Cross modulation in wide band receivers

Article Notes

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Introduction

In his paper *Cross-modulation in a CDMA Mobile Phone Receiver* [1], Y. Zhang derives a simple formula for calculating the cross-modulation power level, in a scenario where a Tx leakage signal and a close-by narrow-band jamming signal mix to provide interference in an Rx band.

When compared with empirical equations [2], Zhang's formula is shown to underestimate measured results by about 4-5%.

In this note, by using the same methodology and information as Zhang, we show that there is a small mathematical error in Zhang's formula. After correcting this error, we show that Zhang's formula underestimates measured results by only about 1-2%.

Crossmodulation power

Consider the spectral scenario shown in Figure 1. Due to nonlinearities in a mutual processing block, the Tx leakage signal and the narrow-band jamming signal may mix to produce interference in the desired Rx band. This interference is known as cross modulation. While it doesn't have to be, the Tx leakage signal is assumed to be wide-band, while the Jammer signal is assumed to be narrow-band.

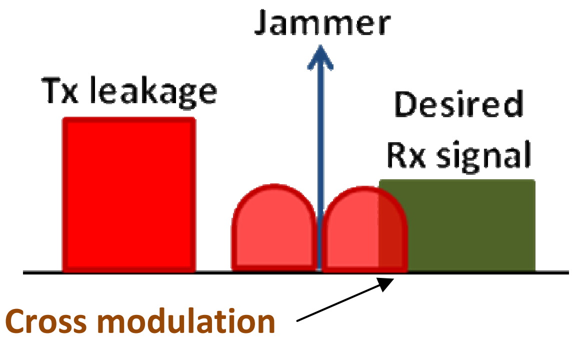

Figure 1: Spectral scenario for cross modulation

An example of a configuration where cross modulation may occur is illustrated in Fig. 2.

Due to finite isolation of the duplexer, some of the wideband Tx signal leaks into the Rx path. If the (wide-band) Rx band is located close to a narrow-band system, a portion of the narrow-band signal will also leak into the Rx path. These two leakage signals will mix to form cross modulation products in the Rx band.

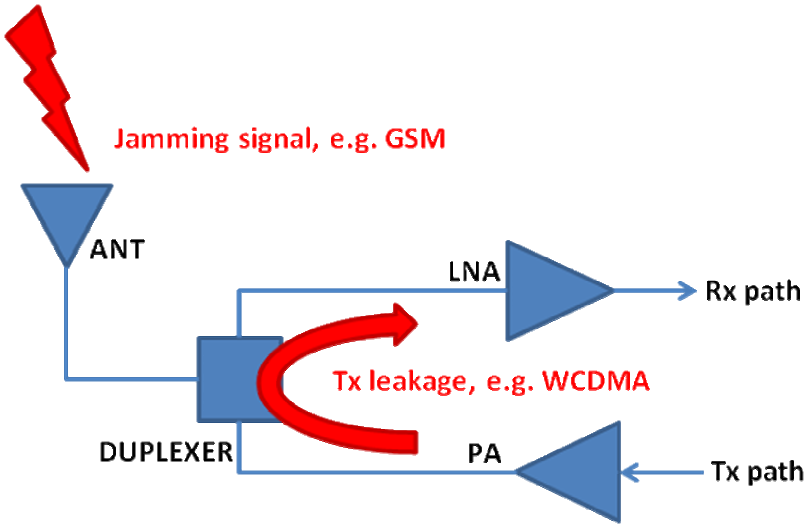

Figure 2: An example where cross modulation may occur

The amount of crossmodulation depends on:

- Leakage Tx (e.g. WCDMA) signal power
- Narrow-band Jammer power
- LNA IIP3.

Since the Tx signal is spread-spectrum, its spectral density can approximately be expressed as a rectangular function, as shown in Fig. 3, where B is the frequency bandwidth.

