MASTER 1, Time Series

Homework # 2

Due date: Tuesday May 7

Problem I: Consider two independent ARMA(1,1) processes x_t and y_t. Define z_t as their sum ($z_t = x_t + y_t$). Prove that z_t is (in general) an ARMA(2,2).

Problem II: Consider two time series that you think that they are possibly co-integrated. Test the co-integration and estimate the co-integration relation.

Problem III: (Reading the paper Stambauch (1999) in the Journal of Financial Economics could help.) Consider the predictive regression model

$$y_t = \beta x_{t-1} + u_t$$

$$x_t = \rho x_{t-1} + v_t$$

where

$$(u_t, v_t)^\top \quad \text{i.i.d.} \quad \sim \quad \mathcal{N}\left(0, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}\right).$$

We will consider four designs: $\rho = .1$ or $\rho = .9$; $\sigma_{uv} = -.001$ or $\sigma_{uv} = -.009$. We will always assume $\beta = 0$, $\sigma_u^2 = 1, \sigma_v^2 = 0.01$.

1. Simulate each design at least 100 times (the ideal number of replications is 1,000) for the sample sizes 100, 500 and 1,000. For each replication, compute the OLS parameters of the regression of y_t on a constant and x_{t-1} and provide their mean and standard deviation over the replications (if the number of replications is 100, you should have 100 OLS estimators of β; give the mean and standard deviation of these 100 estimators). Comment the results.

2. Consider now the two-step ahead forecast regression, i.e., the regression of $y_{t+1} + y_{t+2}$ on the constant and x_t, with $t = 1, ..., T-2$.

   (a) What is the population value of the slope parameter?

(b) Do again the simulation for one design ($\rho = 0.9$, $\sigma_{uv} = -.009$) where the sample size equals 100, 500, and 1,000, and provide the Monte Carlo results of the OLS estimators. Comment the results.

(c) Compute the standard deviation of the OLS estimators by the naive method and by the Newey-West method (here, the object of interest is not the slope but the variance of the OLS estimator).

(d) Compare the previous results with the the true analytical formula (you have to compute it). Comment the results.