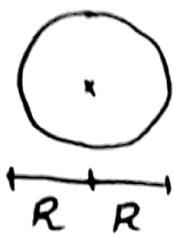


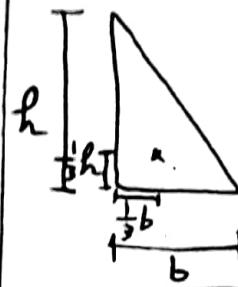
(1)



$$A = \pi R^2$$

$$I_x = \frac{\pi R^4}{4} + A \cdot y^2$$

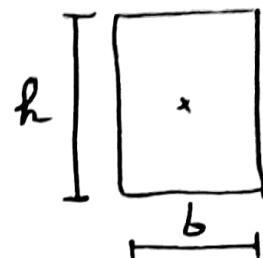
$$I_y = \frac{\pi R^4}{4} + A \cdot x^2$$



$$A = \frac{1}{2} b h$$

$$I_x = \frac{b h^3}{36} + A \cdot y^2$$

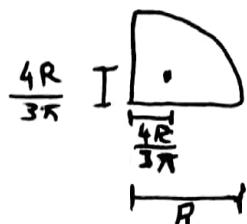
$$I_y = \frac{h b^3}{36} + A \cdot x^2$$



$$A = b h$$

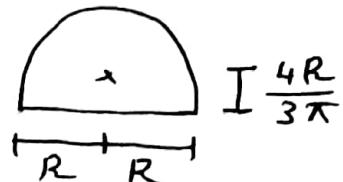
$$I_x = \frac{b h^3}{12} + A \cdot y^2$$

$$I_y = \frac{h b^3}{12} + A \cdot x^2$$



$$A = \frac{1}{4} \pi R^2$$

$$I_x = I_y = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) R^4 + A \cdot y^2$$



$$A = \frac{1}{2} \pi R^2$$

مورياري
القطر

$$I_x = \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) R^4 + A \cdot y^2$$

مورياري
نصف القطر

$$I_y = \frac{\pi R^4}{8} + A \cdot x^2$$

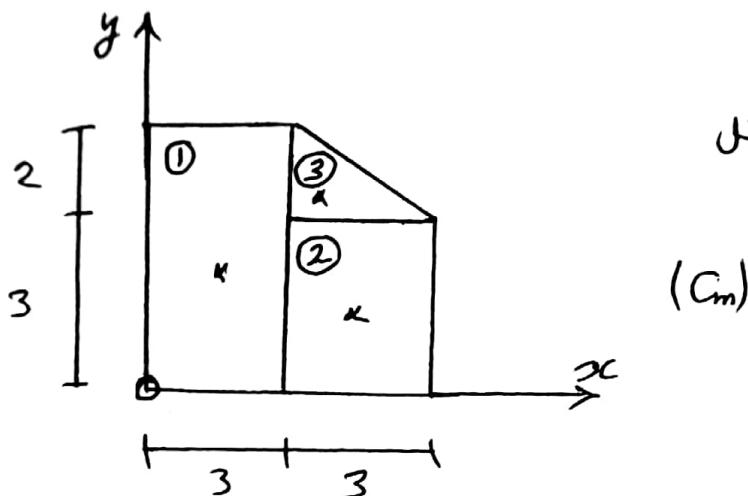
$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum (A \cdot x)}{\sum A}$$

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum (A \cdot y)}{\sum A}$$

اهليات
مركز التعلم

(1)

(2)



أوجد مركز ثقل الشكل

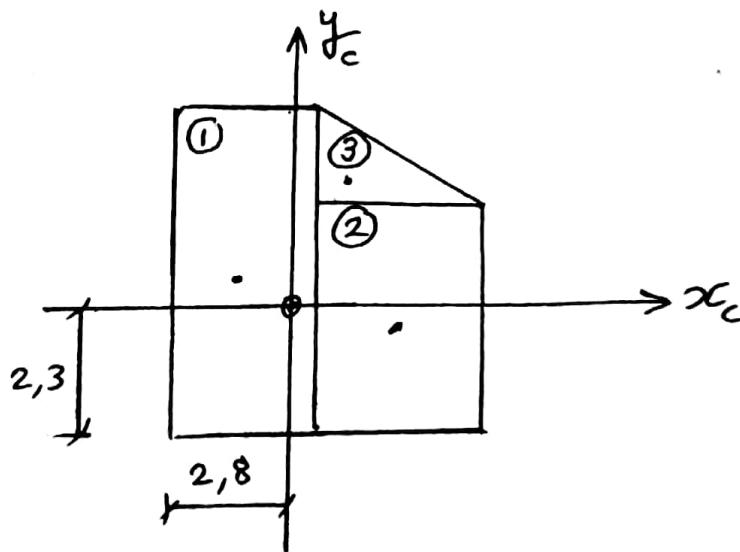
(Cm)

الحل:

رقم المقطع	A المساحة	x	y	$\Sigma y = A \cdot x$	$\Sigma x = A \cdot y$
①	$3 \cdot 5 = 15$	1,5	2,5	22,5	37,5
②	$3 \cdot 3 = 9$	4,5	1,5	40,5	13,5
③	$\frac{1}{2} \cdot 3 \cdot 2 = 3$	4	3,67	12	11
Σ	27			75	62

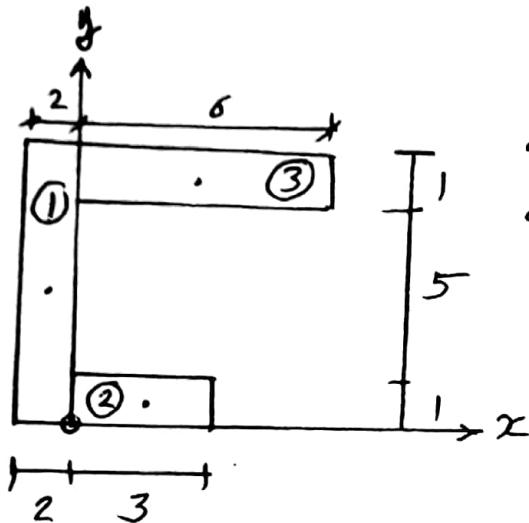
$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum (A \cdot x)}{\sum A} = \frac{75}{27} = 2,8 \text{ cm}$$

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum (A \cdot y)}{\sum A} = \frac{62}{27} = 2,3 \text{ cm}$$



(2)

(3)



مسالة : ① - صد مركز تقل السكك

اذهب \bar{I}_x حول المحاور المفترضة
اذهب \bar{I}_y حول المحاور المركزية

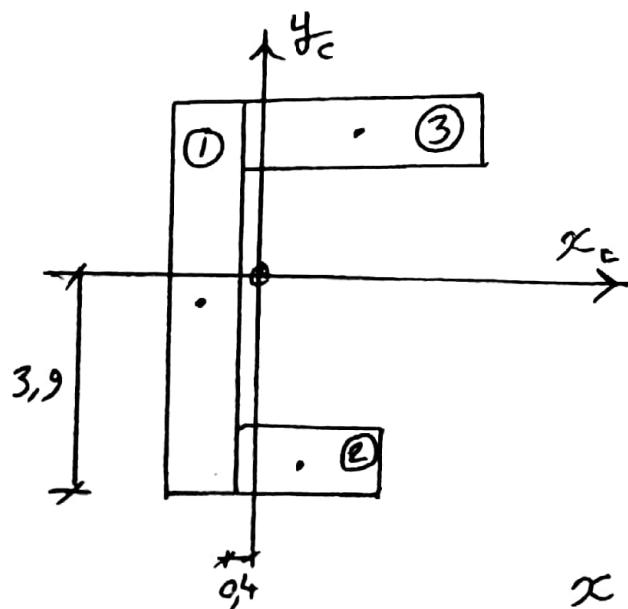
- ① : حل

القسم	A المساحة	x	y	$S_y = A \cdot x$	$S_x = A \cdot y$
①	$7 \cdot 2 = 14$	-1	3,5	-14	49
②	$3 \cdot 1 = 3$	1,5	0,5	4,5	1,5
③	$6 \cdot 1 = 6$	3	0,5	18	39
Σ	[23]			[89,5]	[89,5]

$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum (A \cdot x)}{\sum A} = \frac{89,5}{23} = [0,4]$$

C_m

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum (A \cdot y)}{\sum A} = \frac{89,5}{23} = [3,9]$$



القسم	A	x_c	y_c
①	14	-1,4	-0,4
②	3	1,1	-3,4
③	6	2,6	2,6

$$x = x - x_c$$

$$y = y - y_c$$

التي جربناها قدحه جديده

(3)

(4)

$$I_{x_1} = \frac{2(7)^3}{12} + 14(3,5)^2 = [228,67]$$

- (2)

$$I_{x_2} = \frac{3(1)^3}{12} + 3(0,5)^2 = [1] \quad \text{Cm}^4$$

$$I_{x_3} = \frac{6(11)^3}{12} + 6(6,5)^2 = [254]$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = [483,67 \text{ Cm}^4]$$

$$I_{y_1} = \frac{7(2)^3}{12} + 14(-1)^2 = [18,67]$$

$$I_{y_2} = \frac{1(3)^3}{12} + 3(1,5)^2 = [9] \quad \text{Cm}^4$$

$$I_{y_3} = \frac{1(16)^3}{12} + 6(3)^2 = [72]$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} = [99,67 \text{ Cm}^4]$$

$$I_{x_1} = \frac{2(7)^3}{12} + 14(-0,4)^2 = [59,4]$$

- (3)

$$I_{x_2} = \frac{3(1)^3}{12} + 3(-3,4)^2 = [34,85] \quad \text{Cm}^4$$

$$I_{x_3} = \frac{6(1)^3}{12} + 6(2,6)^2 = [41,1]$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = [136,25 \text{ Cm}^4]$$

$$I_{y_1} = \frac{7(2)^3}{12} + 14(-1,4)^2 = [32,1]$$

$$I_{y_2} = \frac{1(3)^3}{12} + 3(1,1)^2 = [5,88] \quad \text{Cm}^4$$

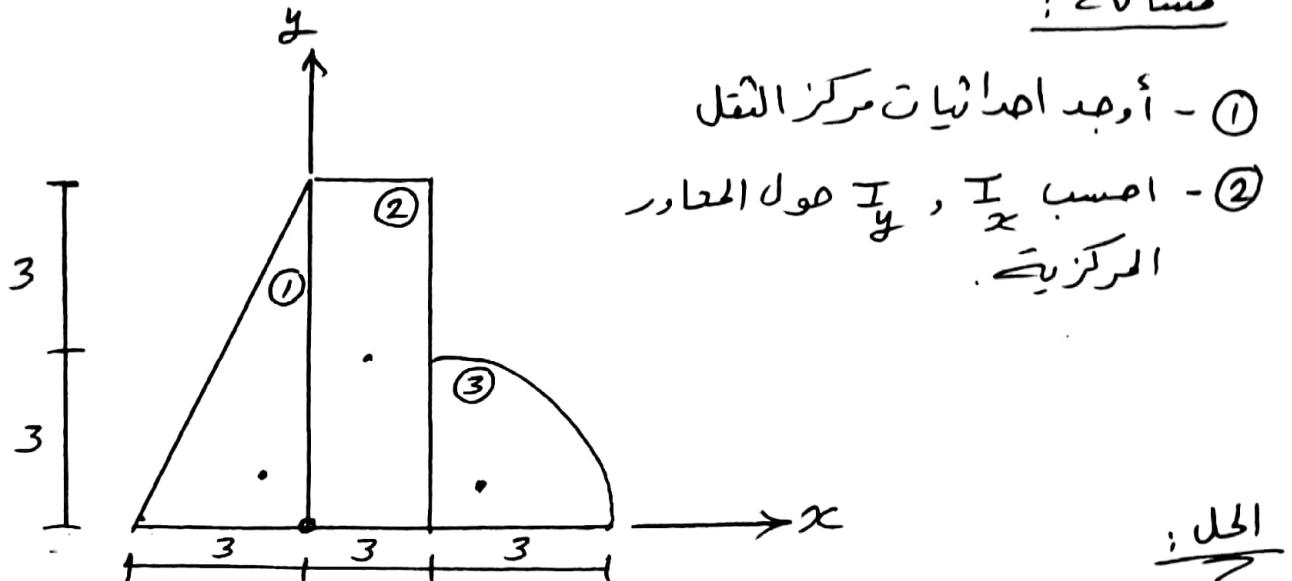
$$I_{y_3} = \frac{1(16)^3}{12} + 6(2,6)^2 = [58,6]$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} = [96,54 \text{ Cm}^4]$$



(4)

(5)

مُسأَلَة:

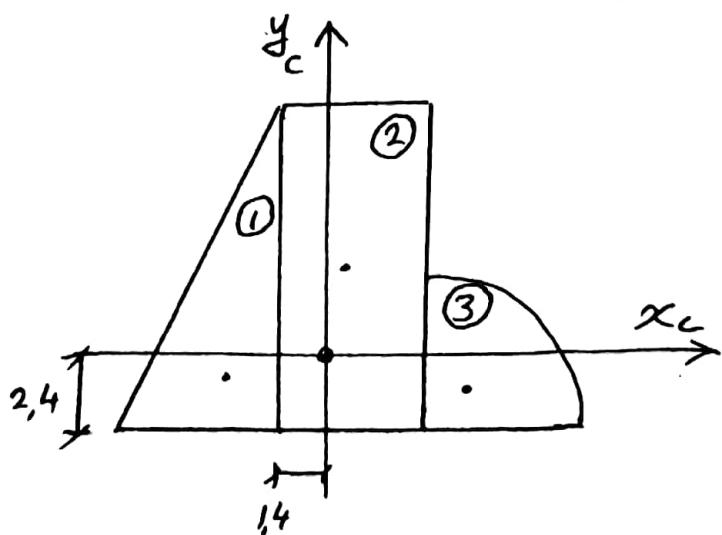
① - أوجد احداثيات مركز الثقل

- احسب I_x , I_y حول المحاور
المركزية.

رقم المقطمة	A المقطمة	x	y	$S_y = A \cdot x$	$S_x = A \cdot y$
①	$\frac{1}{2} \cdot 3 \cdot 6 = 9$	-1	2	-9	18
②	$3 \cdot 6 = 18$	1,5	3	27	54
③	$\frac{1}{4} \pi (3)^2 = 7,07$	$3 + \frac{4(3)}{3\pi}$	$\frac{4(3)}{3\pi}$	30,21	9
Σ	$34,07$			$48,21$	81

$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum (A \cdot x)}{\sum A} = \frac{48,21}{34,07} = 1,4 \text{ cm}$$

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum (A \cdot y)}{\sum A} = \frac{81}{34,07} = 2,4 \text{ cm}$$



رقم المقطمة	A	x	y
①	9	-2,4	-0,4
②	18	0,1	0,6
③	7,07	2,9	-1,1

(5)

(6)

$$I_{x_1} = \frac{3(6)^3}{36} + 9(-94)^2 = 19,44$$

$$I_{x_2} = \frac{3(6)^3}{12} + 18(0,6)^2 = 60,5$$

(2)

 Cm^4

$$I_{x_3} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (3)^4 + 7,07(-1,1)^2 = 13 \text{ } Cm^4$$

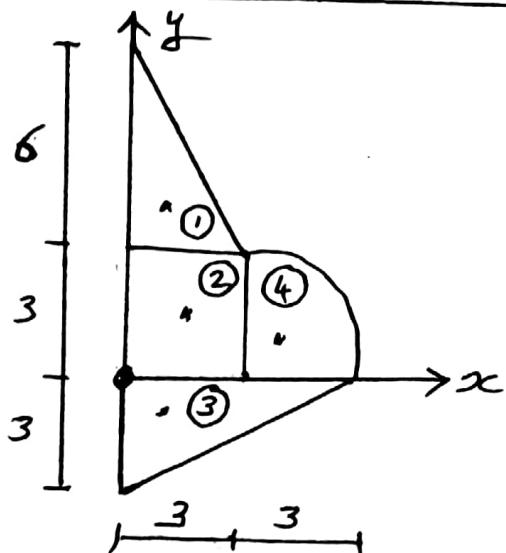
$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = 92,9 \text{ } Cm^4$$

$$I_{y_1} = \frac{6(3)^3}{36} + 9(-2,4)^2 = 56,3$$

$$I_{y_2} = \frac{6(3)^3}{12} + 18(0,1)^2 = 13,68 \text{ } Cm^4$$

$$I_{y_3} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (3)^4 + 7,07(2,9)^2 = 63,9$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} = 133,8 \text{ } Cm^4$$



مسألة:

- أوجد احداثيات مركز الثقل ①

- احسب I_x , I_y حول المحاور المفترضة.- احسب I_x , I_y حول المحاور المفترضة.

اصل: ①

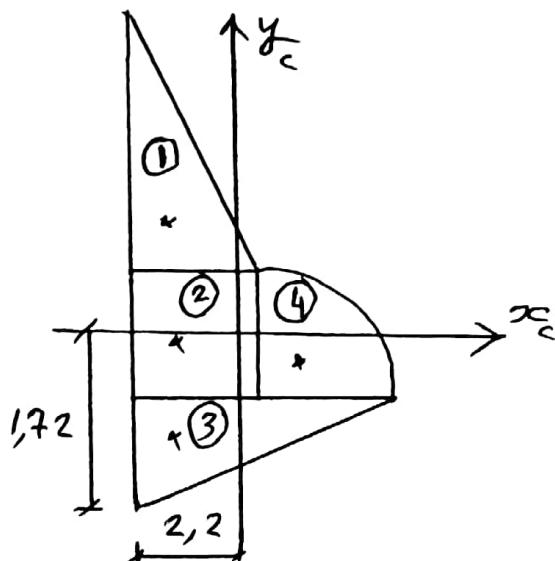
نوع القطعة	$A =$ المساحة	x	y	$S_x = A \cdot x$	$S_y = A \cdot y$
①	$\frac{1}{2}3 \cdot 6 = 9$	1	5	9	45
②	$3 \cdot 3 = 9$	1,5	1,5	13,5	13,5
③	$\frac{1}{2} \cdot 3 \cdot 6 = 9$	2	-1	18	-9
④	$\frac{1}{4} \pi (3)^2 = 7,07$	$3 + \frac{4(3)}{3\pi}$	$\frac{4(3)}{3\pi}$	30,21	9
Σ	(34,07)			(75,21)	(58,5)

(6)

(7)

$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum A \cdot x}{\sum A} = \frac{75,21}{34,07} = [2,2 \text{ cm}]$$

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum A \cdot y}{\sum A} = \frac{58,5}{34,07} = [1,72 \text{ cm}]$$



teil	A	x _c	y _c
①	9	-1,2	3,28
②	9	-0,7	-0,22
③	9	-0,2	-2,72
④	7,07	2,1	-0,44

$$I_{x_1} = \frac{3(6)^3}{36} + 9(5)^2 = [243]$$

$$I_{x_2} = \frac{3(3)^3}{12} + 9(1,5)^2 = [27]$$

$$I_{x_3} = \frac{6(3)^3}{36} + 9(-1)^2 = [13,5]$$

$$I_{x_4} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (3)^4 + 7,07 (1,27)^2 = [15,85]$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} + I_{x_4} = [299,35 \text{ cm}^4]$$

$$I_{y_1} = \frac{6(3)^3}{36} + 9(1)^2 = [13,5]$$

$$I_{y_2} = \frac{3(3)^3}{12} + 9(1,5)^2 = [27]$$

$$I_{y_3} = \frac{3(6)^3}{36} + 9 \cdot (2)^2 = [54]$$

$$I_{y_4} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (3)^4 + 7,07 (4,27)^2 = [573]$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} + I_{y_4} = [227,8 \text{ cm}^4]$$

(2)

 cm^4 cm^4

(7)

(8)

$$I_{x_1} = \frac{3(6)^3}{36} + 9(3,28)^2 = [114,8]$$

$$I_{x_2} = \frac{3(13)^3}{12} + 9(-0,22)^2 = [7,2] \quad Cm^4$$

$$I_{x_3} = \frac{6(3)^3}{36} + 9(-2,72)^2 = [71,1]$$

$$I_{x_4} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{\pi \cdot 18} \right) (3)^4 + 7,07 (-0,44)^2 = [5,8]$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} + I_{x_4} = [198,9 \text{ } Cm^4]$$

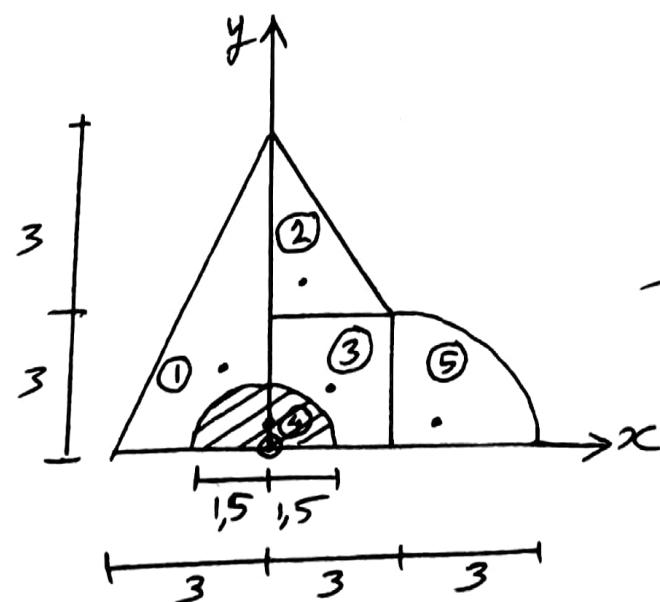
$$I_{y_1} = \frac{6(3)^3}{36} + 9(-1,2)^2 = [17,5]$$

$$I_{y_2} = \frac{3(3)^3}{12} + 9(-0,7)^2 = [11,2] \quad Cm^4$$

$$I_{y_3} = \frac{3(6)^3}{36} + 9(-0,2)^2 = [4,9]$$

$$I_{y_4} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{\pi \cdot 18} \right) (3)^4 + 7,07 (2,1)^2 = [35,6]$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} + I_{y_4} = [82,7 \text{ } Cm^4]$$



- مساحة :
- ① - حدد مركز العقل
 - ② - احسب I_x , I_y حول المحاور المفروضة
 - ③ - استخرج I_x , I_y حول العاور المركبة

(8)

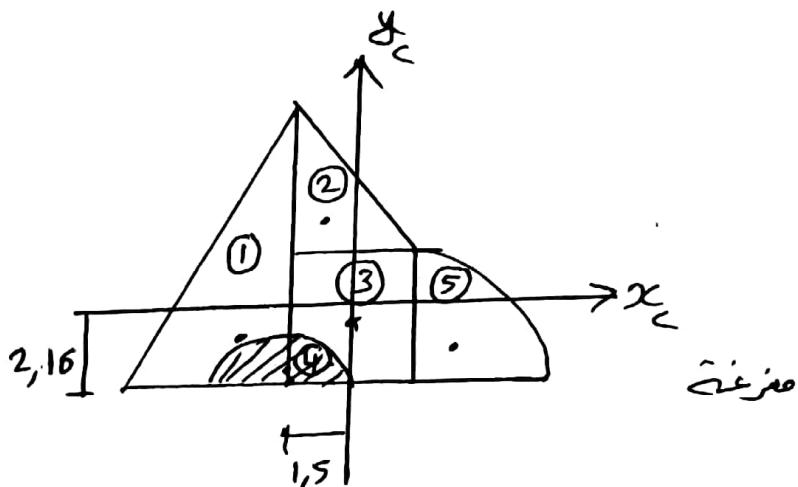
٩

نوع المقطع	A المساحة	x	y	$\Sigma y = A \cdot x$	$\Sigma x = A \cdot y$
١	$\frac{1}{2} 3 \cdot 6 = 9$	-1	2	-9	18
٢	$\frac{1}{2} 3 \cdot 3 = 4,5$	1	4	4,5	18
٣	$3 \cdot 3 = 9$	1,5	1,5	13,5	13,5
٤ مفرغة	$-\frac{1}{2} \pi (1,5)^2 = -3,5$	0	0,64	0	-2,2
٥	$\frac{1}{4} \pi (3)^2 = 7,07$	4,27	1,27	30,2	9
Σ	(26,1)	3		(39,2)	(56,3)

$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum A \cdot x}{\sum A} = \frac{39,2}{26,1} = 1,5$$

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum A \cdot y}{\sum A} = \frac{56,3}{26,1} = 2,16$$

Cm



نوع المقطع	A	x_G	y_G
١	9	-2,5	-0,16
٢	4,5	-0,5	1,84
٣	9	0	-0,66
٤	3,5	-1,5	-1,52
٥	7,07	2,77	-0,89

$$I_{x_1} = \frac{3(6)^3}{36} + 9(2)^2 = 54$$

- (2)

$$I_{x_2} = \frac{3(3)^3}{36} + 4,5(4)^2 = 74,25$$

$$I_{x_3} = \frac{3(3)^3}{12} + 9(1,5)^2 = 27$$

Cm⁴

$$I_{x_4} = \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (1,5)^4 + 3,5(0,64)^2 = 1,99$$

$$I_{x_5} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{18\pi} \right) (3)^4 + 7,07(1,27)^2 = 15,8$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} - I_{x_4} + I_{x_5} = 169,1 \text{ Cm}^4$$

٩

(10)

$$I_{y_1} = \frac{6(3)^3}{36} + 9(-1)^2 = 13,5$$

$$I_{y_2} = \frac{3(3)^3}{36} + 4,5(1)^2 = 6,75$$

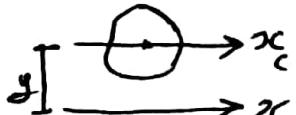
$$I_{y_3} = \frac{3(3)^3}{12} + 9(1,5)^2 = 27 \quad Cm^4$$

$$\text{مذكرة } I_{y_4} = \frac{\pi(1,5)^4}{8} + 3,5(0)^2 = 1,99$$

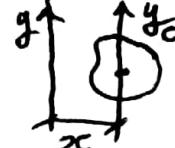
$$I_{y_5} = \frac{1}{2} \left(\frac{\pi}{8} - \frac{16}{\pi} \right) (3)^4 + 7,07 (4,2)^2 = 133,3$$

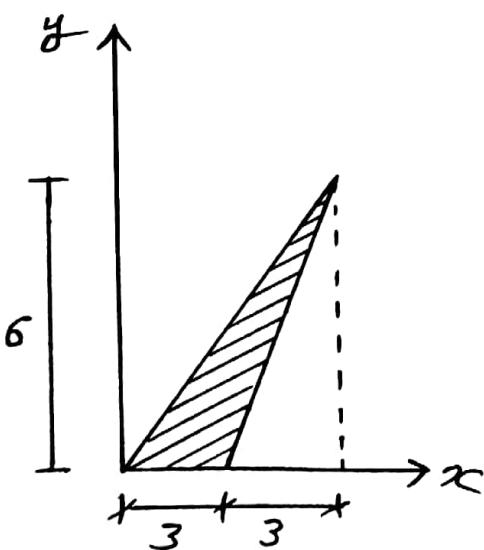
$$I_y = I_{y_1} + I_{y_2} + I_{y_3} - I_{y_4} + I_{y_5} = 178,56 \quad Cm^4$$

$$\frac{I}{x} = \frac{I}{x_c} + A \cdot y^2 \quad : \text{- نطبق هويتير} \quad (3)$$

$$169,1 = I_{x_c} + 26,1 \cdot (2,16)^2 \Rightarrow \frac{I}{x_c} = 47,3 \quad Cm^4$$


$$I_y = I_{y_c} + A \cdot x^2$$

$$178,56 = I_{y_c} + 26,1 \cdot (1,5)^2 \Rightarrow I_{y_c} = 119,8 \quad Cm^4$$




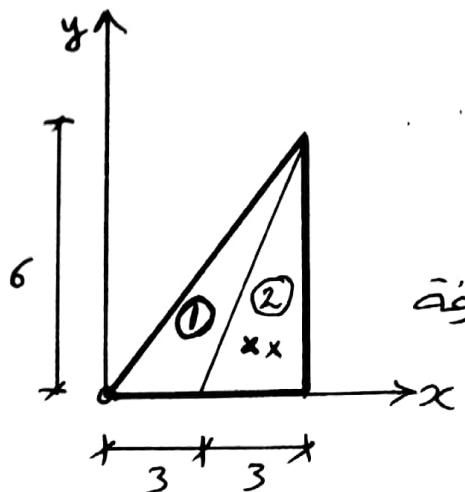
مسألة: ① - أوجد احداثيات مركز الثقل.