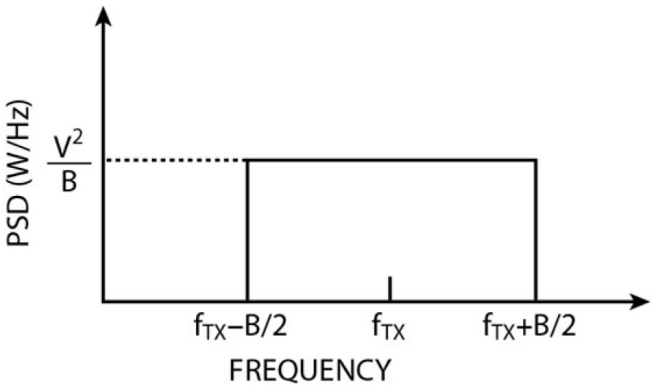

Figure 3

We know that in order to obtain a spectrum as shown in Fig. 3, signal in the time domain must be a *Sinc* function:

$$\mathrm{Sinc}(\omega_{\mathrm{Tx}} \cdot t) = \frac{\sin(\omega_{\mathrm{Tx}} \cdot t)}{\omega_{\mathrm{Tx}} \cdot t}$$

Equation 1

When the Tx signal is present at the LNA input along with a one-tone interference signal, the Tx signal is squared by the nonlinearity of the LNA, thus producing a low-frequency (baseband) product that has twice the bandwidth (B) of the original Tx signal.

This baseband product has power spectral density (PSD) in the form of a triangular function, because it is a result of the convolution of the rectangular PSD by itself. [3]

Crossmodulation is produced when this baseband product is modulated onto the one-tone interference signal, as shown in Fig. 4, where f_C is the centre frequency of the desired channel and f_I is the frequency of the interference signal. Also:

$$\Delta f = f_I - f_C$$

Equation 2

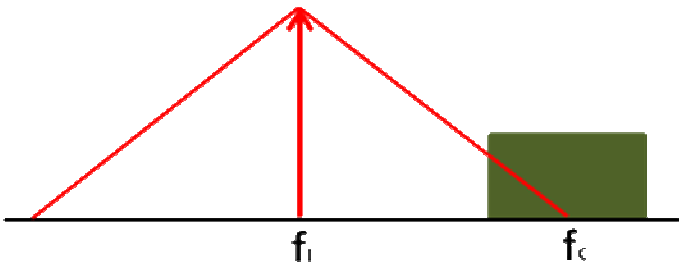

Figure 4

Crossmodulation product of a CDMA signal will not fall completely in the band of the desired signal, because of the frequency offset (Δf), and because the crossmodulation product has a bandwidth that is twice the bandwidth of the desired (e.g. WCDMA) signal.

The PSD of crossmodulation product may be expressed mathematically as:

$$PSD_{\mathrm{X-Mod}}(f) = \frac{A(P_I, IIP3)\cdot V^2}{4\cdot B^2}\left[f - (f_I - B)\right] \qquad IFF \quad (f_I - B) \le f \le f_I$$

Equation 3

$$PSD_{\mathrm{X-Mod}}(f) = \frac{A(P_I, IIP3)\cdot V^2}{4\cdot B^2}\left[(f_I + B) - f\right] \qquad IFF \quad f_I < f \le (f_I + B)$$

Equation 4

$$PSD_{\mathrm{X-Mod}}(f) = 0 \qquad \text{otherwise}$$

Equation 5

where

A = function of the interferer power level P_I (describes the nonlinearity of the LNA),

IIP3 = input third-order LNA intercept point.

