An Analytical Framework for Assessing Asset Pricing Models and Predictability *

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Abstract

Consumption-based equilibrium asset pricing models have regained some momentum with new insights about the connections between stock market volatility and returns, the pricing of long-run claims, or return predictability. Links are established between risk premiums and different types of preferences, where separation between the elasticity of intertemporal substitution and risk aversion, and habit formation take center stage. Often, the solution of these models necessitates an approximation and quantities of interest are computed through simulations. We propose a model that delivers closed-form formulas for many of the statistics usually computed to assess the ability of the models to reproduce stylized facts. The proposed model is flexible enough to capture the various dynamics for consumption and dividends as well as the different types of preferences that have been assumed in consumption-based asset pricing models. The availability of closed-form formulas enhances our understanding of the economic mechanisms behind empirical results and of the limits of validity for the usual approximations.

1 Introduction

In the last twenty years or so, financial economists have devoted a lot of energy to solving two unyielding puzzles, the equity premium puzzle and the risk-free rate puzzle. The specification of preferences in the basic consumption CAPM model introduced by Lucas (1978) and Breeden (1979) has been modified to accommodate a large equity premium and a rather low risk-free rate. The two most popular models are without a doubt the recursive utility model of Epstein and Zin (1989, 1991) and the external habit model of Campbell and Cochrane (1999). Recently, these models have been used to reproduce new facts about the connections between stock market volatility and returns, the pricing of long-run claims, or return predictability (see in particular Bansal and Yaron, 2004, Bansal, Gallant and Tauchen, 2004, Hansen, Heaton and Li, 2004, Lettau, Ludvigson and Wachter, 2004). The effort then has been centered on the specification of the endowment process. New joint dynamic models have been proposed for consumption and dividend growth, while at the beginning the equality of consumption and dividend was often assumed. Often, the solution of these new full-fledged models necessitates an approximation and quantities of interest are computed through simulations.

In this paper we propose a model that delivers closed-form formulas for many of the statistics usually computed to assess the ability of the models to reproduce stylized facts. The proposed model is flexible enough to capture the various dynamics for consumption and dividends as well as various types of preferences that have been postulated in consumption-based asset pricing models.

To derive analytical formulas, we assume that the logarithms of real per capita consumption and dividend growth follow a bivariate process where both the means, variances and covariances change according to a Markov variable s_t which takes the values $1, \ldots, N$ (if N states of nature are assumed for the economy), where s_t is a stationary and homogenous Markov chain. Several asset pricing models have been built with constrained versions of this general process, but the main reason of this choice is that it leads to closed-forms formulas for many of the statistics that researchers have attempted to reproduce: the first and second moments of the equity premium and of the risk-free rate, the mean of and the volatility of the price-dividend ratio, the predictability of returns and excess returns by the dividend-price ratio, the predictability of consumption volatility by the dividend-price ratio and the consumption-wealth ratio, and the negative autocorrelation of returns and excess returns at long horizons. We also use this model to match some moments of the consumption and dividend processes implied by other dynamic models. This is the approach taken by Mehra and Prescott (1985) in their seminal paper that puts forward the equity premium puzzle.

In the formulas we will develop for the various statistics we will assume that we have solved the model for the price of the asset of interest or a ratio of the payoff of the asset to its price. As we will see, the structure of the endowment process implies that there will be one such payoff-price ratio per regime and this will help in computing closed-form analytical formulas.

Of course the price of any asset is dependent upon the stochastic discount factor which will be model-dependent. We solve for the prices in a Markov-switching economy with recursive preferences (Epstein and Zin, 1989) and with external habit (Campbell and Cochrane, 1999). These models deliver two fundamental payoff-price ratios: the consumption-market portfolio price ratio and the dividend-equity price ratio. The first ratio is unobservable but Lettau and Ludvigson (2001 a,b) have proposed a close parent with the consumption-wealth ratio. Once we differentiate consumption and dividends, these models deliver a measure of this important economic quantity. Moreover, in the recursive utility framework, the consumption-price ratio enters the stochastic discount factor. Once a solution to the nonlinear Euler set of equations defining this ratio in the various states is found, all other asset prices can be obtained analytically.

The importance of deriving closed-form formulas should not be underestimated. Lettau, Ludvigson and Wachter (2004), who use precisely a Markov-switching model for their endowment, remark that their two-state model takes very long to solve and that a three-state model would be computationally infeasible. They use a learning model that they must solve at each time period given their new assessment of the transition probabilities of the Markov process. Our formulas can be adapted to this approach and will ease considerably the process. Another considerable saving of processing time comes potentially from the simulations researchers run to compute predictability regressions. The usual procedure is to try to replicate the actual statistics with the same number of observations as in the sample as well with a much larger number of observations to see if the model can produce predictability in population. The last exercise, the most costly in computing time, is avoided by using the formulas we provide. The same is true for the variance ratios.

Another useful contribution is to use these formulas to assess the impact of approximations that researchers apply to solve models. One pervasive approximation in asset pricing is the log-linearization of Campbell and Shiller (1988). We provide formulas for several approximations of the payoff-price quantities in the Epstein and Zin (1989) model.

We apply our analytical framework to two prominent recent papers by Lettau, Ludvigson and Wachter (2004) and Bansal and Yaron (2004). Both promote the role of macroeconomic uncertainty measured by the volatility of consumption as a determining factor in the pricing of assets. The first paper models consumption growth as a Markov switching process and uses Epstein and Zin (1989) preferences, and so fits directly our framework. The second paper uses the same preferences but models the consumption-dividend endowment as an autoregressive process with time-varying volatility. For this model, we propose a moment-matching procedure with our Markov-switching process. By putting the two models in the same framework, we are able to point out their similarities and differences for asset pricing implications and predictability. Our analytical formulas allow us to explore a much wider set of preference parameters than in the original papers and thus to better understand their role in determining the financial quantities of interest. We also match the consumption surplus dynamics specified by Campbell and Cochrane (1999) with a Markov switching model and provide analytical results for many of the quantities generated by simulation in the original paper.

This paper extends considerably the closed-form price-dividend formulas provided in Bonomo and Garcia (1994) for the Lucas (1978) CCAPM model. Recently, two papers have also proposed to develop analytical formulas for asset pricing models. Abel (2005) calculates exact expressions for risk premia, term premia, and the premium on levered equity in a framework that includes habit formation and consumption externalities (keeping up or catching up with the Joneses). The formulas are derived under lognormality and an i.i.d. assumption for the growth rates of consumption and dividends. We also assume lognormality but after conditioning on a number of states and our state variable capture the dynamics of the growth rates. Eraker (2006) produces analytic pricing formulas for stocks and bonds in an equilibrium CCAPM with Epstein-Zin preferences, under the assumption that consumption and dividend growth rates follow affine processes. However, he uses the Campbell and Shiller (1988) approximation to maintain a tractable analytical form of the pricing kernel.

The rest of the paper is organized as follows. Section 2 describes the Markov-switching model for consumption and dividend growth. Section 3 enumerates several empirical facts and provides analytical formulas for the statistics reproducing these stylized facts. In Section 4, we solve for the price-dividend ratio in asset pricing models. Section 5 provides applications to several asset pricing models for the US post-war economy. Section 6 concludes. A technical appendix collects the proofs of propositions.

2 A Markov-Switching Model for Consumption and Dividends

We follow the approach pioneered by Mehra and Prescott (1985) by specifying a stochastic process for the endowment process and solving the model for the prices of the market portfolio, an equity and the risk-free asset in the economy. The goal in this branch of the empirical asset pricing literature is to determine if equilibrium models with reasonable preferences are able to reproduce some stylized facts associated with returns, consumption and dividends.

Contrary to the original model in Lucas (1978)), we make a distinction between consumption and dividends. Consumption is the payoff on the market portfolio while dividends accrue to equity owners. This distinction is nowadays almost always made (see Bansal and Yaron, 2004, Hansen, Heaton and Li (2004) and Lettau, Ludvigson and Wachter (2005) among others), but was introduced originally by Tauchen (1986) and pursued further by Cecchetti, Lam and Mark (1993) and Bonomo and Garcia (1994, 1996). [1]

The main reason for disentangling the consumption and dividend processes is first and foremost an empirical one: the series are very different in terms of mean, variance, and other moments.

We postulate that the logarithms of consumption and dividends growth follow a bivariate process where both the means, variances and covariances change according to a Markov variable s_t which takes the values $1, \ldots, N$ (if N states of nature are assumed for the economy). The sequence $\{s_t\}$ of Markov variables evolves according to the following transition probability matrix P.

We assume that

$$\zeta_t = \begin{cases} (1,0,0,...,0)^\top & \text{when } s_t = 1 \\ (0,1,0,...,0)^\top & \text{when } s_t = 2 \\ \quad\quad . & \quad\quad . \\ \quad\quad . & \quad\quad . \\ \quad\quad . & \quad\quad . \\ (0,0,0,...,1)^\top & \text{when } s_t = N \end{cases}$$

[1] Abel (1992) formulates a model with production, but where the labor supply is inelastic and the stock of capital is fixed and does not depreciate, and randomness comes from technology shocks. Then, consumption is equal to the total income of the economy, which is the sum of dividends - the capital income - with labor income. The disentanglement of consumption and dividends appears naturally in an asset pricing model of a production economy. However, usually total income is different from consumption, since there is investment, and although the Euler condition for asset returns still involves discounting the return by the intertemporal marginal rate of substitution in consumption, the latter depends also on leisure (see Brock, 1982, and Danthine and Donaldson, 1995). In Abel's (1992) simple version, labour supply is fixed and there is no investment. Thus, his version of a production economy fits perfectly our empirical framework.

where s_t is a stationary and homogenous Markov chain. We also assume

$$x_{c,t+1} \equiv \log(C_{t+1}) - \log(C_t) = c_{t+1} - c_t = \mu_c^\top \zeta_t \ + \ (\omega_c^\top \zeta_t)^{1/2} \ \varepsilon_{c,t+1} \tag{2.1}$$

$$x_{d,t+1} \equiv \log(D_{t+1}) - \log(D_t) = d_{t+1} - d_t = \mu_d^\top \zeta_t \ + \ (\omega_d^\top \zeta_t)^{1/2} \ \varepsilon_{d,t+1}, \tag{2.2}$$

where

$$\begin{pmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{d,t+1} \end{pmatrix} \mid \sigma(\varepsilon_{c,\tau}, \varepsilon_{d,\tau}, \tau \leq t; \zeta_m, m \in \mathbf{Z}) \ \sim \ \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho^\top \zeta_t \\ \rho^\top \zeta_t & 1 \end{bmatrix} \right) \tag{2.3}$$

We define the matrix P by

$$P^\top = [p_{ij}]_{1 \leq i,j \leq N}, \quad p_{ij} = P(s_{t+1} = j \mid s_t = i). \tag{2.4}$$

We assume that the Markov chain is stationary with an ergodic distribution Π, $\Pi \in \Re^N$, i.e.,

$$\Pi = E[\zeta_t]. \tag{2.5}$$

We have

$$E[\zeta_t \zeta_t^\top] = Diag(\Pi_1, ..., \Pi_N) \ \text{ and } \ Var[\zeta_t] = Diag(\Pi_1, ..., \Pi_N) - \Pi\Pi^\top. \tag{2.6}$$

Bonomo and Garcia (1994, 1996) use the specification (2.1,2.2) with constant correlations for the joint consumption-dividends process to investigate if an equilibrium asset pricing model with different types of preferences can reproduce various features of the real and excess return series. [2] The heteroscedasticity of the endowment process measures economic uncertainty as put forward by Bansal and Yaron (2004).

In the following, we adopt the notation:

$$\forall u \in \Re^N, \quad A(u) = Diag(\exp(u_1), ..., \exp(u_N))P. \tag{2.7}$$

$$I_t = \sigma(D_\tau, \tau \leq t), \quad J_t = \sigma(D_\tau, s_\tau, \tau \leq t) = \sigma(D_\tau, \zeta_\tau, \tau \leq t). \tag{2.8}$$

We also note

$$P^h = [P_{i,j}(h)]_{1 \leq i,j \leq N}.$$

[2] Cecchetti, Lam, and Mark (1991) use a two-state homoskedastic specification of (11) for the endowment and similar preferences to try to match the first and second moments of the return series. The authors use two models, one with a leverage economy, another with a pure exchange economy without bonds. In both instances, they are unable to replicate the first and second moments taken together.

The vector e denotes the $N \times 1$ vector whose all components equal one, while e_i denotes the vector whose i-th component equals zeros and the others equal zero , i.e.,

$$e = (1, , , , 1)^\top, \ e_1 = (1, 0, ..., 0)^\top, \ e_2 = (0, 1, 0, ..., 0)^\top, ..., \text{ and } e_N = (0, ..., 0, 1)^\top. \quad (2.9)$$

Finally, $\odot$ denotes the element by element multiplication operator, i.e.,

$$A \odot B = (a_1 b_1, ..., a_N b_N)^\top, \text{ where} A = (a_1, ..., a_N)^\top \text{ and} B = (b_1, ..., b_N)^\top.$$

3 Analytical Formulas for Statistics Reproducing Stylized Facts

In this section we start by recalling a series of stylized facts that researchers have tried to reproduce with consumption-based equilibrium models. In the formulas we will develop for the various statistics we will assume that we have solved the model for the price of the asset of interest or a ratio of the payoff of the asset to its price. As we will see, the structure of the endowment process implies that there will be one such payoff-price ratio per regime and this will help in computing closed-form analytical formulas. Of course these prices are model-dependent and in the next section we will solve for the prices in a Markov-switching economy with recursive preferences (Epstein and Zin, 1989).

3.1 The Stylized Facts

In his survey on consumption-based asset pricing Campbell (2002) enumerates a number of stylized facts about the stock market and its relation to short-term interest rates and consumption growth. We report these stylized facts and others computed with a post-war data set of quarterly consumption, dividends and returns data for the US economy (1947:1 to 2002:4). The empirical predictability results for the quarterly US data from 1947 to 2002 are reported in table 1.

1. The average return on stock is high (7.43% per year).

2. The average riskless real interest rate is low (1.20% per year).

3. Real stock returns are volatile (standard deviation of 16.93% per year).

4. The real interest rate is much less volatile (standard deviation of 2.28% per year) and much of the volatility is due to short-run inflation risk. Note however that there might be regimes as shown in Garcia and Perron (1996).

5. Real consumption growth is very smooth (standard deviation of 1.33% per year).

6. Real dividend growth is extremely volatile at short horizons because of seasonality in dividend payments (annualized quarterly standard deviation of 22.50%). At longer horizons it is intermediate between the volatility of stock return and the volatility of consumption growth.

7. Quarterly real consumption growth and real dividend growth have a very weak correlation of 0.15 but the correlation increases at lower frequencies.

8. Real consumption growth and real stock returns have a quarterly correlation of 0.16. The correlation increases at 0.31 at a 1-year horizon and declines at longer horizons.

9. Quarterly real dividend growth and real stock returns have a very weak correlation of 0.11, but correlation increases dramatically at lower frequencies.

10. Real US consumption growth not well forecast by its own history or by the stock market. The first-order autocorrelation of the quarterly growth rate of real nondurables and services consumption is 0.22. The log price-dividend ratio forecasts less than 4.5% of the variation of real consumption growth at horizons of 1 to 4 years.

11. Real US dividend growth has some short-run forecastability arising from the seasonality of dividend payments (autocorrelation of -0.44). But it is not well forecast by the stock market. The log price-dividend ratio forecasts no more than 1.5% of the variation of real dividend growth at horizons of 1 to 4 years.

12. The real interest rate has some positive serial correlation; its first-order autocorrelation is 0.63. However the real interest rate is not well forecast by the stock market.

13. Excess returns of US stock over Treasury bills are highly forecastable. The log price-dividend ratio forecasts 10% of the variance of the excess return at a 1-year horizon, 19% at a 3-year horizon and 26% at a 5-year horizon. Real returns exhibit a lower predictability, also increasing with the horizon (9% at a 1-year horizon, 15% at a 3-year horizon and 22% at a 5-year horizon).

To reproduce these stylized facts one needs three main types of formulas: formulas for expected returns, formulas for variance ratios of returns, formulas for predictability of returns.

3.2 Formulas for Expected Returns

3.2.1 Expected Returns on a Dividend-Producing Asset

We define the return process R_{t+1} as

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{3.1}$$

while the aggregated return over h periods is given by

$$R_{t+1:t+h} = \sum_{j=1}^{h} R_{t+j}. \tag{3.2}$$

We define the return process $R_{M,t+1}$ on the (unobservable) market portfolio as

$$R_{M,t+1} = \frac{P_{M,t+1} + C_{t+1}}{P_{M,t}}. \tag{3.3}$$

One important property that we will use in deriving our analytical formulas is the Markov property of the model. We will show that the variables P_t/D_t, $P_{M,t}/C_t$ and $P_{F,t}/1$ (where the $P_{F,t}$ is the price of a bond), are (non-linear) functions of the state variable ζ_t. On the other hand, the state ζ_t takes a finite number values. Consequently, any real non-linear function $g(\cdot)$ of ζ_t is a linear function of ζ_t. The reason is the following: The function $g(\zeta_t)$ takes the values g_1 is state 1, g_2 is state 2,..., g_N is state N; hence,

$$g(\zeta_t) = \bar{g}^\top \zeta_t \quad \text{where} \quad \bar{g} = (g_1, g_2, ..., g_N)^\top.$$

This property will allow us to characterize analytically the variables P_t/D_t, $P_{M,t}/C_t$ and $P_{F,t}/1$ while other data generating processes need either linear approximations or numerical models to solve the model.

In the rest of the paper, we will adopt the following notation:

$$\frac{P_t}{D_t} = \lambda_1^\top \zeta_t, \tag{3.4}$$

$$\frac{P_{M,t}}{C_t} = \lambda_{1c}^\top \zeta_t, \tag{3.5}$$

$$R_{F,t+1} = \frac{1}{P_{f,t}} = b^\top \zeta_t. \tag{3.6}$$

Observe also that one can write

$$\frac{D_t}{P_t} = \lambda_2^\top \zeta_t \quad \text{with} \quad \lambda_2 = (\lambda_{11}^{-1}, ..., \lambda_{1N}^{-1})^\top, \quad \text{where} \quad \lambda_1 = (\lambda_{11}, ..., \lambda_{1N})^\top. \tag{3.7}$$

Likewise,

$$\frac{C_t}{P_{M,t}} = \lambda_{2c}^\top \zeta_t \quad \text{with} \quad \lambda_2 = (\lambda_{1c1}^{-1}, ..., \lambda_{1cN}^{-1})^\top, \quad \text{where} \quad \lambda_{1c} = (\lambda_{1c1}, ..., \lambda_{1cN})^\top. \tag{3.8}$$

In Section 4, we will use the asset pricing models to characterize the vectors λ_1, λ_2, λ_{1c}, and b as functions of the parameters of the consumption and dividend growth dynamics and the utility function of the representative agent. In the rest of this section, we will characterize the predictability of the returns and excess returns as well as some other population moments by assuming that λ_1, λ_2, λ_{c1}, and b are known. These formulas depends only on the previous vectors and the dynamics of the dividend growth and the Markov chain.

