MASTER M2
Econometrics I: Examination
December 14, 2011. 9h00-12h00

Question 1: 5 Points. Consider the model $y = x'\beta^0 + u$ with $E[u \mid x] = 0$ and (x_i', y_i) an i.i.d. sample. Explain, with details, the different steps of the inference procedure. You have to give the assumptions, the formulaes of the different estimators; you have to give the properties of your estimators (bias, consistency, efficiency, etc...), and consider the possible case of heteroskedasticity. Likewise, you have to explain how to build a confidence interval of a particular parameter, how to test a hypothesis on a parameter and on two parameters.

Question 2: 1 Point. Softwares often provide an F-test in the output of a linear regression. What is the null and alternative hypotheses? What is the distribution of the test?

Question 3: 4 Points. Assume that

$$y_i = \beta_0^0 + \beta_1^0 x_{i1} + \beta_2^0 x_{i2} + u_i, \text{ with } E[u_i \mid x_{i1}, z_{i1}, z_{i2}] = 0,$$

where z_{i1} and z_{i2} are instruments. We assume that we have an i.i.d. sample $(y_i, x_{i1}, x_{i2}, z_{i1}, z_{i2})$, $i = 1, 2, ..., N$.

1. What are the properties that should have the instruments (z_1, z_2)?
2. Explain how do you get the two-stage least-squares?
3. What are the properties of this estimator in finite sample? Asymptotically?
4. Give the formula of the optimal GMM estimator, as well as its asymptotic distribution.
5. How do you implement the optimal GMM estimator?
6. What is a weak instrument? How do you check that z_1 and z_2 are not weak instruments?

Question 4: 3 Points. Consider the predictive regression model

$$\begin{aligned} y_t &= \beta x_{t-1} + u_t \\ x_t &= \rho x_{t-1} + v_t \end{aligned}$$

where

$$(u_t, v_t)^\top \quad \text{i.i.d.} \quad \sim \quad \mathcal{N}\left(0, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}\right).$$

You want to estimate β by the OLS method by regressing y_t on x_{t-1}.

1. What are the properties of this OLS estimator in finite sample? Why?
2. What are the properties of this OLS estimator asymptotically? Why?
3. Provide the asymptotic variance of the OLS estimator. How do you estimate it?

4. Consider now the three-step ahead forecast regression, i.e., the regression of $y_{t+3} + y_{t+2} + y_{t+1}$ on a constant and x_t.

   (a) What are the population values of the slope and constant parameters?

   (b) How do you estimate the variance of the OLS estimator? Give the details.

Question 5: 3 Points. Consider an i.i.d. sample (y_i), i=1,2,...,N, from a random variable Y whose distribution is exponential with a density function

$$f(y) = c\exp(-y/\theta^0),\ y > 0.$$

1. Compute c such that $f(y)$ is a proper density function.
2. Compute the MLE of θ^0 and its asymptotic distribution.
3. Assume N=1,000 and $\sum_{i=1}^{1000} y_i = 2053$. Provide a 95% confidence interval of θ^0.
4. We want to test at the 5% level $H_0 :\ \theta^0 = 2$ against $H_a :\ \theta^0 \neq 2$. Do it by both the Wald and LR tests. For each test, provide the p-value.

Question 6: 4 Points. Consider a time series of an interest rate variable y_t assumed to follow the model

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t,\ \varepsilon_t \text{ i.i.d. } \mathcal{N}(0, \sigma^2).$$

1. You want to test whether the interest rate is stationary or not. How do you proceed?
2. In what follows, we assume that you rejected the non-stationarity assumption. Compute $E[y_t]$, $Var[y_t]$, $Cov[y_t, y_{t-1}]$.
3. Compute the autocorrelation function (ACF) and partial autocorrelation function (PACF) of y_t.
4. How do you estimate (μ, ρ, σ^2)? We do not need details; provide a brief explanation on how you proceed.
5. After the estimation of (μ, ρ, σ^2), how do you check that the model describes well the data? Be precise.
6. In what follows, assume that (μ, ρ, σ^2) are known perfectly. Compute the forecast of y_{t+1} at time t, the forecasting error, as well as the 95% confidence interval of the forecast.
7. Compute the forecast of y_{t+2} at time t, the forecasting error, as well as 95% confidence interval of the forecast.
8. Compute the forecast of y_{t+h} at time t, where $h > 0$. Derive the limit of the forecast when $h \to +\infty$. Comment on the result. Is the result specific to the AR(1) case?