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HW 2

Question 5.A.1: Exercise 1.12.2, section B

$p \to (q \wedge r)$

$\neg q$

$\therefore \neg p$

1.	$p \to (q \wedge r)$	Hypothesis
2.	$\neg p \vee (q \wedge r)$	Conditional Identity
3.	$(\neg p \vee q) \wedge r$	Associative Law
4.	$(q \vee \neg p) \wedge r$	Commutative Law
5.	$q \vee (\neg p \wedge r)$	Associative Law
6.	$\neg q$	Hypothesis
7.	$\neg p \wedge r$	Disjunctive Syllogism 5, 6
8.	$\neg p$	Simplification 7

Question 5.A.1: Exercise 1.12.2, section E

$p \vee q$

$\neg p \vee r$

$\neg q$

$\therefore\ r$

1.	$p \vee q$	Hypothesis
2.	$\neg p \vee r$	Hypothesis
3.	$q \vee r$	Resolution 1, 2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive Syllogism 3, 4

Question 5.A.2: Exercise 1.12.3, section C

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore\ q \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg p \to q$	Conditional Identity
3.	$\neg p$	Hypothesis
4.	q	Modus Ponens 2, 3

Question 5.A.3: Exercise 1.12.5, section C

I will buy a new car and a new house only if I get a job.

I am not going to get a job.

∴I will not buy a new car.

j: I will get a job
c: I will buy a new car
h: I will buy a new house

The form of the argument is:

$$(c \land h) \to j$$

$$\neg j$$

$$\therefore\ \neg c$$

The argument is **NOT VALID**. When h = j =F and c = T, the hypotheses are both true, but the conclusion is false.

c	h	j	$(c \land h) \to j$	$\neg j$	$\neg c$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

Question 5.A.3: Exercise 1.12.5, section D

I will buy a new car and a new house only if I get a job.

I am not going to get a job.

I will buy a new house.

∴I will not buy a new car.

j: I will get a job
c: I will buy a new car
h: I will buy a new house

The form of the argument is:

$$(c \land h) \rightarrow j$$

$$\neg j$$

$$h$$

$$\therefore \neg c$$

The argument is **valid**.

c	h	j	$(c \land h) \rightarrow j$	$\neg j$	h	$\neg c$
T	T	T	T	F	T	F
T	T	F	F	T	T	F
T	F	T	T	F	F	F
T	F	F	T	T	F	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

Proof:

1.	$(c \land h) \to j$	Hypothesis
2.	$\neg j$	Hypothesis
3.	$\neg(c \land h)$	Modus Tollens 1, 2
4.	$\neg c \lor \neg h$	De Morgan's Law 3
5.	h	Hypothesis
6.	$\neg c$	Disjunctive Syllogism 4, 5

Question 5.B.1: Exercise 1.13.3, section B

$\exists x\,(P(x)$
$\vee\, Q(x))$

$\exists x\,\neg Q(x)$

$\therefore\ \exists x\,P(x)$

	P	Q
a	F	T
b	F	F

$\exists x\,(P(x) \vee Q(x))$ is true because $Q(x)$ is true for a. $\exists x\,\neg Q(x)$ is true since b is an example. However, since $P(a)$and $P(b)$are both false, $\exists x\,P(x)$is false.

Therefore, both hypotheses are true but the conclusion is false.

Question 5.B.2: Exercise 1.13.5, section D

Every student who missed class got a detention.

Penelope is a student in the class.

Penelope did not miss class.

Penelope did not get a detention.

D(x): x got a detention
M(x): x missed class
S(x): x is a student

$$\forall x\ ((S(x) \vee M(x)) \rightarrow D(x))$$

$$S(Penelope)$$

$$\neg M(Penelope)$$

$$\neg D(Penelope)$$

The argument is **NOT VALID**. There is no possible way for all three hypotheses to be true and for the conclusion to also be true.

D	M	S	$(S \vee M) \rightarrow D$	S	$\neg M$	$\neg D$
T	T	T	T	T	F	F
T	T	F	T	F	F	F
T	F	T	T	T	T	F
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	T	F	F	F	F	T
F	F	T	F	T	T	T
F	F	F	T	F	T	T

Question 5.B.2: Exercise 1.13.5, section E

Every student who missed class or got a detention did not get an A.

Penelope is a student in the class.

Penelope got an A.

Penelope did not get a detention.

D(x): x got a detention
M(x): x missed class
A(x): x got an A

$$\forall x\ ((M(x) \lor D(x)) \to \neg A(x))$$

$Penelope$ is a student in the class.

$$A(Penelope)$$

$$\neg D(Penelope)$$

The argument is **VALID.**

1.	$\forall x\ ((M(x) \lor D(x)) \to \neg A(x))$	Hypothesis
2.	$Penelope$ is a student in the class.	Hypothesis
3.	$(M(Penelope) \lor D(Penelope)) \to \neg A(Penelope)$	Universal Instantiation, 1, 2
4.	$A(Penelope)$	Hypothesis
5.	$\neg(M(Penelope) \lor D(Penelope))$	Modus Tollens, 3, 4
6.	$\neg M(Penelope) \land \neg D(Penelope)$	De Morgan's Law, 5
7.	$\neg D(Penelope) \land \neg M(Penelope)$	Commutative Law, 6
8.	$\neg D(Penelope)$	Simplification, 7

Question 6: Exercise 2.2.1, section C

If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

Direct proof.

