A LINEAR PROGRAMMING MODEL FOR REAL-TIME PRICING OF ELECTRIC POWER SERVICE

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Real-time pricing (RTP) is an electric power service offering in which prices vary over time, based on projected supply and demand conditions. Prices may vary hourly or in larger time blocks and are typically announced several hours to one day in advance. RTP is an attractive option because it approximates the economic efficiency of spot pricing, while allowing customers time to react to prices. Currently, most RTP programs are experimental and price is set at approximately marginal cost. This approach does not consider how customers respond to the prices, as well as how their responses impact system costs and capacity requirements. As RTP begins to constitute a more significant share of the utility's load, this "open loop" price calculation is no longer appropriate. This paper develops a comprehensive model for RTP that balances supply and demand under various operating scenarios, while addressing utility capacity commitment decisions and customer subscription decisions. The methodology can be used to determine real-time prices that minimize the sum of supply costs and customer curtailment costs, both in a short-run operational mode and in a longer run planning mode.

Real-time pricing (RTP) is one of a number of "differentiated" electric power service offerings that are becoming increasingly attractive. Advances in metering and control technologies have made it possible to offer differentiated service to a greater base of customers. By smoothing the load and reducing the need for reserve capacity, these service offerings allow generation capacity expansion to be postponed or avoided in some cases. A recent study by Caves, Herriges and Windle (1989, Table 3-3) listed 12 utilities in the U.S. with real-time pricing. Details concerning programs differ somewhat, and can be obtained in publicly available documents from a number of utilities, e.g., Pacific Gas and Electric's Real-Time Pricing Project and Niagara Mohawk's Hourly Integrated Pricing Program. The analysis in this paper applies equally well to a service offering in which customers specify a maximum threshold price above which their service is to be automatically curtailed. This sort of RTP variation has been offered in the United Kingdom and France for a number of years.

Real-time pricing is conceptually equivalent to flexible pricing, as described by Vickery (1971) and spot pricing for electric power, as defined by Caramanis, Bohn and Schweppe (1982). Instantaneous market clearing spot prices are not feasible for electric power because they do not provide the utility or customers sufficient time to react. Supply planning requires the utility to make short- and medium-term adjustments, such as unit commitments, while major supply changes often require negotiated power purchases or long-term capital investments. Similarly, customers must often make short-term adjustments in order to curtail their demand in response to higher prices, and may also require capital investments and other long-term modifications to realize their full potential to respond to price changes. Thus, RTP, which typically announces prices a day ahead coupled with a contract that specifies limits on yearly price fluctuations, is an attractive option for many electric power applications.

Electric utilities' costs and ability to respond to changing supply requirements have been analyzed extensively. Customers' short- and long-term costs and ability to curtail demand are much less understood and are not currently included in a comprehensive manner in setting real-time prices. Thus, despite the

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sound economic foundations for RTP, comprehensive methods for its planning and implementation are currently incomplete. To date, RTP programs at the retail or customer level have been largely experimental, in that they represent only a small fraction of the utility's load. This allows real-time prices to be determined by supply cost forecasts alone, because the load reduction responses that they achieve are not sufficient to change the utility's marginal costs. However, if RTP customers become a significant portion of load, as is hoped by many utilities, this "open loop" approach to pricing will no longer be valid.

This paper provides a methodology for analyzing and planning real-time prices that reflect both utility and customer costs, including the short- and long-term options available to each. A piecewise-linear model is proposed for both utility supply costs and customer demand curtailment costs that is consistent with currently available data on these cost structures. This permits a linear programming formulation for minimizing the sum of utility supply costs and customer welfare losses from curtailments. This formulation can also include linear constraints that capture features such as stored hydrosupplies and customer load shifting options. The linear program is shown to satisfy the incentive compatibility and welfare maximizing properties of spot prices. The linear programming optimum may not be achievable when the utility unit commitment decisions and customer curtailment decisions are 0, 1 variables. For this case, a greedy algorithm is presented that obtains approximately optimal real-time prices that satisfy the required supply/demand balancing and incentive compatibility properties.

1. RELATED MODELS IN THE LITERATURE

The real-time pricing analysis in this paper employs many of the principles of optimal spot pricing developed by Caramanis, Bohn and Schweppe (1982) and Schweppe et al. (1987). This research derived general, first-order necessary conditions for the spot prices that maximize social welfare, subject to given constraints on generation and customer benefit. Optimal investment conditions are derived in Caramanis (1982) and Schweppe et al. (1987, chapter 10) for an electric power system operating under spot pricing. The general first-order conditions are complex to solve, and thus the specification of customer benefit and utility cost functional forms that can be estimated and optimized remains a difficult problem. The analysis of the supply side has been extended to include storage by Nguyen (1976), who considered a supply system with energy storage capability and a piecewise-linear cost structure similar to the one used in this paper. Daryanian, Bohn and Tabors (1989) considered the impacts of customer storage options on real-time pricing. These models did not present an integrated framework for considering both the customers' decisions to subscribe to the pricing programs and the utility's investment decisions. Anderson (1972) developed linear programming and dynamic programming models for analyzing the utility's investment decisions, but did not address the spot pricing aspects.

A number of articles have developed pricing structures and analyzed the economic benefits of unbundling service offerings for electric power, e.g., Chao (1983), Woo and Toyama (1986), Chao et al. (1986), Chao and Wilson (1987), and Smith (1989). These studies demonstrated that when supply and demand fluctuate over time a menu of service options based on both the utility's supply costs and customers' interruption costs can achieve efficiency gains for both consumers and the utility. Chao and Wilson show that a preannounced service menu with appropriately chosen prices can, in principle, achieve the same efficiency as instantaneous spot pricing. Chao (1983) and Woo and Toyama (1986) note that efficient prices under fluctuating conditions are expressed as the sum marginal operating costs and the cost of additional capacity, which corresponds to the result demonstrated for peak-load pricing by Crew and Kleindorfer (1978), and which holds in this paper as well. These papers use nonlinear demand models, but do not treat explicit fixed costs and the discrete choice options for customers and the utility that arise operationally.

A number of studies (Caves, Herriges and Windle 1989, Gott, McFadden and Woo 1989, and Woo and Train 1989) have proposed measures and collected data on customer outage costs and value of service. These studies indicate that customers are quite diverse in their service valuations and that fixed costs are a significant part of service outage costs. This paper develops a piecewise-linear customer cost function that is consistent with these empirical findings. The current and future availability of better information on customer curtailment costs creates opportunities to design RTP programs that include predicted demand responses.

