MECHANICAL VIBRATIONS
HW#10

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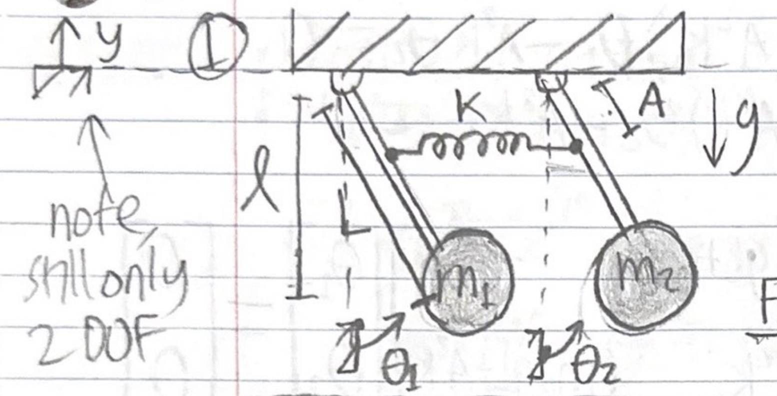


- $k = 20\ \text{N/m}$
- $m_1 = m_2 = 10\text{kg}$
- $A = 0.1\text{m}$
- $l = 0.5\text{m}$, so $L = \dfrac{l}{\cos(\theta_1)}$ ← at screen shot

☆nothing opposes gravity, so must take into accont

Find: equations of motion, and write in matrix form

- $q_1 = \theta_1$ • $q_2 = \theta_2$ • $Q_1 = 0$ • $Q_2 = 0$
- $KE = \frac{1}{2} m_1 L^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 L^2 \dot{\theta}_2^2$ • $PE = m_1 g(-L\cos\theta_1) + m_2 g(-L\cos\theta_2) + \frac{1}{2}k(A\sin\theta_1 - A\sin\theta_2)^2$
- $R = 0$

- $\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{\theta}_1}\right) - \frac{\partial KE}{\partial \theta_1} + \frac{\partial PE}{\partial \theta_1} + \frac{\partial R}{\partial \dot{\theta}_1} = Q_1$

$\frac{d}{dt}(m_1 L^2 \dot{\theta}_1) - 0 + (m_1 g L\sin\theta_1 + Ak(A\sin\theta_1 - A\sin\theta_2)\cos\theta_1) + 0 = 0$

$m_1 L^2 \ddot{\theta}_1 + m_1 g L\sin\theta_1 + A^2 k\sin\theta_1\cos\theta_1 + A^2 k\sin\theta_2\cos\theta_1 = 0$

☆applying small angles $\sin\theta \to \theta$, $\cos\theta \to 1$

$m_1 L^2 \ddot{\theta}_1 + m_1 g L\theta_1 + A^2 k\theta_1 + A^2 k\theta_2 = 0$

- $\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{\theta}_2}\right) - \frac{\partial KE}{\partial \theta_2} + \frac{\partial PE}{\partial \theta_2} + \frac{\partial R}{\partial \dot{\theta}_2} = Q_2$ ⇨ CONT

$\frac{d}{dt}(m_2 L^2 \ddot{\theta}_2) - 0 + (m_2 g L\sin\theta_2 - Ak(A\sin\theta_1 - A\sin\theta_2)\cos\theta_2) + 0 = 0$

$m_2 L^2 \ddot{\theta}_2 + m_2 g L\sin\theta_2 - A^2 k\sin\theta_1\cos\theta_2 + A^2 k\sin\theta_2\cos\theta_2 = 0$

☆applying small angles, $\sin\theta \to \theta$, $\cos\theta \to 1$

$m_2 L^2 \ddot{\theta}_2 + m_2 g L\theta_2 - A^2 k\theta_1 + A^2 k\theta_2 = 0$

Just calling U since seems to be all potential

(1 cont) $\bullet\ m_1 L^2 \ddot{\theta}_1 + (m_1 g L + A^2 K)\theta_1 - A^2 K \theta_2 = 0$

