Question6

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December 23, 2018

In the newton method, the iterative formula is

$$a_{k+1} = a_k - \frac{f(a_k)}{f'(a_k)} \tag{1}$$

In this case our function is

$$f(a) = a^n - q \tag{2}$$

since f is polynomial, then it is differentiable and also we need to guarantee that $f^{'}(a_k) \neq 0 \quad \forall k$.

The function is set to zero in this question in (2).

And the method

$$a_{k+1} = a_k + \frac{1}{n}\left(\frac{q}{a_k^{n-1}} - a_k\right) \tag{3}$$

It is special type of newton, because if we substitute the function (2) in (1) we get

$$\begin{aligned} a_{k+1} &= a_k - \frac{f(a_k)}{f'(a_k)} \\ &= a_k - \left(\frac{a_k^n - q}{n \cdot a_k^{n-1}}\right) \\ &= a_k + \frac{1}{n} \cdot \left(\frac{q}{a^{n-1}} - a_k\right). \end{aligned}$$

To answer the question, which function is minimized by it, I would say this formula derived from taylor series

$$f(p) = f(p_0) + (p - p_0)f(p_0) + \frac{(p - p_0)^2}{2} f^{''}(\xi(p))$$

where $\xi(p)$ lie between p and p_0. Since $f(p) = 0$, this equation gives

$$0 = f(p_0) + (p - p_0)f(p_0) + \frac{(p - p_0)^2}{2} f^{''}(\xi(p))$$

in Newton method we assume that $|p - p_0|$ is small, so

$$0 \approx f(p_0) + (p - p_0)f(p_0)$$

solving for p gives

$$p \approx p_0 - \frac{f(p_0)}{f'(p_0)}$$

this gives us the iterative formula

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$