

Question 1

Geometric approach

1 • 3 •• 5 •••

Combine 1 and 3 to get ••

$$2^2 = 4$$

Combine them with 5 to get •••

$$3^2 = 9$$

So sum of odd numbers is

$$S(n) = 1 + 3 + 5 + \dots + n = \left(\frac{n+1}{2}\right)^2 \quad \text{where } n \text{ is odd}$$

proof

$$\rightarrow \text{if } n=1 \Rightarrow \left(\frac{1+1}{2}\right)^2 = 1$$

let let us assume true at k

$$S(k) = \left(\frac{k+1}{2}\right)^2$$

now

$$S(k+2) = (1 + 3 + 5 + \dots + k) + (k+2)$$

$$= \left(\frac{k+1}{2}\right)^2 + (k+2) = \frac{(k+1)^2 + 4(k+2)}{2}$$

$$= \frac{k^2 + 2k + 1 + 4k + 8}{2} = \frac{k^2 + 6k + 9}{2} = \left(\frac{k+3}{2}\right)^2$$

So this true $\forall n \in \mathbb{N}$

$$(k+3)^2 = k^2 + 6k + 9$$