Considerable success has been achieved in developing mathematical programming algorithms for optimizing unit commitment decisions for large numbers of supply units over short- and medium-time horizons up to approximately one week. Notable contributions include Muckstadt and Koenig (1977), Lauer et al. (1982), Merlin and Sandrin (1983), Cohen and Wan (1987), and Zhuang and Galiana (1988). These algorithms consider the startup and shutdown costs of generating units, reserve requirements, and maintenance costs, and solve the constrained cost minimization problem by Lagrangian relaxation. When customers' subscription decisions must be analyzed over a time horizon of one year or more, the level of detail of unit commitment models seems inappropriate for setting real-time prices, and the number of time periods would exceed the limits of even the best computational approaches. This paper uses a piecewise-linear cost structure with discrete jumps to approximate the system marginal cost curves for a variety of daily scenarios. The contract period then consists of a weighted aggregation of these scenarios. Zahavi, Vardi and Avi-Itzhak (1980) and Feiler and Zahavi (1981) presented methods for calculating expected marginal cost curves consistent with this approach, when supply is subject to random outages. The LP model of Anderson provides another example of this type of cost approximation. Alternatively, existing unit commitment models could be used to determine the marginal supply cost curves in advance for the various daily operating scenarios and these could then be approximated by the appropriate piecewise-linear functions. For short time horizons, the customer curtailment cost structure developed here could be incorporated into existing unit commitment models to obtain optimal economic dispatching solutions.

The piecewise-linear structure in this paper permits the use of a greedy algorithm for approximate optimization of prices that is more efficient than the Lagrangian relaxation methods used to solve nonlinear unit commitment problems. This algorithm is analogous to the one developed by Dobson and Kalish (1988) for solving the problem of matching a variety of products with fixed production costs to the needs of customers with diverse preferences. This algorithm is appropriate for planning real-time prices over medium and long time horizons, but is not intended for optimal short-term dispatching of generation units.

2. MODEL SPECIFICATION

For convenience of terminology, it will be assumed that the contract period, which equals the planning horizon, is one year, that real-time prices are to be announced one day in advance, and that prices vary hourly. These time scales correspond roughly to a number of the current service offerings, but can be set to other values as needed. Over the planning horizon, the model includes uncertainty in both supply and demand through the use of different daily scenarios, $k = 1, 2, \ldots, N$ that can occur with varying frequencies. In practice, the different scenarios would correspond to different weather forecasts, in combination with different availabilities of generating units, different yearly snow pack levels for hydrogeneration, etc. Sets of prespecified scenarios are commonly used in current generation planning. It is assumed that the utility and customers both estimate the same

n_k = the expected number of days of scenario type k during the contract period.

Obviously, the utility can help achieve consistency in these estimates by describing its scenarios and announcing its estimated $\{n_k\}$ for the coming year. If there are asymmetries of information, e.g., certain customers have better forecasts than the utility, the problem becomes considerably more complex, and this case is not analyzed.

Given that a certain day is of type k, it is assumed that all factors affecting customer load and utility generation are deterministic for the purpose of specifying real-time prices 24 hours ahead. As the day actually progresses, variations from the forecast will, of course, occur (which causes the given prices to be slightly suboptimal), but the corresponding shortfalls or surpluses are simply compensated for by the utility's dispatching procedures, as is currently done for other load variations from forecast. Consequently, pricing related decisions by both the utility and the customer are made on an expected value basis using the best information available at each stage.

A piecewise-linear cost function is used for both utility supply cost and customer outage cost. For notational compactness, the same symbols will be used for both customers and utilities, but the variables have different interpretations. In general, the expected cost per year has the form:

$$\text{expected cost per year} = \bar{C}_i \bar{X}_i + \sum_k n_k C_{ik} X_{ik} + \sum_{k,t} n_k c_i(t) x_{ik}(t),$$

$$\text{where } x_{ik}(t) \leq G_{ik} X_{ik}. \tag{2}$$

The last two terms in (1) are *expected* costs for the contract period, based on the varying fractions of days of each scenario type. For symmetry, all decision variables are denoted by either x or X variables, and cost coefficients are denoted by c or C. Roman letters are used for other constants. Greek letters will be used for the shadow prices obtained from the dual optimization problem.

The definitions of the variables and coefficients for the utility are:

$x_{ik}(t)$ = the energy supplied from unit i on day type k during time period t;
$c_i(t)$ = the marginal cost per kWh of energy supplied from unit i during time period t;
X_{ik} = the fraction of supply type i that is committed on day type k;
C_{ik} = the cost of committing supply type i for day type k;
$\bar{X}_i$ = the fraction of supply type i that is committed for the contract period;
$\bar{C}_i$ = the fixed cost of committing the maximum possible amount of supply type i for the contract period;
$G_{ik}(t)$ = the maximum available energy from supply source i for time period t on day type k;
S_i = the maximum available energy from supply type i for the contract period.

The interpretations of the variables are as follows. Given that a day is forecasted to be of type k, the utility has two levels of decisions to make for each supply source i: The fraction X_{ik} of the supply unit to commit for the day and the level $x_{ik}(t)$ at which to operate in each time period. In many cases X_{ik} is a 0, 1 commitment variable, as we discuss subsequently. Figure 1 illustrates the available supplies $G_{ik}(t)$ for a fixed k, which are divided into discrete types i, e.g., base, intermediate, and peak. Available supply may vary gradually over the day, e.g., Base 1 in Figure 1, or there may be step fluctuations in the $G_{ik}(t)$ due to shutdowns for maintenance. Also, for some sources of supply, the utility may be required to make a capacity level decision at the start of the contract period, which is specified by $\bar{X}_i$. For example, for bulk power purchases, a maximum power level needs to be determined, in addition to energy quantities. As noted in the literature review, this supply cost structure is simpler than the one normally considered for unit commitment problems involving thermal and hydrounits. However, the algorithms for unit commitment could be used to generate an optimal supply cost function having approximately the form in Figure 3, for each given generation type i and given daily scenario k. These optimal supply cost functions could then be approximated by the linear generation cost functions in (1).

The parameter S_i is relevant for supply sources such as hydro that operate subject to a fixed storage constraint on total energy. This appears as a constraint

$$\sum_{k,t} n_k x_{ik}(t) \leq S_i$$

for all i which are constrained on energy. (3)

This treatment of storage is simpler than that of Nguyen because no holding cost per period is considered. However, constraints of the form (3) can be used to impose constraints on the total energy supplied by source i for periods shorter than the complete contract period, without increasing the complexity of the linear program.

