Inspire…Educate…Transform.

Stat Skills

Discrete Distributions, Normal Distribution, Sampling Distribution

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Flashback

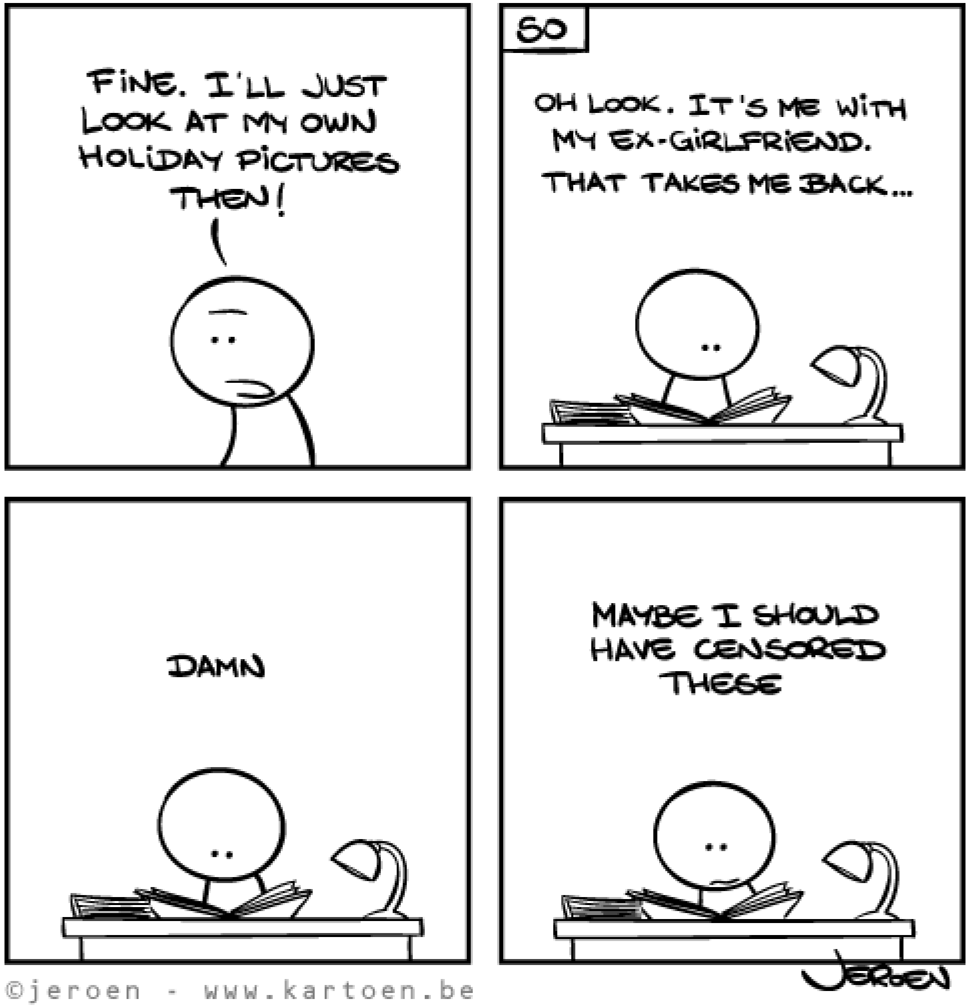

Image source: http://www.kartoen.be/wp/2010/11/
Last accessed: May 03, 2014

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Wake

Up

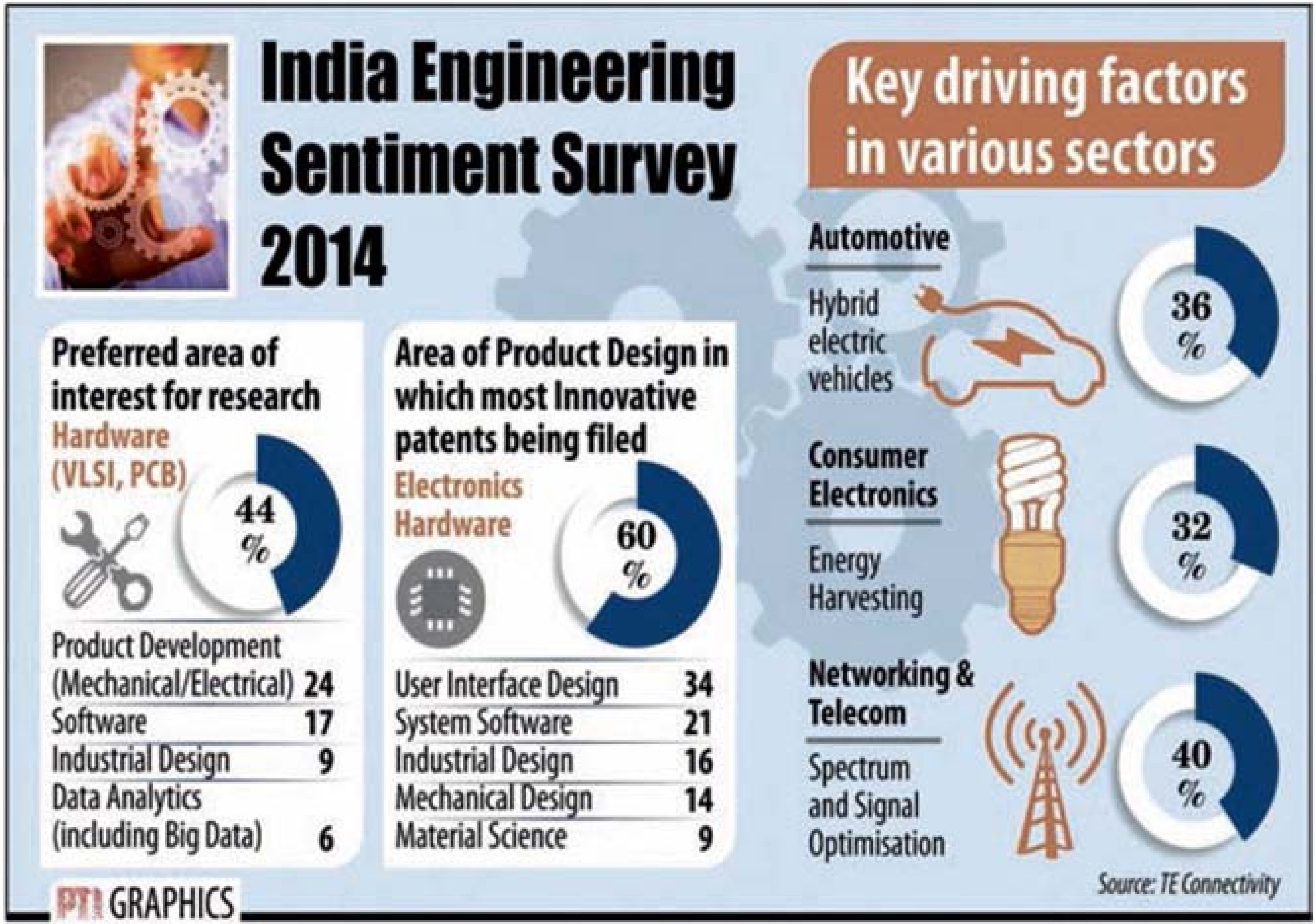

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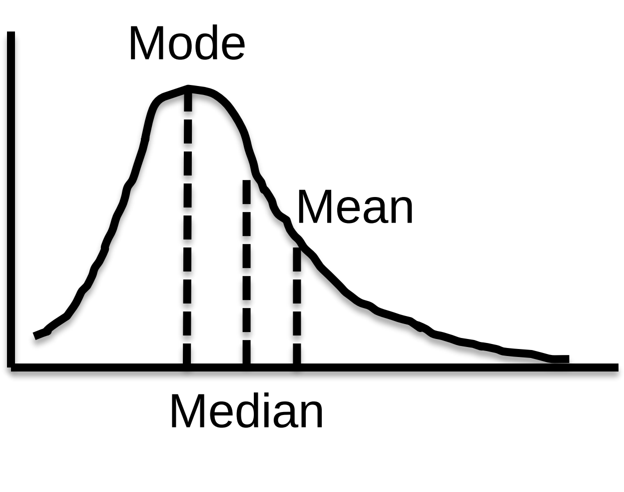
Image source: http://epaper.deccanchronicle.com/articledetailpage.aspx?id=1341004
Last accessed: November 09, 2014

The Central Tendencies

Identify where the MODE, MEDIAN and MEAN lie in the below distributions.



Measures of Spread (Dispersion)

We studied Quartiles in depth and mentioned Deciles and Percentiles in passing. However, just as Quartiles divide data into 4 equal parts, Deciles divide it into 10 equal parts and Percentiles into 100 equal parts.

Given the above, find the 25^{th}, 50^{th}, 75^{th} and the 90^{th} percentiles for the top 16 global marketing sectors for advertising spending for a recent year according to *Advertising Age*. Also, find Q2 and IQR. Data in next slide.

Sector	Ad spending (in $ million)
Automotive	22195
Personal Care	19526
Entertainment and Media	9538
Food	7793
Drugs	7707
Electronics	4023
Soft Drinks	3916
Retail	3576
Restaurants	3553
Cleaners	3571
Computers	3247
Telephone	2448
Financial	2433
Beer, Wine and Liquor	2050
Candy	1137
Toys	699

Distributions – Properties

Var(-X) = ?

Options:

- Var(X) ✓
- -Var(X)
- Variance of negative numbers cannot be calculated
- None of the above (Explain)

Present Day

SOME COMMON DISTRIBUTIONS

Bernoulli

- There are two possibilities (loan taker or non-taker) with probability p of success and $1-p$ of failure
  - Expectation: p
  - Variance: $p(1-p)$ or pq, where $q=1-p$

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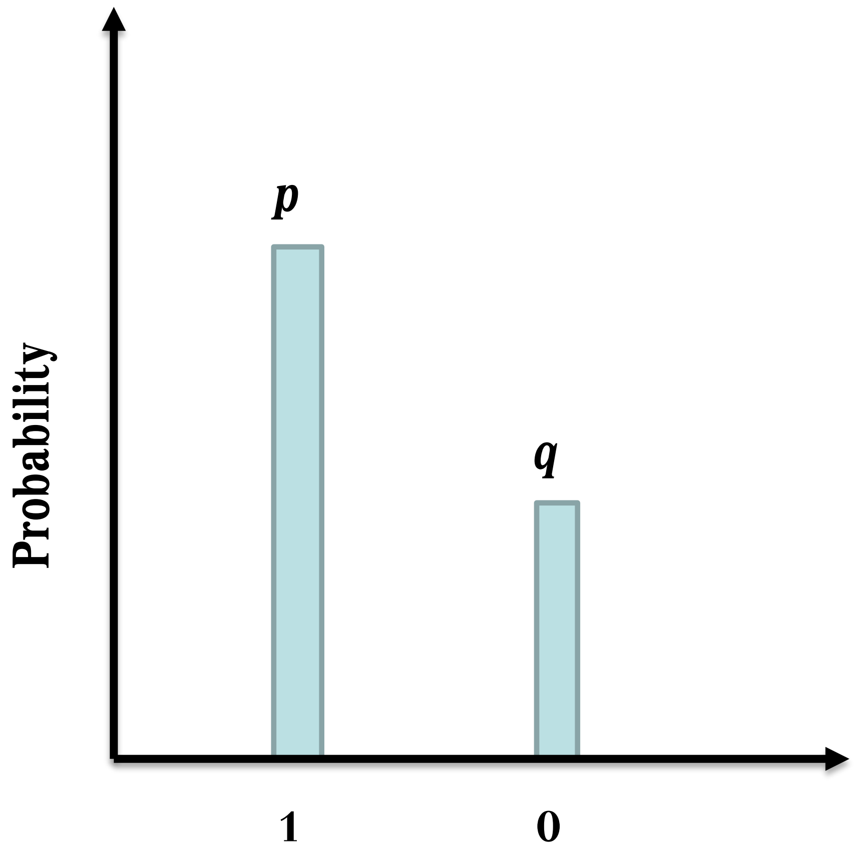
Bernoulli



Geometric distribution

- Number of independent and identical Bernoulli trials needed to get ONE success, e.g., number of people I need to call for the first person to accept the loan.

Geometric distribution

PMF*, $P(X = r) = q^{r-1}p$ — *(r-1)* failures followed by ONE success.

$P(X > r) = q^r$ — Probability you will need more than *r* trials to get the first success.

