Lab 7b: Box Models and Random Variables

Stat 131A, Fall 2018

Learning Objectives:

- Introduction to Box Models.
- Thinking about drawing tickets from a box.

General Instructions

- Write your solutions in an `Rmd` (R markdown) file.
- Name this file as `lab07b-first-last.Rmd`, where `first` and `last` are your first and last names (e.g. `lab07b-gaston-sanchez.Rmd`).
- Knit your `Rmd` file as an html document (default option).
- Submit your `Rmd` and `html` files to bCourses, in the corresponding lab assignment.

Problem 1

One hundred draws are made at random with replacement from the box `[1  2]`. Forty-seven draws turn out to be `1`, and the remaining fifty-three are `2`. How much is the sum?

Problem 2

One hundred draws are made at random with replacement from a box.

a. If the sum of the draws is 7611, what is their mean?

b. If the mean of the draws is 73.94, what is their sum?

Problem 3

Consider the following three situations given below: i), ii), and iii).

i) A box contains one ticket marked `0` and nine marked `1`. A ticket is drawn at random. If it shows `1` you win a panda bear.

ii) A box contains ten tickets marked `0` and ninety marked `1`. One ticket is drawn at random. If it shows `1` you win the panda.

iii) A box contains one ticket marked `0` and nine marked `1`. Ten draws are made at random with replacement. If the sum of the draws equals 10, you win the panda.

Assume you want the panda.

a. Which is better: (i) or (ii)? Or are they the same?

b. What about (i) and (iii)?

Problem 4

There are two options:

a. A die will be rolled 60 times. Each time it shows an ace or a six, you win \$1; on the other rolls, you win nothing.

b. Consider the box `[1 1 1 0 0 0]`; sixty draws will be made at random with replacement from this box. On each draw, you will be paid the amount shown on the ticket, in dollars.

Which option is better? Or are they the same? Explain briefly.

Problem 5

Three cards are dealt from a well-shuffled deck.

a. Find the chance that all of the cards are diamonds.

b. Find the chance that none of the cards are diamonds.

c. Find the chance that the cards are not all diamonds.

Problem 6

In the game of Monopoly, a player rolls two dice, counts the total number of spots, and moves that many squares. Find the chance that the player moves 11 squares.

Problem 7

Choose a college student at random. Decide whether each of the following is a discrete or continuous random variable:

1. The number of courses the student takes this semester:
2. The student's height:
3. The student's (exact) body temperature:
4. The number of siblings the student has:
5. The (exact) time the student spends doing school work during a week:

6. The number of alcoholic beverages that the student drinks in a typical week:

Problem 8

Match the terms with their definitions:

(a) P(A|B) = P(A and B) / P(B)
(b) P(A or B) = P(A) + P(B) - P(A and B)
(c) P(A|B) = P(A)
(d) P(A) = P(B)
(e) P(A and B) = P(B | A) P(A)
(f) P(A and B) = 0
(g) P(A | B) = P(B|A) P(A) / P(B)

i) equally likely:

ii) mutually exclusive:

iii) conditional probability:

iv) Bayes' rule:

v) multiplication rule:

vi) independence:

vii) addition rule:

Problem 9

A coin is tossed 10 times. True or False, and explain:

a. The chance of getting 10 tails in a row is 1/1024.

b. Given that the first 9 tosses were tails, the chance of getting 10 tails in a row is 1/2.

Problem 10

A bag contains 3 white and 2 black balls; another contains 2 white and 1 black ball.

a. If a bag is chosen at random and then a ball selected from it, what is the probability of a white ball?

b. If a white ball is selected, what is the probability it came from the first bag.

c. If all the balls are combined into one bag and a ball is drawn, what is the probability of getting a white ball?

Problem 11

One hundred draws are made at random with replacement from the box `[1  2  9]`.

a. How small can the sum be?

b. How large can the sum be?

c. The following R code simulates a hundred draws from the given box, and looking at the sum of draws `sum_draws`. Rerun the code several times and see what happens to `sum_draws`:

```
# box with tickets, and number of draws
box = c(1, 2, 9)
num_draws = 100

# 100 draws at random
random_draws = sample(box, size = num_draws, replace = TRUE)
sum_draws = sum(random_draws)
```

d. The following R code simulates 1000 repetitions of a hundred draws from the given box (and looking at the sum of draws). Run the code and check the `summary()` statistics of the 1000 sum of draws, as well as a graph of their distribution.

```
# 1000 repetitions of a hundred draws
num_reps = 1000
sum_draws = rep(0, num_reps)
for (i in 1:num_reps) {
  random_draws = sample(box, size = num_draws, replace = TRUE)
  sum_draws[i] = sum(random_draws)
}

# get a summary(), and make a graph of the vector sum_draws
```

e. About how much you expect the sum to be?

Problem 12

Consider the box `[1  2  2  3  3]`. Four draws are going to be made at random with replacement from this box.

a. Find the chance that `2` is drawn at least once.

b. Let's see what could happen by applying the frequency theory. The code below simulates the given box, four draws with replacement, and counting how many of them are euqal to 2. Rerun the code various times to see what could happen empirically:

```r
box = c(1, 2, 2, 3, 3)
num_draws = 4

# four draws
draws = sample(box, size = num_draws, replace = TRUE)

# number of draws equal to 2
sum(draws == 2)
```

```
## [1] 2
```

c. The following R code simulates 1000 repetitions of four draws from the given box (and looking at the number of twos). Run the code and check the `summary()` statistics, and a graph of vector `count_two`. Find out the proportion of `2` obtained at least once? Does your answer matches the theoretical probability?

```r
# 1000 repetitions of a hundred draws
num_reps = 1000
count_two = rep(0, num_reps)
for (i in 1:num_reps) {
  random_draws = sample(box, size = num_draws, replace = TRUE)
  count_two[i] = sum(random_draws == 2)
}

# get a summary(), and make a graph of the vector count_two
```

Problem 13

Suppose the occurrence of A makes it more likely that B will occur. In that case, show that the occurrence of B makes it more likely that A will occur. That is, show that if

$$P(B|A) > P(B)$$

then it is also true that

$$P(A|B) > P(A)$$