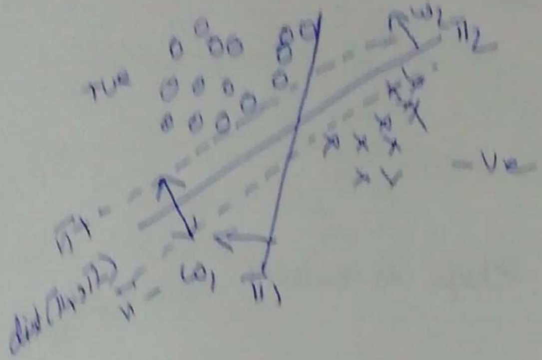
# Support Vector Machines (SVM)

→ SVM can do classification & Regression.

→ we have bunch of negative & positive points




→ Key Idea of SVM

we have to find the hyperplane ($\pi$) that separates +ve & -ve pts as widely as ~~positive~~. possible.

$\pi_+$ & $\pi_-$ are parallel to $\pi$

$\pi_+$ & $\pi_-$ are also parallel to each other

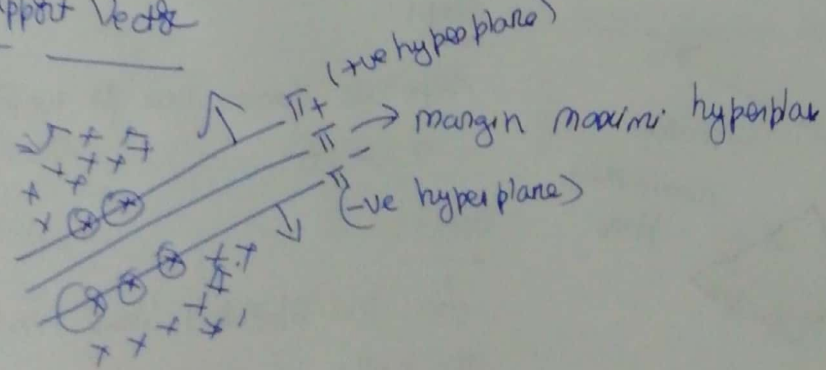
$\pi$: margin maximizing hyper plane.

SVM:- Try to find a $\pi$ that maximizes the margin = dist($\pi^+$, $\pi^-$)

margin↑ ; generalization acc↑

↳ Error on unseen data.

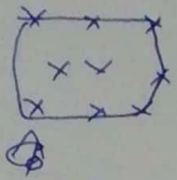
Support Vector




→ pts through which $\pi^+$ & $\pi^-$ pass through are support vectors!

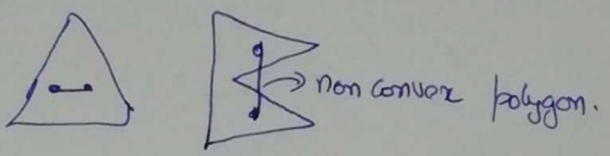
## Alternative geometry intuition of SVM :-

### Convex-hull



Convex-polygon

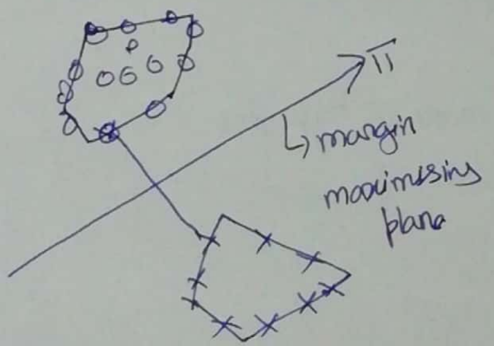
An points which is passing through the shape is called convex polygon.




Convex polygon :- If i want to connect the two points, the line connecting to both the points are inside the polygon is called convex polygon

### Convex-hull

A convex hull is given a bunch of pts, If we can build the smallest convex polygon, that has every point inside the convex polygon or ~~inside~~ on the convex polygon.




