physics

density: mass/volume

Dimensions are often denoted with square brackets.

• Length [L]

• Mass [M]

• Time [T]

•Multiply original value by a ratio equal to one.

•Example:

$$15.0\,in = ?\,cm$$

$$15.0\,\cancel{in}\left(\frac{2.54\,cm}{1\,\cancel{in}}\right) = 38.1cm$$

- Note the value inside the parentheses is equal to 1, since 1 inch is defined as 2.54 cm.

• Represented as 👎x

👎x ≡ xf - xi

Vector: quantities need both magnitude and direction

•Scalar quantities are completely described by magnitude only

Average Velocity

•The **average velocity** is rate at which the displacement occurs.

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The x indicates motion along the x-axis.

•The dimensions are length / time [L/T]

•The SI units are m/s

•Is also the slope of the line in the position – time graph

Average Speed

•Speed is a scalar quantity.

- Has the same units as velocity
- Defined as total distance / total time: $v_{avg} \equiv \dfrac{d}{t}$

Instantaneous Velocity

- Definition: The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero.

- Purpose: Indicates what is happening at every point in time.

Instantaneous Velocity, equations

•The general equation for instantaneous velocity is:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

•The instantaneous velocity can be positive, negative, or zero.

Note

•"Velocity" and "speed" will indicate instantaneous values.

•Average will be used when the average velocity or average speed is indicated.

Average Acceleration

•Acceleration is the rate of change of the velocity.

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

•Dimensions are L/T^2

•SI units are m/s^2

•In one dimension, positive and negative can be used to indicate direction.

Instantaneous Acceleration

•The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0.

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

•The term acceleration will mean instantaneous acceleration.

- If average acceleration is wanted, the word average will be included.

Notes

•When an object's velocity and acceleration are in the same direction,

the object is speeding up.

•When an object's velocity and acceleration are in the opposite

direction, the object is slowing down.

•Negative acceleration does not necessarily mean the object is slowing

down.

- If the acceleration and velocity are both negative, the object is speeding up.

Constant Velocity=Acceleration equals zero.

Constant Acceleration=Acceleration is uniform (acceleration is increasing or decreasing constantly)

Kinematic Equations

- can be used with any particle under uniform acceleration.

TABLE 2.2 *Kinematic Equations for Motion of a Particle Under Constant Acceleration*

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

When $a = 0$

- When the acceleration is zero,
  - $v_{xf} = v_{xi} = v_x$
  - $x_f = x_i + v_x t$
- The constant acceleration model reduces to the constant velocity model.

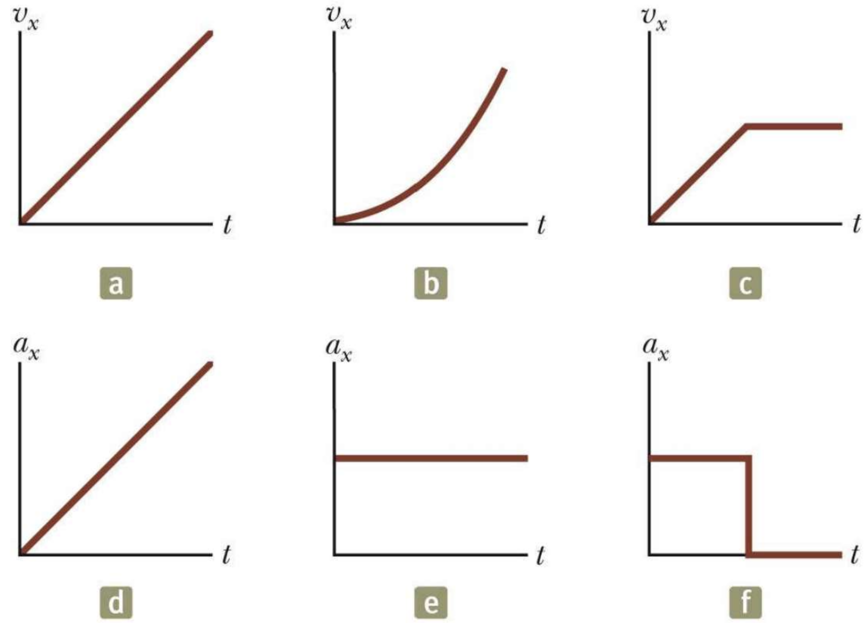
Velocity – Time curve

- The slope gives the acceleration.
- The straight line indicates a constant acceleration.

Acceleration –Time curve

- The zero slope indicates a constant acceleration.