In order to study the predictability of the returns and excess returns, we need to connect them to the state variable ζ_t and to the dividend growth. We show in the appendix that

$$R_{t+1} = (\lambda_2^\top \zeta_t) \exp(x_{d,t+1})(\lambda_3^\top \zeta_{t+1}) \quad \text{with} \quad \lambda_3 = \lambda_1 + e, \tag{3.9}$$

where the vectors λ_1 and e are defined in (3.4) and (2.9) respectively. Finally, we denote the excess return by R_{t+1}^e, i.e.,

$$R_{t+1}^e = R_{t+1} - R_{F,t+1}. \tag{3.10}$$

Proposition 3.1 ***Characterization of the expected values of returns and excess returns.*** *We have*

$$E[R_{t+1} \mid J_t] = (\lambda_2^\top \zeta_t) \;\; \exp(\mu_d^\top \zeta_t + \omega_d^\top \zeta_t / 2) \;\; \lambda_3^\top P \zeta_t = \psi^\top \zeta_t, \tag{3.11}$$

where $\psi = (\psi_1, ..., \psi_N)^\top$ *and*

$$\psi_i = \lambda_{2i} \;\; \exp(\mu_{d,i} + \omega_{d,i}/2) \;\; \lambda_3^\top P e_i, \quad i = 1, ..., N. \tag{3.12}$$

Likewise,

$$E[R_{t+1}^e \mid J_t] = (\psi - b)^\top \zeta_t. \tag{3.13}$$

Consequently, $\forall j \geq 2$

$$E[R_{t+j} \mid J_t] = \psi^\top P^{j-1} \zeta_t \text{ and } E[R_{t+j}^e \mid J_t] = (\psi - b)^\top P^{j-1} \zeta_t. \tag{3.14}$$

Finally,

$$E[R_{t+1:t+h} \mid J_t] = \psi_h^\top \zeta_t \quad \text{and} \quad E[R_{t+1:t+h}^e \mid J_t] = (\psi_h - b_h)^\top \zeta_t \tag{3.15}$$

where

$$\psi_h = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top \psi \quad \text{and} \quad b_h = \left(\sum_{j=1}^{h} P^{j-1}\right)^\top b. \tag{3.16}$$

3.2.2 Expected Risk-Free Rate

In the sequel, we will also compute in the application section the frequency with which models produce negative interest rates. The probability that the risk-free rate is negative is given by

$$P\left(R_{F,t+1} < 1\right) = E\left[\mathbf{1}_{\left\{R_{F,t+1}<1\right\}}\right] = E\left[g^T\zeta_t\right] = g^T\Pi, \tag{3.17}$$

where $g = \left(\mathbf{1}_{\{b_1<1\}}, ..., \mathbf{1}_{\{b_N<1\}}\right)^T$.

3.3 Variance Ratios for Returns

In this section we provide variance formulas for the dividend price ratio as well as some covariance formulas between this ratio and the returns at various horizons. We conclude by a formula for the variance ratio that measures the autocorrelation in returns. Cecchetti, Lam and Mark (1990) were the first to reproduce the autocorrelation in returns with a Lucas-type model where the growth rate of the endowment process (represented either by consumption, income or dividends) followed a two-state Markov-switching model in the mean. Bonomo and Garcia (1994) showed that a two-state model with one mean and two variances is closer to the data but cannot reproduce the autocorrelation in returns.

Proposition 3.2 ***Some Population Parameters.***

$$Var\left[\frac{D_t}{P_t}\right] = \lambda_2^\top Var[\zeta_t]\lambda_2 \quad \text{and} \quad Var\left[\frac{C_t}{P_{M,t}}\right] = \lambda_{2c}^\top Var[\zeta_t]\lambda_{2c}. \tag{3.18}$$

In addition, we have

$$Cov\left(R_{t+1:t+h}, \frac{D_t}{P_t}\right) = \psi_h^\top Var(\zeta_t)\lambda_2, \tag{3.19}$$

$$Cov\left(R_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right) = \psi_h^\top Var(\zeta_t)\lambda_{2c}, \tag{3.20}$$

$$Cov\left(R^e_{t+1:t+h}, \frac{D_t}{P_t}\right) = (\psi_h - b_h)^\top Var(\zeta_t)\lambda_2, \tag{3.21}$$

$$Cov\left(R^e_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right) = (\psi_h - b_h)^\top Var(\zeta_t)\lambda_2. \tag{3.22}$$

We also have

$$
\begin{aligned}
Var[R_{t+1:t+h}] = h\theta_2^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\theta_3. & \\
+ h(\theta_1 \odot \theta_1)^\top E[\zeta_t\zeta_t^\top]P^\top(\lambda_3 \odot \lambda_3) - h^2(\theta_1^\top E[\zeta_t\zeta_t^\top]P^\top\lambda_3)^2 & \\
+ 2\sum_{j=2}^{h}(h-j+1)\theta_1^\top E[\zeta_t\zeta_t^\top]P^\top(\lambda_3 \odot ((P^{j-2})^\top(\theta_1 \odot (P^\top\lambda_3)))), &
\end{aligned}
\tag{3.23}
$$

where

$$\theta_1 = \lambda_2 \odot (\exp(\mu_{d,1} + \omega_{d,1}/2), ..., \exp(\mu_{d,N} + \omega_{d,N}/2))^\top, \tag{3.24}$$

$$\theta_2 = (\theta_1 \odot \theta_1 \odot (\exp(\omega_{d,1}), ..., \exp(\omega_{d,N}))^\top) - (\theta_1 \odot \theta_1), \tag{3.25}$$

$$\theta_3 = \lambda_3 \odot \lambda_3. \tag{3.26}$$

Likewise,

$$
\begin{aligned}
Var\left[R^e_{t+1:t+h}\right] = h\theta_2^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\theta_3 & \\
+ h\left((\theta_1 \odot \theta_1)^\top E\left[\zeta_t\zeta_t^\top\right] P^\top(\lambda_3 \odot \lambda_3) - 2(\theta_1 \odot b)^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\lambda_3\right) & \\
+ h(b \odot b)^\top \Pi - h^2\left(\theta_1^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\lambda_3 - b^\top\Pi\right)^2 & \\
+ 2\sum_{j=2}^{h}(h-j+1)\,q_j &
\end{aligned}
\tag{3.27}
$$

where

$$
\begin{aligned}
q_j = \theta_1^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\left(\lambda_3 \odot \left(\left(P^{j-2}\right)^\top\left(\theta_1 \odot \left(P^\top\lambda_3\right)\right)\right)\right) & \\
- \theta_1^\top E\left[\zeta_t\zeta_t^\top\right] P^\top\left(\lambda_3 \odot \left(\left(P^{j-2}\right)^\top b\right)\right) & \\
- b^\top E\left[\zeta_t\zeta_t^\top\right]\left(P^{j-1}\right)^\top\left(\theta_1 \odot \left(P^\top\lambda_3\right)\right) + b^\top E\left[\zeta_t\zeta_t^\top\right]\left(P^{j-1}\right)^\top b. &
\end{aligned}
\tag{3.28}
$$

Observe that by using (3.23), on gets the variance ratio of aggregate returns which is given by

$$Ratio(h) \equiv \frac{1}{h}\frac{Var[R_{t+1:t+h}]}{Var[R_{t+1:t+1}]}. \tag{3.29}$$

One also gets a similar formula for the excess returns by using (3.27).

3.4 Predictability of Returns: An Analytical Evaluation

As mentioned in the previous section on stylized facts there appears to be a strong predictability of returns by the dividend-price ratio, which increases with the horizon. It is important to establish if this predictability measured inevitably in finite samples is reproduced in population by the postulated model. Therefore, we provide below the formulas for the population coefficients of the regressions of aggregated returns over a number of periods on the price-dividend ratio. In the section on applications below we will investigate by simulation to what extent some models produce predictability in finite samples but not in population. Several papers proposed models to reproduce predictability in returns. Bonomo and Garcia (1994) showed by simulation that a model with disappointment averse preferences (a recursive utility model with a Chew-Deckel certainty equivalent, see Epstein and Zin, 1989) and a Markov switching endowment for consumption and dividends was able to reproduce predictability in finite samples. More recently, Bansal and Yaron (2004) also reproduced this predictability with a recursive utility model with a Kreps and Porteus certainty equivalent.

It is common in the asset pricing literature to predict future (excess) returns by the dividend-price ratio. In doing so, one computes the regression of the aggregate returns onto the dividend-price ration and a constant. In the following, we will use the analytical formulas derives above in order to study these predictive ability in population.

When one does the linear regression of a variable, say $y_{t+1:t+h}$, onto by another one, say x_t, and a constant, one gets

$$y_{t+1:t+h} = a_{y,1}(h) + b_{y,1}(h)x_t + \eta_{y,1,t+h}(h)$$

where

$$b_{y,1} = \frac{Cov(y_{t+1:t+h}, x_t)}{Var[x_t]}$$

while the corresponding population coefficient of determination denoted R^2 is given by

$$R^2 = \frac{(Cov(y_{t+1:t+h}, x_t))^2}{Var[y_{t+1:t+h}]Var[x_t]}.$$

We will use these formulas in the following Proposition in order to characterize the predictive ability of the dividend-price ratio.

Proposition 3.3 ***Regression of the Aggregated Returns onto the Dividend-Price ratio and a constant.*** *Define the population regressions*

$$R_{t+1:t+h} = a_1(h) + b_1(h)\frac{D_t}{P_t} + \eta_{1,t+h}(h) \text{ and } R^e_{t+1:t+h} = a^e_1(h) + b^e_1(h)\frac{D_t}{P_t} + \eta^e_{1,t+h}(h). \quad (3.30)$$

Denote the population coefficients of determination by $R^2(h, D/P)$ and $R^2_e(h, D/P)$. Then,

$$b_1(h) = \frac{Cov\left(R_{t+1:t+h}, \frac{D_t}{P_t}\right)}{Var\left[\frac{D_t}{P_t}\right]}, \quad b^e_1(h) = \frac{Cov\left(R^e_{t+1:t+h}, \frac{D_t}{P_t}\right)}{Var\left[\frac{D_t}{P_t}\right]}, \quad (3.31)$$

$$R^2(h,D/P)=\frac{\left(Cov\left(R_{t+1:t+h},\frac{D_t}{P_t}\right)\right)^2}{Var[R_{t+1:t+h}]Var\left[\frac{D_t}{P_t}\right]},\text{ and } R_e^2(h,D/P)=\frac{\left(Cov\left(R^e_{t+1:t+h},\frac{D_t}{P_t}\right)\right)^2}{Var[R^e_{t+1:t+h}]Var\left[\frac{D_t}{P_t}\right]},\tag{3.32}$$

where $Cov\left(R_{t+1:t+h},\frac{D_t}{P_t}\right)$, $Cov\left(R^e_{t+1:t+h},\frac{D_t}{P_t}\right)$,$Var\left[\frac{D_t}{P_t}\right]$, $Var[R_{t+1:t+h}]$ *and* $Var[R^e_{t+1:t+h}]$ *are given in (3.19), (3.21), (3.18), (3.23) and (3.27) respectively.*

The following Proposition characterizes the predictive ability of the consumption-price ratio:

Proposition 3.4 ***Regression of the Aggregated Returns onto the Consumption-Price ratio and a constant.*** *Define the population regressions*

$$R_{t+1:t+h}=a_{1c}(h)+b_{1c}(h)\frac{C_t}{P_{M,t}}+\eta_{1c,t+h}(h),\;\; R^e_{t+1:t+h}=a^e_{1c}(h)+b^e_{1c}(h)\frac{C_t}{P_{M,t}}+\eta^e_{1c,t+h}(h).\tag{3.33}$$

Denote the population coefficients of determination by $R^2(h,C/P_M)$ *and* $R_e^2(h,C/P_M)$. *Then,*

$$b_{1c}(h)=\frac{Cov\left(R_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)}{Var\left[\frac{C_t}{P_{M,t}}\right]},\quad b^e_{1c}(h)=\frac{Cov\left(R^e_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)}{Var\left[\frac{C_t}{P_{M,t}}\right]},\tag{3.34}$$

$$R^2(h,C/P_M)=\frac{\left(Cov\left(R_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)\right)^2}{Var[R_{t+1:t+h}]Var\left[\frac{C_t}{P_{M,t}}\right]},\text{ and } R_e^2(h,C/P_M)=\frac{\left(Cov\left(R^e_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)\right)^2}{Var[R^e_{t+1:t+h}]Var\left[\frac{C_t}{P_{M,t}}\right]},\tag{3.35}$$

where $Cov\left(R_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)$, $Cov\left(R^e_{t+1:t+h},\frac{C_t}{P_{M,t}}\right)$,$Var\left[\frac{C_t}{P_{M,t}}\right]$, $Var[R_{t+1:t+h}]$ *and* $Var[R^e_{t+1:t+h}]$ *are given in (3.20), (3.22), (3.18), (3.23) and (3.27) respectively.*

The two previous propositions characterize the predictive ability of the dividend-price and consumption-price ratios. However, it is common in the literature to use jointly these two variables in the predictive regressions. The following proposition characterizes the joint predictive ability of the dividend-price and consumption-price ratios. However, we will not study in the current version of the paper the empirical counterpart of these joint predictive ability.

Proposition 3.5 ***Regression of the Aggregated Returns onto the Dividend-price and Consumption-Price ratios and a constant.*** *Define the population regressions*

$$\begin{aligned} R_{t+1:t+h} &= \tilde{a}_1(h) + \left(\frac{D_t}{P_t}, \frac{C_t}{P_{M,t}}\right)\tilde{b}_1(h) + \tilde{\eta}_{1,t+h}(h), \\ R^e_{t+1:t+h} &= \tilde{a}^e_1(h) + \left(\frac{D_t}{P_t}, \frac{C_t}{P_{M,t}}\right)\tilde{b}^e_1(h) + \tilde{\eta}^e_{1,t+h}(h). \end{aligned} \tag{3.36}$$

Denote the population coefficients of determination by $R^2(h, D/P, C/P_M)$ *and* $R^2_e(h, D/P, C/P_M)$. *Then,*

$$\tilde{b}_1(h) = \Omega^{-1}\left(Cov\left(R_{t+1:t+h}, \frac{D_t}{P_t}\right), Cov\left(R_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right)\right)^\top, \tag{3.37}$$

$$\tilde{b}^e_1(h) = \Omega^{-1}\left(Cov\left(R^e_{t+1:t+h}, \frac{D_t}{P_t}\right), Cov\left(R^e_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right)\right)^\top, \tag{3.38}$$

$$R^2(h, D/P, C/P_M) = \frac{\tilde{b}_1(h)^\top\ \Omega\ \tilde{b}_1(h)}{Var[R_{t+1:t+h}]}, \text{ and } R^2_e(h, D/P_M, C/P_M) = \frac{\tilde{b}^e_1(h)^\top\ \Omega\ \tilde{b}^e_1(h)}{Var[R^e_{t+1:t+h}]} \tag{3.39}$$

where $Cov\left(R_{t+1:t+h}, \frac{D_t}{P_t}\right)$, $Cov\left(R_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right)$, $Cov\left(R^e_{t+1:t+h}, \frac{C_t}{P_{M,t}}\right)$, $Var\left[\frac{D_t}{P_t}\right]$, $Var\left[\frac{C_t}{P_{M,t}}\right]$, $Var[R_{t+1:t+h}]$ *and* $Var[R^e_{t+1:t+h}]$ *are given in (3.19), (3.20), (3.21), (3.22), (3.18), (3.18), (3.23) and (3.27) respectively, while the matrix* Ω *is defined by*

$$\Omega = \begin{bmatrix} Var[\frac{D_t}{P_t}] & Cov[\frac{D_t}{P_t}, \frac{C_t}{P_{M,t}}] \\ Cov[\frac{D_t}{P_t}, \frac{C_t}{P_{M,t}}] & Var[\frac{C_t}{P_{M,t}}] \end{bmatrix} \tag{3.40}$$

where

$$Cov\left[\frac{D_t}{P_t}, \frac{C_t}{P_{M,t}}\right] = \lambda_2^\top Var[\zeta_t]\lambda_{2c}. \tag{3.41}$$

3.5 Predictability of Consumption Volatility

Bansal and Yaron (2004) provide empirical evidence for fluctuating consumption volatility. They also provide some evidence that realized consumption volatility predicts and is predicted by the price– dividend ratio.

We start this subsection by characterizing some moments and then we will study the predictability of the aggregate consumption volatility in a subsequent proposition. The consumption variance σ^2_{ct} defined in (2.1) equals $\omega_c^T\zeta_t$. Therefore, we have:

Proposition 3.6 *We have*

$$Cov\left(\sigma^2_{c,t+1:t+h}, \frac{D_t}{P_t}\right) = \omega_{ch}^T Var\left[\zeta_t\right]\lambda_2 \tag{3.42}$$

$$Cov\left(\sigma^2_{c,t+1:t+h}, \frac{C_t}{P_{M,t}}\right) = \omega_{ch}^T Var\left[\zeta_t\right]\lambda_{2c}, \tag{3.43}$$

where

$$\omega_{ch} = \left(\sum_{j=1}^{h} P^j\right)^T \omega_c. \tag{3.44}$$

In addition,

$$Var\left[\sigma^2_{c,t+1:t+h}\right] \;\; = \;\; \omega_c^T Var\left[\zeta_{t+1:t+h}\right]\omega_c \tag{3.45}$$

where

$$Var\left[\zeta_{t+1:t+h}\right] = \left(hI + 2\sum_{j=2}^{h}\left(h-j+1\right)P^{j-1}\right) Var\left[\zeta_t\right]. \tag{3.46}$$

We are now able to study the predictability of the aggregate consumption volatility:

Proposition 3.7 ***Regression of Aggregate Consumption Volatility onto Dividend-Price Ratio.*** *Define the population regression*

$$\sigma^2_{c,t+1:t+h} = a_3\left(h\right) + b_3\left(h\right)\frac{D_t}{P_t} + \eta_{3,t+h}\left(h\right), \tag{3.47}$$

and denote the population coefficient of determination by $R^2\left(h, \sigma_c^2, \frac{D}{P}\right)$. Then,

$$b_3\left(h\right) \;\; = \;\; \frac{Cov\left(\sigma^2_{c,t+1:t+h}, \frac{D_t}{P_t}\right)}{Var\left[\frac{D_t}{P_t}\right]}, \tag{3.48}$$

$$R^2\left(h, \sigma_c^2, \frac{D}{P}\right) \;\; = \;\; \frac{\left(Cov\left(\sigma^2_{c,t+1:t+h}, \frac{D_t}{P_t}\right)\right)^2}{Var\left[\sigma^2_{c,t+1:t+h}\right] Var\left[\frac{D_t}{P_t}\right]}, \tag{3.49}$$

where $Cov\left(\sigma^2_{c,t+1:t+h}, \frac{D_t}{P_t}\right)$, $Var\left[\sigma^2_{c,t+1:t+h}\right]$, and $Var[\frac{D_t}{P_t}]$ are given by (3.42), (3.45), and (3.18) respectively.

We can also characterize the predictive ability of the consumption-price ratio:

Proposition 3.8 ***Regression of Aggregate Consumption Volatility onto Consumption-Price Ratio.*** *Define the population regression*

$$\sigma^2_{c,t+1:t+h} = a_{3c}(h) + b_{3c}(h)\frac{C_t}{P_{M,t}} + \eta_{3c,t+h}(h), \tag{3.50}$$

and denote the population coefficient of determination by $R^2\left(h, \sigma^2_c, \frac{C}{P_M}\right)$. *Then,*

$$b_{3c}(h) = \frac{Cov\left(\sigma^2_{c,t+1:t+h}, \frac{C_t}{P_{M,t}}\right)}{Var\left[\frac{C_t}{P_{M,t}}\right]}, \tag{3.51}$$

$$R^2\left(h, \sigma^2_c, \frac{C}{P_M}\right) = \frac{\left(Cov\left(\sigma^2_{c,t+1:t+h}, \frac{D_t}{P_t}\right)\right)^2}{Var\left[\sigma^2_{c,t+1:t+h}\right] Var\left[\frac{D_t}{P_t}\right]}, \tag{3.52}$$

where $Cov\left(\sigma^2_{c,t+1:t+h}, \frac{C_t}{P_{M,t}}\right)$, $Var\left[\sigma^2_{c,t+1:t+h}\right]$, *and* $Var[\frac{C_t}{P_{M,t}}]$ *are given by (3.43), (3.45), and (3.18) respectively.*

The two previous propositions characterize the predictive ability of the dividend-price and consumption-price ratios in forecasting aggregate volatility. We will study in the next version of the paper the joint ability of the two ratios in predicting consumption volatility.

