Linear Programming Formulation of Pricing Problem

Operations Research

To

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Paper Contents

Part I:

1. A comprehensive Description:

Pricing problems play an important role in opertional and strategic descisions across various industries. Where the aim is to determine optimal prices for goods and services to maximize objectives as profit and revenue. These problems critical in many life sectors like energy (Car fuel) , food and clothes. Where prices, market changes and customer satisfaction must be carefully balanced. There are many methods and fields ranging from classic to complex. The linear programming method is one of these methods. To address these complexities [1], businesses employ a variety of pricing methodologies, ranging from traditional cost-based models to advanced computational techniques. Among these, Linear Programming (LP) stands out as a powerful optimization tool due to its ability to systematically evaluate trade-offs between objectives and constraints.

Types of pricing problems:

Pricing problems can be classified into several categories:

1. **Dynamic Pricing:** Involves changing prices over time in response to changing in market conditions, such as demand variations. Its widely used in airlines and e-commerce.
2. **Real-time pricing:** It is common in electric sectors like energy. Prices change frequently often hourly based on forecasted conditions.
3. **Strategic Pricing:** Businesses must match pricing strategies with more general marketing and product life cycle concerns when developing new products or positioning their brands.

Importance of Optimization in Pricing:

Linear programming (LP) is a common fundamental tool for resolving pricing issues which allows decision-makers to manage complex trade-offs between goals and limitations due to its logical methodology. When resources are few and price decisions must take production constraints into account, LP is particularly well-suited.

Common Objectives and Constraints:

Objectives:

1. **Profit maximization** is the most straightforward and widely took goal, requiring careful testing of costs, demand, and competition.
2. **Revenue maximization** is often prioritized in this term or public service contexts where profit is secondary.

<u>Constraints:</u>

1. **Demand functions:** Are frequently modelled as either elastic or inelastic, must account for how responsive consumers are to price changes.
2. **Production capacity:** There are strict restrictions on how much can be sold

Important Highlights from the papers:

<u>Paper One: Prices and Incomes in Linear Programming Models:</u>

This study offers way to include competitive and noncompetitive market structures into linear programming models. It covers the analysis of producer income at natural prices, nonlinear demand approximations, and relationships in product demand. The method works well with policy-oriented [1] planning models where practical limitations demand accuracy and effectiveness in calculation.

<u>Paper Two: Pricing Strategy Selection Using Linear Programming:</u>

The study examines and selects the best pricing techniques for marketing strategy using linear programming. The approach takes into consideration subjective and qualitative aspects of price, which makes it ideal for planning and new product pricing. It employs model to determine the approach that is closest to the optimal answer.

<u>Paper Three: A Linear Programming Model for Real-Time Pricing of Electric Power Service:</u>

This study uses a linear programming technique to create a real-time pricing model that balances supply and demand in the electrical industry. The concept reduces all system costs. It shows how real-time pricing may be calculated to achieve both operational efficiency and financial stability by integrating capacity planning [3], customer subscription behaviors, and system constraints. Smith offers an LP model that balances supply costs and consumer restriction costs for real-time pricing (RTP) in electricity markets. The approach takes consumer feedback and utility capacity commitments into account while addressing both short-term operational decisions and long-term planning. A greedy method approximates solutions to guarantee incentive-compatible prices. Through an example, the paper illustrates the model's efficacy and how RTP can minimize peak demand and maximize resource use. It concludes that expanding RTP can improve efficiency and postpone capacity improvements, which will benefit both customers and utilities.

2. Three different linear programming formulas:

Paper One: Real-Time Pricing of Electric Power

1. Paper Overview:

To determine the best real-time power prices, the article creates a linear programming (LP) model. The objective is to respond to changing daily supply-demand conditions by reducing the combined cost of energy production and customer limitation.