Step1

separate convex hull for +ve pts & -ve pts

Step2

find the shortest lines connecting the hulls

Step3

bisect the line (Equal parts)

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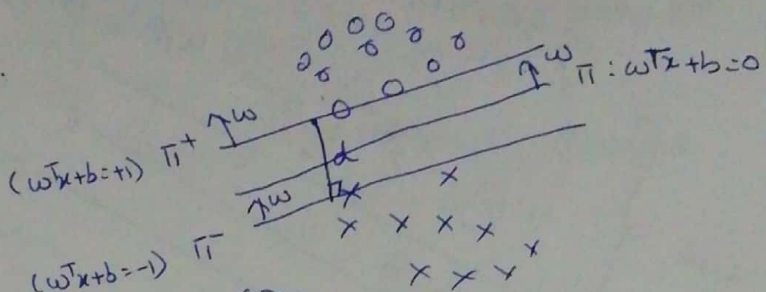
28.2 Mathematical derivation of SVM

$\pi$ : margin - maximization.

$\pi = w^T x + b = 0$




$w$ is perpendicular to $\pi$

$(w^*, b^*) = \underset{(w,b)}{\text{argmax}} = 2/||w||$

Since $\pi^+$ & $\pi^-$ are perpendicular, these are also perpendicular to $w$

If $\pi^+ : w^T x + b = \underline{1}$

$\pi^- : w^T x + b = -1$

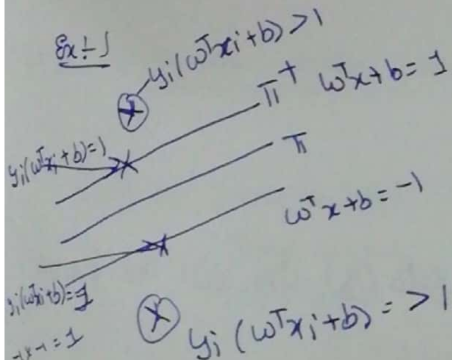
Margin :

$d = \dfrac{2}{||w||}$

$$w^*, b^* = \underset{(w,b)}{\text{argmax}} \; \frac{2}{||w||} = \text{margin.} \quad \text{S.t } y_i (w^T x_i + b) \geq 1 \text{ for all } x_i$$

hard margin SVM

Ex - 1




Constraint

All the +ve points should be one side & -ve points on the other side.

There should be any value in the margin area.

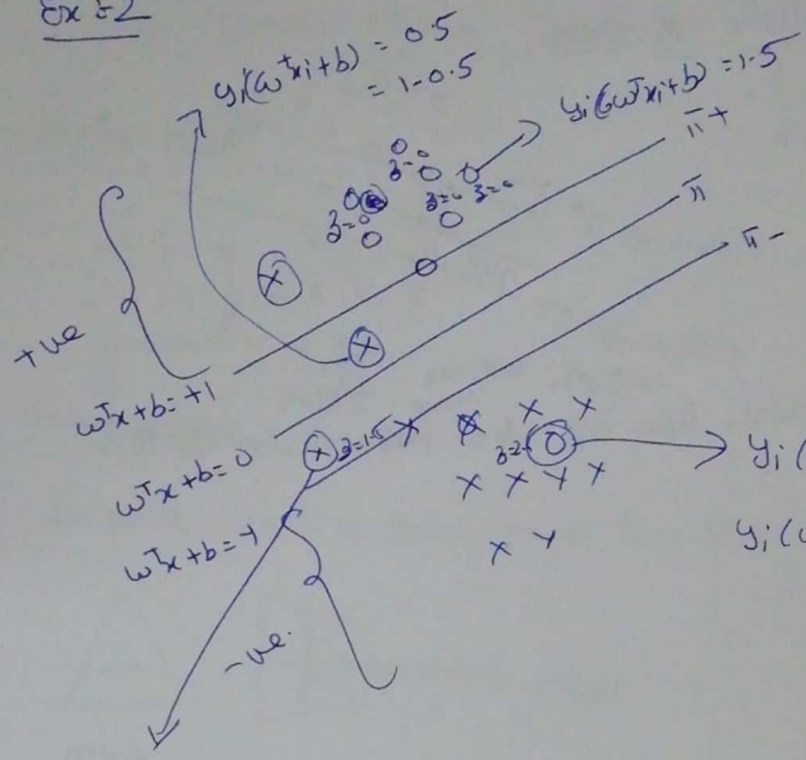
-) data should be linearly separable

$$w^* b^* = \underset{w,b}{\text{argmax}} \; \frac{2}{||w||}$$

S.t $\forall_i$ $\quad y_i (w^T x + b) \cancel{\geq} 1$

for each $\quad\quad \leq$

## Ex :- 2




{not linearly separable

& almost "

$y_i(w^Tx_i+b) = 1.5$

$y_i(w^Tx_i+b) = 1-2.5$

half way b/w $\pi$ & $\pi^-$

$$\boxed{\begin{aligned} y_i(w^Tx_i+b) &= -0.5 \\ y_i(w^Tx_i+b) &= (1-\underbrace{(1.5)}_{\xi_i}) \end{aligned}}$$

(zeta)

→ we will create a new variable called zeta ($\xi$) for all the points

~~such~~ that

$\xi_i \uparrow$, pt is furth away from correct $\pi$ in the incorrect direction.