② - احسب عزوم المطالع I_y , I_x حول المحاور الفرعية.

③ - استنتج عزوم المطالع I_y , I_x حول المحاور المركزية.

(10)

(11)



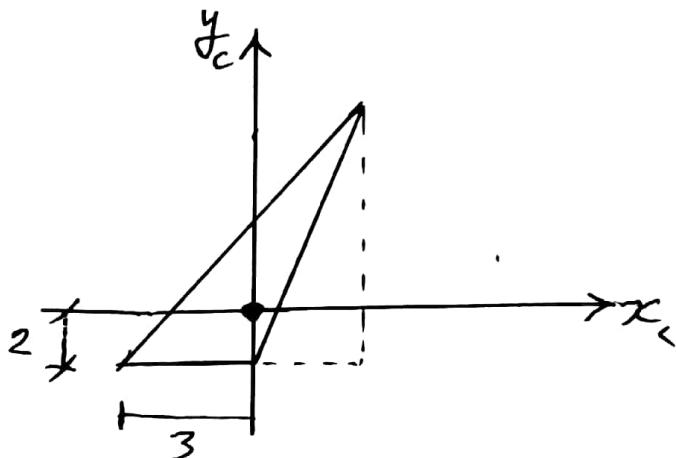
الرقم	A	x	y	$\Sigma y = A \cdot x$	$\Sigma x = A \cdot y$
(1)	$\frac{1}{2} \cdot 6 \cdot 6 = 18$	4	2	72	36
(2)	$\frac{1}{2} \cdot 3 \cdot 6 = 9$	5	2	-45	-18
Σ	9			27	18

$$x_c = \frac{\sum S_y}{\sum A} = \frac{\sum A \cdot x}{\sum A} = \frac{27}{9} = 3$$

cm

$$y_c = \frac{\sum S_x}{\sum A} = \frac{\sum A \cdot y}{\sum A} = \frac{18}{9} = 2$$

(2)



$$I_{x_1} = \frac{6 \cdot (6)^3}{36} + 18 \cdot (2)^2 = 108$$

cm⁴

$$\text{مثلاً } I_{x_2} = \frac{3 \cdot (6)^3}{36} + 9 \cdot (2)^2 = 54$$

$$I_x = I_{x_1} - I_{x_2} = 54 \text{ cm}^4$$

$$I_{y_1} = \frac{6 \cdot (6)^3}{36} + 18 \cdot (4)^2 = 324$$

cm⁴

$$I_{y_2} = \frac{6 \cdot (3)^3}{36} + 9 \cdot (5)^2 = 229,5$$

$$I_y = I_{y_1} - I_{y_2} = 94,5 \text{ cm}^4$$

(11)

(12)

$$I_x = I_{x_c} + A \cdot y^2$$

$$54 = I_{x_c} + 9 \cdot (2)^2 \Rightarrow$$

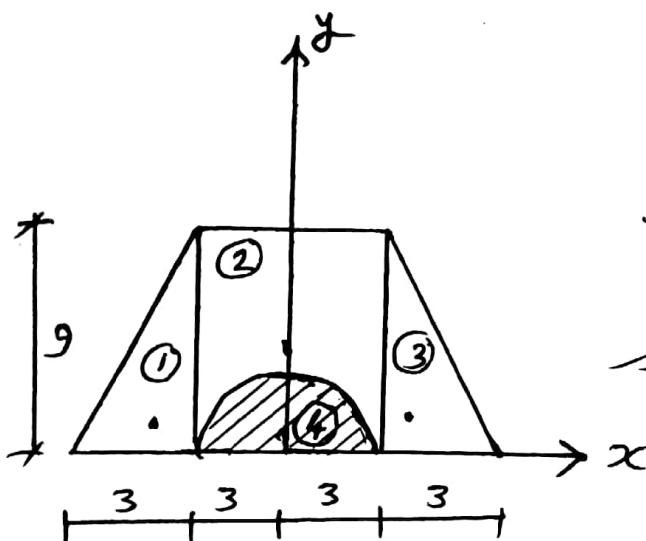
نقطة هويفتر - ③

$$I_{x_c} = 18 \text{ cm}^4$$

$$I_y = I_{y_c} + A \cdot x^2$$

$$94,5 = I_{y_c} + 9 \cdot (3)^2 \Rightarrow$$

$$I_{y_c} = 13,5 \text{ cm}^4$$



مسألة ١ - احسب احداثيات مركز العقل.

مسألة ٢ - احسب I_y , I_x حول المحور المفترض.مسألة ٣ - استخرج I_x , I_y حول المحاور المرئية.

$$x_c = 0$$

الدالة:

$$y_c =$$

$$I_x =$$

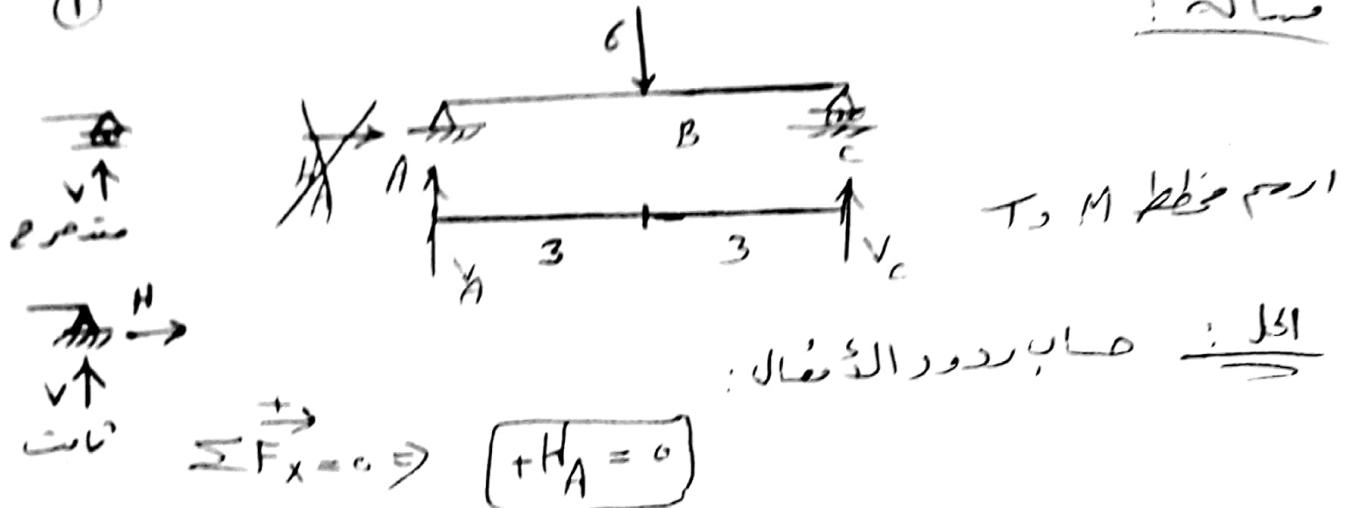
$$I_y =$$

$$I_{x_c} =$$

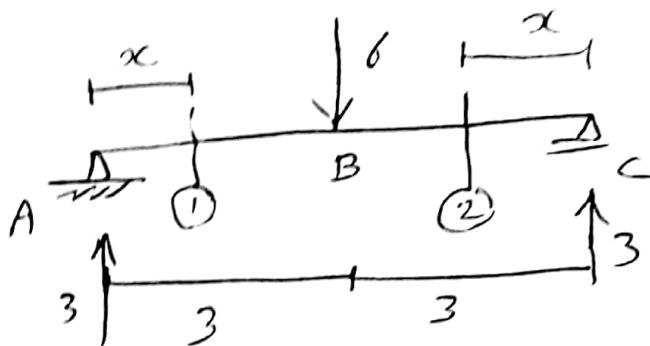
$$I_{y_c} =$$

(12)

①

مُسَأَّلَةٌ

$$V_A = V_C = \frac{1}{2} \cdot 6 = [3] \quad \text{بسبب المعاشر}$$



$$M_1 = +3x$$

$$T_1 = +3$$

$$x=0 \quad \boxed{A}$$

$$x=3 \quad \boxed{B}$$

$$M=0$$

$$M=9$$

$$T=3$$

$$T=3$$

T هو متغير M وهو الميلار لا تغير المترارة
من العبره تغير المترارة

$$M_2 = +3 \cdot x$$

$$T_2 = -3$$

$$x=0 \quad \boxed{C}$$

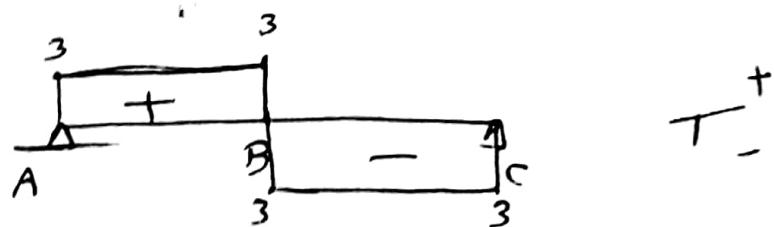
$$x=3 \quad \boxed{B}$$

$$M=0$$

$$M=9$$

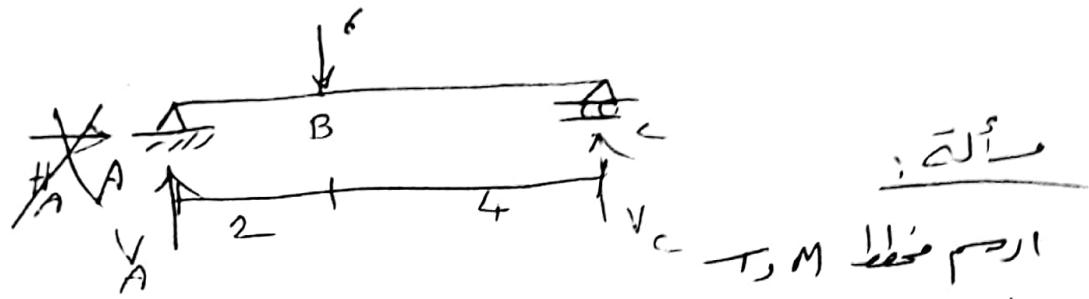
$$T=-3$$

$$T=-3$$

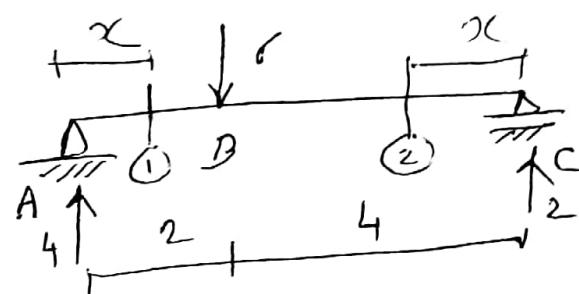


(13)

②



مبدأ الدوران

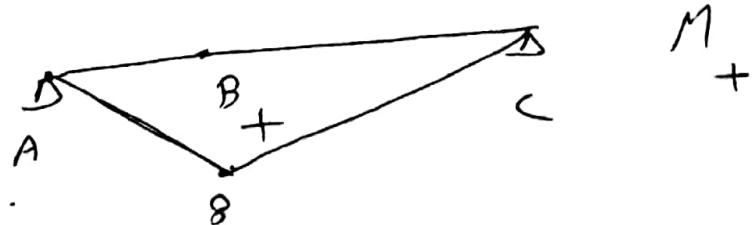


$$M_1 = +4 \cdot x \quad x=0 \quad M=0 \quad T=+4$$

$$T_1 = +4 \quad x=2 \quad M=8 \quad T=+4$$

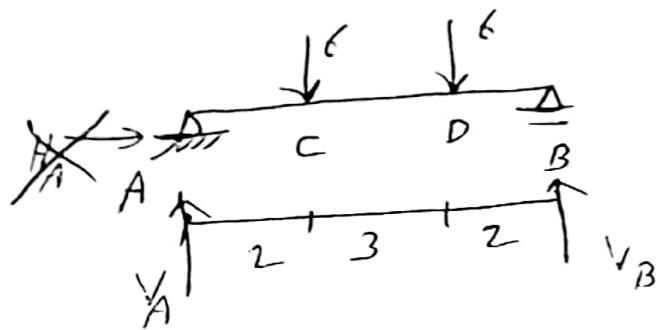
$$M_2 = +2 \cdot x \quad x=0 \quad M=0 \quad T=-2$$

$$T_2 = -2 \quad x=4 \quad M=8 \quad T=-2$$



14

(3)



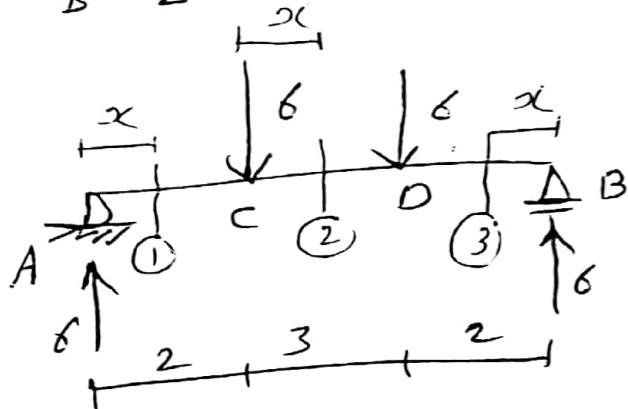
T, M لجهی مرسی

: جلسہ

$$\sum \vec{F}_x = 0 \Rightarrow H_A = 0$$

 $\frac{\text{جلسہ}}{7}$

$$V_A = V_B = \frac{1}{2}(6+6) = 6$$



$$M_1 = 6 \cdot x$$

$$T_1 = 6$$

$$x=0 \quad \boxed{A} \quad M=0 \quad T=6$$

$$x=2 \quad \boxed{C} \quad M=12 \quad T=6$$

$$M_2 = +6(2+x) - 6x$$

$$T_2 = +6 - 6 = 0$$

$$x=0 \quad \boxed{E} \quad M=12 \quad T=0$$

$$x=3 \quad \boxed{D} \quad M=12 \quad T=0$$

$$M_3 = 6 \cdot x$$

$$T_3 = -6$$

$$x=0$$

$$x=2$$

$$\boxed{B}$$

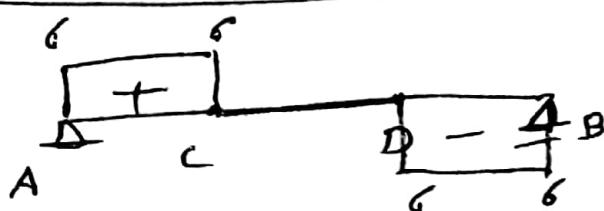
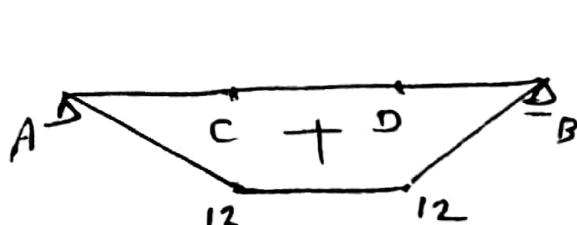
$$\boxed{D}$$

$$M=0$$

$$M=12$$

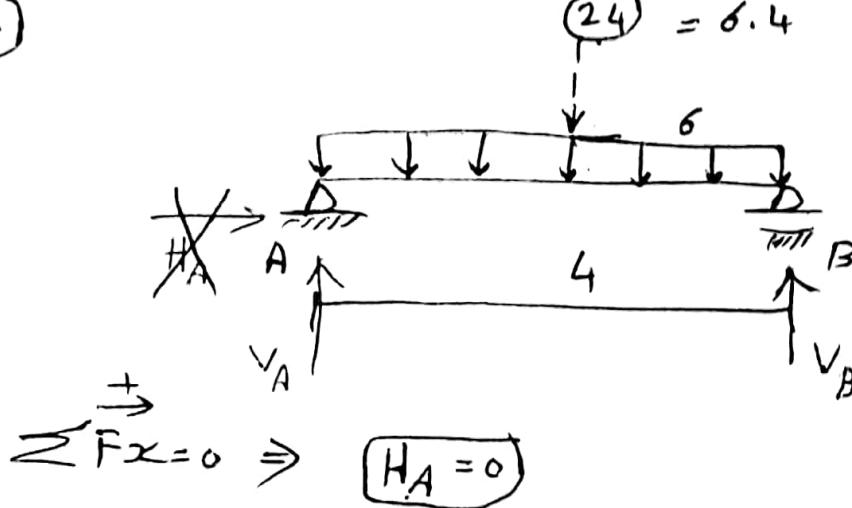
$$T=-6$$

$$T=-6$$

 T_+  M_-

(15)

٤

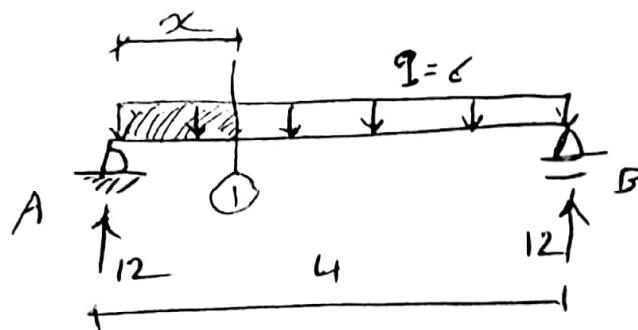


مسألة:

$$24 = 6 \cdot 4$$

$$V_A = V_B = \frac{1}{2} 24 = 12$$

بب الساطر



$$M_1 = 12 \cdot x - \frac{6 \cdot x^2}{2}$$

$$T_1 = 12 - 6x$$

$$x=0 \quad A \quad M=0 \quad T=12$$

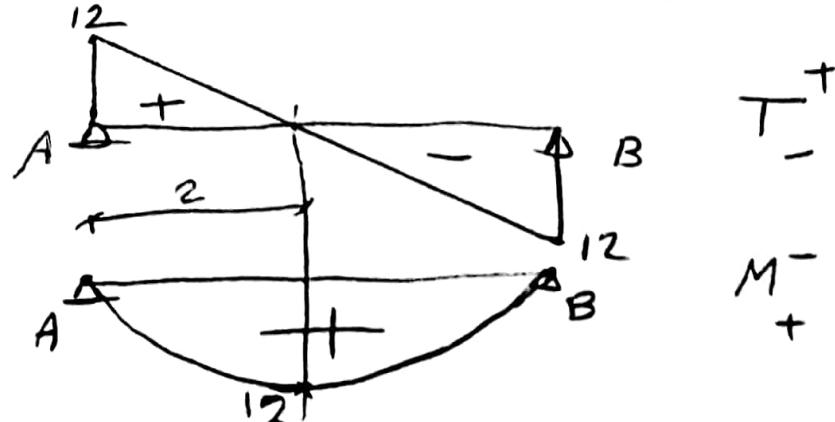
$$x=4 \quad B \quad M=0 \quad T=-12$$

كل حال يجعل صورة معززة يكتبه خلفه M له ميئاً لرسمه يجب
أن دروته يجعل $T=0$

$$T_1=0 \Rightarrow 12 - 6x = 0 \Rightarrow x = 2 \quad \text{موقع الذروة}$$

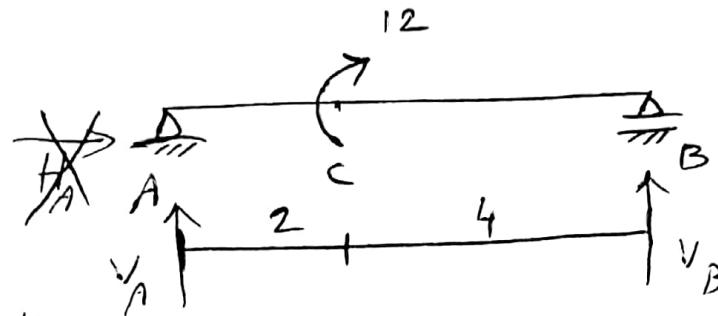
$$M_{\max} = 12 \cdot 2 - \frac{6 \cdot 12^2}{2} = 12 \quad \text{قيمة الذروة}$$

عند الدمام
ووجه ذروة كـ



١٥

(5)



مسأله:

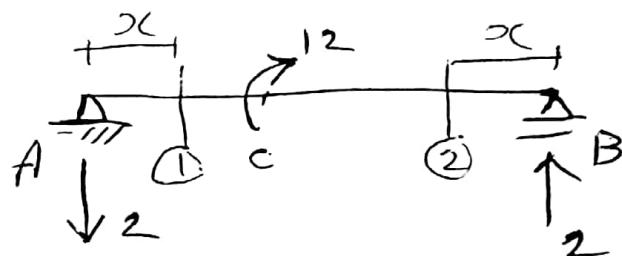
العنصر المترافق
بذراع

$$\sum F_x = 0 \Rightarrow H_A = 0$$

$$\sum M_A = 0 \Rightarrow +12 - V_B \cdot 6 = 0 \Rightarrow V_B = 2$$

$$\sum F_y = 0 \Rightarrow V_A + 2 = 0 \Rightarrow V_A = -2$$

عكس المفروض



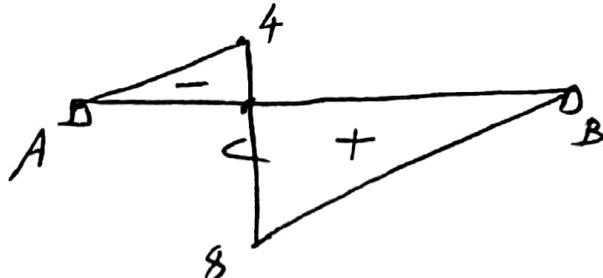
$$M_1 = -2x \quad x=0 \quad \boxed{A} \quad M=0 \quad T=-2$$

$$T_1 = -2 \quad x=2 \quad \boxed{C} \quad M=-4 \quad T=-2$$

$$M_2 = +2x \quad x=0 \quad \boxed{B} \quad M=0 \quad T=-2$$

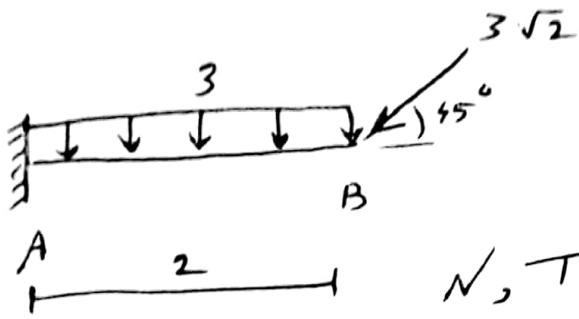
$$T_2 = -2 \quad x=4 \quad \boxed{C} \quad M=8 \quad T=-2$$

عند وجود عنصر
فرعي يوجد مفترزة
في خطأه ثم قيمها
تساوي قيمة العنصر


 T^+
 T^-

 M^-
 M^+

(17)

(6)



مسأله:

ارسم محظوظ N, T, M

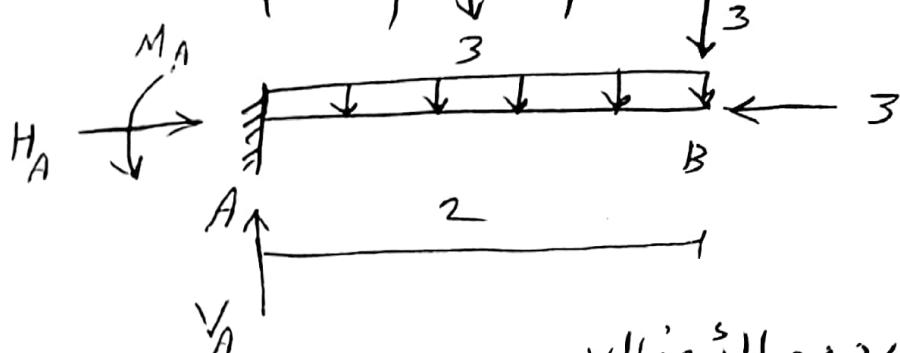
المراد المقصود بـ \cos
هي التي تقرب $\frac{\text{جذر}}{2}$

الحل: خذ القوة المائلة إلى سرقتها

$$3 = 3\sqrt{2} \sin(45)$$

$$2 \cdot 3 = 6$$

$$3\sqrt{2} \cos(45) = 3$$

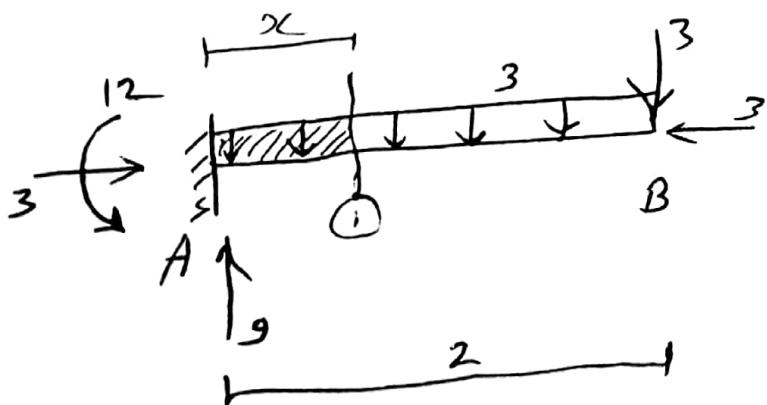


حساب ردود الأفعال:

$$\sum F_x = 0 \Rightarrow +H_A - 3 = 0 \Rightarrow H_A = 3$$

$$\sum M_A = 0 \Rightarrow -M_A + 6 \cdot 1 + 3 \cdot 2 = 0 \Rightarrow M_A = 12$$

$$\uparrow + \sum F_y = 0 \Rightarrow +V_A - 6 - 3 = 0 \Rightarrow V_A = 9$$



(18)

(7)

$$M_1 = +9x - 12 - \frac{3x^2}{2}$$

$$T_1 = +9 - 3x$$

قواري العنصر
 $N_1 = -3$

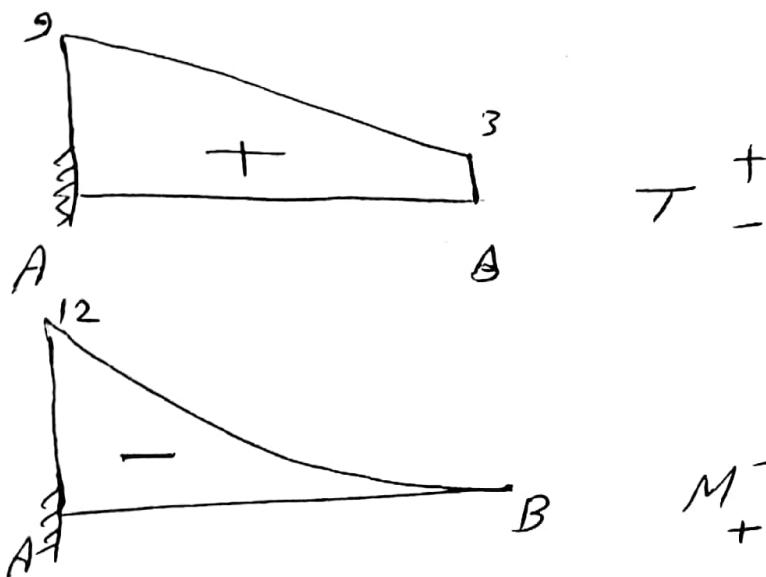
+ سين
- سين

$x=0$ [A] $M=-12$ $T=9$ $N=-3$

$x=2$ [B] $M=0$ $T=3$ $N=-3$

$$T_1 = 0 \Rightarrow 9 - 3x = 0 \Rightarrow x = 3$$

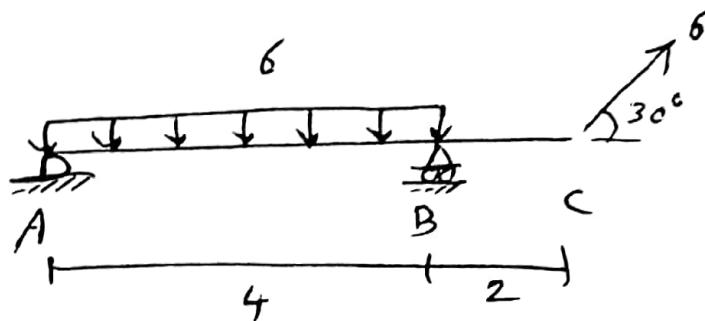
نهاية الم الحال اذا تم توجيه ذروة



(19)