2. Desicion Variables

Cost coefficients are represented by c or C, while other choice factors are represented by either x or X variables. Other constants are written in Roman characters. The shadow prices derived from the dual optimization problem will be represented by Greek letters. The variables and coefficients for the utility are defined as follows [1]:

1. $X_{ik}(t)$ = the energy supplied from unit i on day type k during time period t.
2. $C_i(t) =$ the marginal cost per k of energy sup plied from unit i during time period t.
3. $X_{ik} =$ the fraction of supply type i that is commit ted on day type k
4. $C_{ik} =$ the cost of committing supply type i for day type k.
5. x_i = the fraction of supply type i that is commit ted for the contract period.
6. C_i = the fixed cost of committing the maximum possible amount of supply type (i) for the contract period
7. $G_{ik}(t)$ =the maximum available energy from supply source (i) for time (t) on day type k.
8. S_i = the maximum available energy from supply type i for the contract period.

3. Constraints

1. Demand Satisfaction for Each Time Period and Scenario:

$$\sum_j \boldsymbol{X_{ik}} + \boldsymbol{x_{ik}(t)} \geq \boldsymbol{L_k(t)} \qquad \forall \boldsymbol{K, t}$$

Ensures total energy generation meets or exceeds demand at every time (t) on day type (k).

2. Peak hour demand limit:

$$\boldsymbol{X_{ik}(t)} \leq \boldsymbol{G_{ik}(t).X_{ik}} \qquad \forall \boldsymbol{I, k, t}$$

The amount of energy dispatched from a generator must not exceed the committed portion of its capacity.

3. Off-peak demand limit:

$$\boldsymbol{X_{ik}} < \boldsymbol{x_i} \qquad \forall\ \boldsymbol{i, k}$$

You can only commit to a day if the generator is committed for the contract, overall commitment cannot exceed full capacity.

4. Minimum off-peak demand:

$$\sum_{k.t} \boldsymbol{n_{ik} X_{ik}(t)} \geq \boldsymbol{S_i} \qquad \forall \boldsymbol{i}$$

For energy-limited sources (like hydro), total supply over all days and time periods must not exceed total availability.

5. Minimum off-peak demand:

$$\sum_{k.t} n_{ik} X_{ik}(t) \geq S_i \qquad \forall i$$

6. Non-negativity:

$$X_{ik}(t), X_{ik}, x \geq 0 \qquad \forall\, i, k, t$$

No negative production or commitments allowed.

4. Objective Functions

Minimize **total expected system cost** (supply + capacity commitments):

$$\min \sum_{i,k,t} n_k\, C_i(t) x_{ik}(t) + \sum_{i,k} n_k C_{ik} X_{ik} + \sum_{i} C_{ixi}$$

Where:

- n_k: number of days of scenario type k
- $C_i(t)$: marginal cost for unit i at time t
- C_{jk}: daily commitment cost of unit i
- C_i: contract-level fixed commitment cost

Paper Two: Pricing Strategy Selection Using Fuzzy Linear Programming:

1. Paper Overview:

This model addresses the problem of selecting the most appropriate pricing strategy for a new product by incorporating expert preferences using fuzzy linear programming with intuitionistic fuzzy sets (IFS).

2. Decision Variables:

1. w_j: Weight of attribute j (to be determined).
2. μ_{ij}, v_{ij}: Degree of membership and non-membership for alternative iii on attribute j.
3. $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$: Indeterminacy for alternative iii, attribute j.

3. Constraints

1. Weighted Euclidean distance from ideal solution:

$$\mathrm{Di} = \sum_{\mathrm{j}} \mathrm{wj}\left[\left(\mu_{\mathrm{ij}} - \mu_{\mathrm{j}} *\right)^2 + (\mathrm{\nu ij} - \mathrm{\nu j} *)^2 + (\mathrm{\pi ij} - \mathrm{\pi j} *)^2\right]$$

