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HW6

Question 5:

Use the definition of Θ in order to show the following:

(a) $5n^3 + 2n^2 + 3n = \Theta(n^3)$

$5n^3 + 2n^2 + 3n = O(n^3)$.
Proof.

Let $c = 10$ and $n_0 = 1$. For $1 \leq n$, $n \leq n^3$, so:

$$5n^3 \leq 5n^3$$
$$2n^2 \leq 2n^3$$
$$3n \leq 3n^3$$

Adding the inequalities, the resulting expression is:

$$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3$$

Therefore, $5n^3 + 2n^2 + 3n = O(n^3)$ because $5n^3 + 2n^2 + 3n \leq 10n^3$ for $n \geq 1$.

$5n^3 + 2n^2 + 3n = \Omega(n^3)$.
Proof.

Let $c = 5$ and $n_0 = 1$. If we drop every term except for $5n^3$, the resulting expression is smaller:

$$5n^3 + 2n^2 + 3n \geq 5n^3$$

Therefore, $5n^3 + 2n^2 + 3n = \Omega(n^3)$ because $5n^3 + 2n^2 + 3n \geq 5n^3$ for $n \geq 1$.

(b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

$\sqrt{7n^2 + 2n - 8} = O(n)$.
Proof.

Let $c = 3$ and $n_0 = 1$. For $n \geq 1$, $n \leq n^2$, so:

$$7n^2 \leq 7n^2$$
$$2n \leq 2n^2$$
$$-8 \leq 0$$

Adding the inequalities, the resulting expression is:

$$7n^2 + 2n - 8 \leq 7n^2 + 2n^2$$

Since the square root function is strictly increasing, we take the square root of both sides and the resulting expression is:

$$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2} = \sqrt{9n^2} = 3n$$

Therefore, $\sqrt{7n^2 + 2n - 8} = O(n)$ because $\sqrt{7n^2 + 2n - 8} \leq 3n$ for $n \geq 1$.

$\sqrt{7n^2 + 2n - 8} = \Omega(n)$.
Proof.

Let $c = \sqrt{7}$ and $n_0 = 4$. If we drop every term except for $7n^2$, the resulting expression is smaller:

$$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2} = n\sqrt{7}$$

$\sqrt{7n^2 + 2n - 8}$ is greater than or equal to $n\sqrt{7}$ <u>if and only if</u> $2n - 8 \geq 0$. Solving for n we get:

$$2n - 8 \geq 0$$
$$n \geq 4$$

Therefore, $\sqrt{7n^2 + 2n - 8} = \Omega(n)$ because $\sqrt{7n^2 + 2n - 8} \geq n\sqrt{7}$ for $n \geq 4$.