Match the v_x-t graphs with their respective a_x-t graphs



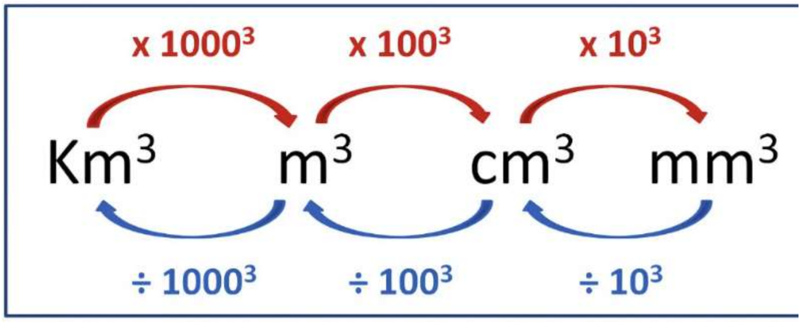
a with e

b with d

c with f

Acceleration of Freely Falling Object

- The acceleration of an object in free fall is directed downward, regardless of the initial motion.
- The magnitude of free fall acceleration is g = 9.80 m/s2 = 32.2 ft/s2



The formula to find time is:

$t = \frac{d}{v}$

where:

- t is the time,
- d is the distance (0.3 meters in this case),
- v is the speed of light ($299,792,458\,\mathrm{m/s}$).

Vectors

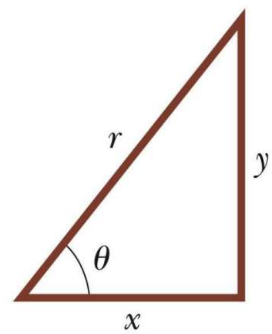
Polar to Cartesian Coordinates

- Based on forming a right triangle from r and θ
- $x = r\cos\theta$
- $y = r\sin\theta$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



- If the Cartesian coordinates are known:

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

Vectors and Scalars

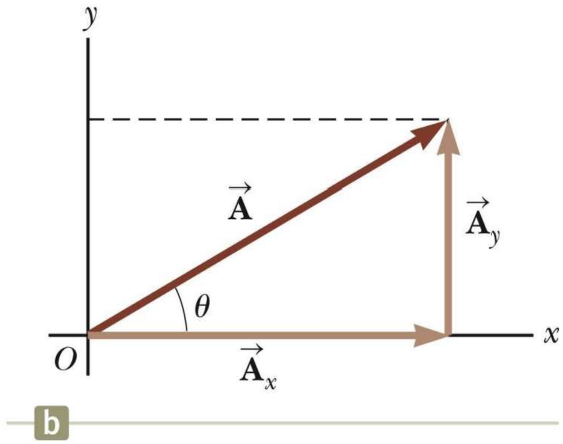
- scalar quantity is completely specified by a single value with an appropriate unit and has no direction.
- A vector quantity is completely described by a number and appropriate units plus a direction.

Equality of Two Vectors

- Two vectors are equal if they have the same magnitude and the same direction.
- if A = B and they point along parallel lines

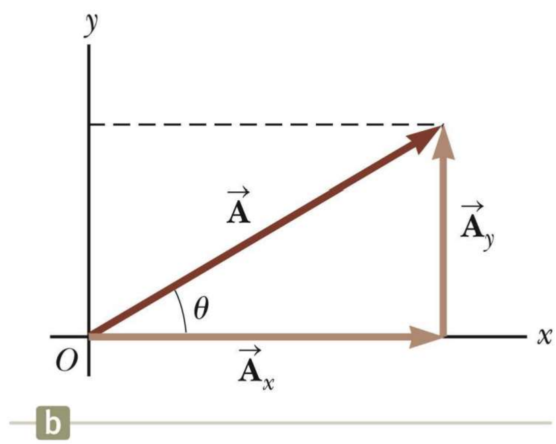
Components of a Vector

- Assume you are given a vector $\vec{\mathbf{A}}$
- It can be expressed in terms of two other vectors, $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$
- These three vectors form a right triangle.
- $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$



Components of a Vector, 2

- The *y*-component is moved to the end of the *x*-component.
- This is due to the fact that any vector can be moved parallel to itself without being affected.
  - This completes the triangle.



Components of a Vector, 3

•The x-component of a vector is the projection along the *x*-axis.

$$A_x = A\cos\theta$$

•The y-component of a vector is the projection along the *y*-axis.

$$A_y = A\sin\theta$$

•This assumes the angle θ is measured with respect to the *x*-axis.

- If not, do not use these equations, use the sides of the triangle directly.

Components of a Vector, 4

•The components are the legs of the right triangle whose hypotenuse is the length of *A*.