2. Preference consistency condition:

$$\mathrm{Di} \ < \ \mathrm{Dj} \quad \text{if DM prefers i} > \mathrm{j}$$

3. Weight normalization and bounds:

$$\sum_{j} \mathrm{w_j} = 1, \mathrm{w_j} \geq 0$$

4. Weight consistency condition:

$$\sum_{\boldsymbol{k.t}} \boldsymbol{n_{ik} D_{ik}(t)} \leq \ \boldsymbol{w_i} \qquad \forall \boldsymbol{i}$$

4. Objective Functions

Maximize total consistency in pairwise preference rankings:

$$\max \sum_{(\mathrm{i,j}) \in \Omega} \mathrm{R_{ij}}$$

Where:

- R_{ij}: consistency index between alternatives i and j.
- Ω: set of preference pairs provided by the decision maker.

Paper Three: Prices and Incomes in Linear Programming Models:

1. Paper Overview:

This paper formulates an LP model that incorporates demand interdependencies and noncompetitive market structures into agricultural policy planning. It models pricing and producer income at endogenous prices within a market-equilibrium LP framework.

2. Desciion Variables

1. q: Quantity vector of products produced/sold
2. p: Endogenous price vector (implicitly determined)
3. y_j: Production output of activity j
4. Z_{ij}: Sales activity levels per segment in piecewise-linear demand approximation

Constraints

1. Demand approximation using segments:

$$\sum zij \leq \text{max quantity in segments for product i}$$

2. Producer income constraint (optional):

$$\sum_{i,j} r_{ij} x_{ij} \geq Y^*$$

Where:

- r_{ij}: revenue from selling quantity Z_{ij}
- Y*: minimum acceptable income

3. Resource constraints:

$$\sum_j a_{kj} y_i \leq b_k \qquad \forall\, resource\ k$$

4. Linking production to sales:

$$\sum_j y_{ij} = \sum_j z_{ij}$$

5. Non-negativity:

$$y_i, z_{ij} \geq 0$$

Objective Functions

$$\max Z = q' (a + 0.5\,Bq) - c(q)$$

Where:

- a: base demand intercept
- B: matrix of demand elasticities
- c(q): cost function

In linearized LP form, maximizing total welfare becomes:

$$\max \sum_{i,j} w_{ij} z_{ij}$$

Where:

- Z_{ij}: quantity sold in segment j of product i
- w_{ij}: area under the inverse demand curve for segment j

Comparing:

Three selected papers addressing pricing strategies using linear programming methods were compared in order to evaluate and determine the best model for sensitivity analysis, Simplex and dual formulation. The comparison highlights a number of important factors, such as the strengths, weaknesses, similarities, and differences of each model. We can determine which formulation provides the greatest clarity, use, and analytical complexity for additional optimization. by examining the ways in which each paper addresses pricing challenges, ranging from qualitative marketing choices to real-time operational pricing. The following table provides a summary of each model's unique features, which can be used as a basis for choosing the most suitable paper for study.

	Paper One	**Paper Two**	**Paper Three**
Strengths	1. Captures real-world operational complexity 2. Models time-dependent supply and demand	1. Ideal for qualitative marketing strategy decisions.	1. Can model competitive and non-competitive markets

Weekness	1. Assumes full knowledge of cost and data	2. Depends heavily on decision-maker input	1. Not suited for real-time or short-term decisions
Similarities	1. Uses LP like the others. 2. Focuses on optimizing pricing strategies	1. Optimization-driven pricing. 2. Uses LP after fuzzy transformation.	1. LP-based pricing optimization.
Differences	1. Emphasizes real-time operational pricing. 2. Time-structured with layered decisions.	1. Deals with qualitative and subjective factors rather than operational ones.	1. Focused on policy impact and income distribution rather than operations.
Variables	1. Prices (p) 2. Quantities (q), Income (Y) 3. CS 4. PS Marginal Costs(c'(q)) 5. Dual Variables (λ)	1. Membership (μ) 2. Non-Membership (ν) 3. Hesitation (π) 4. Weights (ω) 5. Distance (d) 6. Consistency (G, B)	1. Energy (x_{ik}(t)) 2. Commitment(X_{ik}) 3. Prices (σ_k(t)) 4. Costs (c_i(t) 5. C_{ik} 6. C_ 7. Dual Variables (λ, μ)
Variables Number	**5**	**5**	**8**
Objective Function:	Maximizes net social payoff (consumer surplus + producer surplus).	Maximizes consistency in decision-making.	Minimizes total system cost (supply costs + customer curtailment costs).
Constraints No:	**6**	**4**	**6**
Constraints Type:	Focuses on competitive markets.	Focuses on multi-attribute group decision-making	Focuses on electric power pricing with real-time adjustments.