CDF**, $P(X \leq r) = 1 - q^r$ — Probability you will need *r* trials or less to get your first success.

$$E(X) = \frac{1}{p} \qquad Var(X) = \frac{q}{p^2}$$

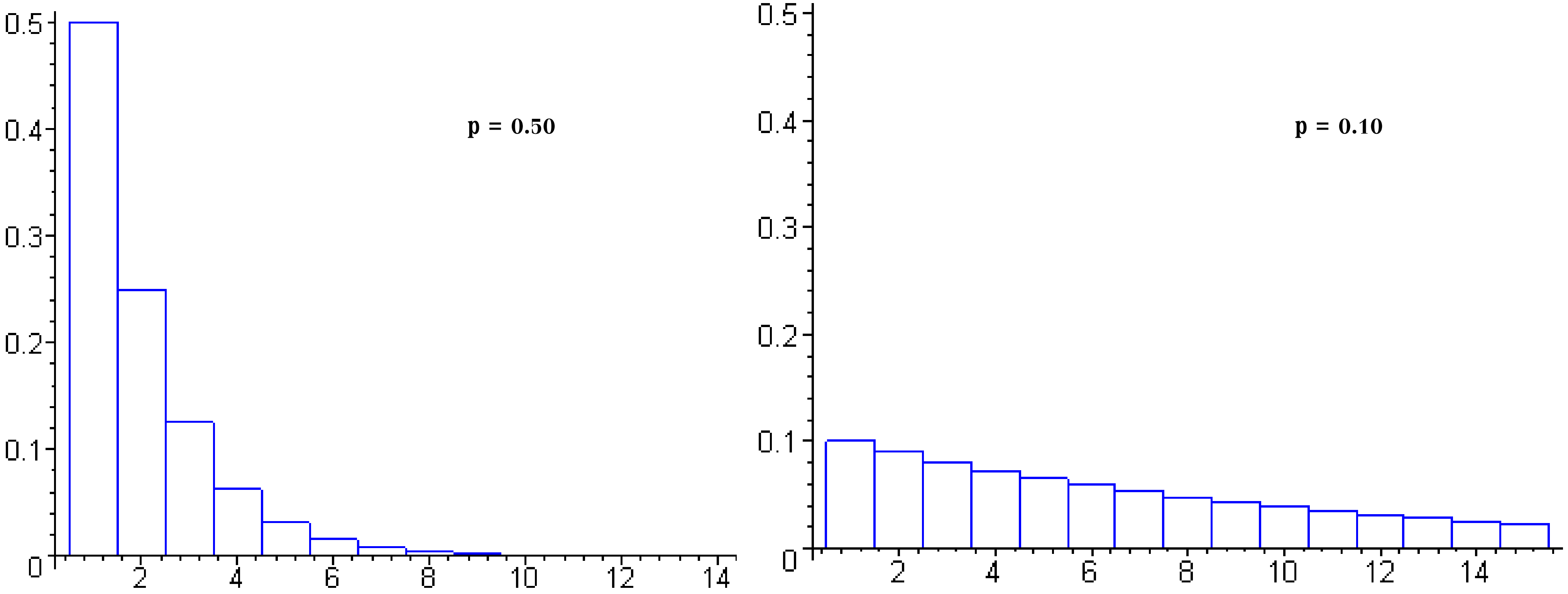
* Probability Mass Function

** Cumulative Distribution Function

Geometric distribution

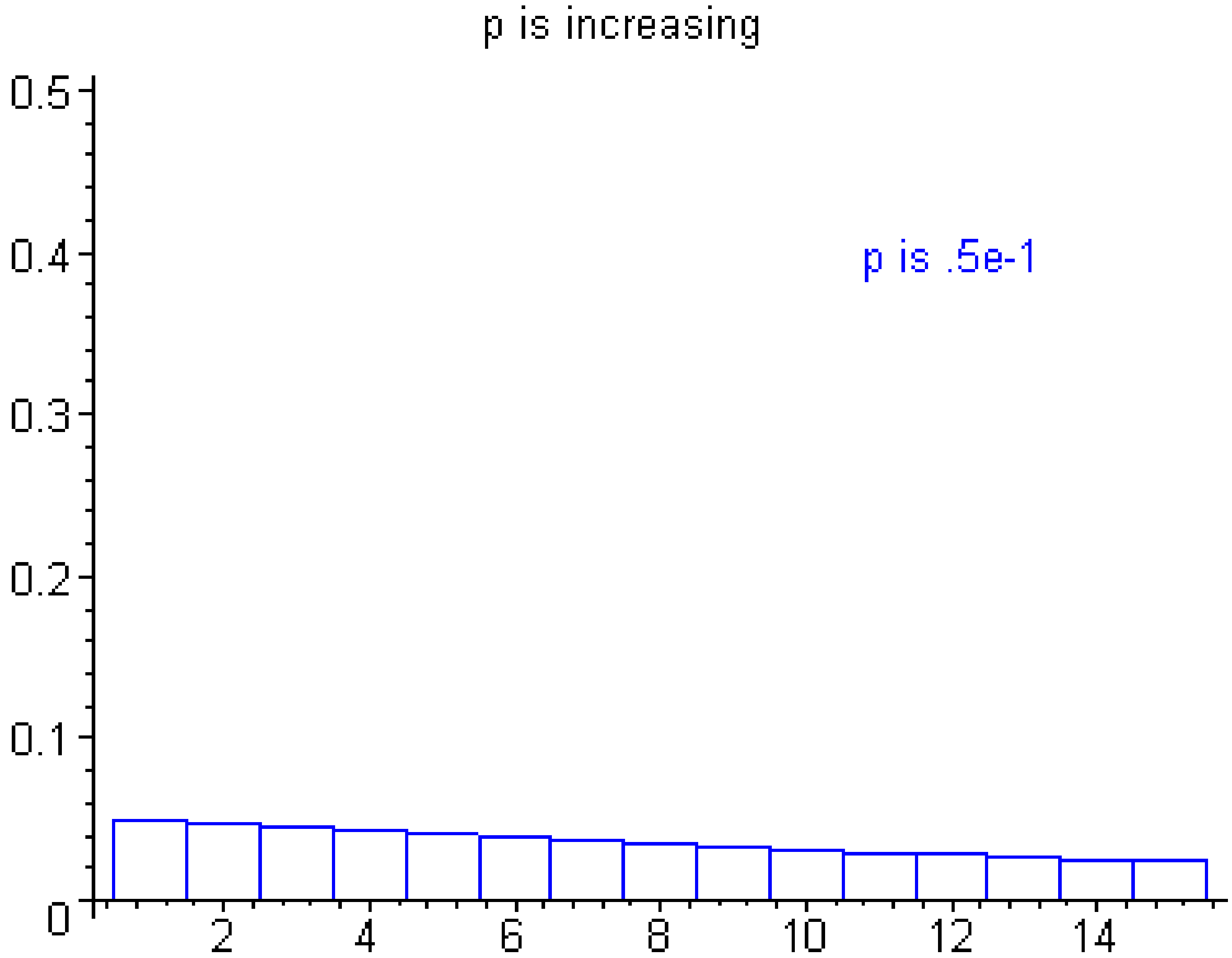
- You run a series of independent trials.
- There can be either a success or a failure for each trial, and the probability of success is the same for each trial.
- The main thing you are interested in is how many trials are needed in order to get the first successful outcome.

X~Geo(p)



Ref: http://personal.kenyon.edu/hartlaub/MellonProject/Geometric2.html
Last accessed: June 12, 2015

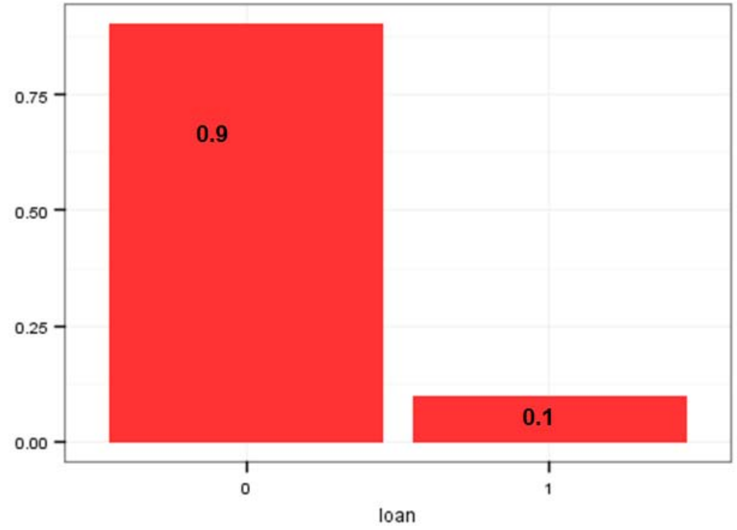
X~Geo(p)



Ref: http://personal.kenyon.edu/hartlaub/MellonProject/Geometric2.html
Last accessed: June 12, 2015

Binomial distribution

- If I randomly pick 10 people, what is the probability that I will get exactly
  - 0 loan takers = 0.9^{10}
  - 1 loan taker = $10 * 0.9^{9} * 0.1^{1}$
  - 2 loan takers = $C_2^{10} * 0.9^{8} * 0.1^{2}$



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Binomial distribution

- If there are two possibilities with probability p for success and q for failure, and if we perform n trials, the probability that we see r successes is