$\bullet\ m_2 L^2 \ddot{\theta}_2 + (m_2 g L + A^2 K)\theta_2 - A^2 K \theta_1 = 0$

$$\begin{bmatrix} m_1 L^2 & 0 \\ 0 & m_2 L^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} (m_1 g L + A^2 K) & -A^2 K \\ -A^2 K & (m_2 g L + A^2 K) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$J \ddot{\vec{\theta}} + U \vec{\theta} = \vec{0}$$

☆Because small angles:

$L \approx \frac{l}{\cos\theta_1}$, $\theta_1 \approx 0$, so $\cos\theta_1 \approx 1$, so $L \approx l \approx 0.5m$

$$\begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 44.25 & -0.2 \\ -0.2 & 49.25 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Find: natural frequencies

☆note, masses are coupled from spring on rod

☆Is a system of linear DEs

$J \ddot{\vec{\theta}} + U \vec{\theta} = \vec{0}$, transient homogeneous solution $\vec{\theta} = \vec{u} e^{rt}$

• will end up with sines and cosines, since

$r = \pm j\sqrt{\frac{U}{J}}$, and will oscillate forever $\Rightarrow r = \pm j\omega$ $\Rightarrow$ CONT

$\bullet\ \vec{\theta} = \vec{u} e^{\pm j\omega t}$ $\bullet\ \dot{\vec{\theta}} = \vec{u} j\omega e^{j\omega t}$ $\bullet\ \ddot{\vec{\theta}} = -\vec{u}\omega^2 e^{j\omega t}$

1 cont

- must ensure that $\vec{\theta} \neq \vec{0}$, since $\vec{0}$ is trivial
- so plugging in $\vec{\theta}$ & $\ddot{\vec{\theta}}$ to DE

$$J(-\omega^2 \vec{u} e^{j\omega t}) + U(\vec{u} e^{j\omega t}) = \vec{0}$$

☆ $e^{j\omega t} = \vec{0}$ is trivial, or will never be 0

$$e^{j\omega t}\left[U\vec{u} - \omega^2 J\vec{u}\right] = \vec{0}$$

☆ note, $\vec{u} = \vec{0}$ is trivial

- so for a non trivial solution

$$\left[U\vec{u} - \omega^2 J\vec{u}\right] = \vec{0} \Rightarrow \left[U - \omega^2 J\right]\vec{u} = \vec{0}$$

- If $\left[U - \omega^2 J\right]^{-1}$ exists, then can get $\vec{u} = 0$, which is trivial
- need to ensure $\left[U - \omega^2 J\right]^{-1}$ doesnt exist

$$\det\left[U - \omega^2 J\right] = 0$$

- for 2x2 matrix, $\det[B] = B\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\det[B] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = ad - bc$$

⇒ CONT

- $\det\left[U - \omega^2 J\right] = \det\left[\begin{bmatrix} m_1 gL + A^2 K & -A^2 K \\ -A^2 K & m_2 gL + A^2 K \end{bmatrix} - \begin{bmatrix} \omega^2 m_1 L^2 & 0 \\ 0 & \omega^2 m_2 L^2 \end{bmatrix}\right]$

⇩

$$\det\begin{bmatrix} m_1 gL + A^2 K - m_1 L^2 \omega^2 & -A^2 K \\ -A^2 K & m_2 gL + A^2 K - m_2 L^2 \omega^2 \end{bmatrix} = 0$$

$$☆\ m_1 = m_2 = m$$

1 cont

$$\det\begin{bmatrix} mgL + A^2k - mL^2\omega^2 & -A^2k \\ -A^2k & mgL + A^2k - mL^2\omega^2 \end{bmatrix} = 0$$

$$(mgL + A^2k - mL^2\omega^2)(mgL + A^2k - mL^2\omega^2) - (-A^2k)(-A^2k) = 0$$

$$m^2g^2L^2 + 2mgLA^2k - 2gm^2L^3\omega^2 + \cancel{A^4k^2} - 2A^2kmL^2\omega^2 + m^2L^4\omega^4 - \cancel{A^4k^2} = 0$$

