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HW #1

Question 1:

A. *Convert the following numbers to their decimal representation:*

1. 10011011_2 =

$$2^7 + 2^4 + 2^3 + 2^1 + 2^0 =$$
$$128 + 16 + 8 + 2 + 1 =$$

$\boxed{155_{10}}$

2. 456_7 =

$(4 * 7^2) + (5 * 7^1) + (6 * 7^0) =$

$$196 + 35 + 6 =$$

$\boxed{237_{10}}$

3. $38A_{16}$ =

$$(3 * 16^2) + (8 * 16^1) + (10 * 16^0) =$$
$$768 + 128 + 10 =$$

$\boxed{906_{10}}$

4. 2214_5 =

$$(2 * 5^3) + (2 * 5^2) + (1 * 5^1) + (4 * 5^0) =$$
$$250 + 50 + 5 + 4 =$$

$\boxed{309_{10}}$

B. *Convert the following numbers to their binary representation:*

1. 69_{10} =

$$69 \div 2 = 34\ R\ 1$$
$$34 \div 2 = 17\ R\ 0$$
$$17 \div 2 = 8\ R\ 1$$
$$8 \div 2 = 4\ R\ 0$$
$$4 \div 2 = 2\ R\ 0$$
$$2 \div 2 = 1\ R\ 0$$
$$1 \div 2 = 0\ R\ 1$$

$\boxed{1000101_2}$

2. 485_{10}=

$$485 \div 2 = 242\ R1$$
$$242 \div 2 = 121\ R\ 0$$
$$121 \div 2 = 60\ R\ 1$$
$$60 \div 2 = 30\ R\ 0$$
$$30 \div 2 = 15\ R\ 0$$
$$15 \div 2 = 7\ R\ 1$$
$$7 \div 2 = 3\ R\ 1$$
$$3 \div 2 = 1\ R\ 1$$
$$1 \div 2 = 0\ R\ 1$$

$\boxed{111100101_2}$

3. $6D1A_{16}$ =

Conversion Table

$$6_{16} = 0110_2$$
$$D_{16} = 1101_2$$
$$1_{16} = 0001_2$$
$$A_{16} = 1010_2$$

$\boxed{0110\ 1101\ 0001\ 1010_2}$

C. *Convert the following numbers to their hexadecimal representation:*

1. 1101011_2 =

$$2^6 + 2^5 + 2^3 + 2^1 + 2^0 =$$
$$64 + 32 + 8 + 2 + 1 = 107_{10}$$
$$107 \div 16 = 6\ R\ 11 \rightarrow B$$
$$6 \div 16 = 0\ R\ 6$$

$\boxed{6B_{16}}$

2. 895_{10} =

$$895 \div 16 = 55\ R\ 15 \rightarrow F$$
$$55 \div 16 = 3\ R\ 7$$
$$3 \div 16 = 0\ R\ 3$$

$\boxed{37F_{16}}$

Question 2:

Solve the following, do all calculations in the given base:

1. $7566_8 + 4515_8 =$

   7566_8

   $+4515_8$

   $\boxed{14303_8}$

2. $10110011_2 + 1101_2 =$

   10110011_2

   $+ \quad 1101_2$

   $\boxed{11000000_2}$

3. $7A66_{16} + 45C5_{16} =$

   $7A66_{16}$

   $+45C5_{16}$

   $\boxed{C02B}$

4. $3022_5 - 2433_5 =$

   3022_5

   $- 2433_5$

   $\boxed{34_5}$

Question 3:

A. *Convert the following numbers to their 8-bits two's complement representation:*

1. $124_{10} =$

128	64	32	16	8	4	2	1
0	1	1	1	1	1	0	0

$\boxed{01111100_{8\ bit\ 2's\ com}}$

2. $-124_{10} =$

$$124_{10} = 01111100_2$$
$$124_{10}\ complement = 10000011_2$$
$$124_{10}\ complement + 1 =$$