PMF, $P(X = r) = C_r^n p^r q^{n-r}$

CDF, $P(X \leq r) = \sum_{i=0}^{r} C_i^n p^i q^{n-i}$

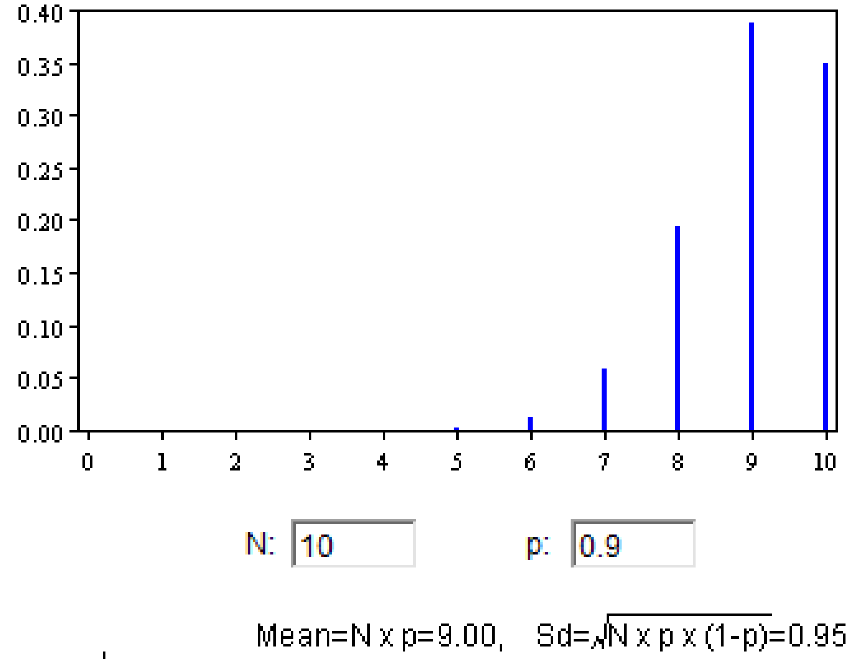
Binomial distribution

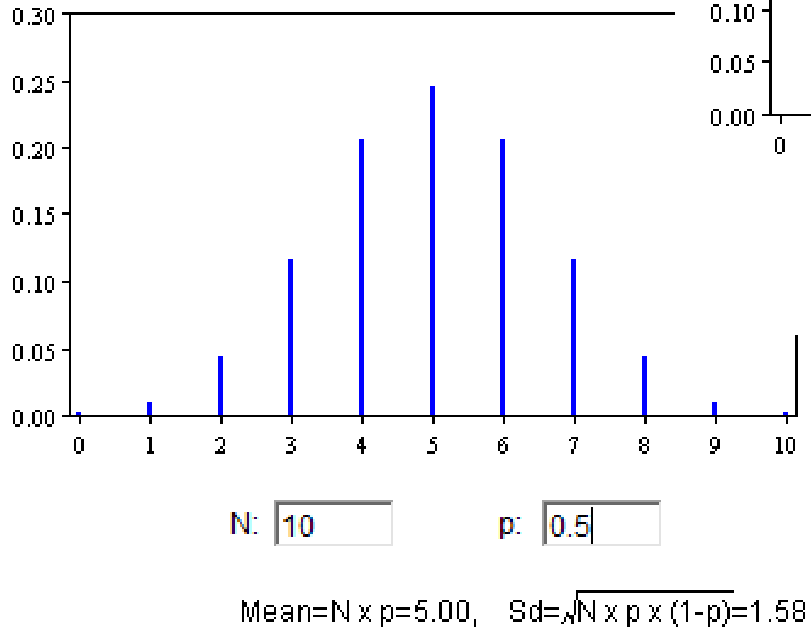
$$E(X) = np \qquad Var(X) = npq$$

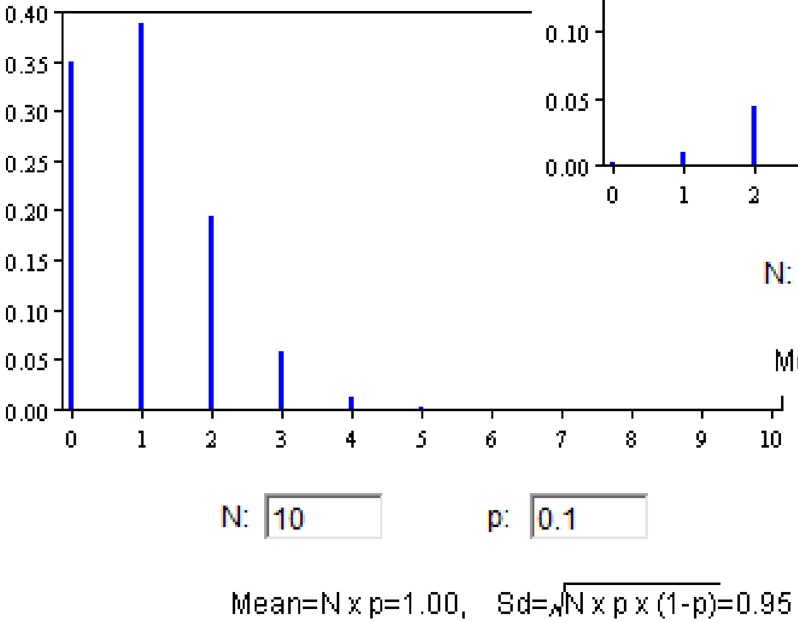
When to use?

- You run a series of independent trials.
- There can be either a success or a failure for each trial, and the probability of success is the same for each trial.
- There are a finite number of trials, and you are interested in the number of successes or failures.

X~B(n,p)







Ref: http://onlinestatbook.com/2/probability/binomial_demonstration.html
Last accessed: June 12, 2015

Poisson distribution

- Probability of getting 15 customers requesting for loans in a given day given on average we see 10 customers

$$\lambda = 10 \; and \; n = 15$$

$$\text{PMF, P(X = n)} = \frac{e^{-\lambda}\lambda^{n}}{n!}$$

Poisson distribution

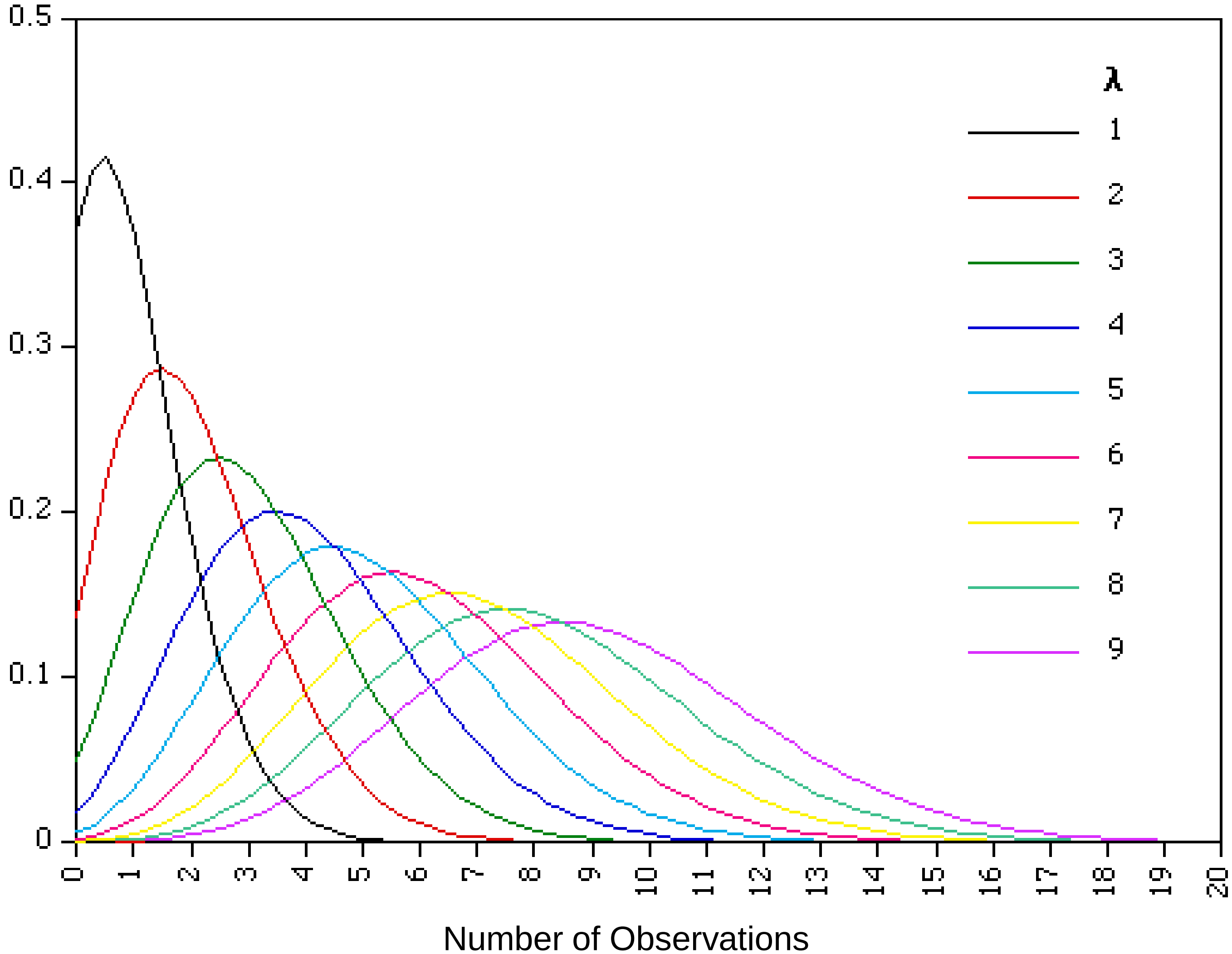
$E(X) = \lambda$ Can be equated to *np* of Binomial if *n* is large (>50) and *p* is small (<0.1)

$Var(X) = \lambda$ Can be equated to *npq* of Binomial in the above situation.

When to use?

- Individual events occur at random and independently in a given interval (time or space).
- You know the mean number of occurrences, λ, in the interval or the rate of occurrences, and it is finite.

X~Po(λ)



Ref: http://www.umass.edu/wsp/resources/poisson/
Last accessed: June 12, 2015

Exponential distribution (Poisson Process)

The probability that no customer will visit the store in one day

$$P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

Probability that she will not have a customer for *n* days

$$e^{-n\lambda}$$

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Exponential distribution

- Probability that a customer will visit in n days:

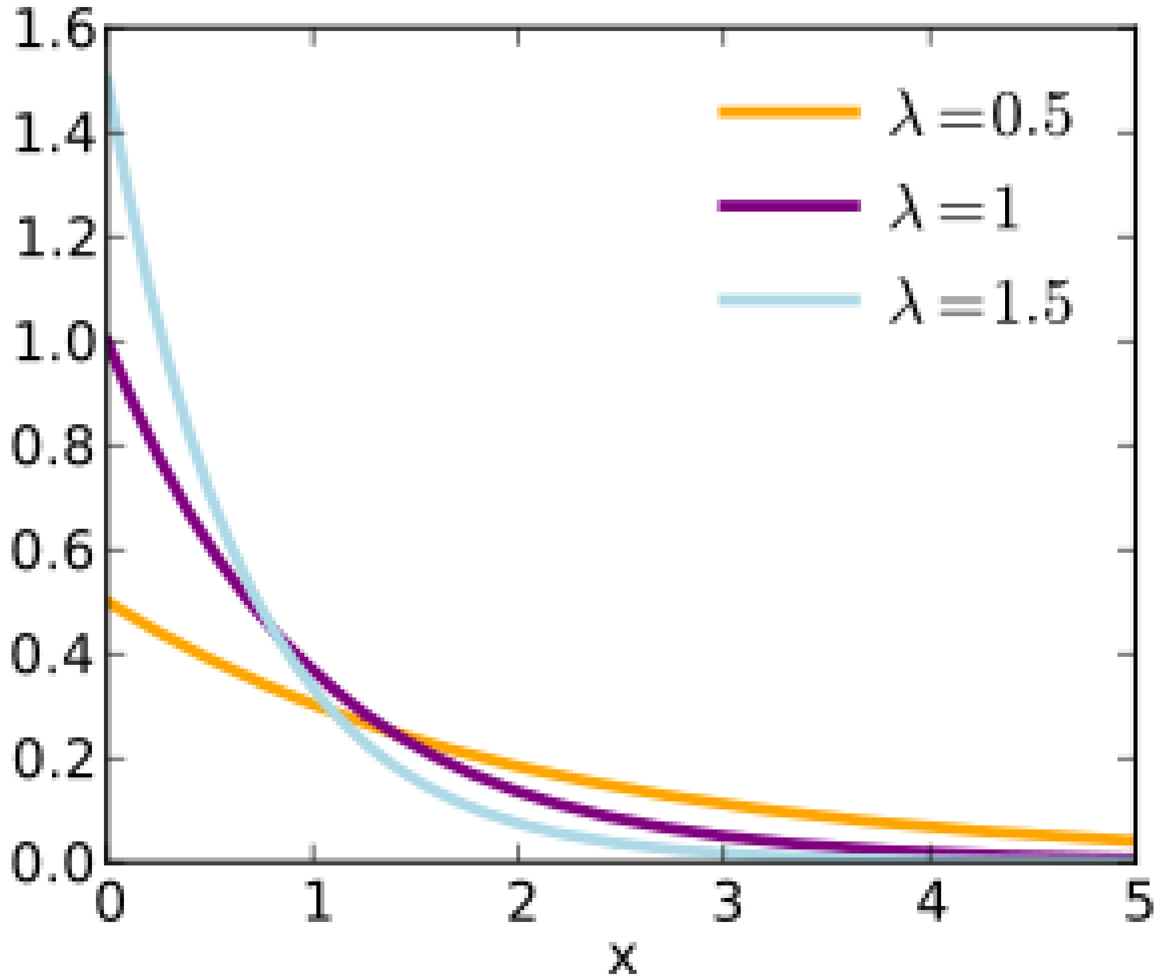
$$1 - e^{-n\lambda}$$

$$CDF = 1 - e^{-n\lambda}, n \geq 0$$

$$PDF^{*} = \lambda e^{-n\lambda}, n \geq 0$$

* Probability Density Functio

X~Exp(λ)



Ref: http://en.wikipedia.org/wiki/Exponential_distribution
Last accessed: June 12, 2015

Exponential distribution

- Poisson process
- Continuous analog of Geometric distribution

Distributions

Geometric:	For estimating number of attempts before first success
Binomial:	For estimating number of successes in n attempts
Poisson:	For estimating n number of events in a given time period when on average we see m events
Exponential:	Time between events

Probability Distributions

Here are a few scenarios. Identify the distribution and calculate expectation, variance and the required probabilities.

Q1. A man is bowling. The probability of him knocking all the pins over is 0.3. If he has 10 shots, what is the probability he will knock all the pins over less than 3 times?

Q2. On average, 1 bus stops at a certain point every 15 minutes. What is the probability that no buses will turn up in a single 15 minute interval?

Q3. 20% of cereal packets contain a free toy. What is the probability you will need to open fewer than 4 cereal packets before finding your first toy?

Probability Distributions

Solutions

A man is bowling. The probability of him knocking all the pins over is 0.3. If he has 10 shots, what is the probability he will knock all the pins over less than 3 times?

$X \sim B(10,0.3)$; $n=10$, $p=0.3$, $q=1-0.3=0.7$, $r=0, 1, 2$ (< 3)

$E(X) = np = 3$

$Var(X) = npq = 2.1$

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$P(X=0) = 0.028$; $P(X=1) = 0.121$; $P(X=2) = 0.233$

$\therefore P(X<3) = 0.028 + 0.121 + 0.233 = 0.382$

Probability Distributions

Solutions

On average, 1 bus stops at a certain point every 15 minutes. What is the probability that no buses will turn up in a single 15 minute interval?

