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Assignment NO (2)

· Question NO 1 :-

Part "a"

$$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$$\lim_{(x,y)\to(2,1)} \frac{(2)^2-2(2)(1)}{(2)^2-4(1)^2}$$

$$= \frac{4-4}{4-4} = \frac{0}{0}$$

$$\therefore x^2 = 4y^2$$

Taking square root on Both side

$$x = 4y \rightarrow y = \frac{x}{4}$$

$$\frac{x^2 - 16 - 2(x)(x/4)}{x^2 - 4(x^2/16)}$$

$$\frac{x^2 - x^2}{16 \quad 2} = \frac{(x^2 - 8x^2)4}{4\ 16(4x^2 - x^2)}$$

$$\frac{x^2 - x^2}{4}$$

$$= \frac{-7x^2}{4(3x^2)} \Rightarrow \frac{-7x^2}{12x^2} \Rightarrow \frac{-7}{12}$$

Part "b"

$$\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x} = \frac{0}{0}$$

$$6y = 7x$$

$$y = \frac{7x}{6}$$

$= x - \frac{4}{7x/6}$

$7x + 7x$

$= x - \frac{28x}{6}$

$14x$

$= 6x - 28x \qquad = -22x$

$6(14x) \qquad 6(14x)$

$= -11$

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Part "c"

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^6}{xy^3}$$

Let $y = mx$

$$\frac{x^2 - m^6 x^6}{x m^3 x^3}$$

$$\frac{x^2(1 - m^6 x^4)}{x^4 m^3}$$

$$\frac{1 - m^6 x^4}{x^2 m^3}$$

Part "d"

$$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 7e^2 y}{6x + 2y - 3z}$$

$$\Rightarrow \frac{(-1)^3 - 4(0)e^2}{6(-1) + 2(0) - 3(4)}$$

$$\Rightarrow \frac{-1 - 0}{-6 + 2 - 12}$$

$$\Rightarrow \frac{1}{16} \qquad \Rightarrow \frac{1}{16}$$

$\frac{1}{16}$

Question NO "2"

Part "a"

$$f(x,y) = \cos\left(\frac{x}{y}\right) \quad \text{in} \quad v = (3,-4)$$

$$\nabla f = \frac{d}{dx}\left(\cos\frac{x}{y}\right)i + \frac{d}{dy}\left(\cos\frac{x}{y}\right)j$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)i + x\left(-\sin\frac{x}{y}\right)\left(\frac{-1}{y^2}\right)j$$

$$= \frac{-1}{y}\sin\left(\frac{x}{y}\right)i + \frac{x}{y}\sin\left(\frac{x}{y}\right)j$$

$$= \frac{3i-4j}{\sqrt{9+16}} \Rightarrow \frac{3}{\sqrt{25}}i - \frac{4}{\sqrt{25}}j$$

$$\text{Diff} = \frac{-3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y}\sin\left(\frac{x}{y}\right)$$

$$\text{Diff} = \frac{1}{5y}\sin\left(\frac{x}{y}\right)\left(\frac{4x}{5y} - 3\right)$$

Part "b"

$$f(x,y,z) = x^2y^3 - 4xz \qquad \bar{v} = (-1,2,0)$$

$$\nabla f = (2y^3x\,i + (-4z)i) + 3y^2x^2j - 4x\,k$$

$$\hat{v} = \frac{-i+2j+0k}{\sqrt{1+4}}$$

$$\hat{v} = \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}} + 0k$$

$$\text{Diff} = \frac{-1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^2) + 0$$

Question NO "3"

$f(x,y,z) = 4xy^2 e^{3xz}$

$\nabla f = 4 - y^2 e^{3x2}(3z)i - 2ye^{3x0}j + y^2 e^{3xz}\ 3xk$

$= (4-0)i - 2j - 9k$

$v = (-1, 4, 2)$

$\hat{v} = \frac{-1i + 4j + 2k}{\sqrt{1+16+9}} \rightarrow \frac{-1}{\sqrt{21}}i + \frac{+4}{\sqrt{21}}j + \frac{+2k}{\sqrt{21}}$

$\frac{-1(4)}{\sqrt{21}} \quad \frac{-2(4)}{\sqrt{21}} \quad \frac{-9(2)}{\sqrt{21}}$

$\Rightarrow \frac{-4}{\sqrt{21}} \quad \frac{-8}{\sqrt{21}} \quad \frac{-18}{\sqrt{21}}$

$\Rightarrow \frac{-4-8-18}{\sqrt{21}}$

$\Rightarrow \frac{-32}{\sqrt{21}}$

Question NO 4

$f(x,y) = \sqrt{x^2+y^3}$ at $(-2,3)$

$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)j$

$= \frac{-2}{\sqrt{13}}i + \frac{27}{2\sqrt{13}}j$

Part "b"

