MECHANICAL VIBRATIONS

HW #11

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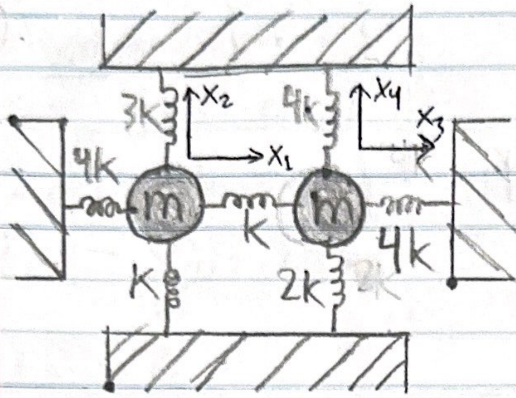
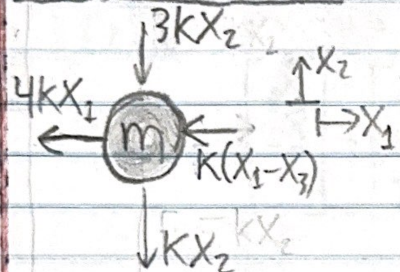
(1)



- masses free to move in $X_1 - X_2$ plane
- small angles (so springs dont slant)
- $m = 10kg$
- $k = 100N/m$

- FBD of left m

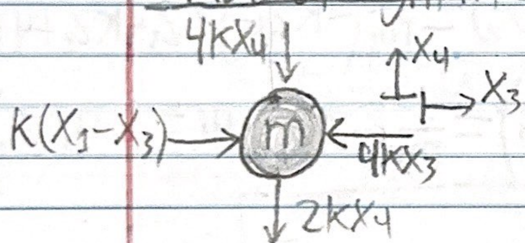


- $-4kX_1 - k(X_1 - X_3) = m\ddot{X}_1$
- $-3kX_2 - kX_2 = m\ddot{X}_2$

$-m\ddot{X}_1 - 5kX_1 + kX_3 = 0$

$-m\ddot{X}_2 - 4kX_2 = 0$

- FBD of right m



- $k(X_1 - X_3) - 4kX_3 = m\ddot{X}_3$
- $-4kX_4 - 2kX_4 = m\ddot{X}_4$

$kX_1 - m\ddot{X}_3 - 5kX_3 = 0$

$-m\ddot{X}_4 - 6kX_4 = 0$

- $m\ddot{X}_1 + 5kX_1 - kX_3 = 0$
- $m\ddot{X}_2 + 4kX_2 = 0$
- $m\ddot{X}_3 + 5kX_3 - kX_1 = 0$
- $m\ddot{X}_4 + 6kX_4 = 0$

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \ddot{X}_4 \end{bmatrix} + \begin{bmatrix} 5k & 0 & -k & 0 \\ 0 & 4k & 0 & 0 \\ -k & 0 & 5k & 0 \\ 0 & 0 & 0 & 6k \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- with matlab:

$\omega_1 = 6.3245 \frac{rad}{s}$ $\omega_3 = 7.746 \frac{rad}{s}$

$\omega_2 = 6.3245 \frac{rad}{s}$ $\omega_4 = 7.746 \frac{rad}{s}$

$\Rightarrow$ CONT TO SEE WHY

$\vec{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{u}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(I cont) • eigenvalues are natural frequencies
• eigenvectors are $\vec{u}$ vectors
• But must be in form: $\vec{y}'' = A\vec{y}$
• rearranging matrix

$$\begin{bmatrix} 5k & 0 & -k & 0 \\ 0 & 4k & 0 & 0 \\ -k & 0 & 5k & 0 \\ 0 & 0 & 0 & 6k \end{bmatrix} \vec{x} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \ddot{\vec{x}}$$

• Because is purely oscillating (no dampers)

$$\vec{x} = \vec{u} e^{j\omega t} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{j\omega t}$$

so means

$$\ddot{\vec{x}} = -\omega^2 \vec{u} e^{j\omega t} = -\omega^2 \vec{x}, \text{ since } \vec{x} = \vec{u} e^{j\omega t}$$

• now, put into the form

$$A^{n\times n} \vec{x} = \lambda \vec{x}, \text{ where } \lambda = -\omega^2$$ (which is why eigenvalues are ω_n)

identity matrix

$$|A^{n\times n} + \omega^2 I| = \vec{0}$$

• finding eigenvectors (1 means first one)