$X \sim Po(1)$; $\lambda=1$, $r=0$

$E(X) = \lambda = 1$

$Var(X) = \lambda = 1$

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$P(X=0) = 0.368$

Probability Distributions

Solutions

20% of cereal packets contain a free toy. What is the probability you will need to open fewer than 4 cereal packets before finding your first toy?

X ~ Geo(0.2); p=0.2, q=1-0.2=0.8, r<4 or ≤3

$E(X) = \frac{1}{p} = 5$

$Var(X) = \frac{q}{p^2} = 20$

$P(X \leq r) = 1 - q^r$

$P(X \leq 3) = 0.488$

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Poisson Distribution Formula Differences?

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad or \quad \frac{e^{-\lambda t}(\lambda t)^r}{r!} \ ?$$

Suppose births in a hospital occur randomly at an average rate of 1.8 births per hour. What is the probability of 5 births in a given 2 hour interval?

What is λ?

$$P(X = 5) = \frac{e^{-3.6}3.6^5}{5!} \quad or \quad \frac{e^{-1.8*2}(1.8*2)^5}{5!} \ ?$$

If you use 1.8, use t=2 in the second formula. Alternatively, you could say that since the average is 1.8 per hour, it is 3.6 per 2 hours (the interval of interest).

Poisson Distribution Formula Differences?

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!} \ or \ \frac{e^{-\lambda t}(\lambda t)^r}{r!} \ ?$$

Now suppose head injury patients (due to not wearing helmets) arrive in Hospital A randomly at an average rate of 0.25 patients per hour, and in Hospital B randomly at an average rate of 0.75 per hour. What is the probability of more than 3 such patients arriving in a given 2 hour interval in both hospitals together?

What is the probability distribution?

$$X \sim Po(\lambda_1) \ and \ Y \sim Po(\lambda_2)$$
$$X + Y \sim Po(\lambda_1 + \lambda_2)$$

What are λ_1 and λ_2 if we use first formula?

$\lambda_1 = 0.5$ and $\lambda_2 = 1.5$

$$P(X + Y > 3) = P(X + Y = 4) + P(X + Y = 5) + P(X + Y = 6) + \cdots$$
$$= 1 - P(X + Y \leq 3) = 1 - (P(X + Y = 0) + P(X + Y = 1) + P(X + Y = 2) + P(X + Y = 3))$$

Poisson or Exponential?

Given a Poisson process:

- The *number* of events in a given time period — Poisson
- The *time* until the first event
- The *time* from now until the next occurrence of the event
- The *time interval* between two successive events

$\left.\begin{array}{l}\text{The } \textit{time} \text{ until the first event}\\ \text{The } \textit{time} \text{ from now until the next occurrence of the event}\\ \text{The } \textit{time interval} \text{ between two successive events}\end{array}\right\}$ Exponential

Poisson or Exponential?

The tech support centre of a computer retailer receives 5 calls per hour on an average. What is the probability that the centre will receive 8 calls in the next hour? What is the probability that more than 30 minutes will elapse between calls?

$$P(X = 8) = \frac{e^{-5}5^8}{8!} = 0.065$$

$$P(Time\ between\ calls > 0.5) = \int_{0.5}^{\infty} \lambda e^{-\lambda T} dT$$

$$= -e^{-\lambda T}]_{0.5}^{\infty} = e^{-5*0.5} = 0.082$$

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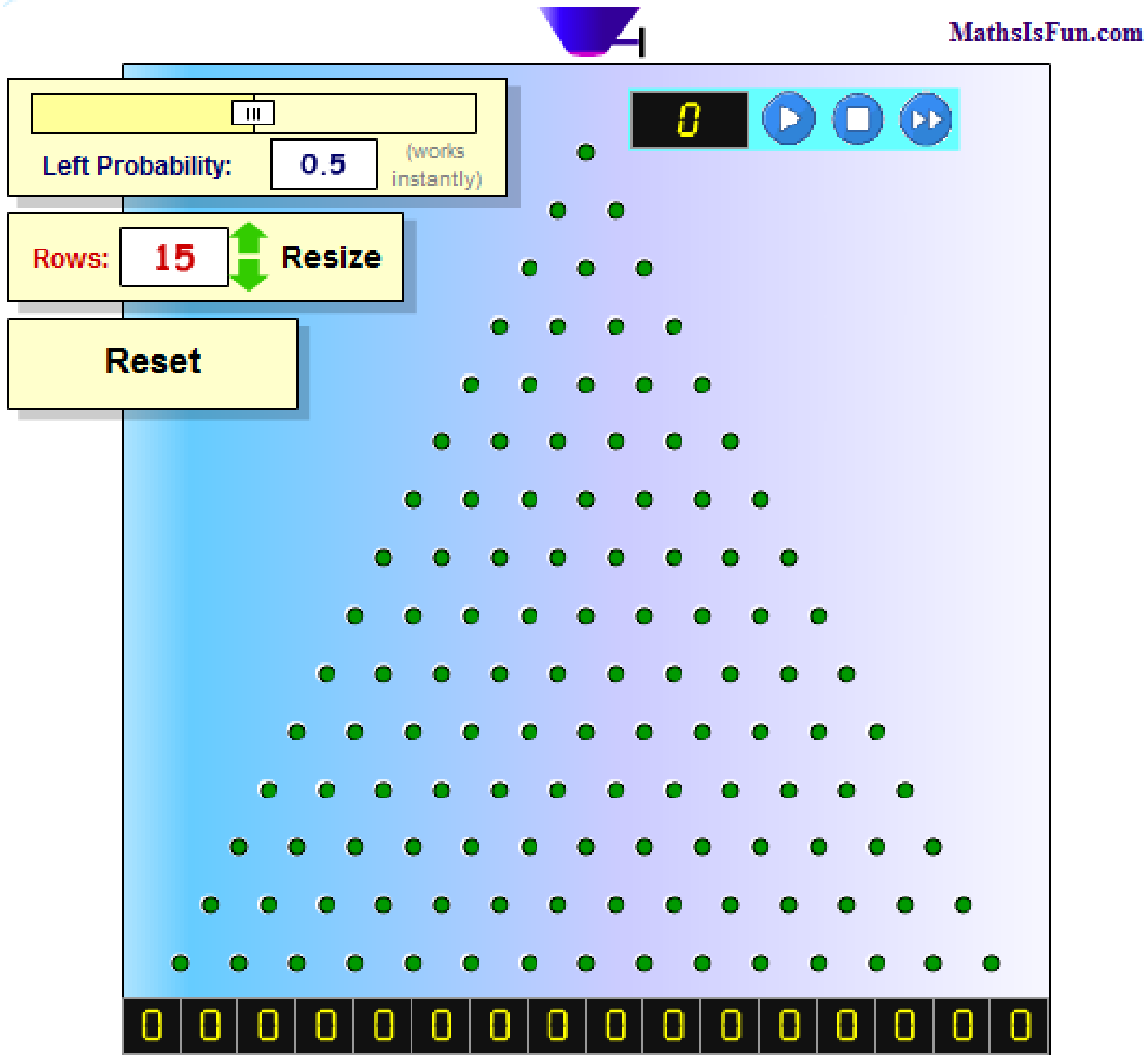
Quincunx Demo

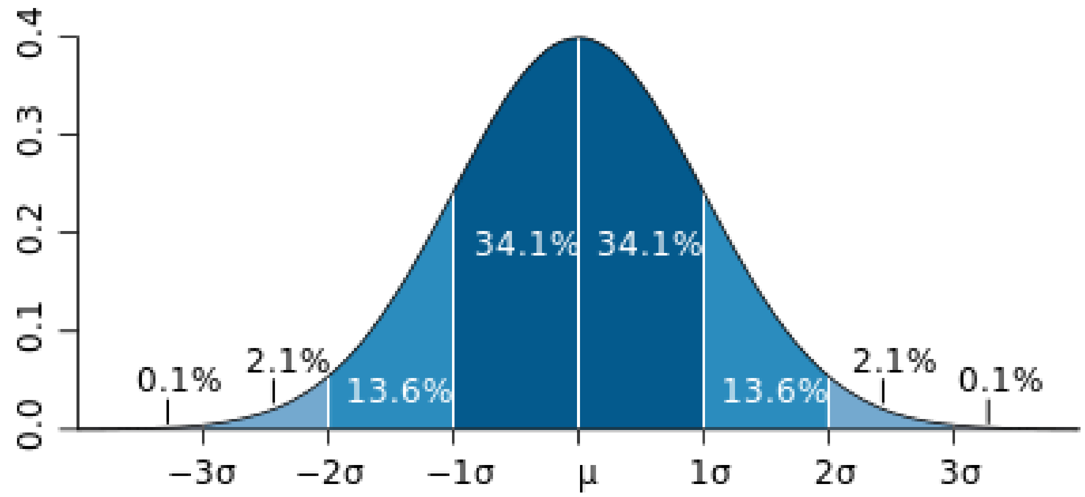


Source: http://www.mathsisfun.com/data/quincunx.html

NORMAL DISTRIBUTION

Normal (Gaussian) Distribution

- Mean = Median = Mode
- 68-95-99.7 empirical rule
- Zero Skew and Kurtosis
- $X \sim N(\mu, \sigma^2)$

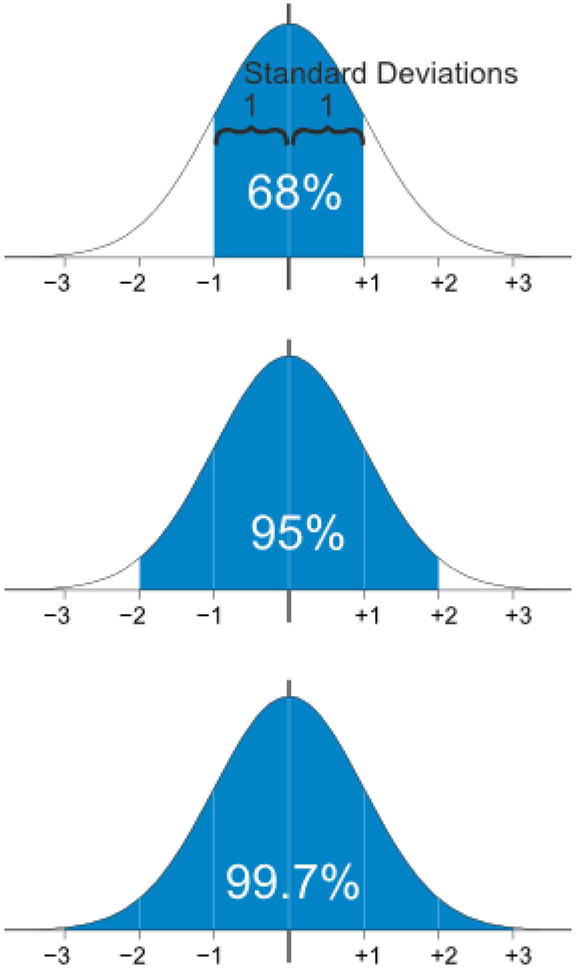


- Shaded area gives the probability that X is between the corresponding values

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Measures of Spread (Dispersion)

You know the 68-95-99.7 rule.



A company produces a lightweight valve that is specified to weigh 1365g, but there are imperfections in the process. While the mean weight is 1365g, the standard deviation is 294g.

Q1. What is the range of weights within which 95% of the valves will fall?

Q2. Approximately 16% of the weights will be more than what value?

Q3. Approximately 0.15% of the weights will be less than what value?

