CBCS SCHEME

USN

18MAT31

Third Semester B.E. Degree Examination, Aug./Sept.2020 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find L{e-21 cos 2t}.

(06 Marks)

b. Express the function in terms of unit step function and hence find Laplace transform of:

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ t & 1 < t \le 2 . \\ t^2 & t > 2 \end{cases}$$

(07 Marks)

c. Solve the equation y''(t) + 3y'(t) + 2y(t) = 0 under the condition y(0) = 1, y'(0) = 0. (07 Marks)

OR

2 a. Find:

i)
$$L^{-1}\left\{\frac{s+3}{s^2-4s+13}\right\}$$
 ii) $L^{-1}\left\{\log\frac{(s^2+1)}{s(s+1)}\right\}$

(06 Marks)

b. Find
$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$$
 using convolution theorem

(07 Marks)

c. A periodic function of period 2a is defined by

$$f(t) = \begin{cases} E & 0 \le t \le a \\ -E & a < t \le 2a \end{cases}$$

Where E is a constant and show that trim
$$L\{f(t)\} = \frac{E}{S} \tan h \left(\frac{as}{2}\right)$$
.

(07 Marks)

Module-2

3 a. Express $f(x) = x^2$ as a Fourier series in the interval $-\pi < x < \pi$. Hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \dots = \frac{\pi^2}{12} . \tag{07 Marks}$$

b. Obtain the Fourier seires expression of $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$ (07 Marks)

c. Obtain the half range cosine series for the function $f(x) = (x - 1)^2$ $0 \le x \le 1$. (06 Marks)

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OR

4 a. Obtain the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)$ $0 < x < 2\pi$. Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \frac{\pi}{4}.$$
 (07 Marks)

b. Obtain the half range cosine series of $f(x) = x \sin x$ $0 \le x \le \pi$. (07 Marks)

c. Express f(x) as a Fourier series upto first harmonic.

x	0	1	2	3	4	5	
f(x)	4	8	15	7	6	2	
		-				-	(06 Marks)

Module-3

5 a. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ (2-x) & \text{for } 1 < x < 2. \\ 0 & \text{for } x > 2 \end{cases}$$
 (07 Marks)

b. Find the Fourier transform by $f(x) = e^{-|x|}$. (07 Marks)

c. Obtain the inverse Z – transform by $u(z) = \frac{z}{(z-2)(z-3)}$. (06 Marks)

OF

6 a. Find the Fourier transform by

$$f(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

and show that $\int_{0}^{\infty} \frac{\sin^{2} t}{t^{2}} dt = \frac{\pi}{2}$ (07 Marks)

b. Find the z-transform of: i) $\cos n\theta$ ii) $\sin n\theta$. (06 Marks)

c. Solve using Z -transform $u_{n+2} - 4u_n = 0$ given that $u_0 = 0$ and $u_1 = 2$. (07 Marks)

Module-4

7 a. Using Taylor's series method solve y(x) = x + y, y(0) = 1 then find y at x = 0.1, 0.2 consider upto 4^{th} degree. (07 Marks)

b. Solve $y'(x) = 1 + \frac{y}{z}$, y(1) = 2 then find y(1.2) with n = 0.2 using modified Euler's method.

(06 Marks)

c. Solve $y'(x) = x - y^2$ and the data is y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762 then find y(0.8) by applying Milne's method and applying corrector formula twice.

(07 Marks)

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OR

8 a. Solve $y'(x) = 3x + \frac{y}{2}$, y(0) = 1 then find y(0.2) with n = 0.2 using modified Euler's method.

(06 Marks)

b. Solve $y(x) = 3e^x + 2y$, y(0) = 0 then find y(0.1) with h = 0.1 using Runge-Kutta method of fourth order.

c. Solve $y'(x) = 2e^x - y$ and data is

X	0	0.1	0.2	0.3
У	2	2.010	2.040	2.090

Then find y(0.4) by using Adam's Bash forth method

(07 Marks)

Module-5

9 a. By applying Milne's predictor and corrector method to compute y(0.4) give the differential equation $\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx}$ and the following table by initial value. (07 Marks)

X	0	0.1	0.2	0.3	
у	1	1.1103	1.2427	1.3990	
y'	1	1.2103	1.4427	1.6990	

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.

(06 Marks)

c. Find the extremal of the functional $\int_{x_1}^{x_2} (y' + x^2 y'^2) dx$.

(07 Marks)

OR

10 a. By Runge Kutta method solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct to four decimal places.

Using initial condition y(0) = 1, y'(0) = 0.

(07 Marks)

b. Prove that the shortest distance between two points in a plane is a straight line. (06 Marks)

Find the curve on which the functional $\int_{0}^{1} [y'^2 + 12xy] dx$ with y(0) = 0, y(1) = 1. (07 Marks)

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