

# System ID: Thermal Quench Experiment

Due online: Tuesday February 21 at 11pm

## Experiment

A thermocouple affixed to the center of a metal sphere is submersed in ice water and left to attain thermal equilibrium. Experimentally, it is determined that just prior to being immersed in boiling water, the cylinder temperature is uniform,  $T_0$ . Taking this as the initial condition, the billet is then submerged very quickly into boiling water (ideally) at  $100^\circ\text{C}$ . The actual temperature of the heated water is independently measured. The thermocouple temperature data has been provided to you on the course website.

You can watch the experiment at <https://youtu.be/0mMjAzS4mCc>

Note: The data in the video is in  $^\circ\text{F}$ , while the data given to you is in  $^\circ\text{C}$ .

## System Identification

Your goal is to estimate the time constant,  $\tau$ , for the lumped thermal mass model of a quenched sphere using the thermocouple data from

1. the ice bath to the boiling water
2. the boiling water to the ice water
3. the boiling water to the air.

## Import Data into MATLAB

First you will need to import the data into MATLAB. For help see [https://www.mathworks.com/help/matlab/import\\_export/ways-to-import-spreadsheets.html](https://www.mathworks.com/help/matlab/import_export/ways-to-import-spreadsheets.html). I recommend using the `readtable` command because with the import tool, if you clear your workspace you have to do it all over again, whereas with the `readtable` command, you just run your file again.

## Determining $\tau$

To determine  $\tau$ , use the following steps:

1. First, determine the initial and final water bath temperatures:  $T_0$  and  $T_{final}$ .

**In MATLAB:** Determine the initial and final water bath temperatures from the data and save them as MATLAB variables (remember to use meaningful variable names). For example for determining the initial temperature, does the first data point work or can you take an average of values over a certain time?

2. As found in class, the solution for the response of a 1st order system to a step input change in temperature is exponential in nature, depending upon the values for  $T_0$ ,  $T_{final}$ , and a most important parameter called the system time constant,  $\tau$

$$T(t) = T_{final} + (T_0 - T_{final}) e^{-\frac{t}{\tau}}$$

This equation can be placed in the form

$$\phi(t) = \ln \left( \frac{T(t) - T_{final}}{T_0 - T_{final}} \right) = -\frac{1}{\tau} t$$

In this way, if you plot the natural logarithm of so-called dimensionless temperature difference,  $\phi(t)$ , versus time  $t$  (on the x-axis), the slope of this log-linear plot will simply be  $-\frac{1}{\tau}$ .

**In MATLAB:** Calculate the dimensionless temperature difference using the data. Then plot  $\phi(t)$  versus  $t$ . Next, estimate the slope by performing a trend-line linear curve-fit in MATLAB. Finally, calculate your estimated time constant for the thermal system from the slope.

**Important Note:** Only use the linear region of the log-linear plot to determine the slope, i.e. exclude the data from the nonlinear regions on the log-linear plot when calculating the linear curve-fit.

**Helpful:** Look into the MATLAB function `polyfit` to help with this part

3. Compare the model

$$T(t) = T_{final} + (T_0 - T_{final}) e^{-\frac{t}{\tau}}$$

using the experimentally determined values of  $T_0$ ,  $T_{final}$ , and  $\tau$  by plotting the model with the experimental data. Be sure to label your plot appropriately.

**In MATLAB:** Plot the experimental data and the model on the same plot. Make sure that they start at approximately the same time. You can achieve this by shifting the experimental data or shifting the theoretical model.

**Note:** Remember that when we plot experimental data we use discrete markers and when we plot a theoretical model we use lines.

## Deliverables:

You may work in a group of up to 2 students. Per group submit the following to Gradescope (remember to select all the members of the team when submitting!)

1. For each of the 3 data sets there should be at a minimum
  - (a) A plot of  $\phi(t)$  vs  $t$  with a linear trend-line
  - (b) Estimated time constant for the system
  - (c) A plot with the model and the experimental data
2. MATLAB code (ALL of your calculations should be in your MATLAB code and your code should be sufficiently commented so that anyone from the class can follow and understand your code)

Do not write a formal lab report. Just present all figures formally (see the document on the course website about formal reports, it includes a section about figures and tables).