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

- May still have to find θ with respect to the positive *x*-axis

•In a problem, a vector may be specified by its components or its magnitude and direction.

Unit Vectors

•A unit vector is a dimensionless vector with a magnitude of exactly 1.

•Unit vectors are used to specify a direction and have no other physical significance.

Adding Vectors Using Unit Vectors

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

- So $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Three-Dimensional Extension

- Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$
- Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

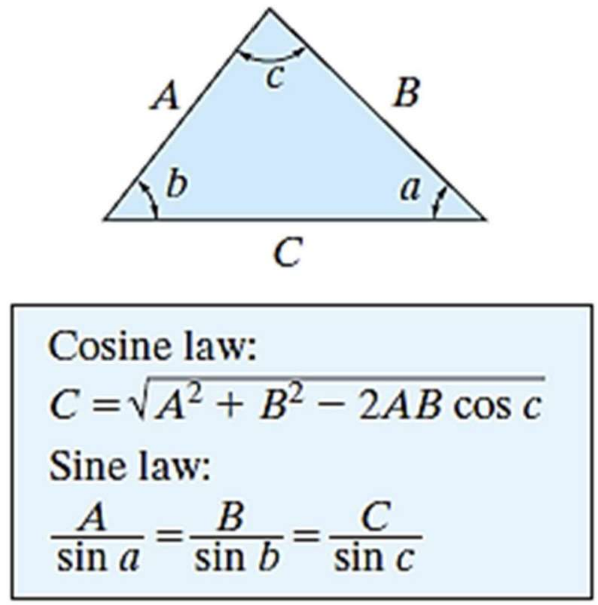
$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

- So $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \boldsymbol{\theta_x = \cos^{-1}\frac{R_x}{R}},\ etc.$$

Useful trigonometry identities



(c)

$$
\begin{aligned}
-\vec{A} - \vec{B} = -(\vec{A} + \vec{B}) = \\
&= -[(0\,\hat{\imath} + (-8.00)\hat{\jmath}) + (7.5\,\hat{\imath} + 13.0\,\hat{\jmath})] \\
&= -7.5\,\hat{\imath} - 5.00\,\hat{\jmath}
\end{aligned}
$$

$$
\begin{aligned}
\vec{B} - \vec{A} = -(\vec{A} - \vec{B}) = \\
&= -[(0\,\hat{\imath} + (-8.00)\hat{\jmath}) - (7.5\,\hat{\imath} + 13.0\,\hat{\jmath})] \\
&= 7.5\,\hat{\imath} + 21.0\,\hat{\jmath}
\end{aligned}
$$

Motion in two dimensions

Average Velocity

•The average velocity is the ratio of the displacement to the time interval for the displacement.

$$\vec{\mathbf{v}}_{avg} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

•The direction of the average velocity is **the direction of the displacement vector.**

•The average velocity between points is *independent of the path* taken.

- This is because it is dependent on the displacement, which is also independent of the path.

Instantaneous Velocity

•The instantaneous velocity is the limit of the average velocity as Δt approaches zero.

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

- As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve.

Average Acceleration

•The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

$$\vec{\mathbf{a}}_{avg} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$$

Instantaneous Acceleration

•The instantaneous acceleration is the limiting value of the ratio $\Delta\vec{\mathbf{v}}/\Delta t$ as Δt approaches zero.

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

- The instantaneous acceleration equals the **derivative of the velocity vector with respect to time.**

Analyzing Projectile

- The initial velocity can be exp
  - $v_{xi} = v_i \cos\theta$ and $v_{yi} = v_i \sin\theta$
- The x-direction has constant v
  - $a_x = 0$
- The y-direction is free fall.
  - $a_y = -g$

Key Equations

1. **Horizontal Range (R):**
   $R = \frac{v_0^2 \sin(2\theta)}{g}$
2. **Maximum Height (H):**
   $H = \frac{v_0^2 \sin^2(\theta)}{2g}$

Chapter 5: The laws of motion

- $\Sigma F_x = m\ a_x$
- $\Sigma F_y = m\ a_y$
- $\Sigma F_z = m\ a_z$

- 1 N = 1 kg·m/s power 2

Weight = Fg= mg

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Analysis Model: The Particle in Equilibrium

- If the acceleration of an object that can be modeled as a particle is **zero**, the object is said to be in **equilibrium.**
  - The model is the *particle in equilibrium model.*

- Mathematically, the net force acting on the object is zero.

$$\sum \vec{\mathbf{F}} = 0$$

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

Newton's Second Law, Example 1, cont.

