HISTORY OF MATHEMATICS

PROJECT OF THE FIRST SEMESTER

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Introduction

Throughout history, mathematics has played a pivotal role in understanding the world around us and developing science and technology. Amidst this intellectual momentum, names emerged that made a difference and contributed to shaping the foundations of mathematics as we know it today. Among these scientists stands the Prince of Mathematics as a symbol of creativity and innovation, as his achievements were not just theories or equations, but rather a scientific legacy that inspired generations throughout the ages. In this research, we delve into the life of this scientist, review the stations of his intellectual career, and shed light on his fingerprints in mathematics.

Johann Carl Friedrich Gauss

Johann Carl Friedrich Gauss (April 30, 1777 – February 23, 1855) was a German mathematician and scientist, recognized as one of history's greatest minds. His genius appeared early; he was able to calculate his exact date of birth based on family records, as it was not initially recorded. At age 7, he solved a complex arithmetic problem, and at 11, he began formal studies in mathematics. At 19 (1796), he constructed a 17-sided polygon using only a ruler and compass, a notable mathematical feat. Over his lifetime, Gauss made groundbreaking contributions to number theory, geometry, algebra, and astronomy, including .calculating the orbit of the asteroid Ceres in 1801

In 1807, Gauss was appointed director of the Göttingen Observatory, where he formulated the method of least squares. In the 1830s, he collaborated with Wilhelm Weber to study magnetism, inventing the first electromagnetic telegraph in 1833. Gauss also contributed to Earth's magnetic field surveys and mapping techniques . Despite his brilliance, he published selectively, often delaying discoveries to ensure precision. He was multilingual, an expert mapmaker, and actively engaged in scientific advancements until his passing in ..1855

Gauss Works and Achievements :

Gauss's achievement in geometry:

Non Euclidean geometries of Gauss:

Gauss had early doubts about the completeness of Euclidean geometry, particularly regarding the concept of a plane and the parallel postulate. He questioned whether Euclid's axioms were sufficient to fully describe the nature of space “Apart from the well-known gap in Euclid’s geometry, there is another that, to my knowledge no one has noticed and which is in no way easy to alleviate (although possible).

This is the definition of a plane as a surface that contains the line joining any two of its points. This definition contains more than is necessary for the determination of the surface, and tacitly involves a theorem which must first be proved…” (Gauss to Bessel, VIII, p. 200). "Gauss hinted at the possibility of non-Euclidean geometry, where the sum of the angles of a triangle could differ from 180 degrees, but he did not publish these ideas openly, perhaps due to concerns about their acceptance.". "The assumption that the sum of the angles of a triangle is less than 180 degrees leads to a geometry that is entirely different from Euclidean geometry, and it is perfectly logical, and I am entirely satisfied with it." (Gauss to Bessel, VIII, p. 187) .

The remarkable theorem of GAUSS:

Gauss searched for a formula to characterize the curvature of a surface at any point. When he eventually found it and published it in his Disquisitiones Generales Circa Superficies Curva ("General Research on Curved Surfaces") in 1828, he named it the "remarkable theorem". What was so remarkable? Gauss started with the naive view of curvature: embedding the surface in three-dimensional space and calculating how bent it is.

But the answer showed him that the surrounding space didn’t matter; it didn’t enter into the formula. He wrote: "The formula... leads itself to the remarkable theorem: If a curved surface is developed upon any other surface whatsoever, the measure of curvature at each point remains unchanged." (Seventeen Equations that Changed the World by Ian Stewart · 2012 .p 17) .

Gauss'sTopology :

Gauss published General investigations of curved surfaces, which in section 3 defines the curved surface in a similar manner to the modern topological understanding: "A curved surface is said to possess continuous curvature at one of its points A, if the direction of all the straight lines drawn from A to points of the surface at an infinitesimal distance from A are deflected infinitesimally from one and the same plane passing through A." (Topology By Dr .S.Kohila ,Dr.B.Komala Durga) .

due to this Gauss discovered the following definitions : Continuous Curvature: This refers to the idea that the curvature of the surface changes smoothly and continuously from point to point. There are no sudden jumps or discontinuities in the curvature. Infinitesimal Distance: Gauss is considering points on the surface that are infinitely close to the point A. Deflection from a Plane: The key idea is that the directions of lines drawn from A to nearby points on the surface deviate slightly from a single plane. This deviation is what characterizes the curvature of the surface at point A .

The impact of his discoveries is the following :

Topological Perspective: Gauss's definition is essentially a topological one. It focuses on the local properties of the surface, such as the way lines are deflected, rather than on specific metric properties like length and angle. Foundation for Differential Geometry: This definition laid the foundation for the development of differential geometry, a branch of mathematics that studies the geometry of curved surfaces. Influence on Later Mathematicians: Gauss's work inspired later mathematicians, such as Bernhard Riemann, to further develop the ideas of curvature and topology .