4 Solving for the Price-Dividend Ratio in Asset Pricing Models

The benchmark model for equilibrium consumption-based asset pricing is the Lucas (1978) model. We will reserve below the acronym CCAPM for this model. It will appear as a particular case of the so-called Epstein and Zin (1989) model that we will analyze in depth in this paper. In fact this model is a particular case of the general recursive specification used by Epstein and Zin (1989) in which a representative agent derives his utility by combining current consumption with a certainty equivalent of future utility through an aggregator. Depending on how this certainty equivalent is specified, the recursive utility concept can accommodate several classes of preferences. A class that is used extensively used in empirical work is the so-called Kreps-Porteus, where the certainty equivalent conforms with expected utility for ranking timeless gambles, but with a different parameter than the aggregator's parameter. This is what it is usually called the Epstein and Zin (1989) model. We will keep below with this tradition.[3]

[3]Epstein and Zin (1989) go further by integrating in a temporal setting a large class of atemporal non-expected utility theories, in particular homogeneous members of the class introduced by Chew (1985) and

Another very influential model is the Campbell and Cochrane (1999) model which extends the basic external habit formation literature. In habit formation models, an investor derives utility not from the absolute level of consumption but from its level relative to a benchmark which is related to past consumption.[4] When this reference level depends on past aggregate per capita consumption, the *catching up* with the Joneses specification of Abel (1990), or on current per capita consumption, the *keeping up* with the Joneses of Abel (1999)[5], it captures the idea that the individual wants to maintain his relative status in the economy. Campbell and Cochrane (1999) specify a slow-moving habit and impose a nonlinear dynamics on the surplus consumption with respect to the habit.

The main goal of this section is to characterize the vectors λ_1, λ_{1c} and b defined in (3.4), (3.5) and (3.6) as function of the parameters describing the dynamics of the consumption and dividend growths and the utility function of the representative agent. We provide analytical formulas for these quantities for the three models just described : CCAPM (Lucas, 1978), Epstein and Zin (1989) and Campbell and Cochrane (1999).

4.1 The Price-Dividend Ratio in the CCAPM

4.1.1 Consumption equals dividend

We start by assuming that the consumption equals the dividend as in Lucas (1978), which implies

$$\mu_c = \mu_d, \quad \omega_c = \omega_d, \quad \rho = (1, 1, ..., 1)^\top. \tag{4.1}$$

Proposition 4.1 ***Characterization of the Asset Prices.*** *We have*

$$\frac{P_t}{D_t} = \delta e^\top \left[Id - \delta A((1-\gamma)\mu_d + (1-\gamma)^2 \omega_d/2) \right]^{-1} \exp\left(((1-\gamma)\mu_d + (1-\gamma)^2\omega_d/2)^\top \zeta_t \right) \zeta_t, \tag{4.2}$$

where the matrix $A(\cdot)$ is defined in (2.7). Consequently, the i-th component, i=1,...,N, of the vector λ_1 defined in (3.4) are given by

$$\lambda_{1,i} = \delta \exp((1-\gamma)\mu_{d,i} + (1-\gamma)^2 \omega_{d,i}/2) e^\top \left[Id - \delta A((1-\gamma)\mu_d + (1-\gamma)^2 \omega_d/2) \right]^{-1} e_i. \tag{4.3}$$

Dekel (1986). The certainty equivalent is then defined implicitly. It includes in particular a disappointment aversion specification, see Bonomo and Garcia (1994).

[4]See among others Abel (1990, 1996), Campbell and Cochrane (1999), Constantinides (1990), Ferson and Constantinides (1991), Heaton (1995), and Sundaresan (1989).

[5]It generalizes Gali's (1994) specification of consumption externalities whereby agents have preferences defined over their own consumption as well as current per capita consumption in the economy.

In addition, $\lambda_{1c} = \lambda_1$ while the i-th component of the vector b defined in (3.6) is given by

$$b_i = \delta^{-1} \exp(\gamma \mu_{c,i} - \frac{\gamma^2}{2} \omega_{c,i}). \tag{4.4}$$

The formulas in the previous proposition are not new. Cecchetti, Lam and Mark (1990) derived them for homoskedastic models while Bonomo and Garcia (1993) did it for the same model as us. It is also worth noting that the matrix $[Id - \delta A((1-\gamma)\mu_d + (1-\gamma)^2 \omega_d/2)]$ might be singular or leads to negative prices for some parameters (of the consumption growth and utility function). Such cases happen when the maximization problem does not admit a solution. We will see in the results that such examples happen and that one can detect them. Note however that an approximation of the model (e.g., log-linearization) may lead to different results, for instance, provide prices that make sense while the true maximization problem does not admit a solution. We will discuss in more details this issue in the next version of the paper.

4.1.2 Consumption and dividend are different

Here, we still consider the CCAPM model but we assume that consumption and dividend are different. In the following proposition, we use the vectors μ_{cd} and ω_{cd} defined by

$$\mu_{cd} = -\gamma \mu_c + \mu_d, \quad \omega_{cd} = \gamma^2 \omega_c + \omega_d - 2\gamma \left(\rho \odot (\omega_c)^{1/2} \odot (\omega_d)^{1/2} \right).$$

Proposition 4.2 ***Characterization of the Asset Prices.*** *The i-th component, i=1,...,N, of the vector λ_1 defined in (3.4) is given by*

$$\lambda_{1,i} = \delta \exp(\mu_{cd,i} + \omega_{cd,i}/2)) e^{\top} \left[Id - \delta A(\mu_{cd} + \omega_{cd}/2) \right]^{-1} e_i, \tag{4.5}$$

where $A(\cdot)$ is defined in (2.7). In addition, the i-th component, i=1,...,N, of the vector λ_{1c} defined in (3.5) is given by

$$\lambda_{1c,i} = \delta \exp\left((1-\gamma)\mu_{c,i} + \frac{(1-\gamma)^2}{2} \omega_{c,i} \right) e^T \left[Id - \delta A \left((1-\gamma)\mu_c + \frac{(1-\gamma)^2}{2} \omega_c \right) \right]^{-1} e_i \tag{4.6}$$

Finally, the components of the vector b defined in (3.6) are given by

$$b_i = \delta^{-1} \exp(\gamma \mu_{c,i} - \frac{\gamma^2}{2} \omega_{c,i}). \tag{4.7}$$

4.2 The Epstein and Zin (1989) Model

The stochastic discount factor of Epstein and Zin (1989) model given by

$$E\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{C,t+1}^{-(1-\theta)}\ R_{i,t+1} \mid J_t\right]=1, \tag{4.8}$$

where

$$\theta=\frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{4.9}$$

4.2.1 Market Return

We start our analysis by characterizing the vector λ_{1c} defined in (3.5) that characterizes the consumption price ratio. The characterization of this vector is the main difference between Epstein-Zin and CCAPM models. We will show below that when one has the vector λ_{1c}, one gets the vectors dividend price ratio (i.e. the vector λ_1) and the risk-free rate (i.e., the vector b) as for the CCAPM.

Writing equation (4.8) for the return on the unobservable market portfolio which pays off aggregate consumption, one obtains:

$$E\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{C,t+1}^{\theta}\ \mid J_t\right]=1. \tag{4.10}$$

The following proposition characterizes the vector λ_{1c}.

Proposition 4.3 *The components $\lambda_{1c,i}$, i=1,...,N, of the vector λ_{1c} are the solution of the following nonlinear equation*

$$\lambda_{c1,i}=\delta\ \left(\sum_{j=1}^{N} p_{ij}(\lambda_{c1,j}+1)^{\theta}\right)^{\frac{1}{\theta}}\quad \exp\left(\left(1-\frac{1}{\psi}\right)\mu_{c,i}+(1-\gamma)\left(1-\frac{1}{\psi}\right)\frac{\omega_{c,i}}{2}\right). \tag{4.11}$$

Equation (4.11) is highly nonlinear when $\theta\neq 1$, that is, when Epstein-Zin model is not the CCAPM. Consequently, one cannot get an analytic formula for the components of λ_{1c}. However, it is easy to solve it numerically by using numericla algorithms. We did by using the nonlinear equation solver in GAUSS.

One can also use some approximations instead of using numerical methods. The first simple approximation is to linearize this function around $\lambda^{*}e$ where λ^{*} is a positive number. It leads to

$$\left(\sum_{j=1}^{N} p_{ij}\left(\lambda_{c1,j}+1\right)^{\theta}\right)^{\frac{1}{\theta}}\approx\sum_{j=1}^{N} p_{ij}\left(\lambda_{c1,j}+1\right). \tag{4.12}$$

Consequently, one gets for $i = 1, ..., N$,

$$\lambda_{1c,i} = \delta \exp\left(\frac{(1-\gamma)}{\theta}\mu_{c,i} + \frac{(1-\gamma)^2}{2\theta}\omega_{c,i}\right) e^T \left[Id - \delta A\left(\frac{(1-\gamma)}{\theta}\mu_c + \frac{(1-\gamma)^2}{2\theta}\omega_c\right)\right]^{-1} e_i. \tag{4.13}$$

Another approximation is the log-linearization of Campbell and Shiller, which leads to

$$r_{C,t+1} = \ln R_{C,t+1} \approx k_0 + k_1 v_{c1}^T \zeta_{t+1} - v_{c1}^T \zeta_t + \Delta c_{t+1}. \tag{4.14}$$

Consequently, one gets

$$\lambda_{1c,i} = \exp\left(v_{c1,i}\right), \ i = 1, .., N \tag{4.15}$$

where

$$v_{c1,i} = (\ln\delta + k_0) + \frac{(1-\gamma)}{\theta}\mu_{c,i} + \frac{(1-\gamma)^2}{2\theta}\omega_{c,i} + \frac{1}{\theta}\ln\left[\sum_{j=1}^{N} p_{ij}\exp\left(\theta k_1 v_{c1,j}\right)\right].$$

These approximations can also serve to obtain starting values for a numerical algorithm.

4.2.2 Equity Return

Interestingly, when one has the consumption price ratio, i.e., the vector λ_{1c}, one gets analytically the dividend price ratio:

Proposition 4.4 *We have*

$$\lambda_1 = [Id - C]^{-1} V \tag{4.16}$$

where

$$C = [c_{ij}], \quad c_{ij} = \delta^\theta (\lambda_{c2,i})^{\theta-1} \exp((\mu_{cd,i} + \omega_{cd,i}/2))\lambda_{c3,j}^{(\theta-1)} p_{ij} \tag{4.17}$$

$$V = [v_i], \quad v_i = \delta^\theta (\lambda_{c2,i})^{\theta-1} \exp((\mu_{cd,i} + \omega_{cd,i}/2))\left(\sum_{j=1}^{N} \lambda_{c3,j}^{(\theta-1)} p_{ij}\right).$$

Likewise, one can also use the log-linearization method to get the dividend price ratio. We present below the formulas when one uses the log-linearization for the equity return (simple log-linearization) and one uses the log-linearization for both the market and equity returns (double log-linearization).

In the double log-linearization[6], one gets

$$\lambda_{1,i} = \exp\left(v_{1,i}\right), \ i = 1, .., N \tag{4.19}$$

[6]The log-linearization of the equity return is given by:

$$r_{t+1} = \ln R_{t+1} \approx k_{m0} + k_{m1} v_1^T \zeta_{t+1} - v_1^T \zeta_t + \Delta d_{t+1}. \tag{4.18}$$

where

$$v_{1,i} = \theta \ln \delta + (\theta - 1) k_0 + k_{m0} - (\theta - 1) v_{c1,i} + \mu_{cd,i} + \frac{1}{2}\omega_{cd,i}$$
$$+ \log \left[\sum_{j=1}^{N} p_{ij} \exp\left((\theta - 1) k_1 v_{c1,j} + k_{m1} v_{1,j} \right) \right].$$

In contrast, the simple log-linearization leads to

$$\lambda_{1,i} = \exp\left(v_{1,i}\right), \ i = 1, .., N \tag{4.20}$$

where

$$v_{1,i} = \theta \ln \delta + k_{m0} + \mu_{cd,i} + \frac{1}{2}\omega_{cd,i} + \ln \left[\sum_{j=1}^{N} p_{ij} \left(\frac{\lambda_{c1,j} + 1}{\lambda_{c1,i}} \right)^{\theta - 1} \exp\left(k_{m1} v_{1,j}\right) \right].$$

4.2.3 Risk-Free Rate

The i-th component, i=1,...,N, of the vector b defined in (3.6) is given by

$$b_i = \delta^{-\theta} \lambda_{1c,i}^{\theta-1} \exp(\gamma \mu_{c,i} + \frac{\gamma^2}{2} \omega_{c,i}) \left[\sum_{j=1}^{N} (\lambda_{1c,j} + 1)^{\theta - 1} \right]^{-1}. \tag{4.21}$$

4.3 The Campbell and Cochrane (1999) Model

The Euler equation in Campbell and Cochrane (1999) is given by

$$1 = E\left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{S_{t+1}}{S_t} \right)^{-\gamma} \ R_{t+1} \mid J_t \right]. \tag{4.22}$$

If we assume that,

$$x_{s,t+1} \equiv \log(S_{t+1}) - \log(S_t) = \mu_s^\top \zeta_t + (\omega_s^\top \zeta_t)^{1/2} \varepsilon_{c,t+1} \tag{4.23}$$

where

$$\begin{pmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{d,t+1} \\ \varepsilon_{s,t+1} \end{pmatrix} \mid \sigma(\varepsilon_{c,\tau}, \varepsilon_{d,\tau}, \varepsilon_{s,\tau}, \tau \le t; \zeta_m, m \in \mathbf{Z}) \ \sim \ \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho^\top \zeta_t & \rho_{cs}^\top \zeta_t \\ \rho^\top \zeta_t & 1 & \rho_{ds}^\top \zeta_t \\ \rho_{cs}^\top \zeta_t & \rho_{ds}^\top \zeta_t & 1 \end{bmatrix} \right). \tag{4.24}$$

Observe that Campbell and Cochrane assumed

$$\rho_{cs} = e = (1, 1, ..., 1)^\top \ \text{ and } \ \rho_{ds} = \rho. \tag{4.25}$$

Under (4.24), (4.22) becomes

$$1 = E\left[\delta \exp(-\gamma x_{cs,t+1})\ R_{t+1} \mid J_t\right]. \tag{4.26}$$

with

$$x_{cs,t+1} \equiv x_{c,t+1} + x_{s,t+1} = \mu_{cs}^\top \zeta_t \ + \ (\omega_{cs}^\top \zeta_t)^{1/2}\ \varepsilon_{cs,t+1} \tag{4.27}$$

where

$$\mu_{cs} = \mu_c + \mu_s,\ \omega_{cs} = \omega_c + \omega_d + 2\rho_{cs} \odot (\omega_c)^{1/2} \odot (\omega_s)^{1/2}, \tag{4.28}$$

$$\varepsilon_{cs,t+1} \mid \sigma(\varepsilon_{c,\tau}, \varepsilon_{d,\tau}, \varepsilon_{x,\tau}, \tau \le t; \zeta_m, m \in \mathbf{Z}) \quad \sim \quad \mathcal{N}(0,1).$$

Consequently, their model is like a CCAPM model where one has x_{cs} in the SDF instead of having x_c. In order to compute the price-payoff ratios with these preferences, it is important to derive the joint dynamics of $(\varepsilon_{cs,t+1}, \varepsilon_{d,t+1}, \varepsilon_{c,t+1})^\top$. It will then be sufficient to plug these formulas in the CCAPM model to derive the vectors λ.
We have

$$\begin{pmatrix} \varepsilon_{cs,t+1} \\ \varepsilon_{d,t+1} \\ \varepsilon_{c,t+1} \end{pmatrix} \mid \sigma(\varepsilon_{c,\tau}, \varepsilon_{d,\tau}, \varepsilon_{s,\tau}, \tau \le t; \zeta_m, m \in \mathbf{Z}) \ \sim \ \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho_{csd}^\top \zeta_t & \rho_{csc}^\top \zeta_t \\ \rho_{csd}^\top \zeta_t & 1 & \rho^\top \zeta_t \\ \rho_{csc}^\top \zeta_t & \rho^\top \zeta_t & 1 \end{bmatrix}\right), \tag{4.29}$$

where

$$\rho_{csc} = (w_c + \rho_{cs} \odot w_c^{1/2} \odot w_s^{1/2}) \odot w_c^{-1/2} \odot \omega_{cs}^{-1/2}, \tag{4.30}$$

$$\rho_{csd} = (\rho_{cd} \odot w_c^{1/2} \odot w_d^{1/2} + \rho_{ds} \odot w_d^{1/2} \odot w_s^{1/2}) \odot w_d^{-1/2} \odot \omega_{cs}^{-1/2}. \tag{4.31}$$

Define

$$\mu_{csd} = -\gamma \mu_{cs} + \mu_d,\ \omega_{csd} = \gamma^2 \omega_{cs} + \omega_d - 2\gamma(\rho_{csd} \odot \omega_{cs}^{1/2} \odot \omega_d^{1/2}) \tag{4.32}$$

$$\mu_{csc} = -\gamma \mu_{cs} + \mu_c =,\ \omega_{csc} = \gamma^2 \omega_{cs} + \omega_c - 2\gamma(\rho_{csc} \odot \omega_{cs}^{1/2} \odot \omega_c^{1/2}) \tag{4.33}$$

Proposition 4.5 ***Characterization of the Asset Prices.*** *The i-th component, i=1,...,N, of the vector λ_1 defined in (3.4) is given by*

$$\lambda_{1,i} = \delta \exp(\mu_{csd,i} + \omega_{csd,i}/2)) e^\top \left[Id - \delta A(\mu_{csd} + \omega_{csd}/2)\right]^{-1} e_i, \tag{4.34}$$

where $A(\cdot)$ is defined in (2.7). In addition, the i-th component, i=1,...,N, of the vector λ_{1c} defined in (3.5) is given by

$$\lambda_{1c,i} = \delta \exp(\mu_{csc,i} + \omega_{csc,i}/2)) e^{\top} \left[Id - \delta A(\mu_{csc} + \omega_{csc}/2) \right]^{-1} e_i, \tag{4.35}$$

Finally, the components of the vector b defined in (3.6) are given by

$$b_i = \delta^{-1} \exp(\gamma \mu_{cs,i} - \frac{\gamma^2}{2} \omega_{cs,i}). \tag{4.36}$$

5 Applications to Models of the Post-War US Economy

In this section, we apply the derived formulas in three contexts. First, we estimate a Markov-switching model directly on the quarterly growth rates of real consumption and dividend per capita for the US postwar period. Then we can apply the formulas derived in the two previous sections for the CCAPM and the Epstein-Zin model. In a second application, we analyze the Markov-switching model with Epstein and Zin (1989) preferences proposed by Lettau, Ludvigson and Wachter (2004). In the last application, we calibrate a Markov-switching model in order to match the endowment process used by Bansal and Yaron (2004). The goal is to easily compute population values for several statistics that have been obtained by numerical techniques or by simulation, as well as to produce results for a larger parameter set than the one in the last two papers. This way we will hopefully better understand the economic intuition behind results and assess robustness to changes in the values of preference and endowment parameters.

5.1 A Two-State Markov Switching Model with Epstein-Zin Preferences

We start with a simple model, a two-state Markov switching model in both means and variances previously estimated by Bonomo and Garcia (1994, 1996) with annual secular data on consumption and dividends. The estimated parameters are reported in table 2. The first state is a low-mean high-variance state for consumption. Dividend growth is also low in this state while variance is not very different form the variance in the high state. Both states have about the same degree of persistence and consequently the unconditional probabilities are close to 0.5.