$☆\ W = \omega^2$

$$m^2L^4W^2 + (2gm^2L^3 - 2A^2kmL^2)W + m^2g^2L^2 + 2mgLA^2k = 0$$

$$W^2 + 2\frac{(gm^2L^3 - A^2kmL^2)}{m^2L^4} = \frac{-m^2g^2L^2 - 2mgLA^2k}{m^2L^4}$$

$$\left(W - \frac{gm^2L^3 - A^2kmL^2}{m^2L^4}\right)^2 = \frac{-m^2g^2L^2 - 2mgLA^2k}{m^2L^4} + \left(\frac{gm^2L^3 - A^2kmL^2}{m^2L^4}\right)^2$$

$$W = \frac{gm^2L^3 - A^2kmL^2}{m^2L^4} \pm \sqrt{\frac{g^2m^4L^6 - 2A^2km^3L^5g + A^4k^2m^2L^4}{m^4L^8} - \frac{m^2g^2L^2 - 2mgLA^2k}{m^2L^4}}$$

$$\omega = \pm\sqrt{\frac{gm^2L^3 - A^2kmL^2}{m^2L^4} \pm \sqrt{\frac{g^2m^4L^6 - 2A^2km^3L^5g + A^4k^2m^2L^4}{m^4L^8} - \frac{m^2g^2L^2 - 2mgLA^2k}{m^2L^4}}}$$

$$\boxed{\omega = \pm 4.429\ \text{rad/s}, \text{or } \pm 4.447\ \text{rad/s} = \omega_n}$$

$$\boxed{\vec{\omega_n} = \begin{bmatrix} \omega_{n1} \\ \omega_{n2} \end{bmatrix} = \begin{bmatrix} 4.429 \\ 4.447 \end{bmatrix} \text{rad/s}}$$

⇨ CONT

(1 cont) Find: mode shapes for system

- now that have $\omega_{n1} = \pm 4.429$ rad/s & $\omega_{n2} = \pm 4.447$ rad/s
- can find associated $\vec{u}$
- from $[U - \omega^2 J]\vec{u} = \vec{0}$,
  - for $\vec{u_1}$, $[U - \omega_{n1}^2 J]\vec{u_1} = \vec{0}$

$$\begin{bmatrix} mgL + A^2k - mL^2\omega_{n1}^2 & -A^2k \\ -A^2k & mgL + A^2k - mL^2\omega_{n1}^2 \end{bmatrix}\begin{bmatrix} u_{1,1} \\ u_{2,1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

is about 0.2, since 4.429 isn't the full number

$$\left.\begin{aligned} 0.2u_{1,1} - 0.2u_{2,1} &= 0 \\ -0.2u_{1,1} + 0.2u_{2,1} &= 0 \end{aligned}\right\} u_{1,1} = u_{2,1}$$

- possible $\vec{u_1}$ mode shape:

$$\boxed{\vec{u_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

☆ Also from the quartic, $(mgL + A^2k - mL^2\omega^2)^2 = (A^2k)^2$ and $A^2k = 0.2$

  - for $\vec{u_2}$, $[U - \omega_{n2}^2 J]\vec{u_2} = \vec{0}$

$$\begin{bmatrix} mgL + A^2k - mL^2\omega_{n2}^2 & -A^2k \\ -AK^2 & mgL + A^2k - mL^2\omega_{n2}^2 \end{bmatrix}\begin{bmatrix} u_{1,2} \\ u_{2,2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

is about −0.2, since 4.447 isn't the full number.

$$\left.\begin{aligned} -0.2u_{1,2} - 0.2u_{2,2} &= 0 \\ -0.2u_{1,2} - 0.2u_{2,2} &= 0 \end{aligned}\right\} u_{1,2} = -u_{2,2}$$

- possible $\vec{u_2}$ mode shape

$$\boxed{\vec{u_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

☆ means $(mgL + A^2k - mL^2\omega^2)^2 - (KA^2)^2 = 0$

$$mgL + A^2k - mL^2\omega^2 = \pm KA^2 = \pm 0.2$$

② Given:

$$\begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- where the solution is

$$\vec{x}(t) = A_1 \sin(\omega_{n1} t + \phi_1)\vec{u}_1 + A_2 \sin(\omega_{n2} t + \phi_2)\vec{u}_2$$