•Apply Newton's Second Law in component form:

$$\sum F_x = T = ma_x$$

$$\sum F_y = n - F_g = 0 \rightarrow n = F_g$$

•Solve for the unknown(s)

•If the tension is constant, then *a* is constant and the kinematic equations can be used to more fully describe the motion of the crate.

Dot Product, cont.

•The dot product is commutative.

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

•The dot product obeys the distributive law of multiplication.

- $\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{B}} + \vec{\mathbf{C}} \right) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

Dot Products of Unit Vectors

- $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$

  $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$

•Using component form with vectors:

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

•In the special case where

$$\vec{\mathbf{A}} = \vec{\mathbf{B}};$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Kinetic Energy, cont

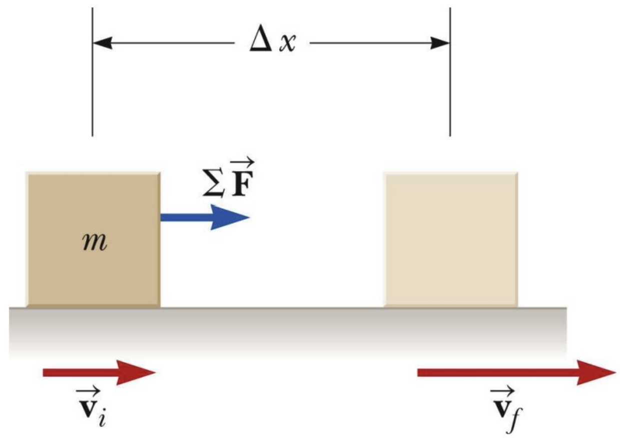
•Calculating the work:

$$W_{ext} = \int_{x_i}^{x_f} \sum F\, dx = \int_{x_i}^{x_f} ma\, dx$$

$$W_{ext} = \int_{v_i}^{v_f} mv\, dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{ext} = K_f - K_i = \Delta K$$



Newton's Second Law and Momentum

•Newton's Second Law can be used to relate the momentum of a particle to the resultant force acting on it.

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\left(m\vec{\mathbf{v}}\right)}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$$

with constant mass

•The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

- This is the form in which Newton presented the Second Law.
- It is a more general form than the one we used previously.
- This form also allows for mass changes.

Conservation of Momentum, 2

•Conservation of momentum can be expressed mathematically in various ways:

- $\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = \text{constant}$
- $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

•In component form, the total momenta in each direction are independently conserved.

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$ $\quad p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$ $\quad p_{1iz} + p_{2iz} = p_{1fz} + p_{2fz}$

•Conservation of momentum can be applied to systems with any number of particles.

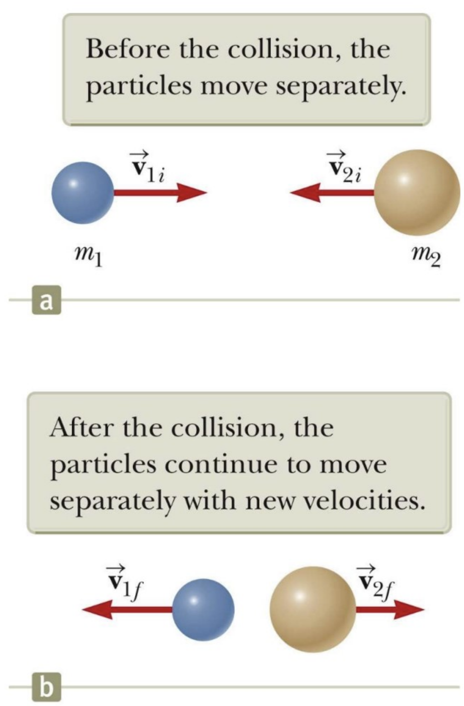
•The momentum version of the isolated system model states *whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.*

Elastic Collisions

•Both momentum and kinetic energy are conserved.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$



Conversions

•Comparing degrees and radians

$$1\,rad = \frac{360^\circ}{2\pi} = 57.3^\circ$$

•Converting from degrees to radians

$$\theta\,(rad) = \frac{\pi}{180^\circ}\,\theta\,(degrees)$$

Angular Acceleration

•The **average angular acceleration, α_{avg}**, of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change.

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

•The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0.

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

TABLE 10.1 *Kinematic Equations for Rotational and Translational Motion*

Rigid Body Under Constant Angular Acceleration	Particle Under Constant Acceleration
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Centripetal Acceleration

•An object traveling in a circle, even if it moves with a constant speed, will have an acceleration.

- Therefore, each point on a rotating rigid object will experience a centripetal acceleration.

$$a_C = \frac{v^2}{r} = r\omega^2$$

Resultant Acceleration

•The tangential component of the acceleration is due to changing speed.

