—— Problem 3 ——

$$\mathrm{L} = \begin{bmatrix} s+2 & -8 & -7 \\ -5 & s+4 & -2 \\ 9 & 3 & s+1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathrm{M}_1(1/(s+2))...InMatlab : R(1,:) = (1/L(1,1)) * R(1,:); L(1,:) = 1/L(1,1)) * L(1,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ -5 & s+4 & -2 \\ 9 & 3 & s+1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathrm{A}_1 toA_3(-9)...InMatlab : R(3,:) = -1 * L(3,1) * R(1,:); L(3,:) = -1 * L(3,1) * L(1,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ -5 & s+4 & -2 \\ 0 & \frac{3\,(s+26)}{s+2} & \frac{s^2+3\,s+65}{s+2} \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{9}{s+2} & 0 & 1 \end{bmatrix}$$

$\mathrm{M}_3(1/(3 * (s+26))/(s+2))...InMatlab : R(3,:) = (1/L(3,)) * R(3,:); L(3,:) = (1/L(3,)) * L(3,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ -5 & s+4 & -2 \\ 0 & 1 & \frac{s^2+3\,s+65}{3\,(s+26)} \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{3}{s+26} & 0 & \frac{s+2}{3\,(s+26)} \end{bmatrix}$$

$\mathrm{A}_1 toA_2(5)...InMatlab : R(2,:) = -1 * L(2,1) * R(1,:); L(2,:) = -1 * L(2,1) * L(1,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ 0 & \frac{s^2+6\,s-32}{s+2} & -\frac{2\,s+39}{s+2} \\ 0 & 1 & \frac{s^2+3\,s+65}{3\,(s+26)} \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ \frac{5}{s+2} & 1 & 0 \\ -\frac{3}{s+26} & 0 & \frac{s+2}{3\,(s+26)} \end{bmatrix}$$

$\mathrm{M}_2(1/(6 * s + s^2 - 32)/(s+2))...InMatlab : R(2,:) = (1/L(2,)) * R(2,:); L(2,:) = (1/L(2,)) * L(2,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ 0 & 1 & -\frac{2\,s+39}{s^2+6\,s-32} \\ 0 & 1 & \frac{s^2+3\,s+65}{3\,(s+26)} \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ \frac{5}{s^2+6\,s-32} & \frac{s+2}{s^2+6\,s-32} & 0 \\ -\frac{3}{s+26} & 0 & \frac{s+2}{3\,(s+26)} \end{bmatrix}$$

$\mathrm{A}_2 toA_3(-1)...InMatlab : R(3,:) = -1 * L(3,2) * R(2,:); L(3,:) = -1 * L(3,2) * L(2,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ 0 & 1 & -\frac{2\,s+39}{s^2+6\,s-32} \\ 0 & 0 & \frac{s^4+9\,s^3+57\,s^2+567\,s+962}{3\,(s+26)\,(s^2+6\,s-32)} \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ \frac{5}{s^2+6\,s-32} & \frac{s+2}{s^2+6\,s-32} & 0 \\ -\frac{3\,s^2+23\,s+34}{(s+26)\,(s^2+6\,s-32)} & -\frac{s+2}{s^2+6\,s-32} & \frac{s+2}{3\,(s+26)} \end{bmatrix}$$

$\mathrm{M}_3(1/(567 * s + 57 * s^2 + 9 * s^3 + s^4 + 962)/(3 * (s+26) * (6 * s + s^2 - 32)))...InMatlab : R(3,:) = (1/L(3,)) * R(3,:); L(3,:) = (1/L(3,)) * L(3,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & -\frac{8}{s+2} & -\frac{7}{s+2} \\ 0 & 1 & -\frac{2\,s+39}{s^2+6\,s-32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{1}{s+2} & 0 & 0 \\ \frac{5}{s^2+6\,s-32} & \frac{s+2}{s^2+6\,s-32} & 0 \\ -\frac{3\left(3\,s^2+23\,s+34\right)}{s^4+9\,s^3+57\,s^2+567\,s+962} & -\frac{3\,(s+2)\,(s+26)}{s^4+9\,s^3+57\,s^2+567\,s+962} & \frac{(s+2)\left(s^2+6\,s-32\right)}{s^4+9\,s^3+57\,s^2+567\,s+962} \end{bmatrix}$$