The simple triangular function described by Eq. 3, Eq. 4, and Eq. 5 is illustrated in Fig. 5. Note that

$$PSD_{\mathrm{Xmod,max}} = \frac{A(P_I, IIP3)\cdot V^2}{4\cdot B}$$

Equation 6

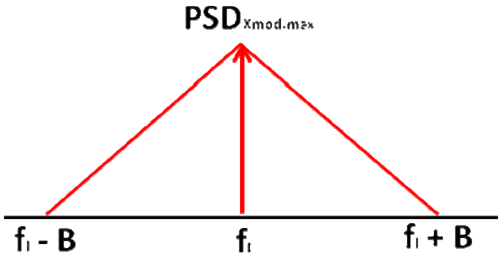

Figure 5: Illustration of Eq. 6

Cross modulation power that falls in the Rx band can be calculated from the overlap integral, where the two overlapping areas are: the crossmodulation power density, and the desired signal power density, as shown in Fig. 6 (orange triangle area).

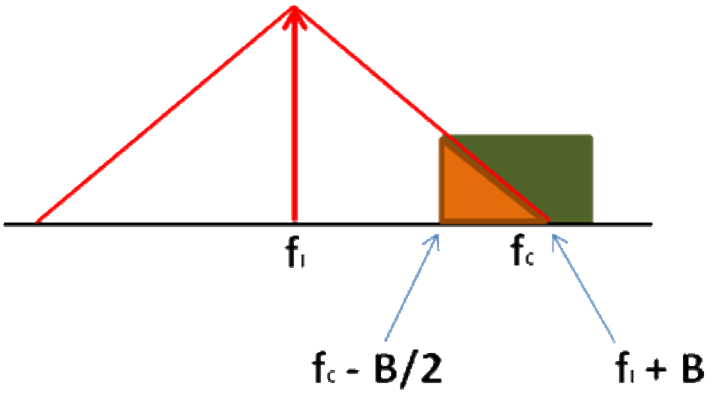

Figure 6: Calculating cross modulation power from the overlap integral

The overlap integral is written as:

$$P_{\text{Xmod, tot,Rx band}} = \int_{f_C-\frac{B}{2}}^{f_I+B} PSD_{\text{Xmod}}(f)\cdot df =$$

$$= \int_{f_C-\frac{B}{2}}^{f_I+B} \frac{A(P_I, IIP3)\cdot V^2}{4\cdot B^2}\left[(f_I + B) - f\right]\cdot df$$

Equation 7

This is in effect the area of the orange triangle in Fig. 6.

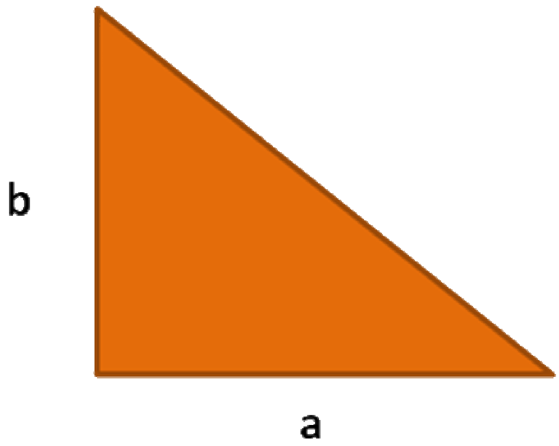

Figure 7

With reference toFig. 7, the area of this right-angle triangle is given by

$$A_{\text{triangle}} = \frac{a \cdot b}{2}$$

Equation 8

In this case we have:

$$\begin{aligned} a &= (f_{\mathrm{I}} + B) - \left(f_{\mathrm{C}} - \frac{B}{2} \right) \\ &= (f_{\mathrm{I}} + B) - \left(f_{\mathrm{C}} - \frac{B}{2} \right) \\ &= f_{\mathrm{I}} - f_{\mathrm{C}} + \frac{3B}{2} \\ &= \Delta f + \frac{3B}{2} \end{aligned}$$