•The centripetal component of the acceleration is due to changing direction.

•Total acceleration can be found from these components:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Properties of the Vector Product

•The vector product is *not* commutative. The order in which the vectors are multiplied is important.

- To account for order, remember $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$

•If $\vec{\mathbf{A}}$ is parallel to $\vec{\mathbf{B}}$ ($\theta = 0^\circ$ or 180°), then $\boxed{\vec{\mathbf{A}} \times \vec{\mathbf{B}} = 0}$

- Therefore $\vec{\mathbf{A}} \times \vec{\mathbf{A}} = 0$

•If $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}}$, then $\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB$

•The vector product obeys the distributive law.

$$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \vec{\mathbf{C}}$$

Vector Products of Unit Vectors

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

- Signs are interchangeable in cross products
  - $\vec{\mathbf{A}} \times \left(-\vec{\mathbf{B}}\right) = -\vec{\mathbf{A}} \times \vec{\mathbf{B}}$
  - and $\hat{\mathbf{i}} \times \left(-\hat{\mathbf{j}}\right) = -\hat{\mathbf{i}} \times \hat{\mathbf{j}}$

Using Determinants

- The cross product can be expressed as

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

- Expanding the determinants gives

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_y B_z - A_z B_y\right) \hat{\mathbf{i}} - (A_x B_z - A_z B_x)\hat{\mathbf{j}} + \left(A_x B_y - A_y B_x\right)\hat{\mathbf{k}}$$

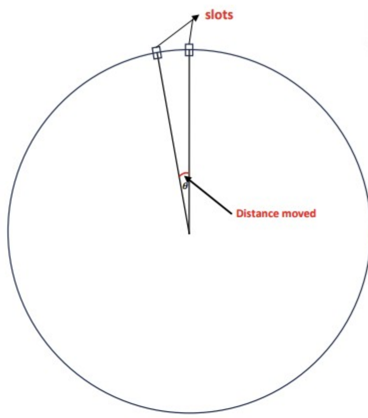
$$\boldsymbol{\omega_f} = \mathbf{2.54 \times 10^4} \left[\frac{\boldsymbol{rev}}{\boldsymbol{min}}\right] \times \left[\frac{\mathbf{2}\boldsymbol{\pi}\ \boldsymbol{rad}}{\mathbf{1}\ \boldsymbol{rev}}\right] \times \left[\frac{\mathbf{1}\ \boldsymbol{min}}{\mathbf{60}\ \boldsymbol{sec}}\right]$$

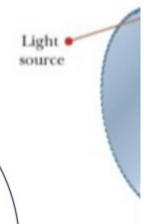
a) $\omega = \frac{\theta}{t}$

- **To get θ(distance moved from one slot to another)**

$$\theta = \frac{2\pi}{N}$$

$$\theta = \frac{2\pi}{500} = 1.26 \times 10^{-2}\ [rad]$$





Page **3** of **5**

- **To get the time :**

$$time = \frac{distance}{speed\ of\ light}$$

$$time = \frac{2\,L}{speed\ of\ light} = \frac{2 \times (500)}{3 \times 10^{8}} = 3.33 \times 10^{-6}\ [sec]$$

$$\omega = \frac{1.26 \times 10^{-2}}{3.33 \times 10^{-6}} = 3.8 \times 10^{3}\ [{}^{rad}/_{sec}]$$

Conservation Law Summary

•For an isolated system -

(1) Conservation of Energy:

- $E_i = E_f$
- If there is no energy transfers across the system boundary

(2) Conservation of Linear Momentum:

- $\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$
- If the net external force on the system is zero

(3) Conservation of Angular Momentum:

- $\vec{\mathbf{L}}_i = \vec{\mathbf{L}}_f$
- If the net external torque on the system is zero

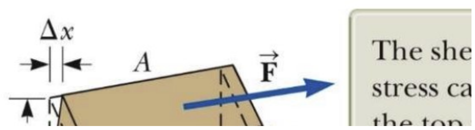
Three Types of Moduli

- Young's Modulus
  - Measures the resistance of a solid to a change in its length.
- Shear Modulus
  - Measures the resistance of motion of the planes within a solid parallel to each other.
- Bulk Modulus
  - Measures the resistance of solids to changes in their volume.

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

- The shear modulus is the ratio of the shear stress to the shear strain.

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$



Bulk Modulus, cont.

- The bulk modulus is the ratio of the volume stress to the volume strain.

$$B = \frac{\textit{volume stress}}{\textit{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$