$x_i \to \xi_i$

$\xi_i = 0$ if $y_i(w^Tx_i+b) \geq 1$

↑ correctly classified $\pi^+$ & $\pi^-$

$\xi_i > 0$ & it is equal to the some units of distance away from the correct hyperplane in the incorrect direction.

$$(w^*, b^*) = \underset{(w,b)}{\operatorname{argmax}} \frac{2}{\|w\|} = \underset{(w,b)}{\operatorname{argmin}} \frac{\|w\|}{2}$$

$$w^*, b^* = \underset{(w,b)}{\operatorname{argmin}} \ \frac{\|w\|}{2} + C \cdot \frac{1}{n}\sum_{i=1}^{n} \xi_i$$

→ regularizat

→ hyperparameter

→ loss

→ average distance of misclassification pts for correct $\pi$'s

st $y_i(w^T x_i + b) \geq 1 - \xi_i \quad \forall i$ → for each

$\boxed{\because \xi_i > 0}$

For correctly classified points $\xi_i = 0$

we have to minimize misclassification, means $\sum_{i=1}^{n} \xi_i$ should be less.

→ As C ↑, tendency to make mistake on $D_{train}$ ↓

⇒ overfit ⇒ high variance.

→ As C ↓ ; tendency to underfit ⇒ high bias

→ This is called soft margin SVM.

28.3 why we take values +1 & −1 for support vector plane.

$\|w\| \neq 1$ (any vector)
need not be
unit vector

margin $= \frac{2}{\|w\|}$

whole task is to maximize the margin.

① $\pi^+ : w^T x + b = k_1$ ; $\pi^- : w^T x + b = k_2$ $\boxed{k>0}$

why we taking only $k_1$ & $-k_2$, the $\pi^+$ & $\pi^-$ should be equally separable

margin $= \frac{2k}{\|w\|}$ ; $\underset{(w,b)}{\operatorname{argmax}} \frac{2}{\|w\|} = \underset{w,b}{\operatorname{argmax}} \frac{2k}{\|w\|} = \frac{8}{\|w\|}$ → k=4

we took +1 & −1 for convinience.

Case 2 $\pi^+$: $w^T x + b = k$

$$\left(\frac{w}{k}\right)^T x + \left(\frac{b}{k}\right) = 1$$

$w \perp \pi$

$\|w\|$ need not be 1

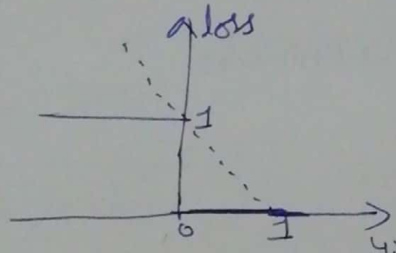
$$(w')^T x + b' = 1$$

## 28.4 Loss function (Hinge Loss) based interpretation

logistic regression → logistic loss + reg

lr. regression → lr. loss + reg

svm : hinge loss + reg




$y_i f(x_i) = y_i (w^T x_i + b) = z_i$

when $z_i > 0$ : $x_i$ correctly classified

$z_i < 0$ : $x_i$ is incorrectly classified.

hinge loss

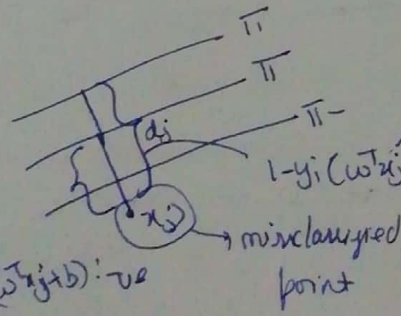
$z_i \geq 1$ ; hinge loss is 0

$z_i < 1$ ; hinge loss $= 1 - z_i$

→ $\max(0, 1 - z_i)$

case 1 : $z_i \geq 1$ ; $1 - z_i$ is a negative value $\Rightarrow \max(0, 1-z_i) = 0$

