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HW3

Question 7 A: Exercise 3.1.1, sections A - G

$$A = \{x \in Z: x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in Z: x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

A.	$27 \in A$	**True**
B.	$27 \in B$	**False**
C.	$100 \in B$	**True**
D.	$E \subseteq C$ or $C \subseteq E$	**False**
E.	$E \subseteq A$	**True**
F.	$A \subset E$	**False**
G.	$E \in A$	**True**

Question 7 B: Exercise 3.1.2, sections A - E

$A = \{x \in Z: x \text{ is an integer multiple of } 3\}$
$B = \{x \in Z: x \text{ is a perfect square}\}$
$C = \{4, 5, 9, 10\}$
$D = \{2, 4, 11, 14\}$
$E = \{3, 6, 9\}$
$F = \{4, 6, 16\}$

A.	$15 \subset A$	**False**
B.	$\{15\} \subset A$	**True**
C.	$\emptyset \subset A$	**True**
D.	$A \subseteq A$	**True**
E.	$\emptyset \in B$	**False**

Question 7 C: Exercise 3.1.5, sections B, D

B. $\{3, 6, 9, 12, \ldots.\}$

$\{x \in N: x\ is\ a\ multiple\ of\ 3\}$; set is infinite

D. $\{0, 10, 20, 30, \ldots., 1000\}$

$\{x \in N: x\ is\ a\ multiple\ of\ 10\ and\ 0 \leq x \leq 1000\}$; the cardinality is 101

Question 7 D: Exercise 3.2.1, sections A - K

$$X = \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$$

A.	$2 \in X$	**True**
B.	$\{2\} \subseteq X$	**True**
C.	$\{2\} \in X$	**False**
D.	$3 \in X$	**False**
E.	$\{1, 2\} \in X$	**True**
F.	$\{1, 2\} \subseteq X$	**True**
G.	$\{2, 4\} \subseteq X$	**True**
H.	$\{2, 4\} \in X$	**False**
I.	$\{2, 3\} \subseteq X$	**False**
J.	$\{2, 3\} \in X$	**False**
K.	$\|X\| = 7$	**False**

Question 8: Exercise 3.2.4, section B

Let $A = \{1, 2, 3\}$. What is $\{X \in P(A): 2 \in X\}$?

$$\boxed{\{2, \{2,3\}\}}$$

Question 9 A: Exercise 3.3.1, sections C - E

$$A = \{-3, 0, 1, 4, 17\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in Z: x\ is\ odd\}$$
$$D = \{x \in Z: x\ is\ positive\}$$

C. $A \cap C$

$$\boxed{\{-3, 1, 17\}}$$

D. $A \cup (B \cap C)$

$$B \cap C = \{-5, 1\}$$
$$A \cup (B \cap C) = \{-3, 0, 1, 4, 17, -5\}$$

E. $A \cap B \cap C$

$$\boxed{\{1\}}$$

Question 9 B: Exercise 3.3.3, sections A, B, E, F

$A_i = \{i^0, i^1, i^2\}$ (Recall that for any number x, $x^0 = 1$.)
$B_i = \{x \in \mathbf{R} : -i \leq x \leq 1/i\}$
$C_i = \{x \in \mathbf{R} : -1/i \leq x \leq 1/i\}$

A.	$\bigcap_{i=2}^{5} A_i$	$A_2 = \{2^0,\ 2^1,\ 2^2\} = \{1, 2, 4\}$ $A_3 = \{3^0,\ 3^1,\ 3^2\} = \{1, 3, 9\}$ $A_4 = \{4^0,\ 4^1,\ 4^2\} = \{1, 4, 16\}$ $A_5 = \{5^0,\ 5^1,\ 5^2\} = \{1, 5, 25\}$ $A_2 \cap A_3 \cap A_4 \cap A_5 = \{1\}$
B.	$\bigcup_{i=2}^{5} A_i$	$A_2 = \{2^0,\ 2^1,\ 2^2\} = \{1, 2, 4\}$ $A_3 = \{3^0,\ 3^1,\ 3^2\} = \{1, 3, 9\}$ $A_4 = \{4^0,\ 4^1,\ 4^2\} = \{1, 4, 16\}$ $A_5 = \{5^0,\ 5^1,\ 5^2\} = \{1, 5, 25\}$ $A_2 \cup A_3 \cup A_4 \cup A_5 = \{1, 2, 4, 3, 9, 16, 5, 25\}$
E.	$\bigcap_{i=1}^{100} C_i$	
F.	$\bigcup_{i=1}^{100} C_i$	$C_1 = \{-1/1 \leq x \leq 1/1\}$ $C_2 = \{-1/2 \leq x \leq 1/2\}$ $C_3 = \{-1/3 \leq x \leq 1/3\}$ $C_4 = \{-1/4 \leq x \leq 1/4\}$ … $C_{100} = \{x \in R: -1/100 \leq x \leq 1/1\}$

