MASTER 2, Econometrics I

Homework # 2

Due date: Friday November 30

You have to work in groups of students (maximum number of students is 4).

Grading: 10% of the final mark.

Problem I: Do the Empirical Exercise, from page 250 until page 254 (questions (a) until (g)), of Hayashi's book.

Problem II: (Reading the paper Bontemps and Meddahi (2012) could help.)

Let X be a continuous random variable with density function $q(\cdot)$.

1) Let $\phi(\cdot)$ a real function which is continuous and differentiable. Prove that under some conditions

$$E[\phi'(X) + \phi(X)(\ln q)'(X)] = 0. \tag{1}$$

2) Why is Eq. (1) interesting?

3) Assume that $X \sim \mathcal{N}(\mu, \sigma^2)$. By using Eq. (1), compute recursively $E[X^n]$ for $n = 1, 2, ..., 8$.

4) Assume that one has an i.i.d. sample $x_1, x_2, ..., x_n$ of X who follows $X \sim \mathcal{N}(0, 1)$. Derive the normality test statistic based on the third and fourth moment of X.

5) Assume that one has an i.i.d. sample $x_1, x_2, ..., x_n$ of X who follows $X \sim \mathcal{N}(\mu, \sigma^2)$ where μ and σ^2 are unknown. Derive the normality test statistic based on the third and fourth moment of X.

7) Check by simulations the size and power against a student of these two tests.

8) Is the latest test statistic robust to more general conditional mean and variance specifications?

Problem III: Consider the cross-section model

$$y_i = \beta_0 + \beta_1 x_i + \exp(a_0 + a_1 x_i + a_2 z_i) u_i$$

where u_i is independent from (x_i, z_i), is i.i.d. and follows a normal distribution $\mathcal{N}(0,1)$. We assume that (x_i, z_i) is i.i.d. and follows a normal $\mathcal{N}\left(0, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$.

We observe (y_i, x_i) and not z_i and assume that $E[y_i \mid x_i] = \beta_0 + \beta_1 x_i$. We are interested in estimating β_0 and β_1. We can do the OLS estimator or GMM estimators when one uses other instruments than x_i. One could also try to have a guess of the conditional variance of y_i given x_i in order to use the optimal instrument. Implement the different estimators in a simulation study to compare the different methods.

Sample sizes: 100 and 250 (and more if you can).

Number of replication: 1000 (or more if you can).

$(\beta_0, \beta_1, a_0, a_1, a_2, \rho) = (1, 1, 1/2, 1/4, 1/2, 1/2)$.