Expected Value and Standard Error

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What's happening

Chapters 3-5	Chapters 17-18
• Data • (list of numbers) • Histogram • Average • Standard Deviation • Normal Curve	

What's happening

Chapters 3-5	Chapters 17-18
• Data	• Chance processes
• (list of numbers)	• (Sum of draws)
• Histogram	• Probability histograms
• Average	• Expected Value
• Standard Deviation	• Standard Error
• Normal Curve	• Normal approximation

Box Model Reminder

A chance problem is like drawing (with replacement) from a box with numbered tickets and looking at the sum of the draws

Example

Roll a die 5 times, and add up the points

1 2 3 4 5 6

5 draws

Example

Toss a coin 5 times, and count # heads

0 1

0 = tails 1 = heads

```
T H T T H
0 1 0 0 1
```

sum(Heads) = 2

Making a Box Model

What numbers go in the box?

What is the quantity of interest?

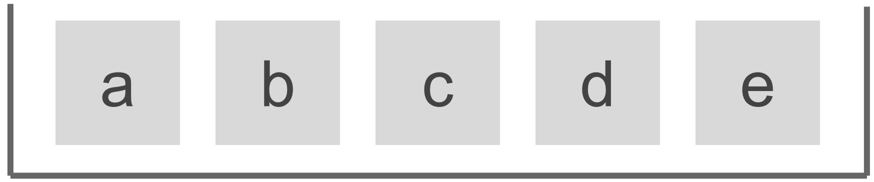
What could happen to that quantity on each draw?

How many tickets of each number?

What are the chances for each draw?

How many draws?

Example



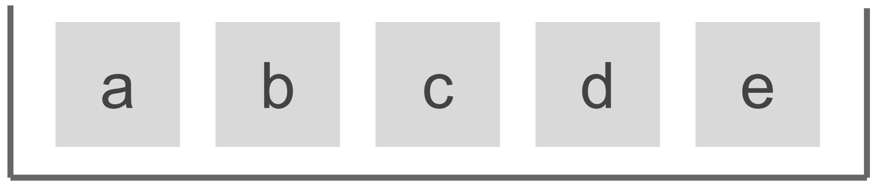
Theoretical Probability of selecting ticket “a”

P(ticket “a”) = ⅕ = 20%

Draw a ticket (with replacement) a large # of times

As the draws increase, |%a - 20%| gets close to zero

Example



c, e, a, e, c, c, d, c, a, a, d, d, c, b, b, e, c, b, e, a, e, e, e, e, a, e, d, c, d, a, b, d, b, e, b, b, d, c, d, a, b, a, a, b, a, a, c, e

Average of the list should be close to

(a + b + c + d + e) / 5 = Expected Value

Expected Value

Chapter 17

Expected Value:

is intuitively the

long run average
































Expected Value

What proportion of heads should you expect in 100 tosses?

In 100 tosses, we should expect **50%** heads

Although there is going to be chance variability

Expected Value for ***sum of draws*** from a box model is:

(# of draws) x (avg of box)

Example

Toss a coin 5 times, and count # heads

0 = tails | 0 | 1 | 1 = heads

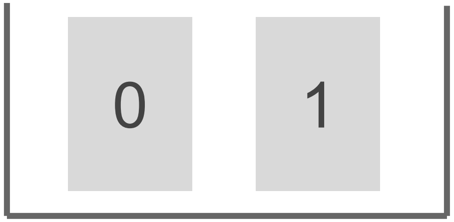
# of draws = **5**

Average of box = (0 + 1) / 2 = **0.5**

EV = (5) x (0.5) = 2.5

Example

Toss a coin 100 times, and count # heads



100 draws

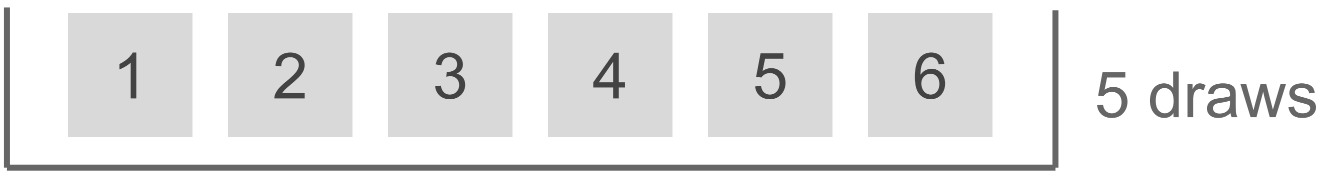
\# of draws = **100**

Average of box = (0 + 1) / 2 = **0.5**

EV = (100) x (0.5) = 50

EV example

Roll a die 5 times, and add up the points



# of draws = **5**

Average of box = (1+2+3+4+5+6) / 6 = **3.5**

EV = (5) x (3.5) = 17.5

EV does not have to be one of the possible values

(it's more a theoretical value)

Standard Error

Chapter 17

Standard Error:

Measures the typical size of the chance error

How far off we expect to be from the expected value

$$SE = \sqrt{\#\ \text{draws}}\ (\text{SD of box})$$

SE example

Roll a die 5 times, and add up the points

1	2	3	4	5	6

5 draws

# of draws = 5

SD of box = 1.67

SE = $\sqrt{5}$ x (1.67) = 3.741

SE example

Toss a coin 5 times, and count # heads

0 = tails | 0 | 1 | 1 = heads

# of draws = 5

SD box = 0.5

SE = $\sqrt{5}$ x (0.5) = 1.11

Discussion about the Standard Error formula

$$SE = \sqrt{\#\ \text{draws}}\ (\text{SD of box})$$

As we increase the number of draws,
the SE becomes larger

Discussion about the Standard Error formula

$$SE = \sqrt{\#\ \text{draws}}\ (\text{SD of box})$$

The more draws, the larger the std error

How spread out the stuff of the box is

SE is a measure of the likely size of the chance error

Standard Error

In 100 tosses, we should expect 50 heads

but there will be variability (chance error)

We expect to get 50 heads, give or take 5 (between 45-55 heads)

Kerrich's Coin Tosses

# tosses	# heads	Expected Value	Chance error	Standard Error
10	4			
50	25			
100	44			
500	255			
1000	502			
5000	2533			
10000	5067			

Kerrich's Coin Tosses

# tosses	# heads	Expected Value	Chance error	Standard Error
10	4	5		
50	25	25		
100	44	50		
500	255	250		
1000	502	500		
5000	2533	2500		
10000	5067	5000		

Kerrich's Coin Tosses

# tosses	# heads	Expected Value	Chance error	Standard Error
10	4	5	-1	
50	25	25	0	
100	44	50	-6	
500	255	250	5	
1000	502	500	2	
5000	2533	2500	33	
10000	5067	5000	67	

Kerrich's Coin Tosses

# tosses	# heads	Expected Value	Chance error	Standard Error
10	4	5	-1	1.58
50	25	25	0	3.53
100	44	50	-6	5
500	255	250	5	11.1
1000	502	500	2	15.81
5000	2533	2500	33	35.55
10000	5067	5000	67	50

SD shortcut formula

Chapter 17: SD of box Shortcut

When a box has only two different numbers ("big" and "small"), the SD can be computed as:

$$\left[\text{big number} - \text{small number}\right] \sqrt{\text{fraction with big number} \times \text{fraction with small number}}$$

Toss a coin 5 times, and count # of heads

0	1

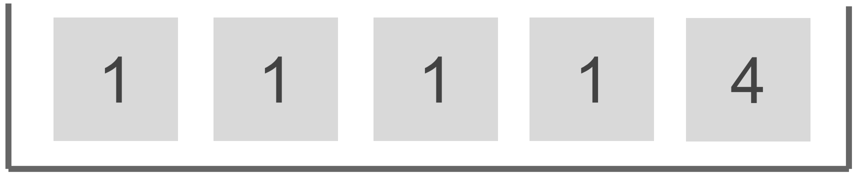
5 draws

$$SD = (1 - 0)\sqrt{\tfrac{1}{2} \times \tfrac{1}{2}}$$

$$SD = (1)\sqrt{\tfrac{1}{4}}$$

$$SD = (1) \times (\tfrac{1}{2}) = 0.5$$

Example 2



100 draws

$$SD = (4 - 1)\sqrt{\tfrac{1}{5} \times \tfrac{4}{5}}$$

$$SD = (3)\sqrt{4/25}$$

$$SD = (3) \times \left(\tfrac{2}{5}\right) = 1.2$$

Roulette Example

Roulette example

Bet $1 on red. 100 bets.

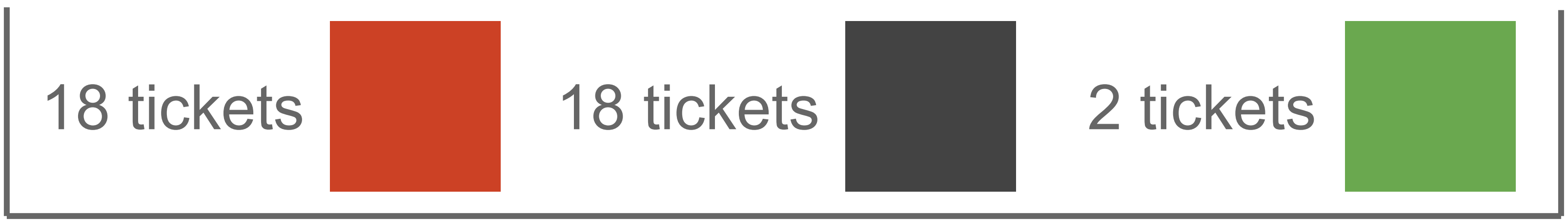
38 slots: 18 red, 18 black, 2 green.