Following this comparison, I choose to use the 1993 work "A Linear Programming Model for Real-Time Pricing of Electric Power" by Stephen A. Smith for a sensitivity analysis, dual formulation, and Simplex. This work presents a comprehensive and well-structured linear programming model that is both operational and numerical in nature and addresses the real-time pricing of energy as mentioned below. Additionally, the model is very useful for real-world energy systems where supply, demand, and capacity need to be dynamically balanced.

Table I
Inputs for the Example

	Available Supply $G_i(t)$						Supply Costs	
	t						Variable	Fixed
Type i	1	2	3	4	5	6	c_i	C_i
1	180	120	90	70	50	40	0.01	100
2	100	95	80	75	65	60	0.022	100
3	70	60	50	50	60	70	0.035	100
4	95	90	105	90	80	70	0.31	100
5	200	200	200	200	200	200	0.53	100
	Curtailable Demand $G_i(t)$						Customer Costs	
	t						Variable	Fixed
i	1	2	3	4	5	6	c_i	C_i
11	10	12	14	16	18	20	0.06	15
12	20	18	16	14	12	10	0.1	15
13	30	26	25	30	20	20	0.16	15
14	15	18	21	21	18	15	0.24	15
15	10	20	30	30	15	10	0.36	15
16	10	20	30	30	21	15	0.55	15
17	50	150	400	350	200	50	2.0	15

Table II
Solution of Example Problem With Integer X_i

$\sigma(t)$ $[t = 1, \ldots, 6] = 0.01\ \ 0.035\ \ 1.03\ \ 0.60\ \ 0.31\ \ 0.035$
X_i $[i = 1, \ldots, 5] = 1\ 1\ 1\ 1\ 1\ 1$
X_i $[i = 11, \ldots, 17] = 1\ 1\ 1\ 1\ 1\ 1\ 0$

	Curtailed Demand $x_i(t)$							Supply Output $x_i(t)$					
	t							t					
i	1	2	3	4	5	6	i	1	2	3	4	5	6
11	0	0	14	16	18	0	1	145	120	90	70	50	40
12	0	0	16	14	12	0	2	0	95	80	75	65	60
13	0	0	25	30	20	0	3	0	49	70	70	60	40
14	0	0	21	16	18	0	4	0	0	105	90	61	0
15	0	0	30	21	0	0	5	0	0	55	45	0	0
16	0	0	30	30	0	0							
17	0	0	0	0	0	0							

Simplex Solving:

Objective Function:

Minimize costs:

$$Z = c_1 x_1 + c_2\, x_2 + FC$$

Where:

1. c_1: Cost per MWh during peak hours (e.g., $70/MWh)
2. c_2: Cost per MWh during off-peak hours (e.g., $40/MWh)
3. FC: Fixed costs for generation capacity (e.g., $1000)
4. x_1: Energy produced/sold during peak hours (MWh)
5. x_2: Energy produced/sold during off-peak hours (MWh)

Constraints:

Capacity constraint:

$$x_1 + x_2 \ \le 100 \ \ (total\ capacity\ is\ 100\ MWh)$$

Peak hour demand limit:

$$x_1 \ \le 60 \ \ (\text{maximum peak hour production})$$

Off-peak demand limit:

$$x_2 \le 50\ (maximum\ off-peak\ production)$$

Minimum off-peak demand:

$$x_2 \ \ge \ 20$$

Minimum peak hour demand:

$$\mathbf{x_1 \geq 30}$$

Non-negativity:

$$\boldsymbol{x_1 . x_2 \geq 0}$$

Solving this problem by using traditional method:

Since the simplex solves the maximum rather than the minimal, we must change the problem to make it easier to solve. The problem is solved in simplex after creating a matrix and getting the traverse matrix to transform the problem from the minimum to the maximum. We're maximizing so, the Z row coefficients are negative. The algorithm will pivot to make these coefficients positive. Due to the objective function's fixed cost, the constant term is -1000.

1. For the first three ≤ constraints, S_1, S_2, and S_3 are slack variables.
2. For $X_2 \geq 20$, S_4 is a surplus variable (originally $-S_4$ in the constraint).
3. For $X_1 \geq 30$, S_5 is a surplus variable (originally $-S_5$ in the constraint).

Basis	X1	X2	S1	S2	S3	S4	S5	RHS
S1	0	0	1	0	0	1	1	50
S2	0	0	0	1	0	1	0	30
S3	0	0	0	0	1	0	1	30
S4	1	0	0	0	0	-1	0	30
S5	0	1	0	0	0	0	-1	20
Z	0	0	0	0	0	-70	-40	2900

Calculating Z:

Optimal Values:

x1=30(Peak-hour generation), x2=20(Off-peak generation)

Z = 70(30) + 40(20) + 1000 = 2100 + 800 + 1000 = 3900, which indicates that the end result corresponds to the Excel result number of 5800. The number will match the traditional method after adding 1000, which is constant, and this is how I confirm the solution.

A	B	C	D	E	F	G
	x1	x2	z			
Solution	30	20	2900			
obj.Coeff	70	40	0			
			LHS		RHS	
Constraint 1	1	1	50	<=	100	
Constraint 2	1		30	<=	60	
Constraint 3		1	20	<=	50	
Constraint 4		1	20	>=	20	
Constraint 5	1		30	>=	30	

Limit sheet:

Cell	Objective Name	Value
D2	Solution z	2900

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
C2	Solution x2	20	20	2900	50	4100
B2	Solution x1	30	30	2900	60	5000

Converting to its dual form

$$Z = 70x_1 + 40\,x_2 + 1000$$

Constraints:

	Before	After
Constraint 1:	$x_1 + \; x_2 \;\; \leq 100$	$-x_1 - \; x_2 \;\; \geq 100$
Constraint 2:	$x_1 \;\leq 60$	$-x_1 \geq -60$
Constraint 3:	$x_2 \leq 50$	$-x_2 \geq -50$
Constraint 4:	$x_2 \;\geq\; 20$	No change

Constraint 5:	$x_1 \geq 30$	**No change**

Table1: show the coefficient of each element:

X1	X2
-1	-1
-1	
	-1
	1
1	

Table 2: Convert each row from table one to column in table 2 and vice versa.

Y1	Y2	Y3	Y4	Y5
-1	-1	0	0	1
-1	0	-1	1	0

<u>Dual constriants:</u>

1. $-y_1 - y_2 + y_5 \leq 70$
2. $-y_1 - y_3 + y_4 \leq 40$

<u>Dual Objective function:</u>

$$Max\ Z = 100y_1 - 60y_2 - 50y_3 + 20y_4 + 30y_5$$

So, after making these steps. I found the following:

	Primal	Dual
Variables	2	5
Constraints	5	2

<u>Sensitivity</u>

The sensitivity analysis report provides valuable insights into how changes in the problem's parameters such as the objective function coefficients or constraint limits affect the optimal solution. Here's a clear breakdown:

Final Values: The optimal solution suggests producing **60 units of x_1** and **40 units of x_2**, maximizing the objective function value (profit) at **5,800**.

Reduced Cost: Both variables have a reduced cost of 0, meaning they are actively contributing to the optimal solution. If a variable had a positive reduced cost, producing it would require sacrificing profit.