Case 2 : $z_i \leq 1$ ; $1 - z_i > 0 \Rightarrow \max(0, 1 - z_i) = 1 - z_i$




$1 - y_i (w^T x_j + b) =$ → assuming negative, negative * negative will become positive

→ misclassified point

$(w^T x_j + b)$ : -ve

$d_j = 1 - y_j(w^T x_j + b) = z_j$

$\xi$ = dist of $x_j$ to $\pi^+ = d_j = 1 - z_j$

$\xi_j = 1 - z_j \rightarrow$ when $x_j$ is misclassified

$\max(0, 1 - z_i) = \xi_i$

Soft svm : $\min_{w,b} \frac{\|w\|}{2} + \underbrace{\boxed{c}}_{\rightarrow \text{loss}} \sum_{i=1}^{n} \xi_i$

s.t $\left.\begin{array}{l}(1 - y_i(w^T x_i + b) \leq \xi_i \\ \xi \geq 0\end{array}\right\}$

$c\uparrow$ = overfit

$c\downarrow$ = underfit

loss min : $\min_{w,b} \sum_{i=1}^{n} \max(0, 1 - \underbrace{y_i(w^T x_i + b)}_{z_i}) + \overset{\rightarrow \text{reg}}{\boxed{\lambda}} \|w\|^2$

$\|w\| \geq 0 \Rightarrow \min \frac{\|w\|}{2}$ is same as $\min \|w\|^2$

$\lambda\uparrow$ = underfit

$\lambda\downarrow$ = overfit

$\xi = 0$ ; for correctly classified points

$\xi > 0$ ; for incorrectly classified points.

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## 28.5 Dual form of SVM formulation

$$\left\{\begin{array}{l} \min_{w,b} \frac{1}{2}\|w\| + C\sum_{i=1}^{n}\xi_i \\ \text{s.t } y_i(w^T x_i + b) \geq 1-\xi_i \ \forall i \\ \xi_i \geq 0 \end{array}\right\} = \underbrace{\max_{\alpha_i} \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j x_i^T x_j}_{\text{dual form of SVM}}$$

Primal of SVM

s.t $\alpha_i \geq 0$ → $\alpha_i = 0$ for non SVs, $\alpha_i > 0$ for support

$\sum_{i=1}^{n}\alpha_i y_i = 0$

### dual form

① for every $x_i$ there is an corresponding $\alpha_i$

② $x_i^T x_j \Rightarrow x_i$'s only occur in the form.

### End of the day

$x_q$ : $(w^T x_q + b) = f(x_q)$

③ $f(x_q) = \sum_{i=1}^{n}\alpha_i y_i x_i^T x_q + b$

④ $\alpha_i > 0$ only for support vectors

$\alpha_i = 0$ for non support vectors.

$f(x_q)$: only pts that matter are support vectors

$x_i^T x_j = x_i \cdot x_j = \text{cosine.sim}(x_i, x_j)$

If $\|x_i\| = 1$ ; $\|x_j\| = 1$

→ We can replace $x_i^T x_j$ with any similarity function $\text{sim}(x_i, x_j)$

→ This is what made SVM super popular

→ $\text{Sim}(x_i, x_j) = K(x_i, x_j)$

updated Eq

$$\max_{\alpha_i} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j K(x_i, x_j)$$

Kernal - fn.

$$\text{s.t } \sum_{i=1}^{n} \alpha_i y_i = 0 \; ; \alpha_i \geq 0$$

## 28.6 Kernel Trick

→ one class of similarity function is Kernal function.

The most important idea in svm is Kernal.

Soft svm hyperplane ≈ log-reg
↳ margin-max.

If we ~~Ki~~ did not apply Kernal trick and just leave it as $x_i^T x_j$.
Then it is called linear svm

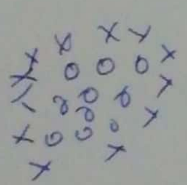
lr. svm : $x_i^T x_j$

Kernal svm : $K(x_i, x_j)$

In a linear svm we are finding margin maximizing hyperplane in the space of $x_i$'s

Log-reg we are ~~fi~~ finding a hyperplane which minimize logistic loss in the space of $x_i$'s

lr. svm & log. reg will in below case.