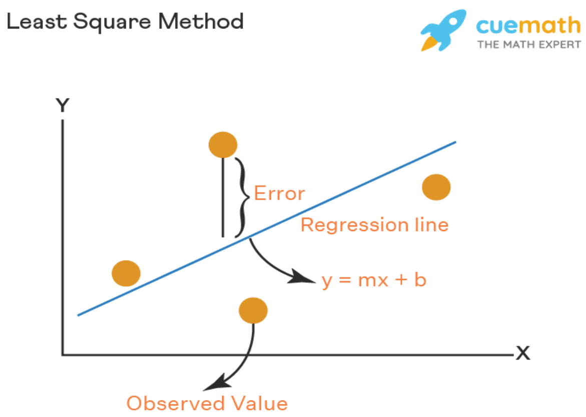
Gauss's Pentagramma Mirificum :

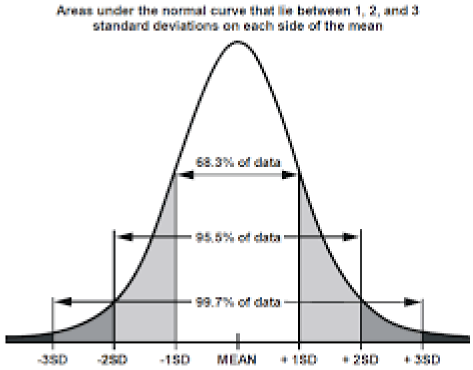
Gauss showed that the five "parts" in the above order are the supplements of the angles of a pentagon whose five pairs of alternate sides are perpendicular. They are also the halves (Non Euclidean Geometry By H ,S . M. Coxeter · 1998 p 234) . This discovery highlights the deep connections between different geometric figures and the power of mathematical reasoning. It shows how seemingly unrelated shapes can be linked through elegant mathematical relationships. Gauss's work on the Pentagramma Mirificum contributed to the development of non-Euclidean geometry and the understanding of the properties of hyperbolic space .

Gauss' s achievement in statistics :

When Gauss left the Caroline College in October, 1795 at the age of eighteen to enter the University of Gottingen . He had already invented (when he was eighteen) the least-squares method (the least-squares method is a statistical method used to find the line of best fit of the form of an equation such as $y = mx + b$ to the given data and it's formula : $\sum XY = a\sum X + b\sum X2$. (course, 2011)) which today is indispensable in geodetic surveying, in the reduction of observations and indeed in all work where the “most probable” value of anything that is measured is to be inferred from a large number of measurements. (The most probable value is furnished by making the sum of the squares of the “residuals’’—roughly, divergences from assumed exactness—a minimum.) Gauss shares this honor with Legendre who published the method independently in 1806.

This work was the beginning of Gauss' interest in the theory of errors of observation. The Gaussian law of normal distribution of errors and its accompanying bell-shaped curve (The term "bell curve" is used to describe a graphical depiction of a normal probability distribution whose underlying standard deviations from the mean create the curved bell shape (Bloomenthal, 2024)) is familiar today to all who handle statistics, from high-minded intelligence testers to unscrupulous market manipulators. (Bell, 1937).





Gauss's Achievements in Number Theory:

In the field of numbers theory gauss made a profound contributions. Proving several important theorems laying the foundations of the modern mathematics, especially when he published at 1801 his main mathematical work "Disquisitions arithmeticae" (Glitsch, 2005, p. 59) , and which explain with details the most important theories of numbers theories. And mention "several conjectures that continue to occupy us to this day" (Tschinkel, 2007, p. 247)'.

Quadratic Reciprocity:

The Quadratic Reciprocity Law was first formulated by Euler and Legendre and proved by Gauss and partly by Legendre, (FULLTEXT01) , Such that he says that the quadratic equation has integer solution and it gives the conditions for a prime p to be a quadratic residue modulo a prime q in terms of whether or not q is a quadratic residue modulo p .This law help us to prove the existence of primes in certain arithmetic progressions, plus each primes can be expressed in terms of some quadratic forms (Lemmermeyer, 2000, p. 9)

Prime numbers:

He worked on it and made important conjectures on prime numbers that helped the mathematician after him to discover it, which describes the distribution of primes. It states that, the number of primes ≤ n approximately equals n/ln (n). To answer fundamental questions about the structure of integers and enables the development of algorithms in various fields (Ore, 1948).

gauss's circle problem:

This investigates the distribution of lattice points within a circle centered at the origin and with radius. And the first progress on solution was by gauss, such that it asks to estimate the number of lattice points g(r) = πr2 + E(r) enclosed by the circle r. (department, 7/26/05)

Diophantine Equations:

These are equations that seek integer solutions, named after the ancient Greek mathematician Diaphantus and Gauss made significant contributions to their study particularly through modular arithmatice and congruence.

