Ratio may seem a bit higher than usual since the stocks rebound (increases) faster than the decreasing interest rate.

e. Out-of-Sample Performance Evaluation

In this section, we will compare the performance of the portfolios on the testing set (July - Novemeber 2020). The first 6 daily returns of the portfolios are as follows:

```
##         Date      Tau = 0   Tau = 0.01    Tau = 0.02   Tau = 0.05    Tau = 0.1
## 1 2020-07-08  0.016497679  0.017451766  0.0184058532  0.021268115  0.026038550
## 2 2020-07-09  0.001781461  0.004323196  0.0068649304  0.014490134  0.027198807
## 3 2020-07-10  0.005731418  0.002970947  0.0002104762 -0.008070937 -0.021873292
## 4 2020-07-13 -0.010388537 -0.012348419 -0.0143083018 -0.020187949 -0.029987361
## 5 2020-07-14  0.012192483  0.012681428  0.0131703724  0.014637207  0.017081931
## 6 2020-07-15  0.012640531  0.012199179  0.0117578266  0.010433769  0.008227007
##      Tau = 0.2   Tau = 0.5           1/N
## 1  0.035579422  0.06420204  0.0009036191
## 2  0.052616154  0.12886819  0.0008805917
## 3 -0.049478002 -0.13229213 -0.0017014365
## 4 -0.049586184 -0.10838265  0.0017459324
## 5  0.021971379  0.03663972  0.0040828401
## 6  0.003813483 -0.00942709  0.0026861879
```

Here are the first 6 daily returns of S&P 500 on the testing set

```
##         Date      S&P 500
## 1 2020-07-08  0.007827462
## 2 2020-07-09 -0.005643606
## 3 2020-07-10  0.010466201
## 4 2020-07-13 -0.009362541
## 5 2020-07-14  0.013406371
## 6 2020-07-15  0.009082051
```

We can compare the performance of these portfolios using some financial metrics as follows:

```
##                                 Tau = 0   Tau = 0.01  Tau = 0.02  Tau = 0.05
## Mean Returns (Daily)       0.0010340495 0.0012359503 0.001437851 0.002043553
## Standard Deviation (Daily) 0.0097544731 0.0101859444 0.010989853 0.014911918
## Cumulative Returns (4m)    0.0643745641 0.0819861241 0.099597684 0.152432364
## Sharpe Ratio               1.8671431513 2.2037908352 2.447752570 2.795196108
## Treynor Ratio              0.9645765000 1.0473161456 1.121542530 1.317364030
## Jensen's Alpha             0.0006921282 0.0008515176 0.001010907 0.001489076
## Beta                       0.2985484069 0.3388939998 0.379239593 0.500276371
##                              Tau = 0.1   Tau = 0.2   Tau = 0.5     S&P 500
## Mean Returns (Daily)       0.003053057 0.005072065 0.011129088 0.001327403
## Standard Deviation (Daily) 0.023665867 0.043208406 0.104112743 0.011656010
## Cumulative Returns (4m)    0.240490164 0.416605764 0.944952563 0.107099821
## Sharpe Ratio               3.035670484 3.711393560 9.052814647 2.096508592
## Treynor Ratio              1.618107060 2.293676280 6.435039429 0.386381715
## Jensen's Alpha             0.002286023 0.003879918 0.008661602 0.000000000
## Beta                       0.702004335 1.105460264 2.315828049 1.000000000
##                                    1/N
## Mean Returns (Daily)       0.0008823369
## Standard Deviation (Daily) 0.0085473648
## Cumulative Returns (4m)    0.0635364072
```

```
## Sharpe Ratio                1.7745826628
## Treynor Ratio               0.4589528022
## Jensen's Alpha              0.0003043865
## Beta                        0.5225531310
```

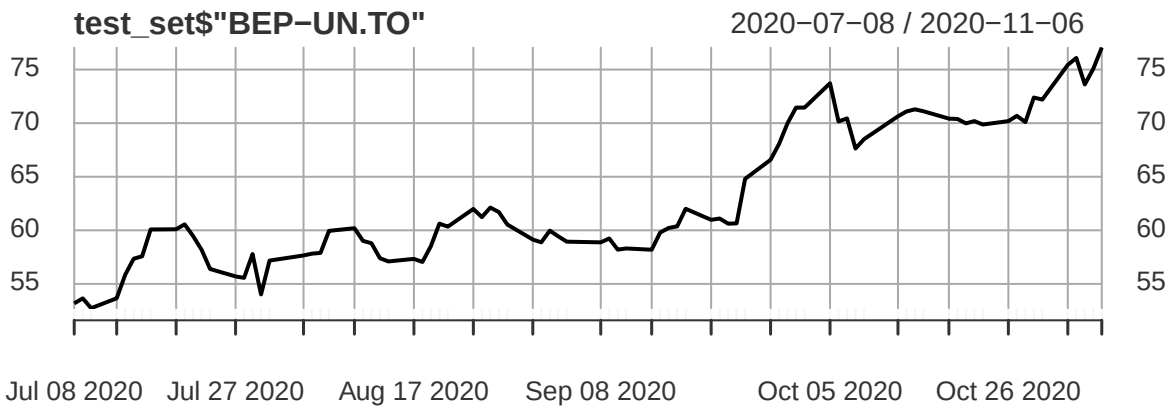
Price comparison between first day and last day of testing set

