CPPSP Assignment 02

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1) Poisson distribution

A Geiger counter is used to count γ rays from a long lived radio-active source. Whenever a γ ray is detected, the Geiger counter makes a "click" sound. In a ten minute long counting interval, 1200 counts are observed. Now the detector counts for a five second long counting interval i) What is the probability that there are 10 counts in the five second long counting interval.

The Solution

In 10 minutes we hear 1200 click. So the average of counting per second is

$$\lambda = \frac{1200}{10 * 60} = 2$$

i. The probability that there are 10 counts in the five second long counting interval.

$$\begin{aligned} P(10 \text{ counts in 5 seconds}) =& \frac{e^{(-\lambda t)}(\lambda t)^x}{x!} \\ =& \frac{e^{(-2*5)}(2*5)^{10}}{10!} = 0.1251100357211333 \end{aligned}$$

ii. The probability that there are no counts in the five second long counting interval

$$\begin{aligned} P(\text{no count in 5 seconds}) =& \frac{e^{(-\lambda t)}(\lambda t)^x}{x!} \\ =& \frac{e^{(-2*5)}(2*5)^{0}}{0!} = 4.5399929762484854e-05 \end{aligned}$$

iii. Time interval between one click and the next click is measured. What is the probability that second click is heard between 2.00 s and 2.01 s later.

In this case I will use the relationship between the Poisson and Exponential distribution. If we expect a certain clicks on average for each unit of time, then the average waiting time between clicks is Exponentially distributed, with parameter λ .

$$P(2.0 \leq x \leq 2.01) = \int_{2}^{2.01} \lambda e^{-\lambda t} dt = \int_{2}^{2.01} 2e^{-2t} dt = 0.00036267394923131246$$

Spacing of charges on a line: Python program

(a) Consider a three point positive electric charges which are able to move on a straight line of length L. The left hand end of the line is at the origin (x = 0), and the right hand end is at x = L The charges will move around until they are in an equilibrium configuration.

(i) Explain why you think, or why you do not think, that the first charge will now be at x = 0, the second charge will be at x = L/2 and the third charge will be at x = L.
Solution
The charges will be in the positions $0\frac{L}{2}, L$ respectfully.
On the equilibrium the force between q_1 and q_2 must equal the force between q_3 and q_2. Let us assume the distance between q_1 and q_2 is x, so the distance between q_2 and q_3 is $L - x$.
Now,

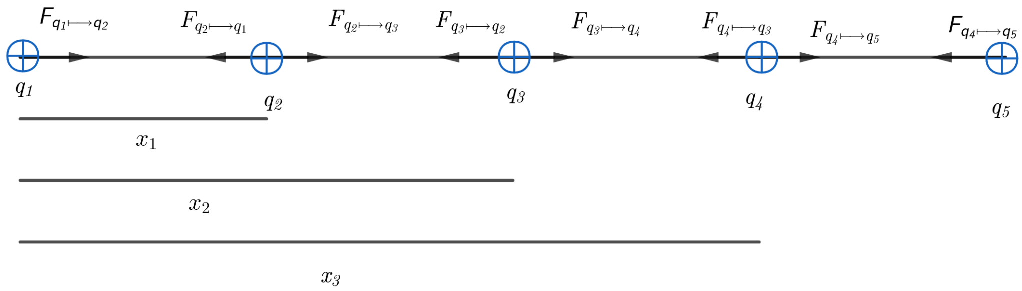
$$\begin{aligned} F_{12} =& F_{23} \\ \frac{kq_1q_2}{x^2} =& \frac{kq_2q_3}{(L-x)^2} \\ \frac{1}{x^2} =& \frac{1}{(L-x)^2} \\ x^2 =& (L-x)^2 \\ x =& L - x \Rightarrow x = \frac{L}{2} \end{aligned}$$

That is mean the second charge must be in the middle.

(ii) Make an argument to show that this equilibrium distribution has the lowest potential energy.
Solution
Please see Python code section a.2

(b) Now consider that five point positive charges can move on the line. They will come to an equilibrium configuration. Do you think that the five charges will be at positions $x = 0$, $x = L/4$, $x = L/2$, $x = 3L/4$ and $x = L$? If so, give reasons. If not, give reasons why not.
Solution
The charges on the equilibrium state, so the force on the right side cancels the force on the left side. We will assume the charges on the positions $0, x_1, x_2, x_3, L$. Now our job to find the unknowns x's.
We will take the 3 forces on the middle to construct three equations in 3 unknowns as follows:

$$\begin{cases} F_{q_2 \longmapsto q_1} = -F_{q_3 \longmapsto q_2} \\ F_{q_3 \longmapsto q_2} = -F_{q_4 \longmapsto q_3} \\ F_{q_4 \longmapsto q_3} = -F_{q_5 \longmapsto q_4} \end{cases} \begin{cases} \frac{1}{x_1^2} = \frac{1}{(x_2-x_1)^2} \\ \frac{1}{(x_2-x_1)^2} = \frac{1}{(x_3-x_2)^2} \\ \frac{1}{(x_3-x_2)^2} = \frac{1}{(L-x_3)^2} \end{cases}$$



$$x_1^2 = (x_2 - x_1)^2 \Rightarrow x_1 = x_2 - x_1 \qquad \boxed{x_2 = 2x_1}$$

$$(x_2 - x_1)^2 = (x_3 - x_2)^2 \Rightarrow x_1^2 = (x_3 - 2x_1)^2 \Rightarrow x_1 = x_3 - 2x_1 \qquad \boxed{x_3 = 3x_1}$$

$$(x_3 - x_2)^2 = (L - x_3)^2 \Rightarrow (3x_1 - 2x_1)^2 = (L - 3x_1)^2$$

$$\Rightarrow x_1^2 = (L - 3x_1)^2 \Rightarrow x_1 = L - 3x_1 \qquad \boxed{x_1 = \frac{L}{4}}$$

$$x_2 = 2x_1 = 2 * L/4 = L/2$$

$$x_3 = 3x_1 = 3 * L/4$$