In table (3), we report the asset pricing implications of this endowment when the agent has Epstein-Zin preferences. As expected, a high risk aversion is needed to arrive at equity premium values comparable to what is observed in the data. The equity premium increases in both risk and intertemporal substitution, while the risk-free decreases sharply with the elasticity of intertemporal substitution. The reduction of the interest rate may come either from the variance of the market portfolio if θ is negative or from the variance of consumption if θ is positive. To see that, it is easier to look at the Euler condition in a model with jointly lognormal and homoskedastic asset returns and consumption, where the risk-free interest is given by:

$$r_{f,t+1} = -log\delta + \frac{1}{\psi}E_t[\Delta c_{t+1}] + \frac{\theta - 1}{2}\sigma_w^2 - \frac{\theta}{2\psi^2}\sigma_c^2 \tag{5.1}$$

The sign of θ is determined, for a given γ, by the value of ψ. If ψ is less than one, θ is positive, if it is greater than one, θ is negative.

It is interesting to note that the expected price-dividend ratio takes very large values when γ is 10 and ψ is greater than one. These values reflect a lack of convergence. The matrix C in (4.17) becomes nearly singular and this inflates the value of the price-dividend ratio.

The value of the volatility of the dividend-price ratio appears to be very high for all preference parameter pairs and it increases with the risk aversion.

We compute the R^2 of the regressions of multiperiod future returns on the current dividend-price or consumption-price ratio but we do not find any significant predictability at any horizon for any pair of preference parameters. Neither can this model reproduce the negative autocorrelation observed in both returns and excess returns as reported in table (1).

In the next two sections we will look at two models that have been proposed recently by Bansal and Yaron (2004) and Lettau, Ludvigson and Wachter (2005) to advocate the determining role of economic uncertainty (volatility of consumption) in the formation of asset prices. The latter model uses a Markov-switching endowment process and Epstein-Zin preferences. It is therefore a direct application of our framework and we will be able to compute directly all quantities of interest analytically. In the former model, the endowment follows an autoregressive process, but the preferences are also based on Epstein and Zin (1989). We will see how to set this model in our framework by matching the autoregressive endowment process with a Markov-switching process.

5.2 The Lettau, Ludvigson and Wachter (2005) Model

The endowment process in this paper is a constrained version of the general process (2.1), (2.2). They assume a consumption process (2.1) where the mean and the variance are governed by two different Markov chains. For the dividend process they simply assume that $D_t = (C_t)^\lambda$. Therefore the mean and the standard deviation of dividend growth is simply

λ times the mean and the standard deviation of consumption growth, and the correlation parameter is one. We report in table (4) the corresponding values of the resulting four-state Markov chain based on the estimates reported in their paper.

In their model, they assume that investors do not know the state they are in but they know the parameters of the process. Therefore at each period they update their estimate of the probability of being in a state given their current information. In other words they compute filtered probabilities. Based on the latter, they compute numerically the price-consumption and price-dividend ratios that are solutions of the Euler conditions of the equilibrium model. We have seen that given the parameters of the endowment process we could calculate the price-consumption ratio by solving a nonlinear equation and the price-dividend ratio analytically in the Epstein-Zin model. Following this procedure at each point in time by using the filtered probabilities for the Markov chain along the way, we can reproduce easily the full trajectory of the price-dividend ratio. We intend to carry out this exercise in future versions of this paper. Instead we will assume that investors know the state and compute the various statistics corresponding to the stylized facts we presented earlier.

Since Lettau, Ludvigson and Wachter (2005) focused on the trajectory of the price-dividend ratio and its relationship with consumption volatility, they did not report the values for these statistics and the sensitivity of the various quantities to the values of preference parameters. We include a large set of preference parameters to see how the various economic and financial quantities change as a function of preference parameters.

5.2.1 Asset Pricing Implications for the LLW Model

We report in table 5 the values of the first two moments of the equity premium and the risk-free rate, as well as the means of the price-dividend and the price-consumption ratios and the standard deviations of the consumption-price and the dividend-price ratios. We have limited the risk aversion parameter γ to this range of values because for values below 15 we obtain negative prices for large ψ values and for values above 30 we start having problems solving the nonlinear system for the price-consumption ratios.

Several comments can be made. While the equity premium can be matched with a risk aversion of 25 to 30, the risk-free rate remains high. Negative prices appear with a γ of 15 and even at 20 convergence problems occur. The expected value of the price-dividend ratio takes very large values. At around a maximum of 11 percent, the volatility of the equity premium is low compared to the data, but the volatility of the risk-free rate matches well the actual value.

A comparison with the previous two-state model is instructive. While a higher risk aversion is needed to increase the equity premium it matches better the level of the risk-free rate and the volatility of the equity premium and produces less convergence problems at similar levels of risk aversion. The key parameters to understand these differences are the mean and volatility of dividend growth. Limited at 13.5 percent in the high-volatility state (a direct result of setting λ to 4.5), the volatility is much lower than the 20 percent estimated with the dividend data. Moreover it falls at around 7 percent in the low volatility state. In the two-state model it remained at 16 percent. For the mean, it is the same multiple of the mean of consumption growth in low and high states. This does not seem to be coherent with the data, especially in the high mean state. This state is the most frequently visited with an overall probability of 86 percent. We will come back to these remarks later when we analyze the Bansal and Yaron (2004) model. It will be also a four-state model but the parameters of the dividend process will be based on the data.

5.2.2 Asset Returns and Consumption Volatility Predictability in the LLW Model

We report the R^2 values of the regression of future returns on the current consumption-price ratio in table 6 and the same regression on the current dividend-price ratio in table 7. Before we compare the results with the data, it is important to emphasize that the statistics we compute in a quarterly model is the predictability of future returns at several horizons (in the table we report 1 to 20 years) based on the current quarterly price-dividend ratio, that is computed with the dividend of the current quarter. In the regressions we carried out in the data and reported in table 1, the independent variable was a price-dividend ratio with dividends cumulated over a year. This adds persistence to the regressor and increases the R^2 of the regression. However this difference will not affect our ability to detect the ability of a model to generate predictability. To say it in a few words, the models do not seem to produce predictability at any horizon for any parameter configuration.

It is not the case with excess returns. We report the R^2 values of the regression of future excess returns on the current consumption-price ratio in table 8 and the same regression on the current dividend-price ratio in table 9. Even if is not very high, there is a non-negligible predictability, which increase with risk aversion. The fact that dividends are perfectly correlated with consumption plays certainly a role in the higher predictability for excess returns than for returns.

The other important predictability concerns the volatility of consumption, which plays a key role in explaining asset prices in both Lettau, Ludvigson and Wachter (2005) and

Bansal and Yaron (2004). We report the R^2 values of the regression of future consumption volatilities on the current consumption-price ratio in table 10 and the same regression on the current dividend-price ratio in table 11. As expected in this model, consumption volatility is highly predictable since both regressors depend only on the consumption states. It is more predictable by the dividend-price ratio since there is more variability in this ratio than in the consumption-price ratio.

5.2.3 Variance Ratios in the LLW Model

The last point we analyzed is the capacity of the models to produce the negative autocorrelation at long horizons. The variance ratios of returns and excess returns on the stock are reported in table 12 and table 13 respectively. When ψ is greater than one, the models are able to produce variance ratios less than one, declining with the horizon, for both returns and excess returns. For excess returns there is negative autocorrelation even for values of ψ less than one, but it is more pronounced above one.

5.3 Reproducing the Bansal and Yaron (2004) Model with a Markov-Switching Model

The model of Bansal and Yaron (2004) for the endowment is:

$$\begin{aligned} x_{t+1} &= (1-\rho_x)\,\mu_x + \rho_x x_t + \varphi_e \sqrt{h_t} e_{t+1} && (5.2) \\ h_{t+1} &= (1-\nu_1)\,\sigma^2 + \nu_1 h_t + \sigma_w w_{t+1} && (5.3) \\ x_{c,t+1} &= x_t + \sqrt{h_t}\eta_{t+1} && (5.4) \\ x_{d,t+1} &= \mu_{xd} + \phi\,(x_t - \mu_x) + \varphi_d \sqrt{h_t} u_{t+1} && (5.5) \end{aligned}$$

with $e_{t+1}, w_{t+1}, \eta_{t+1}, u_{t+1} \sim N.i.i.D.(0,1)$.

Our goal here is to characterize a Markov Switching (MS) model described in Section 2 that has the same features than the endowment model chosen by Bansal and Yaron (2004). The main characteristics of the later endowments are: 1) The expected means of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted x_t. 2) The conditional variances of the consumption and dividend growth rates are a linear function of the same autoregressive process of order one denoted h_t. 3) The variables x_{t+1} and h_{t+1} are independent conditionally to their past. 4) The innovations of the consumption and dividend growth rates are independent given the state variables.

5.3.1 Characterizing the Matching Markov-Switching Model

In the MS case, the first characteristic of Bansal and Yaron (2004) Model implies that one has to assume that the expected means of the consumption and dividend growth rates are a linear function of the same Markov chain with two states given that a two-state Markov chain is an AR(1) process. Likewise, the second one implies that the conditional variances of the consumption and dividend growth rates are a linear function of the same two-state Markov chain. The third characteristic implies that the mean and variance Markov chains should be independent. Consequently, we should assume that the Markov chain described in Section 2 has 4 states, two states for the means and two states for the variances and that the transition matrix P is restricted such as the means and variance states are independent; see Table 4. Finally, the last characteristic implies that the vector ρ defined in (2.3) equals zero.

In the rest of this subsection, our goal is two approximate an AR(1) process, say z_t, like x_t or h_t by a two states Markov chain. Without loss of generality, we assume that the Markov chain y_t takes the values 0 (first state) and 1 (second state) while the transition matrix P_y is given by

$$P_y^\top = \begin{pmatrix} p_{y,11} & 1-p_{y,11} \\ 1-p_{y,22} & p_{y,22} \end{pmatrix}.$$

The stationary distribution is

$$\pi_{y,1} = P(y=0) = \frac{1-p_{y,22}}{2-p_{y,11}-p_{y,22}}, \quad \pi_{y,2} = P(y=1) = \frac{1-p_{y,11}}{2-p_{y,11}-p_{y,22}} \tag{5.6}$$

In addition, we assume that $z_t = a + by_t$. Without loss of generality, we assume that $b > 0$, that is, the second state corresponds to this high value of z_t. Our goal is to characterize the vector $\theta = (p_{y,11}, p_{y,22}, a, b)^\top$ that matches the characteristic of the process z_t. The first characteristics that we want to match or the mean, the variance and the first order autocorrelation of the process z_t denoted μ_z, σ_z^2 and ρ_z respectively. Given that the dimension of θ is four, another restriction is needed. For instance, Mehra and Prescott (1985) assumed $p_{y,11} = p_{y,22}$. In contrast, we will focus on matching the kurtosis of the process z_t denoted κ_z. We will show below that matching the mean, variance, kurtosis and first autocorrelation does not fully identify the parameters. However, knowing the sign of the skewness of z_t (denotes sk_z) and the other four characteristics will fully identify the vector θ.

Proposition 5.1 Moments of a two-state Markov chain.

We have

$$\mu_z = a + b\mu_y = a + b\pi_{y,2} \tag{5.7}$$
$$\sigma_z^2 = b^2\sigma_y^2 = b^2\pi_{y,1}\pi_{y,2}$$
$$sk_z = sk_y = \left(-\frac{\pi_{y,2}}{\pi_{y,1}} + \frac{\pi_{y,1}}{\pi_{y,2}}\right)$$
$$\kappa_z = \kappa_y = \frac{\pi_{y,1}^2}{\pi_{y,2}} + \frac{\pi_{y,2}^2}{\pi_{y,1}}$$
$$\rho_z = \rho_y = p_{y,11} + p_{y,22} - 1$$

The previous proposition, combined with (5.6), characterizes the moments of a Markov chain in terms of the vector θ. As pointed out above, Mehra and Prescott (1985) assumed that $p_{y,11} = p_{y,22}$, which implies $sk_z = 0$ and $\kappa_z = 1$. The empirical evidence reported in Cecchetti, Lam and Mark (1990) suggests that the kurtosis of the expected consumption growth is higher than one and that its skewness is negative.[7]

We will now invert this characterization, that is, we will determine the vector θ in terms of the moments of z_t:

Proposition 5.2 Matching an AR(1) process by a two-state Markov chain.
We have

$$\text{if } sk_z \leq 0,\ p_{y,11} = \frac{1+\rho_z}{2} - \frac{1-\rho_z}{2}\sqrt{\frac{k_z-1}{k_z+3}},\ p_{y,22} = \frac{1+\rho_z}{2} + \frac{1-\rho_z}{2}\sqrt{\frac{k_z-1}{k_z+3}} \tag{5.8}$$
$$\text{if } sk_z > 0,\ p_{y,11} = \frac{1+\rho_z}{2} + \frac{1-\rho_z}{2}\sqrt{\frac{k_z-1}{k_z+3}},\ p_{y,22} = \frac{1+\rho_z}{2} - \frac{1-\rho_z}{2}\sqrt{\frac{k_z-1}{k_z+3}} \tag{5.9}$$

and

$$b = \frac{\sigma_z}{\sqrt{\pi_{y,1}\pi_{y,2}}},\ a = \mu_z - b\pi_{y,2} \tag{5.10}$$

where $\pi_{y,1}$ and $\pi_{y,2}$ are connected to $p_{y,11}$ and $p_{y,22}$ through (5.6).

We will now characterize the moments of the process x_t and h_t of the Bansal and Yaron (2004) model.

[7] Strictly speaking, the process x here is the expected mean of the consumption growth and not the growth. Therefore, the skewness and kurtosis of these two processes are different but connected.

Proposition 5.3 Moments of the Bansal and Yaron (2004)Model. *The mean μ_x and the first autocorrelation ρ_x of x_t are given in (5.2). The variance, skewness and kurtosis of x_t are given by*

$$\sigma_x^2 = \frac{\varphi_e^2 \sigma^2}{1-\rho_x^2},\ sk_x = 0,\ \kappa_x = 3\frac{(1-\rho_x^2)^2}{1-\rho_x^4}\left(1 + 2\frac{\rho_x^2}{1-\rho_x^2}\frac{\nu_1}{\sigma^2} + \frac{\sigma_w^2}{\sigma^2(1-\nu_1^2)}\right). \tag{5.11}$$

Likewise,

$$\mu_h = \sigma^2,\ \sigma_h^2 = \frac{\sigma^2}{1-\nu_1^2},\ sk_h = 0,\ \kappa_h = 3,\ \rho_h = \nu_1. \tag{5.12}$$

Observe that the skewness of the expected mean of the growth consumption equals zero in Bansal and Yaron (2004) model as in Mehra and Prescott (1985) model. In contrast, in order to generate a kurtosis higher than one, the Markov switching needs some skewness. Given that the skewness of consumption growth is empirically negative, we will take this identification assumption, that is, we will use (5.8) to identify the transition probabilities $p_{x,11}$ and $p_{x,22}$.

Likewise, the skewness of the variance process is zero in Bansal and Yaron (2004) model which is somewhat unrealistic given that the variance is a positive random variable. A popular variance model is the Heston (1993) model where the stationary distribution of the variance process is a Gamma distribution. Given that the skewness of a Gamma distribution is positive, we make the same assumption on h_t and we therefore use (5.9) to identify the transition probabilities $p_{h,11}$ and $p_{h,22}$.

We do have now the two independent Markov chains that generate the expected mean and variance of consumption growth. Putting together these two processes leads to a four-state Markov chain (low mean and low variance, low mean and high variance, high mean and low variance, high mean and high variance) whose transition probability matrix is given by

$$P^\top = \begin{bmatrix} p_{x,11}p_{h,11} & p_{x,11}p_{h,12} & p_{x,12}p_{h,11} & p_{x,12}p_{h,12} \\ p_{x,11}p_{h,21} & p_{x,11}p_{h,22} & p_{x,12}p_{h,21} & p_{x,12}p_{h,22} \\ p_{x,21}p_{h,11} & p_{x,21}p_{h,12} & p_{x,22}p_{h,11} & p_{x,11}p_{h,12} \\ p_{x,21}p_{h,21} & p_{x,21}p_{h,22} & p_{x,22}p_{h,21} & p_{x,22}p_{h,22} \end{bmatrix} \tag{5.13}$$

where $p_{\cdot,12} = 1 - p_{\cdot,11}$ and $p_{\cdot,21} = 1 - p_{\cdot,2}$, while the vectors μ_c, ω_c, μ_d, and ω_d defined in (2.1) and (2.2) are given by

$$\begin{aligned} \mu_c &= (a_c, a_c, a_c + b_c, a_c + b_c)^\top \\ \omega_c &= (a_h, a_h + b_h, a_h, a_h + b_h)^\top \\ \mu_d &= (\mu_{xd} - \phi\mu_x)e + \phi\mu_c \\ \omega_d &= \varphi_d^2\, \omega_c. \end{aligned} \tag{5.14}$$

The parameters of the resulting Markov-switching model are given in Table 16.

5.3.2 Reproducing the Stylized Facts

We are now able to reproduce some stylized facts that were considered in Bansal and Yaron (2004), that is the first two moments of the equity premium and the risk-free rate and some statistics about the price-dividend ratio, predictability of returns by the price-dividend ratio, predictability of the variance of consumption by the price-dividend ratio and the ratio of variances.

***(a)* Asset Pricing Implications** The set of statistics reproduced by Bansal and Yaron (2004) is given in Table 17. We present an equivalent table generated with the analytical formulas reported in the previous sections and the parameter values of the matching MS process in Table 16. We include a larger spectrum of preference parameters than in Bansal and Yaron (2004)to better understand the variation of economic and financial quantities as a function of preference parameters. To gauge the usefulness of analytical formulas it is essential to remember that in the case of the Bansal and Yaron's model, finding these quantities means either solving the model numerically for each configuration of the preference parameters or computing these quantities by simulation. Numerical solutions take time to achieve a reasonable degree of precision. For simulations, long trajectories are needed to obtain population parameters. Determining which length is appropriate is not a trivial issue, especially when coupled with time considerations.

Table 18 is based on the value of 0.998 chosen by Bansal and Yaron (2004) for the time discount parameter. We observe that the values for the first two moments of the equity premium are close to the values found by Bansal and Yaron with their model reported in 17, but the average risk-free rate is higher. Several interesting observations can be made from this table. First and foremost, the table shows clearly that it is through values greater than 1 for the ψ parameter that the equity premium puzzle is solved. The expected value of the equity return is about equal (around 9%) at $\gamma = 10$ for all values of ψ. However, the risk-free rate drops five points of percentage when ψ goes from 0.5 to 1.5. At a low risk aversion, the magnitude of the drop is less pronounced. In fact, at $\gamma = 10$ the expected value of the price-consumption ratio decreases in a significant way. A second observation concerns the price-dividend ratio. At low values of the risk aversion parameter γ the expectation of the price-dividend ratio increases significantly with the elasticity of intertemporal substitution ψ, while the volatility of the price-dividend does not change much. At low values of the risk aversion parameter γ it is exactly the opposite.

Thanks to analytical formulas it is immediate to reproduce the same table for a slightly larger δ of 0.999. The results are presented in table 19. Again several instructive conclusions can be drawn. Looking only at the moments, one does not see much difference with the previous table, except maybe for the fact that the expected risk-free rate decreases, which is an expected result. However a look at the left side of the table shows that the expected values for the price-consumption ratio and the price-dividend change drastically and take in certain configurations of the preference parameters very large implausible values.