Two aspects considered in previous research are omitted in this supply model. Minimum operating levels for $x_{ik}(t)$, which appear in unit commitment models, could be included within the LP formulation in this paper, but this is omitted to simplify the presentation. The transmission and distribution

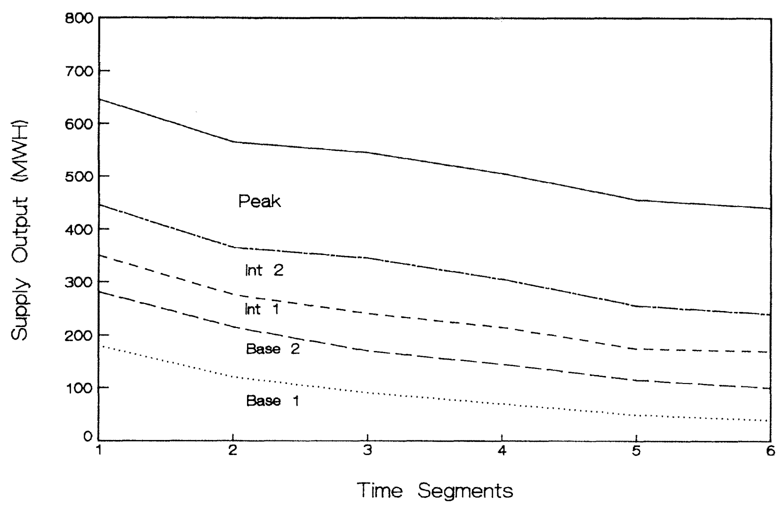

Figure 1. Available supply versus time.

component of spot prices, which is treated by Schweppe et al. is a much more difficult issue, and generally requires prices that differ explicitly by customer. Since customer-dependent pricing seems problematic from a regulatory standpoint as well, this aspect has not been developed.

The definitions of the variables for the customer are:

$x_{ik}(t)$ = the energy curtailed of load type i on day type k during time period t;

$c_i(t)$ = the marginal outage cost per kWh of energy curtailed of load type i during time period t;

$G_{ik}(t)$ = the maximum curtailable load of type i for time period t on day type k;

X_{ik} = the fraction of curtailable load type i that is available on day type k;

C_{ik} = the fixed cost of curtailing load type i for day type k;

$\bar{X}_i$ = the fraction of load type i that is committed to RTP for the contract period;

$\bar{C}_i$ = the fixed cost to commit load type i to RTP for the contract period;

S_i = the maximum total curtailed energy for load type i for the contract period;

$L_{ik}(t)$ = the "normal" load of type i on day k in time period t.

The rationale for the customer is as follows. First, curtailment costs are considered, as opposed to customer benefits for consuming energy. This is because essentially all data on curtailments is of this form. Indeed, this seems appropriate, because the customer's cost of a curtailment may differ from the corresponding incremental benefit of consuming the energy. Some recent outage cost surveys suggest that customer costs have approximately the form in (1). For example, in Caves, Herriges and Windle (Figure 2-4), studies of industrial outage costs per KW appear to fit a function of this type fairly well with the ratio $C_{ik}/c_i(t) = 7/3$. No data are given on how $c_i(t)$ may vary with t, but the model can include this variation without loss of generality. The selection of the different customer types k can be tailored to the information that the utility has on different customer classes who may participate in the RTP program. Classification should be such that all load increments of the same type i have similar curtailment costs.

Figure 2 illustrates curtailable customer loads $L_{ik}(t)$ that are disaggregated by load type on the vertical axis. Since RTP is an incremental service offering, specific model inputs are needed only for those customer loads that are impacted by the program. Each type may correspond to a given customer or to a group of similar customer applications. For each type i, the normal load $L_{ik}(t)$ corresponds to the load that would be consumed with no curtailments. A first cut estimate of $L_{ik}(t)$ is load type i's historical load pattern in response to current pricing programs. However, if real-time pricing is projected to generate new demand through lower overall prices, then $L_{ik}(t)$ should be increased accordingly. Since customers experience a fixed cost of curtailment on each occurrence, they tend to maintain their planned consumption patterns $L_{ik}(t)$ unless the price becomes too high for a period of time. Consumption tends to fall in preplanned discrete jumps, as the real-time price exceeds certain threshold values.

Total energy curtailment constraints of the form (3) are relevant for certain types of customers. For

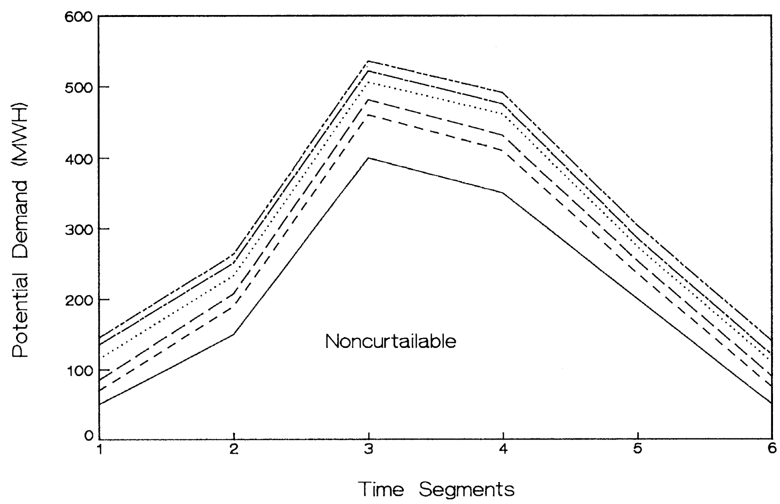

Figure 2. Potential demand versus time.

example, manufacturers with high energy costs such as aluminum and air products producers often suspend operations when prices exceed a certain level or produce ahead when prices are low. Total output tends to remain constant, which corresponds to a fixed energy constraint. Constraints similar to (3) could also apply as separate constraints over shorter time periods, as noted in the case of generation. When load shifting is possible, the constraints (3) can be used to capture the customer's total energy usage requirements during certain time windows, while $c_i(t)$ corresponds to the cost of shifting load away from the planned period t.

2.1. Linear Program for Socially Optimal Prices

The piecewise-linear costs for the utility and its customers can be used to formulate a linear program for maximizing total welfare, defined as consumer benefit minus supply costs, subject to the production and curtailment constraints. The dual problem yields the optimal real-time prices. In Caramanis, Bohn and Schweppe this was formulated as a general, nonlinear, welfare maximization problem, which specified first-order necessary conditions for the spot prices. This paper's linear program minimizes the sum of supply costs and curtailment costs, to be consistent with the form of the available customer data discussed above. The two LPs are as follows.

Primal Problem

$$\text{Min} \sum_{i,k,t} n_k c_i(t) x_{ik}(t) + \sum_{i,k} n_k C_{ik} X_{ik} + \sum_i \bar{C}_i \bar{X}_i \tag{4}$$

subject to

$$\sum_i x_{ik}(t) \geq L_k(t) \quad \text{for all } k,\ t \tag{5}$$

$$x_{ik}(t) \leq G_{ik} X_{ik}(t) \quad \text{for all } i,\ k,\ t \tag{6}$$

$$X_{ik} \leq \bar{X}_i \quad \text{for all } k \tag{7}$$

$$\bar{X}_i \leq 1 \quad \text{for all } i \tag{8}$$

$$\sum_{k,t} n_k x_{ik}(t) \leq S_i \quad \text{for all } i \tag{9}$$

$$x_{ik}(t),\ X_{ik},\ \bar{X}_i \geq 0 \quad \text{for all } i,\ k,\ t, \tag{10}$$

where $L_k(t) = \sum_i L_{ik}(t)$.