(8)

مسأله:

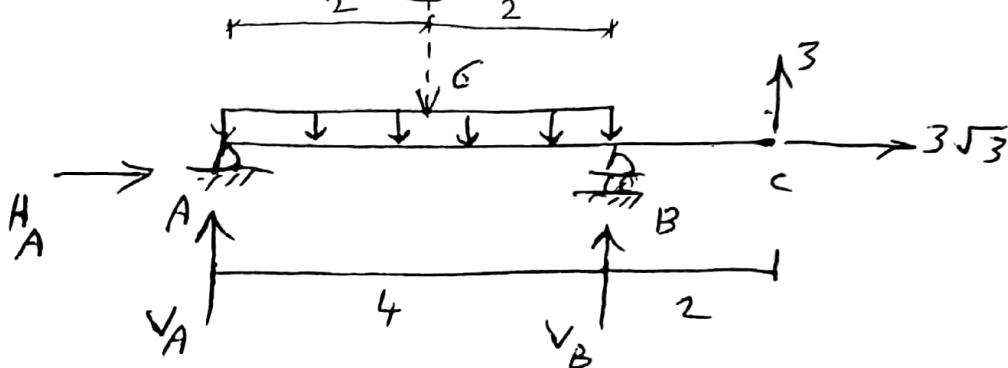


حل:

$$6 \sin(30) = 3$$

كلل القوة المائلة الى مركبة

$$6 \cos(30) = 3\sqrt{3}$$

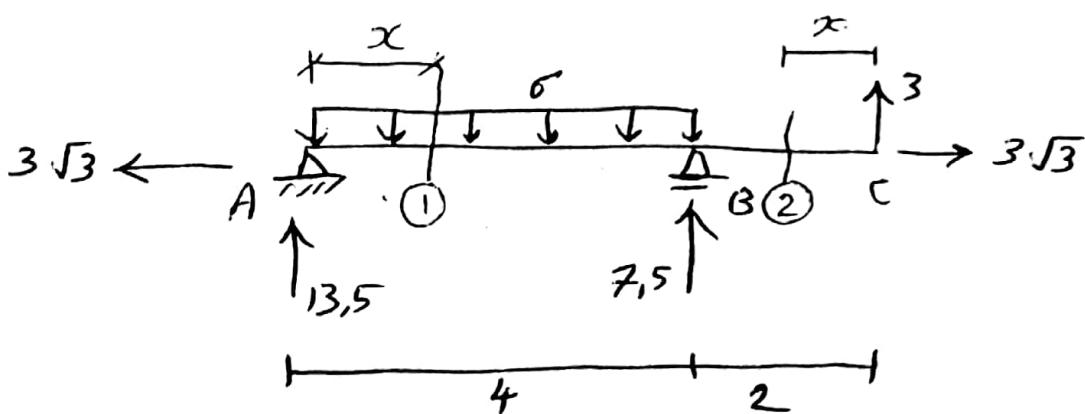


$$\sum F_{xc} = 0 \Rightarrow +H_A + 3\sqrt{3} = 0 \Rightarrow H_A = -3\sqrt{3}$$

الب عکس المترافق

$$\sum M_A = 0 \Rightarrow 24 \cdot 2 - V_B \cdot 4 - 3 \cdot 6 = 0 \Rightarrow V_B = 7,5$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_A + 7,5 + 3 - 24 = 0 \Rightarrow V_A = 13,5$$



(20)

(9)

$$M_1 = 13,5x - 6 \frac{x^2}{2}$$

$$T_1 = 13,5 - 6x$$

$$N_1 = +3\sqrt{3}$$

$$x=0 \quad A \quad M=0 \quad T=13,5 \quad N=3\sqrt{3}$$

$$x=4 \quad B \quad M=6 \quad T=-10,5 \quad N=3\sqrt{3}$$

$$T_1 = 0 \Rightarrow 13,5 - 6x = 0 \Rightarrow x = 2,25 \quad \text{موقع الذرة}$$

$$M_{max} = 13,5 \cdot 2,25 - 6 \frac{(2,25)^2}{2} = 15,18 \quad \text{قيمة الذرة}$$

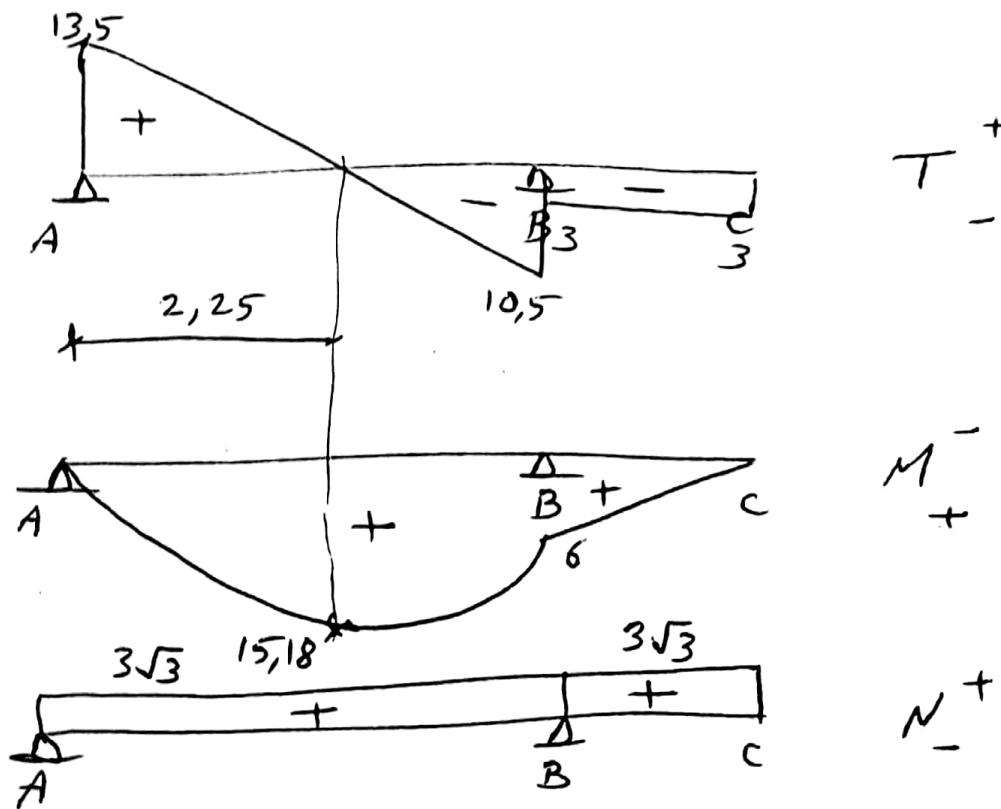
$$M_2 = 3x$$

$$x=0 \quad C \quad M=0 \quad T=-3 \quad N=3\sqrt{3}$$

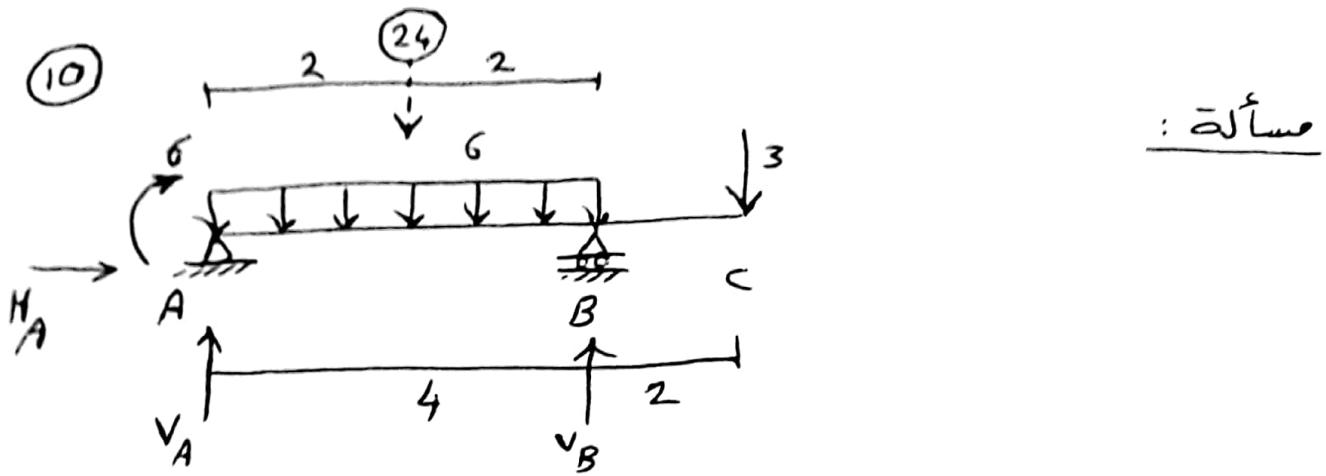
$$T_2 = -3$$

$$x=2 \quad B \quad M=6 \quad T=-3 \quad N=3\sqrt{3}$$

$$N_2 = +3\sqrt{3}$$



(21)

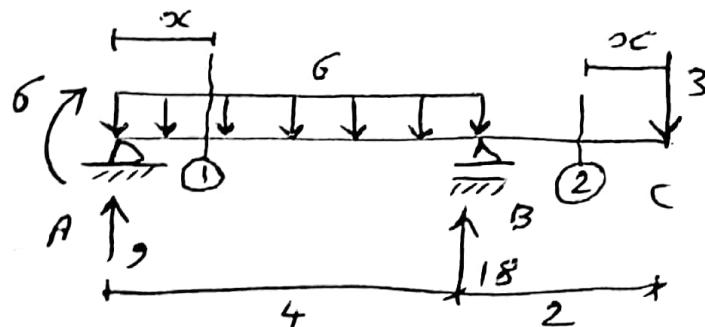


: مسأله

$$\sum \vec{F}_{2C} = 0 \Rightarrow H_A = 0$$

$$\sum \vec{M}_A = 0 \Rightarrow 6 + 24 \cdot 2 + 3 \cdot 6 - V_B \cdot 4 = 0 \Rightarrow V_B = 18$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_A + 18 - 24 - 3 = 0 \Rightarrow V_A = 9$$



$$M_1 = 9 \cdot x + 6 - 6 \frac{x^2}{2} \quad x=0 \quad A \quad M=6 \quad T=9$$

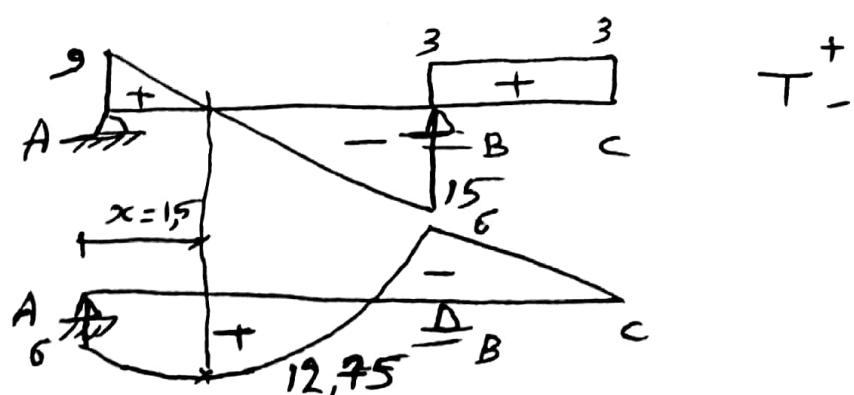
$$T_1 = 9 - 6x \quad x=4 \quad B \quad M=-6 \quad T=-15$$

$$T_1 = 0 \Rightarrow 9 - 6x = 0 \Rightarrow x = 1,5 \quad \text{موقع الدررة}$$

$$M_{max} = 9 \cdot 1,5 + 6 - \frac{6 \cdot (1,5)^2}{2} = 12,75 \quad \text{قيمة الدررة}$$

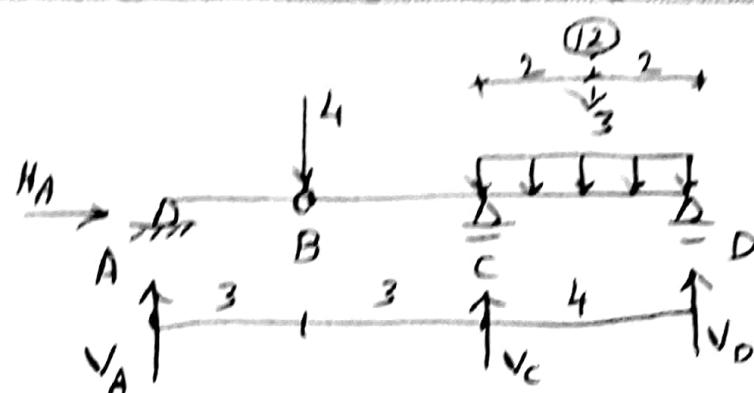
$$M_2 = -3x \quad x=0 \quad C \quad M=0 \quad T=3$$

$$T_2 = +3 \quad x=2 \quad B \quad M=-6 \quad T=3$$



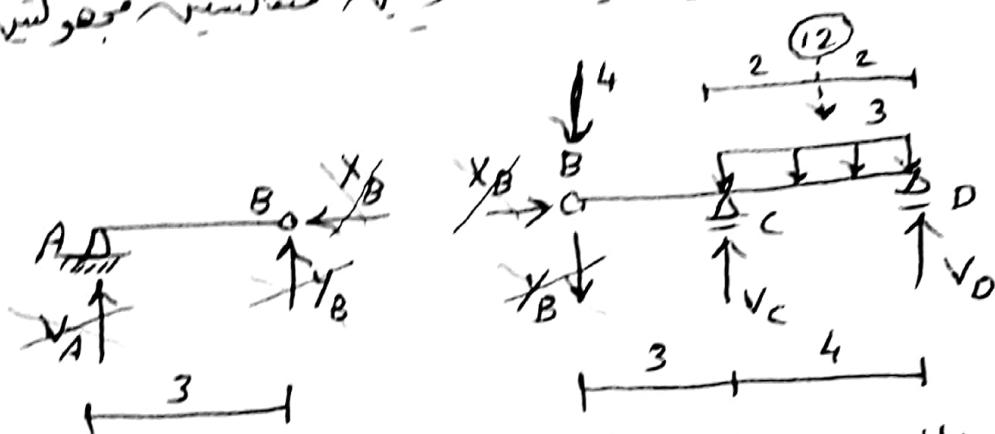
(22)

(11)



$$\sum F_x = 0 \Rightarrow [H_A = 0]$$

نحصل على المغادرة ونضع متغير متساوين معه ونحل



نبدأ بالجزء الذي عدد محاهيل أتم.

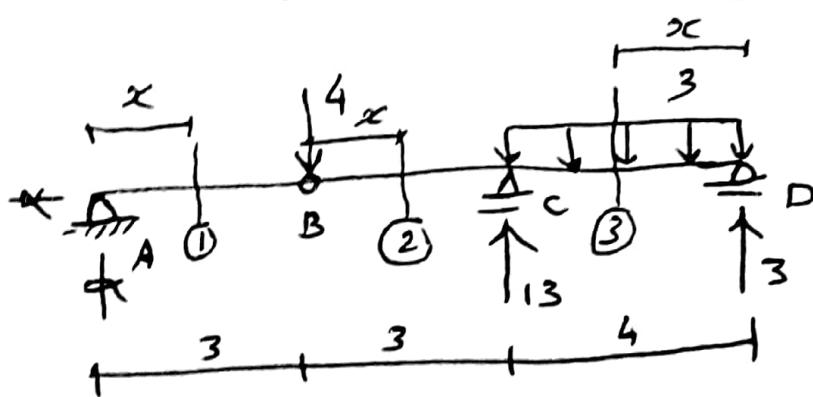
$$\sum F_x = 0 \Rightarrow -X_B = 0 \Rightarrow [X_B = 0] \quad \text{الجزء A-B}$$

$$\sum M_A = 0 \Rightarrow -Y_B \cdot 3 = 0 \Rightarrow [Y_B = 0]$$

$$+\uparrow \sum F_y = 0 \Rightarrow [V_A = 0]$$

$$\sum M_D = 0 \Rightarrow -12 \cdot 2 + V_C \cdot 4 - 4 \cdot 7 = 0 \Rightarrow [V_C = 13] \quad \text{الجزء B-C-D}$$

$$+\uparrow \sum F_y = 0 \Rightarrow -4 + V_D + 13 - 12 = 0 \Rightarrow [V_D = 3]$$



(23)

(12)

$$M_1 = 0$$

$$T_1 = 0$$

$$x = 0 \quad \boxed{A} \quad M = 0$$

$$x = 3 \quad \boxed{B} \quad M = 0$$

$$T = 0$$

$$T = 0$$

$$M_2 = -4x$$

$$x = 0 \quad \boxed{B} \quad M = 0$$

$$x = 3 \quad \boxed{C} \quad M = -12$$

$$T = -4$$

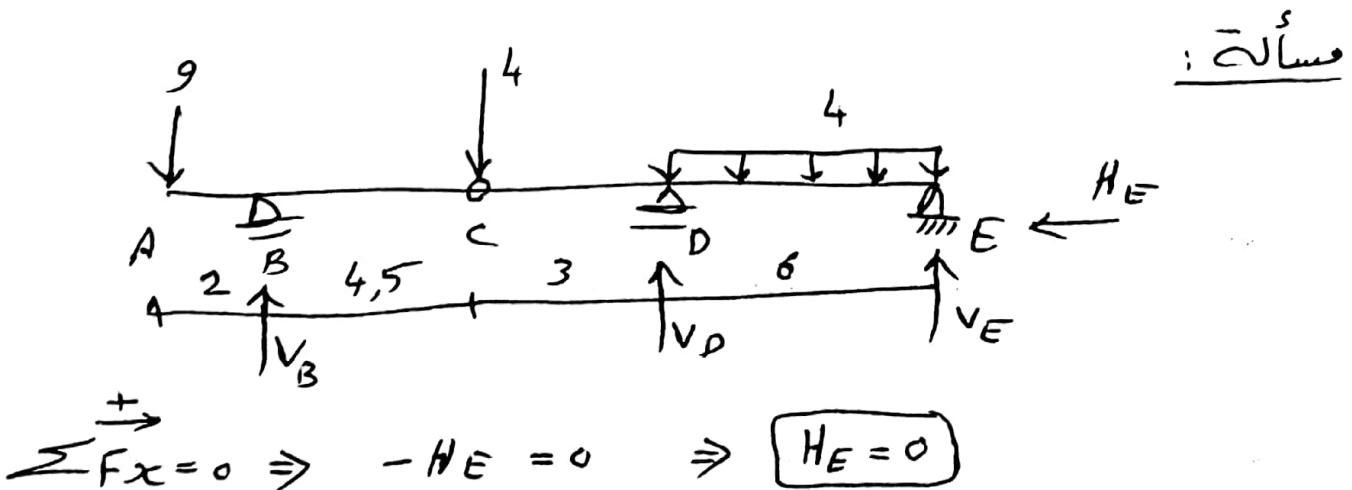
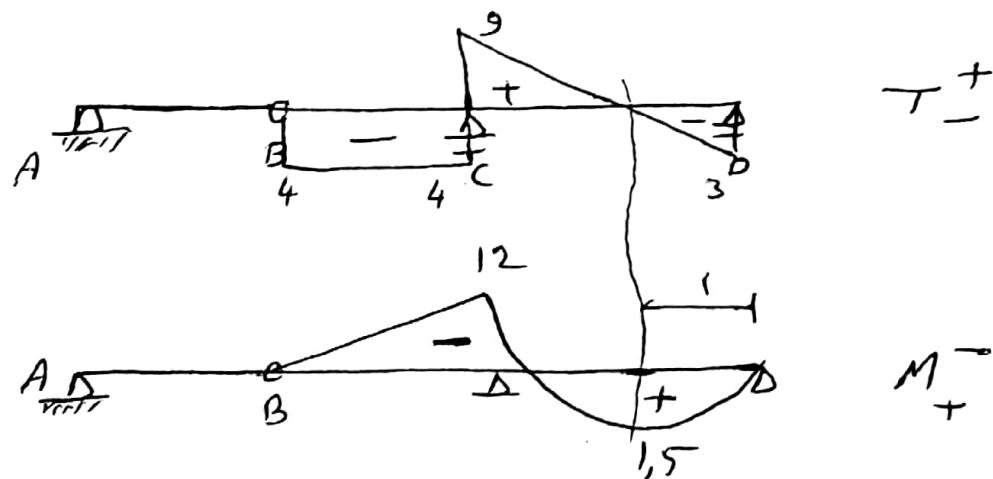
$$T = -6$$

$$M_3 = 3x - 3\frac{x^2}{2} \quad x = 0 \quad \boxed{D} \quad M = 0 \quad T = -3$$

$$T_3 = -3 + 3x \quad x = 4 \quad \boxed{C} \quad M = -12 \quad T = +9$$

$$T_3 = 0 \Rightarrow -3 + 3x = 0 \Rightarrow \boxed{x = 1} \quad \text{موقع النزوة}$$

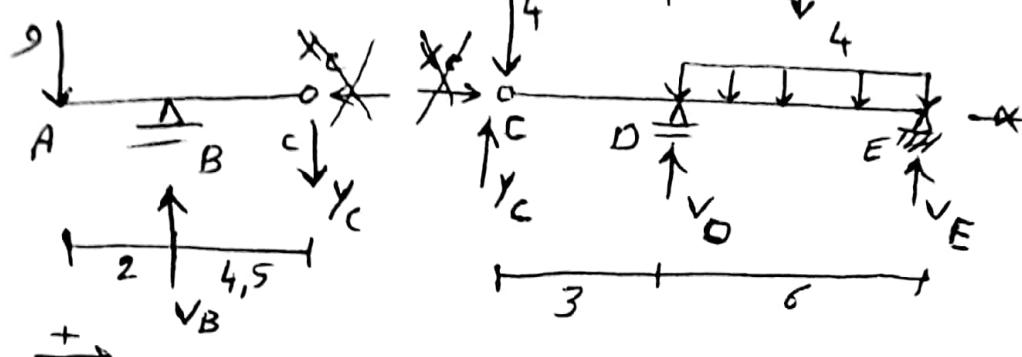
$$M_{max} = 3 \cdot (1) - \frac{3 \cdot (1)^2}{2} = \boxed{1,5} \quad \text{قيمة النزوة}$$



نفصل عند المفصل ونضع قويبه صاوبيه مما لاكتبه فهو ليس

(24)

(13)



$$\sum F_x = 0 \Rightarrow -X_C = 0 \Rightarrow X_C = 0 \quad ; \text{ABC} \rightarrow \text{jed!}$$

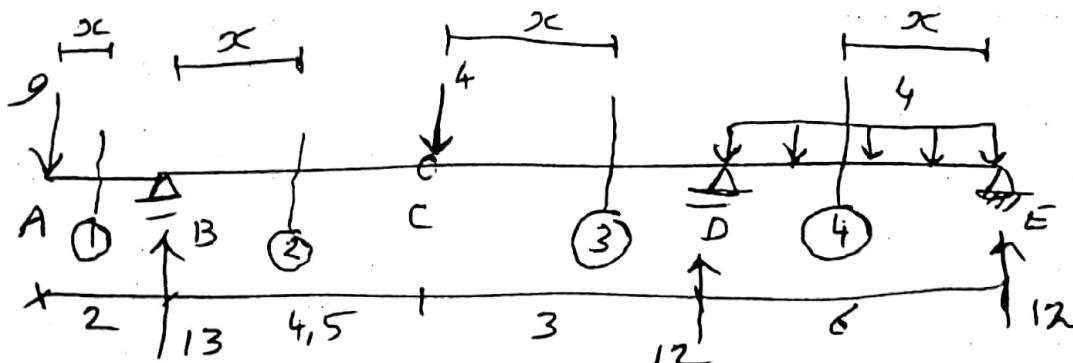
$$\sum M_B = 0 \Rightarrow -9 \cdot 2 + Y_C \cdot 4,5 = 0 \Rightarrow Y_C = 4$$

$$+ \uparrow \sum F_y = 0 \Rightarrow -9 - 4 + V_B = 0 \Rightarrow V_B = 13$$

$$\sum M_E = 0 \Rightarrow V_D \cdot 6 - 24 \cdot 3 - 4 \cdot 9 + 4 \cdot 9 = 0 \quad ; \text{CDE} \rightarrow \text{jed!}$$

$$V_D = 12$$

$$+ \uparrow \sum F_y = 0 \Rightarrow 4 - 4 + 12 - 24 + V_E = 0 \Rightarrow V_E = 12$$



$$M_1 = -9x \quad x = 0 \quad \boxed{A} \quad M = 0 \quad T = -9$$

$$T_1 = -9 \quad x = 2 \quad \boxed{B} \quad M = -18 \quad T = -9$$

$$M_2 = 13x - 9(2+x) \quad x = 0 \quad \boxed{B} \quad M = -18 \quad T = 4$$

$$T_2 = 13 - 9 = 4 \quad x = 4,5 \quad \boxed{C} \quad M = 0 \quad T = 4$$

$$M_3 = -9(6,5+x) + 13(4,5+x) - 4x \quad x = 0 \quad \boxed{C} \quad M = 0 \quad T = 0$$

$$T_3 = -9 + 13 - 4 = 0 \quad x = 3 \quad \boxed{D} \quad M = 0 \quad T = 0$$

(25)

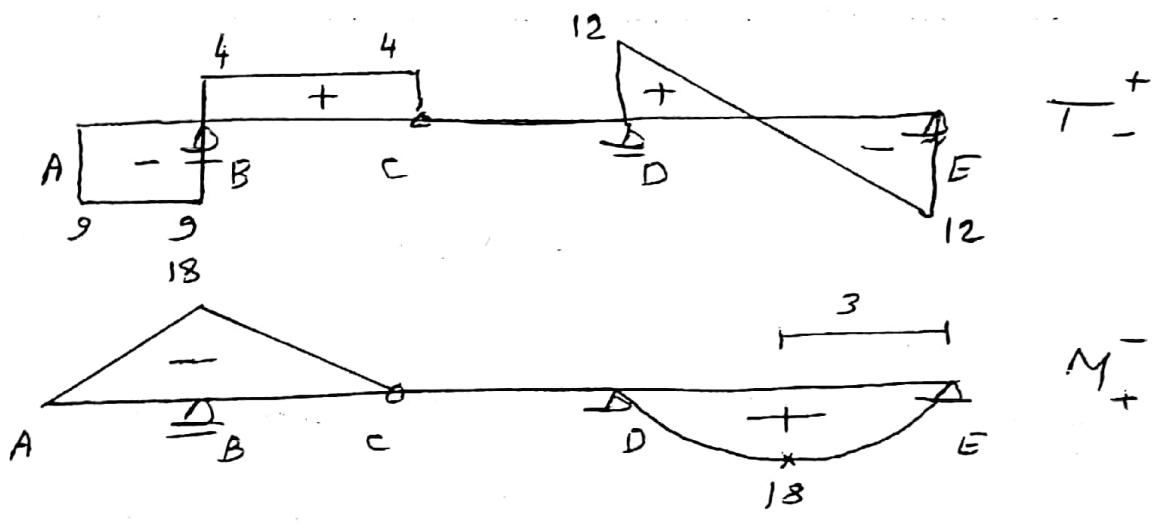
(14)

$$M_4 = 12x - 4 \frac{x^2}{2} \quad x=0 \quad \text{E} \quad M=0 \quad T=-12$$

$$T_4 = -12 + 4x \quad x=6 \quad \text{D} \quad M=0 \quad T=12$$

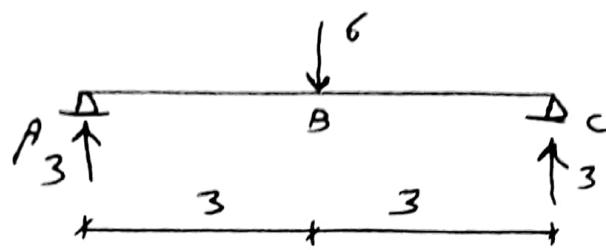
$$T_4 = 0 \Rightarrow -12 + 4x = 0 \Rightarrow x = 3 \quad \text{موقع النزعة}$$

$$M_{\max} = 12 \cdot 3 - \frac{4(3)^2}{2} = 18 \quad \text{قيمة النزعة}$$



(26)