Another interesting issue is the effect of log-linearizing the returns on the market portfolio (equation (4.15)) and on the equity (equation (4.19)). We present two sets of results. First, in table 20, we choose reasonable yet arbitrary values for the k parameters in the approximating formulas. Second, in Table 22, we set for the k parameters the values implied by the preference parameters. Indeed, in the Campbell and Shiller approximation, the parameter k is a function of the parameters and cannot be set arbitrarily [8]. With arbitrary values, the very large values present in table 19 disappear and one may think that the model is acceptable for all configurations of the preference parameters. However, when we compute the statistics with the k values corresponding to the preference parameters (reported in table 21), the values obtained for all moments are close to the analytical values shown in table 19. This illustrates the fact that log-linearizations must be conducted very carefully. One major obstacle is to compute the k parameters when one does not have analytical formulas for the price-payoff ratios. When the model is solved numerically the values have to be chosen by successive trials. The exact way to proceed and the stopping rule remain unclear to us. Therefore, even if one does not want to model the endowment with a Markov-switching structure, the values obtained with this modeling strategy for these crucial parameters could definitely help for finding the right values of the k parameters.

An important message of Bansal and Yaron (2004) concerns the role played by time-varying volatility in consumption, a proxy for economic uncertainty. To gauge the sensitivity of the results to time-varying volatility we recompute the same moments by keeping the volatility constant in the Markov-switching model. We now have a two-state model with the parameters reported in table (23). The corresponding asset return moments are given in 24. We find that they are almost identical to the results we obtained with time-varying volatility reported in table 18. This result is different from the result reported in Bansal and Yaron (2004) and suggests that the action is more in the time-varying mean than in

[8] The formula for k_0 is given by: $k_0 = -log(k_1) - (1-k_1)log(1/k_1 - 1)$ where $k_1 = 1/(1+exp(\overline{(d_t - p_t)}))$, with $\overline{(d_t - p_t))}$ the mean log dividend-price ratio. The expressions for k_{0m} and k_{1m} are similar but for the consumption price ratio instead of the dividend-price ratio

the variance. This point deserves further investigation.

***(b)* Predictability of Returns** Bansal and Yaron (2004) computed by simulation the R^2 of regressions of the cumulative excess returns from t to $t+h$ on the dividend-price ratio at t. They found that their model with a risk aversion parameter of 10 and an elasticity of intertemporal substitution of 1.5 was able to reproduce some of the predictability observed in the data. The simulation was run with 840 observations as in their data sampling period.

We have derived analytically the R^2 of the same regression in population. In table (25) we report the corresponding results with the same configurations of preference parameters that we selected before for asset pricing implications.

The first striking result is the total lack of predictability of excess returns by the dividend-price ratio. This is in contrast with the predictability found in Bansal and Yaron (2004). They report R^2 of 5, 10 and 16 percent at horizons of 1, 3 and 5 years respectively. To identify the source of the difference between these results, we first reproduce by simulation the same statistics both for the original Bansal and Yaron model (2004) and the matching Markov-switching model we have built. Another word of caution is in order before we look at the results. The regression that we run has as a dependent variable the cumulative monthly returns over *yearly* periods (1 to 20) and the *monthly* dividend-price ratio as an independent variable. In Bansal and Yaron (2004) it is a yearly dividend (cumulated monthly dividends over twelve months). Cumulating the dividends will certainly increase the R^2 but would not change the evidence over the actual presence of predictability.

In tables 30 and 26, we can see that there is strong predictability both in the original Bansal and Yaron (2004) model and the matching Markov switching, so it is not due to a perverse effect of our matching procedure. These results seem to point strongly towards a small sample explanation. Predictability appears in finite sample due to the presence of a very persistent variable on the right hand side[9] but disappears in population regressions. Abel (2005) also finds little or no predictability of excess returns by the dividend-price ratio in a model of preferences with a benchmark level of consumption (habit formation or consumption externalities such as keeping up or catching up with the Joneses) and i.i.d. growth rates of consumption and dividends[10]. However Abel (2005) finds that the return

[9] cite literature on problems in predictability regressions.

[10] Abel (2005) finds that the dividend-price ratio cannot predict the excess rate of return on stock relative to one-period riskless bills, when the excess rate of return is defined as the ratio of the gross rates of return on the two assets. He finds a very small R^2 for plausible values of the preference parameters when the excess rate of return is defined as the arithmetic difference between the rates of return on stocks and one-period riskless bills.

on stock is predictable by the dividend-price ratio.

Table 27 reports the analytical R^2 of the regression of returns on equity on the dividend-price ratio for the matching MS model. There seems to be some predictability for values of the elasticity of intertemporal substitution (ψ parameter) below one, but that it disappears for values above one. This is true for all values of the risk aversion parameter γ, the only difference being that predictability increases with γ for all values of ψ. This result about the pivotal value of one for ψ is the opposite of what was found in the previous section for asset pricing implications. The asset return moments were better reproduced for values of ψ greater than one.

To contrast these regression results in population with the finite sample results, we simulate the Markov-switching model over periods of 840 observations, the sample length in Bansal and Yaron (2004), and compute the R^2 of the same regression. The results are reported in table 28. The R^2 of the finite sample regressions are not too different from the analytical R^2 for $\psi = 0.5$. As the value of the elasticity of intertemporal substitution increases the gap between the population and finite sample statistics increases[11]. Therefore, for some values of the preference parameters, predictability appears to be a finite sample phenomenon while for some others it seems to be a feature of the model.

Lettau and Ludvigson (2001a,b) have put forward that a measure of consumption over wealth has a greater predicting power than the dividend-price ratio. We present in table 29 results of the regression of cumulative returns on the consumption-price ratio in the Epstein-Zin economy. Indeed, we find higher predictability for all preference parameter pairs. In particular, for $\gamma = 10$ and $\psi = 0.5$ the R^2 for the consumption-price ratio is equal to 9.39, 14.19 and 13.53 for 1, 3 and 5 years, as opposed to 7.46, 11.22 and 10.65 for the dividend-price ratio. The remarks made above about the finite sample results for the dividend-price ratio apply equally to the consumption-price ratio. In particular, there is no predictability of excess returns by the price-consumption ratio.

Some predictability of excess returns appears if risk aversion increases for values of ψ greater than one. It is interesting to note that for higher risk aversion the BY model behaves more like the LLW model. The volatility of the stock decreases as well as the level of the price-dividend ratio. These results are illustrated in tables 32 and 31.

***(c)* Predictability of Volatility** Another important message found in Bansal and Yaron (2004) is the predictability of consumption volatility by the dividend-price ratio. In table 33, we report the R^2 of the regression of cumulative future consumption volatility over

[11]We have checked that results similar to the analytical are obtained when we simulate with a sample of 2,000 observations

several horizons on the current price-dividend ratio. Results are similar to those obtained for future returns predictability. Not all preference configurations are able to produce predictable volatility. Again only low values of the elasticity of intertemporal substitution are able to generate predictability ($\psi = 0.5$). There is no predictability at all for values of ψ above one. Predictability is the strongest at a one-year horizon.

(*d*) Variance Ratios There is negative autocorrelation at long horizons in returns. Evidence is provided by the variance ratios computed at several horizons in table 1. The variance ratios are less than one and decrease from year 2 up to year 4.

The corresponding analytical quantities are reported in table 34. Most of the preference parameter combinations produce strong positive autocorrelations increasing with the horizons. Only one set, $\gamma = 5$ and $\psi = 1.5$ produce slight negative autocorrelation. The same results would have been visible in a simulated finite sample setting with 840 observations (see table 35). However, predictability would have appeared stronger for the above-mentioned particular set of parameters and other candidate sets would have appeared.

6 Conclusion

Equilibrium asset pricing models have become harder to solve. To reproduce resilient stylized facts, researchers have assumed that the representative investor is endowed with more sophisticated preferences. The fundamentals in the economy, consumption and dividends, have also been modeled with richer dynamics. Often the time required to solve the model numerically or to simulate it to compute the statistics of interest is prohibitive. Therefore, researchers lean towards simpler models, making simplifying assumptions as a compromise between reality and feasibility.

In this paper, we have provided analytical formulas that should be of great help to assess the ability of these models to reproduce the stylized facts. We have chosen a flexible model for the endowment that can be applied directly to the data, as already done by several researchers, or used to match other processes that are contemplated. In terms of preferences, we have chosen the recursive framework of Epstein and Zin (1989), widely used in the asset pricing literature. We have limited our analysis to the Kreps and Porteus (1978) certainty equivalent. In future research we intend to try to find analytical formulas for other certainty equivalents in the recursive framework and other types of preferences.

Table 1: **Predictability of Returns and Growth Rates: Data.**

This table shows estimates of slope coefficients, and R-squared of regressions $y_{t+1:t+h} = a_y(h) + b_y(h)\frac{D_t}{P_t} + \eta_{y,t+h}(h)$, where the variable y is return, excess return, consumption growth rate or dividend growth rate. Standard errors are Newey and West (1987) corrected using 10 lags. Lines 6 and 11 show variance ratios of aggregate returns and aggregate excess returns respectively. The horizon h is quarterly in regressions and converted into annual in the table. Estimates and standard deviations of slope coefficients are multiplied by 10^{-4} in the table.

h	1	2	3	4	5
			Returns		
Estimate	0.1416	0.2415	0.3027	0.3747	0.5128
Std. Dev.	0.0502	0.0930	0.1166	0.1277	0.1498
R-squared	9.0192	13.5480	15.1060	17.5200	22.3720
Var. Ratio	1.0271	0.9623	0.8660	0.8209	0.9199
			Excess Returns		
Estimate	0.1527	0.2617	0.3247	0.3875	0.5126
Std. Dev.	0.0458	0.0858	0.1066	0.1156	0.1354
R-squared	10.9800	17.1750	19.5180	21.8600	26.2010
Var. Ratio	1.0028	0.9105	0.7880	0.7189	0.8017
			Consumption Growth		
Estimate	-0.0047	-0.0058	-0.0117	-0.0163	-0.0214
Std. Dev.	0.0041	0.0073	0.0098	0.0117	0.0136
R-squared	1.6140	1.0546	2.6146	3.6245	4.6223
			Dividend Growth		
Estimate	0.0045	0.0184	0.0233	0.0370	0.0623
Std. Dev.	0.0176	0.0337	0.0431	0.0503	0.0536
R-squared	0.0488	0.4107	0.4435	0.9214	2.3147

Table 2: **Parameters of a Two-State Markov-Switching Model for Quarterly US Data on Consumption and Dividends - 1947:3-2002:4.**

This table shows parameters of the two-state Markov-Switching Model estimated on actual data with $N = 2$. μ_c and μ_d are conditional means of consumption and dividend, ω_c and ω_d are conditional variances of consumption and dividend. ρ is the conditional correlation between consumption and dividend shocks. P^T is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent.

	State 1	State 2
μ_c^T	1.647	2.798
μ_d^T	-12.075	13.868
$\left(\omega_c^T\right)^{\frac{1}{2}}$	2.669	1.587
$\left(\omega_d^T\right)^{\frac{1}{2}}$	16.976	19.369
ρ^T	0.003	0.003
	P^T	
State 1	0.687	0.313
State 2	0.301	0.699
Π^T	0.490	0.510

Table 3: **Asset Pricing Implications of the Two-State Markov Switching Model: $\delta = 0.98$**

The entries are model population values of asset prices. The price-consumption ratio is given in 4.11 and the price-dividend ratio in 4.16. The input parameters for the model are given in Table 2. The expressions $E[R_m - R_f]$ and $E[R_f]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The subjective factor of discount is set to 0.98.

γ	ψ	$E[R_m - R_f]$	$E[R_f]$	$\sigma(R_m)$	$\sigma(R_f)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
10.0	0.5	1.38	6.04	26.90	1.56	24.52	34.79	0.044	0.553
10.0	0.7	1.56	4.80	27.25	1.14	34.38	57.20	0.014	0.350
10.0	1.3	1.79	3.38	27.69	0.66	63.41	210.88	0.004	0.100
10.0	1.5	1.83	3.16	27.77	0.59	72.91	359.03	0.005	0.059
20.0	0.5	3.42	5.37	26.58	1.93	26.67	23.21	0.047	0.790
20.0	0.7	3.73	4.23	26.96	1.43	36.12	29.39	0.015	0.653
20.0	1.3	4.12	2.93	27.44	0.86	60.53	41.98	0.005	0.481
20.0	1.5	4.19	2.73	27.52	0.78	67.57	44.94	0.006	0.453
30.0	0.5	5.50	4.74	26.09	2.31	29.52	17.04	0.048	0.996
30.0	0.7	5.94	3.68	26.45	1.72	38.26	19.35	0.016	0.924
30.0	1.3	6.48	2.47	26.93	1.06	57.63	22.83	0.006	0.829
30.0	1.5	6.57	2.28	27.01	0.96	62.51	23.48	0.008	0.813
40.0	0.5	7.21	4.13	25.49	2.70	33.39	14.02	0.047	1.090
40.0	0.7	7.77	3.14	25.83	2.02	40.88	15.11	0.016	1.074
40.0	1.3	8.44	2.01	26.28	1.25	54.81	16.56	0.007	1.045
40.0	1.5	8.55	1.84	26.35	1.13	57.85	16.80	0.009	1.040

Table 4: **Parameters of the Four-State Quarterly Markov-Switching Model of Lettau, Ludvigson and Wachter (2006).**

In this table, we report the parameters of the Markov-Switching Model (2.1), (2.2) with $N = 4$, constructed using estimates reported in Lettau, Ludvigson and Wachter (2006). μ_c and μ_d are conditional means of consumption and dividend, ω_c and ω_d are conditional variances of consumption and dividend. ρ is the conditional correlation between consumption and dividend shocks. P^T is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent.

	State 1	State 2	State 3	State 4
μ_c^T	0.62	0.62	-0.32	-0.32
μ_d^T	2.80	2.80	-1.45	-1.45
$\left(\omega_c^T\right)^{\frac{1}{2}}$	0.75	0.40	0.75	0.40
$\left(\omega_d^T\right)^{\frac{1}{2}}$	3.36	1.82	3.36	1.82
		P^T		
State 1	0.960	0.006	0.034	0.000
State 2	0.009	0.957	0.000	0.034
State 3	0.205	0.001	0.789	0.005
State 4	0.002	0.204	0.007	0.787
Π^T	0.515	0.343	0.085	0.057

Table 5: **Asset Pricing Implications: LLW**

The entries are model population values of asset prices. The price-consumption ratio is given by (4.11) and price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The expressions $E[R_m - R_f]$ and $E[R_f]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	$E[R_m - R_f]$	$E[R_f]$	$\sigma(R_m)$	$\sigma(R_f)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
15.0	0.5	2.74	6.93	8.42	1.27	21.76	243.31	0.036	0.013
15.0	0.7	3.21	5.70	9.38	0.92	27.08	-243.93	0.013	0.016
15.0	1.3	3.83	4.26	10.69	0.53	37.54	-75.01	0.005	0.067
15.0	1.5	3.94	4.03	10.91	0.47	39.92	-67.84	0.007	0.077
20.0	0.5	4.11	6.99	8.69	1.26	22.73	55.92	0.038	0.062
20.0	0.7	4.81	5.69	9.69	0.92	27.72	90.73	0.014	0.046
20.0	1.3	5.70	4.14	11.00	0.53	36.89	296.39	0.006	0.017
20.0	1.5	5.85	3.90	11.22	0.47	38.87	451.93	0.008	0.012
25.0	0.5	5.57	7.11	8.71	1.28	23.93	30.16	0.038	0.116
25.0	0.7	6.50	5.69	9.69	0.93	28.47	36.30	0.014	0.115
25.0	1.3	7.68	4.01	10.96	0.53	36.20	46.92	0.006	0.106
25.0	1.5	7.87	3.75	11.17	0.47	37.77	49.10	0.009	0.104
30.0	0.5	6.92	7.27	8.49	1.32	25.38	20.77	0.038	0.163
30.0	0.7	8.09	5.72	9.40	0.95	29.32	22.95	0.015	0.174
30.0	1.3	9.54	3.87	10.60	0.53	35.49	25.99	0.007	0.182
30.0	1.5	9.78	3.58	10.80	0.46	36.67	26.53	0.009	0.183

Table 6: **Predictability of Returns by the Consumption-Price Ratio: LLW**

This table shows the R-squared of the regression $R_{t+1:t+h} = a_2(h) + b_2(h)\frac{C_t}{P_t} + \eta_{2,t+h}(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	0.97	0.71	0.46	0.31	0.21	0.14	0.10	0.07	0.05	0.04	0.01	0.00
15.0	0.7	0.05	0.02	0.01	0.00	0.00	0.00	0.01	0.01	0.02	0.02	0.04	0.06
15.0	1.3	0.30	0.35	0.34	0.34	0.33	0.33	0.33	0.33	0.33	0.32	0.32	0.30
15.0	1.5	0.43	0.48	0.47	0.45	0.43	0.42	0.42	0.41	0.40	0.40	0.38	0.36
20.0	0.5	0.40	0.25	0.12	0.06	0.02	0.01	0.00	0.00	0.00	0.01	0.04	0.06
20.0	0.7	0.01	0.04	0.06	0.09	0.12	0.14	0.17	0.18	0.20	0.22	0.26	0.28
20.0	1.3	0.70	0.82	0.82	0.81	0.81	0.80	0.80	0.80	0.80	0.80	0.78	0.75
20.0	1.5	0.87	1.01	1.01	0.98	0.96	0.95	0.94	0.93	0.93	0.92	0.89	0.84
25.0	0.5	0.24	0.11	0.03	0.00	0.00	0.02	0.03	0.06	0.08	0.10	0.18	0.22
25.0	0.7	0.05	0.12	0.18	0.24	0.29	0.34	0.38	0.41	0.44	0.47	0.56	0.58
25.0	1.3	0.87	1.08	1.13	1.16	1.18	1.19	1.21	1.23	1.24	1.25	1.26	1.23
25.0	1.5	1.06	1.29	1.34	1.35	1.36	1.37	1.38	1.39	1.40	1.40	1.40	1.35
30.0	0.5	0.29	0.11	0.02	0.00	0.01	0.04	0.08	0.12	0.16	0.19	0.32	0.39
30.0	0.7	0.02	0.10	0.18	0.27	0.36	0.43	0.50	0.56	0.61	0.65	0.79	0.85
30.0	1.3	0.74	0.99	1.12	1.21	1.28	1.34	1.40	1.44	1.48	1.52	1.60	1.60
30.0	1.5	0.92	1.20	1.32	1.40	1.47	1.53	1.58	1.62	1.65	1.68	1.76	1.74

Table 7: **Predictability of Returns by the Dividend-Price Ratio: LLW**

This table shows the R-squared of the regression $R_{t+1:t+h} = a_1(h) + b_1(h)\frac{D_t}{P_t} + \eta_{1,t+h}(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	0.38	0.20	0.08	0.03	0.00	0.00	0.01	0.02	0.03	0.04	0.10	0.13
15.0	0.7	0.00	0.01	0.04	0.08	0.11	0.15	0.18	0.21	0.23	0.25	0.33	0.36
15.0	1.3	0.34	0.46	0.52	0.57	0.61	0.64	0.67	0.69	0.71	0.73	0.78	0.78
15.0	1.5	0.44	0.57	0.64	0.68	0.71	0.74	0.77	0.79	0.81	0.83	0.87	0.86
20.0	0.5	0.10	0.02	0.00	0.02	0.05	0.09	0.13	0.17	0.20	0.24	0.35	0.41
20.0	0.7	0.06	0.15	0.23	0.31	0.38	0.44	0.49	0.54	0.58	0.61	0.73	0.77
20.0	1.3	0.70	0.91	1.01	1.07	1.12	1.17	1.21	1.24	1.27	1.29	1.35	1.34
20.0	1.5	0.84	1.08	1.17	1.23	1.27	1.31	1.34	1.37	1.40	1.42	1.47	1.45
25.0	0.5	0.04	0.00	0.02	0.08	0.15	0.22	0.29	0.35	0.40	0.45	0.63	0.71
25.0	0.7	0.12	0.25	0.38	0.49	0.59	0.67	0.75	0.82	0.88	0.92	1.08	1.13
25.0	1.3	0.88	1.17	1.31	1.40	1.48	1.54	1.60	1.65	1.69	1.72	1.81	1.80
25.0	1.5	1.04	1.36	1.50	1.58	1.65	1.71	1.76	1.80	1.84	1.87	1.95	1.92
30.0	0.5	0.07	0.00	0.03	0.10	0.20	0.29	0.38	0.46	0.54	0.60	0.83	0.94
30.0	0.7	0.08	0.22	0.37	0.51	0.64	0.76	0.86	0.95	1.03	1.10	1.32	1.39
30.0	1.3	0.78	1.10	1.29	1.43	1.55	1.65	1.74	1.82	1.88	1.94	2.09	2.10
30.0	1.5	0.94	1.29	1.48	1.62	1.73	1.82	1.91	1.98	2.04	2.09	2.23	2.23