$f_1, f_2 \rightarrow f_1^2, f_2^2$

In the above case log-reg + feature transfd i succeed.

Kernal svm will be successful.
↳ It will transform $x_i \rightarrow x_i'$, It tries to find hyperplane in the space of $x_i'$

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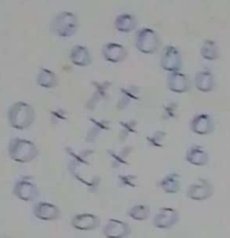
Kernel SVM : non-lr separable data

25.7 <u>Kernel</u> <u>Polynomial</u>



$\rightarrow (f_1, f_2) \xrightarrow{FT} (f_1^2, f_2^2)$

$\downarrow$

log reg

Let's see how Kernalization solve this problem.

$$K(x_1, x_2) = (x_1^T x_2 + c)^d$$

(e.g) $K(x_1, x_2) = \underbrace{(1 + x_1^T x_2)^2}_{\text{Quadratic Kernel}}$

$x_1 = \langle x_{11}, x_{12} \rangle$

$x_2 = \langle x_{21}, x_{22} \rangle$

$$= (1 + x_{11}x_{21} + x_{12}x_{22})^2$$

$$= 1 + x_{11}^2 x_{21}^2 + x_{12}^2 x_{22}^2 + 2x_{11}x_{21} + 2x_{12}x_{22} + 2x_{11}x_{21}x_{12}x_{22}$$

$$[1, x_{11}^2, x_{12}^2, \sqrt{2}x_{11}, \sqrt{2}x_{12}, \sqrt{2}x_{11}x_{12}] = x_1'$$

$$[1, x_{21}^2, x_{22}^2, \sqrt{2}x_{21}, \sqrt{2}x_{22}, \sqrt{2}x_{21}x_{22}] = x_2'$$

$$= (x_1')^T (x_2')$$

$$\begin{array}{ccc} x_1 & & x_2 \\ \downarrow & FT & \downarrow \\ x_1' & & x_2' \end{array} \qquad x_1^T x_2 \rightarrow x_1'^T x_2'$$

Kernalization takes data (d) $\xrightarrow[\text{Internally Implicitly}]{FT}$ $d'$ $\boxed{d' > d}$

Mercer's Theorem :-

Says what Kernel-trick is doing

$d \longrightarrow d' \longrightarrow$ lr-separable

typically $d' > d$

not
linearly separable.

SVM → Explicitly FT → logr-reg

↳ finding the right kernal.

## 28.7 RBF - Kernel

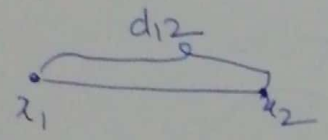
→ Radial Basis Function (RBF)

→ most popular / general purpose kernal : RBF

$(x_1, x_2)$

$$K_{RBF}(x_1, x_2) = \exp\left(\frac{-\|x_1 - x_2\|^2}{2\sigma^2}\right)$$

↑ hyper parameter

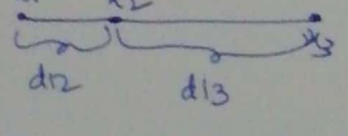



$\|x_1 - x_2\|^2 = d_{12}^2$

$$K_{RBF}(x_1, x_2) = \exp\left(\frac{-d_{12}^2}{2\sigma^2}\right) = \frac{1}{e^{d^2/2\sigma^2}}$$

$d_{12} \uparrow$ ; $K(x_1, x_2) \downarrow$

Similarity




$K(x_1, x_2) > K(x_1, x_3)$

(2) Impact of 'σ'

$\sigma = 1$, $\sigma = 0.1$, $\sigma = 10$

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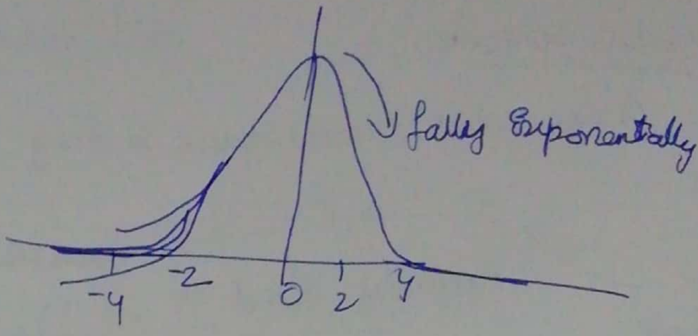
① $d = 0$

$k = 1$

② $d \uparrow$ ; $k \downarrow$, Exponentially




Kernal → Similar

dist → dissimilar.