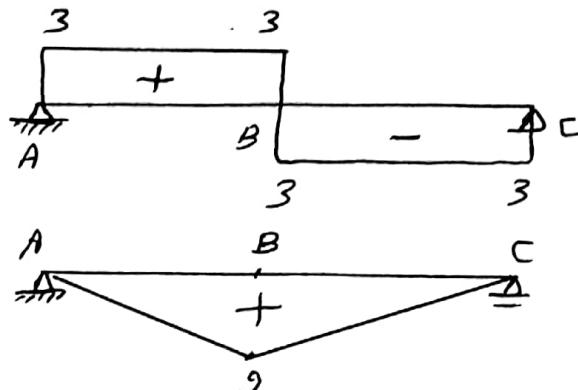
(15)

EVLine

$$M_A = \boxed{0}$$

$$M_B = 3 \cdot 3 = \boxed{9}$$

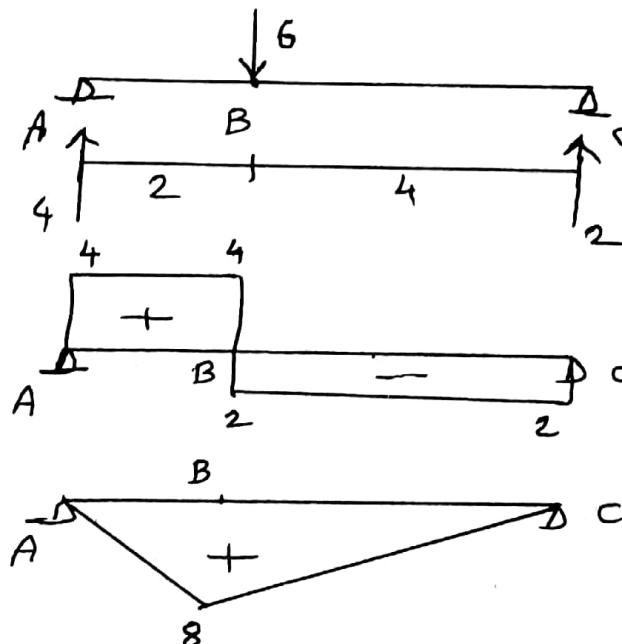
$$M_C = \boxed{0}$$

 T_-^+ M_-^-

$$M_A = \boxed{0}$$

$$M_B = \boxed{8}$$

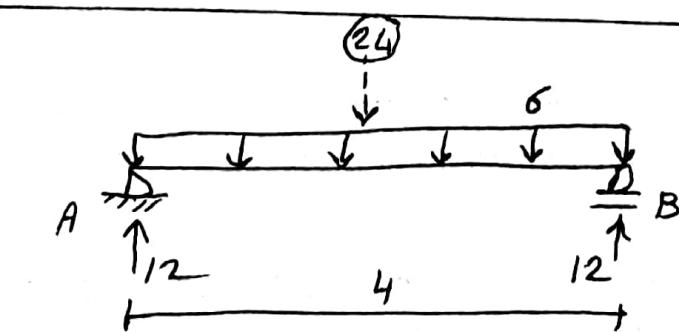
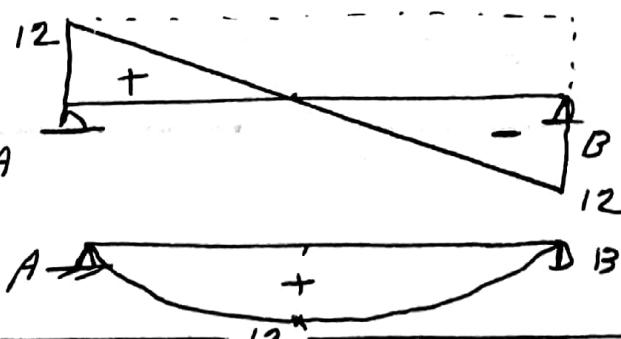
$$M_C = \boxed{0}$$

 T_-^+ M_-^-

$$M_A = \boxed{0}$$

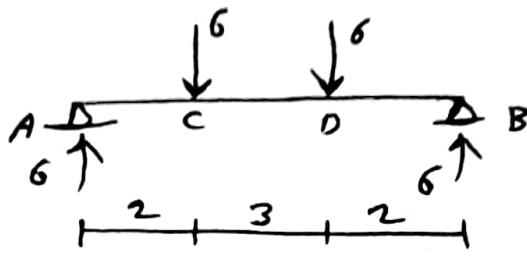
$$M_B = \boxed{0}$$

$$M_{\max} = \frac{9L^2}{8} = \frac{6(4)^2}{8} = \boxed{12}$$

EVLine T_-^+ M_-^-

(27)

(16)



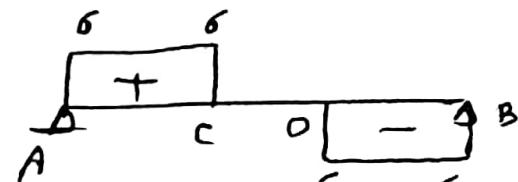
: حاصل

$$M_A = 0$$

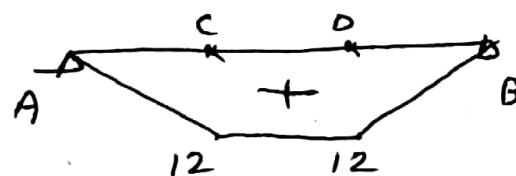
$$M_C = 6 \cdot 2 = 12$$

$$M_D = 6 \cdot 2 = 12$$

$$M_B = 0$$



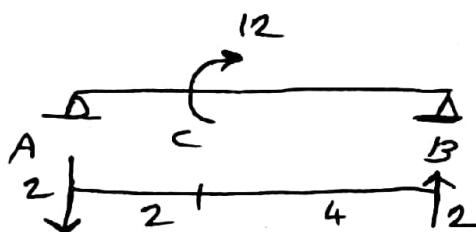
T_-^+



M_-^-

M_+^+

: حاصل

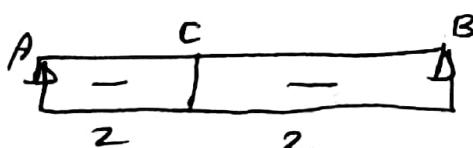


$$M_A = 0$$

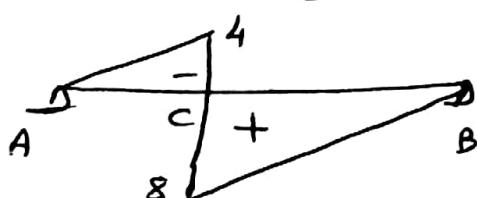
$$- M_C = -2 \cdot 2 = -4$$

$$M_C = 2 \cdot 4 = 8$$

$$M_B = 0$$



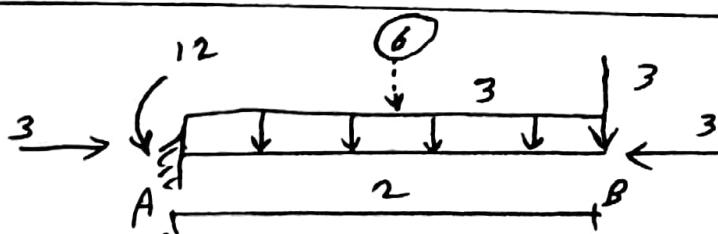
T_-^+



M_-^-

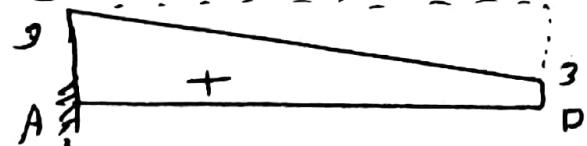
M_+^+

: حاصل



$$M_A = -12$$

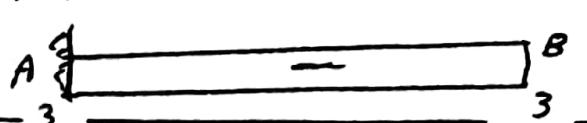
$$M_B = 0$$



T_-^+



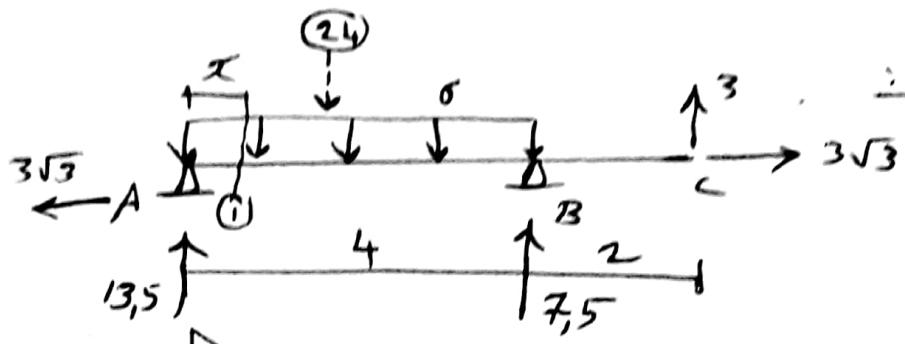
M_-^-



N_+^+

(28)

(17)



$$M_A = 0$$

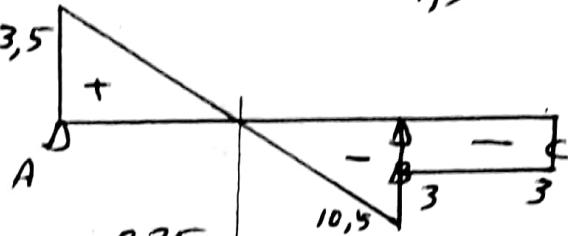
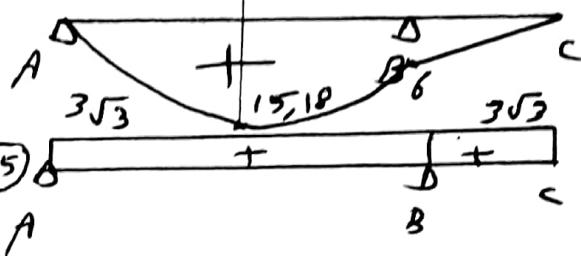
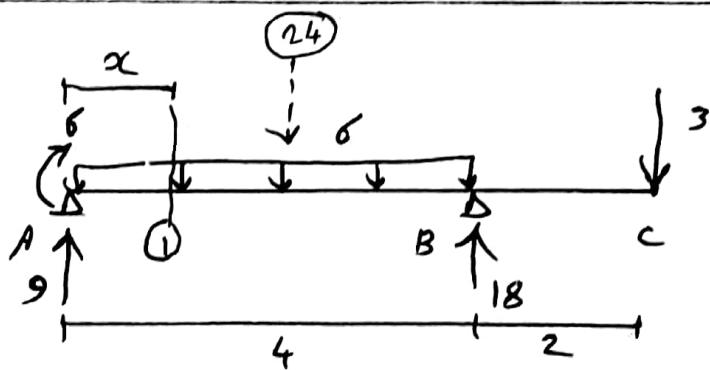
$$M_B = 3 \cdot 2 = 6$$

$$M_C = 0$$

$$M_1 = 13,5x - 6 \frac{x^2}{2}$$

$$M_1' = 13,5 - 6x = 0 \Rightarrow$$

$$M_{max} = 15,18$$

 T_-  M_- N_+ 

$$M_A = 6$$

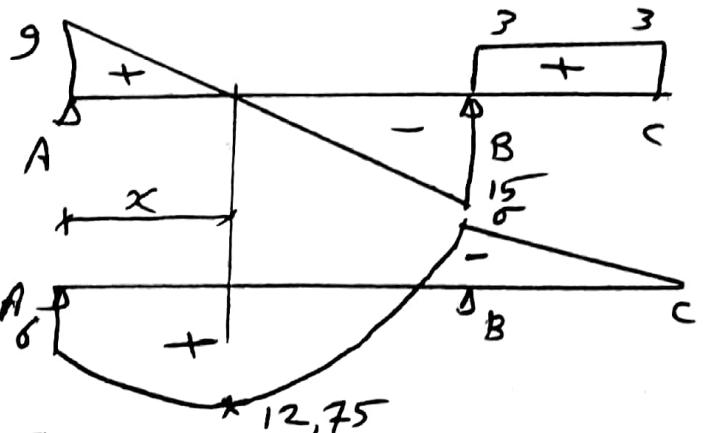
$$M_B = -3 \cdot 2 = -6$$

$$M_C = 0$$

$$M = 9x - 6 \frac{x^2}{2} + 6$$

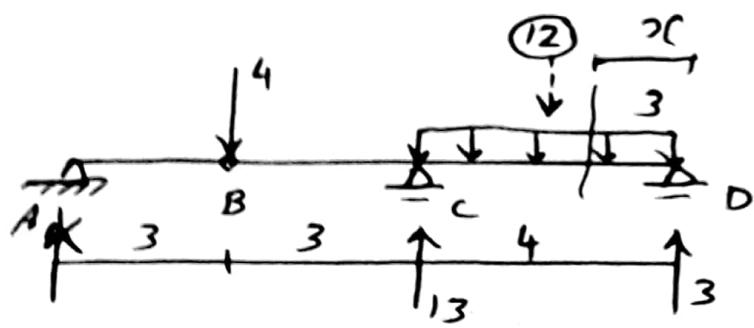
$$M' = 9 - 6x = 0 \Rightarrow x = 3$$

$$M_{max} = 12,75$$

 T_- M_-

(29)

(18)



مسار

$$M_A = 0$$

$$M_B = 0$$

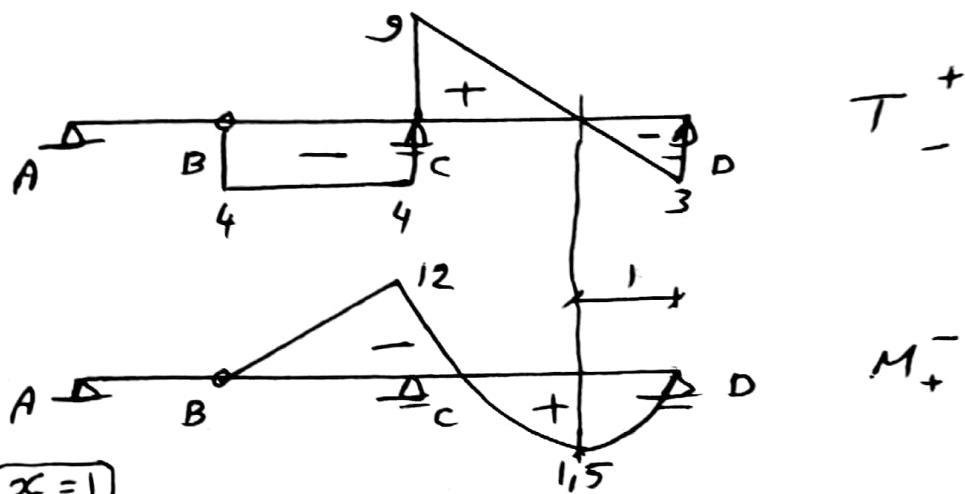
$$M_C = -4 \cdot 3 = -12$$

$$M_D = 0$$

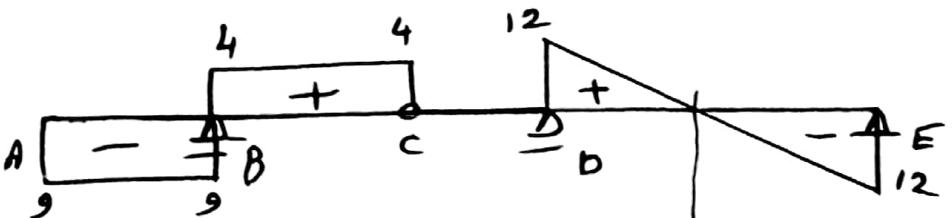
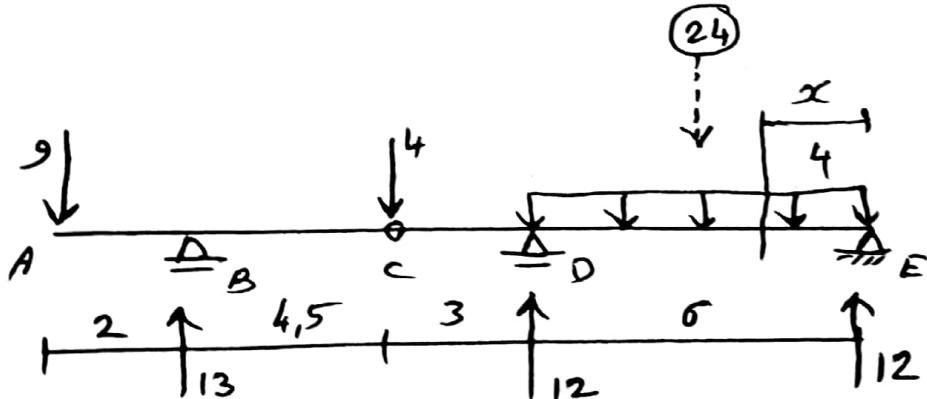
$$M = 3x - \frac{3x^2}{2}$$

$$M = 3 - 3x = 0 \Rightarrow x = 1$$

$$M_{\max} = 1,5$$



مسار



$$M_A = 0$$

$$M_B = -9 \cdot 2 = -18$$

$$M_C = 0$$

$$M_D = -9 \cdot 9,5 + 13 \cdot 7,5 - 4 \cdot 3 = 0$$

$$M_E = 0$$

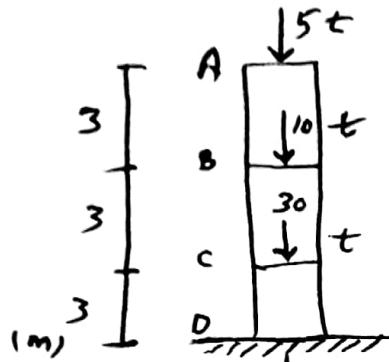
$$M = 12 \cdot x - 4 \frac{x^2}{2}$$

$$M = 12 - 4x = 0 \Rightarrow x = 3$$

$$M_{\max} = 18$$

(30)

١



- المقوى المحوري -

مسائل

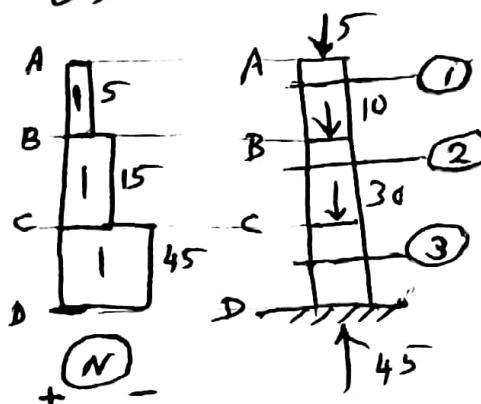
$\uparrow \downarrow$
0 45

45

$$E = 2 \cdot 10^6 \text{ Kg/cm}^2$$

عامل الرونة الطري

جهاز مع الصيغة
و متوجه افتراض



$$A_1 = A_2 = A_3 = 30 \cdot 30 = 900 \text{ cm}^2$$

حساب N - ٢

$$N_1 = -5t \quad \begin{matrix} \text{منطقة} \\ + \end{matrix}$$

$$N_2 = -5 - 10 = -15t \quad \begin{matrix} \text{منطقة} \\ - \end{matrix}$$

$$N_3 = -5 - 10 - 30 = -45t \quad \begin{matrix} \text{منطقة} \\ - \end{matrix}$$

الامتداد الناتجي - ٣

$$\sigma = \frac{N}{A}$$

$$t \xrightarrow{\times 1000} \text{kg}$$

$$KN \xrightarrow{\times 1000} N$$

$$\sigma_1 = \frac{-5 \cdot 1000}{900} = -5,6 \text{ MPa}$$

$$\sigma_2 = \frac{-15 \cdot 1000}{900} = -16,7 \text{ MPa}$$

$$\sigma_3 = \frac{-45 \cdot 1000}{900} = -50 \text{ MPa}$$

$$\Delta L = \frac{N \cdot L}{E A}$$

لدينا: ٤

$$\Delta L_1 = \frac{-5 \cdot 1000 \cdot 300}{2 \cdot 10^6 \cdot 900} = -9,00083 \text{ mm}$$

$$m \xrightarrow{\times 100} \text{cm}$$

$$\Delta L_2 = \frac{-15 \cdot 1000 \cdot 300}{2 \cdot 10^6 \cdot 900} = -0,0025 \text{ mm}$$

$$\text{نطوال} +$$

$$\Delta L_3 = \frac{-45 \cdot 1000 \cdot 300}{2 \cdot 10^6 \cdot 900} = -0,0075 \text{ mm}$$

$$\text{نطوال} -$$

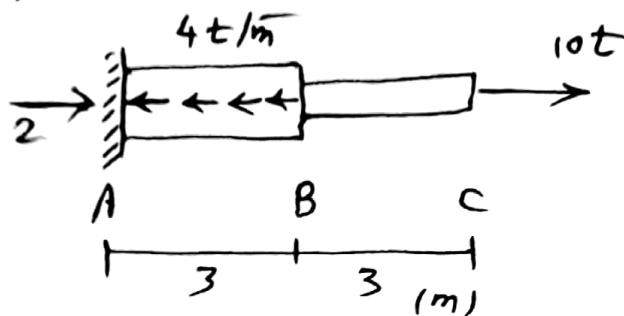
$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3 = -0,01083 \text{ cm}$$

(31)

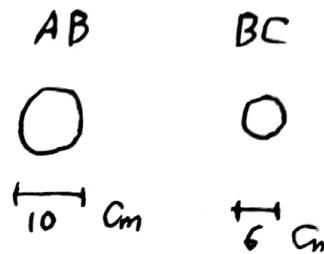
2

النرخ = القدرة × الطول

$$\begin{array}{c} \leftarrow \\ 12 \\ \rightarrow \\ 10 \end{array}$$

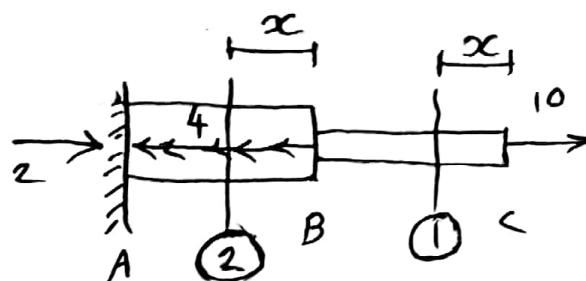


: مسأله



$$E = 8 \cdot 10^5 \text{ Kg/cm}^2$$

: حل

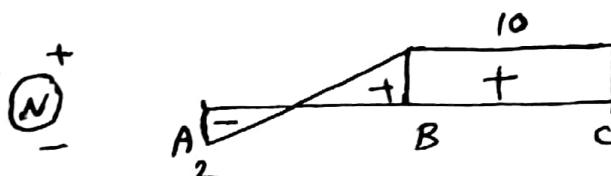


: نسبه -

$$N_1 = [+10] + \dots$$

$$N_2 = [+10 - 4x]$$

$$\begin{array}{ll} x=0 & N=10 \\ x=3 & N=-2 \end{array}$$



$$\sigma = \frac{N}{A}$$

: σ نسبه -

$$A_2 = \pi (5)^2 = 78,5 \text{ cm}^2$$

$$A_1 = \pi (3)^2 = 28,3 \text{ cm}^2$$

$$\sigma_1 = \frac{10 \cdot 1000}{28,3} = 350,9$$

$$\sigma_2 = \frac{(10-4x) \cdot 1000}{78,5}$$

Kg/cm^2

$$\begin{array}{ll} x=0 & \sigma = 127,4 \\ x=3 & \sigma = -25,5 \end{array}$$

Kg/cm^2

$$\Delta L = \frac{N \cdot L}{E \cdot A}$$

: ΔL نسبه -

(32)

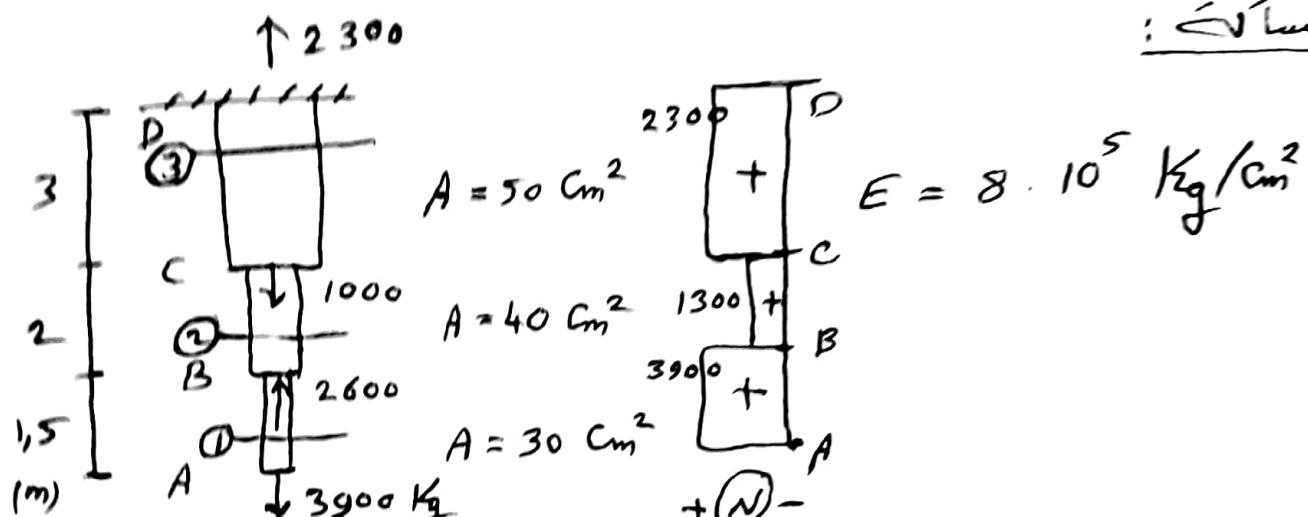
(3)

$$\Delta L_1 = \frac{10 \cdot 1000 \cdot 300}{8 \cdot 10^5 \cdot 28,3} = [+0,132] \text{ cm طاول}$$

$$\Delta L_2 = \frac{\left(\frac{10+(-2)}{2}\right) \cdot 1000 \cdot 300}{8 \cdot 10^5 \cdot 78,5} = [+0,019] \text{ cm طاول}$$

$$\Delta L = \Delta L_1 + \Delta L_2 = [+0,151] \text{ cm طاول}$$

مسار



$$N_1 = [+3900] \text{ N} \quad : N \text{ قوى -}$$

$$N_2 = +3900 - 2600 = [+1300] \text{ N} \quad : N \text{ قوى - Kg}$$

$$N_3 = [+2300] \text{ N} \quad : N \text{ قوى -}$$

$$\sigma = \frac{N}{A} \quad : \sigma \text{ قوى -}$$

$$\sigma_1 = \frac{3900}{30} = [+130] \text{ N/mm}^2$$

$$\sigma_2 = \frac{1300}{40} = [+32,5] \text{ N/mm}^2 \quad \text{Kg/cm}^2$$

$$\sigma_3 = \frac{2300}{50} = [+46] \text{ N/mm}^2$$

$$\Delta L = \frac{N \cdot L}{E \cdot A} \quad : \Delta L \text{ قوى -}$$

$$\Delta L_1 = \frac{3900 \cdot 150}{8 \cdot 10^5 \cdot 30} = [+0,024] \text{ cm طاول}$$