Equation 9

and:

$$\begin{aligned} b &= P_{\text{Xmod, Rx band}}\left(f_{\mathrm{C}} - \frac{B}{2} \right) \\ &= \frac{A(P_{\mathrm{I}}, IIP3) \cdot V^2}{4 \cdot B^2} \cdot \left[(f_{\mathrm{I}} + B) - \left(f_{\mathrm{C}} - \frac{B}{2} \right) \right] \\ &= \frac{A(P_{\mathrm{I}}, IIP3) \cdot V^2}{4 \cdot B^2} \cdot \left[f_{\mathrm{I}} - f_{\mathrm{C}} + \frac{3B}{2} \right] \\ &= \frac{A(P_{\mathrm{I}}, IIP3) \cdot V^2}{4 \cdot B^2} \cdot \left[\Delta f + \frac{3B}{2} \right] \end{aligned}$$

Equation 10

The total power is then

$$
\begin{aligned}
P_{\text{Xmod, tot,Rx band}} &= A_{\text{triangle}} \\
&= \frac{a \cdot b}{2} \\
&= \frac{1}{2} \cdot \left[\Delta f + \frac{3B}{2} \right] \cdot \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{4 \cdot B^2} \cdot \left[\Delta f + \frac{3B}{2} \right] \\
&= \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{8 \cdot B^2} \cdot \left[\Delta f + \frac{3B}{2} \right]^2
\end{aligned}
$$

Equation 11

with B^2 taken out of the bracket above, we obtain:

$$
\begin{aligned}
P_{\text{Xmod, tot,Rx band}} &= \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{8 \cdot B^2} \cdot B^2 \cdot \left[\frac{\Delta f}{B} + \frac{3}{2} \right]^2 \\
&= \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{8} \cdot \left[\frac{\Delta f}{B} + \frac{3}{2} \right]^2 \\
&= \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{8} \cdot \left[\frac{3}{2} + \frac{\Delta f}{B} \right]^2
\end{aligned}
$$

Equation 12

Equation 12 gives an expression for the total cross modulation power that falls in the desired Rx band, as a function of:

- narrow-band interferer (jammer) power, P_{I}
- input third-order LNA intercept point, *IIP*3
- location of the jamming signal, relative to the desired-band centre frequency, Δf, and
- wide-band Tx signal bandwidth, B

Cross modulation power ratio

It may not always be possible to know or measure factor V, which also features in Eq. 12. This factor is used to describe, as per Fig. 3, the amplitude of the power spectral density function of the wide-band Tx signal.

It is therefore interesting to calculate the ratio of the total crossmodulation power to the portion of this power that falls into the Rx band. This is a convenient way to avoid using V in calculations. If we call this portion C, we obtain:

$$
C = \frac{P_{\text{X-Mod}}(\text{in Rx band})}{P_{\text{X-Mod}}(\text{total})}
$$

Equation 13

$P_{\text{X-Mod}}$(*in Rx band*) is equal to $P_{\text{X-Mod,tot,Rx band}}$ and is given by Eq. 12.

The total crossmodulation power can be worked out by considering the area of triangle shown in Fig. 8 (same as Fig. 5).

