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Multi - variable Calculus

Assignment NO 1

• Question NO "3"

Given vector

$\vec{u_1} = \vec{r} = (t_2 + y) + \lambda(-2\vec{z} - \vec{y})$

$u_2 = (\bar{y} + t\bar{k}) + u(-2\bar{j} + \vec{k})$

Shortest distance between the time t_1 and t_2

$\vec{a_1} = t_2 + j$ $\qquad$ $b_1 = -2j - j$

$a_2 = j + t\vec{k}$ $\qquad$ $b_2 = -2j + \vec{k}$

$\vec{a_2} - \vec{a_1} = j(0) + t\bar{k} - t\vec{j}$

$= -\vec{t_2} + t\vec{k}$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$\rightarrow i(-1 - 0) - j(-2-0) + k(4-0)$

$= i + 2j + 4k$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{1+4+16} = \sqrt{21}$

• Shortest Distance b/w points

$$d = \left| \frac{(b_1 \times b_2) \cdot (a_2 - a_2)}{b_1 \times b_2} \right|$$

$$= \frac{(i + 2j + 4k) \cdot (-ti + tk)}{\sqrt{21}}$$

$$= \frac{-t + 4t}{\sqrt{21}}$$

Given

$$\left|\frac{3t}{\sqrt{21}}\right| = \sqrt{21}$$

$$3t = 21$$

$$t = 7$$

Question NO 12

Part "a"

$$AB \times CD = (\lambda - 2)i + (4\lambda - 12)j - 4k$$

$$(-5i + 2j + 4k) \cdot [(\lambda-2)i + (4\lambda - 12)j - 4k] = 3\lambda - 30$$

$$\frac{|3\lambda - 30|}{\sqrt{(\lambda-2)^2 + (4\lambda-12)^2 + 16}} = 3$$

$$\lambda^2 - 5\lambda + 4 = 0 \qquad (A \cdot G)$$

Part "b"

$$\lambda = 1, \quad n_1 = 5i + 13j - 7k$$

$$\lambda = 4, \quad n_2 = 8i + 25j - 7k$$

For line 2 { 3 $= 10 \dfrac{-5\lambda - 24 + 8\lambda - 16}{\sqrt{(2-\lambda)^2 + (12-4\lambda)^2 + 4^2}}$

$= 3$

$(\lambda - 10)^2 = 164 - 100\lambda + 17\lambda^2$

$= 17\lambda^2 - 100\lambda + 164$

Question NO "5"

Part $\left(\dfrac{x}{25}\right)^2 + \left(\dfrac{y}{16}\right)^2 = 1$

$a = 25$ $\qquad$ $b = 16$

$a^2 = (5)^2$ $\qquad$ $b = (4)^2$

$a > b$

$\dfrac{x^2}{a^2} + \dfrac{y}{b^2} = 0$

Major $= v(\pm a, 0)\ (\pm 5, 0)$

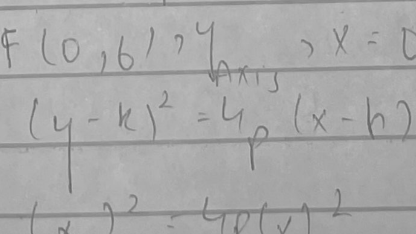
Minor $= (0, \pm b)\ (0, \pm 4)$

$F = (\pm c, 0) = (\pm 3, 0)$

$c = a^2 - b^2$

$c^2 = 9$, $c = 3$

Part "b"



$x^2 = 24y$

$p = 6$

$V(0, 0)$

$F(0, 6)$, y_{Axis}, $x = 0$

$(y-k)^2 = 4_p (x-h)^2$

$(x)^2 = 4p(y)^2$

$y^2 = 100x$

$4P_x = 100x$

$P = \frac{100}{4}$

$P = 25$

$F = (25, 0)$

Axis of Parabole x Ais $y = 0$

D.E $= (x = -25)$

Part "c"

Major = 10

Minor = 8

$C = 0$

$c^2 = a^2 - b^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$

$a > b$

$a^2 = 10$ $b^2 = 8$

$a = 3.21$ $b = 2.21$

$a > b$

$V = (\pm 3.21, 0)$

$V = (0, \pm 2.20)$

$F = (\pm C, 0)$

$F = (\pm 0.62)$

Part "d"

$x^2 + y^2 = r^2$

$x^2 + (y - b)^2 = r^2$

$(4)^2 + (-b)^2 = r^2$

And

$(0)^2 + (2 - b)^2 = r^2$

comparing

$(b - 2)^2 = b^2 + 4^2$

$(-2)(2b - 2) = 16$

$b = -3$

So

$r^2 = 4^2 + 3^2$

$r^2 = 25$

$r = 5$

Question NO 09:-

The Points A, B, C from Position vectors form of $ax+by+cz=0$

· Solution:-

$A(4,-4,1)$
$B(-4,3,-4)$
$C(4,-1,-2)$

$AB = OB - OA$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix} = -8i + 7j - 5k$$

$AC = OC - OA$

$$= \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} = 3j - 3k$$

$$AB \times AC = \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

By Taking Deter
$= i + 4j + 4k$

$d = a \cdot n = (4i - 4j + k) \cdot (i + 4j + 4k)$
$= 4 - 16 + 4$
$= -8$

if plane,
$r \cdot n = d$

$r \cdot n = d$

$(xi + yj + zk)(i + 4j + 4k) = -8$

$x + 4y + 4z = -8$

$x + 4y + 4z + 8 = 0$

Part "b"

Find Perpendicular:-

$$\text{Perp dist} = \frac{d}{|n|} = \frac{8}{\sqrt{1^2 + 4^2 + 4^2}}$$

$$= \frac{8}{33}$$

$$= 1.39$$

Part "c"

line OD : $r = a + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -3 \end{pmatrix}$$

Same value of λ

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$2\lambda + 12\lambda - 12\lambda = -8$

$2\lambda = -8$

$\lambda = -4$

$$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix}$$

$= (-8, -12, 12)$