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Effective Decision Making: Optimization Simulation and Statistical Methods

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Types of models

	Model Characteristics		
Category	**Form of $f(\cdot)$**	**Values of Independent Variables**	**Management Science Techniques**
Predictive Models	unknown, ill-defined	known or under decision maker's control	Regression Analysis, Time Series Analysis, Discriminant Analysis
Descriptive Models	known, well-defined	unknown or uncertain	Simulation, Queuing, PERT, Inventory Models
Prescriptive Models	known, well-defined	known or under decision maker's control	Linear Programming, Networks, Integer Programming, CPM, Goal Programming, EOQ, Nonlinear Programming

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LP

The problem is usually expressed in matrix form and then it becomes:

$$\begin{aligned} \text{Maximize} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \end{aligned}$$

where A is a $m \times n$ matrix.

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$$
\begin{array}{lrcrcr}
\text{MAX:} & 350X_1 & + & 300X_2 & & \\
\text{Subject to:} & 1X_1 & + & 1X_2 & \leq & 200 \\
 & 9X_1 & + & 6X_2 & \leq & 1{,}566 \\
 & 12X_1 & + & 16X_2 & \leq & 2{,}880 \\
 & 1X_1 & & & \geq & 0 \\
 & & & 1X_2 & \geq & 0
\end{array}
$$

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INTEGER AND BINARY PROGRAMMING

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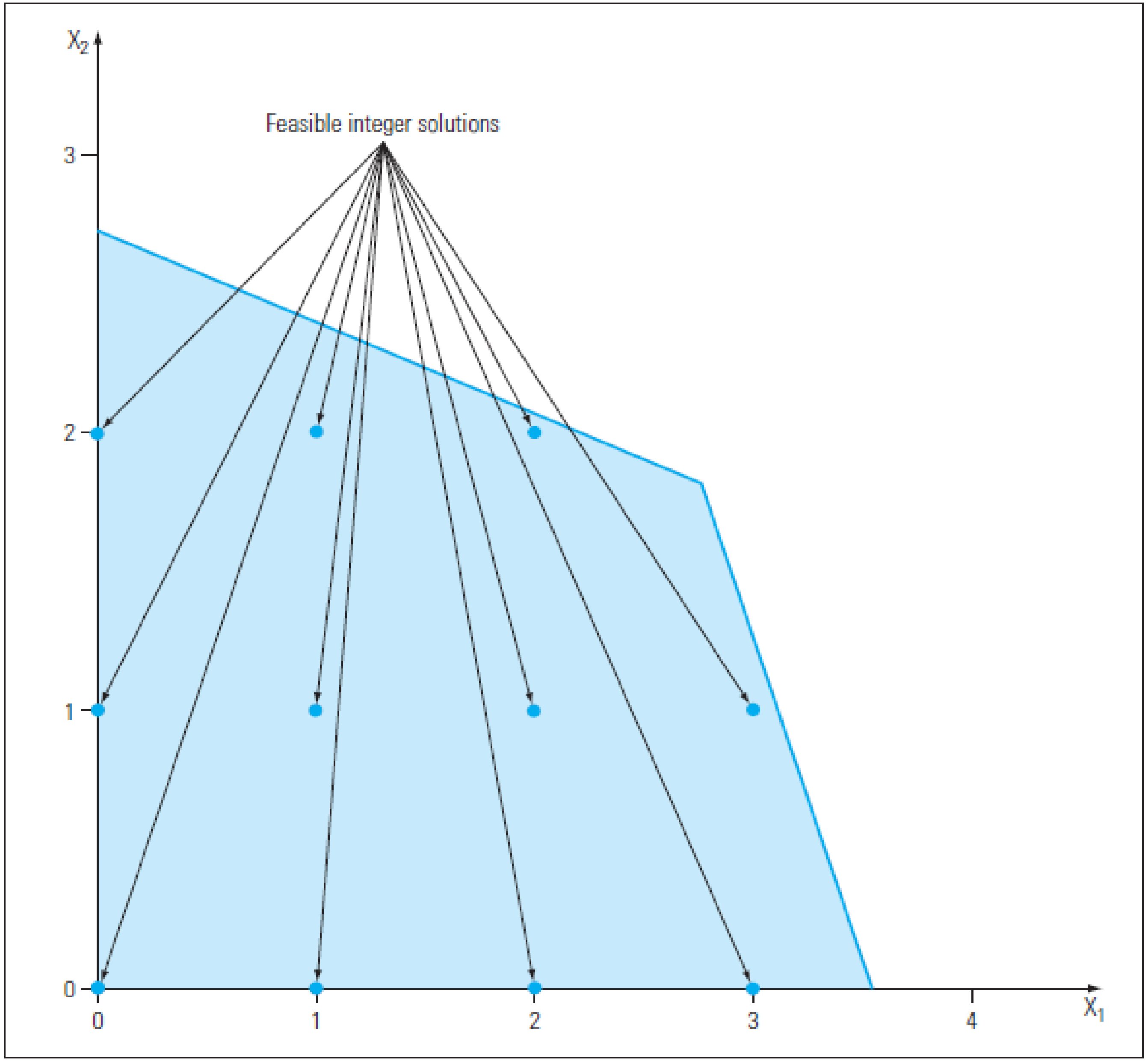