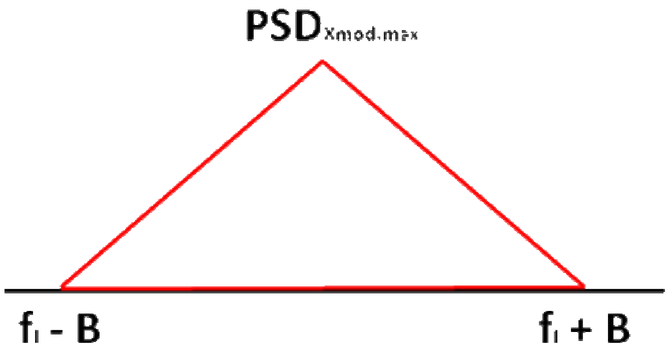

Figure 8

This area is given by

$$A_{\text{triangle}} = \frac{\text{base} \cdot \text{height}}{2}$$

Equation 14

Therefore,

$$\begin{aligned} P_{\text{Xmod, tot}} &= A_{\text{triangle}} \\ &= \frac{1}{2} \cdot [(f_{\text{I}} + B) - (f_{\text{I}} - B)] \cdot PSD_{\text{Xmod}}(f_{\text{I}}) \\ &= \frac{1}{2} \cdot [2 \cdot B] \cdot PSD_{\text{Xmod,max}} \\ &= \frac{1}{2} \cdot [2B] \cdot \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{4 \cdot B} \\ &= \frac{A(P_{\text{I}}, IIP3) \cdot V^2}{4} \end{aligned}$$

Equation 15

Factor C therefore becomes:

$$\begin{aligned} C &= \frac{P_{\text{X-Mod}}(\text{in Rx band})}{P_{\text{X-Mod}}(\text{total})} \\ &= \frac{\frac{A(P_{\text{I}}, IIP3) \cdot V^2}{8} \cdot \left[\frac{3}{2} + \frac{\Delta f}{B}\right]^2}{\frac{A(P_{\text{I}}, IIP3) \cdot V^2}{4}} \\ &= \frac{1}{2} \cdot \left[\frac{3}{2} + \frac{\Delta f}{B}\right]^2 \end{aligned}$$

Equation 16

Since $f_I < f_C$, $\Delta f < 0$ Eq. 16 becomes:

$$C = \frac{1}{2} \cdot \left[\frac{3}{2} - \frac{|\Delta f|}{B}\right]^2$$

Equation 17

Equation 17 allows one to calculate the portion of the total crossmodulation power that falls in the Rx band, as a function of two simple parameters:

- location of the jamming signal, relative to the desired-band centre frequency, Δf, and
- wide-band Tx signal bandwidth, B.

Power level

The last remaining step required to find the absolute crossmodulation power in the Rx band is to calculate the total crossmodulation power produced. The two are related by factor C given by Eq. 17.

The same nonlinearity of the LNA is responsible for both intermodulation and crossmodulation products. Therefore, the total crossmodulation power can be calculated by using:

$$P_{(2\mathrm{A}-\mathrm{B})} = 2P_{\mathrm{A}} + P_{\mathrm{B}} - 2 \cdot IIP_3$$

Equation 18

where

- P_{A} = power level (dBm) of the signal that is squared, which is the wide-band Tx signal in the case of cross modulation = P_{Tx}
- P_{B} = single-tone interference (dBm), which is the narrow-band jamming signal in the case of cross modulation = P_{I}
- $IIP3$ = input third-order intercept point of the nonlinear device (dBm), the low-noise amplifier in the case of both intermodulation and cross modulation.

All power levels in Eq. 18 are referred to the input of the device. Note that for a single-tone intermodulation interference, all the interference power falls in the band of the desired signal.

From previous discussion we know that

$$P_{(2\mathrm{A}-\mathrm{B})} = P_{\mathrm{Xmod}}(\mathrm{total}) = P_{\mathrm{Xmod}}(\mathrm{in\ Rx\ band}) - C(\mathrm{dB})$$

Equation 19

Finally, we obtain:

$$\begin{aligned} P_{\mathrm{Xmod}}(\mathrm{in\,Rx\,band}) &= P_{\mathrm{Xmod}}(\mathrm{total}) + C(\mathrm{dB}) = \\ &= 2P_{\mathrm{A}} + P_{\mathrm{B}} - 2 \cdot IIP_3 + C(\mathrm{dB}) \end{aligned}$$

Equation 20

In the fully expanded form we have:

$$P_{\mathrm{Xmod}}(\mathrm{in\,Rx\,band}) = 2P_{\mathrm{Tx}} + P_{\mathrm{I}} - 2 \cdot IIP_3 + 10 \cdot \log\left(\frac{1}{2} \cdot \left[\frac{3}{2} - \frac{|\Delta f|}{B}\right]^2\right)$$