$\boxed{10000100_{8\ bit\ 2's\ com}}$

3. $109_{10} =$

128	64	32	16	8	4	2	1
0	1	1	0	1	1	0	1

$\boxed{01101101_{8\ bit\ 2's\ com}}$

4. $-79_{10} =$

128	64	32	16	8	4	2	1
0	1	0	0	1	1	1	1

$$79_{10} = 01001111_2$$
$$79_{10}\ complement = 10110000_2$$
$$79_{10} complement + 1 =$$

$\boxed{10110001_{8\ bit\ 2's\ com}}$

B. *Convert the following numbers (represented as 8-bit two's complement) to their decimal representation:*

1. $00011110_{8\ bit\ 2's\ comp} =$

$$2^4 + 2^3 + 2^2 + 2^1 =$$
$$16 + 8 + 4 + 2 =$$

$\boxed{30_{10}}$

2. $11100110_{8\ bit\ 2's\ comp} =$

$$flipped = 000101101_2$$

$$\begin{array}{r} 00011001_2 \\ +\quad 1_2 \\ 00011010_2 \end{array}$$

$$2^4 + 2^3 + 2^1 =$$
$$16 + 8 + 2 = 26_{10}$$
$$26_{10}\ negated =$$

$\boxed{-26_{10}}$

3. $00101101_{8\ bit\ 2's\ comp} =$

$$2^5 + 2^3 + 2^2 + 2^0 =$$
$$32 + 8 + 4 + 1 =$$

$\boxed{45_{10}}$

4. $10011110_{8\ bit\ 2's\ comp} =$

$$flipped = 01100001_2$$

$$\begin{array}{r} 01100001_2 \\ +\quad 1_2 \\ 01100010_2 \end{array}$$

$$2^6 + 2^5 + 2^1 =$$
$$64 + 32 + 2 = 98_{10}$$
$$98_{10}\ negated =$$

$\boxed{-98_{10}}$

Question 4:

1. Exercise 1.2.4, sections b, c

b. $\neg(p \lor q)$

p	q	$\neg(p \lor q)$
T	T	F
T	F	F
F	T	F
F	F	T

c. $r \lor (p \land \neg q)$

p	q	r	$r \lor (p \land \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2. Exercise 1.3.4, sections b, d

b. $(p \to q) \to (q \to p)$

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 5:

1. Exercise 1.2.7, sections b, c

   B*: Applicant presents a birth certificate.*
   D*: Applicant presents a driver's license.*
   M*: Applicant presents a marriage license.*

   b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

   $\boxed{(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)}$

   c. Applicant must present either a birth certificate or both a driver's license and a marriage license.

   $\boxed{B \vee (D \wedge M)}$

2. Exercise 1.3.7, sections b - e

   s*: a person is a senior*
   y*: a person is at least 17 years of age*
   p*: a person is allowed to park in the school parking lot*

   b. A person can park in the school parking lot if they are a senior or at least seventeen years of age.

   $\boxed{(s \vee y) \rightarrow p}$

   c. Being 17 years of age is a necessary condition for being able to park in the school parking lot.

   $\boxed{p \rightarrow y}$

   d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

   $\boxed{p \leftrightarrow (s \wedge y)}$

   e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

   $\boxed{p \rightarrow (s \vee y)}$

3. Exercise 1.3.9, sections c, d

y: *the applicant is at least eighteen years old*
p: *the applicant has parental permission*
c: *the applicant can enroll in the course*

c. The applicant can enroll in the course only if the applicant has parental permission.

$\boxed{c \to p}$

d. Having parental permission is a necessary condition for enrolling in the course.

$\boxed{c \to p}$

Question 6:

1. Exercise 1.3.6, sections b - d

   b. Maintaining a B average is necessary for Joe to be eligible for the honors program.

   If Joe is eligible for the honors program, then he maintains a B aver

   c. Rajiv can go on the roller coaster only if he is at least four feet tall.