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LET US SOLVE SOME PROBLEMS

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Capital budget allocation

- In his position as vice president of research and development (R&D) for CRT Technologies, Mark Schwartz is responsible for evaluating and choosing which R&D projects to support. The company received 18 R&D proposals from its scientists and engineers, and identified six projects as being consistent with the company’s mission.

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- However, the company does not have the funds available to undertake all six projects. Mark must determine which of the projects to select. The funding requirements for each project are summarized in the following table along with the NPV the company expects each project to generate.

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Project	Expected NPV (in $1,000s)	Capital (in $1,000s) Required in Year 1	Year 2	Year 3	Year 4	Year 5
1	$141	$ 75	$25	$20	$15	$10
2	$187	$ 90	$35	$ 0	$ 0	$30
3	$121	$ 60	$15	$15	$15	$15
4	$ 83	$ 30	$20	$10	$ 5	$ 5
5	$265	$100	$25	$20	$20	$20
6	$127	$ 50	$20	$10	$30	$40

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- The company currently has $250,000 available to invest in new projects. It has budgeted $75,000 for continued support for these projects in year 2 and $50,000 per year for years 3, 4, and 5. Surplus funds in any year are re-appropriated for other uses within the company and may not be carried over to future years.

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$$
\begin{array}{ll}
\text{MAX:} & 141X_1 + 187X_2 + 121X_3 + 83X_4 + 265X_5 + 127X_6 \\
\text{Subject to:} & 75X_1 + 90X_2 + 60X_3 + 30X_4 + 100X_5 + 50X_6 \le 250 \\
& 25X_1 + 35X_2 + 15X_3 + 20X_4 + 25X_5 + 20X_6 \le 75 \\
& 20X_1 + 0X_2 + 15X_3 + 10X_4 + 20X_5 + 10X_6 \le 50 \\
& 15X_1 + 0X_2 + 15X_3 + 5X_4 + 20X_5 + 30X_6 \le 50 \\
& 10X_1 + 30X_2 + 15X_3 + 5X_4 + 20X_5 + 40X_6 \le 50 \\
& \text{All } X_i \text{ must be binary}
\end{array}
$$

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Assignment problem

- *Air-Express is an express shipping service that guarantees overnight delivery of packages anywhere in the continental United States. The company has various operations centers, called hubs, at airports in major cities across the country. Packages are received at hubs from other locations and then shipped to intermediate hubs or to their final destinations.*

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- *The manager of the Air-Express hub in Baltimore, Maryland, is concerned about labor costs at the hub and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles each day varies from one day to the next. Using historical data on the average number of packages received each day, the manager estimates the number of workers needed to handle the packages as:*

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Day of Week	Workers Required
Sunday	18
Monday	27
Tuesday	22
Wednesday	26
Thursday	25
Friday	21
Saturday	19

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- The package handlers working for Air-Express are unionized and are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $655 per week. Because most workers prefer to have Saturday or Sunday off, the union has negotiated bonuses of $25 per day for its members who work on these days. The possible shifts and salaries for package handlers are:

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Shift	Days Off	Wage
1	Sunday and Monday	$680
2	Monday and Tuesday	$705
3	Tuesday and Wednesday	$705
4	Wednesday and Thursday	$705
5	Thursday and Friday	$705
6	Friday and Saturday	$680
7	Saturday and Sunday	$655

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- The manager wants to keep the total wage expense for the hub as low as possible. With this in mind, how many package handlers should be assigned to each shift if the manager wants to have a sufficient number of workers available each day?