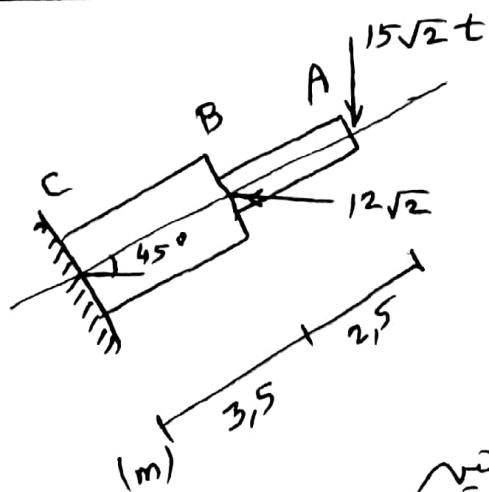
(33)

(4)

$$\Delta L_2 = \frac{1300 \cdot 200}{8 \cdot 10^5 \cdot 40} = [+0,008] \text{ تطاول}$$

$$\Delta L_3 = \frac{2300 \cdot 300}{8 \cdot 10^5 \cdot 50} = [+0,017] \text{ تطاول}$$

الكلية, $\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3 = [+0,049 \text{ cm}]$ تطاول



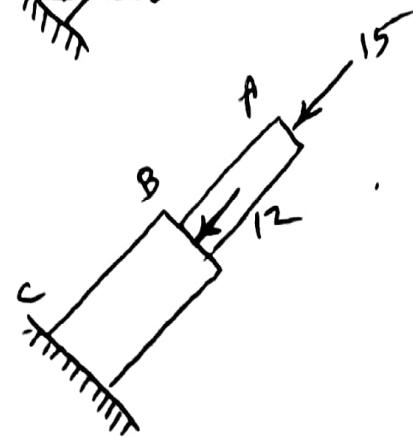
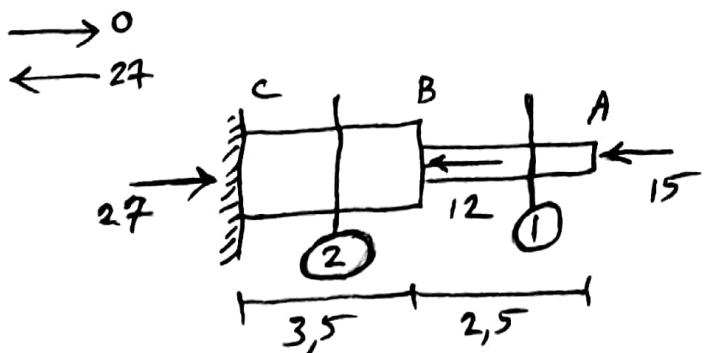
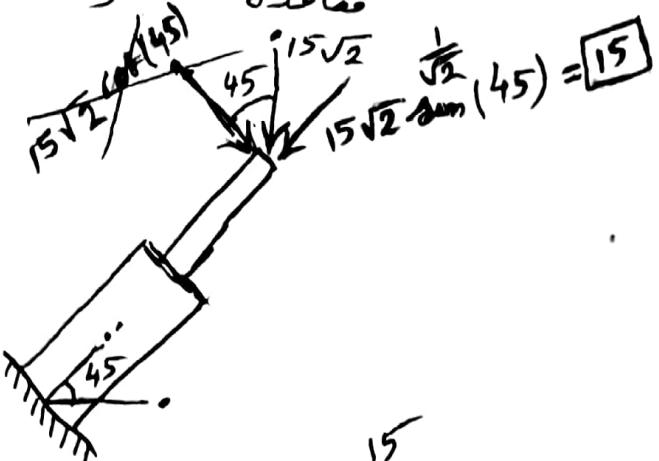
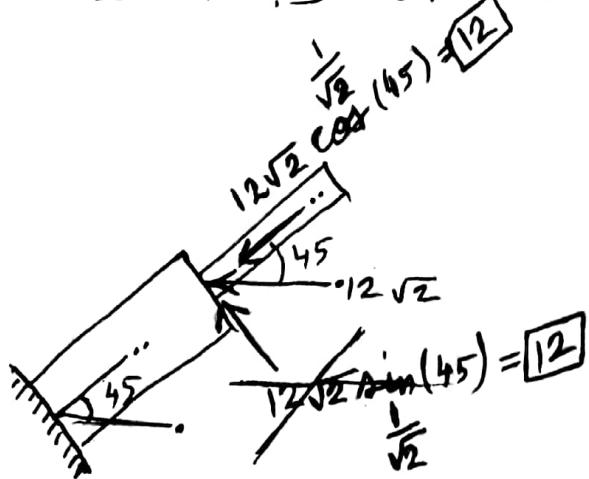
$$A = 50 \text{ cm}^2 \quad BA$$

$$A = 65 \text{ cm}^2 \quad CB$$

$$E = 2 \cdot 10^5 \text{ kg/cm}^2$$

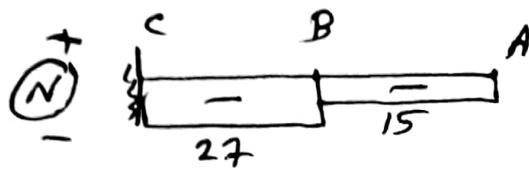
: مسالة

حلل العمود المكون من اجزاء A-B-C الى مركبة
معادلة للعنصر وموازية له. ونحو المركبة المعاوقة للعنصر



(34)

(5)



$$N_1 = \boxed{-15} \text{ kip} \quad \frac{N \downarrow \text{Längs}}{t}$$

$$N_2 = \boxed{-27} \text{ kip}$$

$$\sigma = \frac{N}{A}$$

: σ ↓ Längs -

$$\sigma_1 = \frac{-15 \cdot 1000}{50} = \boxed{-300} \text{ kip} \quad \text{kg/cm}^2$$

$$\sigma_2 = \frac{-27 \cdot 1000}{65} = \boxed{-415,3} \text{ kip}$$

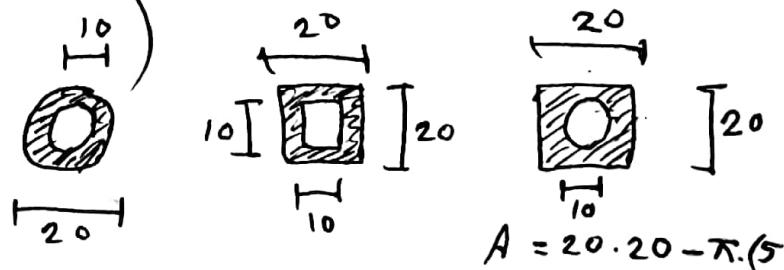
$$\Delta L = \frac{N \cdot L}{E \cdot A}$$

: ΔL ↓ Längs -

$$\Delta L_1 = \frac{-15 \cdot 1000 \cdot 250}{2 \cdot 10^5 \cdot 50} = \boxed{-0,375 \text{ cm}} \quad \text{رطاخ}$$

$$\Delta L_2 = \frac{-27 \cdot 1000 \cdot 350}{2 \cdot 10^5 \cdot 65} = \boxed{-0,727 \text{ cm}} \quad \text{رطاخ}$$

الإجابة $\Delta L = \Delta L_1 + \Delta L_2 = \boxed{-1,102 \text{ cm}}$ رطاخ

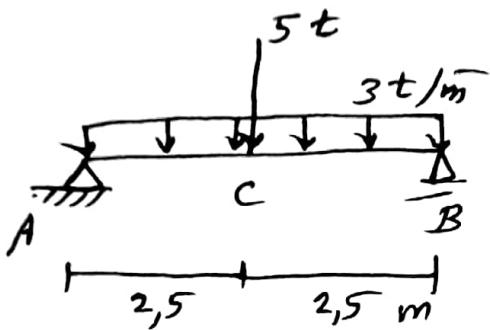


$$A = 20 \cdot 20 - \pi \cdot (5)^2$$

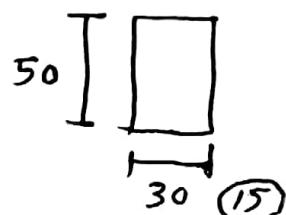
$$A = \pi \cdot (10)^2 - \pi \cdot (5)^2 \quad A = 20 \cdot 20 - 10 \cdot 10$$

(35)

٦



مسألة: ١ - ارسم عرض مقطع عن المقطع
٢ - احسب سعى لين على مسافة ٣٠ سم اسفل المحور السليم عند C.



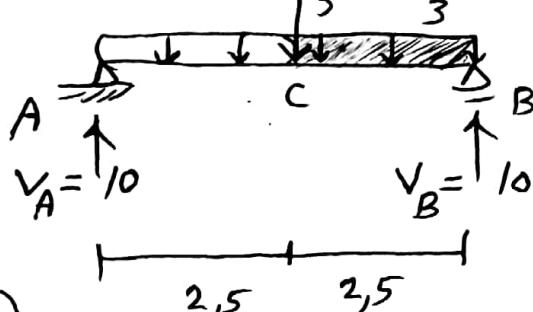
الجهاد
الوطني

$$\sigma = \frac{Mz}{Iz} y$$

١ - لدينا

الحل:

- حساب العزم M عند C :

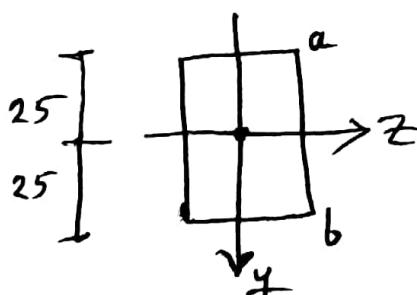


بسبب التمازن

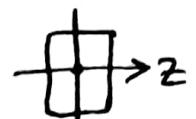
$$V_A = V_B = \frac{1}{2} (5 + 15) = 10$$

$$M_C = 10 \cdot 2,5 - \frac{3(2,5)^2}{2} = 15,625 \text{ t.m}$$

- حساب I_Z :



$$I_Z = \frac{30 \cdot (50)^3}{12} = 112500 \text{ cm}^4$$



$$I_Z = \frac{bh^3}{12}$$

- نوصي في معادلة س:

$$\sigma = \frac{15,625 \cdot 10^5}{112500} y \Rightarrow \sigma = 13,89 y$$

$$\begin{aligned} \text{t.m} &\xrightarrow{\times 10^5} \text{kg/cm} \\ \text{kN.m} &\xrightarrow{\times 10^5} \text{N.cm} \end{aligned}$$

$$\sigma_a = 13,89 (-25) = -347,2$$

$$\sigma_b = 13,89 (25) = 347,2$$

Kg/cm²

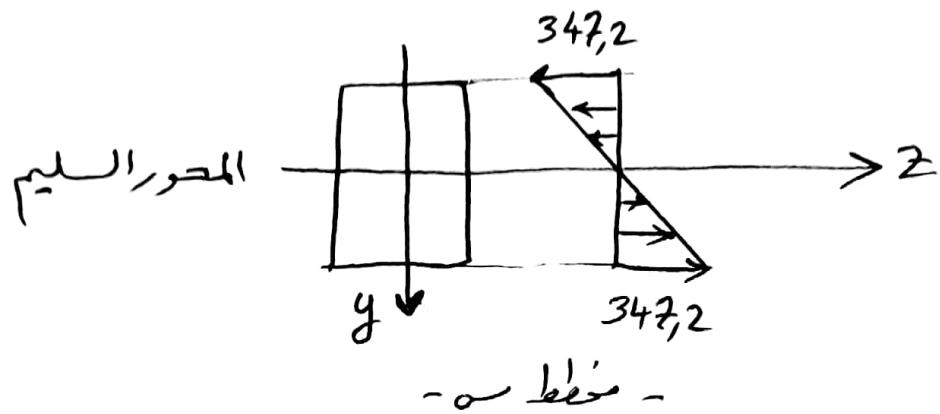
ارسم المحور السليم نيلقي
على المحور Z

$$y = 0$$

$$13,89 \cdot y = 0 \Leftrightarrow \sigma = 0$$

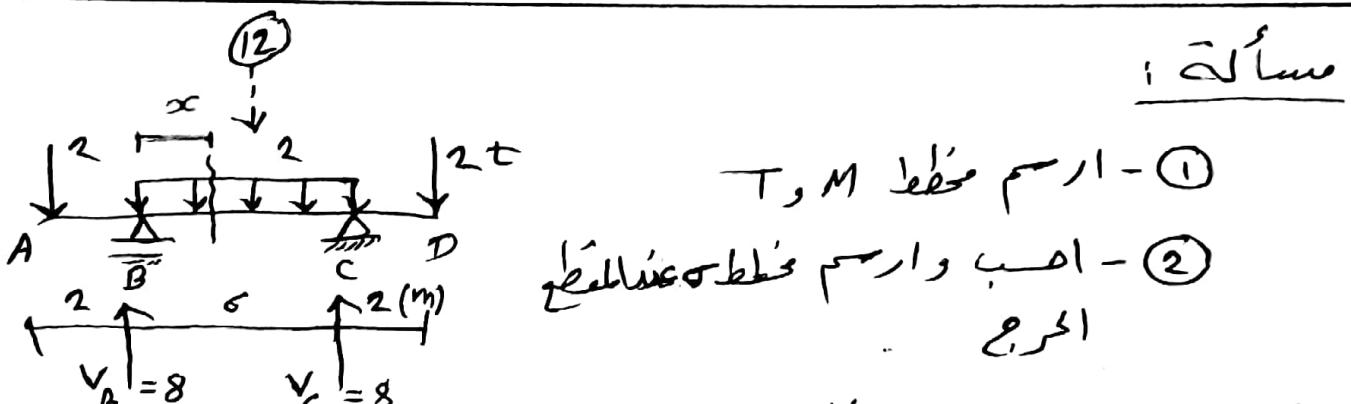
(36)

(7)



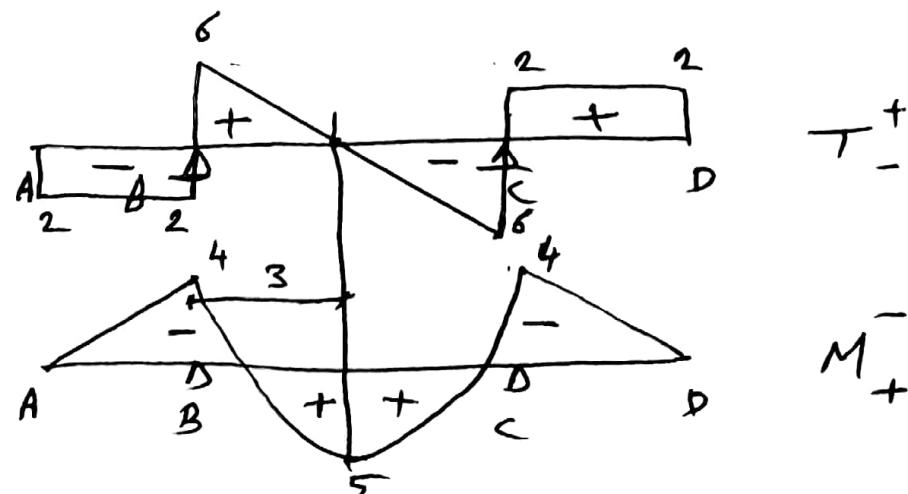
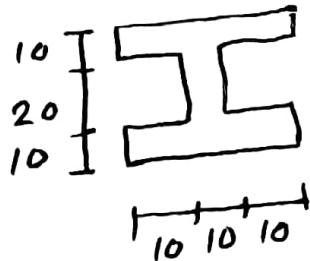
$$y = 10 \Rightarrow \sigma = 13,89 \cdot (10) = 138,9 \text{ kg/cm}^2$$

(2)



الحل: ① بسب السياق

$$V_B = V_C = \frac{1}{2}(2+12+2) = 8$$



$$M_A = 0$$

$$M_B = -2 \cdot 2 = -4$$

$$M_C = -2 \cdot 2 = -4$$

$$M_D = 0$$

$$M = -2(2+x) + 8 \cdot x - 2 \cdot \frac{x^2}{2}$$

$$M' = -2 + 8 - 2x = 0 \Rightarrow x = 3 \quad \text{موقع الضرورة}$$

$$M_{max} = -2(2+3) + 8 \cdot 3 - 2 \cdot \frac{(3)^2}{2} = 5 \quad \text{نقطة الضرورة}$$

(37)

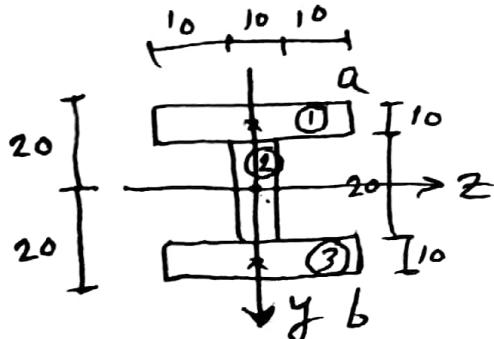
(8)

$$\sigma = \frac{Mz}{I_z} y$$

- لدينا: (2)

نـ المـعـصـعـ المـرـجـعـ يـقـعـ عـنـ الـبـرـجـ زـمـ بـالـعـصـعـةـ الـمـطـلـعـةـ

$$M = 5 t \cdot m$$



: I_z بـسـابـعـ

الـمـحـورـ الـمـرـجـعـ يـاـهـ هـمـاـ صـوـرـ الـسـاطـعـ

نـمـمـعـةـ	A	z	y
①	$30 \cdot 10 = 300$	0	-15
②	$20 \cdot 10 = 200$	0	0
③	$30 \cdot 10 = 300$	0	15

$$I_{z_1} = \frac{30 \cdot (10)^3}{12} + 300 \cdot (-15)^2 = 70000$$

$$I_{z_2} = \frac{10 \cdot (20)^3}{12} + 200 \cdot (0)^2 = 6666,67 \text{ Cm}^4$$

$$I_{z_3} = \frac{30 \cdot (10)^3}{12} + 300 \cdot (15)^2 = 70000$$

$$I_z = I_{z_1} + I_{z_2} + I_{z_3} = 146666,67 \text{ Cm}^4$$

نـعـصـ فـيـ مـعـادـلـهـ سـ

$$\sigma = \frac{5 \cdot 10^5}{146666,67} \cdot y \Rightarrow \sigma = 3,41 \cdot y$$

علـويـ a ; y = -20 \Rightarrow \sigma = 3,41 \cdot (-20) = -68,18

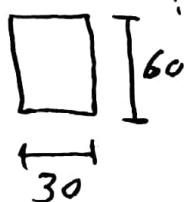
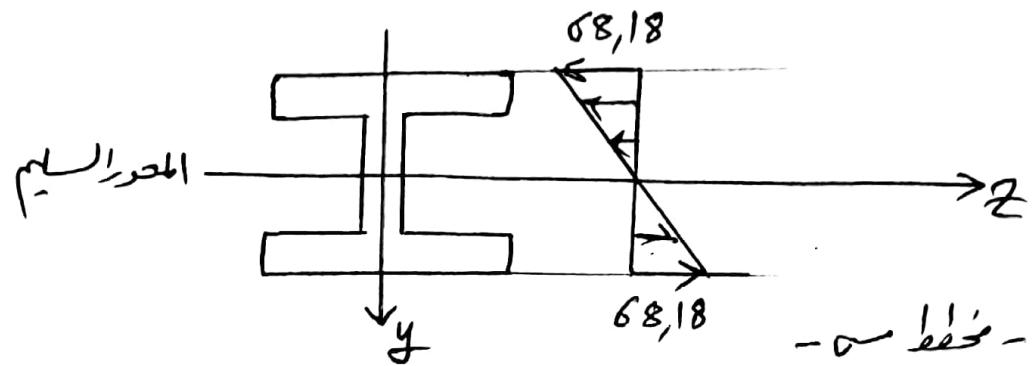
kg/cm²

سـفـنـيـ b ; y = 20 \Rightarrow \sigma = 3,41 \cdot (20) = 68,18

(38)

لـرـسـ الـمـحـرـ الـلـيـمـ نـقـتـ 0 \Leftrightarrow 3,41y = 0 \Leftrightarrow \sigma = 0

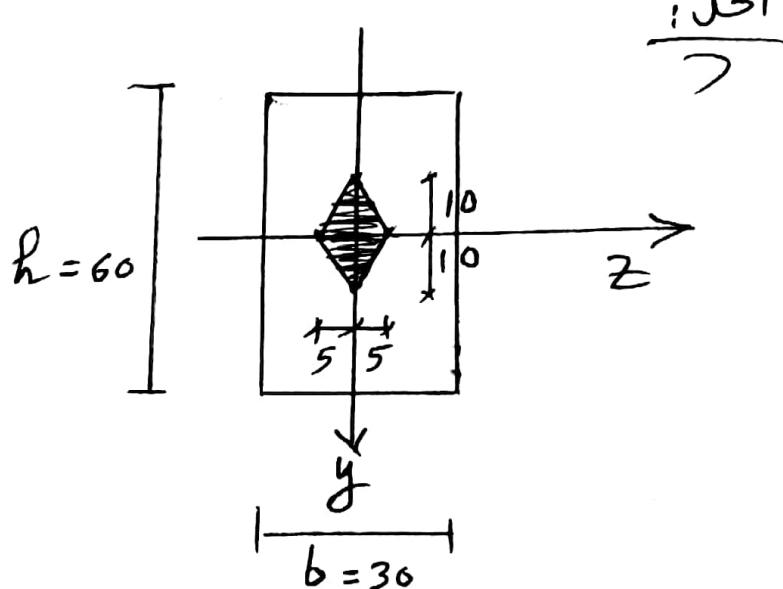
6



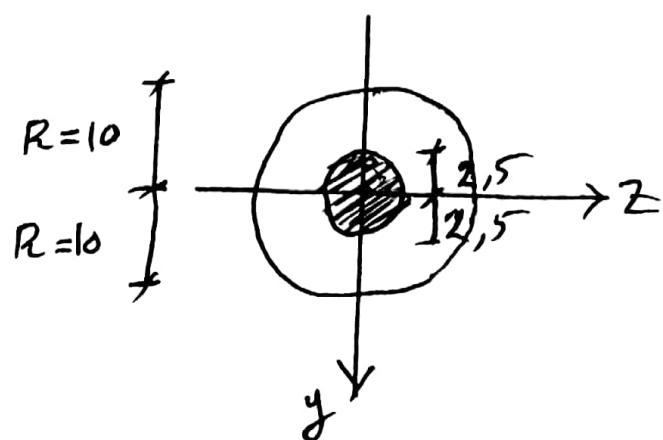
مسأله: ارسم النواة المركبة للعاصف الماليت:

$$\frac{h}{6} = \frac{60}{6} = 10$$

$$\frac{b}{6} = \frac{30}{6} = 5$$

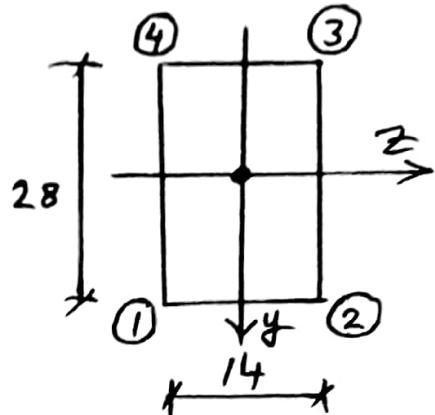


$$\frac{R}{4} = \frac{10}{4} = 2,5$$



(10)

مكعب



الملحوظ رسم مرفق الاجهادات و
وهي في المحو السليم ورسم

$$N = 8t$$

$$M_z = 1 t \cdot m$$

$$\sigma = \frac{N}{A} + \frac{M_z}{I_z} y + \frac{\cancel{M_y}}{\cancel{I_y}} \cancel{z}$$

الكل: $\frac{N}{A} + \frac{M_z}{I_z} y$ لدينا:

$$A = 14 \cdot 28 = 392 \text{ cm}^2$$

: A, I_z مساحت -

$$I_z = \frac{14 (28)^3}{12} = 25610,67 \text{ cm}^4$$

$$\begin{aligned} t &\xrightarrow{= 10^3} \text{kg} \\ t \cdot \text{m} &\xrightarrow{= 10^5} \text{kg} \cdot \text{cm} \end{aligned}$$

$$\sigma = \frac{8 \cdot 10^3}{392} + \frac{1 \cdot 10^5}{25610,67} y$$

لعمد

$$\sigma = 20,4 + 3,9 y$$

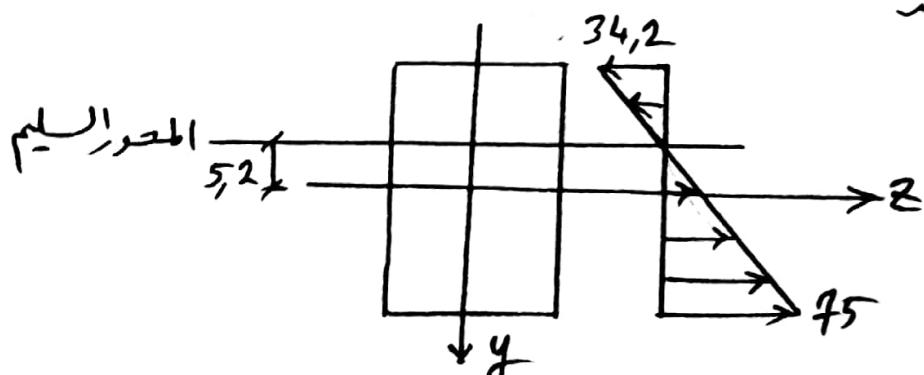
معادلة σ

$$20,4 + 3,9 y = 0 \Leftrightarrow \sigma = 0$$

$$y = -5,2 \quad \text{معادلة المحو السليم مستقيم بواري 2}$$

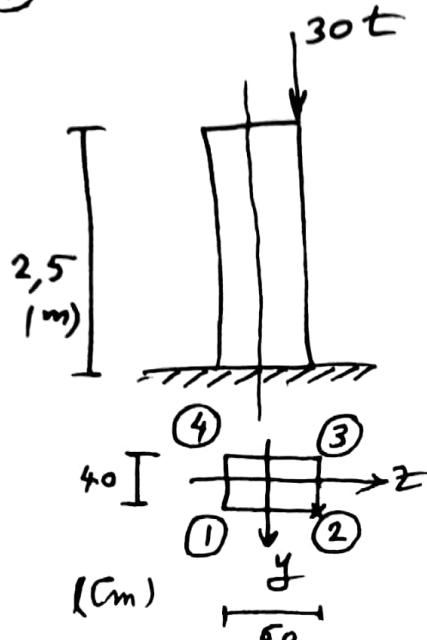
$$\text{علوي } ③, ④; \quad y = -14 \Rightarrow \sigma = 20,4 + 3,9(-14) = -34,2$$

$$\text{سفلي } ①, ②; \quad y = 14 \Rightarrow \sigma = 20,4 + 3,9(14) = 75 \quad \text{kg/cm}^2$$



(40)

(11)

مسألة:

العواد المبسوط يقمع لغزة ضد محور ز
قدرها $N = 30t$ أو جهد تميم الدمجارات ٥
في زوايا المقطع وارسم المحور السليم
علماً أن الفوة N هي فوة مطبقة في ②
الحل ١ لدينا: الدمجارات الناظمية

$$\sigma = \frac{N}{A} + \frac{M_z}{I_z} \cdot y + \frac{M_y}{I_y} \cdot z$$

-حساب I_y, I_z, A -

$$A = 40 \cdot 60 = 2400 \text{ cm}^2$$

$$I_z = \frac{60 \cdot (40)^3}{12} = 320000 \text{ cm}^4$$

$$I_y = \frac{40 \cdot (60)^3}{12} = 720000 \text{ cm}^4$$

-حساب M_y, M_z, N -

$$N = -30t \quad \text{صيغة}$$

$$② (z, y) \text{ cm}$$

$$M_z = N \cdot y = (-30) \left(\frac{20}{100} \right) = -6t \text{ m}$$

$$M_y = N \cdot z = (-30) \left(\frac{30}{100} \right) = -9t \text{ m}$$

$$\sigma = \frac{-30 \cdot 10^3}{2400} + \frac{-6 \cdot 10^5}{320000} y + \frac{-9 \cdot 10^5}{720000} z$$

$$\begin{aligned} \text{Cm} &\xrightarrow{\div 100} \text{m} \\ t &\xrightarrow{\cdot 10^3} \text{kg} \\ tm &\xrightarrow{\cdot 10^6} \text{kg.m} \end{aligned}$$