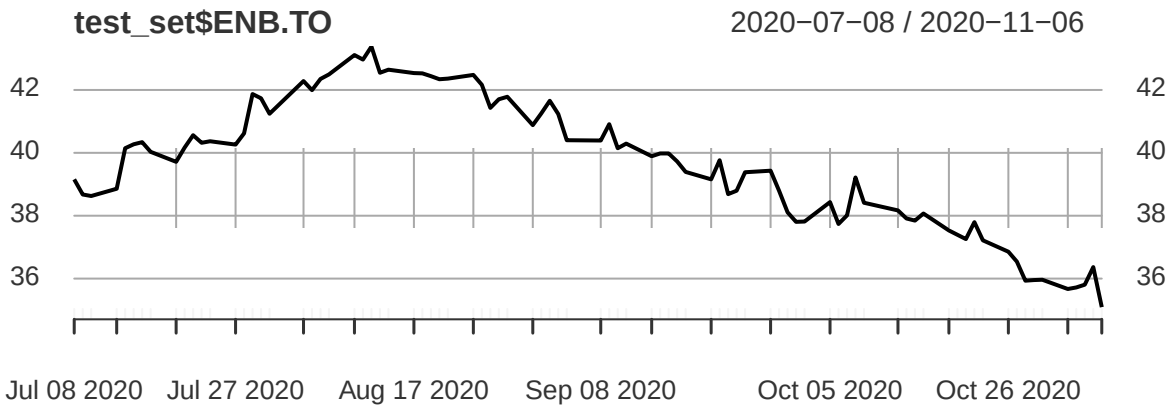
```
##          Date   RY.TO   MFC.TO SHOP.TO   CSU.TO   DOL.TO   CTC.TO   CNR.TO
## 1  2020-07-08 89.9267 17.99939 1372.88 1623.552 46.15732 213.6276 120.7948
## 85 2020-11-06 96.7400 18.83000 1361.45 1572.590 49.02000 204.6700 137.1200
##        T.TO   ENB.TO BEP-UN.TO
## 1  22.14702 39.15604   53.1831
## 85 23.32000 35.09155   77.0500
```

As seen from the above tables, our efficient portfolio can still outperform the market portfolio in the testing (out-of-sample) period. This can be seen from the returns (daily mean and cumulative), risk (standard deviation and beta), and risk-reward comparisons (Sharpe, Treynor, Jensen). In general, the optimum portfolios have maintained lower risk while reaching better return compared to the broad market index. One of the best optimum portfolios is the one with $\tau = 0.05$. With a relatively small standard deviation and beta (as compared to 1/N strategy), it is able to return 15.2% within the last 4 months. Our most aggressive strategy ($\tau = 0.5$) can yield up to 94% in expected return in just 4 months.

The 1/N strategy performed worse than the market portfolio, mainly due to the portfolio not being able to "short" or not invest in some of the stocks. This causes the high return stock to be netted down by the negative return stock that is invested with the same proportion. Another thing to note here is that our efficient portfolio return is actually very dependent on only some stocks. If we see the table that compares between the first and last day, we can see that some of the expectedly "good" stocks such as Shopify and Constellation Software went down, mainly due to the tech sell-off in September 2020. Conversely, stocks that have been shorted such as Manulife and Canadian Rail actually went up, further diminishing the return. However, the trade that really lift up the return is by leveraging on BEP and shorting on ENB. The portfolio leverages BEP by 50 to 260%, and fortunately the stock did actually went up by almost 50% in the last 4 months, mostly due to the recent US election news, where investors are expecting the future government to use renewable energy services by BEP. On the other side of the correlation, oil and gas stocks like Enbridge are predicted to face a gloomy future. Overall, while the trade has generated a huge amount of return in just 4 months, it is worth noting that it is still relatively risky an may advanced models such as machine learning and preddictive models might be needed to better understand other factors that can contribute to stock prices.

Visualized below is the sharp contrast between BEP-UN and ENB during the testing period. The optimum portfolio would be able to capitalize twice by leveraging on BEP-UN while shorting on ENB. On the other hand, 1/N strategy is forced to invest equally in both stocks.


test_set$"BEP−UN.TO"
2020−07−08 / 2020−11−06
75
70
65
60
55
Jul 08 2020
Jul 27 2020
Aug 17 2020
Sep 08 2020
Oct 05 2020
Oct 26 2020


test_set$ENB.TO
2020−07−08 / 2020−11−06
42
40
38
36
Jul 08 2020
Jul 27 2020
Aug 17 2020
Sep 08 2020
Oct 05 2020
Oct 26 2020