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Decision variables

- X1 _ the number of workers assigned to shift 1
- X2 _ the number of workers assigned to shift 2
- X3 _ the number of workers assigned to shift 3
- X4 _ the number of workers assigned to shift 4
- X5 _ the number of workers assigned to shift 5
- X6 _ the number of workers assigned to shift 6
- X7 _ the number of workers assigned to shift 7

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Objective and constraints

The LP model for the Air-Express scheduling problem is summarized as:

MIN: $680X_1 + 705X_2 + 705X_3 + 705X_4 + 705X_5 + 680X_6 + 655X_7$ } total wage expense

Subject to:

$0X_1 + 1X_2 + 1X_3 + 1X_4 + 1X_5 + 1X_6 + 0X_7 \geq 18$ } workers required on Sunday

$0X_1 + 0X_2 + 1X_3 + 1X_4 + 1X_5 + 1X_6 + 1X_7 \geq 27$ } workers required on Monday

$1X_1 + 0X_2 + 0X_3 + 1X_4 + 1X_5 + 1X_6 + 1X_7 \geq 22$ } workers required on Tuesday

$1X_1 + 1X_2 + 0X_3 + 0X_4 + 1X_5 + 1X_6 + 1X_7 \geq 26$ } workers required on Wednesday

$1X_1 + 1X_2 + 1X_3 + 0X_4 + 0X_5 + 1X_6 + 1X_7 \geq 25$ } workers required on Thursday

$1X_1 + 1X_2 + 1X_3 + 1X_4 + 0X_5 + 0X_6 + 1X_7 \geq 21$ } workers required on Friday

$1X_1 + 1X_2 + 1X_3 + 1X_4 + 1X_5 + 0X_6 + 0X_7 \geq 19$ } workers required on Saturday

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 0$

All X_i must be integers

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Product mix problems

- National Petroleum produces two types of unleaded gasoline: regular and premium. It sells these at Rs. 600 and 800 per barrel. These are blended from their internal domestic oil and foreign oil and must meet the following constraints

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	Maximum vapor pressure	Minimum octane rating	Maximum demand (barrels/wk)	Minimum deliverables (barrels/wk)
Regular	23	88	100,000	50,000
Premium	23	93	20,000	5000

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	vapor pressure	Octane rating	Inventory(barrels)	Cost(barrels)
Domestic	27	87	40,000	400
Foreign	15	98	60,000	500

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Decision variables

- Let us say we use d1 barrels of domestic for regular oil and d2 barrels for premium oil. Similarly, we use f1 barrels of foreign for regular and f2 barrels for premium.

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Objective function

- Domestic oil consumed = d1+d2
- Foreign oil consumed: f1+f2
- Regular blended: d1+f1
- Premium blended: d2+f2

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- Costs
  - The cost of domestic oil: (d1+d2)400;
  - The cost of foreign oil: 500(f1+f2)
- Price
  - The price of regular oil: 600(d1+f1);
  - The price of premium oil: 800(d2+f2)

- The profit: total price - total cost = 600(d1+f1) + 800(d2+f2) − [400(d1+d2)+500(f1+f2)]

200d1+100f1+400d2+300f2

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- Amount of domestic oil consumed: d1+d2

- Inventory available: 40,000 barrels
  - $d1+d2 \leq 40{,}000$
  - $f1+f2 \leq 60{,}000$

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- Amount of regular produced: d1+f1
- The maximum demand: 100,000 and minimum deliverables = 50000 barrels

$$50{,}000 \leq d1+f1 \leq 100{,}000$$

$$5000 \leq d2+f2 \leq 20{,}000$$

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- Vapor pressure is based on the weight fractions

- Vapor pressure of d1+f1 of regular = (weight fraction of domestic)* vapor pressure of domestic + (weight fraction of foreign)*vapor pressure of foreign