Image source: http://www.mathsisfun.com/data/standard-normal-distribution.html

Last accessed: May 03, 2014

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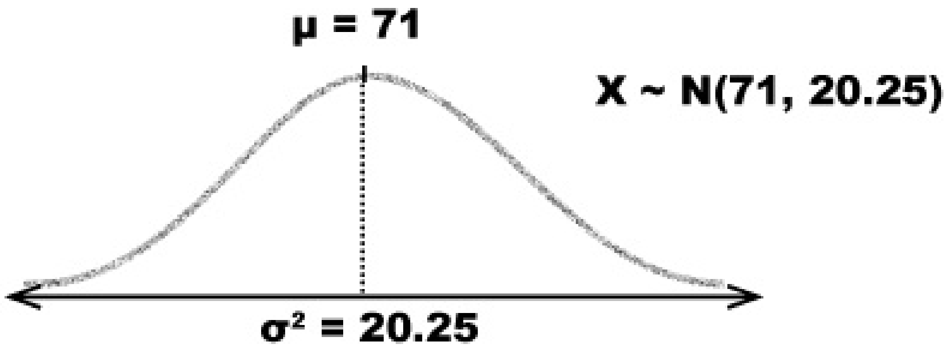
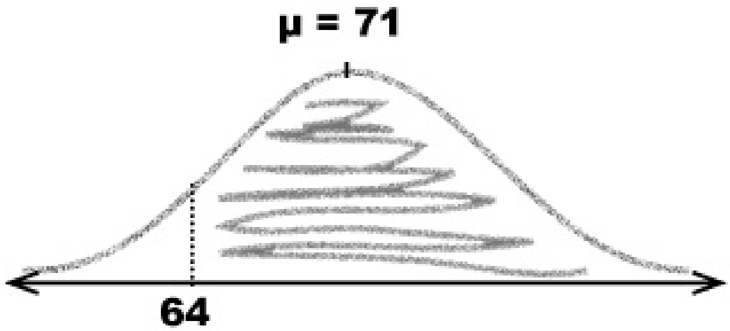
Calculating Normal Probabilities

Step 1: Determine the distribution

Julie wants to marry a person taller than her and is going on blind dates. The mean height of the 'available' guys is 71" and the variance is 20.25 inch^2 (yuck!). Oh! By the way, Julie is 64" tall.





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Calculating Normal Probabilities

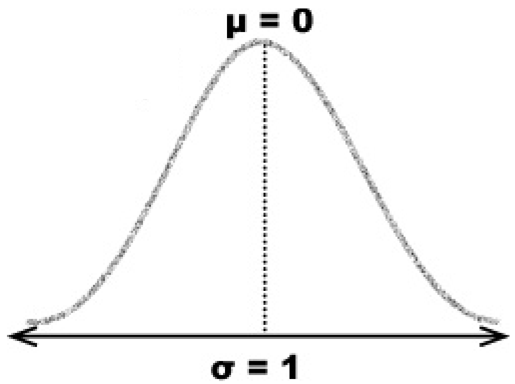
Step 2: Standardize to Z ~ N(0,1)

1. Move the mean

   This gives a new distribution X-71 ~ N(0,20.25)

2. Squash the width by dividing by the standard deviation

   This gives us $\frac{X-71}{4.5} \sim N(0,1)$



$Z = \frac{X-\mu}{\sigma}$ is called the Standard Score or the z-score.

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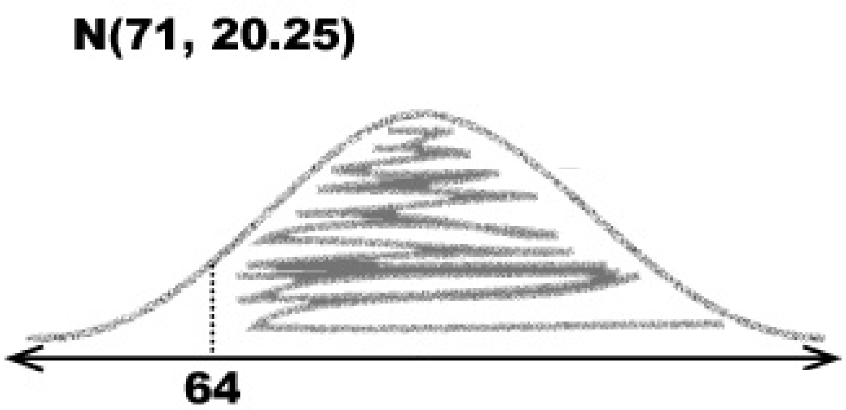
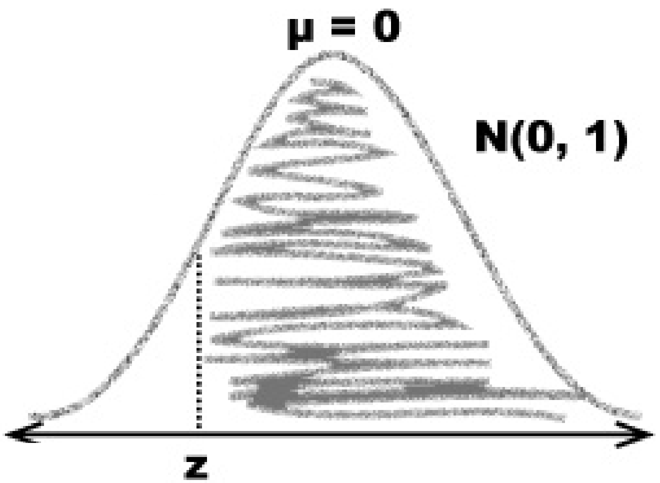
Calculating Normal Probabilities

Step 2: Standardize to Z ~ N(0,1)



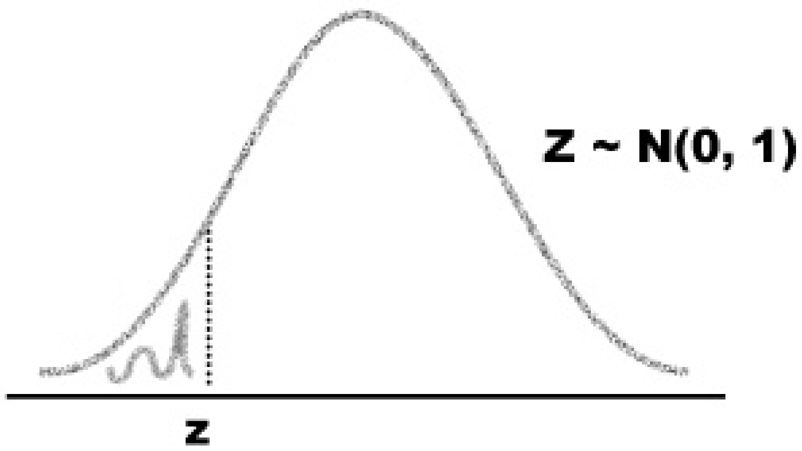


$Z = \frac{64-71}{4.5} = -1.56$ in the case of our problem.

Calculating Normal Probabilities

Step 3: Look up the probability in the tables

Note the tables give P(Z<z).



Calculating Normal Probabilities

Step 3: Look up the probability in the tables

Note the tables give P(Z<z).

$$Z = \frac{64-71}{4.5} = -1.56 \text{ in the}$$

case of our problem.

P(Z>-1.56) = 1- P(Z<-1.56)

= 1-0.0594 = 0.9406

Here's the column for .06, the second decimal place for z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	[illegible]	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	[illegible]	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	[illegible]	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	[illegible]	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	[illegible]	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	[illegible]	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	[illegible]	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	[illegible]	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	[illegible]	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	[illegible]	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	[illegible]	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	[illegible]	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	[illegible]	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	[illegible]	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	[illegible]	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	[illegible]	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	[illegible]	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	[illegible]	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	[illegible]	.0475	[illegible]	.0455
-1.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170

Here's the row for z = -1.5x, where x is some number.

Probability

Attention Check

Q. What is the standard score for N(10,4), value 6?

A. $Z = \frac{6-10}{2} = -2$

Q. The standard score of value 20 is 2. If the variance is 16, what is the mean?

A. $2 = \frac{20-\mu}{4}$. $\therefore\ \mu = 20 - 8 = 12$

Attention Check

Q. Julie just realized that she wants her date to be taller when she is wearing her heels, which are 5” high. Find the new probability that her date will be taller.

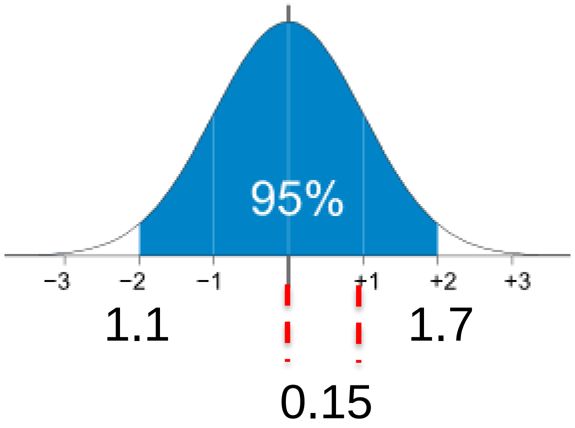
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A. $Z = \frac{69-71}{4.5} = -0.44$; P(Z<-0.44) = 0.33, ∴ P(Z>-0.44) = 0.67 or 67%

CSE 7315c

Attention Check

95% of students at a school are between 1.1m and 1.7m tall. Assuming this data is normally distributed, calculate the mean and the standard deviation.

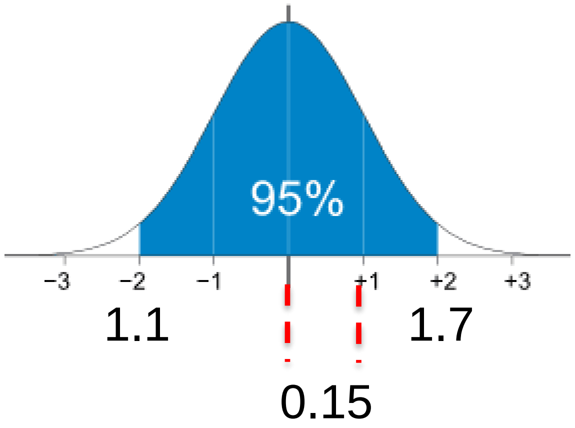


- Mean is halfway between 1.1 and 1.7, and is equal to (1.1+1.7)/2 = 1.4m
- 95% is the area covered between -2σ and +2σ. That is, this spread is equal to 4 standard deviations. Therefore, 1 standard deviation is equal to (1.7-1.1)/4 = 0.15m.

CSE 7315c

Attention Check

In the same school, one of your friends is 1.85m tall. What is the z-score of your friend's height?



- Step 1: How far is 1.85 from the mean?
  - It is 1.85 – 1.4 = 0.45m from the mean.
- Step 2: How many standard deviations is that?
  - If 0.15m is 1σ, 0.45m is 0.45/0.15 = 3 standard deviations. Therefore, the z-score of your friend's height is 3.

Outlier detection – Excel

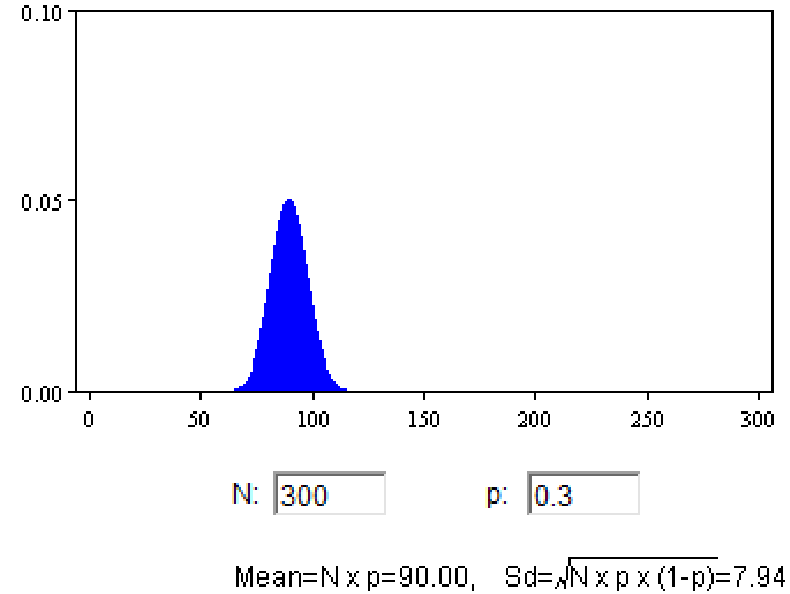
Hadlum vs Hadlum case

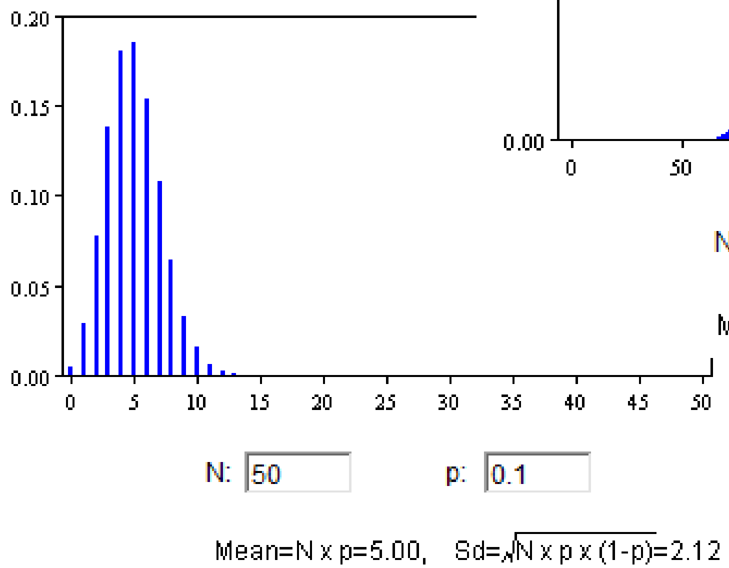
Source: http://www.alphamom.com/legacy/pregnancy-calendar/week36.jpg
Last accessed: November 01, 2014