If we assume $x \leq 3$, then we can rewrite it as:

$$0 \leq 3 - x$$

Factoring $12 - 7x + x^2 \geq 0$, we get:

$$(3 - x) * (4 - x) \geq 0$$

$$3 - x \geq 0$$
$$4 - x \geq 1$$

Since $3 - x$ is positive, and $4 - x$ is 1 more than $3 - x$, it is also positive. The product of two positive numbers is positive, therefore, $12 - 7x + x^2 \geq 0$.

Question 6: Exercise 2.2.1, section D

The product of two odd integers is an odd integer.

Direct proof.

Assume x and y to be two odd integers. We will prove that $x * y$ is also an odd integer.

Since x and y are odd, they can be expressed as $2k + 1$ and $2m + 1$, for some integers k and m. We can now rewrite $x * y$ as:

$(2k + 1) * (2m + 1)$=
$4km + 2k + 2m + 1$=

$$2(2km + k + m) + 1$$

$2(2km + k + m) + 1$ can be expressed as 2 times an integer, plus 1, or $2c + 1$, which is an odd number.

Therefore, $2(2km + k + m) + 1$ is odd.

Question 7: Exercise 2.3.1, section D

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

Proof by contrapositive.

We assume that n is an even integer, and show that $n^2 - 2n + 7$ is odd.

If n is an even integer, $n = 2k$, for some integer k. We can then express $n^2 - 2n + 7$ as:
$(2k)^2 - 2(2k) + 7$=

$$4k^2 - 4k + 6 + 1 =$$
$$2(2k^2 - 2k + 3) + 1$$

Since k is an integer, $2(2k^2 - 2k + 3) + 1$ is also an integer, so $2(2k^2 - 2k + 3) + 1$ can be expressed as 2 times an integer, plus 1, or $2m + 1$, which is an odd number.

Therefore, $n^2 - 2n + 7$ is odd.

Question 7: Exercise 2.3.1, section F

For every non-zero real number x, if x is irrational, then $1/x$ is also irrational.

Proof by contrapositive.

We assume that $1/x$ is rational, and show that x is rational.

Since every rational number can be expressed as a/b, we can rewrite $1/x$ as:

$$1/x = a/b =$$

$$ax = b =$$

$x = b/a$

Since $1/x \neq 0$, and $1/x = a/b$, then $a \neq 0$.

Therefore, x is rational.

Question 7: Exercise 2.3.1, section G

For every pair of real numbers, x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.

Proof by contrapositive.

We assume that $x > y$, and show that $x^3 + xy^2 > x^2y + y^3$.

We can rewrite $x^3 + xy^2 > x^2y + y^3$ as:

$$x(x^2 + y^2) > y(x^2 + y^2)$$

Since $x(x^2 + y^2) > y(x^2 + y^2)$ can be expressed as $x(c) > y(c)$, and $x > y$, x will always be greater than y for any number c.

Therefore, $x^3 + xy^2 > x^2y + y^3$.

Question 7: Exercise 2.3.1, section L

For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.

Proof by contrapositive.

We assume that $x \leq 10$ and $y \leq 10$, and show that $x + y \leq 20$.

If we assign x and y their maximum possible values we get:

$$x = 10$$
$$y = 10$$

We can now rewrite $x + y \leq 20$ as:
$10 + 10 \leq 20$=

$$20 \leq 20$$

Therefore, $x + y \leq 20$.

Question 8: Exercise 2.4.1, section C

The average of three real numbers is greater than or equal to at least one of the numbers.

Proof by contradiction.

Suppose that the average of three real numbers a, b, c is less than all three numbers.

Let v be the average:

$$(a + b + c)/3 \ = \ v$$

$v < a$ and $v < b$ and $v < c$ can be expressed as:

$$a + b + c < 3v$$

However, v is already defined as $(a + b + c)/3$, so $a + b + c = 3v$. However, now we have $3v < 3v$, which contradicts itself.

Question 8: Exercise 2.4.1, section E

There is no smallest integer.

Proof by contradiction.

Suppose that there is a smallest integer, x.
If x is an integer, then $x - 1$ is an integer as well.
However $x - 1 < x$, which contradicts that x is the smallest integer.

Question 9: Exercise 2.5.1, section C

If integers x and y have the same parity, then $x + y$ is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Proof.
We consider two cases: x and y are both even, or x and y are both odd.

Case 1:
x and y are both even.
If x and y are both even, $x = 2k$, and $y = 2k$, for some integer k.

$2k + 2k = 4k$ =

$$2(2k)$$

Since k is an integer, $2(2k)$ is also an integer. Therefore, $x + y$ is equal to 2 times an integer, which is even.

Case 2:
x and y are both odd.
If x and y are both odd, $x = 2k + 1$, and $y = 2k + 1$, for some integer k.

$(2k + 1) + (2k + 1) = 4k + 2$=

$$2(2k + 1)$$

Since k is an integer, $2(2k + 1)$ is also an integer. Therefore, $x + y$ is equal to 2 times an integer, which is even.