نوعها

$$\boxed{\sigma = -12,5 - 1,875y - 1,25z}$$

$$\boxed{-12,5 - 1,875y - 1,25z = 0} \Leftrightarrow \sigma = 0$$

معادلة العواد السليم

$$\begin{matrix} z \\ y \end{matrix} \begin{pmatrix} -10 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -6,67 \end{pmatrix}$$

$$\begin{matrix} y \\ z \end{matrix} \begin{pmatrix} 0 \\ -6,67 \end{pmatrix} \Rightarrow \begin{matrix} z \\ y \end{matrix} \begin{pmatrix} -10 \\ -6,67 \end{pmatrix}$$

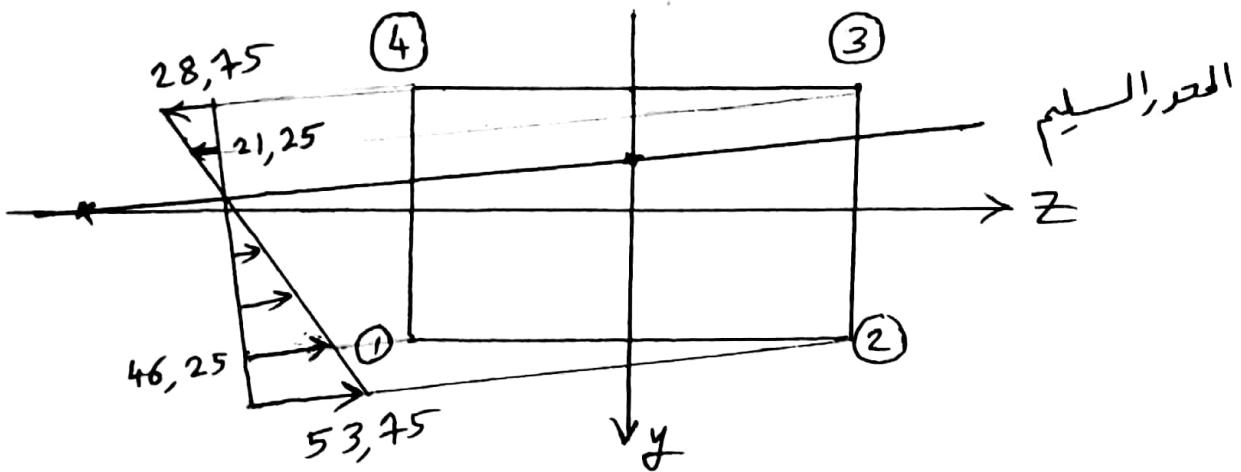
رسم المحور السليم

(41)

(12)

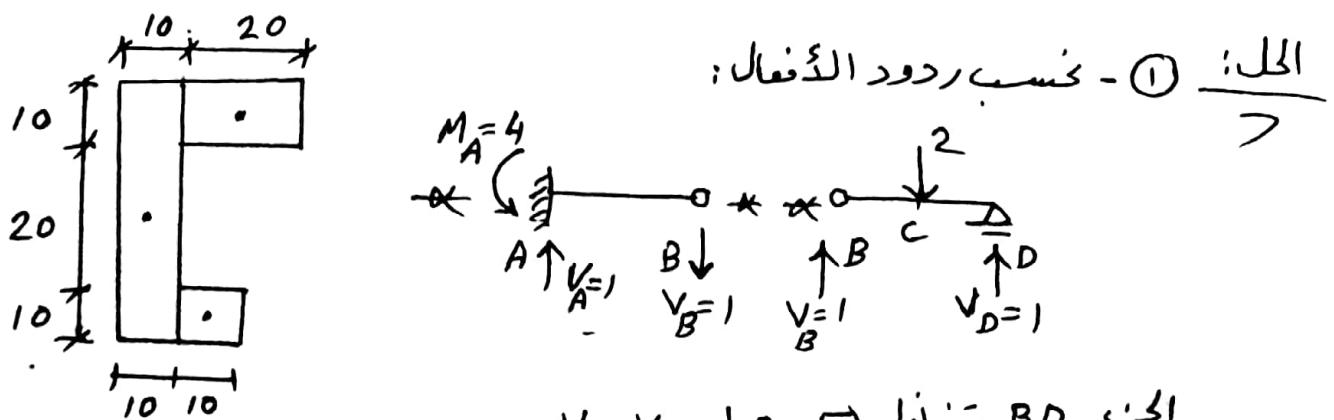
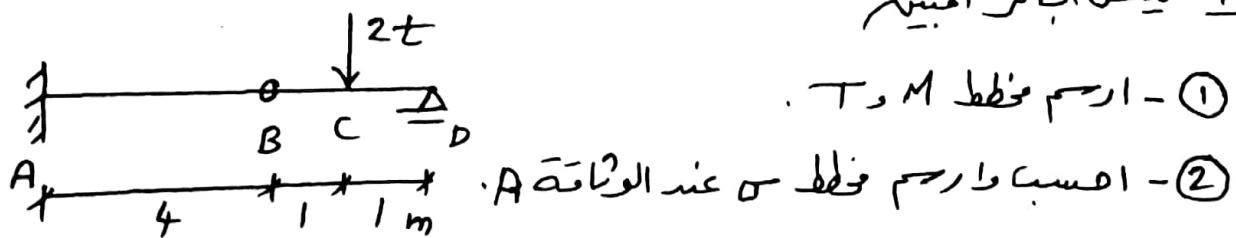
خسب الدعامات في زوايا المطبع

النقطة	Z	y	σ
①	-30	+20	-46,25 صم
②	+30	+20	-53,75 صم
③	+30	-20	+21,25 نس
④	-30	-20	+28,75 نس



مسألة: ليكن الجائز المبين

- 1 - ارسم محظط M و T .
 2 - احسب وارسم محظط σ عند الوجهة A .



$$V_B = V_D = \frac{1}{2} 2 = 1$$

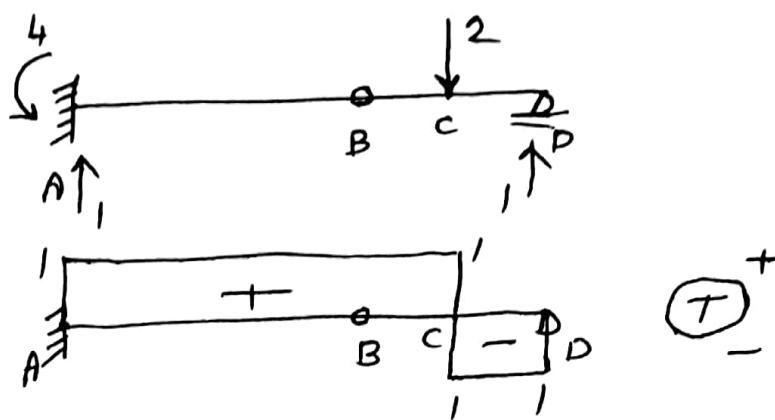
$$\sum M_A = 0 \Rightarrow -M_A + 1 \cdot 4 = 0 \quad ; \quad \underline{\text{الجزء } AB}$$

$$M_A = 4$$

(42)

(13)

$$+\uparrow \sum F_y = 0 \Rightarrow V_A - 1 = 0 \Rightarrow V_A = 1$$



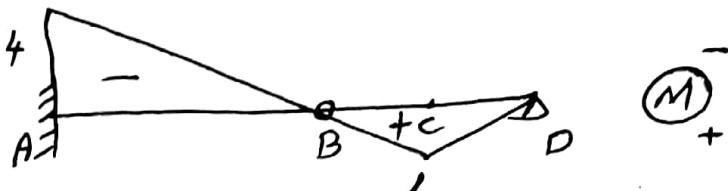
$$M_A = -4$$

$$M_B = 0$$

$$M_C = 1 \cdot 1 = 1$$

$$M_D = 0$$

يوجد لفسم رجود مدةً أقصى

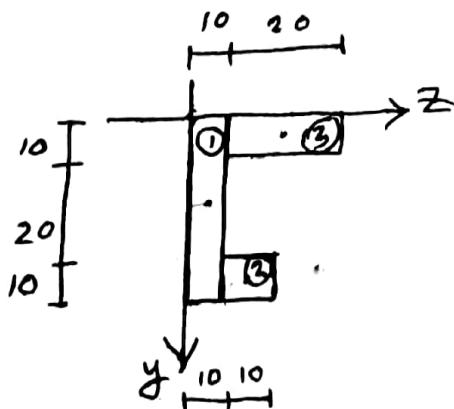


$$\sigma = \frac{M_z}{I_z} y$$

: لـ ٢

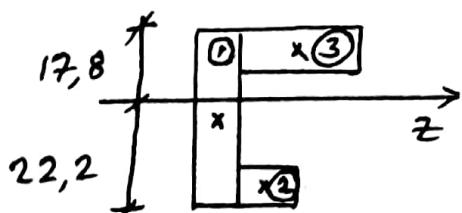
$$M_A = -4 \text{ t.m}$$

: I_z عـ



\bar{z}_{ab} (الـ)	A	z	y	$S_z = A \cdot y$	\bar{y}_{ab} (الـ)
(1)	400	5	20	8000	2,2
(2)	100	15	35	3500	17,2
(3)	200	20	5	1000	-12,8

$$y_c = \frac{\sum S_z}{\sum A} = \frac{12500}{700} = 17,8$$



$$I_{z_1} = \frac{10 \cdot (40)^3}{12} + 400(2,2)^2 = 55269$$

$$I_{z_2} = \frac{10 \cdot (10)^3}{12} + 100(17,2)^2 = 30417 \text{ cm}^4$$

$$I_{z_3} = \frac{20 \cdot (10)^3}{12} + 200(-12,8)^2 = 34434$$

$$I_z = I_{z_1} + I_{z_2} + I_{z_3} = 120120 \text{ cm}^4$$

(43)

(14)

$$\sigma = \frac{-4 \cdot 10^5}{120120} y \Rightarrow \boxed{\sigma = -3,33 \cdot y}$$

نوع

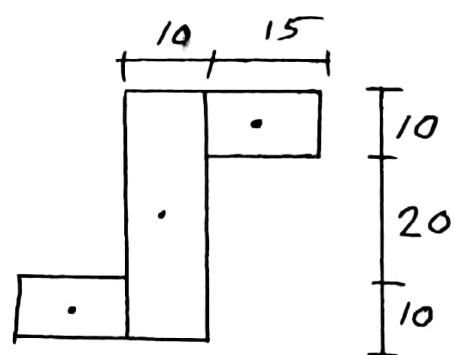
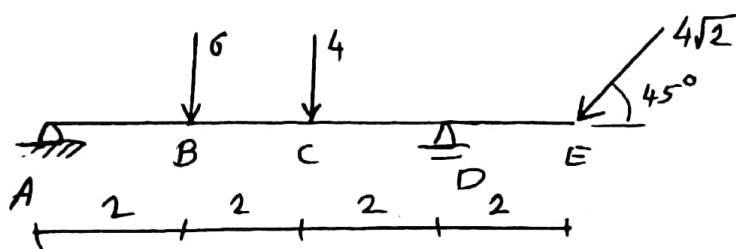
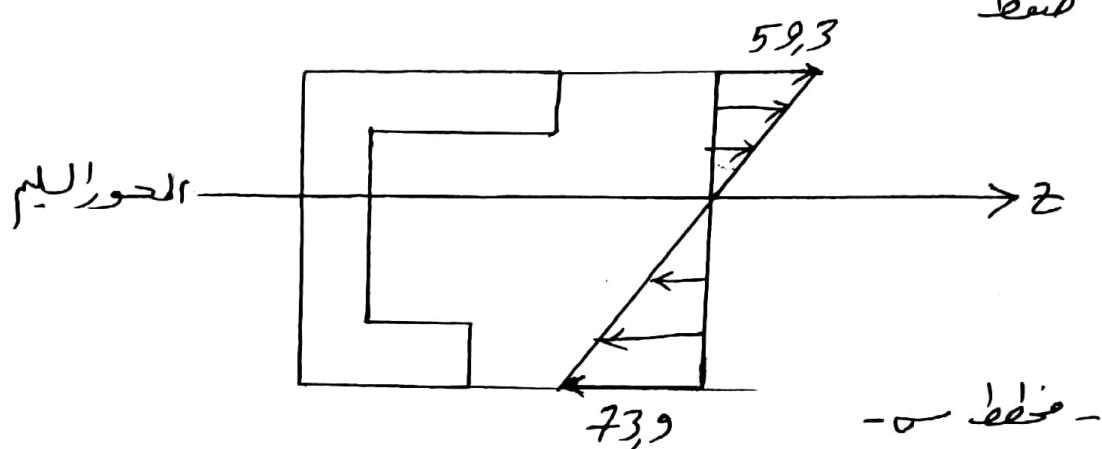
-3,33y = 0 \Leftrightarrow \sigma = 0 \quad \text{لورم المدral السليم نفع = 0}

معادلة المدral $\boxed{y = 0} \Leftrightarrow$

وهو مستقيم منطبق على 2

علوي $y = -17,8 \Rightarrow \sigma = -3,33(-17,8) = \boxed{+59,3}$

سفلي $y = 22,2 \Rightarrow \sigma = -3,33(22,2) = \boxed{-73,9}$ kg/cm^2



N, T, M - حجم منطبق - ①

B inc متساوٍ، دار حجم منطبق - ②

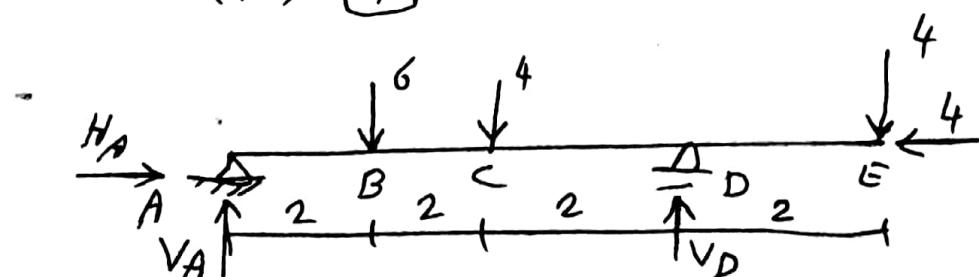
$$4\sqrt{2} \sin(45^\circ) = \boxed{4}$$

$4\sqrt{2}$

45°

- خال القوة المائلة لركبتها - ① $\frac{1}{2}$

$$4\sqrt{2} \cos(45^\circ) = \boxed{4}$$



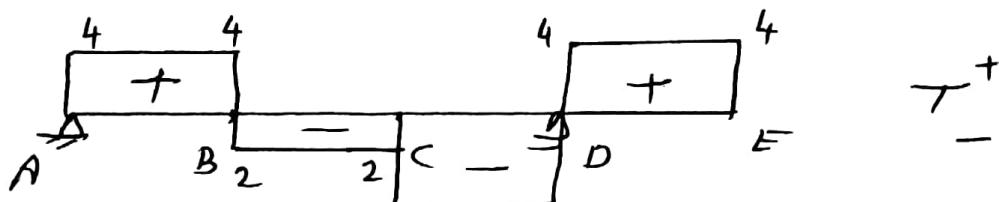
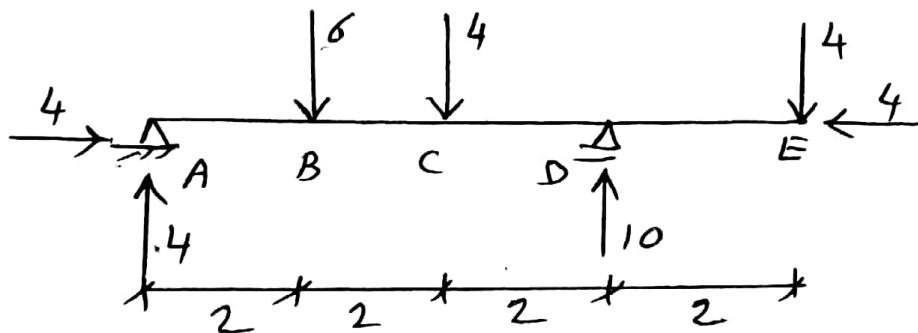
(44)

15

$$\sum F_x = 0 \Rightarrow H_A - 4 = 0 \Rightarrow H_A = 4$$

$$\sum M_A = 0 \Rightarrow 6 \cdot 2 + 4 \cdot 4 + 4 \cdot 8 - V_D \cdot 6 = 0 \Rightarrow V_D = 10$$

$$\uparrow + \sum F_y = 0 \Rightarrow V_A - 6 - 4 - 4 + 10 = 0 \Rightarrow V_A = 4$$



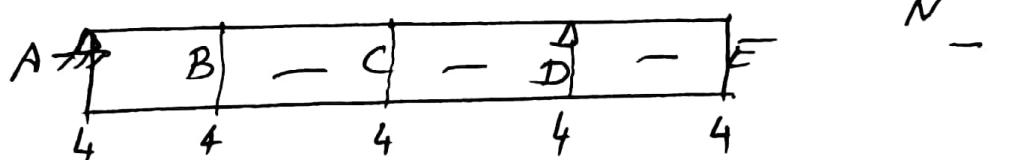
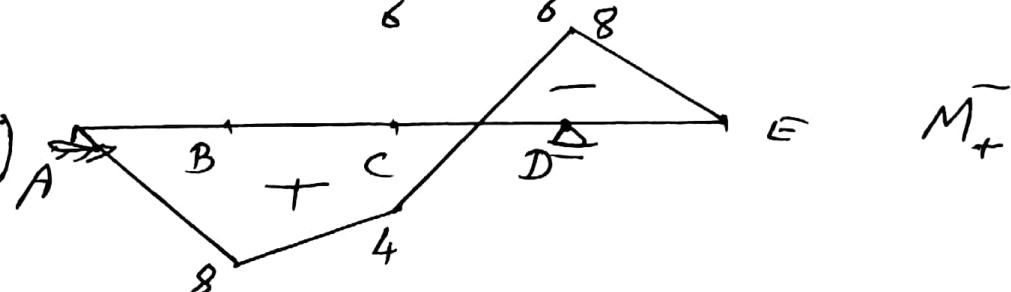
$$M_A = 0$$

$$M_B = 4 \cdot 2 = 8$$

$$M_C = 4 \cdot 4 - 6 \cdot 2 = 4$$

$$M_D = -4 \cdot 2 = -8$$

$$M_E = 0$$



$$\sigma = \frac{N}{A} + \frac{M_z}{I_z} y \quad \text{Liu-W - (2)}$$

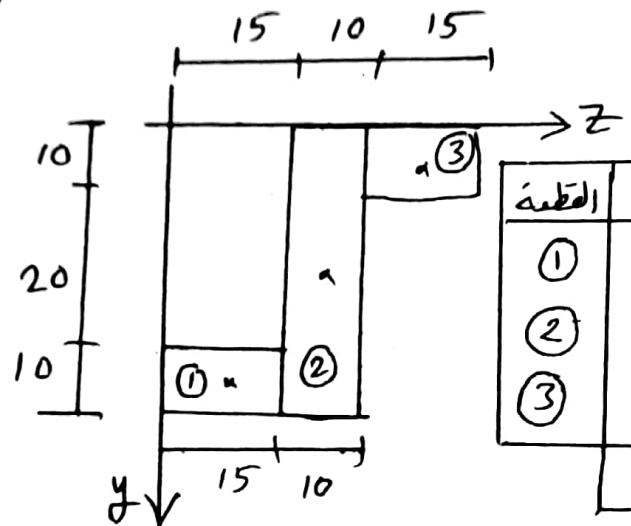
$$N_B = -4t$$

: \rightarrow Elastizitätseig.

$$M_B = 8t \cdot m$$

$$A = 15 \cdot 10 + 10 \cdot 40 + 15 \cdot 10 = 700 \text{ cm}^2 \quad \frac{\frac{T}{2} \cdot A \cdot u_{\max}}{(45)}$$

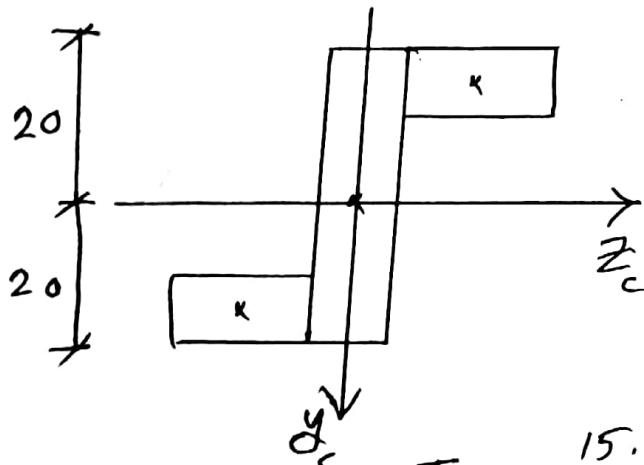
(16)



القطعة	A	Z	Y	$A \cdot Z$	$A \cdot Y$
①	150	7,5	35	1125	5250
②	400	20	20	8000	8000
③	150	32,5	5	4875	750
	700			14000	14000

$$Z_c = \frac{\sum A Z}{\sum A} = \frac{14000}{700} = 20$$

$$Y_c = \frac{\sum A Y}{\sum A} = \frac{14000}{700} = 20$$



القطعة	A	Z	Y
①	150	-12,5	15
②	400	0	0
③	150	12,5	-15

$$I_{Z_1} = \frac{15 \cdot (10)^3}{12} + 150 \cdot (15)^2 = 35000$$

$$I_{Z_2} = \frac{10 \cdot (40)^3}{12} + 400 \cdot (0)^2 = 53333,3 \text{ Cm}^4$$

$$I_{Z_3} = \frac{15 \cdot (10)^3}{12} + 150 \cdot (-15)^2 = 35000$$

$$I_Z = I_{Z_1} + I_{Z_2} + I_{Z_3} = 123333,3 \text{ Cm}^4$$

$$\sigma = \frac{-4 \cdot 10^3}{700} + \frac{8 \cdot 10^5}{123333,3} y$$

نواص نوي

$$\sigma = -5,7 + 6,48 y$$

(46)

(17)

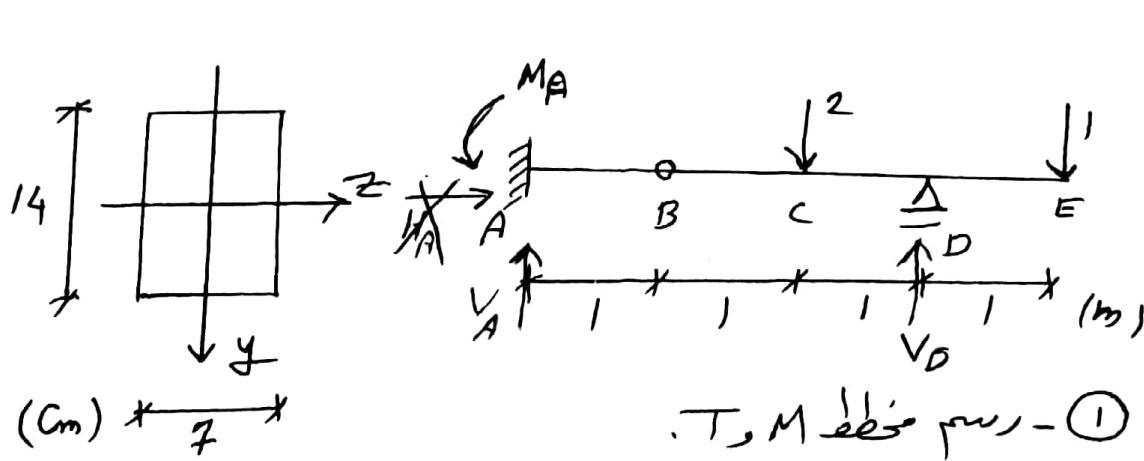
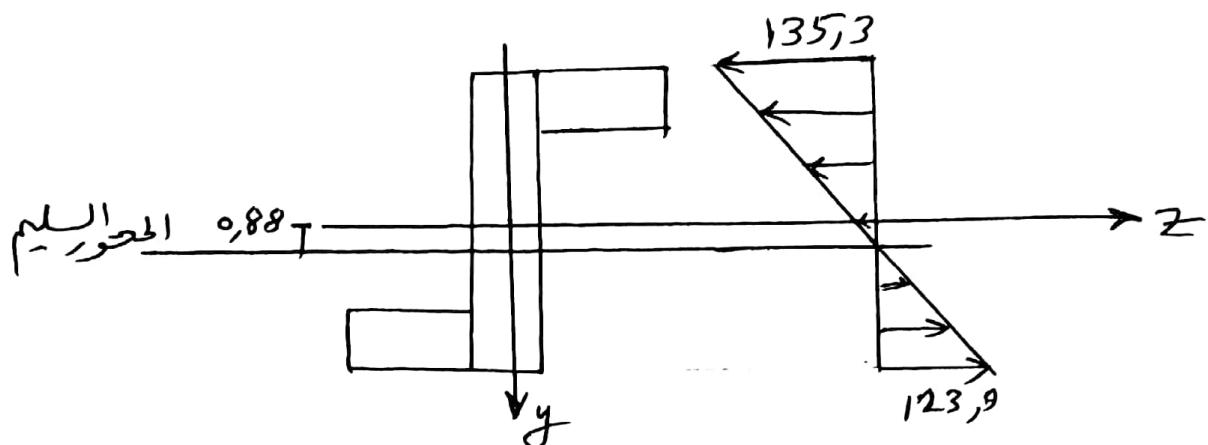
رسم المحور الليم نفع $\sigma = 0$

$$-5,7 + 6,48 y = 0 \Rightarrow y = 0,88$$

معادلة المحور
الليم
وهو مستقيم يوازي Z.

علوي $y = -20 \Rightarrow \sigma = -5,7 + 6,48 \cdot (-20) = -135,3$ kg/cm^2

سفلي $y = +20 \Rightarrow \sigma = -5,7 + 6,48 \cdot (20) = 123,9$ kg/cm^2



المطلوب: ① - رسم مخطط M و T .

..... ② - حساب σ عن العزم الموجب الأقصى

..... ③ - حساب T_{max} عن المسافة D .

④ - 1- حساب ابعاد الصاس في الباقي الذي يقع على مسافة 5 cm
أ أسفل المحور الليم عند المقطع D.