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- Vapor pressure of regular =

$$\frac{d1}{d1+f1}(Vapor\ pressure\ of\ domestic) + \frac{f1}{d1+f1}\ (vapor\ pressure\ of\ foreign)$$

$$\boldsymbol{\frac{d1}{d1+f1}(27) + \frac{f1}{d1+f1}\ (15) \leq 23,}$$

27d1+15f1 = 23d1+23f1 -> 4d1-8f1 $\leq$0

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- $\frac{d2}{d2+f2}(27) + \frac{f2}{d2+f2}(15) = 23$

27d2+15f2 ≤ 23d2+23f2 ->4d2-8f2≤0

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- Extending the same logic to octane rating

$$\frac{d1}{d1+f1}(87)+\frac{f1}{d1+f1}(98)\geq 88$$

$$-d1+10f1\geq 0$$

- $\frac{d2}{d2+f2}(87)+\frac{f2}{d2+f2}(98)=93$

$$-6d2+5f2\geq 0$$

- Hidden constraints: d1, d2, f1, f2 ≥0

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Assignment

- A 400-meter medley relay involves four different swimmers, who successively swim 100 meters of the backstroke, breaststroke, butterfly and freestyle. A coach has six very fast swimmers whose expected times (in seconds) in the individual events are given in following

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Assignment

	Event 1 (backstroke)	Event 2 (breaststroke)	Event 3 (butterfly)	Event 4 (freestyle)
Swimmer 1	65	73	63	57
Swimmer 2	67	70	65	58
Swimmer 3	68	72	69	55
Swimmer 4	67	75	70	59
Swimmer 5	71	69	75	57
Swimmer 6	69	71	66	59

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Transportation Problem

- Tropicsun currently has 275,000 bags of citrus at Mt. Dora, 400,000 bags at Eustis, and 300,000 bags at Clermont. Tropicsun has citrus processing plants in Ocala, Orlando, and Leesburg with processing capacities to handle 200,000, 600,000, and 225,000 bags, respectively.

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- Tropicsun contracts with a local trucking company to transport its fruit from the groves to the processing plants. The trucking company charges a flat rate for every mile that each bushel of fruit must be transported. Each mile a bushel of fruit travels is known as a bushel-mile. The following table summarizes the distances (in miles) between the groves and processing plants:

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	Distances (in miles) Between Groves and Plants		
Grove	Ocala	Orlando	Leesburg
Mt. Dora	21	50	40
Eustis	35	30	22
Clermont	55	20	25

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$$
\begin{aligned}
\text{MIN:}\quad & \left.\begin{aligned} & 21X_{14} + 50X_{15} + 40X_{16} + \\ & 35X_{24} + 30X_{25} + 22X_{26} + \\ & 55X_{34} + 20X_{35} + 25X_{36} \end{aligned}\right\} && \text{total distance fruit is shipped (in bushel-miles)} \\
\text{Subject to:}\quad & X_{14} + X_{24} + X_{34} \le 200{,}000 && \} \text{ capacity restriction for Ocala} \\
& X_{15} + X_{25} + X_{35} \le 600{,}000 && \} \text{ capacity restriction for Orlando} \\
& X_{16} + X_{26} + X_{36} \le 225{,}000 && \} \text{ capacity restriction for Leesburg} \\
& X_{14} + X_{15} + X_{16} = 275{,}000 && \} \text{ supply available at Mt. Dora} \\
& X_{24} + X_{25} + X_{26} = 400{,}000 && \} \text{ supply available at Eustis} \\
& X_{34} + X_{35} + X_{36} = 300{,}000 && \} \text{ supply available at Clermont} \\
& X_{ij} \ge 0, \text{ for all } i \text{ and } j && \} \text{ nonnegativity conditions}
\end{aligned}
$$

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Method 2

You start with the cost matrix as above but add dummy source or receiver to ensure that demand = supply

	Ocala	Orlando	Leesburg	Supply available
Mt. Dora	21	50	40	275000
Eustis	35	30	22	400000
Clermont	55	20	25	300000
Dummy	0	0	0	50000
Capacities	200000	600000	225000	

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Production scheduling

- An industrial firm must plan for each of the four seasons over the next year. The company’s production capacities and the expected demands (all in units) are as follows:

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	Spring	Summer	Fall	Winter
Demand	250	100	400	500
Regular Capacity	200	300	350	---
Overtime Capacity	100	50	100	150

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- Regular production costs for the firm are $7.00 per unit. The unit cost of overtime varies seasonally being $8.00 in spring and fall, $9.00 in summer and $10.00 in winter.