   If Rajiv goes on the rollercoaster, then he is at least four feet

   d. Rajiv can go on the roller coaster if he is at least four feet tall.

   If Rajiv is at least four feet tall, then he can go on the roller coa

2. Exercise 1.3.10, sections c - f

   p = *T*
   q = *F*
   r = *?*

   c. $(p \lor r) \leftrightarrow (q \land r)$

   $\boxed{False}$

   d. $(p \land r) \leftrightarrow (q \land r)$

   $\boxed{False}$

   e. $p \rightarrow (r \lor q)$

   $\boxed{Unknown}$

   f. $(p \land q) \rightarrow r$

   $\boxed{True}$

Question 7:

Exercise 1.4.5, sections b - d:

j: *Sally got the job.*

l: *Sally was late for her interview*

r: *Sally updated her resume.*

b.

If Sally did not get the job, then she was late for her interview or did not update her resume.

$$\boxed{\neg j \rightarrow (l \vee \neg r)}$$

If Sally updated her resume and was not late for her interview, then she got the job.

$$\boxed{(r \wedge \neg l) \rightarrow j}$$

j	l	r	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge \neg l) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

$$\boxed{Logically\ equivalent.}$$

c.

If Sally got the job then she was not late for her interview.

$\boxed{j \to \neg l}$

If Sally did not get the job, then she was late for her interview.

$\boxed{\neg j \to l}$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F
$NOT\ logically\ equivalent.$			

d.
If Sally updated her resume or she was not late for her interview, then she got the job.

$\boxed{(r \vee \neg l) \rightarrow j}$

If Sally got the job, then she updated her resume and was not late for her interview.

$\boxed{j \rightarrow (r \wedge \neg l)}$

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T
$NOT\ logically\ equivalent.$				

Question 8:

1. Exercise 1.5.2, sections c, f, i

   Use the laws of propositional logic to prove the following:

c. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	$Conditional\ Identity$
$\neg p \lor (q \land r)$	$Distributive\ Law$
$p \to (q \land r)$	$Conditional\ Identity$

f. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	
$\neg((p \lor \neg p) \land (p \lor q))$	$Distributive\ Law$
$\neg(T \land (p \lor q))$	$Complement\ Law$
$\neg((p \lor q) \land T)$	$Commutative\ Law$
$\neg(p \lor q)$	$Identity\ Law$
$\neg p \land \neg q$	$De\ Morgan's\ Law$

i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	$Conditional\ Identity$
$(\neg p \lor \neg q) \lor r$	$De\ Morgan's\ Law$
$r \lor (\neg p \lor \neg q)$	$Commutative\ Law$
$(r \lor \neg p) \lor \neg q$	$Associative\ Law$
$(\neg\neg r \lor \neg p) \lor \neg q$	$Double\ Negation\ Law$
$\neg(\neg r \land p) \lor \neg q$	$De\ Morgan's\ Law$
$(\neg r \land p) \to \neg q$	$Conditional\ Identity$
$(p \land \neg r) \to \neg q$	$Commutative\ Law$

2. Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

c. $\neg r \vee (\neg r \rightarrow p)$

$\neg r \vee (\neg r \rightarrow p)$	
$\neg r \vee (r \vee p)$	$Conditional\ Identity$
$(\neg r \vee r) \vee p$	$Associative\ Law$
$(r \vee \neg r) \vee p$	$Commutative\ Law$
$T \vee p$	$Complement\ Law$
$p \vee T$	$Commutative\ Law$
T	$Domination\ Law$

d. $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(p \rightarrow q) \rightarrow \neg q$	
$\neg(\neg p \vee q) \rightarrow \neg q$	$Conditional\ Identity$
$(\neg\neg p \wedge \neg q) \rightarrow \neg q$	$De\ Morgan's\ Law$
$(p \wedge \neg q) \rightarrow \neg q$	$Double\ Negation\ Law$
$\neg(p \wedge \neg q) \vee \neg q$	$Conditional\ Identity$
$\neg p \vee q \vee \neg q$	$De\ Morgan's\ Law$
$\neg p \vee T$	$Complement\ Law$
T	$Domination\ Law$

Question 9:

1. Exercise 1.6.3, sections c, d

   c. There is a number that is equal to its square.