RBF ≈ gaussian PDF

Case

$\sigma = 0.1$

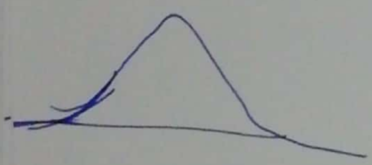
$\sigma^2 = 0.01$

$dis > 1$ ; $k = 0$

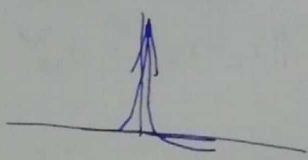
let $\sigma$ increase

$\sigma = 10$, $\sigma^2 = 100$

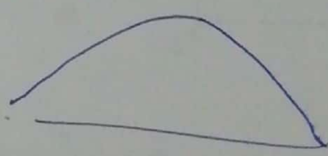
$\sigma = 1$



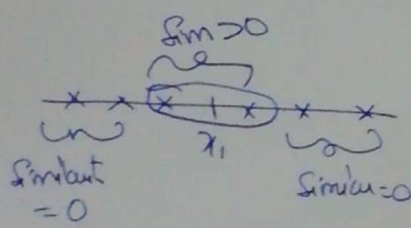
$\sigma = 0.1$



$\sigma = 10$

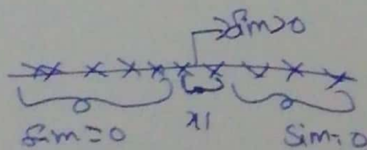


as $\sigma \uparrow$

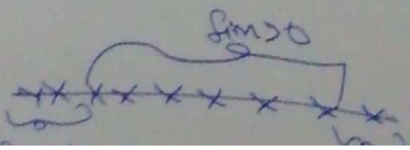



$= \sigma = 1$

$\sigma = 0.1$




$\sigma = 10$



There is a relation b/w k-NN & RBF Kernal.

$\sigma \uparrow \Rightarrow k \uparrow$ in kNN
(in RBF)

RBF-SVM $\approx$ KNN

k-NN : Store all the k pts. & LSH

RBF-SVM : we can just have SV's & its $\alpha_i$'s

# SV's $\ll n$
(number of points)

If we don't have best Kernal simply use RBF-SVM.

Soft margin : $C$ & $\sigma$

$(C, \sigma) \rightarrow$ grid search
random search.

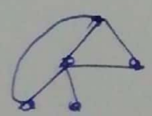
## 289 Domain Specific Kernels

RBF $\rightarrow$ general purpose kernel

→ String Kernels (for text classification)

→ genome Kernels

→ Graph Kernels



→ given the ~~data~~ problem, we have to select the appropiate Kernels

For Amazon data set, String kernel will give better results than RBF

28-10 : Train & runtime Complexities

Train → SGD
↳ Specialized algo (dual) → Sequential minimal optimization (SMO)

libsvm: best libraries for training SVM's.

Training Time ≈ $O(n^2)$ for kernel SVM's

If n is large → $O(n^2)$ is very large.

Typically donot use SVM when n is large

applications

Internet

Runtime Complexity

$$f(x_q) = \sum_{i=1}^{n} \alpha_i y_i K(x_i, x_q) + b$$

$\alpha = 0$ for non SV's

# SV's = K ⇒ $O(Kd)$

# of support vector

20.11 nu-SVM: Control Errors & support vectors.

C-SVM → Original formulation. $(C \geq 0)$

alternative formulation of SVM:

nu-SVM : $0 \leq nu \leq 1$

nu: hyper parameter

nu ≥ fraction of Error

SVM

Dtrain

↓

I don't want 10% Errors

↓

nu=0.1

↓

I don't want 1% Errors

↓

nu=0.01

Let assume

nu=0.01 ⇒ % of Errors ≤ 1%

# SV's ≥ 1% of n

→ For runtime Complexity : fewer SV's is ideal

28.13 Cases for svm's

→ Feature Engineering & FT

↳ Finding the right kernel (svm tends to work well)

→ Decision Surface

lr. svm's : hyperplane

Kernel-svm's : $d$ $x_i$ → non-linear surface

$\downarrow$ $\downarrow$

$d'$ $x_i'$ → linear surface.