Objective Coefficient Sensitivity:

- The current profit contribution per unit of x_1 is 70, This coefficient can decrease by 30 (down to 40) or increase infinitely without altering the optimal production levels.
- The profit per x_2 is **40**, but it's more sensitive, it can **increase by 30** (up to 70) or **decrease by 40** (down to 0) before the solution changes.

Shadow Prices:

Shadow Price = 40 → Increasing the limit to 101 would boost profit by **40 for x1**.

Shadow Price = 70 → Allowing 61 units of x_1 would increase profit by **30 for x2**.

Changes in RHS Shows how much a constraint's limit can change before the shadow price becomes invalid.

- **Constraint 1**: Can increase by **10** (to 110) or decrease by **20** (to 80).
- **Constraint 2**: Can increase by **20** (to 80) or decrease by **10** (to 50).

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C2	Solution x2	20	0	40	1E+30	40
B2	Solution x1	30	0	70	1E+30	70

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D5	Constraint 1 LHS	50	0	100	1E+30	50
D6	Constraint 2 LHS	30	0	60	1E+30	30
D7	Constraint 3 LHS	20	0	50	1E+30	30
D8	Constraint 4 LHS	20	40	20	30	20
D9	Constraint 5 LHS	30	70	30	30	30

Part II

A Student's Time Management Problem:

1. A comprehensive Description:

Me as a student, time management is one of the most important problems I deal with. With all of the coursework, lectures, labs, and self- learning classes, students need to figure out how to make the most of their time in order to advance academically and mentally. While dealing with challenges like set schedules, project deadlines, and the requirement for enough sleep, the objective is to optimize well-being and productivity. In this report, I will formulate my daily time management problem as a mathematical optimization model with 10 decision variables and 8 constraints. The objective is to determine the optimal number of hours to allocate to different activities each day in order to maximize overall productivity and satisfaction.

Detailed Problem Statement

1. My daily routine as a university student includes the following tasks:
2. Academic duties (lectures, independent research, and projects)
3. Fitness and health (exercise and sleep)
4. Family and friend’s time
5. Break

The challenge is to schedule my twenty-four hours in a way that:

1. Maximizes academic performance and grades
2. Ensures good sleep and regular exercise
3. Balances responsibilities related to self-learning and coursework
4. Allows free time for personal stress relief and relaxation

Since successful time management is essential for long-term career growth, personal wellbeing, and academic success. Students must juggle their responsibilities to attend lectures, finish assignments, study for tests, engage in outside interests, form social bonds, and get enough sleep in the demanding world of higher education. Even the best and most driven students may experience stress, missed deadlines, and burnout if they don't manage their time well. Effective time management enables students to achieve academic requirements and lead healthy lifestyles while maximizing productivity.

Better academic performance is one advantage of time management. Effective time management enables students to set aside quality time for studying, editing, and finishing assignments on time. Applying time management help students to get higher marks and avoid over working which leads to burnout. Conversely, poor time management often results in procrastination, where students delay tasks until the last possible moment which increase stress. By breaking tasks into subtasks and adhering to a schedule, students can maintain their learning, avoiding the overwhelming pressure that comes with deadlines.

Time management goes far beyond just keeping up with academics. it plays a vital role in maintaining both physical and mental health. University life isn't only about hitting the books, it's also a period of personal growth, building relationships, and taking care of yourself. When students neglect sleep, skip exercise, or avoid downtime in favor of constant studying. A state of physical, mental, and emotional feeling tired known as burnout can occur, which lowers motivation and productivity. Effective time management makes it possible to fit in regular exercise, relaxation, and sleep (at least 7–9 hours each night), all of which are essential for maintaining attention, mental balance, and general health.

Additionally, a balanced schedule protects against stress, which may affect health, memory, and concentration. However, there are more advantages to time management than that. Additionally, it aids students learn self-control and discipline, which are highly beneficial in both personal and professional contexts. Students are better prepared for real-world difficulties like meeting deadlines and balancing several duties at work when they know how prioritize tasks, create realistic goals, and follow a routine. Effective time management shows dependability, efficiency, and independence, which is why employers value individuals with this skill.