Constraint (5) states that all planned load is "served," either through generation or curtailment and (7) states that long-term commitment $\bar{X}_i$ is required before any daily commitment X_{ik} is possible. Constraints (6) and (9) follow from (2) and (3), respectively.

The dual problem is derived by the standard procedures of linear programming, using the following constraint and dual variable correspondences

(5) $n_k \sigma_k(t)$ (6) $n_k \lambda_{ik}(t)$ (7) $n_k \Lambda_{ik}$ (8) $\bar{\Lambda}_i$ (9) μ_i.

The first three dual variables have been multiplied by the constant n_k so that their units have convenient interpretations.

Dual Problem

$$\text{Max} \sum_{k,t} L_k(t) n_k \sigma_k(t) - \sum_i \bar{\Lambda}_i - \sum_i S_i \mu_i \tag{11}$$

subject to

$$\sigma_k(t) - \lambda_{ik}(t) - \mu_i \leq c_i(t) \quad \text{for all } i,\ k,\ t \tag{12}$$

$$\sum_t G_{ik}(t) \lambda_{ik}(t) - \Lambda_{ik} \leq C_{ik} \quad \text{for all } i,k \tag{13}$$

$$\sum_k n_k \Lambda_{ik} - \bar{\Lambda}_i \leq \bar{C}_i \quad \text{for all } i \tag{14}$$

$$\sigma_k(t),\ \lambda_{ik}(t),\ \Lambda_{ik},\ \bar{\Lambda}_i \geq 0 \quad \text{for all } i,\ k,\ t. \tag{15}$$

Interpretation of the dual variables defines the real-time prices and consumer surpluses. The supply and demand constraints (5) imply that the real-time prices are:

$\sigma_k(t)$ = the real-time price in time period t for day type k.

As observed by Vickery, the prices $\{\sigma_k(t)\}$ need not equal any particular marginal costs $\{c_i(t)\}$ and can be larger than any of them. Caramanis, Bohn and Schweppe and others have observed that the complementary slackness conditions for the primal and dual variables are equivalent to conditions for incentive compatibility of the prices for all suppliers of electricity. By symmetry, the analogous statements can be made for customer curtailments. Since the customer case is treated less frequently, that interpretation will be summarized here. This occurs because the fixed commitment costs must be recovered.

At each time t on day k, the customer will curtail available load only if the net surplus $\lambda_{ik}(t)$ derived from (12) is positive, i.e., only if

$$\lambda_{ik}(t) = \{\sigma_k(t) - c_i(t) - \mu_i\}^+ > 0, \tag{16}$$

where $\{x\}^+$ denotes the positive part of x. When $\lambda_{ik}(t) > 0$, the customer will curtail the maximum available load $X_{ik} G_{ik}(t)$, which follows from the complementary slackness condition for (6) and $\lambda_{ik}(t)$. Note in (16) that μ_i acts as an additive cost component. The μ_i may be the dominant cost for the case in which load shifting is not difficult, i.e., $c_i(t)$ is small. On day type k, the customer will choose to curtail, i.e., set $X_{ik} = 1$, only if the net surplus Λ_{ik} from (13) for curtailing is positive, i.e., only if

$$\Lambda_{ik} = \left\{\sum_t G_{ik}(t)\lambda_{ik}(t) - C_{ik}\right\}^+ > 0. \tag{17}$$

Finally, the customer will subscribe to RTP only if the total expected surplus $\overline{\Lambda}_i$ for the contract period is positive, i.e.,

$$\overline{\Lambda}_i = \left\{\sum_k n_k\Lambda_{ik} - \overline{C}_i\right\}^+ > 0. \tag{18}$$

The analogous statements for supply types are that available supply is provided at time t and day k only if the real-time price exceeds marginal cost $c_i(t) + \mu_i$, that supply is committed on day k only if the sum producer surplus exceeds the commitment cost C_{ik}, and commitment for the contract period occurs only if the expected net producer surplus exceeds the fixed commitment cost.

One of the benefits of RTP, which was noted by both Vickery and Caramanis, Bohn and Schweppe, is that each source of supply can be managed autonomously by the spot price, based on these incentive compatibility properties. That is, if each supply source has an autonomous manager who responds to forecasted prices, the social optimum will result. Thus, real-time prices are a socially optimal signal for cogenerators as well.

2.2. Deciding Between RTP and Another Service Option

The customer's decision to subscribe to RTP is typically a choice between RTP and one or more other options. The linear programming formulation can be generalized to incorporate this decision as shown below, but this is not carried throughout the paper in order to reduce complexity. Suppose that the choice is between RTP and a service option that has a fixed cost B_i for the contract period and offers prices $p_k(t)$, which may or may not vary with k and t. We will consider the case in which the consumer must allocate fixed load fractions $\overline{X}_i$ to RTP and $\overline{Y}_i$ to the second option. (Typically, either $\overline{X}_i$ or $\overline{Y}_i$ will be zero in the optimal LP solution.) The customer's choice is made based on which option provides the greater consumer surplus. As noted in the LP formulation, consumer surplus is maximized when the customer charges plus interruption costs are minimized. It is assumed that the prices $p_k(t)$ are fixed, which allows the corresponding customer curtailments $y_{ik}(t)$ to be determined in advance.

Treating $L_{ik}(t)$ as the potential load available, the total charges plus interruption costs associated with the alternative service option are given by

$$Z_i = \sum_{k,t} [L_{ik}(t) - y_{ik}(t)]p_k(t) + \sum_{k,t} n_k y_{ik}(t)c_i(t) + B_i, \tag{19}$$

which is treated as a constant in the LP for determining the prices $\sigma_k(t)$. Customer i will select the RTP option only if the charges plus interruption costs associated with the optimal $\sigma_k(t)$ are less than or equal to Z_i. Based on the interpretations of the dual variables $\overline{\Lambda}_i$ and Λ_{ik} in (16), (17), and (18) and constraints (12), (13), and (14), it can be seen that $\overline{\Lambda}_i \geq$ cost savings − interruption cost for customer i, with equality holding at the optimal solution. (The cost savings are the savings in charges that result from the curtailed demand.) Thus,

$$\sum_{k,t} n_k L_{ik}(t)\sigma_k(t) - \overline{\Lambda}_i \leq \text{customer } i\text{'s charges} + \text{interruption costs}, \tag{20}$$

with equality holding at the optimal solution. Therefore, incorporating the constraint

$$\sum_{k,t} n_k L_{ik}(t)\sigma_k(t) - \overline{\Lambda}_i \leq Z_i \tag{21}$$

in the dual objective acts as an additional constraint on the revenue that is generated by the prices $\{\sigma_k(t)\}$. This generates an additional primal variable $\overline{Y}_i$, defined as the fraction of load that customer i allocates to the alternative service option. The corresponding modification of the primal problem is to add a sum of terms $Z_i\overline{Y}_i$ to the objective function and to modify (5) and (8) as

$$\sum_i [x_{ik}(t) + L_{ik}(t)\overline{Y}_i] \geq L_k(t) \quad \text{for all } k, t \tag{5'}$$

$$\overline{X}_i + \overline{Y}_i \leq 1 \quad \text{for all } i. \tag{8'}$$

It can be seen that setting $\overline{Y}_i$ to 1 removes all customer i's load from the supply and demand constraint (5) and substitutes different costs in the objective function.