If ball lands on red you win $1,

Otherwise you lose $1

Roulette example

38 slots: 18 red, 18 black, 2 green.



draws = 100

Not the right box model

Roulette example

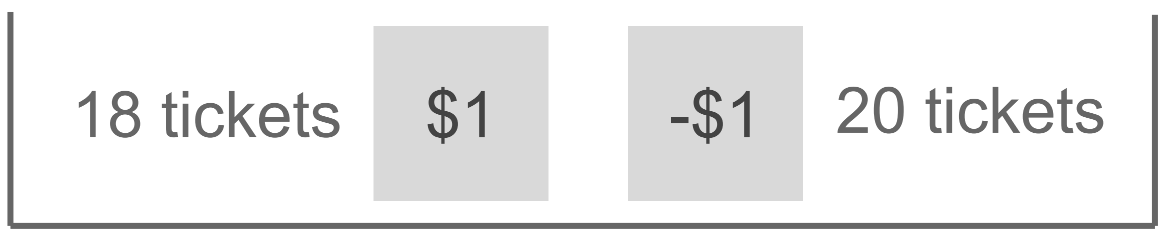
What quantity are we interested in?

How much money I win or lose

Roulette example

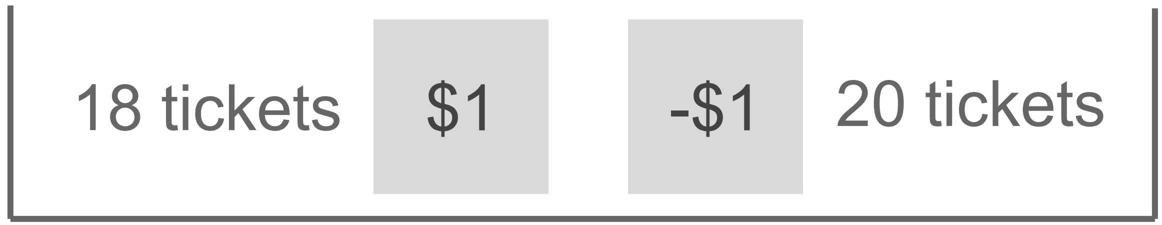
Making a box model

38 slots: 18 red, 18 black, 2 green.



draws = 100

Roulette example



draws = 100

Avg of box = {18 (1) + 20 (-1)} / 38 = -0.0526

EV = (Avg of box) (# draws) = (-0.0526) (100) = - 5.26

If we gamble 100 times, we expect to lose $5.26

Roulette example

18 tickets | $1 | -$1 | 20 tickets

draws = 100

SD of box = (1 - (-1)) sqrt((18/38) (20/38)) = 1

SE = (SD of box) sqrt(# draws) = 10

If we gamble 100 times, we expect to lose $5.26,
give or take $10

Roulette Gain demo

Classifying and Counting

Classifying and Counting

If counting # of times something happens,

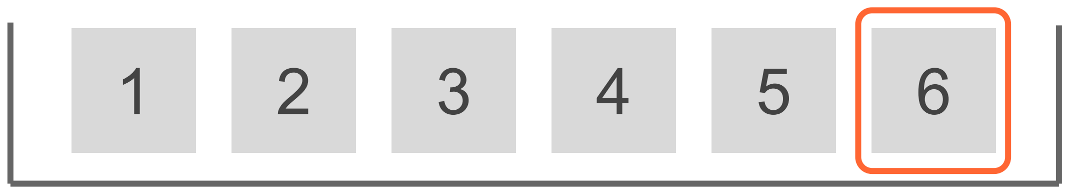
make a box with **1's** for what you are counting,

and **0's** for everything else

Classifying and Counting

Roll a fair die 100 times, and count the number of sixes

This box model is good for **sum of draws**

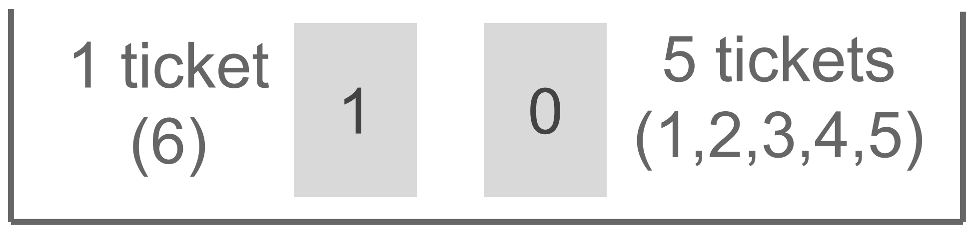


100 draws

But we are interested just in the 6

Classifying and Counting

Roll a fair die 100 times, and count the number of sixes



draws = 100

Classifying and Counting

1 ticket
(6)

1

0

5 tickets
(1,2,3,4,5)

draws = 100

Avg of box = {1 (1) + 5 (0)} / 6 = 1/6

EV = (Avg of box) (# draws) = (1/6) (100) = 16.66

If we roll a die 100 times, we expect get 16.66 sixes

Classifying and Counting

1 ticket (6) | 1 | 0 | 5 tickets (1,2,3,4,5)

draws = 100

SD of box = (1 - 0) sqrt((1/6) (5/6)) = 0.372

SE = (SD of box) sqrt(# draws) = (0.372) (10) = 3.72

If we roll a die 100 times, we expect to get 16.66 sixes, give or take 3.72

Kinds of Boxes

Kinds of Boxes

1) Numbers in box (die rolls, tickets)
2) Gambling (net gain, dollar amounts)
3) Classifying and Counting