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- The company has 200 units of inventory on January 1, but as it plans to discontinue the product at the end of the year, it wants no inventory after the winter season. Units produced on regular shifts are not available for shipment during the season of production; generally, they are sold during the following season.

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- Those that are not are added to inventory and carried forward at a cost of $0.70 per unit per season. In contrast, units produced on overtime shifts must be shipped in the same season as produced. Determine a production schedule that meets all demands at minimum total cost.

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Costs						Suppl y
From/To	Spring	Summer	Fall	Winter	Dummy	
RegSpr	10000	7	7.7	8.4	0	200
RegSum	10000	10000	7	7.7	0	300
RegFall	10000	10000	10000	7	0	350
Initial	0	0.7	1.4	2.1	10000	200
OTSpr	8	10000	10000	10000	0	100
OTSum	10000	9	10000	10000	0	50
OTFall	10000	10000	8	10000	0	100
OTWinter	10000	10000	10000	10	0	150
Demand	250	100	400	500	200	

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DATA ENVELOPMENT ANALYSIS

Efficiency

- What is the efficiency of a unit?

$$\text{efficiency} = \frac{\text{output}}{\text{input}}$$

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Efficiency

When there are multiple outputs and multiple inputs

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Data envelopment analysis

- In DEA, there are a number of producers. The production process for each producer is to take a set of inputs and produce a set of outputs. Each producer has a varying level of inputs and gives a varying level of outputs.

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- A fundamental assumption behind this method is that if a given producer, A, is capable of producing Y(A) units of output with X(A) inputs, then other producers should also be able to do the same if they were to operate efficiently. Similarly, if producer B is capable of producing Y(B) units of output with X(B) inputs, then other producers should also be capable of the same production schedule.

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- Producers A, B, and others can then be combined to form a composite producer with composite inputs and composite outputs. Since this composite producer does not necessarily exist, it is typically called a virtual producer.

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- The heart of the analysis lies in finding the "best" virtual producer for each real producer. If the virtual producer is better than the original producer by either making more output with the same input or making the same output with less input then the original producer is inefficient. The subtleties of DEA are introduced in the various ways that producers A and B can be scaled up or down and combined.

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Simple example

- Consider a set of banks. Each bank has a certain number of tellers, a certain square footage of space, and a certain number of managers (the inputs). There are a number of measures of the output of a bank, including number of checks cashed, number of loan applications processed, and so on (the outputs). DEA attempts to determine which of the banks are most efficient, and to point out specific inefficiencies of the other banks.

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Data Envelopment Analysis

- Bank A: 10 tellers, 1000 checks, 20 loan applications
- Bank B: 10 tellers, 400 checks, 50 loan applications
- Bank C: 10 tellers, 200 checks, 150 loan applications