2.3. Revenue for Capacity Expansion

Caramanis, Bohn and Schweppe, and Schweppe et al. observed that spot prices always generate sufficient revenue to recover the utility's costs and, by including investment decisions, derive the first-order necessary conditions (FONC) for the optimal investment level. Although the LP formulation in this paper did not include investment options explicitly, it can be argued that the revenue generated by the real-time prices is sufficient for socially optimal investments. This discussion will not include any additional constraints for revenue reconciliation, as considered in Schweppe et al. (chapter 8). The result can be stated as follows.

Theorem 1. *For the optimal solutions of the primal and dual problems, we have*

total revenue

$$= \text{total supply cost} + \sum_{i \in \text{supplies}} (\bar{\Lambda}_i + S_i \mu_i).$$

The proof is in the Appendix. Note that the index i in this theorem corresponds only to supply sources and not to the customers.

One example interpretation is as follows. Suppose, for example, that the capital cost of an additional supply unit of type i is K_i and that the cost of capital is 10% per year. Then if $0.1K_i \leq \bar{\Lambda}_i + \mu_i S_i$, it is socially optimal to add another unit of type i, because $\bar{\Lambda}_i + \mu_i S_i$ is the net surplus generated per year for a unit of type i. The total extra revenue obtained is precisely equal to the sum of all net surpluses, based on Theorem 1.

In summary, the linear programming formulation accomplishes three objectives. First, total cost, as defined by the sum of supply costs and customer curtailment costs, is minimized, subject to the available supplies and planned customer loads. This is equivalent to maximizing total welfare subject to these constraints. Second, the solution gives real-time prices that are "incentive compatible," in that they induce customers and cogenerators to make consumption decisions that result in the socially optimal solution. Finally, supply and demand balance in the optimal solution in such a way that the utility recovers its costs and obtains additional revenue for socially optimal capacity expansion.

3. SOLVING THE MIXED INTEGER PROBLEM

If the X_{ik} are restricted to be 0 or 1, the primal problem becomes a mixed integer program. In general, this can be solved by mathematical programming techniques to obtain optimal (cost minimizing) values of supply allocations and load curtailments for the contract period. However, the dual prices that result from this solution correspond to marginal energy costs and do not reflect the commitment costs associated with the 0, 1 variables. Thus, the optimal mixed integer solution does not yield prices that recover the utility's costs. Furthermore, the response by customers and cogenerators to these prices would generally produce inadequate supply. In the case of large numbers of scenarios, time periods, and customer types, the optimal solution of the mixed integer problem may become prohibitive computationally.

This section describes a greedy heuristic solution method that obtains an approximate solution to the mixed integer LP, and at the same time yields incentive compatible real-time prices. This greedy algorithm is analogous to the one used by Dobson and Kalish for a different application with no constraint (5). Their computational experience indicates that the algorithm could solve the case of 128 scenarios having 100 time periods each in less than three minutes on an IBM AT personal computer. By comparison, the more general Lagrangian relaxation method of Zhuang and Galiana for the unit commitment problem required just over one hour on an HP Vectra for one scenario with 100 generating units and a 168 hour time horizon. Dobson and Kalish also reported a high degree of accuracy in the approximation, with zero error in many test cases and very small errors in others. It can be argued that the computational results for the problem in this paper should exhibit roughly the same degree of accuracy as that obtained by Dobson and Kalish, because (5) could be incorporated into the objective function through Lagrangian relaxation to obtain a problem identical to theirs. Computational experience on RTP examples so far has been positive.

The greedy heuristic first obtains a feasible solution for the dual linear program. The resulting prices $\{\sigma_k(t)\}$ are then used to obtain a feasible solution of the mixed integer problem (MIP). Neither of these solutions is typically optimal, because complementary slackness cannot be assured. However, they give upper and lower bounds for the optimal mixed integer solution, as shown by the following sequence of inequalities:

$$\text{Feas. MIP} \geq \text{Opt. MIP} \geq \text{Opt. Primal LP}$$
$$= \text{Opt. Dual LP} \geq \text{Feas. Dual LP}.$$

Thus, the optimal solution of the mixed integer problem (Opt. MIP) is bounded below by the greedy heuristic solution (Feas. Dual LP) and above by its corresponding Feas. MIP.

To develop the algorithm, the dual LP is reformulated as an unconstrained nonlinear optimization problem in $\{\sigma_k(t)\}$ and $\{\mu_i\}$. This is done by using (12), (13), and (14) to substitute for $\lambda_{ik}(t)$, Λ_{ik}, and $\bar{\Lambda}_i$, respectively, in the dual objective function. Define indicator functions of the form

$$I\{\sigma_k(t) \geq c_i(t)\} = \begin{cases} 1 & \text{if } \sigma_k(t) \geq c_i(t) \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

The indicator function allows us to write the following relationships for the solutions of the primal and dual LPs,

$$x_{ik}(t) = G_{ik}(t)I\{\lambda_{ik}(t) > 0\}I\{\Lambda_{ik} > 0\}I\{\overline{\Lambda}_i > 0\} \quad (23)$$

$$\lambda_{ik}(t) = \{\sigma_k(t) - c_i(t) - \mu_i\}^+. \quad (24)$$

For the case in which $\mu_i > 0$, substituting (23) and (24) into (18) and (9), respectively, implies that

$$\overline{\Lambda}_i = \sum_k n_k \sum_t G_{ik}(t)I\{\Lambda_{ik} > 0\}\{\sigma_k(t) - c_i(t) - \mu_i\}^+ \quad (25)$$

$$S_i = \sum_k n_k \sum_t G_{ik}(t)I\{\lambda_{ik}(t) > 0\}I\{\Lambda_{ik} > 0\}I\{\overline{\Lambda}_i > 0\}. \quad (26)$$

Considering the terms $\overline{\Lambda}_i + S_i\mu_i$ in the dual objective function, we see that terms involving μ_i in (25) cancel with $\mu_i S_i$. (Note that by complementary slackness, either $\mu_i = 0$ or (26) holds.) Then the dual objective function can be rewritten as

$$\max \sum_{k,t} L_k(t)n_k\sigma_k(t) - \sum_k n_k \sum_{t,i} G_{ik}(t)I\{\overline{\Lambda}_i > 0\}$$
$$\cdot\, I\{\lambda_{ik}(t) > 0\}I\{\Lambda_{ik} > 0\}[\sigma_k(t) - c_i(t)]. \quad (27)$$

Since $\overline{\Lambda}_i$, Λ_{ik} and $\lambda_{ik}(t)$ can all be expressed in terms of $\{\sigma_k(t)\}$ and $\{\mu_i\}$, (27) is now an unconstrained objective in these variables, although it is no longer linear because of the indicator functions.