Equation 21

Difference with the original paper

In the original paper (Equation 5 in [1]), the following expression was quoted for the total crossmodulation power:

$$P_{X\,\mathrm{mod}}(total) = \frac{1}{2} \cdot A(P_I, IIP3) \cdot V^2$$

Equation 22

It has been shown above that this power should be half of this:

$$P_{\mathrm{Xmod,\,tot}} = \frac{1}{4} \cdot A(P_I, IIP3) \cdot V^2$$

Equation 23

It is believed that this error occurred while working out the area of the triangle representing the crossmodulation spectrum.

This difference also affects the value of *C*, namely the paper states:

$$C = \frac{1}{4} \cdot \left(\frac{3}{2} - \frac{|\Delta f|}{B} \right)^2$$

Equation 24

while here it has been shown:

$$C = \frac{1}{2} \cdot \left[\frac{3}{2} - \frac{|\Delta f|}{B} \right]^2$$

Equation 25

This again is the difference of ½ or 3 dB.

Comparison with measured results

In the last section of their paper Zhang compares his prediction of the crossmodulation power in the Rx band with the prediction based on the Draxler:

$$P_{\mathrm{Xmod}}(\text{in Rx band}) = 1.906 \cdot P_{\mathrm{Tx}} + 0.949 \cdot P_{\mathrm{I}} - 1.852 \cdot IIP_3 - 8.4$$

Equation 26

and

$$P_{\mathrm{Xmod}}(\text{in Rx band}) = 1.913 \cdot P_{\mathrm{Tx}} + 0.949 \cdot P_{\mathrm{I}} - 1.864 \cdot IIP_3 - 12.7$$

Equation 27

Draxler's first equation (Eq. 26) was derived specifically for the cellular band, while Draxler's second equation (Eq. 27) was derived specifically for the PCS band. Parameters of these two bands are shown below.

Draxler's equations are empirical, and they have been derived through a series of discrete time-domain simulations using Omnisys (Agilent Technologies) and curve-fitting with linear regression.

In the cellular band:

$$\Delta f = 900\,kHz$$
$$B = 1.23\,MHz$$

Equation 28

In the PCS band:

$$\Delta f = 1.25\,MHz$$
$$B = 1.23\,MHz$$

Equation 29

Table 1 below shows the difference in the prediction of the crossmodulation power in the Rx band, when three methods are used: Draxler's empirical equations, Zhang's original equations, and Zhang's modified equations (this paper).

	Cellular band	**PCS band**
Draxler's empirical equations	-118.3	-123.0
Zhang's equations	-124.3	-128.3
Zhang's modified equations (this paper)	-121.3	-125.3
Draxler-Original difference	**-4.8 %**	**-4.2 %**
Draxler-Modified difference (this paper)	**-0.8 %**	**-1.8 %**

Table 1

Other parameters used in calculations are: P_{Tx} = 33 dBm, P_{I} = 32 dBm, $IIP3$ = 9 dBm.

Table 1 shows that, when the small mathematical error is corrected, Zhang's equations in fact give better results than originally thought.

References

[1] Y. Zhang, "Cross-modulation in CDMA Mobile Phone Receiver," Microwave Journal, October 2003.

[2] V. Aparin, B. Butler, and P. Draxler, "Cross-modulation Distortion in CDMA Receivers," IEEE Transactions on Microwave Theory and Techniques, vol. 48, no. 12, December 2000, pp. 100 – 109.

[3] E. O. Brigham, "The Fast Fourier Transform and Its Applications," Prentice-Hall, Englewood Cliffs, 1998, pp. 60 – 65.

[4] W. Y. Ali-Ahmad, "RF System Issues Related to CDMA Receiver Specifications," RF Design, September 1999.

Version history

Version B (18 SEP 2010): Correction of small errors and re-formatting.

Version A (21 FEB 2009): First release.