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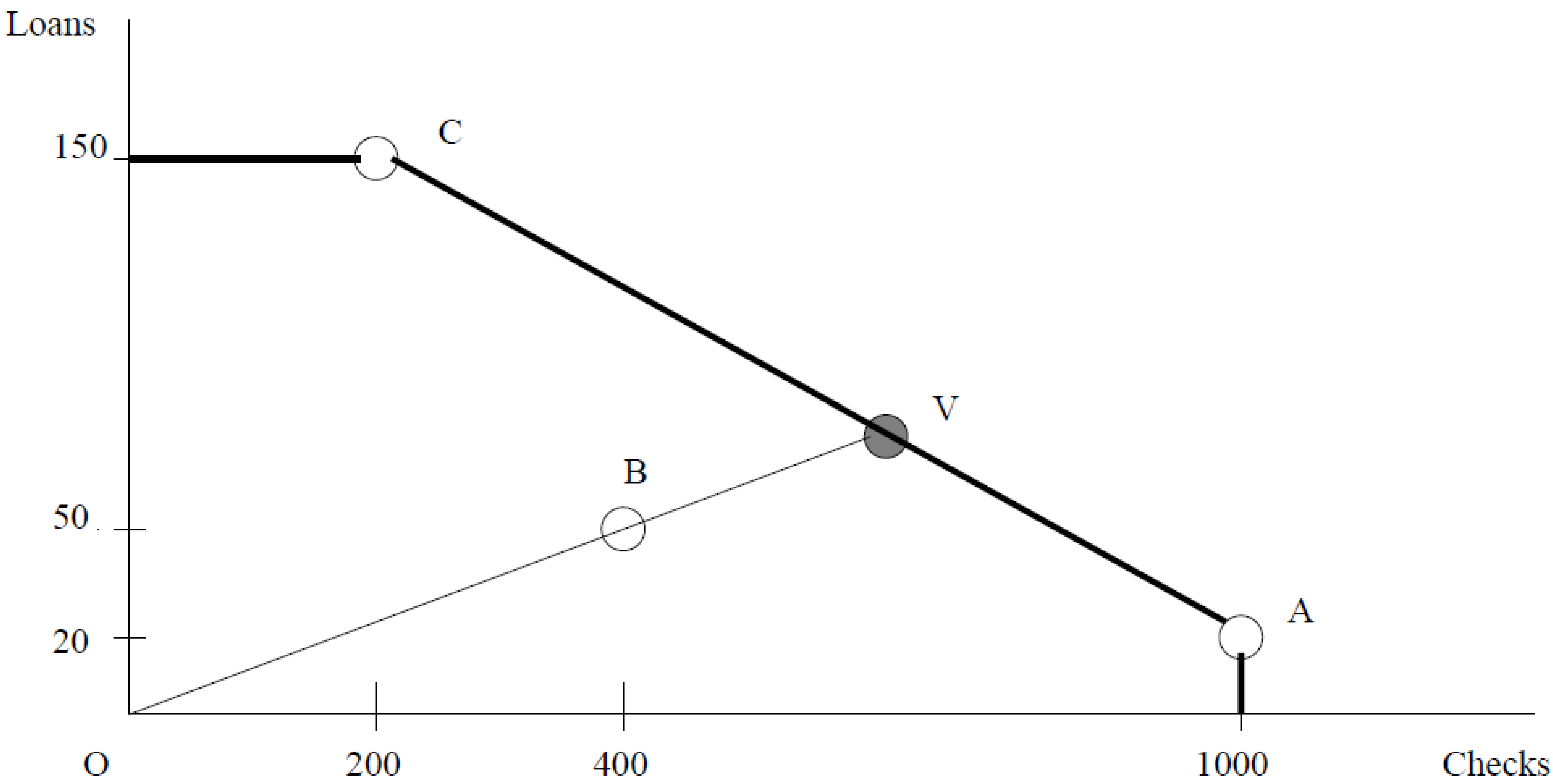
Loans
150
50
20
C
V
B
A
O
200
400
1000
Checks

- Mike Lister is a district manager for the Steak & Burger fast-food restaurant chain. The region Mike manages contains 12 company-owned units. Mike is in the process of evaluating the performance of these units during the past year to make recommendations on how much of an annual bonus to pay each unit's manager. He wants to base this decision, in part, on how efficiently each unit has been operated.

- Mike has collected the data shown in the following table on each of the 12 units. The output she has chosen includes each unit’s net profit (in $100,000s), average customer satisfaction rating, and average monthly cleanliness score. The inputs include total labor hours (in 100,000s) and total operating costs (in $1,000,000s). He wants to apply DEA to this data to determine an efficiency score of each unit.