The partial derivative with respect to $\sigma_k(t)$ is

$$\frac{\partial}{\partial\sigma_k(t)} = n_k\Bigg[L_k(t) - \sum_i G_{ik}(t)I\{\overline{\Lambda}_i > 0\}$$
$$\cdot\, I\{c_i(t) - \sigma_k(t) - \mu_i > 0\}I\{\Lambda_{ik} > 0\}\Bigg]. \quad (28)$$

Note that the partial derivatives are piecewise constant and nonincreasing in $\{\sigma_k(t)\}$ and further that the cross partials $\partial^2/\partial\sigma_s\partial\sigma_u \leq 0$.

3.1. Solution Algorithm

The greedy algorithm determines the $\{\sigma_k(t)\}$, but does not determine the $\{\mu_i\}$. Since (9) are relevant only for supply sources or customers that have storage capability, there are likely to be only a few μ_i variables. One approximate method for selecting $\{\mu_i\}$ is to solve the linear program, and simply use these values in solving the greedy algorithm. If complementary slackness of μ_i and (9) is violated, the μ_i can successively either be increased or reduced to cause the appropriate change in the use of energy type i so that the constraint is satisfied. The linear programming solution to the primal problem can always be used to obtain a feasible solution to the mixed integer problem, by simply rounding up all the positive noninteger X_{ik} and $x_{ik}(t)$ variables. However, this solution is generally inferior to the one obtained by the greedy algorithm.

The greedy algorithm begins with all $\sigma_k(t) = \min_i\{c_i(t)\}$. At this point, all partial derivatives (28) are clearly positive. Pick the t and k with the largest positive partial derivative (28). Increase this $\sigma_k(t)$ step-by-step (increments of 0.01 were used in the example) until (28) becomes nonpositive for that t, k, or until some other t', k' has a larger positive partial derivative. Then fix $\sigma_k(t)$ at its current value and continue the procedure with the new t' k', until all partial derivatives are nonpositive. Cycling is clearly prevented, because the cross partials are always nonpositive.

Once the prices $\{\sigma_k(t)\}$ are determined, economic dispatching can be used to obtain a feasible mixed integer solution to the primal problem. Using these prices, the $x_{ik}(t)$ must satisfy (23). This determines all the 0, 1 primal variables as well, because

$$X_{ik} = \max_t \{I\{x_{ik}(t) > 0\}\} \quad \text{and} \quad \overline{X}_i = \max_k \{X_{ik}\}. \quad (29)$$

Figure 3 illustrates the form of the supply curve for each time period t that results from price $\sigma_k(t)$

$$S(\sigma_k(t)) = \sum_{\{i \mid \mu + c_i(t) < \sigma_k(t)\}} X_{ik}G_{ik}(t). \quad (30)$$

The fact that the primal solution from (23) and (29) satisfies the demand for each day type and each time period follows from the nonnegativity of the partial derivatives (28). To see this, simply substitute (23) into (28) and note that this reduces to the supply and demand constraint (5). Although complementary slackness generally holds for the $\{\sigma_k(t)\}$ and (5), it need not hold for $\lambda_{ik}(t)$ and (6). If it does hold, the greedy algorithm's solution is optimal.

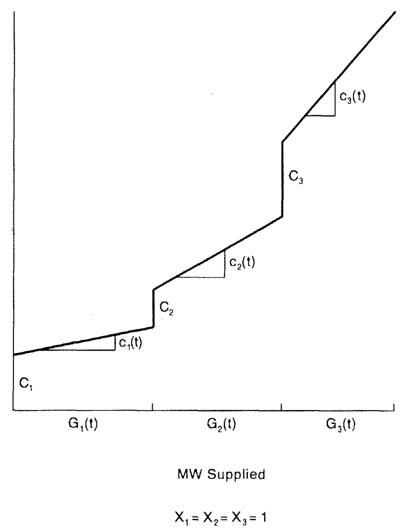

Figure 3. Total supply cost versus load.

The case in which the $\bar{X}_i$ are not required to be 0, 1 but the X_{ik} variables are restricted to the values 0, $\bar{X}_i$ is relevant for certain supply types, e.g., in the case partial supply commitment for the whole contract period. The greedy algorithm can treat this case as well. The values of the $\bar{X}_i$ would need to be obtained in advance from the linear programming solution, without restricting the X_{ik} to 0, $\bar{X}_i$. Then, these $\bar{X}_i$ values are substituted for the $I\{\bar{\Lambda}_i > 0\}$ in (26), (27), and (28). The prices $\{\sigma_i(t)\}$ are obtained from (28) as before. The $x_{ik}(t)$ and X_{ik} are then obtained from

$$x_{ik}(t) = G_{ik}(t) I\{\lambda_{ik}(t) > 0\} I\{\Lambda_{ik} > 0\} \bar{X}_i$$
$$X_{ik} = \bar{X}_i \max_t \{I\{x_{ik}(t) > 0\}\}. \tag{31}$$

Substituting (31) for $x_{ik}(t)$ in (28) shows that supply is adequate in this case as well. Suboptimality arises through the lack of complementary slackness.

4. ANALYSIS OF AN EXAMPLE

To illustrate the application of the model and the algorithm, let us consider an example with six time periods per day, five supply sources, seven load types, and one scenario, i.e., all days are of the same type. This small illustrative example was selected to make it easier to display the results graphically, but larger examples can be analyzed in the same manner. The input assumptions are shown in Table I. Dependence on k is not shown since values are the same for all k. Also, for this example, $c_i(t) = c_i$ for all t. The index i corresponds to the supply sources for $i = 1, 2, 3, 4, 5$ and to customers for $i = 11, \ldots, 17$. The magnitudes of the $G_i(t)$ for supply sources and for customers were given some nonsymmetric variations to create price fluctuations. The last customer (type 17) is a block of load that is considered noncurtailable, because it has a very high interruption cost. This example was first solved as an ordinary LP with no integer constraints. This gave a total cost (objective function value) of 759.0 and

$$\{\sigma(t)\} = 0.01 \quad 0.035 \quad 1.03 \quad 0.53 \quad 0.368 \quad 0.035$$

(real-time prices by time period)

The example was then solved for the case in which the X_i are 0, 1 variables by using the greedy algorithm. The parameter values obtained from the heuristic solution are shown in Table II and are discussed below. The heuristic solution obtained a total cost of 834.0. For comparison, the problem was also solved optimally as a mixed integer program, which resulted in a total cost of 772. However, this solution had a total revenue of 562 and a supply cost of 636. This illustrates that, while the mixed integer optimum achieves a lower total cost value, its dual prices do not recover supply cost. Also, the optimal consumption patterns in the mixed integer solution cannot be achieved by using dual prices to induce customers to make these choices.

