Lab 8a: Expectation and Variance

Stat 131A, Fall 2018

Learning Objectives:

- Expected Value.
- Variance.

General Instructions

- Write your solutions in an `Rmd` (R markdown) file.
- Name this file as `lab08a-first-last.Rmd`, where `first` and `last` are your first and last names (e.g. `lab08a-gaston-sanchez.Rmd`).
- Knit your `Rmd` file as an html document (default option).
- Submit your `Rmd` and `html` files to bCourses, in the corresponding lab assignment.

About Expectation and Variance

Here are some definitions and properties that you may find useful while working on this lab assignment.

Expectation: The *expectation* (also called *expected value*, or *mean*) of a random variable X, is the mean of the distribution of X, denoted $E(X)$. That is:

$$E(X) = \sum_{i=1}^{n} x_i \, P(X = x_i)$$

the average of *all possible values of X, weighted by their probabilities.*

Addition Rule for Expectation: For any two random variables X and Y defined in the same setting,

$$E(X + Y) = E(X) + E(Y)$$

no matter whether X and Y are independent or not. Consequently, for a sequence of random variables $X_1, X_2, \dots, X_n$, however independent,

$$E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n)$$

Constants: The expectation of a constant random variable is its constant value:

$$E(c) = c$$

Scaling and Shifting: For constants a and b,

$$E(aX + b) = aE(X) + b$$

Variance: The variance of a random variable X, denoted $Var(X)$, is the mean squared deviation of X form its expected value $\mu = E(X)$:

$$Var(X) = E\left[(X - \mu)^2\right]$$

Standard Deviation: The standard deviation of X, denoted $SD(X)$, is the square root of the variance of X:

$$SD(X) = \sqrt{Var(X)}$$

Computational Formula: The variance is the mean of the square minus the square of the mean, that is:

$$Var(X) = E(X^2) - [E(X)]^2$$

Scaling and Shifting: For constants a and b,

$$Var(aX + b) = a^2\, Var(X)$$

Addition rule for Variances: If X and Y are independent then:

$$Var(X + Y) = Var(X) + Var(Y)$$

If $X_1, X_2, \ldots, X_n$ are independent, then:

$$Var(X_1 + X_2 + \cdots + X_n) = Var(X_1) + Var(X_2) + \cdots + Var(X_n)$$

Square Root Law: Let S_n be the sum, and $\bar{X}_n = S_n/n$ the average, of n independent random variables $X_1, \ldots, X_n$, each with the same distribution as X. Then:

$$E(S_n) = nE(X), \quad SD(S_n) = \sqrt{n}\, SD(X), \quad E(\bar{X}_n) = E(X), \quad SD(\bar{X}_n) = \frac{SD(X)}{\sqrt{n}}$$

Problem 1

Suppose that 10% of the numbers in a list are 15, 20% of the numbers are 25, and the remaining numbers are 50. What is the average of the numbers in the list?

Problem 2

Let X be the number of heads in three tosses of a fair coin.

a. Display the probability distribution of X.

b. Find the Expected Value of X

c. Find the Variance of X

Problem 3

Consider the random variable X of the previous question.

a. Find the probability distribution of $Y = |X - 1|$

b. Find the Expected Value of Y

Problem 4

It costs one dollar to buy a lottery ticket, which has five prizes. The prizes and the probability that a player wins the prize are listed in the following table. Calculate the expected value of the payoff.

Prize	Probability
1 million	1/10 million
200,000	1/1 million
50,000	1/500,000
10,000	1/50,000
1,000	1/10,000

Problem 5

You have been given the choice of receiving \$500 in cash or receiving a gold coin that has a face value of \$100. However, the actual value of the gold coin depends on its gold content. You are told that the coin has a 40% probability of being worth \$400, a 30% probability of being worth \$900, and a 30% probability of being worth its face value. Basing your decision on expected value, should you choose the coin?

Problem 6

A small taxi company has 4 taxis. In a month's time, each taxi will get 0 traffic tickets with probability 0.3, 1 traffic ticket with probability 0.5, or 2 traffic tickets with probability 0.2.

a. What is the expected number of tickets per month amassed by the fleet of 4 taxis?

b. What is the standard deviation of the number of tickets?

Problem 7

A designer makes a profit of $30 on each item that is produced in perfect condition, and suffers a loss of $6 on each item that is produced in less-than-perfect condition. Fillin the blanks:

If each item produced is in perfect condition with probability 0.4, the designer should expect a profit per item of about ______, give or take ______ or so.

Problem 8

Show that if $E(X) = m$, and $Var(X) = s^2$, then for every constant a:

$$E\left[(X-a)^2\right] = s^2 + (m-a)^2$$

Problem 9

One hundred draws are made at random with replacement from the box:

`[1   1   2   3]`

The draws come out as follows: 45 `1`'s, 23 `2`'s, and 32 `3`'s. Find the value for each phrase.

a. the observed value for the sum of the draws is:

b. the observed value for the number of `3`'s is:

c. the observed value for the number of `1`'s is:

d. the expected value for the sum of the draws is:

e. the expected value for the number of `3`'s is:

f. the expected value for the number of `1`'s is:

g. the standard deviation of the sum of the draws is:

h. the standard deviation for the number of `1`'s is:

Problem 10

One hundered draws are going to be made at random with replacement from the box:

`[1 2 3 4 5 6 7]`

a. Find the expected value and standard deviation for the sum.

b. The sum of the draws will be around ________, give or take ________ or so.

c. Suppose you had to guess what the sum was going to be. What would you guess? Would you expect it to be off by 2, 4, or 20?

Problem 11

The expected value for a sum is 50, with an SD of 5. The chance process generating the sum is repeated 10 times. Which is the sequence of observed values?

a. 51, 57, 48, 52, 57, 61, 58, 41, 53, 48

b. 51, 49, 50, 52, 48, 47 53, 50, 49, 47

c. 45, 50, 55, 45, 50, 55, 45, 50, 55, 45