Table 8: **Predictability of Excess Returns by the Consumption-Price Ratio: LLW**

This table shows the R-squared of the regression $R^e_{t+1:t+h} = a^e_2\,(h) + b^e_2\,(h)\,\frac{C_t}{P_t} + \eta^e_{2,t+h}\,(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.11). The input parameters for the model (2.1)-(2.2) are given in Table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	1.06	1.18	1.13	1.07	1.03	0.99	0.96	0.94	0.93	0.91	0.85	0.79
15.0	0.7	1.32	1.46	1.38	1.29	1.22	1.16	1.12	1.09	1.06	1.04	0.94	0.87
15.0	1.3	1.61	1.76	1.66	1.53	1.43	1.35	1.29	1.25	1.21	1.17	1.05	0.96
15.0	1.5	1.65	1.81	1.70	1.56	1.46	1.38	1.32	1.27	1.23	1.19	1.07	0.97
20.0	0.5	1.76	2.07	2.08	2.05	2.01	1.99	1.98	1.96	1.95	1.94	1.87	1.77
20.0	0.7	2.07	2.40	2.38	2.31	2.24	2.19	2.16	2.13	2.10	2.08	1.97	1.85
20.0	1.3	2.37	2.74	2.68	2.57	2.47	2.39	2.33	2.29	2.25	2.21	2.06	1.92
20.0	1.5	2.42	2.79	2.72	2.60	2.50	2.42	2.36	2.31	2.27	2.23	2.08	1.93
25.0	0.5	2.20	2.74	2.89	2.96	3.00	3.04	3.08	3.11	3.14	3.15	3.15	3.03
25.0	0.7	2.45	3.01	3.12	3.14	3.15	3.16	3.17	3.18	3.19	3.19	3.14	3.01
25.0	1.3	2.69	3.25	3.32	3.29	3.26	3.24	3.22	3.21	3.19	3.18	3.09	2.93
25.0	1.5	2.72	3.28	3.34	3.31	3.27	3.24	3.22	3.21	3.19	3.18	3.07	2.92
30.0	0.5	2.23	2.98	3.34	3.57	3.76	3.92	4.05	4.16	4.25	4.32	4.45	4.35
30.0	0.7	2.37	3.10	3.39	3.57	3.70	3.82	3.91	4.00	4.06	4.11	4.20	4.09
30.0	1.3	2.48	3.16	3.39	3.50	3.58	3.65	3.71	3.76	3.80	3.83	3.86	3.75
30.0	1.5	2.49	3.17	3.38	3.48	3.56	3.62	3.67	3.72	3.76	3.79	3.81	3.69

Table 9: **Predictability of Excess Returns by the Dividend-Price Ratio: LLW**

This table shows the R-squared of the regression $R^e_{t+1:t+h} = a^e_1(h) + b^e_1(h)\frac{D_t}{P_t} + \eta^e_{1,t+h}(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	1.15	1.49	1.64	1.73	1.80	1.87	1.92	1.97	2.01	2.05	2.12	2.09
15.0	0.7	1.28	1.62	1.73	1.79	1.84	1.88	1.92	1.95	1.98	2.00	2.04	2.00
15.0	1.3	1.38	1.72	1.81	1.85	1.87	1.90	1.92	1.94	1.95	1.97	1.98	1.93
15.0	1.5	1.39	1.73	1.82	1.85	1.87	1.89	1.91	1.93	1.95	1.96	1.97	1.92
20.0	0.5	1.81	2.43	2.72	2.92	3.08	3.21	3.33	3.42	3.51	3.57	3.73	3.69
20.0	0.7	1.96	2.55	2.78	2.91	3.02	3.11	3.18	3.25	3.30	3.35	3.44	3.37
20.0	1.3	2.11	2.67	2.84	2.92	2.97	3.02	3.06	3.10	3.13	3.15	3.18	3.09
20.0	1.5	2.13	2.69	2.85	2.92	2.96	3.00	3.04	3.07	3.10	3.12	3.14	3.05
25.0	0.5	2.27	3.15	3.63	3.96	4.23	4.46	4.66	4.82	4.96	5.07	5.34	5.29
25.0	0.7	2.39	3.20	3.56	3.80	3.98	4.14	4.27	4.39	4.48	4.56	4.72	4.65
25.0	1.3	2.51	3.25	3.52	3.66	3.76	3.85	3.92	3.99	4.04	4.09	4.16	4.05
25.0	1.5	2.53	3.26	3.51	3.64	3.73	3.81	3.88	3.94	3.98	4.02	4.08	3.97
30.0	0.5	2.40	3.50	4.19	4.71	5.13	5.49	5.79	6.05	6.26	6.43	6.86	6.82
30.0	0.7	2.41	3.37	3.89	4.27	4.57	4.82	5.04	5.22	5.37	5.49	5.79	5.73
30.0	1.3	2.43	3.25	3.63	3.87	4.06	4.22	4.36	4.47	4.57	4.64	4.81	4.73
30.0	1.5	2.43	3.24	3.59	3.82	3.99	4.14	4.26	4.37	4.46	4.53	4.68	4.60

Table 10: **Predictability of Consumption Volatility by the Consumption-Price Ratio: LLW**

This table shows the R-squared of the regression $\sigma^2_{t+1:t+h} = a_3\,(h)+b_3\,(h)\,\frac{C_t}{P_t}+\eta_{3,t+h}\,(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	9.90	9.52	9.16	8.81	8.48	8.16	7.86	7.57	7.30	7.04	5.90	5.00
15.0	0.7	11.11	10.69	10.28	9.89	9.51	9.16	8.82	8.50	8.19	7.90	6.63	5.61
15.0	1.3	12.85	12.36	11.89	11.44	11.00	10.59	10.20	9.83	9.48	9.14	7.66	6.49
15.0	1.5	13.16	12.66	12.18	11.71	11.27	10.85	10.45	10.07	9.70	9.36	7.85	6.65
20.0	0.5	15.89	15.28	14.70	14.14	13.60	13.10	12.61	12.15	11.71	11.30	9.47	8.03
20.0	0.7	17.33	16.66	16.02	15.41	14.83	14.28	13.75	13.25	12.77	12.32	10.33	8.75
20.0	1.3	19.27	18.53	17.82	17.14	16.49	15.88	15.29	14.73	14.20	13.69	11.49	9.73
20.0	1.5	19.60	18.85	18.13	17.44	16.78	16.15	15.56	14.99	14.45	13.93	11.69	9.90
25.0	0.5	22.11	21.27	20.45	19.67	18.93	18.23	17.55	16.91	16.30	15.72	13.18	11.17
25.0	0.7	23.45	22.55	21.68	20.86	20.07	19.33	18.61	17.93	17.28	16.67	13.98	11.85
25.0	1.3	25.14	24.18	23.25	22.37	21.53	20.72	19.96	19.23	18.54	17.87	14.99	12.70
25.0	1.5	25.42	24.45	23.51	22.62	21.77	20.95	20.18	19.44	18.74	18.07	15.16	12.85
30.0	0.5	28.21	27.13	26.09	25.10	24.15	23.25	22.39	21.58	20.80	20.05	16.82	14.25
30.0	0.7	29.18	28.07	26.99	25.96	24.99	24.05	23.17	22.32	21.51	20.74	17.40	14.75
30.0	1.3	30.36	29.19	28.07	27.01	25.99	25.02	24.10	23.22	22.38	21.58	18.10	15.34
30.0	1.5	30.54	29.37	28.25	27.17	26.15	25.17	24.25	23.36	22.52	21.71	18.21	15.43

Table 11: **Predictability of Consumption Volatility by the Dividend-Price Ratio: LLW**

This table shows the R-squared of the regression $\sigma^2_{t+1:t+h} = a_4\,(h) + b_4\,(h)\,\frac{D_t}{P_t} + \eta_{4,t+h}\,(h)$. The horizon h is quarterly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	41.99	40.38	38.83	37.35	35.95	34.61	33.33	32.11	30.95	29.85	25.03	21.21
15.0	0.7	43.30	41.64	40.05	38.53	37.07	35.69	34.37	33.12	31.92	30.78	25.82	21.88
15.0	1.3	46.92	45.12	43.39	41.74	40.17	38.67	37.24	35.88	34.59	33.35	27.98	23.71
15.0	1.5	47.67	45.85	44.09	42.41	40.82	39.29	37.84	36.46	35.14	33.89	28.43	24.09
20.0	0.5	45.26	43.53	41.86	40.27	38.75	37.30	35.93	34.62	33.37	32.17	26.99	22.87
20.0	0.7	44.33	42.63	41.00	39.44	37.96	36.54	35.19	33.91	32.68	31.51	26.43	22.40
20.0	1.3	44.60	42.89	41.25	39.68	38.19	36.76	35.40	34.11	32.88	31.70	26.59	22.54
20.0	1.5	44.75	43.03	41.38	39.81	38.31	36.88	35.52	34.22	32.99	31.81	26.68	22.61
25.0	0.5	46.92	45.12	43.39	41.74	40.17	38.67	37.25	35.88	34.59	33.35	27.98	23.71
25.0	0.7	44.36	42.66	41.02	39.46	37.98	36.56	35.21	33.92	32.70	31.53	26.45	22.41
25.0	1.3	42.55	40.92	39.36	37.86	36.43	35.07	33.78	32.55	31.37	30.25	25.37	21.50
25.0	1.5	42.36	40.73	39.17	37.68	36.26	34.91	33.62	32.39	31.22	30.11	25.25	21.40
30.0	0.5	49.49	47.59	45.77	44.03	42.37	40.79	39.28	37.85	36.48	35.18	29.51	25.00
30.0	0.7	45.71	43.95	42.27	40.66	39.13	37.67	36.28	34.96	33.69	32.49	27.25	23.09
30.0	1.3	42.60	40.97	39.40	37.90	36.47	35.11	33.82	32.58	31.41	30.28	25.40	21.52
30.0	1.5	42.21	40.59	39.04	37.55	36.14	34.79	33.51	32.28	31.12	30.00	25.17	21.33

Table 12: **Variance Ratios of Aggregate Returns: LLW**

This table shows the variance ratios $\frac{Var(R_{t+1:t+h}}{hVar(R_{t+1})}$, where the horizon h is quarterly and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	1.12	1.21	1.26	1.28	1.30	1.32	1.32	1.33	1.34	1.34	1.35	1.35
15.0	0.7	1.04	1.06	1.08	1.08	1.09	1.09	1.09	1.09	1.10	1.10	1.10	1.09
15.0	1.3	0.96	0.93	0.91	0.90	0.89	0.89	0.88	0.88	0.87	0.87	0.86	0.86
15.0	1.5	0.95	0.91	0.89	0.88	0.87	0.86	0.85	0.85	0.85	0.84	0.83	0.82
20.0	0.5	1.09	1.16	1.19	1.22	1.23	1.24	1.25	1.25	1.26	1.26	1.27	1.27
20.0	0.7	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
20.0	1.3	0.94	0.89	0.87	0.85	0.84	0.83	0.82	0.82	0.82	0.81	0.80	0.79
20.0	1.5	0.93	0.88	0.84	0.83	0.81	0.80	0.80	0.79	0.79	0.78	0.77	0.76
25.0	0.5	1.09	1.15	1.19	1.21	1.22	1.23	1.24	1.24	1.25	1.25	1.26	1.26
25.0	0.7	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01
25.0	1.3	0.94	0.89	0.86	0.84	0.83	0.82	0.82	0.81	0.81	0.80	0.79	0.79
25.0	1.5	0.93	0.87	0.84	0.82	0.81	0.80	0.79	0.78	0.78	0.78	0.76	0.76
30.0	0.5	1.11	1.19	1.23	1.26	1.28	1.29	1.30	1.30	1.31	1.31	1.33	1.33
30.0	0.7	1.02	1.04	1.05	1.06	1.06	1.07	1.07	1.07	1.07	1.07	1.07	1.07
30.0	1.3	0.95	0.91	0.89	0.88	0.87	0.86	0.86	0.85	0.85	0.85	0.84	0.84
30.0	1.5	0.94	0.89	0.87	0.85	0.84	0.83	0.83	0.83	0.82	0.82	0.81	0.81

Table 13: **Variance Ratios of Aggregate Excess Returns: LLW**

This table shows the variance ratios $\frac{Var(R^e_{t+1:t+h}}{hVar(R^e_{t+1})}$, where the horizon h is quarterly and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in table 4. The quarterly subjective factor of discount is set to 0.9925.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
15.0	0.5	0.94	0.89	0.86	0.85	0.84	0.83	0.82	0.82	0.82	0.82	0.81	0.81
15.0	0.7	0.92	0.86	0.83	0.81	0.79	0.78	0.78	0.77	0.77	0.76	0.75	0.75
15.0	1.3	0.91	0.83	0.79	0.77	0.75	0.74	0.73	0.72	0.72	0.71	0.70	0.69
15.0	1.5	0.90	0.83	0.79	0.76	0.74	0.73	0.72	0.72	0.71	0.71	0.69	0.68
20.0	0.5	0.92	0.87	0.83	0.81	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77
20.0	0.7	0.91	0.83	0.79	0.77	0.75	0.74	0.74	0.73	0.73	0.72	0.71	0.70
20.0	1.3	0.89	0.80	0.76	0.73	0.71	0.70	0.69	0.68	0.67	0.67	0.65	0.64
20.0	1.5	0.89	0.80	0.75	0.72	0.70	0.69	0.68	0.67	0.66	0.66	0.64	0.63
25.0	0.5	0.92	0.86	0.83	0.81	0.80	0.79	0.79	0.79	0.79	0.78	0.78	0.78
25.0	0.7	0.90	0.83	0.79	0.77	0.75	0.74	0.73	0.73	0.73	0.72	0.71	0.71
25.0	1.3	0.89	0.80	0.75	0.72	0.70	0.69	0.68	0.68	0.67	0.67	0.65	0.65
25.0	1.5	0.88	0.80	0.75	0.72	0.70	0.69	0.68	0.67	0.66	0.66	0.64	0.64
30.0	0.5	0.93	0.88	0.86	0.85	0.84	0.83	0.83	0.83	0.83	0.83	0.84	0.85
30.0	0.7	0.92	0.85	0.82	0.80	0.79	0.78	0.78	0.77	0.77	0.77	0.77	0.77
30.0	1.3	0.90	0.82	0.78	0.76	0.74	0.73	0.72	0.72	0.71	0.71	0.70	0.70
30.0	1.5	0.90	0.82	0.78	0.75	0.73	0.72	0.72	0.71	0.70	0.70	0.69	0.69

Table 14: **Matching the Mean Process Using Variance and Kurtosis**

This table presents parameters of the MS mean process, obtained so that unconditional variance and kurtosis of the mean of consumption are matched with the corresponding unconditional moments of the BY mean process.

$skew(y_t) > 0$				$skew(y_t) < 0$			
p^y_{11}	p^y_{22}	y_1	y_2	p^y_{11}	p^y_{22}	y_1	y_2
0.99569	0.98331	0.00064	0.00481	0.98331	0.99569	-0.00181	0.00236
$moments$				$moments$			
BY		MS		BY		MS	
$skew(x_t)$	0.00000	$skew(y_t)$	1.45838	$skew(x_t)$	0.00000	$skew(y_t)$	-1.45838
$kurt(x_t)$	3.12688	$kurt(y_t)$	3.12688	$kurt(x_t)$	3.12688	$kurt(y_t)$	3.12688
$skew(g_t)$	0.00000	$skew(\Delta c_t)$	0.01370	$skew(g_t)$	0.00000	$skew(\Delta c_t)$	-0.01370
$kurt(g_t)$	3.16251	$kurt(\Delta c_t)$	3.15178	$kurt(g_t)$	3.16251	$kurt(\Delta c_t)$	3.15178

Table 15: **Matching the Volatility Process Using Variance and Kurtosis**

This table presents parameters of the MS volatility process, obtained so that unconditional variance and kurtosis of the volatility of consumption are matched with the corresponding moments of the BY volatility process.

$skew(z_t) > 0$				$skew(z_t) < 0$			
p^z_{11}	p^z_{22}	z_1	z_2	p^z_{11}	p^z_{22}	z_1	z_2
0.99725	0.98975	$5.34E-5$	$8.85E-5$	0.98975	0.99725	$3.32E-5$	$6.82E-5$
$moments$				$moments$			
BY		MS		BY		MS	
$skew(h_t)$	0.00000	$skew(z_t)$	1.41421	$skew(h_t)$	0.00000	$skew(z_t)$	-1.41421
$kurt(h_t)$	3.00000	$kurt(z_t)$	3.00000	$kurt(h_t)$	3.00000	$kurt(z_t)$	3.00000

Table 16: **Parameters of the Markov-Switching Model.**

This table shows parameters of the Markov-Switching Model calibrated to match the model of Bansal and Yaron (2004). Calibration is made such that unconditional variance and kurtosis of MS mean and volatility of consumption matched similar moments in the BY model, whereas the implied skewness of MS mean is negative and that of MS volatility is positive. μ_c and μ_d are conditional means of consumption and dividend, ω_c and ω_d are conditional variances of consumption and dividend. P^T is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent. The model is calibrated at the monthly frequency. The correlation vector ρ is set to zero.

	State 1	State 2	State 3	State 4
μ_c^T	-0.181	-0.181	0.236	0.236
μ_d^T	-0.843	-0.843	0.407	0.407
$\left(\omega_c^T\right)^{\frac{1}{2}}$	0.731	0.941	0.731	0.941
$\left(\omega_d^T\right)^{\frac{1}{2}}$	3.289	4.233	3.289	4.233
		P^T		
State 1	0.981	0.003	0.017	0.000
State 2	0.010	0.973	0.000	0.017
State 3	0.004	0.000	0.993	0.003
State 4	0.000	0.004	0.010	0.985
Π^T	0.162	0.043	0.627	0.168

Table 17: **Asset Pricing Implications: Table IV of Bansal and Yaron (2004)**

The entries are model population values of asset prices. The expressions $E\left[r_m - r_f\right]$ and $E\left[r_f\right]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma\left(r_m\right)$, $\sigma\left(r_f\right)$, and $\sigma\left(p-d\right)$ are respectively the annualized volatilities of market return, risk-free rate and price-dividend ratio. The monthly subjective factor of discount is set to 0.998 and the elasticity of intertemporal substitution to $\psi = 1.5$.

	Data		Model	
Variable	Estimate	Std.dev.	$\gamma = 7.5$	$\gamma = 10$
		Returns		
$E\left[r_m - r_f\right]$	6.33	(2.15)	4.01	6.84
$E\left[r_f\right]$	0.86	(0.42)	1.44	0.93
$\sigma\left[r_m\right]$	19.42	(3.07)	17.81	18.65
$\sigma\left[r_f\right]$	0.97	(0.28)	0.44	0.57
		Price-Dividend Ratio		
$E\left[\exp\left(p-d\right)\right]$	26.56	(2.53)	25.02	19.98
$\sigma\left[p-d\right]$	0.29	(0.04)	0.18	0.21
$AC1\left[p-d\right]$	0.81	(0.09)	0.80	0.82
$AC2\left[p-d\right]$	0.64	(0.15)	0.65	0.67

Table 18: **Asset Pricing Implications:** $\delta = 0.998$

The entries are model population values of asset prices.The price-consumption ratio is given in 4.11 and the price-dividend ratio in 4.16. The input parameters for the monthly model are given in Table (16). The expressions $E\left[R_m - R_f\right]$ and $E\left[R_f\right]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma\left(R_m\right)$, $\sigma\left(R_f\right)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The monthly subjective factor of discount is set to 0.998.