Question 9 C: Exercise 3.3.4, sections B, D

Let $A = \{a, b\}$ and $B = \{b, c\}$

B. $P(A \cup B)$

$$A \cup B = \{a, b, c\}$$
$$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

D. $P(A) \cup P(B)$

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$
$$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$$
$$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$$

Question 10 A: Exercise 3.5.1, sections B, C

$$A = \{tall, grande, venti\}$$
$$B = \{foam, no - foam\}$$
$$C = \{non - fat, whole\}$$

B. Write an element from the set $B * A * C$.

$$(foam, tall, non - fat)$$

C. Write the set $B * C$ using roster notation.

$$\{(foam, non - fat), (foam, whole), (no - foam, non - fat), (no - foam, whole)\}$$

Question 10 B: Exercise 3.5.3, sections B, C, E

B. $Z^2 \subseteq R^2$

True

C. $Z^2 \cap Z^3 = \emptyset$

False

E. For any three sets, A, B, and C, if $A \subseteq B$, then $A * C \subseteq B * C$.

Ex:

$$A = \{1, 2\}$$
$$B = \{1, 2, 3\}$$
$$C = \{4\}$$

$$A * C = \{(1, 4), (2, 4)\}$$
$$B * C = \{(1, 4), (2, 4), (3, 4)\}$$

$$A * C \subseteq B * C$$

True

Question 10 C: Exercise 3.5.6, sections D, E

D. $\{xy\text{: } where\ x \in \{0\} \cup \{0\}^2\ and\ y \in \{1\} \cup \{1\}^2\}$

$$x = \{0, 00\}$$
$$y = \{1, 11\}$$

$$xy = \{01, 011, 001, 0011\}$$

E. $\{xy\text{: } x \in \{aa, ab\}\ and\ y \in \{a\} \cup \{a\}^2\}$

$$x = \{aa, ab\}$$
$$y = \{a, aa\}$$

$$xy = \{aaa, aaaa, aba, abaa\}$$

Question 10 D: Exercise 3.5.7, sections C, F, G

$$A = \{a\}$$
$$B = \{b, c\}$$
$$C = \{a, b, d\}$$

C. $(A * B) \cup (A * C)$

$$A * B = \{ab, ac\}$$
$$A * C = \{aa, ab, ad\}$$

$$(A * B) \cup (A * C) = \{ab, ac, aa, ad\}$$

F. $P(A * B)$

$$\{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$$

G. $P(A) * P(B)$. Use ordered pair notation for elements of the Cartesian product.

$$P(A) = \{\emptyset, \{a\}\}$$
$$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$P(A) * P(B) = \{\{\emptyset, \emptyset\}, \{\emptyset, \{b\}\}, \{\emptyset, \{c\}\}, \{\emptyset, \{b, c\}\}, \{\{a\}, \emptyset\}, \{\{a\}, \{b\}\}, \{\{a\}, \{c\}\}, \{\{a\}, \{b, c\}\}\}$$

Question 11 A: Exercise 3.6.2, sections B, C

B. $(B \cup A) \cap (\underline{B} \cup A) = A$

$(B \cup A) \cap (\underline{B} \cup A) = A$	
$(B \cap \underline{B}) \cup A$	Distributive Law
$A \cup (B \cap \underline{B})$	Commutative Law
$A \cup \emptyset$	Complement Law
A	Identity Law

C. $\underline{A \cap \underline{B}} = \underline{A} \cup B$

$\underline{A \cap \underline{B}} = \underline{A} \cup B$	
$\underline{A} \cup B$	De Morgan's Law

Question 11 B: Exercise 3.6.3, sections B, D

B. $A - (B \cap A) = A$

If $A = \{a, b\}$ and $B = \{b\}$ then $A - (B \cap A) = \{a\}$ which is not equal to B.

D. $(B - A) \cup A = A$

If $A = \{b\}$ and $B = \{a, b\}$, then $(B - A) \cup A = \{a, b\}$ which does not equal A.

Question 11 C: Exercise 3.6.4, sections B, C

B. $A \cap (B - A) = \emptyset$

$A \cap (B - A)$	
$A \cap (B \cap \underline{A})$	Set Subtraction Law
$A \cap (\underline{A} \cap B)$	Commutative Law
$(A \cap \underline{A}) \cap B$	Associative Law
$\emptyset \cap B$	Complement Law
$\emptyset$	Domination Law

C. $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	
$A \cup (B \cap \underline{A})$	Set Subtraction Law
$(A \cup B) \cap (A \cup \underline{A})$	Distributive Law
$(A \cup B) \cap U$	Complement Law
$A \cup B$	Identity Law