Lab 13a: One Sample t-test

Stat 131A, Fall 2018

Learning Objectives:

- One sample t-test

Problem 1

True or False and explain.

a) The P-value of a test is the chance that the null hypothesis is true.

b) If a result is statistically significant, there are only 5 chances in 100 for it to be due to chance, and 95 chances in 100 for it to be real.

Problem 2

The National Assessment of Educational Progress (NAEP) gives a math assessment every year to 12th graders in the U.S. On the math test, a score above 275 indicates that a student has the skills to balance a checkbook. For a random sample of 500 young men, the mean NAEP math score is 272 with a standard deviation of 78. Do we have evidence to support that claim that young men nationwide have a mean score below 275?

The null and alternative hypotheses are $H_0 : \mu = 275$, $H_a : \mu < 275$. The level of significance is 5%.

The t-test statistic is -0.86 with a P-value of 0.20. What is the correct conclusion?

a. We do not have enough evidence to conclude that the mean score is less than 275 for young men nationwide.

b. It is likely that these 500 young men have a mean score less than 275.

c. The evidence suggests that young men nationwide have a mean score less than 275.

d. The evidence suggests that young men nationwide have a mean score equal to 275.

Problem 3

Engineers on the Bay Bridge are measuring tower rods to find out if any rods have been corroded from salt water. There are rods on the east and west sides of the bridge span. One engineer plans to measure the length of an eastern rod 25 times and then calculate the average of the 25 measurements to estimate the true length of the eastern rod. A different

engineer plans to measure the length of a western rod 20 times and then calculate the average of the 20 measurements to estimate the true length of the western rod.

Suppose the engineer who collected 25 measurements for the eastern rod has a mean length of 23.9 feet with a standard deviation of 0.5 feet. The critical T-value for a 90% (one-sample) confidence interval with df = 24 is T=1.71.

Which of the following is the resulting 90% confidence interval?

a. (23.73, 24.07)

b. (23.05, 24.76)

c. (23.80, 24.00)

Problem 4

Commute times in the U.S. are heavily skewed to the right. We select a random sample of 500 people from the 2000 U.S. Census who reported a non-zero commute time. In this sample the mean commute time is 27.6 minutes with a standard deviation of 19.6 minutes. Can we conclude from this data that the mean commute time in the U.S. is less than half an hour?

Conduct a hypothesis test at the 5% level of significance. What can we conclude?

a. Nothing. The distribution of the variable in the population is heavily skewed, so the conditions for use of a t-model are not met. We cannot trust that the p-value is accurate for this reason.

b. With a mean of 27.6 minutes, the data supports the claim that the average commute time is less than 30 minutes, but the difference is not statistically significant. We fail to reject the null hypothesis that the mean commute time in the U.S. in the year 2000 was 30 minutes.

c. With a mean of 27.6 minutes, the data supports the claim that the average commute time is significantly less than 30 minutes. We reject the null hypothesis that the mean commute time in the U.S. in the year 2000 was 30 minutes.

Problem 5

Use R to find the area under Student's curve with 5 degrees of freedom:

a) to the right of 2.02

b) to the left of -2.02

c) between -2.02 and 2.02

d) to the left of 2.02

Problem 6

Many studies have been conducted to determine the concentration of CO (carbon monoxide) near freeways with various conditions of traffic flow. The basic technique involves measuring CO concentrattions with a spectrophotometer. These machines are quite delicate and have to be calibrated every day. If the machine reads close to 70 ppm on the span gas, it's ready for use; if not, it has to be adjusted.

Each data set below represents some readings on span gas. Assume the Gaussian model, with errors following the normal curve. In each case, make a t-test to see whether the instrument is properly calibrated or not.

a) 71, 68, 79

b) 71, 68, 79, 84, 78, 85, 69

c) 71

d) 71, 84

Problem 7

A new spectrophotometer is being calibrated. It is not clear whether the errors follow the normal curve, or even whether the Gauss model applies. In two cases, these assumptions should be rejected. Which two, and why? The numbers replicate measurements on span gas.

a) 71, 70, 72, 69, 71, 68, 93, 75, 68, 61, 74, 67

b) 71, 73, 69, 74, 65, 67, 71, 69, 70, 75, 71, 68

c) 71, 69, 71, 69, 71, 69, 71, 69, 71, 69, 71, 69

Problem 8

A long series of measurements on a checkweight averages out to 253 micrograms above ten grams, and the SD is 7 micrograms. The Gauss model is believed to apply, with negligible bias. At this point, the balance has to be rebuilt, which may introduce bias as well as changing the SD of the error box. Ten measurements on the checkweight, using the rebuilt scale, show an average of 245 micrograms above ten grams, and the SD sample is 9 micrograms. Has bias been introduced? Or is this chance variation? (You may assume that the errors follow the normal curve; use a 5% significance level.)

Problem 9

A study of the GPAs of students at UC Berkeley found that the average GPA was a 3.0 with an SD of 0.3. You wish to test whether students in Stat 2 have a larger average GPA. You

take a simple random sample of 10 students from the Stat 2 class and find that the average was 3.1 with an SD of 0.4. Test the hypothesis at the 5% significance level.

a. Identify the appropriate hypothesis test to use.

b. State the null and alternative hypotheses.

c. Compute the test statistic.

d. What do you conclude from this test?