$A_2 to A_1(8/(s+2))...InMatlab: R(1,:) = -1 * L(1,2)/L(2,2) * R(1,:); L(1,:) = -1 * L(1,2)/L(2,2) * L(1,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & -\frac{7\,s+44}{s^2+6\,s-32} \\ 0 & 1 & -\frac{2\,s+39}{s^2+6\,s-32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{s+4}{s^2+6\,s-32} & \frac{8}{s^2+6\,s-32} & 0 \\ \frac{5}{s^2+6\,s-32} & \frac{s+2}{s^2+6\,s-32} & 0 \\ -\frac{3\,(3\,s+17)}{s^3+7\,s^2+43\,s+481} & -\frac{3\,(s+26)}{s^3+7\,s^2+43\,s+481} & \frac{s^2+6\,s-32}{s^3+7\,s^2+43\,s+481} \end{bmatrix}$$

$A_3 to A_1((7 * s + 44)/(6 * s + s^2 - 32))...InMatlab: R(1,:) = -1 * L(1,3)/L(3,3) * R(1,:); L(1,:) = -1 * L(1,3)/L(3,3) * L(1,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{2\,s+39}{s^2+6\,s-32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{s^2+5\,s+10}{s^3+7\,s^2+43\,s+481} & \frac{8\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{7\,s+44}{s^3+7\,s^2+43\,s+481} \\ \frac{5}{s^2+6\,s-32} & \frac{s+2}{s^2+6\,s-32} & 0 \\ -\frac{3\,(3\,s+17)}{s^3+7\,s^2+43\,s+481} & -\frac{3\,(s+26)}{s^3+7\,s^2+43\,s+481} & \frac{s^2+6\,s-32}{s^3+7\,s^2+43\,s+481} \end{bmatrix}$$

$A_3 to A_2((2 * s + 39)/(6 * s + s^2 - 32))...InMatlab: R(2,:) = -1 * L(2,3)/L(3,3) * R(2,:); L(2,:) = -1 * L(2,3)/L(3,3) * L(2,:)$

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R} = \begin{bmatrix} \frac{s^2+5\,s+10}{s^3+7\,s^2+43\,s+481} & \frac{8\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{7\,s+44}{s^3+7\,s^2+43\,s+481} \\ \frac{5\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{s^2+3\,s+65}{s^3+7\,s^2+43\,s+481} & \frac{2\,s+39}{s^3+7\,s^2+43\,s+481} \\ -\frac{3\,(3\,s+17)}{s^3+7\,s^2+43\,s+481} & -\frac{3\,(s+26)}{s^3+7\,s^2+43\,s+481} & \frac{s^2+6\,s-32}{s^3+7\,s^2+43\,s+481} \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s^2+5\,s+10}{s^3+7\,s^2+43\,s+481} & \frac{8\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{7\,s+44}{s^3+7\,s^2+43\,s+481} \\ \frac{5\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{s^2+3\,s+65}{s^3+7\,s^2+43\,s+481} & \frac{2\,s+39}{s^3+7\,s^2+43\,s+481} \\ -\frac{3\,(3\,s+17)}{s^3+7\,s^2+43\,s+481} & -\frac{3\,(s+26)}{s^3+7\,s^2+43\,s+481} & \frac{s^2+6\,s-32}{s^3+7\,s^2+43\,s+481} \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s^2+5\,s+10}{s^3+7\,s^2+43\,s+481} & \frac{8\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{7\,s+44}{s^3+7\,s^2+43\,s+481} \\ \frac{5\,s-13}{s^3+7\,s^2+43\,s+481} & \frac{s^2+3\,s+65}{s^3+7\,s^2+43\,s+481} & \frac{2\,s+39}{s^3+7\,s^2+43\,s+481} \\ -\frac{3\,(3\,s+17)}{s^3+7\,s^2+43\,s+481} & -\frac{3\,(s+26)}{s^3+7\,s^2+43\,s+481} & \frac{s^2+6\,s-32}{s^3+7\,s^2+43\,s+481} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{5\,s-91}{s^3+7\,s^2+43\,s+481}$$