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	Outputs			Inputs	
Unit	Profit	Satisfaction	Cleanliness	Labor Hours	Operating Costs
1	5.98	7.7	92	4.74	6.75
2	7.18	9.7	99	6.38	7.42
3	4.97	9.3	98	5.04	6.35
4	5.32	7.7	87	3.61	6.34
5	3.39	7.8	94	3.45	4.43
6	4.95	7.9	88	5.25	6.31
7	2.89	8.6	90	2.36	3.23
8	6.40	9.1	100	7.09	8.69
9	6.01	7.3	89	6.49	7.28
10	6.94	8.8	89	7.36	9.07
11	5.86	8.2	93	5.46	6.69
12	8.35	9.6	97	6.58	8.75

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$$\text{Efficiency of unit } i = \frac{\text{Weighted sum of unit } i\text{'s outputs}}{\text{Weighted sum of unit } i\text{'s inputs}} = \frac{\sum_{j=1}^{n_O} O_{ij} w_j}{\sum_{j=1}^{n_I} I_{ij} v_j}$$

$$\text{MAX: } \sum_{j=1}^{n_O} O_{ij} w_j$$

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MAX:	$5.98w_1 + 7.7w_2 + 92w_3$	} weighted output for unit 1
Subject to:	$5.98w_1 + 7.7w_2 + 92w_3 - 4.74v_1 - 6.75v_2 \leq 0$	} efficiency constraint for unit 1
	$7.18w_1 + 9.7w_2 + 99w_3 - 6.38v_1 - 7.42v_2 \leq 0$	} efficiency constraint for unit 2
	and so on to . . .	
	$8.35w_1 + 9.6w_2 + 97w_3 - 6.58v_1 - 8.75v_2 \leq 0$	} efficiency constraint for unit 12
	$4.74v_1 + 6.75v_2 = 1$	} input constraint for unit 1
	$w_1, w_2, w_3, v_1, v_2 \geq 0$	} nonnegativity conditions

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Goal Programming

- Soft constraints or goals
- Let us look at a problem

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DEFINING THE DECISION VARIABLES

- The decision facing the hotel owner is how many small, medium, and large conference rooms to include in the conference center expansion.

- These quantities are represented by X1, X2, and X3, respectively.

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DEFINING THE GOALS

- The expansion should include
  - Goal 1: approximately 5 small conference rooms.
  - Goal 2: approximately 10 medium conference rooms.
  - Goal 3: approximately 15 large conference rooms.
  - Goal 4: approximately 25,000 square feet.
  - Goal 5: approximately $1,000,000

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DEFINING THE GOAL CONSTRAINTS

$$X_1 + d_1^- - d_1^+ = 5 \quad \} \text{ small rooms}$$

$$X_2 + d_2^- - d_2^+ = 10 \quad \} \text{ medium rooms}$$

$$X_3 + d_3^- - d_3^+ = 15 \quad \} \text{ large rooms}$$

where $d_i^-, d_i^+ \geq 0$ for all i

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- The RHS value of each goal constraint (the values 5, 10, and 15 in the previous constraints) is the **target value** for the goal because it represents the level of achievement that the decision maker wants to obtain for the goal.

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- The variables *di*− and *di*+ are called **deviational variables** because they represent the amount by which each goal deviates from its target value. The *di* represents the amount by which each goal's target value is *underachieved*, and the *di*+ represents the amount by which each goal's target value is *overachieved*.

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How deviation variables work

- Suppose that we have a solution where X1 = 3, X2 = 13, and X3 = 15.

$$X_1 + d_1^- - d_1^+ = 5 \qquad \} \text{ small rooms}$$

$$X_2 + d_2^- - d_2^+ = 10 \qquad \} \text{ medium rooms}$$

$$X_3 + d_3^- - d_3^+ = 15 \qquad \} \text{ large rooms}$$

$$\text{where } d_i^-, d_i^+ \geq 0 \text{ for all } i$$

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- To illustrate how deviational variables work, suppose that we have a solution where $X1 = 3$, $X2 = 13$, and $X3 = 15$.

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- To satisfy the first goal constraint listed previously, its deviational variables would assume the values $d1^- = 2$ and $d1^+ = 0$ to reflect that the goal of having 5 small conference rooms is *underachieved* by 2.

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- Similarly, to satisfy the second goal constraint, its deviational variables would assume the values $d2- = 0$ and $d2+ = 3$ to reflect that the goal of having 10 medium conference rooms is *overachieved* by 3.

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- Finally, to satisfy the third goal constraint, its deviational variables would assume the values $d3- = 0$ and $d3+ = 0$, to reflect that the goal of having 15 medium conference rooms is *exactly* achieved.