$f(x,y,z) = e^{2x}\cos(y-2x)$ at $\vec{a} = (4,-2,0)$

$\nabla f = (e^{2x}\cdot 2\cos(y-2x))i + e^{2x}(-\sin(y-2x)(1))j + e^{2x}(-\sin(y-2x)(-2))k$

$\nabla f(4,-2,0) = e^{4x}\cdot 2\cos(-2,0)i + e^{4x}(-\sin(-2)j + e^{4x}(-\sin(-2)(-2)$

$\nabla f = e^{4x}(-2\cos 2i - \sin(-2)j + 2\sin(-2)k$

Question NO "5"

Part "A"

$$F = x^2y\,i - (z^3 - 3x)j + 4y^2k$$

$$\text{Div } f = \nabla f \cdot f$$
$$= (2xy\,i)(x^2y\,i - (z^3 - 3x)j + 4y^2k$$

$$\text{curve} = \nabla f \times f$$
$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & -(z^3-3) & 4y^2 \end{vmatrix}$$

$$(8y + 3z^2)i - 0j + (2x^2)k$$

Part "b"

$$F = (2x + 2z^2)i + \frac{x^3}{2}y^2 j - (27x)k$$

$$\text{Div} \cdot \nabla f = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right)\left(2x + 2z^2 + \frac{x^3}{2}y^2 i - (27x)k\right)$$
$$= 2 + \frac{2x^3y}{2} - 1$$

$$\text{curve} = \nabla f \times f$$
$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 3x + 2z^2 & x^3y^2 - (2 - 7x^2) & z \end{vmatrix}$$

$$\frac{x^3y^2}{z^2}\,i - x - (7 - 4z)j + \left(3x^2y^2 + \frac{}{z}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2) = \cos x^2 \cdot 2x$$
$$= 2x\cos^2$$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= (4x^2y^3 - z)(2x\cos x^2)$$

Question NO "6"

$$F = x^2y\,i - \left(4x^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)j + \left(1 - \frac{2x^3y}{z^3}\right)$$

$$\frac{dM}{dy} = 2N \quad , \frac{dN}{dz} = \frac{dP}{dy} \quad , \frac{dM}{dz} = \frac{dP}{dx}$$

$$F = \left(4y^2 + \frac{3x^2y}{z^2}\,i\right) + \left(8xy + \frac{x^3}{z^3}\right)j + \left(11 - \frac{2x^3y}{z^3}\right)k \cdot P$$

$$\frac{dM}{dy} = 8y + \frac{3x^2}{z} \quad , \frac{dN}{dx} = 8y - \frac{3x^2}{z^2} \quad , \frac{dN}{dz} = x^3z^{-2} = x^3(-2)z^{-3}$$

$$= -\frac{2x^3}{z^3}$$

$$= -\frac{2x3}{z^3}$$

$$\frac{dM}{dz} = \frac{4x^3 + 3x^2y}{z^2}$$

$$= 3x^2y(-2)z$$

$$= -\frac{6x^2y}{z^3}$$

$$\frac{dP}{dx} = \frac{d}{dx}\left(11 - \frac{2x^3y}{z^3}\right)$$

$$= -\frac{6x^2y}{z^3}$$

$$\vec{F} = 6x\hat{i} + (2x - y^2)\hat{j} + (6z - x^3)k\hat{}$$

$\frac{dy}{dz}$ $\frac{dy}{dz}$

$\frac{dM}{dz} = \frac{d}{dz} 6x = 0$

$\frac{dP}{dx} = \frac{d}{dx}(6z - x^3) = -3x^2$

$\frac{dM}{dy} \neq \frac{dN}{dx}$, $\frac{dN}{dz} = \frac{dP}{dy}$; $\frac{dM}{dz} \neq \frac{dP}{dx}$

NOT conservative

Question NO 7

Part " B "

$z = x^2y^4 - 2y$, $y = \sin(x^2)$

$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$

$\frac{dz}{dy} = \frac{d}{dy}(x^2y^4 - 2y)$

$= (4x^3y^3 - 2)$

$\frac{dy}{dx} = \frac{d}{dx} \sin(x^2)$

$= \cos^2 x \, 2x$

$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx}$

$$= (4x^2y^3 \cdot 2)(2x\cos^2 x)$$

$$= 8x^3y^3\cos x^2 - 4x\cos x^2$$

Part "C"

$$x^2y^4 - 3 = \sin(xy)$$

$$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}(\sin(xy))$$

$$2xy^4 + x^2 4y^3 \frac{dy}{dx} = y\cos(xy)$$

$$4x^2y^3 \frac{dy}{dx} = y\cos(xy) - 2xy^4$$

$$\frac{dy}{dx} = y\left(\frac{\cos(xy) - 2xy^3}{4x^2y}\right)$$

$$\frac{dy}{dx} = \left(\frac{\cos(xy) - 2xy^3}{4x^2y}\right)$$