King Saud University
College of Computer and Information Sciences
Software Engineering Department

[SWE485] Selected Topics in Software Engineering Project

Applying Search Techniques in a real-world problem

Phase 1: Problem Definition and Formulation

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Transportation problem, Also called the The Travelling Salesman problem, is a problem where you have to visit *n* cities, each city should be visited only once, where you can start with any city, but you should finish at the same city. The objective is to optimize the total distance.

For this phase, we are defining the problem mathematically as a Constraint Satisfaction Problem by defining the following:

1. Variables and Domains

Assume that we are given a complete graph $G(N, E)$ where N is the set of vertices (or cities) {1,2,...,n} and E is the set of edges (or roads) {1,2,..,e}. For each pair of vertices $i, j \in N$, $i \neq j$ the edge $(i, j) \in E$ is associated with a weight (or distance) $d_{ij} \in R +$

To model the problem, we introduce a binary variable, x_{ij} that indicates if edge (i, j) is part of the tour or not.

$$x_{ij} \in \{0, 1\}, \qquad \forall i, j \in N$$

Where 0 indicates no route from city i to city j, and 1 indicates the presence of a route from city i to city j.

2. Constraints

1. Starting and ending should be in the same city.

2. Every city must be visited exactly once

Mathematically:

$$\sum_{j=1}^{n} x_{ij} = 1 \ \forall i$$ (meaning that from each city i we can go to only one city j)

$$\sum_{i=1}^{n} x_{ij} = 1 \ \forall j$$ (meaning that for each city j we came from only one city i)

These constraints ensure that there is one and only one incoming and one and only one outgoing edge at each vertex, which ensures that we have visited each city exactly once.

Additionally, to ensure that we have an only one entirely connected tour where the start city is the same as the end city, we should eliminate subtours by imposing the following constraint:

$$\sum_{i \in S,\, j \in S,\, i \neq j}^{n} x_{ij} \ \leq \ |S| - 1 \ , \ \forall \, S \subsetneq N \, , \ |S| \geq 2$$

So there is no subset of vertices that could form a subtour which is not connected to the rest of the vertices.[1]

3. Objective Function

The objective function for the transportation problem is to minimize the total distance traveled while visiting all cities exactly once. The total distance traveled is determined by summing the distances of all edges that are part of the tour.

Let's denote d_{ij} as the distance between city i and city j. The objective function can be defined as follows:

Objective Function:

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \cdot x_{ij}$$

In this objective function:

- d_{ij} represents the distance between city i and city j.
- x_{ij} is a binary variable indicating whether there is a route from city i to city j, x_{ij}=1 if there is a route, x_{ij}=0 otherwise.
- The double summation iterates over all pairs of cities, considering their distances and whether there's a route between them.
- The product $d_{ij} \cdot x_{ij}$ computes the distance between cities i and j only if there is a route between them.
- Summing up all such products gives the total distance traveled while considering only the routes that are part of the tour.

Therefore, this objective function guides the optimization process to find the set of routes that minimize the total distance traveled while satisfying the given constraints.

References

[1] “Traveling salesperson problem: Dantzig-Fulkerson-Johnson formulation,” YouTube, https://youtu.be/5IbbUkmHidE?si=8tYmCUf2MpLe5vRK (accessed Mar. 5, 2024).