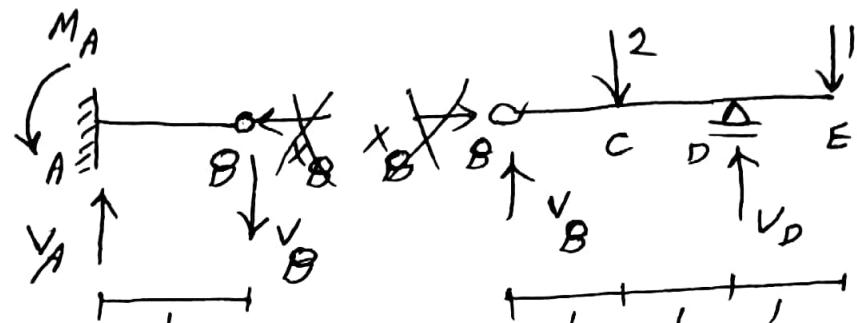
$\frac{\text{أصل}}{\text{ج}} : ① - \text{حساب ردود الأفعال:}$

$$\sum F_x = 0 \Rightarrow H_A = 0$$

نفضل عن المفصل ونضع م亹س مساوية لمسافة 5 cm

(47)

(18)



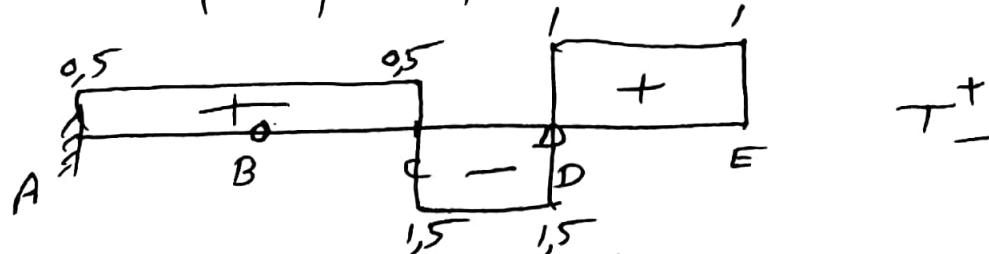
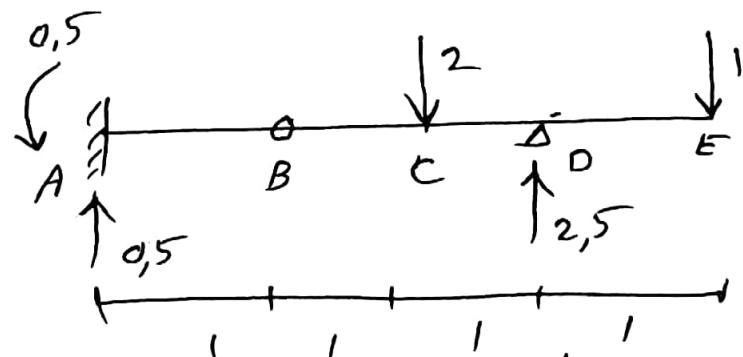
$$\sum F_x = 0 \Rightarrow X_B = 0 \quad : \text{CDE جزء}$$

$$\sum M_B = 0 \Rightarrow 2 \cdot 1 + 1 \cdot 3 - V_D \cdot 2 = 0 \Rightarrow V_D = 2,5$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_B + 2,5 - 2 - 1 = 0 \Rightarrow V_B = 0,5$$

$$\sum M_A = 0 \Rightarrow -M_A + 0,5 \cdot 1 = 0 \Rightarrow M_A = 0,5 \quad : \text{AC سیمی}$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_A - 0,5 = 0 \Rightarrow V_A = 0,5$$



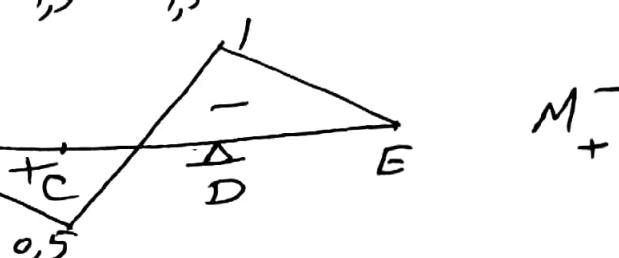
$$M_A = -0,5$$

$$M_B = -0,5 + 0,5 \cdot 1 = 0$$

$$M_C = -0,5 + 0,5 \cdot 2 = 0,5$$

$$M_D = -1 \cdot 1 = -1$$

$$M_E = 0$$

 M_+

(48)

(19)

$$\sigma = \frac{Mz}{I_z} \cdot y$$

لدينا - ②

$$M_{max}^+ = 0,5 t \cdot m \rightarrow M \text{ ليس معروضاً}$$

$$I_z = \frac{\pi \cdot (14)^3}{12} = 1600,67 \text{ cm}^4$$

لدينا I_z

$$\sigma = \frac{0,5 \cdot 10^5}{1600,67} \cdot y \Rightarrow \sigma = 31,2 y$$

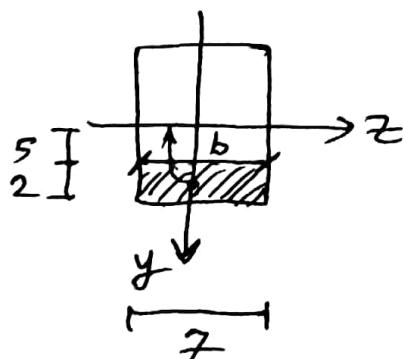
نحو ص

(علوي) $y = -7 \Rightarrow \sigma = 31,2 \cdot (-7) = -218,6 \text{ kg/cm}^2$
 (سفلي) $y = 7 \Rightarrow \sigma = 31,2 \cdot (7) = +218,6 \text{ kg/cm}^2$

نحل اسارة ت بالسقعي طساب $\tau_{max} = \frac{3T}{26H}$ لدینا - ③

$$T_D = -1,5 t \rightarrow T \text{ ليس معروضاً}$$

$$\tau_{max} = \frac{3 \cdot 1,5 \cdot 10^3}{2 \cdot 7 \cdot 14} = 22,9 \text{ kg/cm}^2$$



$$\tau = \frac{T \cdot S_z}{I_z \cdot b}$$

بعد مراعاة المحور z لامنة البروز الآخر

$$S_z = (7 \cdot 2) \left[5 + \frac{1}{2} \cdot 2 \right] = 84 \text{ cm}^3$$

لدينا - ④

$$\tau = \frac{1,5 \cdot 10^3 \cdot 84}{1600,67 \cdot 7} = 11,2 \text{ kg/cm}^2$$

(49)

①

- الفعل -

R هـنـه الـعـلـم

أهمـاـت الـعـصـلـاتـ

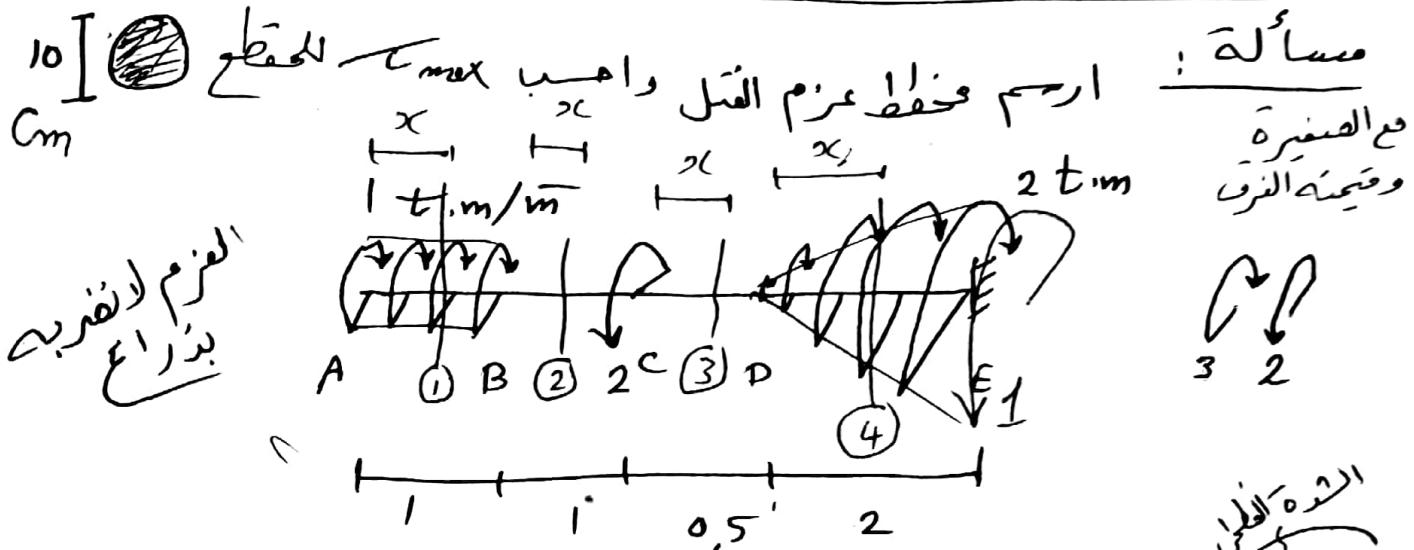
$$T_{max} = \frac{M_t \cdot R}{I_p}$$

$$\frac{I}{P} = \frac{\pi R^4}{2}$$

عزم المـطـالـةـ العـطـبـيـ

$$\frac{I}{P} = \frac{\pi R^4}{2} - \frac{\pi r^4}{2}$$

الـتـرـلـزـ السـيـرـةـ xـ الطـولـ



$$M_{t_1} = +1 \cdot x$$

$$\begin{array}{l} x=0 \quad \boxed{A} \quad M_t = 0 \\ x=1 \quad \boxed{B} \quad M_t = 1 \end{array}$$

$$M_{t_2} = +1 \cdot 1 = \boxed{1} \quad \forall x$$

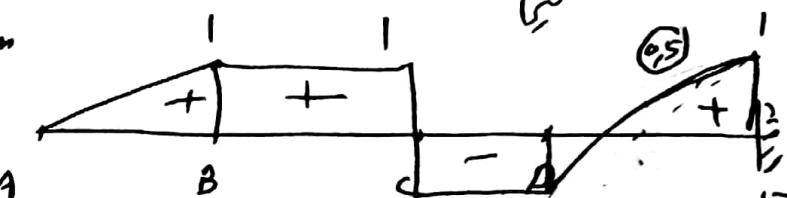
$$M_{t_3} = +1 \cdot 1 - 2 = \boxed{-1} \quad \forall x$$

$$M_{t_4} = 1 \cdot 1 - 2 + \frac{2 \cdot x^2}{2 \cdot 2}$$

$$\begin{array}{l} x=0 \quad \boxed{D} \quad M_t = 1 \\ x=2 \quad \boxed{E} \quad M_t = 1 \end{array}$$



$$\begin{aligned} t \cdot m &\xrightarrow{10^5} \text{kg} \cdot \text{cm} \\ \text{Kw} \cdot \text{m} &\xrightarrow{10^5} \text{N} \cdot \text{cm} \end{aligned}$$



أـلـيـلـ اـسـنـادـ

عـذـبـ الـغـوـرـ

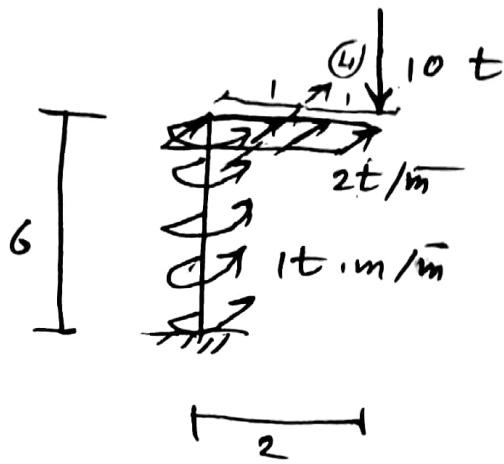
عـذـبـ الفـعـلـ المـتـلـقـ

يـحـكـرـ الـعـلـمـ

$$T_{max} = \frac{M_t \cdot R}{I_p} = \frac{1 \cdot 10^5 \cdot 0.5}{\frac{\pi (5)^4}{2}} = \boxed{509 \text{ kg/cm}^2}$$

(50)

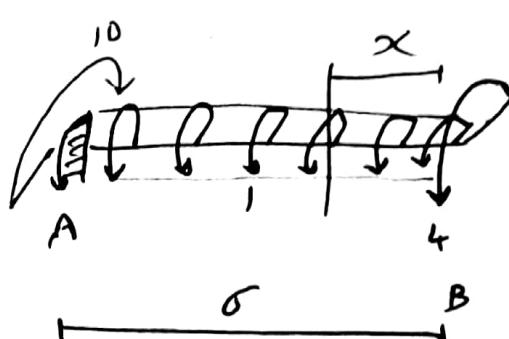
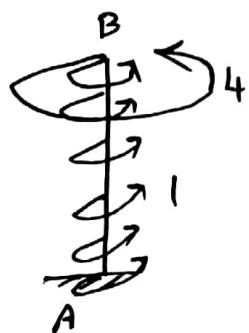
(2)



$$M_t = 1 \text{ متر} : \underline{\text{مُسَلَّه}} \quad \frac{1 \text{ طن}}{2}$$

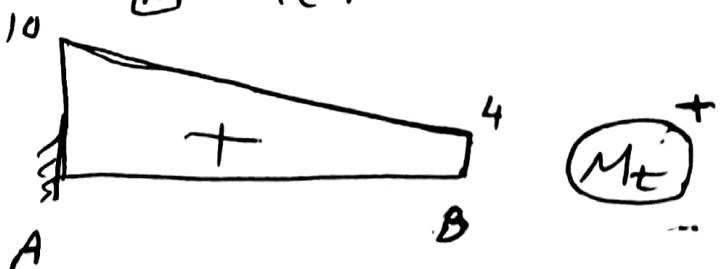
كل حصة توازي عنصر لافعل
ـ ـ ـ تعطى عناصر لافعل
ذراع تركيزه

$$M_t = 4 \cdot 1 = \boxed{4}$$



$$M_t = 4 + 1 \cdot x$$

$$\begin{array}{ll} x=0 & M_t = 4 \\ x=\sigma & M_t = 10 \end{array}$$



طلب اضافي: احسب قطر المقطع على انة يدار للسموع

$$M_t \text{ حول محور} \Rightarrow T_{max} = \frac{M_t \cdot R}{I_p} \Rightarrow 1200 = \frac{10 \cdot 10^5 \cdot R}{\pi R^4 / 2} \Rightarrow R^3 = 424,4 \Rightarrow R = 7,5 \text{ Cm}$$

$$1200 = \frac{10 \cdot 10^5}{\pi R^3} \Rightarrow R^3 = \frac{10 \cdot 10^5}{\pi \cdot 1200} \Rightarrow \boxed{R = 7,5 \text{ Cm}}$$

(51)

(3)

$$r = \frac{1}{2} R$$



طلب اضافي: أعد المقادير اذا كان المقطع بالشكل

$$T_{max} = \frac{M_t \cdot R}{I_p} \Rightarrow 1200 = \frac{10 \cdot 10^5 \cdot R}{\frac{\pi R^4}{2} - \frac{\pi r^4}{2}} \Rightarrow$$

$$1200 = \frac{10 \cdot 10^5 \cdot R}{\frac{\pi}{2} R^4 - \frac{\pi}{2} (\frac{1}{2} R)^4} \Rightarrow 1200 = \frac{10 \cdot 10^5 \cdot R}{\frac{\pi}{2} (1 - \frac{1}{16}) R^4}$$

$$R^3 = \frac{10 \cdot 10^5}{\frac{\pi}{2} (1 - \frac{1}{16}) 1200} \Rightarrow \boxed{R = 7,7 \text{ cm}}$$

$$\boxed{r = 3,85 \text{ cm}}$$

طلب اضافي: احسب نسبة العرض

$$\text{نسبة العرض} = \frac{\text{كبيرة } A - \text{صغيرة } A}{\text{كبيرة } A} \cdot 100$$

$$A = \pi R^2$$

$$\textcircled{1} \quad A = \pi \cdot (7,5)^2 = \boxed{176 \text{ cm}^2}$$

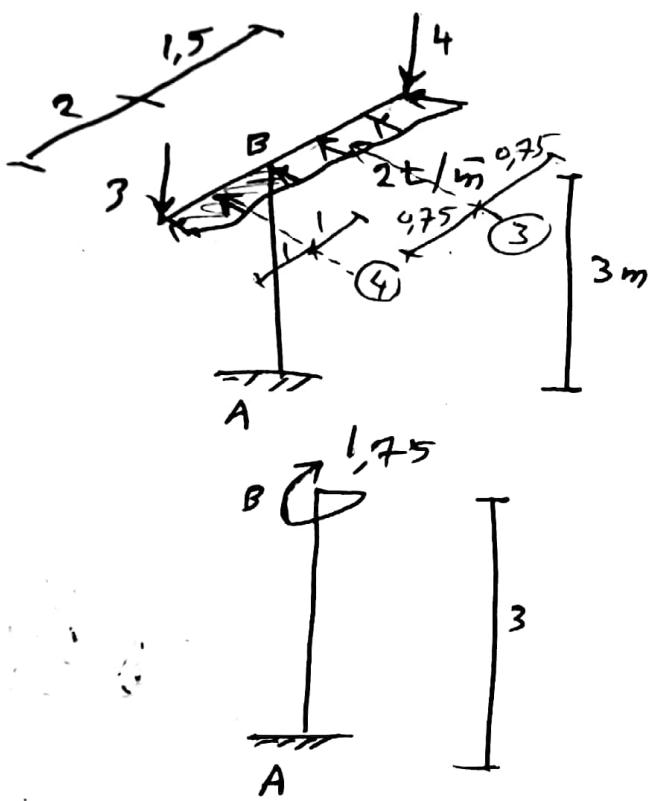
$$\textcircled{2} \quad A = \pi \cdot (7,7)^2 - \pi \cdot (3,85)^2 = \boxed{139 \text{ cm}^2}$$

$$\text{نسبة العرض} = \frac{176 - 139}{176} \cdot 100 = \boxed{21\%}$$



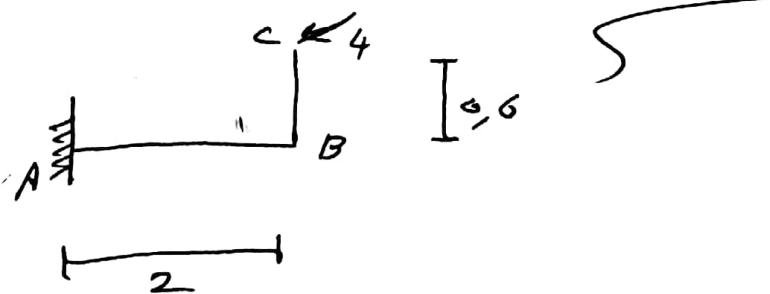
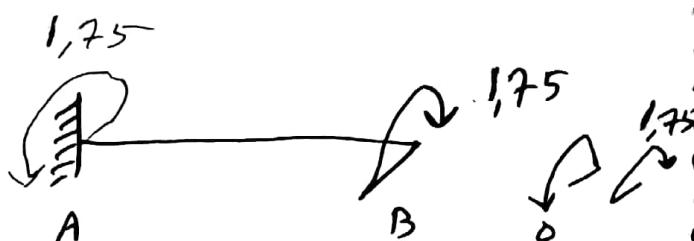
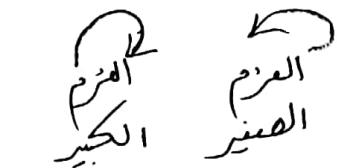
(52)

(4)



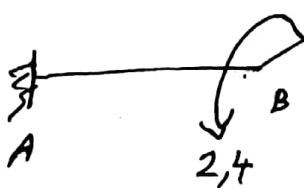
$$M_t = \text{أرجح محصلة} \quad \underline{\text{مسائل}}:$$

$$M_t = 4 \cdot 1 - 3 \cdot 0,75 = \boxed{1,75}$$



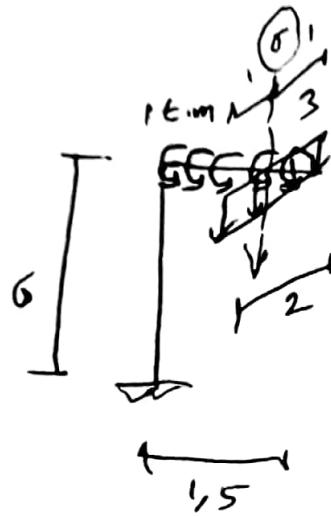
مسائل

$$M_t = 4 \cdot 0,6 = \boxed{2,4}$$

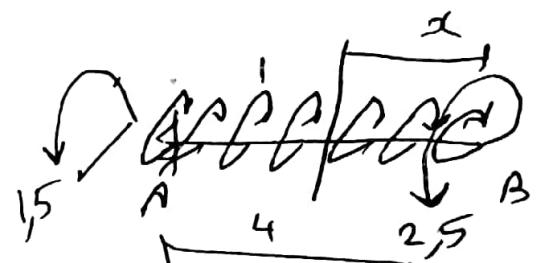
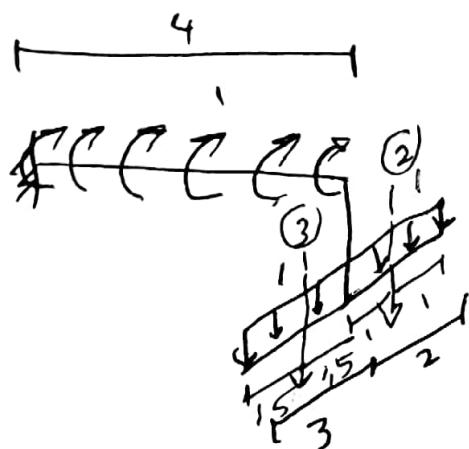
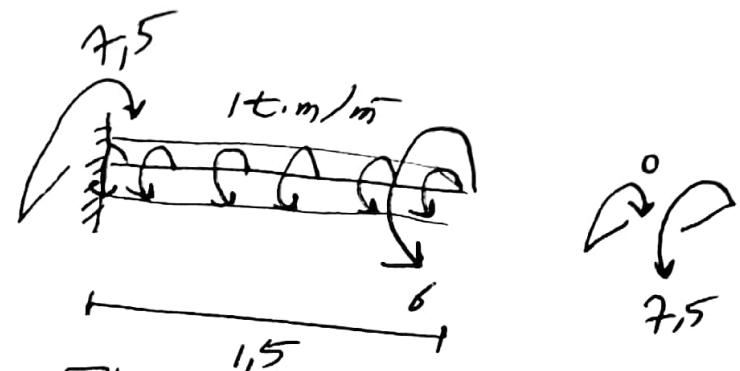


(53)

⑤



$$M_t = 6 \cdot 1 = 6$$

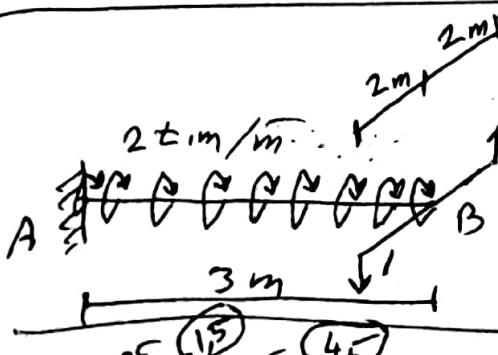
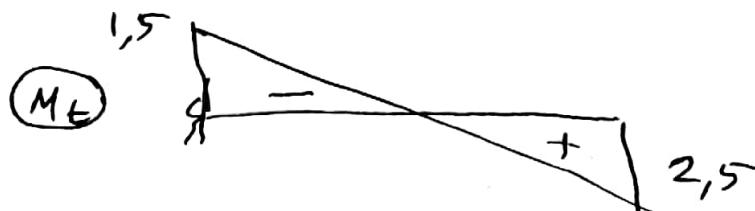


$$M_t = 3 \cdot 1.5 - 2 \cdot 1 = 2.5$$

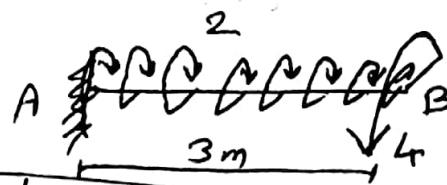
$$M_t = 2.5 - 1 \cdot 2$$

$$x=0 \quad \boxed{B} \quad M_t = 2.5$$

$$x=4 \quad \boxed{A} \quad M_t = -1.5$$



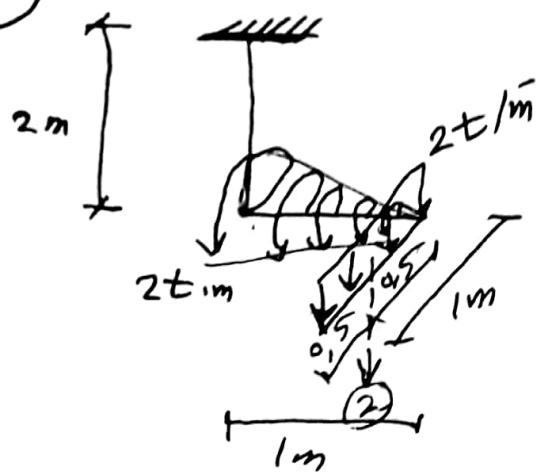
$$M_t = 1 \cdot 2 + 1 \cdot 2 = 4 \text{ t.m}$$



$$\begin{aligned} M_t &= 4.5 \cdot 0.75 - 1.5 \cdot 0.25 \\ M_t &= 3 \text{ t.m} \end{aligned}$$

⑥

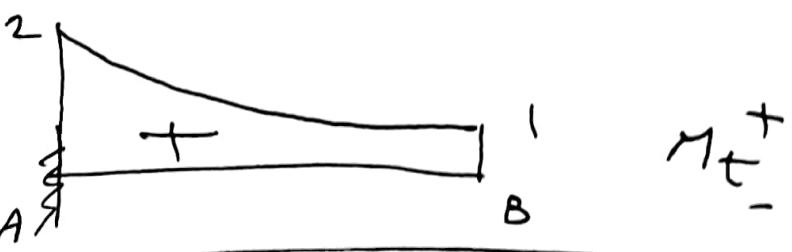
(6)



$$m_t = 1 + \frac{2 \cdot x^2}{2 \cdot 1}$$

$$x=0 \quad B \quad m_t = 1$$

$$x=1 \quad A \quad m_t = 2$$



$$h=30 \quad I \quad \boxed{\square}$$

: t_{\max} case 1 - ②

$\frac{h}{b}$	---	2
K_1		4,068
K_2		0,796

$$K_1 = 4,068$$

$$K_2 = 0,796$$

الشكل مستطيل $\Rightarrow t_{\max} = \frac{h}{b} = \frac{30}{15} = 2$ من الجدول

$$b=15$$

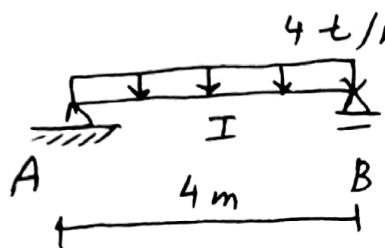


$$t_{\max 1} = K_1 \cdot \frac{M_t}{b^2 \cdot h} = 4,068 \cdot \frac{2 \cdot 10^5}{(15)^2 \cdot (30)} = 120 \text{ kg/cm}^2$$

$$t_{\max 2} = K_2 \cdot t_{\max 1} = 0,796 \cdot 120 = 95,9 \text{ kg/cm}^2$$

(55)

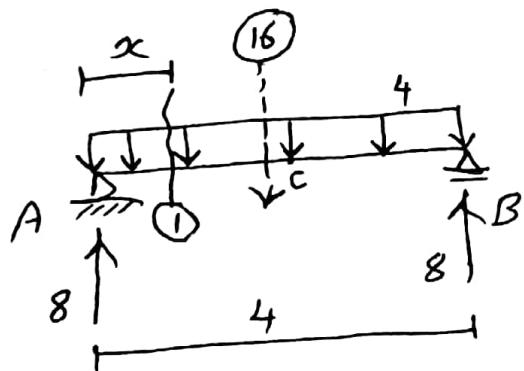
١ دورة $\frac{y}{y=0}$ دورة $\frac{y}{y=0}$ دورة $\frac{y}{y=0}$ مصالحة



أوجد معادلة الدفع والدراجه الجائز ①

حساب الدراجه عن A و B ②

حساب الدفع الذي يحيي من فتحه الجائز ③



الحل: ① - حساب رسم الدفع
ونكتب معادلة الرسم
لكل الحالات:

$$M_1 = 8x - \frac{4x^2}{2}$$

$$EIy'_1 = -M_1$$

$$EIy''_1 = -8x + 2x^2$$

نطبق معادلة الخط المرن

$$EIy'_1 = -8\frac{x^2}{2} + 2\frac{x^3}{3} + C_1 \quad ① \quad \text{تكامل}$$

$$EIy''_1 = -8\frac{x^3}{2 \cdot 3} + 2\frac{x^4}{3 \cdot 4} + C_1x + C_2 \quad ② \quad \text{تكامل ثانية}$$

نحسب C_1 و C_2 من الترسانة:

$$\text{من A: } x_A = 0; y_A = 0 \Rightarrow ② \Rightarrow 0 = 0 + 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$$\text{من B: } x_B = 4; y_B = 0 \Rightarrow ② \Rightarrow 0 = \frac{-8(4)^3}{6} + \frac{2(4)^4}{12} + C_1 \cdot 4 \Rightarrow C_1 = +10,67$$