Gauss and congruence :

Congruence is a very important concept , it is One of Gauss's most important contributions to number theory involved the invention of the idea of congruence (or agreement) in numbers and the use of what he called "modulos" or small measures or sets of numbers. In effect, his theory of congruence allows people to break up the infinite series of whole numbers into smaller, more manageable chunks of numbers and perform computations upon them. This arrangement makes the everyday arithmetic involved in such things as telling time much easier to program into computers. (para 2) .Gauss said that if one number is subtracted from another (a-b), and the remainder of the subtraction can be divided by another number,m , then a and b are congruent to each other by the number m Gauss's formula is as follows: a is congruent to b modulo c. (para 3) **(JRank articles , para 2 - 3)**

Gauss integers :

Gauss integers are complex numbers such that both their real and imaginary parts are integers , where it is written as Z= a + i b ; a ,b $\epsilon \mathbb{Z}$. There is also a subset of the gauss integers called gauss primes "A prime element of the ring of Gaussian integers will be called a gauss prime ." "If p is a prime integer , then either p is a Gauss prime or else it is a product of two complex conjugate Gauss primes ." **(Adhikari, 2001, p. 72)**

Gauss effectively shown that the primes existed in other sets then integers , where in complex numbers , primes of the form 4k-1 are still primes , but 2 and primes of the form 4k+1 can be factored in $\mathbb{C}$ so they aren't primes **(Schroeder,2006 , p. 307)** .

Gauss and Firma's last theorem :

Euler and Karl Friedrich Gauss (1777-1850) greatly extended the work done by Fermat. Gauss's book Disquisitiones Arithmeticae, which was published in 1801, gives a treatment of modular arith- metic that is very close to the present-day version.**(James S.Kraft, Lawrence C. Washington , 2016 , p.4)**

Karl Friedrich Gauss didn't directly prove Fermat's Last Theorem, but his work laid the groundwork for later breakthroughs. His research in number theory, especially on concepts like modular forms and the distribution of prime numbers, influenced the development of tools that were crucial for proving the theorem. While Gauss worked long before Andrew Wiles's eventual proof in 1994, his contributions to areas like Gaussian integers and quadratic reciprocity shaped the mathematical landscape. In many ways, Gauss's ideas helped set the stage for Wiles's success in solving this centuries-old puzzle.

Gauss and cryptography :

Although Gauss never worked directly in this filled, his works above paved the way to today's cryptography , where concepts such as congruence and Gaussian integers are basic and crucial to cryptography , since primes are initial , as the keys to one of the most important coding and decoding methods . **.(James S.Kraft, Lawrence C. Washington , 2016 , p.7-8 .and chapter 8)**

Major Publications :

Disquisitiones Arithmeticae (1801) :

The *Disquisitiones* covers both elementary number theory and parts of the area of mathematics now called :Algebraic Number Theory. Gauss did not explicitly recognize the concept of a group, which is central to modern algebra, so he did not use this term. His own title for his subject was Higher Arithmetic. In his Preface to the *Disquisitiones*, Gauss describes the scope of the book as follows:

The inquiries which this volume will investigate pertain to that part of Mathematics which concerns itself with integers.

Gauss also writes, "When confronting many difficult problems, derivations have been suppressed for the sake of brevity when readers refer to this work."

Carl Friedrich Gauss Werke page :212-223

Theoria Motus Corporum Coelestium (1809) :

A book about the orbital motion of celestial bodies.It deals with astronomical calculation methods, especially predicting the positions of planets and comets.

Theoria Combinationis Observationum Erroribus Minimis Obnoxiae (1823–1828) :

Discusses data analysis methods, including the normal distribution known as the Gaussian distribution.

Allgemeine Theorie des Erdmagnetismus (1839) :

It deals with the study of the Earth's magnetic field and the development of units for measuring magnetism.

5-Dioptrische Untersuchungen (1840) :

It deals with the design of lenses and the calculation of their properties to improve the performance of optical instruments.

Conclusion

Carl Friedrich Gauss, the Prince of Mathematics, was a living example of unparalleled mathematical genius, and left an indelible mark on the history of science. Gauss passed away on February 23, 1855 in Göttingen, leaving behind an exceptional scientific legacy that inspired many scientists, as testified by his contemporaries such as the Duke of Brunswick. His legacy continues to inspire us today to reach new heights in scientific and cognitive ..pursuits

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