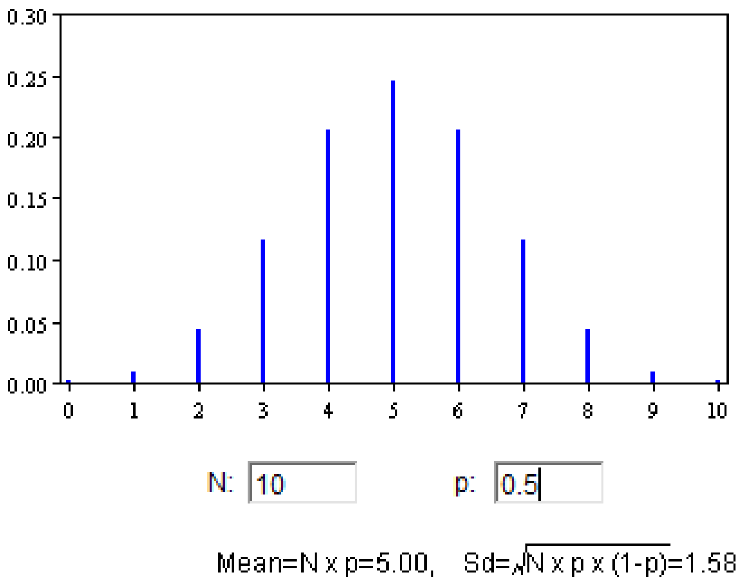
Source: http://3.bp.blogspot.com/-0YwIRjLMWr0/T4DqOwVClgI/AAAAAAAAAgg/Yjf-ttkQLSg/s1600/fishy.jpg
Last accessed: November 01, 2014

Normal distribution

Binomial distribution can be approximated to a Normal distribution if np>5 and nq>5 (Continuity Correction required).

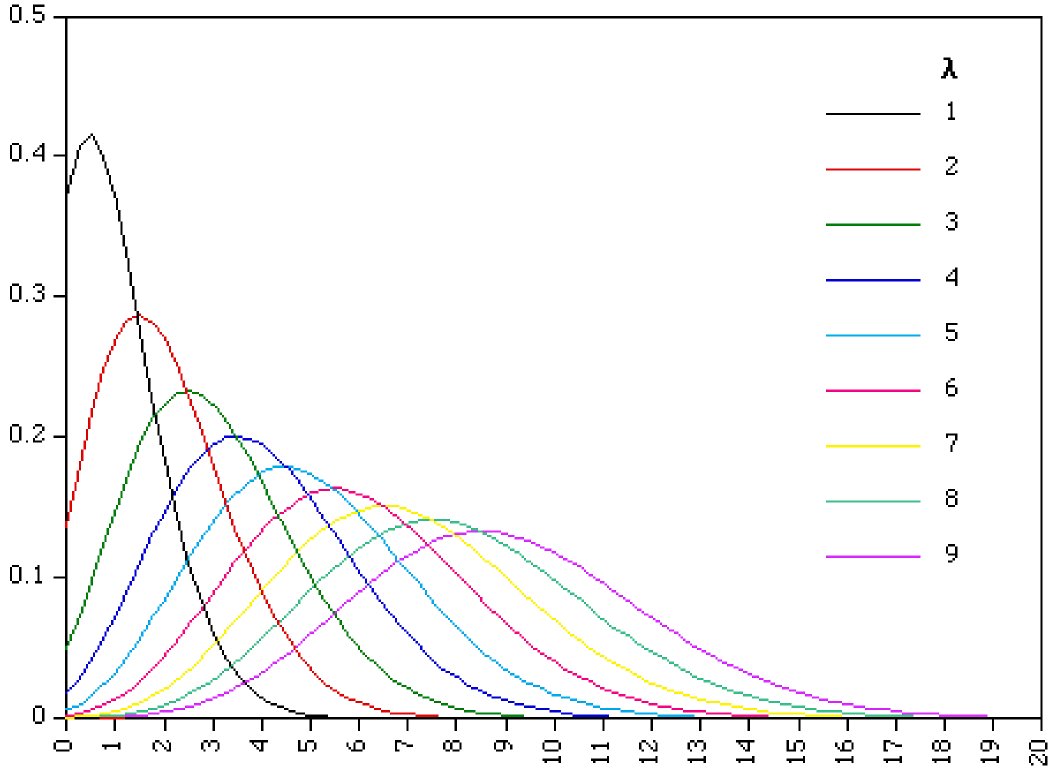






Normal distribution

Poisson distribution can be approximated to a Normal distribution when $\lambda > 15$ (Continuity Correction required).



CSE 7315c

Continuity Correction?

You are playing Who Wants to Win a 

ility you will get 5 or fewer correct out of 12, given that each question has only 2 possible choices? You have no lifelines.

$X \sim B(12, 0.5)$ and we need to find $P(X < 6)$.

$$P(X=0) = {}^{12}C_0 (0.5)^0 (0.5)^{12-0} = 0.5^{12}$$
$$P(X=1) = {}^{12}C_1 (0.5)^1 (0.5)^{12-1} = 12 * 0.5^{12}$$
$$P(X=2) = {}^{12}C_2 (0.5)^2 (0.5)^{12-2} = 66 * 0.5^{12}$$
$$P(X=3) = {}^{12}C_3 (0.5)^3 (0.5)^{12-3} = 220 * 0.5^{12}$$
$$P(X=4) = {}^{12}C_4 (0.5)^4 (0.5)^{12-4} = 495 * 0.5^{12}$$
$$P(X=5) = {}^{12}C_5 (0.5)^5 (0.5)^{12-5} = 792 * 0.5^{12}$$
$$\therefore P(X<6) = (1+12+66+220+495+792) * 0.5^{12} \cong 0.387$$

Continuity Correction?

X ~ B(12, 0.5) can be approximated to X ~ N(6, 3). How/Why?

n = 12, p = 0.5 and q = 0.5. Since np and nq are both > 5, the Binomial distribution can be approximated to a Normal distribution, i.e., X ~ B(n, p) can be approximated to X ~ N(np, npq).

If we want to get P(X < 6), what is the next step to do in the Normal distribution?

Calculate the z-score (or the standard-score).

$$z = \frac{x-\mu}{\sigma} = \frac{6-6}{\sqrt{3}} = 0$$

What do we do with the z-score?

Look it up in the probability tables.

What is the probability corresponding to the z-score of 0?

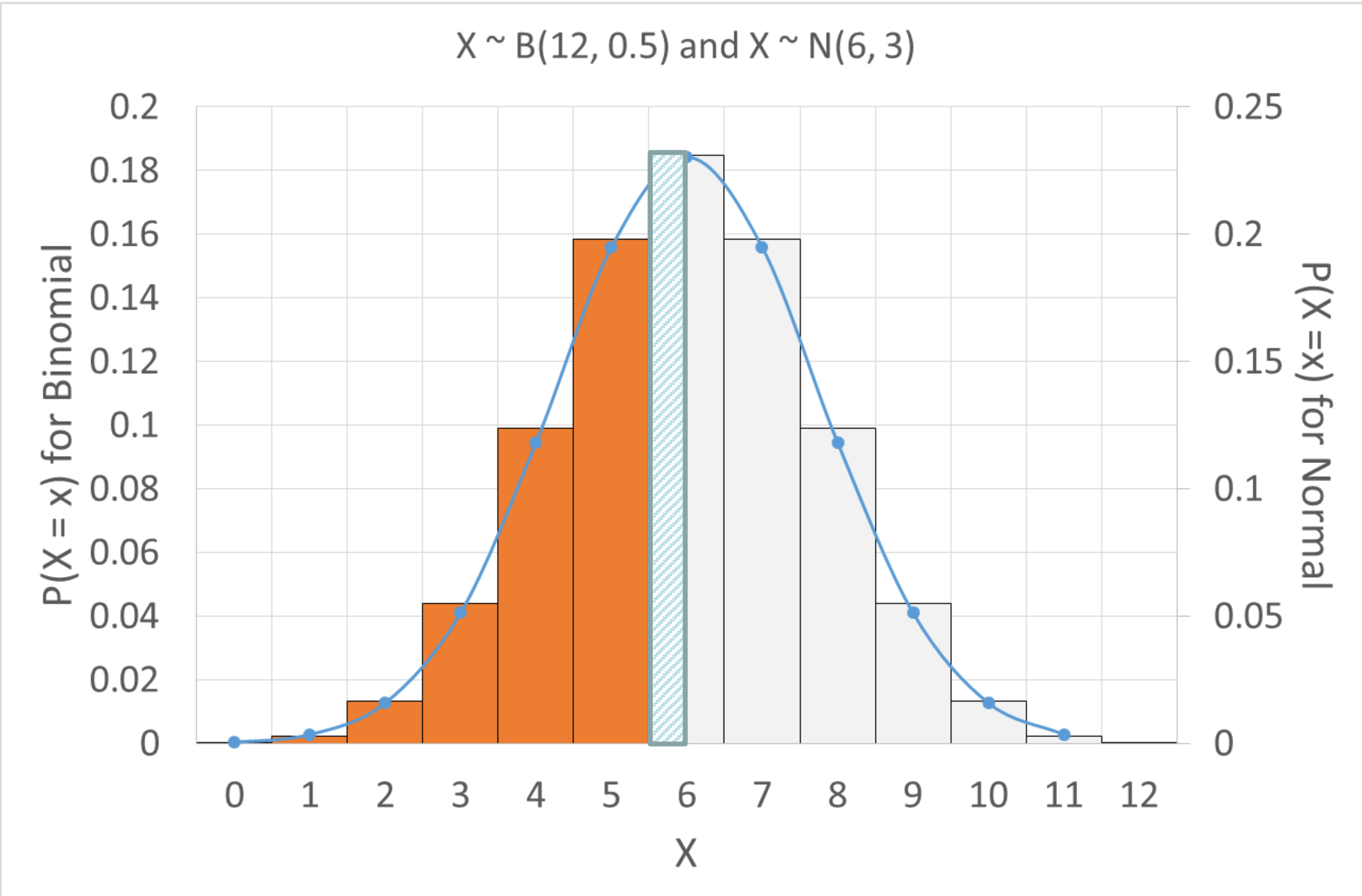
P(X < 6) = 0.5

Z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647

Continuity Correction?

So, $P(X < 6) = 0.387$ for $X \sim B(12, 0.5)$

and $P(X < 6) = 0.5$ for $X \sim N(6, 3)$. Is this a good approximation?



CSE 7315c

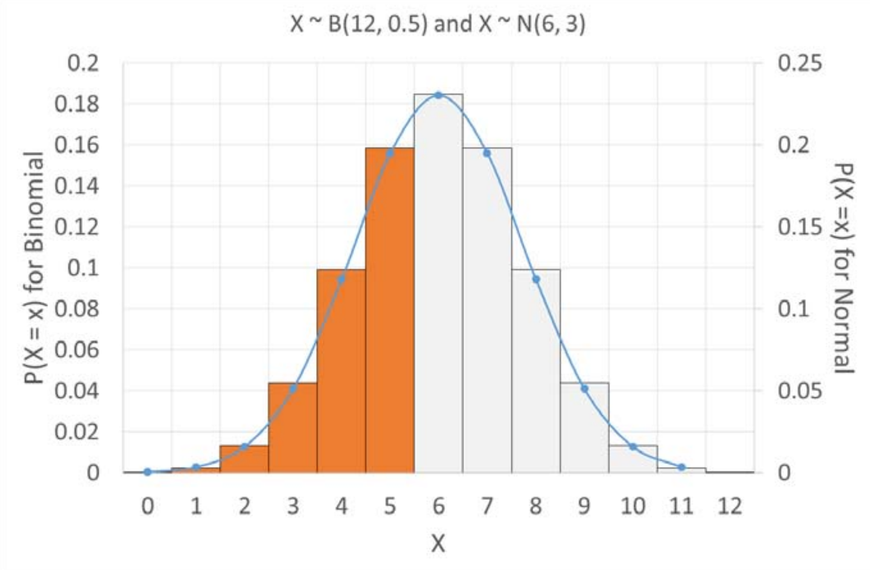
Continuity Correction?