- where $\omega_{n1} = \sqrt{2}$ rad/s, $\omega_{n2} = 2$ rad/s,

$$u_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

we were allowed to change this

Calculate: the response of the system to the initial conditions (find A_1, A_2, ϕ_1, ϕ_2)

$$\vec{x}(0) = \frac{\sqrt{2}}{2}\begin{bmatrix} 1/3 \\ 1 \end{bmatrix}, \quad \dot{\vec{x}}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3}\frac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Solution:

- $$\frac{1}{3}\frac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \vec{x}(0) = A_1 \sin(\cancel{(4.429)(0)} + \phi_1)\begin{bmatrix} 1 \\ 3 \end{bmatrix} + A_2 \sin(\cancel{4.447(0)} + \phi_2)\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} \sqrt{2}/2 \\ 3\sqrt{2}/2 \end{bmatrix} = \begin{bmatrix} A_1 \sin(\phi_1) \\ 3A_1 \sin(\phi_1) \end{bmatrix} + \begin{bmatrix} -A_2 \sin(\phi_2) \\ 3A_2 \sin(\phi_2) \end{bmatrix}$$

$$\frac{1}{3}\left[\frac{\sqrt{2}}{2}\right] = A_1 \sin\phi_1 - A_2 \sin\phi_2$$

$$\frac{1}{3}\left[\cancel{3}\frac{\sqrt{2}}{2}\right] = \cancel{3}A_1 \sin\phi_1 + \cancel{3}A_2 \sin\phi_2$$

⇨ CONT

(2 cont) • finding $\vec{\dot{x}}$

$$\vec{\dot{x}} = \omega_{n1} A_1 \cos(\omega_{n1} t + \phi_1)\vec{u_1} + \omega_{n2} A_2 \cos(\omega_{n2} t + \phi_2)\vec{u_2}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{\dot{x}}(0) = 4.429 A_1 \cos((4.429)\cdot 0 + \phi_1)\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 4.447 A_2 \cos(4.447\cdot 0 + \phi_2)\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_{n1} A_1 \cos(\phi_1) \\ 3\omega_{n1} A_1 \cos(\phi_1) \end{bmatrix} + \begin{bmatrix} -\omega_{n2} A_2 \cos(\phi_2) \\ 3\omega_{n2} A_2 \cos(\phi_2) \end{bmatrix}$$

$$0 = \omega_{n1} A_1 \cos\phi_1 - \omega_{n2} A_2 \cos\phi_2$$

$$\cancel{0 = 3\omega_{n1} A_1 \cos\phi_1 + 3\omega_{n2} A_2 \cos\phi_2}$$

• system of eqns

• $\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right) = A_1 \sin\phi_1 - A_2 \sin\phi_2$

• $\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right) = A_1 \sin\phi_1 + A_2 \sin\phi_2$

smash into eachother: $\frac{1}{3}\sqrt{2} = 2A_1 \sin\phi_1$

• $0 = \omega_{n1} A_1 \cos\phi_1 - \omega_{n2} A_2 \cos\phi_2$

• $0 = \omega_{n1} A_1 \cos\phi_1 + \omega_{n2} A_2 \cos\phi_2$

smash into: $0 = 2\omega_{n1} A_1 \cos\phi_1$

• solving for A_1 & ϕ_1

• $\sqrt{2} = 2A_1 \sin\phi_1 \Rightarrow \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right) = A_1 \sin\phi_1 \Rightarrow A_1 = \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{\sin\phi_1}\right)$

• $0 = 2\cancel{\omega_{n1}} A_1 \cos\phi_1 \quad 0 = A_1 \cos\phi_1$

$\Rightarrow$ CONT

$$\cancel{0 = \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)\frac{\cos\phi_1}{\sin\phi_1}} \Rightarrow \cot\phi_1 = 0 \Rightarrow \tan\phi_1 = \infty, \boxed{\phi_1 = 90^\circ}$$