The effect of real-time pricing on demand and supply decisions is illustrated in Figures 4–6. In Figure 4, the total potential demand is illustrated by the topmost curve. The demand that results from the RTP solution in Table II is shown as a solid line, which matches the potential demand in time segments 1 and 2. For comparison purposes, constant pricing across time segments was also tested, where the price

Table I
Inputs for the Example

	Available Supply $G_i(t)$						Supply Costs	
	t						Variable	Fixed
Type i	1	2	3	4	5	6	c_i	C_i
1	180	120	90	70	50	40	0.01	100
2	100	95	80	75	65	60	0.022	100
3	70	60	50	50	60	70	0.035	100
4	95	90	105	90	80	70	0.31	100
5	200	200	200	200	200	200	0.53	100
	Curtailable Demand $G_i(t)$						Customer Costs	
	t						Variable	Fixed
i	1	2	3	4	5	6	c_i	C_i
11	10	12	14	16	18	20	0.06	15
12	20	18	16	14	12	10	0.1	15
13	30	26	25	30	20	20	0.16	15
14	15	18	21	21	18	15	0.24	15
15	10	20	30	30	15	10	0.36	15
16	10	20	30	30	21	15	0.55	15
17	50	150	400	350	200	50	2.0	15

Table II
Solution of Example Problem With Integer X_i

$\sigma(t)$ $[t = 1, \ldots, 6] = 0.01$ 0.035 1.03 0.60 0.31 0.035
X_i $[i = 1, \ldots, 5] = 1$ 1 1 1 1 1
X_i $[i = 11, \ldots, 17] = 1$ 1 1 1 1 1 0

Curtailed Demand $x_i(t)$							Supply Output $x_i(t)$						
	t							t					
i	1	2	3	4	5	6	i	1	2	3	4	5	6
11	0	0	14	16	18	0	1	145	120	90	70	50	40
12	0	0	16	14	12	0	2	0	95	80	75	65	60
13	0	0	25	30	20	0	3	0	49	70	70	60	40
14	0	0	21	16	18	0	4	0	0	105	90	61	0
15	0	0	30	21	0	0	5	0	0	55	45	0	0
16	0	0	30	30	0	0							
17	0	0	0	0	0	0							

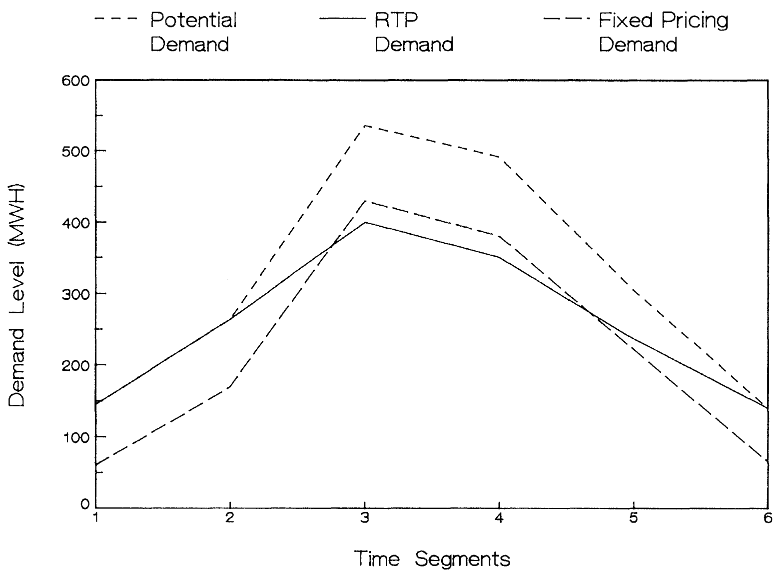

Figure 4. Potential and actual demands with RTP and fixed pricing.

\$0.549 was selected to achieve roughly the same revenue and supply cost as the RTP solution. The total welfare maximizing solution with a single price provided less revenue for the utility, and thus did not seem to be an appropriate comparison. The demand under this pricing program is indicated by the dashed line. Clearly, with RTP, demand in off-peak periods is greater, while demand during the peak period is less than with the fixed price solution.

The relative efficiencies of the two price plans are compared in Table III. Based on the table, we see that only 335 of the 1,535 units consumed in this example choose to subscribe to the RTP program. Thus, the efficiency gains in Table III were obtained by applying RTP to about 20% of the total demand. The impact of the two price plans on supply usage is illustrated by Figures 5 and 6. With RTP, less energy is used from the peaking technology (the topmost area) while more energy is used from base and intermediate technologies.

Table III
Comparison of Real-Time Pricing and Fixed Pricing

Item	Real-Time Pricing	Fixed Pricing
Units Subscribed to RTP	335	0
Supply Cost	656	679
Revenue	711	728
Revenue − Cost	55	49
Consumption	1,535	1,326
Consumer Surplus	1,776	1,741

6. CONCLUSION

Economic analyses have demonstrated that electric utilities can achieve greater efficiency by offering a diversity of service options to their customers. Real-time pricing, which approximates spot pricing and announces prices in advance to provide sufficient time for response, holds great potential for efficiency improvements, if used on a broader scale. To expand RTP beyond its current limited status, a comprehensive methodology is needed that analyzes its impact on customer consumption and commitment decisions. This paper develops a model for analyzing RTP that reflects fixed and variable costs for both the utility and its customers and which minimizes the sum of supply and service curtailment costs. The cost formulations are consistent with existing cost models for supply and with existing customer outage cost data.