نوصي C_1 و C_2 في ①، ②

$$EIy'_1 = -4x^2 + 0,67x^3 + 10,67 \quad ① \quad \text{معادلة الدورانات}$$

$$EIy''_1 = -1,33x^3 + 0,167x^4 + 10,67x \quad ② \quad \text{معادلة الدفعات}$$

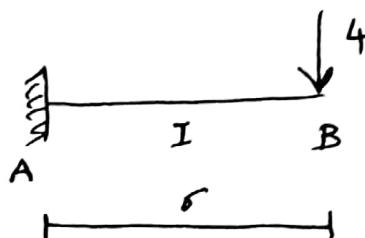
$$x_A = 0 \Rightarrow ① \Rightarrow EIy'_A = 10,67 \Rightarrow y'_A = \frac{10,67}{EI} \quad ②$$

$$x_B = 4 \Rightarrow ① \Rightarrow EIy'_B = -4(4)^2 + 0,67(4)^3 + 10,67 \Rightarrow y'_B = \frac{-10,67}{EI} \quad ②$$

(2)

$$x_c = 2 \Rightarrow (2) \Rightarrow EIy_c = -1,33(2)^3 + 0,167(2)^4 + 10,67/2 \quad - (3)$$

$$\boxed{y_c = \frac{13,37}{EI}} + \downarrow$$



مسألة: أكتب معادلة الدسخال والدوران

$$y_B, y_B'$$

أحل:

حسب ردود الأفعال ونكتب العزم لكل اتجاهات

$$M_A = 24 \quad x$$

$$+ \uparrow \sum F_y = 0 \Rightarrow V_A - 4 = 0 \Rightarrow \boxed{V_A = 4}$$

$$\sum M_A = 0 \Rightarrow -M_A + 4 \cdot 6 = 0 \Rightarrow \boxed{M_A = 24}$$

$$M_1 = 4x - 24$$

لعمد في معادلة المطابق

$$EIy_1'' = -M_1$$

$$EIy_1''' = -4x + 24$$

$$EIy_1' = -4 \frac{x^2}{2} + 24x + C_1 \quad (1)$$

تكامل

$$EIy_1 = -4 \frac{x^3}{2 \cdot 3} + 24 \frac{x^2}{2} + C_1 x + C_2 \quad (2)$$

تكامل ثانية

نجد C_1 و C_2 من الشرط الخبر

$$\text{نهاية } A; x_A = 0; y_A' = 0 \Rightarrow (1) \Rightarrow 0 = 0 + 0 + C_1 \Rightarrow \boxed{C_1 = 0}$$

$$\text{نهاية } A; x_A = 0; y_A = 0 \Rightarrow (2) \Rightarrow 0 = 0 + 0 + 0 + C_2 \Rightarrow \boxed{C_2 = 0}$$

نوصي C_2, C_1 في $(1), (2)$

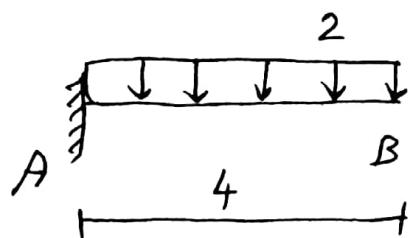
$$\text{معادلة الدوران } \boxed{EIy_1' = -2x^2 + 24x} \quad (1)$$

$$\text{معادلة الدسخال } \boxed{EIy_1 = -0,67x^3 + 12x^2} \quad (2)$$

(57)

$$③ \quad x_B = 6 \Rightarrow ① \Rightarrow EI y'_B = -2(6)^2 + 24(6) \Rightarrow y'_B = \frac{72}{EI} +$$

$$x_B = 6 \Rightarrow ② \Rightarrow EI y_B = -0,67(6)^3 + 12(6)^2 \Rightarrow y_B = \frac{287}{EI} +$$

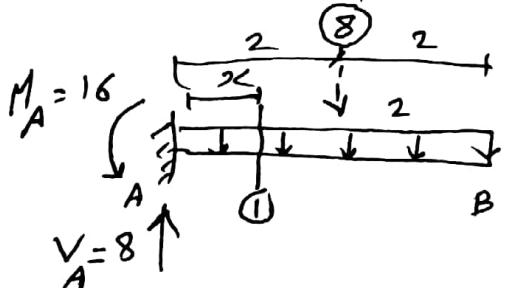


مسأله ③

كتب معادلة الاستئصال والدوران

$$y_B, y'_B$$

اصل: كتب حدود الاتصال ونكتب معادلة الغرض لكل اتجاهات



$$\begin{aligned} \uparrow + \sum F_y &= 0 \Rightarrow V_A - 8 = 0 \Rightarrow V_A = 8 \\ \sum M_A &= 0 \Rightarrow -M_A + 8 \cdot x = 0 \Rightarrow M_A = 16 \end{aligned}$$

$$\begin{aligned} M_1 &= 8 \cdot x - 16 - 2 \frac{x^2}{2} \\ EI y''_1 &= -M_1 \end{aligned}$$

$$EI y'_1 = -8x + 16 + x^2$$

معادلة الحفر المرن

$$EI y'_1 = -8 \frac{x^2}{2} + 16x + \frac{x^3}{3} + c_1 \quad ①$$

تكامل

$$EI y_2 = -8 \frac{x^3}{2 \cdot 3} + 16 \frac{x^2}{2} + \frac{x^4}{3 \cdot 4} + c_1 x + c_2 \quad ②$$

تكامل ثانية

نوبه c_1 و c_2 من التوط الحري

$$\text{وناته } A; x_A = 0; y'_A = 0 \Rightarrow ① \Rightarrow 0 = 0 + 0 + 0 + c_1 \Rightarrow c_1 = 0$$

$$\text{وناته } A; x_A = 0; y_A = 0 \Rightarrow ② \Rightarrow 0 = 0 + 0 + 0 + c_2 \Rightarrow c_2 = 0$$

نوصي c_2, c_1 من

$$EI y'_1 = -4x^2 + 16x + 0,33x^3 \quad ①$$

$$EI y_1 = -1,33x^3 + 8x^2 + 0,0825x^4 \quad ②$$

$$x_B = 4 \Rightarrow ① \Rightarrow EI y'_B = -4(4)^2 + 16(4) + 0,33(4)^3 \Rightarrow$$

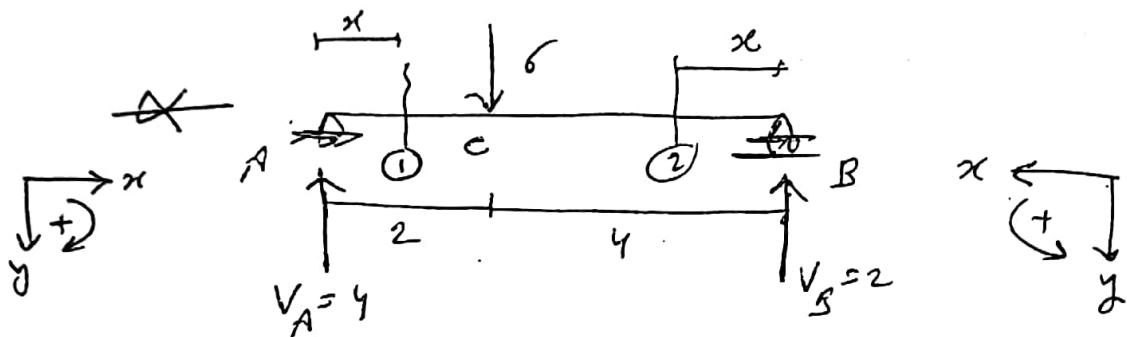
$$y'_B = \frac{21}{EI} +$$

$$x_B = 4 \Rightarrow ② \Rightarrow EI y_B = -1,33(4)^3 + 8(4)^2 + 0,0825(4)^4 \Rightarrow$$

$$y_B = \frac{64}{EI} +$$

٤

مكعب



$$\sum M_A = 0 \Rightarrow 6 \cdot 2 - V_B \cdot 6 = 0 \Rightarrow V_B = 2$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_A + 2 - 6 = 0 \Rightarrow V_A = 4$$

$$M_1 = 4x$$

$$EIy'_1 = -M_1$$

$$EIy''_1 = -4x$$

$$EIy'_1 = -4 \frac{x^2}{2} + C_1 \quad (1)$$

$$EIy_1 = -4 \frac{x^3}{2 \cdot 3} + C_1 x + C_2 \quad (2)$$

A هي المركبة من C1 و C2 كخط مستقيم

و B هي المركبة من C3 و C4 كخط مستقيم

$$M_2 = 2x$$

$$EIy''_2 = -M_2$$

$$EIy'_2 = -2x$$

$$EIy'_2 = -2 \frac{x^2}{2} + C_3 \quad (3)$$

$$EIy_2 = -2 \frac{x^3}{2 \cdot 3} + C_3 x + C_4 \quad (4)$$

B هي المركبة من C3 و C4 كخط مستقيم

و A هي المركبة من C1 و C2 كخط مستقيم

لذلك $\gamma_A = 0$; $y_1 = 0 \Rightarrow (2) \Rightarrow C_2 = 0$

لذلك $\gamma_B = 0$; $y_2 = 0 \Rightarrow (4) \Rightarrow C_4 = 0$

نفترض على النقاد المجالس

$-EIy'_c$	$= EIy'_c$
-----------	------------

(I)

EIy'_c	$= EIy'_c$
----------	------------

(II)

$$x_c = 2 \Rightarrow (1) \Rightarrow EIy'_c = -4 \frac{(2)^2}{2} + C_1$$

$$x_c = 4 \Rightarrow (3) \Rightarrow EIy'_c = -\frac{2(4)^2}{2} + C_3$$

EIy'_c	$= -8 + C_1$
----------	--------------

EIy'_c	$= -16 + C_3$
----------	---------------

نفرض في

59

$$-(-8 + c_1) = (-16 + c_3)$$

$$x_c = 2 \Rightarrow ② \rightarrow EIy_c = -4 \frac{(2)^3}{2 \cdot 3} + c_1 \cdot 2$$

$$x_c = 4 \Rightarrow y_c = -\frac{2(4)^3}{2 \cdot 3} + c_3 \cdot 4$$

$$\underline{\underline{E}} \underline{\underline{I}} \underline{\underline{y}}_c = \underline{\underline{-\frac{16}{3}}} + 2\underline{\underline{C}}_1$$

$$\underline{\underline{E}} I y_c = -\frac{64}{3} + 4 C_3$$

نحوه - ۶

$$\frac{-16}{3} + 2C_1 = \left| \frac{-64}{3} + 4C_3 \right\downarrow \boxed{2C_1 - 4C_3 = 16} \quad (6)$$

$$C_3 = 24 - C_1$$

@ سے
لہٰ مصطفیٰ

$$2c_1 - 4(24 - c_1) = 16$$

$$C_1 = \square$$

جَلْ مَلَكٌ ⑥ - ⑨ صَدِيقٌ

$$C_1 = 18,67$$

$$C_3 = 5,33$$

MODE'S 1

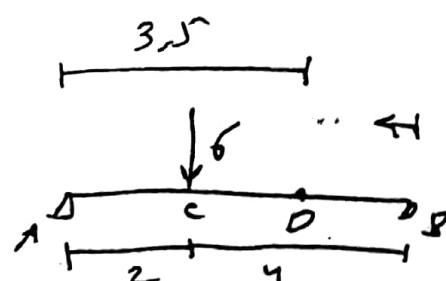
MODE 1

$$AC \quad EIy_1' = -2x^2 + 18,67$$

در راه

$$EIy_1 = -0.667 x^3 + 18.67 x$$

$$EIy_2 = -0,333x^3 + 5,33x \quad (4)$$



②

$$x_B = 6 \Rightarrow ① \Rightarrow$$

$$\underline{\underline{EIy'_B}} = -2,0415(6)^2 + 0,25(6)^3 + 11$$

$$\underline{\underline{EIy'_B}} = -8,494$$

$$x_B = 2 \Rightarrow ③ \Rightarrow$$

$$\underline{\underline{EIy'_B}} = 5 \cdot 2 + C_3$$

$$\underline{\underline{EIy'_B}} = 10 + C_3$$

① جو معادل

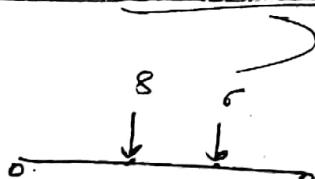
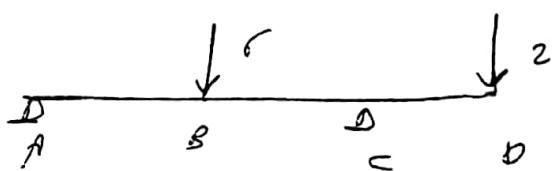
$$- (-8,494) = 10 + C_3 \Rightarrow C_3 = 1,5$$

② جو معادل

$$2(-1,5) + C_1 = -10 \Rightarrow C_1 = -7$$

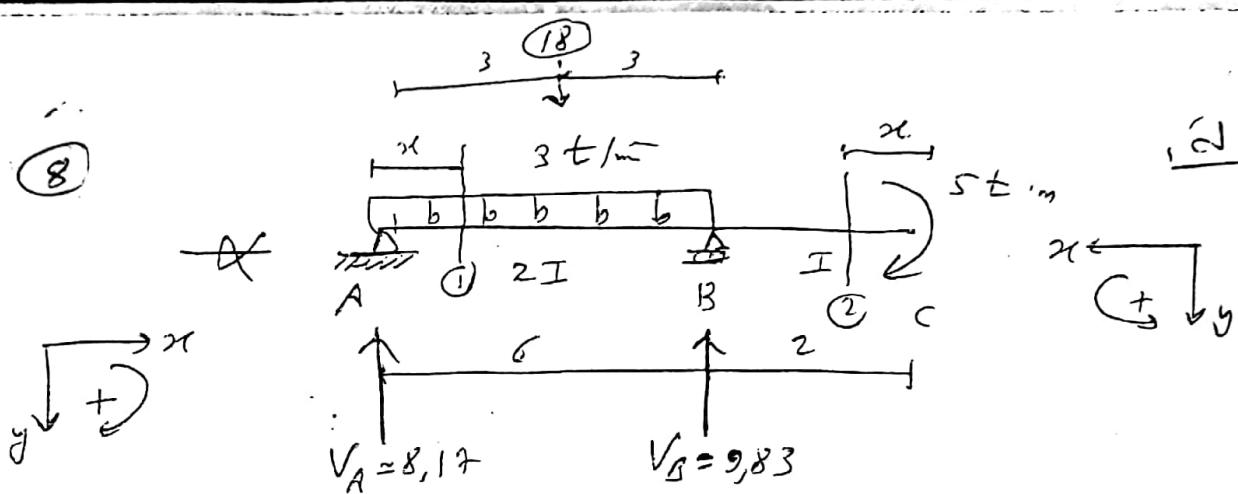
$$EIy'_2 = 5x - 1,5 \quad ③$$

$$EIy'_2 = 2,5x^2 - 1,5x - 7 \quad ④$$



61

(8)



$$\sum M_A = 0 \Rightarrow 18 \cdot 3 - V_B \cdot 6 + 5 = 0 \Rightarrow V_B = 9,83$$

$$+ \sum F_x = 0 \Rightarrow V_A + 9,83 - 18 = 0 \Rightarrow V_A = +8,17$$

$$M_1 = 8,17 \cdot x - 3 \frac{x^2}{2}$$

$$EIy_1'' = -M_1$$

$$EIy_1'' = -8,17x + 1,5x^2$$

$$EIy_1' = -4,083 \frac{x^2}{2} + 0,75 \frac{x^3}{3} + C_1$$

$$EIy_1 = -4,083 \frac{x^3}{2 \cdot 3} + 0,75 \frac{x^4}{3 \cdot 4} + C_1 x + C_2$$

حسب القيمة المطلوبة
نحدد ثوابت C_1 و C_2

$$\text{--- (A)} \quad x_A = 0; y_1 = 0 \Rightarrow C_2 = 0$$

$$\text{--- (B)} \quad x_B = 6; y_1 = 0 \Rightarrow C_2 = 0$$

$$0 = -4,083 \frac{(6)^3}{2 \cdot 3} + 0,75 \frac{(6)^4}{3 \cdot 4} + C_1 \cdot 6$$

أ ب على درجة

$$C_1 = 11$$

$$EIy_1' = -2,045x^2 + 0,75x^3 + 11 \quad (1)$$

$$EIy_1 = -0,68x^3 + 0,0625x^4 + 11x \quad (2)$$

$$M_2 = -5$$

$$EIy_2'' = -M_2$$

$$EIy_2'' = +5$$

$$EIy_2' = 5x + C_3 \quad (3)$$

$$EIy_2 = 5 \frac{x^2}{2} + C_3 x + C_4 \quad (4)$$

نحدد ثوابت C_3 و C_4

$$\text{من (B)} \quad x_B = 2; y_2 = 0 \Rightarrow C_4 = 0$$

$$0 = 5 \frac{(2)^2}{2} + C_3 \cdot 2 + C_4 \Rightarrow$$

$$2C_3 + C_4 = -10 \quad (5)$$

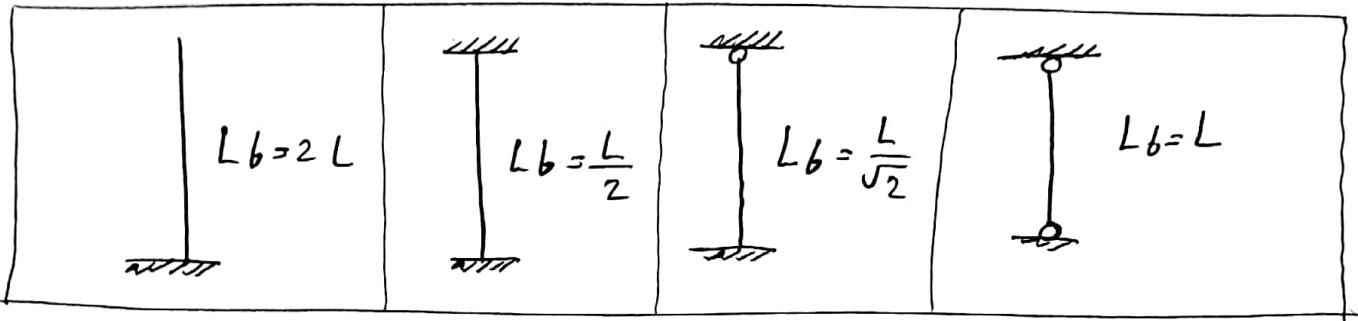
نحدد ثوابت C_3 و C_4

$$-EIy_2' = EIy_1' \quad (6)$$

(62)

١ - التَّحْبِيب -

مُوَلَّ التَّحْبِيب لِلْعُودِ :

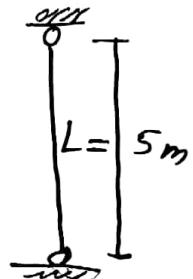
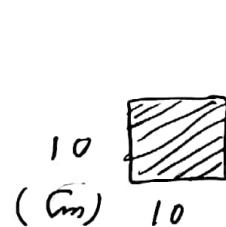


مسأله ١) أوجد العوّة الضربيّة التي يُعَكِّرُ تَحْبِيبَ العُودِ المُتَسْهَلِ
صَطْرُهُ مُشَبِّهُ بِالسَّكَلِ هَذِهِ

عامل المرونة المطابق $E = 2100 \text{ t/cm}^2$

أجلد المرونة $\sigma_y = 2400 \text{ kg/cm}^2$

$m \xrightarrow{*100} \text{cm}$



كل : المحولة التي تسبّب التَّحْبِيب F_{cr}

$$F_{cr} = \frac{\pi^2 E \cdot I}{L_b^2} = \frac{\pi^2 \cdot 2100 \cdot 833,33}{(500)^2} = 69,08 \text{ t}$$

محولة التَّحْبِيب للزنة

$$I = \frac{b H^3}{12} = \frac{10 \cdot (10)^3}{12} = 833,33 \text{ cm}^4$$

كثيف F_y المحولة التي تسبّب التَّلَقُّب $\text{kg} \xrightarrow{*1000} \text{t}$

$$F_y = \sigma_y \cdot A = 2400 \cdot (10 \cdot 10) = 240000 \text{ kg} = 240 \text{ t}$$

محولة التَّلَقُّب

نَفَارِيَّةُ الصُّفَرِ $F_{cr} \approx F_y \approx 69,08 \text{ t}$

العوّة الضربيّة التي يُعَكِّرُ تَحْبِيبَ العُودِ

$$F = 69,08 \text{ t}$$

٢. مساحة: ماحف الطول الذي يعطي لعمود طوله L، مساحة مربع $(10 \times 10) \text{ cm}$
حيث لدى معلم فيه كيس مي إلى المسمى (المالية):

- ممكحل سه صارمه ①
- ممكحل سه صرف وموتوه من العرض الآخر.

$$E = 2100 \text{ t/cm}^2$$

$$\sigma_y = 2400 \text{ kg/cm}^2$$

- ممكحل سه صارمه ②

- ممكحل سه صرف وموتوه من العرض الآخر ③

$$F_{cr} = \frac{\pi^2 E \cdot I}{L_b^2} \quad ①$$

$$I = \frac{b H^3}{12} = \frac{10 \cdot (10)^3}{12} = [833,3 \text{ cm}^4]$$

الكل: $\frac{\text{لغم}}{3}$

~~الآن نحسب المقادير~~

$$F_y = \sigma_y \cdot A$$

وكم

$$F_{cr} = F_y \quad \text{يمثل}$$

$$F_y = \sigma_y \cdot A = 2400 \cdot (10 \cdot 10) = [240000 \text{ kg}] = [240 \text{ t}]$$

① نحوه

$$L_b = L$$

أولاً: ممكحل سه صارمه

$$240 = \frac{\pi^2 \cdot 2100 \cdot 833,3}{L^2} \Rightarrow L = 268,3 \text{ cm}$$

$$L_b = \frac{L}{\sqrt{2}}$$

ثانياً: ممكحل سه صرف

$$240 = \frac{\pi^2 \cdot 2100 \cdot 833,3}{\left(\frac{L}{\sqrt{2}}\right)^2} \Rightarrow L = 379,3 \text{ cm}$$

$$L_b = \frac{L}{2}$$

ثالثاً: ممكحل سه صارمه

$$240 = \frac{\pi^2 \cdot 2100 \cdot 833,3}{\left(\frac{L}{2}\right)^2} \Rightarrow L = 536,3 \text{ cm}$$

64

(3)

مسأله ١ ما هو عول الصنل الأذدي للسعود للربيع الذي صدره $L = 5 \text{ m}$ المتصصل منه العزمينه ويغفرصه طحولة مقدارها $\underline{100 t}$ على تانه.

$$E = 2100 \text{ t/cm}^2 \quad \sigma_y = 2400 \text{ kg/cm}^2$$

$$I = \frac{6H^3}{12} = \frac{a \cdot a^3}{12}$$

$$\boxed{\begin{array}{c} a \\ a \end{array}} \quad F_{cr} = \frac{\pi^2 E \cdot I}{L^2} \Rightarrow 100 = \frac{\pi^2 \cdot 2100 \cdot \frac{a \cdot a^3}{12}}{(500)^2}$$

$$a^4 = \frac{100 \cdot (500)^2}{\pi^2 \cdot 2100} \Rightarrow a^4 = 14474,5 \Rightarrow a = 10,97 \text{ cm}$$

طول الصنل الذي يسبب التكثيف
بمحضه بـ ١٠٠ طن

جذر مربع بالدالة الثالثة

- الدافعه المدار

$$F_y = \sigma_y \cdot A \quad \Rightarrow \quad 100 \cdot 10^3 = 2400 \cdot a \cdot a$$

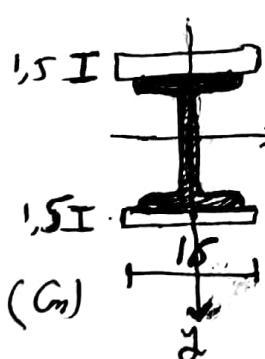


$$a^2 = \frac{100 \cdot 10^3}{2400} \Rightarrow a = 6,45 \text{ cm}$$

$$a = 10,97 \text{ cm}$$

نختار المقطع من ذيل

مسأله ٢ عوول مركب من قطاع I وصفيحتان طاوه سيد



$$\begin{aligned} A &= 53,4 \text{ cm}^2 \\ I_y &= 288 \text{ cm}^4 \end{aligned}$$

(فقطه المقطع I)

$$\sigma_y = 2400 \text{ kg/cm}^2$$

$$E = 2100 \text{ t/cm}^2$$

في الواقع (الحاله):

① - العوول متصصل من العزمين

② - " " متوجوح من العزمين

③ - " " متوجوح ومتصل

④ - العوول متوجوح

الحل: تكثيف أولئك حموله ازدهار المدار

$$F_y = \sigma_y \cdot A$$

$$A = 53,4 + (16 \cdot 1,5) \cdot 2 = 101,4 \text{ cm}^2$$

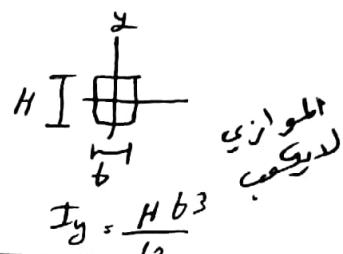
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$$F_y \cdot \sigma_y \cdot A \geq 2400 = \frac{F_y}{101,4} \Rightarrow F_y = [243360 \text{ kg}] = [243,36 t]$$

كتابه معرفة المقاومة بالتحسب ←

$$F_{cr} = \frac{\pi^2 E \cdot I}{L_b^2}$$



$$I = 288 + 2 \cdot \frac{1,5 \cdot (16)^3}{12} = [1312 \text{ cm}^4]$$

$$L_b = L$$

كتابه معرفة المقاومة - ①

$$F_{cr} = \frac{\pi^2 \cdot 2100 \cdot 1312}{(500)^2} = [108,77 t] < F_y = 243,36 t$$

كتابه المقاومة بالتحسب $F_1 = 108,77 t$

$$F_{cr} = \frac{\pi^2 \cdot 2100 \cdot 1312}{\left(\frac{500}{2}\right)^2} = [435,08 t] > F_y = 243,36 t$$

كتابه المقاومة بالتحسب $F_2 = 243,36 t$

$$F_{cr} = \frac{\pi^2 \cdot 2100 \cdot 1312}{\left(\frac{500}{\sqrt{2}}\right)^2} = [217,7 t] < F_y = 243,36 t$$

كتابه المقاومة بالتحسب $F_3 = 217,7 t$

$$F_{cr} = \frac{\pi^2 \cdot 2100 \cdot 1312}{(500 \cdot 2)^2} = [27,18 t] < F_y = 243,36 t$$

كتابه المقاومة بالتحسب $F_4 = 27,18 t$

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الحالة التي يعادل المعدل ①

لتكن $F_y = \sigma_y A$

حيث $F_{ur} = \frac{\pi^2 E I}{L^2 b}$

نهاية الأصفن.

L دليل معندي $F_y = F_{ur}$ ونجد $F_y = \sigma_y A$ حيث الصيغة الأولى ②

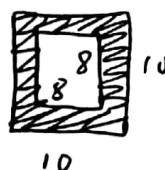
$$F_{ur} = F_y = \frac{\pi^2 E I}{L^2 b}$$

طريق ضلع المربع المربع ③

$$F_y = \frac{F_y}{a^2} \Rightarrow [a = ?]$$

$$F_{ur} = \frac{\pi^2 E \cdot \frac{a \cdot a^3}{12}}{L^2 b} \Rightarrow [d = ?]$$

نهاية ونهاية الأذن.



$$I = \frac{10(10)^3}{12} - \frac{8(8)^3}{12} = \boxed{\quad} \text{ cm}^4$$

$$A = 10 \cdot 10 - 8 \cdot 8 = \boxed{\quad} \text{ cm}^2$$

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