$\boxed{d' > d}$

→ Imagine we are given distance or similarity fn

↳ $k(x_i, x_j)$

→ Interpretability & Feature Importance

↳ no way for us to get feature importance directly

↳ we can use forward feature selection

→ Outlier : very little impact on the model

↳ SV's that matter

↳ RBF with a small $\sigma$ → knn with small 'k'

↳ there too might get impacted with small $\sigma$ & $k$

→ Bias Variance

$C \uparrow$ → overfit ↑

$C \downarrow$ → underfit the model

→ large d → v.good for svm ($d \to d'$)

↳ good kernels ↳ we have otherwise RBF

→ Best cases :

↳ right Kernel

→ worst cases

↳ when 'n' is large → Training time is typically high

Ex: internet based application.

↳ K is large =

↳ we cannot have low latency

when n is v.v large ppl endup using Logistic Regression.

## 28.14 Code Sample

→ on Scikit Learn

→ Sklearn implements SVC, nu-SVC, SVR

//class. sklearn. svm. SVC (C=1.0, Kernal='rbf', degree=3, gamma='auto',

coef0=0.0, shrinking= True, probability = False, tol=0.001,

cache_size=200, class_weight= None, Verbose= False, max_iter=-1,

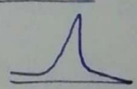
decision_function_shape = 'ovr', random_state= None)

//

→ C: bias & variance

degree is useful only when the kernel is polynomial

→ gamma: In RBF we have sigma ($\sigma$). In SK-learn it is referred to as gamma $\boxed{\gamma = 1/\sigma}$

$\sigma\downarrow$ $\gamma\uparrow$ 

tol: 0.001

→ Tolerance says when we are moving from $i^{th}$ iteration to $i+1^{th}$ iteration

$(w_i - w_{i+1})$

If difference b/w $w_i$ & $w_{i+1}$ is smaller than tolerance, terminate the loop.

→ class weight = None

Imbalance data if we want to do upsampling then we have to give class weight

→ max_iter: -1

it will iterate until the tolerance is reached

→ decision_function_shape

If it is multiclass then we have to provide 'ovr' (one vs rest)

```
import numpy as np
X = np.array([[-1,-1], [-2,-1], [1,1], [2,1]])
y = np.array([1,1,2,2])
from sklearn.svm import SVC
clf = SVC()
clf.fit(X,y)
-> print(clf.predict([[-0.8,-1]]))
```

o/p : [1]

*

<u>gamma</u>

The gamma parameters can be seen as the inverse of the radius of influence of samples selected by the model as support vectors.

gamma = 10^-1, c = $10^{-2}$ → g = $10^0$, c = $10^{-2}$, ⇒ g = 10^1, c = 10^-2

when σ ↓, we are overfitting $\xrightarrow{\text{when moving}}$

(or)

γ ↑

when k = 1, σ ↓ in RBF both are same, they will overfit

<u>Similarly</u>

when c ↑ it will overfit

gamma = $10^{-1}$, c = $10^{-2}$

g = $10^{-1}$, c = $10^{0}$

g = $10^{-1}$, c = $10^{-2}$

Sklearn. svm. NuSVC

Only thing will change is nu (nu=0.5)

nu:

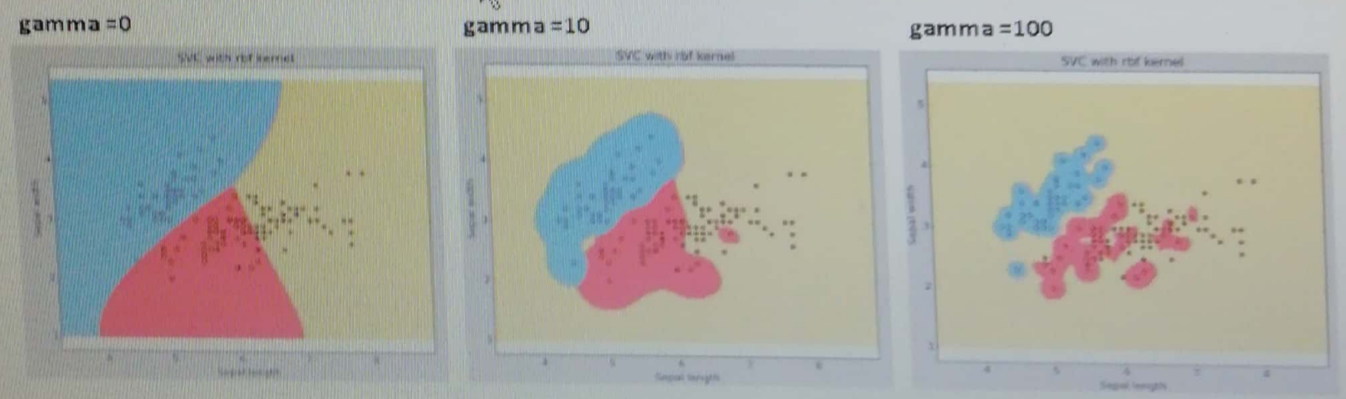
An upper bound on the fraction of training error and a lower bound of the fraction of support vectors.