So, $P(X < 6) = 0.387$ for $X \sim B(12, 0.5)$

and $P(X < 6) = 0.5$ for $X \sim N(6, 3)$.

$$P(X < 5.5) = \frac{5.5 - 6}{\sqrt{3}} = -0.29$$

$P(X < 5.5) = 0.3859$ for $X \sim N(6, 3)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Continuity Correction

Identify the right continuity correction for each discrete probability distribution.

Discrete	Continuous
$X < 3$	$X < 2.5$
$X > 3$	$X > 3.5$
$X \leq 3$	$X < 3.5$
$X \geq 3$	$X > 2.5$
$3 \leq X < 10$	$2.5 < X < 9.5$
$X = 0$	• If there are two possibilities with probability p for success and q for failure, and if we perform n trials, the probability that we see r successes is $C_r^n p^r q^{n-r}$
$3 \leq X \leq 10$	$E(X) = np$ $Var(X) = npq$ When to use? • You run a series of independent trials. • There can be either a success or a failure for each trial, and the probability of success is the same for each trial. • There are a finite number of trials, and you are interested in the number of successes or failures.
$3 < X \leq 10$	$3.5 < X < 10.5$
$X > 0$	$E(X) = \lambda$ Can be equated to np of Binomial if n is large (>50) and p is small (<0.1) $Var(X) = \lambda$ Can be equated to npq of Binomial in the above situation. When to use? • Individual events occur at random and independently in a given interval (time or space). • You know the mean number of occurrences, λ, in the interval or the rate of occurrences, and it is finite.
$3 < X < 10$	• If I randomly pick 10 people, what is the probability that I will get exactly – 0 loan takers $= 0.9^{10}$ – 1 loan taker$=10 * 0.9^9 * 0.1^1$ – 2 loan takers $=C_2^{10} * 0.9^8 * 0.1^2$

Continuity Correction

You are playing Who Wants to Win a 

ability of getting at least 30 out of 40 questions correct, where each question has 2 possible choices?

$$e^{-n\lambda}$$

$$P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{0!} = e^{-\lambda}$$

Normal Distribution

You have designed a new game, Angry Buds. The key to success is that it should not be so difficult that people get frustrated, nor should it be so easy that they don't get challenged. Before building the new level, you want to know what the mean and standard deviation are of the number of minutes people take to complete level 1. You know the following:

1. The # of minutes follows a normal distribution.
2. The probability of a player playing for less than 5 minutes is 0.0045.
3. The probability of a player playing for less than 15 minutes is 0.9641.

Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$P(X < 5) = 0.0045$

$z_1 = -2.61$

Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
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3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$P(X < 15) = 0.9641$

$z_2 = 1.8$

Normal Distribution

$-2.61 = \frac{5-\mu}{\sigma}$ and $1.8 = \frac{15-\mu}{\sigma}$

Solving for the above 2 equations, we get

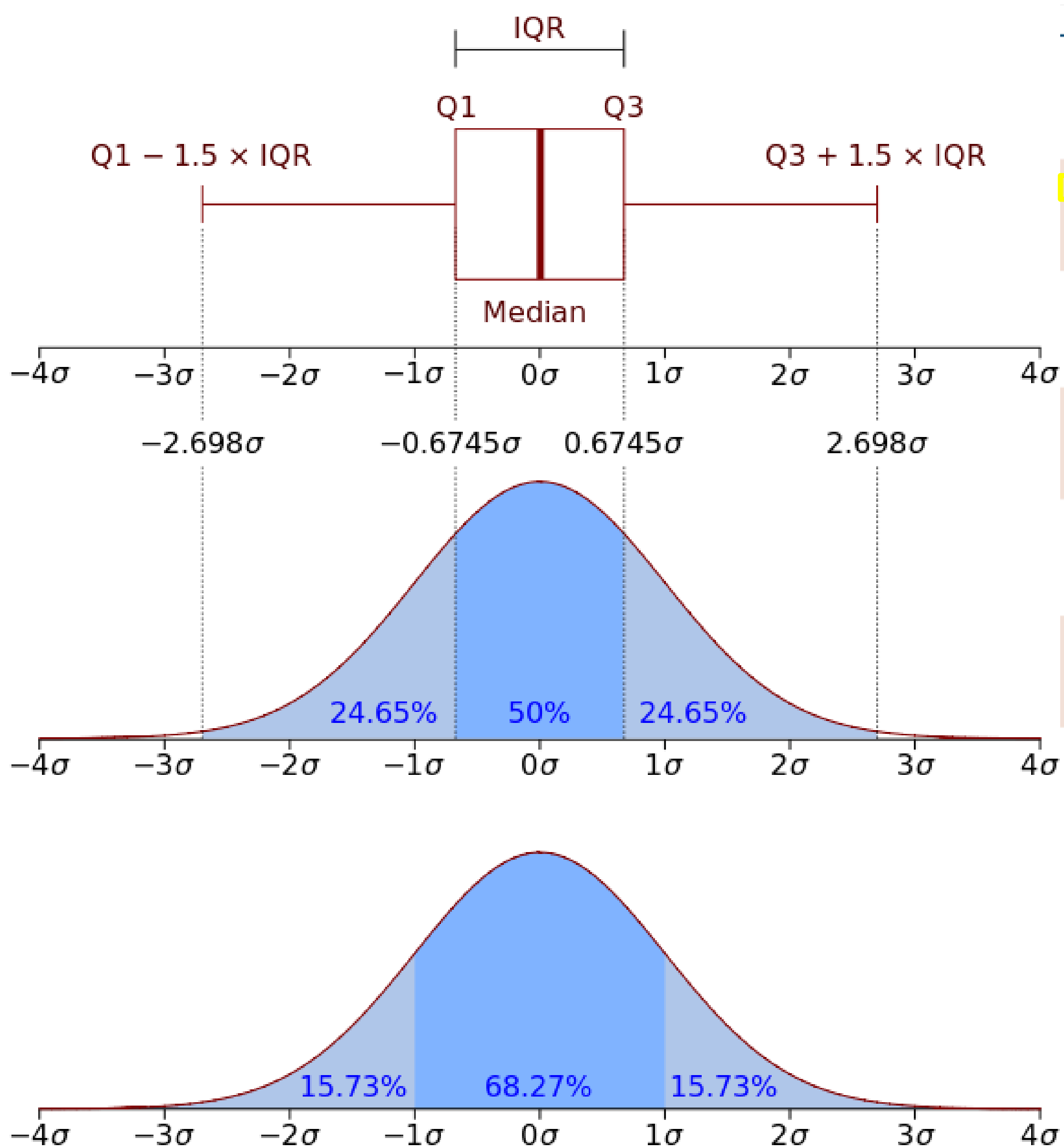
$\mu = 5 + 2.61\sigma$

$\mu = 15 - 1.8\sigma$

Subtracting the two, we get

$0 = -10 + 4.41\sigma \Rightarrow \ \sigma = 10 \div 4.41 = 2.27$

Substituting this value of σ in either of the above 2 equations, we get $\mu = 5 + 2.61 * 2.27 = 10.925$



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2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Source: http://en.wikipedia.org/wiki/Box_plot#mediaviewer/File:Boxplot_vs_PDF.svg; Last accessed: July 01, 2014

Academic exercise

0.75 is not in the z-table. The closest values are 0.7486 corresponding to a z-score of 0.67 and 0.7517 corresponding to a z-score of 0.68.

Method 1: Take the closest value, 0.7486 in this case.

Method 2: Interpolate.

z-value	P(0<Z<z)	Proportional distance from top
0.67	0.7486	0.0000
c (to be found)	0.75 (desired)	0.4516
0.68	0.7517	1.0000

$$\frac{0.75 - 0.7486}{0.7517 - 0.7486} = \frac{0.0014}{0.0031} = 0.4516$$

$$\Rightarrow \frac{c - 0.67}{0.68 - 0.67} = 0.4516$$

$$\therefore c = 0.67 + 0.4516 * 0.01 = 0.6745$$

SAMPLING DISTRIBUTION OF MEANS

Sampling distribution of the means

- The sampling distribution of means is what you get if you consider all possible samples of size n taken from the same population and form a distribution of their means.
- Each randomly selected sample is an independent observation.

Central Limit Theorem

- http://onlinestatbook.com/2/sampling_distributions/clt_demo.html
- As sample size goes large and number of buckets are high, the means will follow a normal distribution with same mean and $\frac{1}{n}$ of variance (σ^2).

Expectation and Variance for $\bar{X}$

$E(\bar{X}) = \mu$

Mean of all sample means of size *n* is the mean of the population.

$Var(\bar{X}) = \dfrac{\sigma^2}{n}$

Standard deviation of $\bar{X}$ tells how far away from the population mean the sample mean is likely to be and is called the **Standard Error of the Mean**, and is given by

$$Standard\ Error\ of\ the\ Mean = \frac{\sigma}{\sqrt{n}}$$

If $X \sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \sigma^2/n)$

When an attribute is not normal

- Let us assume it is a sample from infinite data
- So, if we take many such samples of large sample size (>30 as a thumb rule), the mean values, $\bar{x}$, will be hovering close to the population mean, μ, with a standard deviation, $s = \frac{\sigma}{\sqrt{n}}$, where σ is the population standard deviation and n is the sample size.

Using the Central Limit Theorem

Let us say the mean number of Gems per packet is 10, and the variance is 1. If you take a sample of 30 packets, what is the probability that the sample mean is 8.5 Gems per packet or fewer?

Using the Central Limit Theorem

We know that $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, $\mu = 10$, $\sigma^2 = 1$ and n = 30.

We need the value of $P(\bar{X} < 8.5)$ when $\bar{X} \sim N(10, 0.0333)$.

$$z = \frac{8.5 - 10}{\sqrt{0.0333}} = -8.22$$

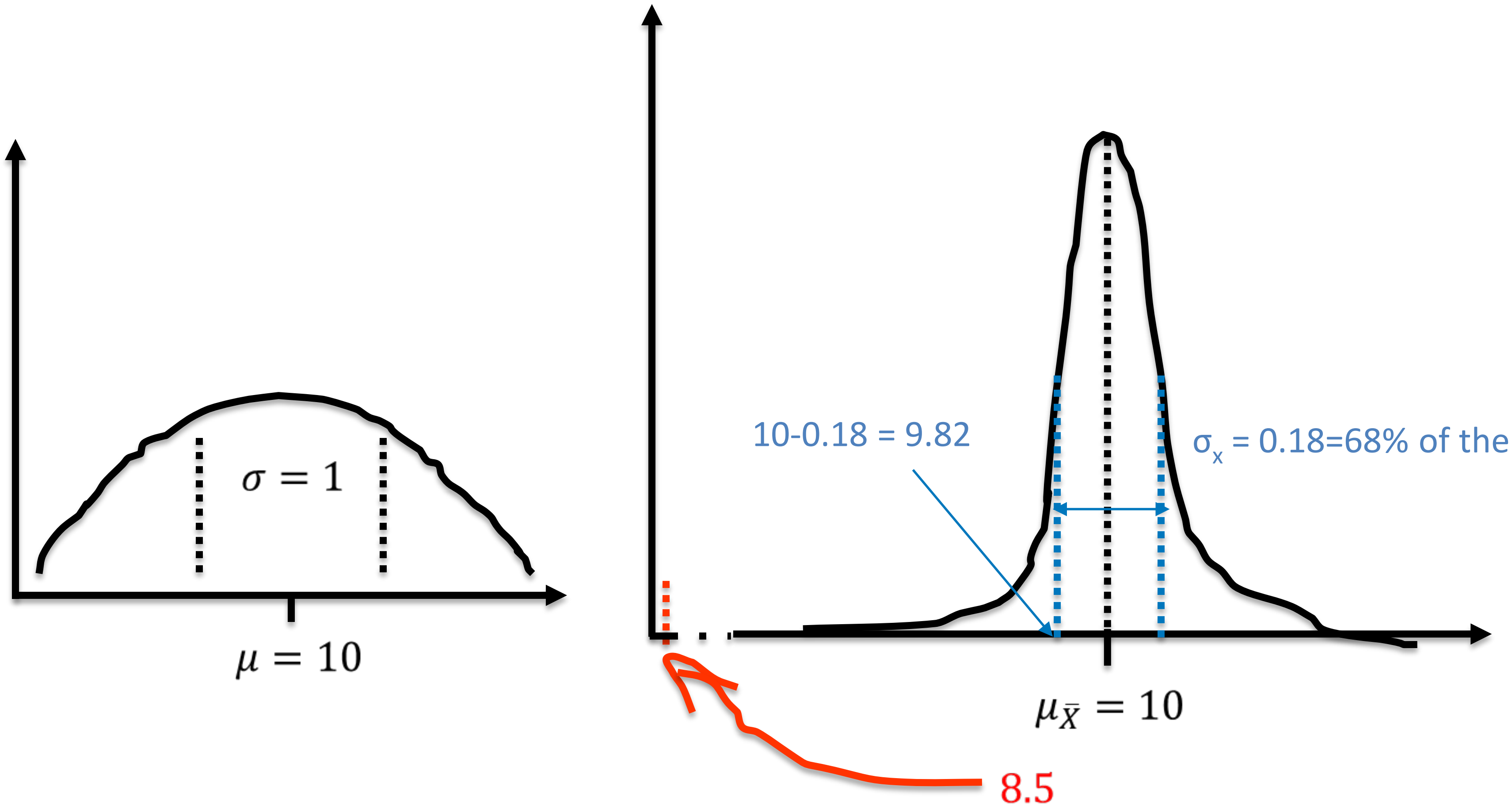
$P(Z < z) = P(Z < -8.22)$

This doesn't exist in probability tables. What does it mean?