   $\boxed{\exists x\ (x = x^2)}$

   d. Every number is less than or equal to its square.

   $\boxed{\forall x\ (x \leq x^2)}$

2. Exercise 1.7.4, sections b - d

   S(x): *x was sick yesterday*
   W(x): *x went to work yesterday*
   V(x): *x was on vacation yesterday*

   b. Everyone was well and went to work yesterday.

   $\boxed{\forall x\ (\neg S(x) \wedge W(x))}$

   c. Everyone who was sick yesterday did not go to work.

   $\boxed{\forall x\ (S(x) \rightarrow \neg W(x))}$

   d. Yesterday someone was sick and went to work.

   $\boxed{\exists x\ (S(x) \wedge W(x))}$

Question 10:

1. Exercise 1.7.9, sections c - i

	P(x)	Q(x)	R(x)
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

c. $\exists x((x = c) \to P(x))$

$\boxed{True, example: R(x)}$

d. $\exists x(Q(x) \land R(x))$

$\boxed{True, example: e}$

e. $Q(a) \land P(d)$

$\boxed{True}$

f. $\forall x\ ((x \neq b) \to Q(x))$

$\boxed{True}$

g. $\forall x\ (P(x) \lor R(x))$

$\boxed{False, counter\ example: c}$

h. $\forall x\ (R(x) \to P(x))$

$\boxed{True}$

i. $\exists x(Q(x) \lor R(x))$

$\boxed{True, example: a}$

2. Exercise 1.9.2, sections b - i

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

b. $\exists x\ \forall y\ Q(x, y)$

$\boxed{True. Example: x = 1}$

c. $\exists x\ \forall y\ P(y, x)$

$\boxed{True. Example: x = 1}$

d. $\exists x\ \exists y\ S(x, y)$

$\boxed{False}$

e. $\forall x\ \exists y\ Q(x, y)$

$\boxed{False}$

f. $\forall x\ \exists y\ P(x, y)$

$\boxed{True. Example: y = 1}$

g. $\forall x\ \forall y\ P(x, y)$

$\boxed{False. Counter\ example: P(2,2)}$

h. $\exists x\ \exists y\ Q(x, y)$

$\boxed{True. Example: Q(2,2)}$

i. $\forall x\ \forall y\ \neg S(x, y)$

$\boxed{True}$

Question 11:

1. Exercise 1.10.4, sections c - g
   Translate each of the following English statements into logical expressions. The domain of discourse is the set of all real numbers.

   c. There are two numbers whose sum is equal to their product.

   $$\boxed{\exists x\, \exists y\, ((x + y) = (x * y))}$$

   d. The ratio of every two positive numbers is also positive.

   $$\boxed{\forall x\, \forall y\, (((x > 0) \land (y > 0)) \to (x/y > 0))}$$

   e. The reciprocal of every positive number less than one is greater than one.

   $$\boxed{\forall x\, ((1 > x > 0) \to (1/x > 1))}$$

   f. There is no smallest number.

   $$\boxed{\neg\exists x\, \forall y\, (x \leq y)}$$

   g. Every number besides 0 has a multiplicative inverse.

   $$\boxed{\forall x\, \exists y\, ((x \neq 0) \to (xy = 1))}$$

2. Exercise 1.10.7, sections c - f
   P(x, y): *x knows y's phone number. (A person may or may not know their own phone number.)*
   D(x): *x missed the deadline.*
   N(x): *x is a new employee.*

   c. There is at least one new employee who missed the deadline.

   $$\boxed{\exists x\, (N(x) \land D(x))}$$

   d. Sam knows the phone number of everyone who missed the deadline.