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$$400X_1 + 750X_2 + 1{,}050X_3 + d_4^- - d_4^+ = 25{,}000 \qquad \}\ \text{square footage}$$

$$18{,}000X_1 + 33{,}000X_2 + 45{,}150X_3 + d_5^- - d_5^+ = 1{,}000{,}000 \qquad \}\ \text{building cost}$$

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What is the objective function

$$\text{MIN:} \qquad \sum_i (d_i^- + d_i^+)$$

7 rooms + 1,500 dollars = 1,507 units of what?

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Option 2

$$\text{MIN:} \quad \sum_i \frac{1}{t_i}(d_i^- + d_i^+)$$

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- suppose that we have a solution where first goal is underachieved by 1 room and the fifth goal is overachieved by $20,000 and all other goals are achieved exactly

$$\frac{1}{t_1}d_1^- + \frac{1}{t_5}d_5^+ = \frac{1}{5} \times 1 + \frac{1}{1{,}000{,}000} \times 20{,}000$$

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- The percentage deviation objective can be used only if all the target values for all the goals are non-zero; otherwise a division by zero error will occur.

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- Is \$20,000 1/10th as important as 1 room?
- Is \$1.1 million as bad as \$900K?

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Minimize the weighted sum of the deviations: MIN: $\sum_i (w_i^- d_i^- + w_i^+ d_i^+)$

or

Minimize the weighted sum of the percentage deviations: MIN: $\sum_i \frac{1}{t_i}(w_i^- d_i^- + w_i^+ d_i^+)$

You need to follow an iterative procedure in which you try a particular set of weights, solve the problem, analyze the solution, and then refine the weights and solve the problem again.

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To summarize, the LP model for our example GP problem is:

MIN: $\frac{w_1^-}{5}d_1^- + \frac{w_2^-}{10}d_2^- + \frac{w_3^-}{15}d_3^- + \frac{w_4^-}{25{,}000}d_4^- + \frac{w_4^+}{25{,}000}d_4^+ + \frac{w_5^+}{1{,}000{,}000}d_5^+$

Subject to:

$$
\begin{array}{rcll}
X_1 + d_1^- - d_1^+ &=& 5 & \text{\} small rooms} \\
X_2 + d_2^- - d_2^+ &=& 10 & \text{\} medium rooms} \\
X_3 + d_3^- - d_3^+ &=& 15 & \text{\} large rooms} \\
400X_1 + 750X_3 + 1{,}050X_3 + d_4^- - d_4^+ &=& 25{,}000 & \text{\} square footage} \\
18{,}000X_1 + 33{,}000X_2 + 45{,}150X_3 + d_5^- - d_5^+ &=& 1{,}000{,}000 & \text{\} building cost}
\end{array}
$$

$d_i^-, d_i^+ \geq 0$ for all i } non-negativity conditions

$X_i \geq 0$ for all i } non-negativity conditions

X_i must be integers

Notice that this objective omits (or assigns weights of 0 to) the deviational variables about which the decision maker is indifferent. Thus, this objective would not penalize a solution where, for example, 7 small conference rooms were selected (and therefore $d_1^+ = 2$) because we assume that the decision maker would not view this as an undesirable deviation from the goal of having 5 small conference rooms.

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Summary

1. Identify the decision variables in the problem.
2. Identify any hard constraints in the problem and formulate them in the usual way.
3. State the goals of the problem along with their target values.
4. Create constraints using the decision variables that would achieve the goals exactly.

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GP Summary contd…

5. Transform the above constraints into goal constraints by including deviational variables.
6. Determine which deviational variables represent undesirable deviations from the goals.
7. Formulate an objective that penalizes the undesirable deviations.
8. Identify appropriate weights for the objective.
9. Solve the problem.
10. Inspect the solution to the problem. If the solution is unacceptable, return to step 8 and revise the weights as needed.

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- Suppose we want to eliminate any solution that exceeds the target building cost by more than $50,000. We could build this requirement

$$d5_{+} \leq 50{,}000$$

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- In some GP problems, one or more goals might be viewed as being infinitely more important than the other goals. We could assign arbitrarily large weights to deviations from these goals

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Can you set up a LP?

- There are many vehicles in a depo
- Many customers with different preferred time windows
- Minimize the travel distance and minimize the number of breaks in the preferred windows

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