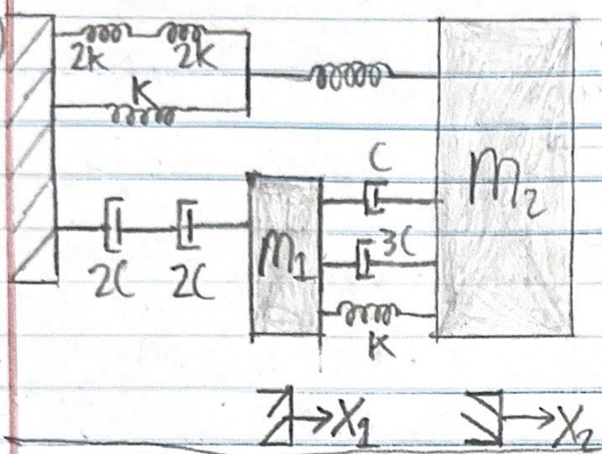
$$(A^{n\times n} + \omega_1^2 I)\vec{u}_1 = 0$$

• so this is why use eig(K,m)

K matrix

m matrix

②



Equations of motion from HW 4:

- $-2kx_2 - 4c\dot{x}_2 + 4c\dot{x}_1 = m_2\ddot{x}_2$
- $kx_2 + 4c\dot{x}_2 = m_1\ddot{x}_1 + 5c\dot{x}_1 + kx_1$

- $z_1 = x_1,\ z_2 = x_2,\ z_3 = \dot{x}_1,\ z_4 = \dot{x}_2$

- $\vec{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$, • $\dot{\vec{z}} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}$

- $\dot{x}_1 = z_3$ • $\dot{x}_2 = z_4$
- $\ddot{x}_1 = \frac{1}{m_1}\left(-kx_1 - 5c\dot{x}_1 + kx_2 + 4c\dot{x}_2\right) = \frac{1}{m_1}\left(-kz_1 - 5cz_3 + kz_2 + 4cz_4\right)$
- $\ddot{x}_2 = \frac{1}{m_2}\left(-2kx_2 - 4c\dot{x}_2 + 4c\dot{x}_1\right) = \frac{1}{m_2}\left(-2kz_2 - 4cz_4 + 4cz_3\right)$

$$\boxed{\vec{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \quad \dot{\vec{z}} = \begin{bmatrix} z_3 \\ z_4 \\ \frac{1}{m_1}\left(-kz_1 - 5cz_3 + kz_2 + 4cz_4\right) \\ \frac{1}{m_2}\left(-2kz_2 - 4cz_4 + 4cz_3\right) \end{bmatrix}}$$

```
clc;
clear;
close all;

res = 1000;
t_max = 60;
t = linspace(0,t_max,res);

m = 10; k = 100;

DOF = 4;

m_matrix = m*eye(DOF);
k_matrix = [[5*k,0,-k,0];[0,4*k,0,0];[-k,0,5*k,0];[0,0,0,6*k]];

[u_vectors,w_n_squared] = eig(k_matrix,m_matrix);

w_n = sqrt(w_n_squared);

%normalizing u vectors
for i = 1:DOF
    u_vec = u_vectors(:,i)
    lowestNum = abs(min(u_vec(u_vec ~=0)))
    u_vectors(:,i) =  u_vectors(:,i)/lowestNum;
end

for i = 1:DOF
    disp("u_"+num2str(i) + "=")
    disp( "u_" + num2str(i) + "," + num2str([1:DOF]')+" = [" +
 num2str(u_vectors(:,i)) + "]" )
end

for i = 1:DOF
    disp("w_n"+num2str(i) + "=" + num2str(w_n(i,i)))
end
```

```
u_vec =

         0
    0.3162
         0
         0


lowestNum =

    0.3162


u_vec =
```

```
   -0.2236
         0
   -0.2236
         0


lowestNum =

    0.2236


u_vec =

   -0.2236
         0
    0.2236
         0


lowestNum =

    0.2236


u_vec =

         0
         0
         0
    0.3162


lowestNum =

    0.3162

u_1=
    "u_1,1 = [0]"
    "u_1,2 = [1]"
    "u_1,3 = [0]"
    "u_1,4 = [0]"

u_2=
    "u_2,1 = [-1]"
    "u_2,2 = [ 0]"
    "u_2,3 = [-1]"
    "u_2,4 = [ 0]"

u_3=
    "u_3,1 = [-1]"
    "u_3,2 = [ 0]"
    "u_3,3 = [ 1]"
    "u_3,4 = [ 0]"
```

```
u_4=
    "u_4,1 = [0]"
    "u_4,2 = [0]"
    "u_4,3 = [0]"
    "u_4,4 = [1]"

w_n1=6.3246
w_n2=6.3246
w_n3=7.746
w_n4=7.746
```

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