γ	ψ	$E\left[R_m - R_f\right]$	$E\left[R_f\right]$	$\sigma\left(R_m\right)$	$\sigma\left(R_f\right)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
2.5	0.5	0.13	5.89	13.32	1.16	25.48	30.11	0.080	0.074
2.5	0.8	0.65	4.55	15.62	0.72	35.91	46.28	0.015	0.093
2.5	1.2	1.13	3.73	17.55	0.49	46.46	65.15	0.008	0.087
2.5	1.5	1.37	3.38	18.47	0.40	52.61	77.52	0.015	0.081
5.0	0.5	1.05	6.11	13.68	1.14	29.89	22.77	0.074	0.115
5.0	0.8	2.36	4.49	16.16	0.72	37.95	27.08	0.016	0.174
5.0	1.2	3.33	3.46	18.09	0.49	44.37	30.09	0.009	0.201
5.0	1.5	3.78	3.02	18.96	0.40	47.51	31.44	0.018	0.210
7.5	0.5	1.82	6.60	13.26	1.21	37.51	17.51	0.060	0.122
7.5	0.8	3.83	4.49	15.18	0.75	40.46	18.87	0.015	0.210
7.5	1.2	5.19	3.15	16.68	0.48	42.37	19.74	0.010	0.257
7.5	1.5	5.79	2.58	17.36	0.36	43.19	20.11	0.019	0.276
10.0	0.5	1.86	7.26	12.73	1.34	48.86	15.42	0.045	0.089
10.0	0.8	4.62	4.51	14.05	0.78	42.94	15.90	0.013	0.187
10.0	1.2	6.37	2.83	15.19	0.45	40.80	16.22	0.009	0.245
10.0	1.5	7.12	2.12	15.73	0.32	40.11	16.36	0.019	0.268

Table 19: **Asset Pricing Implications:** $\delta = 0.999$

The entries are model population values of asset prices. The price-consumption ratio is given in 4.11 and the price-dividend ratio in 4.16. The input parameters for the monthly model are given in Table 16. The expressions $E[R_m - R_f]$ and $E[R_f]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The monthly subjective factor of discount is set to 0.999.

γ	ψ	$E[R_m - R_f]$	$E[R_f]$	$\sigma(R_m)$	$\sigma(R_f)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
2.5	0.5	0.15	4.69	13.40	1.16	36.97	46.98	0.058	0.050
2.5	0.8	0.72	3.35	15.95	0.72	63.48	99.86	0.009	0.046
2.5	1.2	1.26	2.52	18.10	0.49	104.88	248.74	0.004	0.024
2.5	1.5	1.53	2.16	19.12	0.40	141.35	589.05	0.006	0.011
5.0	0.5	1.17	4.97	13.80	1.14	48.38	29.78	0.048	0.092
5.0	0.8	2.62	3.31	16.49	0.72	70.92	36.55	0.009	0.136
5.0	1.2	3.70	2.23	18.56	0.49	93.86	41.43	0.005	0.153
5.0	1.5	4.19	1.77	19.51	0.40	107.21	43.66	0.008	0.159
7.5	0.5	1.92	5.57	13.24	1.22	75.51	20.89	0.031	0.102
7.5	0.8	4.12	3.33	15.22	0.75	81.12	22.55	0.008	0.177
7.5	1.2	5.60	1.91	16.77	0.47	84.75	23.61	0.005	0.218
7.5	1.5	6.25	1.30	17.48	0.35	86.31	24.06	0.010	0.234
10.0	0.5	1.81	6.32	12.67	1.37	147.73	18.15	0.015	0.071
10.0	0.8	4.83	3.37	13.99	0.79	92.11	18.59	0.006	0.157
10.0	1.2	6.73	1.58	15.15	0.45	78.57	18.91	0.005	0.208
10.0	1.5	7.53	0.82	15.70	0.31	74.61	19.05	0.011	0.229

Table 20: **Asset Pricing Implications: Log-linearization** $\delta = 0.999$

The entries are model population values of asset prices. The price-consumption ratio is given by (4.15) and the price-dividend ratio by (4.20). The input parameters for the monthly model are given in Table 16. The coefficient k_1 in the log-linearization (4.14) is set to 0.997 and the coefficient k_{m1} in the log-linearization (4.18) is set to 0.996. The expressions $E\left[R_m - R_f\right]$ and $E\left[R_f\right]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma\left(R_m\right)$, $\sigma\left(R_f\right)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The monthly subjective factor of discount is set to 0.999.

γ	ψ	$E\left[R_m - R_f\right]$	$E\left[R_f\right]$	$\sigma\left(R_m\right)$	$\sigma\left(R_f\right)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
2.5	0.5	0.76	4.69	13.23	1.16	35.36	36.28	0.058	0.059
2.5	0.8	1.78	3.34	15.14	0.72	48.56	46.31	0.011	0.085
2.5	1.2	2.54	2.53	16.59	0.49	58.00	53.10	0.006	0.095
2.5	1.5	2.88	2.19	17.25	0.40	62.29	56.09	0.012	0.098
5.0	0.5	1.29	4.88	13.65	1.14	41.19	29.32	0.053	0.089
5.0	0.8	2.74	3.26	15.96	0.72	50.44	34.56	0.011	0.132
5.0	1.2	3.74	2.27	17.67	0.49	56.54	37.94	0.007	0.152
5.0	1.5	4.19	1.85	18.44	0.40	59.21	39.39	0.013	0.159
7.5	0.5	1.89	5.26	13.40	1.20	48.63	22.59	0.045	0.103
7.5	0.8	3.91	3.24	15.52	0.74	52.58	24.42	0.011	0.173
7.5	1.2	5.26	1.98	17.14	0.48	55.00	25.53	0.007	0.212
7.5	1.5	5.85	1.44	17.87	0.37	56.02	25.99	0.014	0.227
10.0	0.5	2.07	5.75	12.87	1.30	56.37	19.32	0.037	0.084
10.0	0.8	4.84	3.24	14.44	0.77	54.56	19.30	0.010	0.173
10.0	1.2	6.63	1.67	15.74	0.46	53.66	19.31	0.007	0.228
10.0	1.5	7.39	1.00	16.35	0.33	53.32	19.33	0.014	0.251

Table 21: **Coefficients of the Campbell and Shiller (1988)'s log-linearization.** The entries are model implied coefficients of the Campbell and Shiller (1988)'s log-linearization. The price-consumption ratio is given by (4.15) and the price-dividend ratio by (4.20). The input parameters for the monthly model are given in Table 16.

γ	ψ	$\delta = 0.998$				$\delta = 0.999$			
		k_1	k_0	k_{m1}	k_{m0}	k_1	k_0	k_{m1}	k_{m0}
2.5	0.5	0.99673	0.02198	0.99723	0.01906	0.99774	0.01600	0.99822	0.01302
2.5	0.8	0.99768	0.01636	0.99819	0.01325	0.99869	0.01001	0.99916	0.00680
2.5	1.2	0.99821	0.01311	0.99871	0.00990	0.99921	0.00646	0.99966	0.00305
2.5	1.5	0.99842	0.01178	0.99891	0.00853	0.99941	0.00497	0.99986	0.00142
5	0.5	0.99721	0.01920	0.99634	0.02418	0.99827	0.01271	0.99720	0.01926
5	0.8	0.99781	0.01561	0.99690	0.02099	0.99883	0.00909	0.99770	0.01626
5	1.2	0.99813	0.01364	0.99720	0.01927	0.99911	0.00712	0.99796	0.01467
5	1.5	0.99825	0.01287	0.99731	0.01860	0.99922	0.00634	0.99806	0.01406
7.5	0.5	0.99778	0.01581	0.99525	0.03014	0.99889	0.00864	0.99602	0.02599
7.5	0.8	0.99794	0.01477	0.99557	0.02841	0.99897	0.00809	0.99629	0.02447
7.5	1.2	0.99804	0.01420	0.99575	0.02745	0.99902	0.00779	0.99644	0.02361
7.5	1.5	0.99807	0.01397	0.99582	0.02706	0.99904	0.00766	0.99650	0.02327
10	0.5	0.99829	0.01259	0.99462	0.03349	0.99943	0.00480	0.99542	0.02921
10	0.8	0.99806	0.01404	0.99476	0.03273	0.99910	0.00724	0.99552	0.02870
10	1.2	0.99796	0.01466	0.99485	0.03226	0.99894	0.00832	0.99558	0.02837
10	1.5	0.99793	0.01488	0.99489	0.03207	0.99888	0.00870	0.99561	0.02822

Table 22: **Asset Pricing Implications: Log-linearization with $\delta = 0.999$ and Analytical Coefficients**

The entries are model population values of asset prices. The price-consumption ratio is given by (4.15) and the price-dividend ratio by (4.20). The input parameters for the monthly model are given in Table 16. For each combination of preference parameters, the coefficients k_1 and k_0 in the log-linearization (4.14), and the coefficients k_{m1} and k_{m0} in the log-linearization (4.18) are given in Table 21. The expressions $E\left[R^e\right]$ and $E\left[R_f\right]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma\left(R\right)$, $\sigma\left(R_f\right)$, $\sigma\left(\frac{C}{P_M}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of equity return, risk-free rate, consumption-price ratio and dividend-price ratio. The monthly subjective factor of discount is set to 0.999.

γ	ψ	$E\left[R^e\right]$	$E\left[R_f\right]$	$\sigma\left(R\right)$	$\sigma\left(R_f\right)$	$E\left[\frac{P_M}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P_M}\right)$	$\sigma\left(\frac{D}{P}\right)$
2.5	0.5	0.15	4.69	13.41	1.16	36.83	46.89	0.058	0.050
2.5	0.8	0.73	3.35	15.97	0.72	63.46	98.99	0.009	0.046
2.5	1.2	1.27	2.52	18.12	0.49	104.87	244.58	0.004	0.025
2.5	1.5	1.54	2.16	19.13	0.40	141.27	575.38	0.006	0.012
5.0	0.5	1.18	4.96	13.82	1.14	48.00	29.73	0.048	0.093
5.0	0.8	2.69	3.30	16.60	0.72	70.88	35.90	0.009	0.140
5.0	1.2	3.81	2.23	18.75	0.49	93.83	40.14	0.005	0.161
5.0	1.5	4.33	1.77	19.74	0.40	107.09	42.04	0.008	0.169
7.5	0.5	1.95	5.55	13.26	1.22	74.69	20.83	0.032	0.103
7.5	0.8	4.22	3.33	15.31	0.75	81.06	22.13	0.008	0.184
7.5	1.2	5.77	1.91	16.95	0.47	84.72	22.85	0.005	0.231
7.5	1.5	6.46	1.30	17.69	0.35	86.20	23.14	0.010	0.250
10.0	0.5	1.83	6.31	12.68	1.37	146.07	18.12	0.015	0.072
10.0	0.8	4.91	3.37	14.03	0.79	92.04	18.36	0.006	0.162
10.0	1.2	6.87	1.58	15.24	0.45	78.54	18.47	0.005	0.217
10.0	1.5	7.71	0.82	15.80	0.30	74.52	18.50	0.011	0.240

Table 23: **Parameters of the Markov-Switching Model with Constant Volatility.** This table shows parameters of the Markov-Switching Model calibrated to match the model of Bansal and Yaron (2004). Calibration is made such that unconditional variance and kurtosis of MS mean of consumption matched similar moments in the BY model, whereas the implied skewness of MS mean is negative. The MS volatility of consumption is constant as in Case I of Bansal and Yaron (2004). μ_c and μ_d are conditional means of consumption and dividend, ω_c and ω_d are conditional variances of consumption and dividend. P^T is the transition matrix across different regimes and Π is the vector of unconditional probabilities of regimes. Means and standard deviations are in percent. The model is calibrated at the monthly frequency. The correlation vector ρ is set to zero.

	State 1	State 2
μ_c^T	-0.181	0.236
μ_d^T	-0.843	0.407
$\left(\omega_c^T\right)^{\frac{1}{2}}$	0.780	0.780
$\left(\omega_d^T\right)^{\frac{1}{2}}$	3.510	3.510
	P^T	
State 1	0.983	0.017
State 2	0.004	0.996
Π^T	0.205	0.795

Table 24: **Asset Pricing Implications:** $\delta = 0.998$

The entries are model population values of asset prices. The price-consumption ratio is given in 4.11 and the price-dividend ratio in 4.16. The input parameters for the monthly model are given in Table 23. The expressions $E[R_m - R_f]$ and $E[R_f]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The expression $P(R_f < 0)$ denotes the probability that the risk-free rate is negative. The monthly subjective factor of discount is set to 0.998.

γ	ψ	$E[R_m - R_f]$	$E[R_f]$	$\sigma(R_m)$	$\sigma(R_f)$	$P(R_f < 0)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
2.5	0.5	0.13	5.89	13.30	1.16	0.21	25.48	30.09	0.080	0.073
2.5	0.8	0.65	4.55	15.60	0.72	0.21	35.91	46.22	0.015	0.093
2.5	1.2	1.13	3.73	17.54	0.49	0.00	46.46	65.03	0.008	0.087
2.5	1.5	1.37	3.38	18.46	0.40	0.00	52.61	77.34	0.015	0.081
5.0	0.5	1.06	6.11	13.65	1.14	0.21	29.88	22.74	0.074	0.114
5.0	0.8	2.36	4.49	16.14	0.72	0.21	37.95	27.04	0.016	0.174
5.0	1.2	3.34	3.46	18.07	0.49	0.00	44.37	30.03	0.009	0.201
5.0	1.5	3.78	3.02	18.95	0.40	0.00	47.52	31.38	0.018	0.210
7.5	0.5	1.83	6.60	13.23	1.21	0.21	37.49	17.47	0.060	0.119
7.5	0.8	3.84	4.49	15.16	0.75	0.21	40.46	18.83	0.015	0.209
7.5	1.2	5.20	3.15	16.66	0.47	0.21	42.37	19.70	0.010	0.257
7.5	1.5	5.79	2.58	17.34	0.36	0.00	43.19	20.07	0.019	0.276
10.0	0.5	1.88	7.26	12.68	1.34	0.21	48.78	15.36	0.045	0.082
10.0	0.8	4.64	4.51	14.02	0.78	0.21	42.93	15.84	0.013	0.185
10.0	1.2	6.39	2.83	15.17	0.45	0.21	40.81	16.17	0.009	0.244
10.0	1.5	7.13	2.13	15.70	0.32	0.21	40.13	16.31	0.019	0.267

Table 25: **Predictability of Excess Returns by the Dividend-Price Ratio:** $\delta = 0.998$
This table shows the R-squared of the regression $R^e_{t+1:t+h} = a^e_1(h) + b^e_1(h)\frac{D_t}{P_t} + \eta^e_{1,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the monthly model are given in Table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2.5	0.8	0.08	0.13	0.16	0.17	0.17	0.17	0.17	0.16	0.15	0.15	0.11	0.09
2.5	1.2	0.20	0.33	0.40	0.43	0.44	0.44	0.43	0.41	0.40	0.38	0.29	0.23
2.5	1.5	0.27	0.44	0.53	0.58	0.60	0.60	0.59	0.56	0.54	0.52	0.40	0.31
5.0	0.5	0.11	0.17	0.21	0.22	0.23	0.23	0.22	0.21	0.20	0.19	0.14	0.11
5.0	0.8	0.38	0.62	0.75	0.82	0.84	0.84	0.82	0.79	0.76	0.73	0.56	0.44
5.0	1.2	0.60	0.97	1.20	1.31	1.36	1.37	1.34	1.30	1.25	1.20	0.93	0.74
5.0	1.5	0.70	1.13	1.40	1.54	1.60	1.61	1.58	1.53	1.48	1.42	1.11	0.88
7.5	0.5	0.05	0.07	0.09	0.09	0.10	0.09	0.09	0.09	0.08	0.08	0.06	0.05
7.5	0.8	0.14	0.23	0.28	0.30	0.31	0.30	0.29	0.28	0.27	0.26	0.20	0.15
7.5	1.2	0.21	0.33	0.40	0.43	0.45	0.44	0.43	0.42	0.40	0.38	0.29	0.23
7.5	1.5	0.23	0.37	0.45	0.49	0.50	0.50	0.49	0.47	0.45	0.43	0.33	0.26
10.0	0.5	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
10.0	0.8	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.01
10.0	1.2	0.02	0.04	0.04	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.02
10.0	1.5	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.03	0.03

Table 26: **Predictability of Excess Returns by the Dividend-Price Ratio: Simulation with $S = 1000$ and $T = 840$.**

This table shows the R-squared of the regression $R^e_{t+1:t+h} = a^e_1(h) + b^e_1(h)\frac{D_t}{P_t} + \eta^e_{1,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The entries are based on 1000 simulations of consumption and dividend processes, each with 840 monthly observations. The parameter configuration is given in Table 16. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	1.73	2.96	3.95	4.77	5.39	5.93	6.40	6.78	7.12	7.48	8.87	9.85
2.5	0.8	2.70	4.34	5.53	6.46	7.10	7.62	8.06	8.39	8.68	8.99	10.48	11.73
2.5	1.2	3.45	5.46	6.85	7.89	8.58	9.12	9.56	9.87	10.14	10.40	11.89	13.28
2.5	1.5	3.79	5.96	7.46	8.56	9.28	9.83	10.26	10.57	10.83	11.08	12.55	13.96
5.0	0.5	2.02	3.40	4.48	5.37	6.03	6.60	7.08	7.46	7.78	8.13	9.51	10.52
5.0	0.8	3.23	5.17	6.56	7.62	8.35	8.93	9.39	9.72	9.99	10.26	11.66	12.85
5.0	1.2	4.03	6.38	8.01	9.21	10.00	10.61	11.08	11.40	11.64	11.88	13.23	14.48
5.0	1.5	4.37	6.89	8.62	9.88	10.71	11.34	11.81	12.12	12.36	12.57	13.90	15.15
7.5	0.5	1.75	3.02	4.07	4.92	5.59	6.17	6.67	7.08	7.43	7.79	9.19	10.15
7.5	0.8	2.61	4.23	5.44	6.39	7.05	7.60	8.06	8.41	8.71	9.02	10.48	11.66
7.5	1.2	3.18	5.06	6.40	7.42	8.11	8.65	9.10	9.43	9.70	9.98	11.47	12.77
7.5	1.5	3.41	5.41	6.81	7.85	8.55	9.10	9.54	9.86	10.13	10.40	11.88	13.23
10.0	0.5	1.46	2.62	3.63	4.48	5.17	5.82	6.38	6.85	7.25	7.64	9.02	9.90
10.0	0.8	1.91	3.20	4.20	5.00	5.58	6.07	6.49	6.84	7.17	7.52	8.99	10.11
10.0	1.2	2.31	3.73	4.76	5.56	6.10	6.54	6.92	7.22	7.52	7.84	9.34	10.65
10.0	1.5	2.49	3.97	5.02	5.82	6.34	6.76	7.12	7.41	7.69	8.00	9.51	10.89

Table 27: **Predictability of Returns by the Dividend-Price Ratio:** $\delta = 0.998$