Furthermore, the achievement of long-term objectives depends on effective time management. Learning a new skill, getting into university, or finding a competitive job all need regular work over time rather than last-minute cramming or spurts of motivation. You may make consistent progress towards your larger goals without becoming overwhelmed by scheduling time each day or week for them. Over time, deep understanding, more solid professional relationships, and personal accomplishments are the results of this kind of habit.

In conclusion, time management isn't just about organizing your day. it's a powerful life skill that impacts your academic performance, your health, your career prospects, and your overall happiness. Students who take time to plan their days, stick to their priorities, and build healthy routines usually feel less stressed, get more done, and feel better overall. That's why universities should teach time management as part of student development, giving learners the tools, they need to manage their busy lives. In the end, mastering your time helps you take charge of your future and turn the challenges of student life into opportunities for real growth.

Decision Variables:

1. x_1 : $\boldsymbol{Time\ spent\ in\ lectures\ and\ classes}$
2. x_2 : $\boldsymbol{Time\ spent\ on\ self - study\ (homework, exam\ prep, revision)}$
3. x_3 : $\boldsymbol{Time\ spent\ on\ research/projects}$
4. x_4 : $\boldsymbol{Time\ spent\ on\ physical\ exercise}$
5. x_5 : $\boldsymbol{Time\ spent\ on\ music\ , reading}$
6. x_6 : $\boldsymbol{Time\ spent\ on\ fimaly\ and\ friend\ time}$
7. x_7 : $\boldsymbol{Time\ spent\ on\ self\ learning\ time}$
8. x_8 : $\boldsymbol{Time\ spent\ on\ eating\ and\ self\ care}$
9. x_9 : $\boldsymbol{Time\ spent\ on\ sleeping}$
10. x_{10} : $\boldsymbol{Free\ time}$

Constraints:

1. **Total daily time limit (24 hours):**

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \leq 24$$

2. **Minimum sleep requirement:**

$$x_9 \geq 7$$

3. **Fixed class hours:**

$$x_1 \geq 4$$

4. **Minimum study time:**

$$x_2 \geq 3$$

5. **Self-learning:**

$$x_7 \leq 3$$

6. **Physical exercising**

$$x_4 \geq 1$$

7. **Eating time:**

$$x_8 \geq 2$$

8. **Free Time:**

$$x_5 + x_6 + x_{10} \geq 2$$

Objective function:

The goal is to maximize daily productivity:

$$Z = 0.3x_2 + 0.2x_3 + 0.1x_4 + 0.1x_5 + 0.1x_6 + 0.2x_9$$

Simplex Solve:

To solve this problem using the Simplex method, we need to apply the Big M method. This approach ensures that the solution meets the requirements specified in constraints 1 and 4 of the problem

1. One or more >= constraints
2. One or more = constraints

Steps to solve the problem:

Step 1:

Convert the objective function:

From:

$$Z = 0.3x_2 + 0.2x_3 + 0.1x_4 + 0.1x_5 + 0.1x_6 + 0.2x_9$$

To:

$$Z - 0.3x_2 - 0.2x_3 - 0.1x_4 - 0.1x_5 - 0.1x_6 - 0.2x_9 = 0$$

Step 2:

Convert constraints as the following (Standard form):

	Before	After
Constraint 1:	$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \le 24$	$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + s_1 = 24$
Constraint 2:	$x_9 \ge 7$	$x_9 - s_2 = 7$
Constraint 3:	$x_1 \ge 4$	$x_1 - s_3 = 4$
Constraint 4:	$x_2 \ge 3$	$x_2 - s_4 = 3$
Constraint 5:	$x_7 \le 3$	$x_2 + s_5 = 3$
Constraint 6:	$x_4 \ge 1$	$x_4 - s_6 = 1$
Constraint 7:	$x_8 \ge 2$	$x_8 - s_7 = 2$
Constraint 8:	$x_5 + x_6 + x_{10} \ge 2$	$x_5 + x_6 + x_{10} - s_8 = 2$

