Master 2 MIF. Financial Econometrics: Homework # 1

Due date: February 12

Do the work in teams. The maximum (minimum) number of students per team is 3 (2).

1. **Descriptive statistics of financial data.** Consider the files attached to the email. Use the following data: S&P500, US$/Yen, and the US 3-months treasury bonds. You have to use the log-returns of the first two series and the levels of the third one.

   (a) Provide the following sample statistics: mean, variance, skewness, kurtosis, ACF at several lags of the three series, and the ACF of the squares of the first and second series. Comment the results.

   (b) Provide graphs of the rolling means, variances, skewness, kurtosis. Consider several windows and provide the best graphs (the window should be the same for a given asset). Comment the results.

2. **Predictive regressions.** Consider the following model

$$y_t = \beta x_{t-1} + u_t$$
$$x_t = \rho x_{t-1} + v_t$$
$$Cov\left([u_t, v_t]^\top, [u_t, v_t] \right) = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}.$$

   (a) Simulate the first design provided in the first Table of Lecture 1B (third panel). The Table is from the paper by Stambaugh (1999, Predictive Regressions, *Journal of Financial Economics*). For each design, simulate 1,000 samples. Compute the OLS estimator of β. Comment the results. Change ρ to 0.5 and 0; likewise, change σ_{uv} to 0; comment the results. Change the sample size (double the size); comment the results.

   (b) Do again the same regressions when one consider two-step and four-step ahead forecasts. Comment the results of the OLS estimators of the regression's slope. Compute the standard deviation of the OLS estimators by the naive method and by the Newey-West method. Compare them with the the true analytical formula (you have to compute it). Comment the results.

   (c) Go the the web-site of Robert Shiller (Yale University). He posted several data. Use the data he used in his book *Irrational Exuberance*. Do the same type of regressions as in the second and third tables of Lecture 1B. You need to take real prices (and not nominal ones). It might be useful to read the appendix of Hodrick (1992, *Review of Financial Studies*) to help you in using the right variables.

3. **GARCH models**. Consider the model

$$R_t = \mu + \sigma_t z_t,\ z_t\ i.i.d.\ D(0,1),\ \sigma_t^2 = \omega + \alpha(R_{t-1} - \theta\sigma_{t-1})^2 + \beta\sigma_{t-1}^2.$$

We denote $\psi = (\mu, \omega, \alpha, \beta, \theta)$. Simulate the following designs: 1) $\psi = (0; 0.01; 0.05; 0.90; 0)$, $z_t \sim \mathcal{N}(0,1)$, 2) $\psi = (0; 0.01; 0.05; 0.6; 2)$, $z_t \sim \mathcal{N}(0,1)$, with $T = 250$ and $T = 1,000$. (It is always useful to remove the first observations of the simulated model; here remove the 250 observations, meaning that when you want a sample size T, simulate T+250 observations and remove the first 250 observations). For each design, simulate at least 100 replications (ideally 1,000 replications).

   (a) For each design and each sample size, estimate ψ by the ML method. Provide the simulation mean of ψ and their standard deviation (for a given sample, you will estimate a unique ψ; then compute the mean of the estimator across the replications). Comment the results.

   (b) Consider again the same designs but assume that z_t is a standardized $T(10)$ (standardized means that the variance is one). I remind you that the definition of a T(n) is $T(n) = X/\sqrt{Y/n}$, $X \sim \mathcal{N}(0,1)$, $Y \sim \chi^2(n)$, X and Y are independent. Do the same work as in the previous question a). Comment the results and compare with the results in a).

   (c) Estimate the GARCH model (with and without leverage effect) by using the S&P500 and the US$/Yen data of the first question. Test the presence of leverage effect. Provide the diagnostics (in sample and out-of-sample ones).

4. **Value-at-Risk.** Consider again the same two series of the previous equation. For each sample, you have two models (GARCH and GARCH with leverage). Consider also the i.i.d. normal model. If you are able to compute the MLE with student error, do it. So, at the end you have three (or six models if you are able to compute the Student MLEs) for each sample. For each model, compute the 5% conditional VaR. Derive the sample of violations of the VaR. Do the three tests studied in Lecture 3. Comment the results.