The model can be optimized as a linear program over both short and long time horizons. An approximate, mixed integer algorithm provides an efficient method for solving the case of piecewise-linear costs in which capacity commitments and customer curtailments are 0, 1 decision variables. Because of its simplicity, year-long time horizons consisting of various daily scenarios can be analyzed by this algorithm. Furthermore, the greedy heuristic determines real-time prices that satisfy the social efficiency and incentive compatibility properties of spot prices. Although customer outage costs could be incorporated into existing unit commitment optimization models, the solution of these more general models over long time

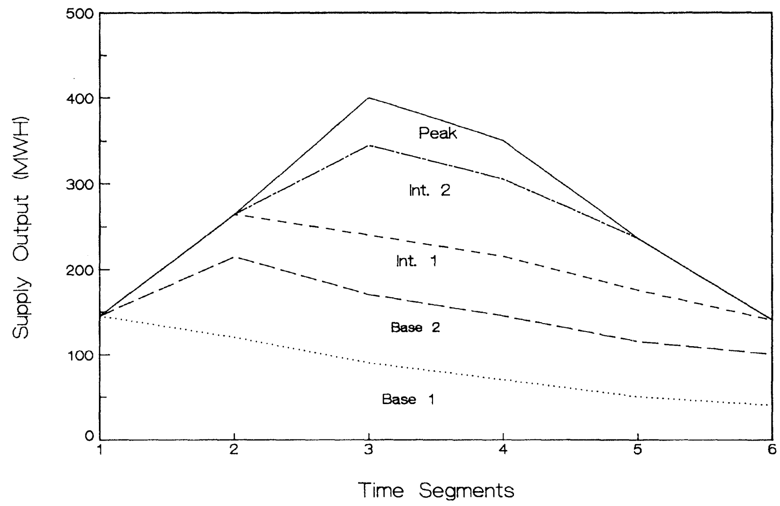

Figure 5. Supply outputs with RTP.

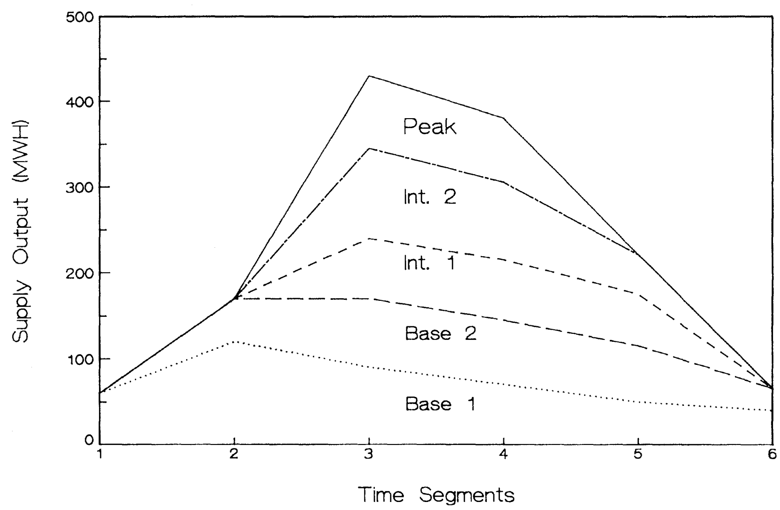

Figure 6. Supply outputs with fixed pricing.

horizons is computationally difficult and involves details for which only rough forecasts can be made.

Since RTP is one of several available service options, it is important to be able to analyze the customer's decision to subscribe. This model can be adapted to include the prices and terms of other available service options in setting real-time prices and analyzing subscription decisions. For customers who can achieve energy storage, either through thermal storage such as heated and cooled structures, or through rescheduling energy intensive applications, the model can reflect both the constraints on storage and the costs of shifting energy consumption away from its planned time period. Analogous constraints can capture approximately the utility's scheduling of hydroreserves. Because of the simplifications made in the model's costs and constraints, it is not a substitute for the more detailed economic dispatching models that optimally schedule generating units. However, it could be used as a first cut analysis in this context for planning purposes.

Interruptible service options based on customer-controlled curtailments can be implemented through a system in which service is curtailed if the price exceeds a predetermined threshold. Real-time prices can thus be used to control this class of service options as well. In this way, RTP methodology provides a unified framework in which to plan and manage a variety of expanded electric power service options. Observing customer responses to time-varying prices provides a rich source of new empirical data on customers' service outage costs and price elasticity. Expansion of current RTP programs will thus yield more complete and reliable data, particularly over the longer term, as customers make the capital investments necessary to participate fully in expanded service offerings.

APPENDIX

Proof of Theorem 1

This proof follows from equating the primal and dual objective functions at the optimal LP solutions. First, separate the supply sources and the customers in the primal objective (4) by letting the index i denote the supplies and j denote the customers. For indices j only, make the following transformations of the cost terms in the objective function. From the complementary slackness relationships for (13) and (14), respectively, it follows that

$$\sum_{j,k} n_k C_{jk} X_{jk} = \sum_{j,k} n_k \left\{ X_{jk} \sum_t G_{jk}(t) \lambda_{jk}(t) - \Lambda_{jk} X_{jk} \right\} \quad \text{(A.1)}$$

$$\sum_j \bar{X}_j \bar{C}_j = \sum_j \bar{X}_j \left\{ \sum_k n_k \Lambda_{jk} - \bar{\Lambda}_j \right\}. \quad \text{(A.2)}$$

Similarly, from (6), (7), and (8), respectively,

$$X_{jk} G_{jk}(t) \lambda_{jk}(t) = x_{jk}(t) \lambda_{jk}(t), \quad \bar{X}_j \Lambda_{jk} = X_{jk} \Lambda_{jk}$$

$$\text{and} \qquad \bar{X}_j \bar{\Lambda}_j = \bar{\Lambda}_j. \quad \text{(A.3)}$$

Adding (A.1) and (A.2) and substituting (A.3) yields

$$\sum_{j,k} n_k C_{jk} X_{jk} + \sum_j \bar{X}_j \bar{C}_j$$
$$= \sum_{j,k,t} n_k x_{jk}(t) \lambda_{jk}(t) - \sum_j \bar{\Lambda}_j. \quad \text{(A.4)}$$

From (14),

$$x_{jk}(t) \lambda_{jk}(t) = x_{jk}(t) \{ \sigma_k(t) - \mu_j - c_j(t) \}.$$

Substituting this into (A.4) and using the remaining primal customer interruption cost terms to cancel with the terms in (A.4) that contain $c_j(t)$, we obtain the following rewritten form of (4)

$$\text{supply costs} + \sum_{j,k,t} n_k x_{jk}(t) \sigma_k(t) - \sum_j S_j \mu_j - \sum_j \bar{\Lambda}_j,$$

$$\text{where } \mu_j S_j = \sum_{k,t} \mu_j n_k x_{jk}(t) \quad \text{(A.5)}$$

from (9) has been substituted. Setting (A.5) equal to the dual objective function, canceling the last two sums in (A.5) and rearranging terms yields

$$\sum_{k,t} \left\{ L_k(t) - \sum_j x_{jk}(t) \right\} n_k \sigma_k(t)$$
$$= \text{supply cost} + \sum_i S_i \mu_i + \sum_i \bar{\Lambda}_i. \quad \text{(A.6)}$$

Since the right-hand side of (A.6) corresponds to actual revenue, this proves the theorem.

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