   $$\boxed{\forall y\, (D(y) \to P(Sam, y))}$$

   e. There is a new employee who knows everyone's phone number.

$$\exists x\ \forall y\ (N(x) \land P(x,y))$$

f. Exactly one new employee missed the deadline.

$$\exists x\ ((N(x) \land D(x)) \land \forall y\ ((x \neq y) \rightarrow (\neg N(x) \land \neg D(y))))$$

3. Exercise 1.10.10, sections c - f

c. Every student has taken at least one class besides Math 101.

$$\forall x\ \exists y\ (y \neq Math\ 101 \land T(x,y))$$

d. There is a student who has taken every math class besides Math 101.

$$\exists x\ \forall y\ (y \neq Math\ 101 \rightarrow T(x,y))$$

e. Everyone besides Sam has taken at least two different math classes.

$$\forall x\ \exists y\ \exists z\ (x \neq Sam \land T(x,y) \land T(x,z) \land y \neq z)$$

f. Sam has taken exactly two math classes.

$$\exists x\ \exists y\ (T(Sam,x) \land T(Sam,y) \land x \neq y \land \forall z\ (T(Sam,z) \rightarrow (z = x \lor z = y)))$$

Question 12:

1. Exercise 1.8.2, sections b - e

   P(x): *x was given the placebo*

   D(x): *x was given the medication*

   M(x): *x had migraines*

   b. Every patient was given the medication or the placebo or both.

   $$\forall x\ (D(x) \vee P(X) \vee (D(x) \wedge P(x)))$$

   Negation: $\neg\forall x((D(x) \vee P(X)) \vee (D(x) \wedge P(x)))$

   Applying De Morgan's Law: $\exists x(\neg(D(x) \vee P(x)) \vee \neg(D(x) \wedge P(x)))$

   English:

   $There\ exists\ a\ patient\ who\ was\ either\ not\ given\ the\ medication\ or\ the\ placebo\ or\ both.$

   c. There is a patient who took the medication and had migraines.

   $$\exists x\ (D(x) \wedge M(x))$$

   Negation: $\neg\exists x\ (D(x) \wedge M(x))$

   Applying De Morgan's Law: $\forall x(\neg D(x) \vee \neg M(x))$

   English: $Every\ patient\ either\ didn't\ take\ the\ medication\ or\ didn't\ have\ migraines.$

   d. Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p → q ≡ ¬p ∨ q.)

   $$\forall x\ (P(x) \to M(x))$$

   Negation: $\neg\forall x\ (P(x) \to M(x))$

   Applying Conditional Identity: $\neg\forall x\ (\neg P(x) \vee M(x))$

   Applying De Morgan's Law: $\exists x\ (P(x) \wedge \neg M(x))$

   English: $There\ is\ a\ patient\ who\ took\ the\ placebo\ and\ did\ not\ have\ migraines.$

   e. There is a patient who had migraines and was given the placebo.

   $$\exists x\ (M(x) \wedge P(x))$$

   Negation: $\neg\exists x\ (M(x) \wedge P(x))$

   Applying De Morgan's Law: $\forall x(\neg M(x) \vee \neg P(x))$

   English: $Every\ patient\ either\ did\ not\ have\ migraines\ or\ was\ not\ given\ the\ placebo.$

2. Exercise 1.9.4, sections c - e
Write the negation of each of the following logical expressions so that all negations immediately precede predicates.

c. ∃x ∀y (P(x, y) → Q(x, y))

$$\boxed{\forall x\ \exists y\ (P(x,y) \wedge \neg Q(x,y))}$$

d. ∃x ∀y (P(x, y) ↔ P(y, x))

$$\boxed{\forall x \exists y\ ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))}$$

e. ∃x ∃y P(x, y) ∧ ∀x ∀y Q(x, y)

$$\boxed{\forall x \forall y\ \neg P(x,y) \vee \exists x \exists y\ \neg Q(x,y)}$$