Should be in the interval of (0, 1)

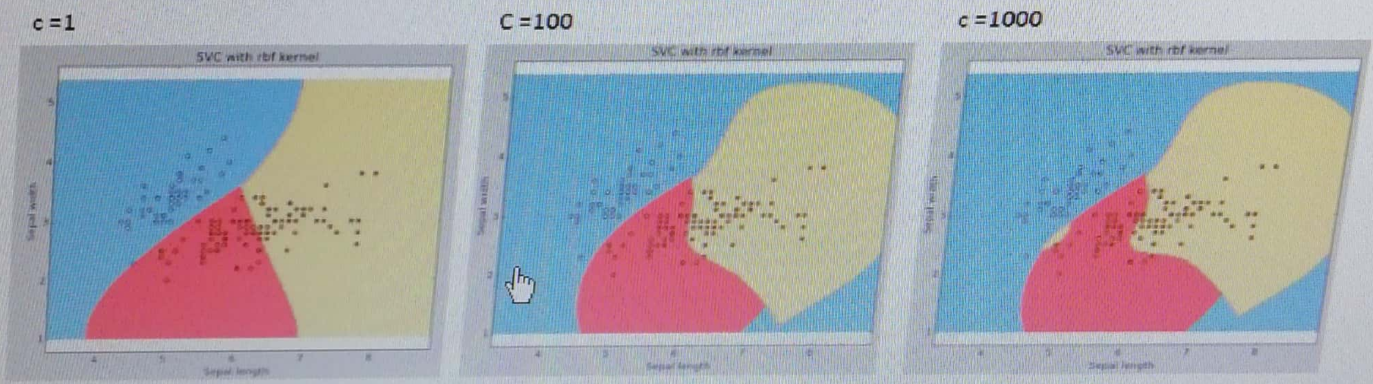
**gamma**: Kernel coefficient for 'rbf', 'poly' and 'sigmoid'. Higher the value of gamma, will try to exact fit the as per training data set i.e. generalization error and cause over-fitting problem.

**Example:** Let's difference if we have gamma different gamma values like 0, 10 or 100.

```
svc = svm.SVC(kernel='rbf', C=1,gamma=0).fit(X, y)
```



**C:** Penalty parameter C of the error term. It also controls the trade off between smooth decision boundary and classifying the training points correctly.




We should always look at the cross validation score to have effective combination of these parameters and avoid over-fitting.

## Pros and Cons associated with SVM

- **Pros:**
    - It works really well with clear margin of separation
    - It is effective in high dimensional spaces.
    - It is effective in cases where number of dimensions is greater than the number of samples.
    - It uses a subset of training points in the decision function (called support vectors), so it is also memory efficient.

- **Cons:**
    - It doesn't perform well, when we have large data set because the required training time is higher
    - It also doesn't perform very well, when the data set has more noise i.e. target classes are overlapping
    - SVM doesn't directly provide probability estimates, these are calculated using an expensive five-fold cross-validation. It is related SVC method of Python scikit-learn library.

JS] | https://medium.com/@aneesha/svm-parameter-tuning-in-scikit-learn-using-gridsearchcv-2413c02125a0

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```
def svc_param_selection(X, y, nfolds):
    Cs = [0.001, 0.01, 0.1, 1, 10]
    gammas = [0.001, 0.01, 0.1, 1]
    param_grid = {'C': Cs, 'gamma' : gammas}
    grid_search = GridSearchCV(svm.SVC(kernel='rbf'), param_grid,
cv=nfolds)
    grid_search.fit(X, y)
    grid_search.best_params_
    return grid_search.best_params_
```