CSE 7315c

Using the Central Limit Theorem

How do we visualize it?



CSE 7315c

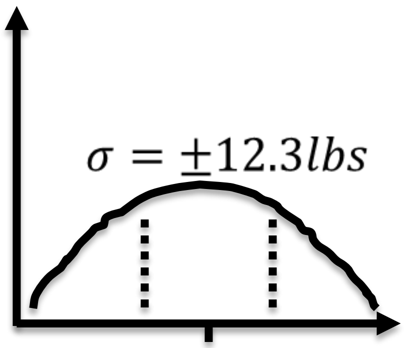
Using the Central Limit Theorem

The Aluminum Association of America reports that the average American household uses 56.8 lbs of aluminium in a year. A random sample of 51 households is monitored for one year to determine aluminium usage. If the population standard deviation of annual usage is 12.3 lbs, what is the probability that the sample mean will be $> 60\ lbs$?

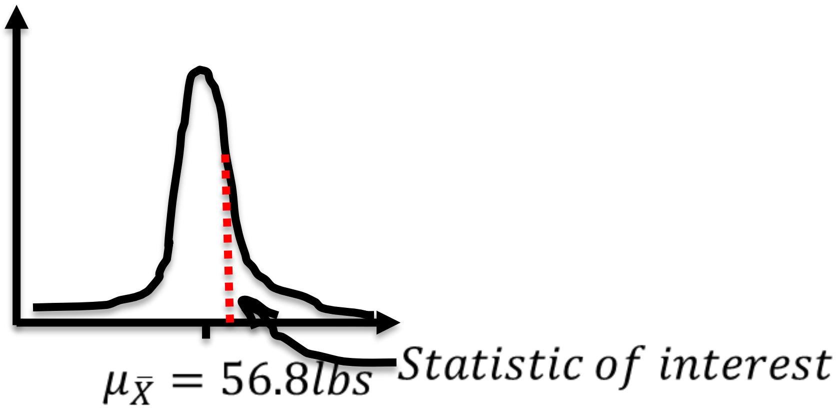
CSE 7315c

Sampling Distribution

Population distribution *Sampling distribution of sample mean when* $n = 51$

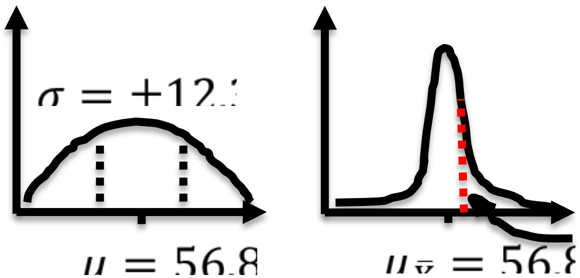


A. $Z = \frac{69-71}{4.5} = -0.44; P(Z<-0.44) = 0.33, \therefore P(Z>-0.44) = 0.67$ or 67%



- Step 1: List all known parameters and values
- Step 2: Calculate others or estimate, if cannot be calculated
- Step 3: Find probabilities using tables, Excel or R

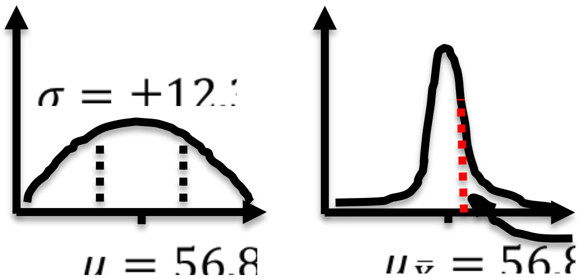
Sampling Distribution



$X < 3.5$

- Step 1: List all known parameters and values
  - Population mean, $\mu = 56.8\ lbs$
  - Population standard deviation, $\sigma = 12.3\ lbs$
  - Sample size, $n = 51$
  - Sample mean, $\bar{x} > 60\ lbs$
  - Mean of sample means, $\mu_{\bar{X}} = \mu = 56.8\ lbs$

Sampling Distribution



$X < 3.5$

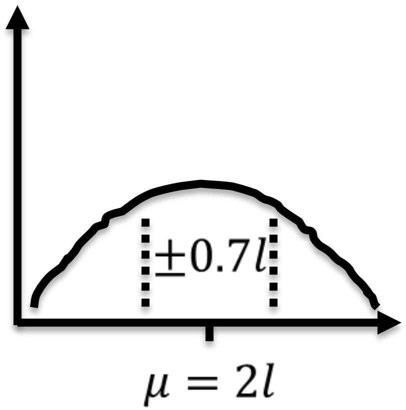
- Step 2: Calculate others or estimate, if cannot be calculated
  - Standard deviation of sample means, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12.3}{\sqrt{51}} = 1.72$
  - $\therefore z = \frac{60-56.8}{1.72} = 1.86$
- Step 3: Find probabilities using tables, Excel or R
  - Excel: 1-NORM.S.DIST(z,TRUE) = 0.0316
  - Please calculate these for:
    - $> 58\ lbs$
    - $> 56\ lbs < 57\ lbs$
    - $< 55\ lbs$
    - $< 50\ lbs$

Sampling Distribution

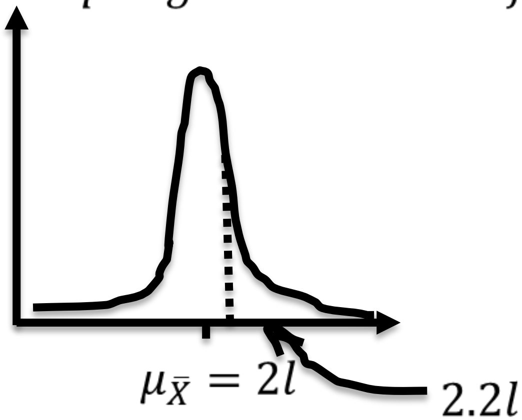
The average male drinks $2l$ of water when active outdoors with a standard deviation of $0.7l$. You are planning a trip for 50 men and bring $110l$ of water. What is the probability that you will run out of water?

$\mu = 2, \sigma = 0.7$

$P(run\ out) \Rightarrow P(use > 110l) \Rightarrow P(average\ water\ use\ per\ male > 2.2l)$



Sampling distribution of sample mean when $n = 50$



$\mu_{\bar{X}} = \mu = 2l, {\sigma_{\bar{X}}}^2 = \dfrac{\sigma^2}{n} = \dfrac{0.49}{50} \Rightarrow \sigma_{\bar{X}}$
$= 0.099$

$z = \dfrac{2.2 - 2}{0.099} = 2.02$

$P(\bar{X} < 2.02) = 0.9783$

The probability of running out is
1-0.9783 = 0.0217 or 2.17%

Source: https://www.khanacademy.org/math/probability/statistics-inferential/sampling_distribution/v/sampling-distribution-example-problem

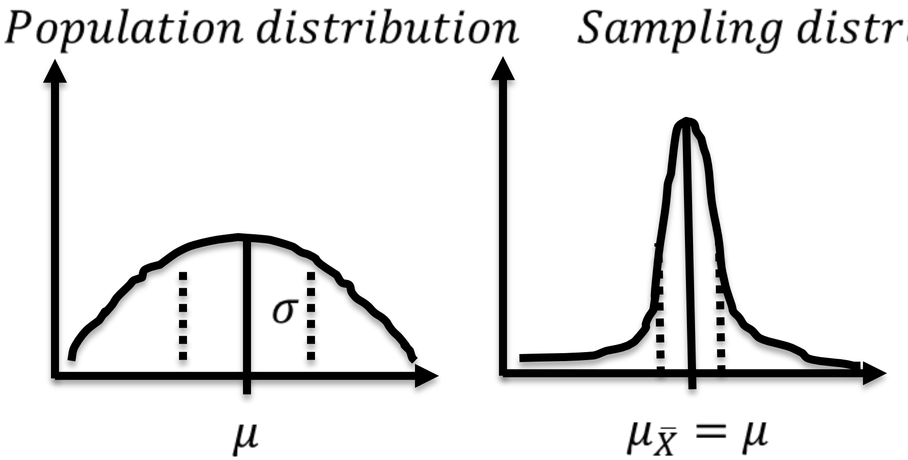
Last accessed: May 9, 2014

Using the Central Limit Theorem

You sample 36 apples from your farm's harvest of 200,000 apples. The mean weight of the sample is 112g with a 40g sample standard deviation. What is the probability that the mean weight of all 200,000 apples is between 100 and 124g?

CSE 7315c

Sampling Distribution



$$\sigma_{\bar{X}}{}^2 = \frac{\sigma^2}{n} = \frac{\sigma^2}{36} \Rightarrow \sigma_{\bar{X}} = \frac{\sigma}{6}$$

What are we trying to find out?

We need to know if population mean, μ, is within $\pm$12g of the sample mean, $\bar{X}$.

This is the same as saying that we need to know if sample mean, $\bar{X}$, is within $\pm$12g of the population mean, μ. Since $\mu = \mu_{\bar{X}}$, we can now use the sampling distribution of the means.

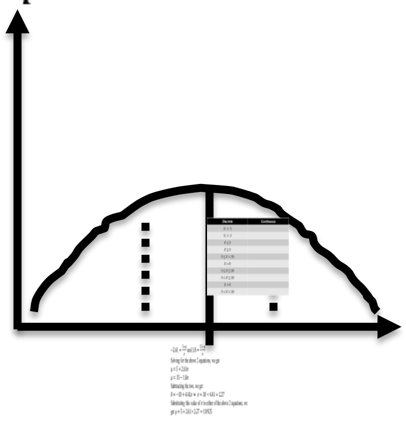
Source: https://www.khanacademy.org/math/probability/statistics-inferential/confidence-intervals/v/confidence-interval-1

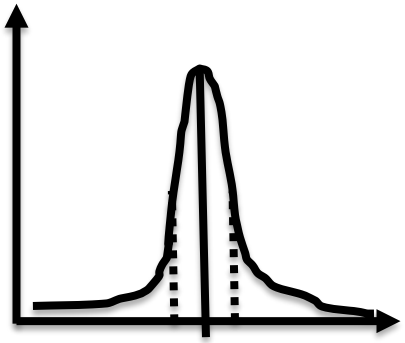
Last accessed: May 9, 2014

Sampling Distribution

Population distribution

$$X > 3.5$$





$$\mu_{\bar{X}} = \mu$$

$$E(X) = np$$
$$Var(X) = npq$$

When to use?

- You run a series of independent trials.
- There can be either a success or a failure for each trial, and the probability of success is the same for each trial.
- There are a finite number of trials, and you are interested in the number of successes or failures.

Find the region between -1.8 and +1.8 z.

0.9282. How would you get this answer if you did not have the negative z table?

Source: https://www.khanacademy.org/math/probability/statistics-inferential/confidence-intervals/v/confidence-interval-1

Last accessed: May 9, 2014

Normal Distribution

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2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

(0.9641-0.5000)*2

= 0.4641*2

= 0.9282

Using the Central Limit Theorem

- **Binomial distribution**

  If n>30 for a population with distribution X ~ B(n, p),

  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ or $\bar{X} \sim N(np, pq)$ since $\mu = np$ and $\sigma^2 = npq$.

- **Poisson distribution**

  If n>30 for a population with distribution X ~ Po(λ),

  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ or $\bar{X} \sim N\left(\lambda, \frac{\lambda}{n}\right)$ since $\mu = \sigma^2 = \lambda$.

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