This table shows the R-squared of the regression $R_{t+1:t+h} = a_1(h) + b_1(h)\frac{D_t}{P_t} + \eta_{1,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the monthly model are given in Table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ						h						
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	5.85	8.27	9.09	9.12	8.79	8.29	7.73	7.17	6.63	6.14	4.28	3.18
2.5	0.8	1.23	1.83	2.10	2.17	2.14	2.06	1.95	1.84	1.72	1.60	1.15	0.87
2.5	1.2	0.17	0.27	0.31	0.33	0.33	0.32	0.31	0.29	0.28	0.26	0.19	0.15
2.5	1.5	0.02	0.04	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.03	0.02
5.0	0.5	4.33	6.21	6.89	6.97	6.75	6.39	5.98	5.56	5.16	4.78	3.35	2.50
5.0	0.8	0.56	0.85	0.99	1.03	1.03	1.00	0.95	0.90	0.84	0.79	0.57	0.44
5.0	1.2	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
5.0	1.5	0.03	0.05	0.06	0.06	0.07	0.07	0.06	0.06	0.06	0.06	0.04	0.03
7.5	0.5	5.52	7.82	8.61	8.65	8.34	7.86	7.34	6.80	6.30	5.83	4.06	3.02
7.5	0.8	1.20	1.79	2.05	2.13	2.10	2.02	1.92	1.80	1.69	1.58	1.13	0.86
7.5	1.2	0.17	0.27	0.32	0.33	0.34	0.33	0.31	0.30	0.28	0.26	0.19	0.15
7.5	1.5	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.03	0.02
10.0	0.5	7.46	10.35	11.22	11.15	10.65	9.98	9.26	8.55	7.88	7.27	5.01	3.70
10.0	0.8	2.94	4.28	4.80	4.90	4.78	4.55	4.27	3.99	3.71	3.45	2.44	1.82
10.0	1.2	1.05	1.58	1.82	1.89	1.87	1.80	1.71	1.61	1.50	1.41	1.01	0.77
10.0	1.5	0.57	0.87	1.01	1.06	1.06	1.02	0.97	0.92	0.86	0.81	0.59	0.45

Table 28: **Predictability of Returns by the Dividend-Price Ratio: Simulation with $S = 1000$ and $T = 840$**

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	5.79	8.25	9.24	9.37	9.10	8.74	8.39	8.04	7.74	7.52	7.08	7.70
2.5	0.8	2.98	4.61	5.54	5.99	6.18	6.32	6.44	6.54	6.67	6.83	7.78	9.27
2.5	1.2	2.97	4.68	5.73	6.35	6.71	7.01	7.27	7.49	7.75	8.01	9.37	11.11
2.5	1.5	3.21	5.06	6.21	6.91	7.33	7.68	7.98	8.23	8.52	8.80	10.24	12.03
5.0	0.5	4.69	6.84	7.82	8.06	7.95	7.76	7.58	7.38	7.23	7.13	7.11	7.97
5.0	0.8	2.74	4.32	5.29	5.83	6.12	6.36	6.58	6.78	6.99	7.22	8.42	10.05
5.0	1.2	3.12	4.95	6.11	6.82	7.27	7.66	7.98	8.26	8.58	8.87	10.37	12.16
5.0	1.5	3.48	5.51	6.79	7.60	8.11	8.54	8.89	9.19	9.52	9.82	11.36	13.16
7.5	0.5	5.60	8.03	9.03	9.21	9.00	8.70	8.41	8.11	7.87	7.68	7.33	7.97
7.5	0.8	2.83	4.42	5.36	5.83	6.05	6.22	6.38	6.51	6.66	6.83	7.79	9.25
7.5	1.2	2.72	4.33	5.35	5.96	6.34	6.67	6.96	7.22	7.50	7.77	9.16	10.88
7.5	1.5	2.90	4.62	5.72	6.40	6.83	7.21	7.53	7.83	8.14	8.44	9.93	11.69
10.0	0.5	7.50	10.43	11.45	11.51	11.13	10.63	10.18	9.72	9.34	9.03	8.23	8.54
10.0	0.8	3.74	5.62	6.58	6.93	6.98	6.95	6.93	6.88	6.87	6.89	7.27	8.36
10.0	1.2	2.74	4.31	5.26	5.74	5.98	6.18	6.36	6.51	6.68	6.87	7.88	9.36
10.0	1.5	2.62	4.16	5.12	5.66	5.96	6.22	6.46	6.67	6.90	7.13	8.34	9.93

Table 29: **Predictability of Returns by the Consumption-Price Ratio:** $\delta = 0.998$
This table shows the R-squared of the regression $R_{t+1:t+h} = a_2(h) + b_2(h)\frac{C_t}{P_t} + \eta_{2,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the monthly model are given in Table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	6.02	8.51	9.35	9.38	9.04	8.53	7.95	7.38	6.83	6.32	4.41	3.28
2.5	0.8	1.24	1.85	2.12	2.20	2.17	2.08	1.98	1.86	1.74	1.62	1.17	0.88
2.5	1.2	0.17	0.27	0.32	0.33	0.34	0.33	0.31	0.30	0.28	0.26	0.19	0.15
2.5	1.5	0.02	0.04	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.03	0.02
5.0	0.5	4.50	6.45	7.16	7.25	7.02	6.65	6.23	5.79	5.37	4.98	3.50	2.61
5.0	0.8	0.57	0.87	1.01	1.06	1.05	1.02	0.97	0.92	0.86	0.81	0.59	0.45
5.0	1.2	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
5.0	1.5	0.03	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.04	0.03
7.5	0.5	5.96	8.45	9.31	9.36	9.03	8.53	7.96	7.39	6.85	6.34	4.43	3.30
7.5	0.8	1.23	1.85	2.12	2.20	2.18	2.10	1.99	1.87	1.75	1.64	1.18	0.89
7.5	1.2	0.18	0.28	0.33	0.35	0.35	0.34	0.33	0.31	0.29	0.28	0.20	0.16
7.5	1.5	0.03	0.04	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.03	0.03
10.0	0.5	9.39	13.06	14.19	14.13	13.53	12.70	11.81	10.92	10.08	9.31	6.46	4.79
10.0	0.8	3.10	4.52	5.08	5.18	5.06	4.82	4.54	4.24	3.95	3.67	2.60	1.95
10.0	1.2	1.09	1.65	1.89	1.97	1.95	1.88	1.79	1.68	1.58	1.48	1.07	0.81
10.0	1.5	0.60	0.91	1.05	1.11	1.10	1.07	1.02	0.96	0.91	0.85	0.62	0.47

Table 30: **Predictability of Excess Returns by the Dividend-Price Ratio: Simulation with $S = 1000$ and $T = 840$**

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	1.40	2.56	3.50	4.34	5.08	5.66	6.21	6.78	7.31	7.83	9.59	10.57
2.5	0.8	1.49	2.74	3.72	4.57	5.31	5.94	6.56	7.19	7.74	8.26	9.96	11.38
2.5	1.2	1.54	2.82	3.82	4.68	5.42	6.08	6.73	7.37	7.94	8.46	10.19	11.87
2.5	1.5	1.55	2.84	3.86	4.72	5.46	6.13	6.79	7.44	8.01	8.53	10.27	12.05
5.0	0.5	1.38	2.53	3.47	4.30	5.03	5.61	6.16	6.73	7.26	7.78	9.53	10.52
5.0	0.8	1.47	2.71	3.69	4.55	5.28	5.92	6.53	7.15	7.71	8.23	9.94	11.36
5.0	1.2	1.53	2.80	3.81	4.67	5.40	6.06	6.71	7.35	7.93	8.45	10.19	11.85
5.0	1.5	1.54	2.83	3.85	4.71	5.45	6.12	6.78	7.43	8.00	8.53	10.28	12.04
7.5	0.5	1.36	2.51	3.46	4.30	5.02	5.62	6.17	6.75	7.29	7.81	9.59	10.61
7.5	0.8	1.47	2.71	3.70	4.57	5.30	5.95	6.57	7.20	7.77	8.30	10.04	11.48
7.5	1.2	1.53	2.81	3.83	4.70	5.44	6.12	6.77	7.42	8.00	8.53	10.31	11.98
7.5	1.5	1.55	2.84	3.87	4.75	5.49	6.18	6.84	7.50	8.09	8.62	10.42	12.17
10.0	0.5	1.35	2.50	3.46	4.31	5.04	5.66	6.24	6.82	7.38	7.90	9.73	10.79
10.0	0.8	1.47	2.72	3.74	4.62	5.36	6.03	6.67	7.30	7.89	8.42	10.21	11.68
10.0	1.2	1.53	2.83	3.88	4.77	5.52	6.22	6.88	7.54	8.13	8.68	10.50	12.17
10.0	1.5	1.56	2.87	3.93	4.83	5.58	6.29	6.96	7.62	8.22	8.77	10.61	12.36

Table 31: **Predictability of Excess Returns by the Dividend-Price Ratio: MS Matching BY, high γ**

This table shows the R-squared of the regression $R^e_{t+1:t+h} = a^e_1(h) + b^e_1(h)\frac{D_t}{P_t} + \eta^e_{1,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in Table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
5.0	0.5	0.11	0.17	0.21	0.22	0.23	0.23	0.22	0.21	0.20	0.19	0.14	0.11
5.0	0.8	0.38	0.62	0.75	0.82	0.84	0.84	0.82	0.79	0.76	0.73	0.56	0.44
5.0	1.2	0.60	0.97	1.20	1.31	1.36	1.37	1.34	1.30	1.25	1.20	0.93	0.74
5.0	1.5	0.70	1.13	1.40	1.54	1.60	1.61	1.58	1.53	1.48	1.42	1.11	0.88
10.0	0.5	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
10.0	0.8	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.01
10.0	1.2	0.02	0.04	0.04	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.02
10.0	1.5	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.03	0.03
15.0	0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15.0	0.8	0.63	0.97	1.13	1.20	1.20	1.17	1.12	1.06	1.00	0.94	0.69	0.53
15.0	1.2	1.29	1.95	2.25	2.34	2.32	2.24	2.13	2.01	1.88	1.76	1.27	0.97
15.0	1.5	1.57	2.34	2.68	2.78	2.75	2.64	2.51	2.36	2.20	2.06	1.48	1.12
20.0	0.5	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
20.0	0.8	0.91	1.38	1.61	1.69	1.69	1.64	1.56	1.47	1.39	1.30	0.95	0.72
20.0	1.2	2.75	4.06	4.61	4.74	4.65	4.45	4.19	3.93	3.66	3.41	2.43	1.83
20.0	1.5	3.57	5.20	5.83	5.94	5.79	5.51	5.18	4.83	4.49	4.17	2.95	2.21

Table 32: **Asset Pricing Implications: MS Matching BY, high γ**

The entries are model population values of asset prices. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the model (2.1)-(2.2) are given in Table 16. The expressions $E\left[R_m - R_f\right]$ and $E\left[R_f\right]$ are respectively the annualized equity premium and mean risk-free rate. The expressions $\sigma\left(R_m\right)$, $\sigma\left(R_f\right)$, $\sigma\left(\frac{C}{P}\right)$ and $\sigma\left(\frac{D}{P}\right)$ are respectively the annualized volatilities of market return, risk-free rate, consumption-price ratio and dividend-price ratio. The monthly subjective factor of discount is set to 0.998.

γ	ψ	$E\left[R_m - R_f\right]$	$E\left[R_f\right]$	$\sigma\left(R_m\right)$	$\sigma\left(R_f\right)$	$E\left[\frac{P}{C}\right]$	$E\left[\frac{P}{D}\right]$	$\sigma\left(\frac{C}{P}\right)$	$\sigma\left(\frac{D}{P}\right)$
5.0	0.5	1.05	6.11	13.68	1.14	29.89	22.77	0.074	0.115
5.0	0.8	2.36	4.49	16.16	0.72	37.95	27.08	0.016	0.174
5.0	1.2	3.33	3.46	18.09	0.49	44.37	30.09	0.009	0.201
5.0	1.5	3.78	3.02	18.96	0.40	47.51	31.44	0.018	0.210
10.0	0.5	1.86	7.26	12.73	1.34	48.86	15.42	0.045	0.089
10.0	0.8	4.62	4.51	14.05	0.78	42.94	15.90	0.013	0.187
10.0	1.2	6.37	2.83	15.19	0.45	40.81	16.22	0.009	0.245
10.0	1.5	7.12	2.13	15.73	0.32	40.11	16.36	0.019	0.268
15.0	0.5	0.69	8.49	12.48	1.62	86.61	15.18	0.022	0.053
15.0	0.8	5.09	4.52	13.02	0.85	47.08	14.44	0.010	0.127
15.0	1.2	7.63	2.23	13.69	0.41	38.74	14.15	0.008	0.185
15.0	1.5	8.67	1.29	14.03	0.24	36.38	14.05	0.018	0.210
20.0	0.5	-0.80	9.37	12.54	1.85	181.33	16.76	0.009	0.060
20.0	0.8	5.03	4.43	12.73	0.91	50.44	14.64	0.008	0.095
20.0	1.2	8.25	1.68	13.14	0.38	37.42	13.83	0.008	0.144
20.0	1.5	9.54	0.58	13.37	0.18	34.14	13.56	0.017	0.167

Table 33: **Predictability of Volatility by the Dividend-Price Ratio:** $\delta = 0.998$

This table shows the R-squared of the regression $\sigma^2_{t+1:t+h} = a_3(h) + b_3(h)\frac{D_t}{P_t} + \eta_{3,t+h}(h)$. The horizon h is monthly in the regression and converted into annual in the table. The price-consumption ratio is given by (4.11) and the price-dividend ratio by (4.16). The input parameters for the monthly model are given in Table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	2.08	1.88	1.70	1.55	1.41	1.29	1.18	1.09	1.00	0.93	0.65	0.48
2.5	0.8	0.58	0.53	0.48	0.44	0.40	0.36	0.33	0.31	0.28	0.26	0.18	0.14
2.5	1.2	0.35	0.32	0.29	0.26	0.24	0.22	0.20	0.18	0.17	0.16	0.11	0.08
2.5	1.5	0.29	0.26	0.24	0.22	0.20	0.18	0.17	0.15	0.14	0.13	0.09	0.07
5.0	0.5	2.24	2.03	1.84	1.67	1.53	1.40	1.28	1.18	1.08	1.00	0.70	0.52
5.0	0.8	0.63	0.57	0.52	0.47	0.43	0.39	0.36	0.33	0.30	0.28	0.20	0.15
5.0	1.2	0.36	0.33	0.30	0.27	0.25	0.23	0.21	0.19	0.18	0.16	0.11	0.08
5.0	1.5	0.30	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.14	0.13	0.09	0.07
7.5	0.5	4.45	4.02	3.65	3.32	3.03	2.77	2.54	2.33	2.15	1.99	1.39	1.03
7.5	0.8	1.05	0.95	0.86	0.79	0.72	0.66	0.60	0.55	0.51	0.47	0.33	0.25
7.5	1.2	0.57	0.51	0.46	0.42	0.39	0.35	0.32	0.30	0.27	0.25	0.18	0.13
7.5	1.5	0.45	0.41	0.37	0.34	0.31	0.28	0.26	0.24	0.22	0.20	0.14	0.10
10.0	0.5	14.47	13.09	11.87	10.79	9.84	9.00	8.25	7.59	7.00	6.47	4.53	3.36
10.0	0.8	2.29	2.08	1.88	1.71	1.56	1.43	1.31	1.20	1.11	1.03	0.72	0.53
10.0	1.2	1.09	0.98	0.89	0.81	0.74	0.68	0.62	0.57	0.53	0.49	0.34	0.25
10.0	1.5	0.83	0.75	0.68	0.62	0.56	0.51	0.47	0.43	0.40	0.37	0.26	0.19

Table 34: **Variance Ratios of Aggregate Returns:** $\delta = 0.998$

This table shows the variance ratios $\frac{Var(R_{t+1:t+h})}{hVar(R_{t+1})}$, where the horizon h is monthly and converted into annual in the table. The price-consumption ratio is given by (4.11) and price-dividend ratio by (4.16). The input parameters for the monthly model are given in table 16. The monthly subjective factor of discount is set to 0.998.

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	1.14	1.27	1.38	1.48	1.56	1.63	1.69	1.74	1.78	1.82	1.96	2.04
2.5	0.8	1.06	1.12	1.17	1.21	1.24	1.27	1.30	1.32	1.34	1.36	1.42	1.46
2.5	1.2	1.02	1.04	1.06	1.07	1.09	1.10	1.11	1.12	1.12	1.13	1.15	1.16
2.5	1.5	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.04	1.05	1.05	1.06	1.06
5.0	0.5	1.12	1.23	1.32	1.40	1.47	1.53	1.58	1.63	1.66	1.70	1.82	1.88
5.0	0.8	1.04	1.08	1.11	1.13	1.16	1.18	1.19	1.21	1.22	1.23	1.27	1.29
5.0	1.2	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.03
5.0	1.5	0.99	0.98	0.98	0.97	0.97	0.96	0.96	0.96	0.95	0.95	0.94	0.94
7.5	0.5	1.13	1.26	1.37	1.46	1.54	1.60	1.66	1.71	1.76	1.79	1.93	2.00
7.5	0.8	1.06	1.11	1.16	1.20	1.23	1.26	1.29	1.31	1.33	1.34	1.40	1.43
7.5	1.2	1.02	1.04	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12	1.14	1.16
7.5	1.5	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.04	1.05	1.05	1.06	1.06
10.0	0.5	1.17	1.32	1.45	1.57	1.66	1.74	1.82	1.88	1.93	1.98	2.14	2.24
10.0	0.8	1.09	1.18	1.26	1.32	1.38	1.42	1.46	1.50	1.53	1.56	1.65	1.70
10.0	1.2	1.05	1.10	1.15	1.18	1.21	1.24	1.26	1.28	1.30	1.32	1.37	1.40
10.0	1.5	1.04	1.08	1.11	1.13	1.15	1.17	1.19	1.21	1.22	1.23	1.27	1.29

Table 35: **Variance Ratios of Aggregate Returns: Simulation with $S = 1000$ and $T = 840$**

γ	ψ	h											
		1	2	3	4	5	6	7	8	9	10	15	20
2.5	0.5	1.11	1.19	1.27	1.33	1.37	1.41	1.43	1.45	1.45	1.46	1.40	1.26
2.5	0.8	1.04	1.07	1.09	1.11	1.12	1.13	1.13	1.13	1.13	1.12	1.05	0.93
2.5	1.2	1.01	1.01	1.01	1.00	1.00	1.00	0.99	0.98	0.97	0.96	0.88	0.77
2.5	1.5	1.00	0.99	0.98	0.97	0.96	0.95	0.94	0.92	0.91	0.90	0.81	0.71
5.0	0.5	1.09	1.16	1.22	1.27	1.30	1.33	1.35	1.36	1.36	1.36	1.30	1.17
5.0	0.8	1.02	1.03	1.04	1.05	1.05	1.05	1.05	1.04	1.04	1.03	0.95	0.84
5.0	1.2	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.90	0.89	0.88	0.79	0.69
5.0	1.5	0.98	0.96	0.94	0.92	0.91	0.89	0.87	0.86	0.84	0.83	0.74	0.64
7.5	0.5	1.10	1.19	1.26	1.31	1.36	1.39	1.41	1.42	1.43	1.44	1.38	1.24
7.5	0.8	1.04	1.06	1.08	1.10	1.11	1.12	1.12	1.12	1.11	1.10	1.03	0.92
7.5	1.2	1.01	1.00	1.00	1.00	1.00	0.99	0.98	0.97	0.96	0.95	0.87	0.76
7.5	1.5	1.00	0.98	0.97	0.96	0.96	0.95	0.93	0.92	0.91	0.89	0.81	0.71
10.0	0.5	1.13	1.24	1.33	1.41	1.46	1.51	1.54	1.56	1.57	1.58	1.53	1.38
10.0	0.8	1.07	1.12	1.16	1.20	1.22	1.24	1.25	1.26	1.26	1.26	1.19	1.07
10.0	1.2	1.03	1.05	1.07	1.09	1.10	1.10	1.10	1.10	1.09	1.09	1.01	0.90
10.0	1.5	1.02	1.03	1.04	1.05	1.05	1.05	1.05	1.04	1.03	1.02	0.95	0.84

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