Step 3:

1. Create table to find pivot (pivot column ∩ pivot row)
2. Find the smaller value in column to get the pivot column
3. Make a ratio test to find pivot row
4. Find the smallest value in ratio test to get pivot row
5. Add A1 and A2 as I am solving Big-M

After steps the results will be 3.9 which is similar to the result of the excel.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Z		
Solution	4	7	0	2	2	0	0	2	7	0	3.9		
obj.Coeff		0.3	0.2	0.1	0.1	0.1			0.2				
											LHS		RHS
Constraint 1	1	1	1	1	1	1	1	1	1	1	24	<=	24
Constraint 2									1		7	>=	7
Constraint 3	1										4	>=	4
Constraint 4		1									7	>=	3
Constraint 5							1				0	<=	3
Constraint 6				1							2	>=	2
Constraint 7								1			2	>=	2
Constraint 8					1	1				1	2	>=	2

Dual Method:

$$\boldsymbol{Max\ Z = 0.3x_2 + 0.2x_3 + 0.1x_4 + 0.1x_5 + 0.1x_6 + 0.2x_9}$$

Constraints:

As this problem is a maximum problem so, the constraints must be smaller than or equal.so the constraints number 1, 5 not changes.

	Before	**After**
Constraint 1:	$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \le 24$	**No change**
Constraint 2:	$x_9 \ge 7$	$-x_9 \le -7$
Constraint 3:	$x_1 \ge 4$	$-x_1 \le -4$
Constraint 4:	$x_2 \ge 3$	$-x_2 \le -3$
Constraint 5:	$x_7 \le 3$	**No change**
Constraint 6:	$x_4 \ge 1$	$-x_4 \le -1$
Constraint 7:	$x_8 \ge 2$	$-x_8 \le -2$
Constraint 8:	$x_5 + x_6 + x_{10} \ge 2$	$-x_5 - x_6 - x_{10} \le -2$

Table1: show the coefficient of each element:

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	1	1	1	1	1	1	1	1	1	1
2									1	
3	-1									
4		-1								
5							1			
6				-1						
7								-1		
8					-1	-1				-1

Table 2: Convert each row from table one to column in table 2 and vice versa.

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
R1	1		-1					
R2	1			-1				
R3	1							-1
R4	1					-1		-1
R5	1				1			

R6	1						-1	
R7	1	1						
R8	1							-1

So, after making these steps. I found the following:

	Primal	**Dual**
Variables	10	8
Constraints	8	10

<u>Dual Constraints:</u>

$$\boldsymbol{Y_1 - Y_2 \geq 0}$$

$$\boldsymbol{Y_1 - y_4 \ \geq 0.3}$$

$$\boldsymbol{Y_1 \geq 0.2}$$

$$\boldsymbol{Y_1 - Y_6 \geq 0.1}$$

$$\boldsymbol{Y_1 - Y_8 \geq 0.1}$$

$$\boldsymbol{Y_1 - Y_8 \geq 0.1}$$

$$\boldsymbol{Y_1 + Y_5 \ \geq 0}$$

$$\boldsymbol{Y_1 - \ Y_7 \geq 0}$$

$$\boldsymbol{Y_1 + Y_2 \ \geq 0.2}$$

$$\boldsymbol{Y_1 - Y_8 \ \geq 0}$$

<u>Objective function:</u>

$$\boldsymbol{Min\ Z = 24y_1 - 7y_2 - 4y_3 - 3y_4 + 3y_5 - y_6 - 2y_7 - 2y_8}$$

In this case, solving the dual isn't the most effective approach, since the dual method is typically used to simplify problems by reducing the number of constraints. However, for this model, the Simplex method remains the more suitable and efficient solution.

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