2cont

- If $\phi_1 = 90°$, then $\frac{1}{3}\frac{\sqrt{2}}{2} = A_1 \sin(\phi_1) \Rightarrow \frac{\sqrt{2}}{6} = A_1 \sin(90)$

$\boxed{A_1 = \frac{\sqrt{2}}{6}}$

Known now

- Solving for A_2 & ϕ_2 ☆ $A_1 \sin\phi_1 = \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)$
  - $\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right) = A_1 \sin\phi_1 + A_2 \sin\phi_2$ ☆ $\omega_{n1} A_1 \cos\phi = 0$
  - $0 = \omega_{n1} A_1 \cos\phi_1 + \omega_{n2} A_2 \cos\phi_2$

$\cancel{\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)} = \cancel{\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)} + A_2 \sin\phi_2 \Rightarrow 0 = A_2 \sin\phi_2$

$\cancel{0} = \cancel{0} + \omega_{n2} A_2 \cos\phi_2 \Rightarrow 0 = A_2 \cos\phi_2$

- $\sin\phi_2$ & $\cos\phi_2$ cant both be 0 at the same time
- only way for this to be true is $\boxed{A_2 = 0}$
- $\boxed{\text{doesnt matter what } \phi_2 \text{ is}}$
- Summary: $\boxed{A_1 = \frac{\sqrt{2}}{2}}$, $\boxed{A_2 = 0}$ $\boxed{\phi_1 = 90°}$ $\boxed{\phi_2 \text{ doesnt matter}}$
- Solution:

$$\vec{X}(t) = \frac{\sqrt{2}}{6}\sin(\omega_{n1} t + 90°)\begin{bmatrix}1\\3\end{bmatrix} + 0\sin(\omega_{n2}t + \phi_2)\begin{bmatrix}-1\\3\end{bmatrix}$$

☆ $\omega_{n1} = \frac{\sqrt{2}}{2}$

$$\boxed{\vec{X}(t) = \frac{\sqrt{2}}{6}\cos\left(\frac{\sqrt{2}}{2}t\right)\begin{bmatrix}1\\3\end{bmatrix}}$$

$\Rightarrow$ CONT

2cont

Calculate: response to initial conditions (find A_1, A_2, ϕ_1, ϕ_2

$$\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \dot{\vec{x}}(0) = \frac{\sqrt{2}}{2}\begin{bmatrix} 1/3 \\ -1 \end{bmatrix} \rightarrow = \frac{1}{3}\frac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Solution:

- $\vec{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = A_1 \sin(\cancel{\omega_{n1} \cdot 0} + \phi_1)\begin{bmatrix} 1 \\ 3 \end{bmatrix} + A_2 \sin(\cancel{\omega_{n2} \cdot 0} + \phi_2)\begin{bmatrix} -1 \\ 3 \end{bmatrix}$
  - $0 = A_1 \sin(\phi_1) - A_2 \sin(\phi_2)$
  - $0 = \cancel{3}A_1 \sin(\phi_1) + \cancel{3}A_2 \sin(\phi_2)$
- $\dot{\vec{x}}(0) = \frac{1}{3}\begin{bmatrix} -\sqrt{2}/2 \\ 3\sqrt{2}/2 \end{bmatrix} = \omega_{n1} A_1 \cos(\cancel{\omega_{n1} \cdot 0} + \phi_1)\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \omega_{n2} A_2 \cos(\cancel{\omega_{n2} \cdot 0} + \phi_2)\begin{bmatrix} -1 \\ 3 \end{bmatrix}$
  - $\frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right) = \omega_{n1} A_1 \cos(\phi_1) - \omega_{n2} A_2 \cos(\phi_2)$
  - $\cancel{\frac{1}{3}}\left(\frac{-\cancel{3}\sqrt{2}}{2}\right) = \cancel{3}\omega_{n1} A_1 \cos(\phi_1) + \cancel{3}\omega_{n2} A_2 \cos(\phi_2)$
- system of eqns
  - $0 = A_1 \sin\phi_1 - A_2 \sin\phi_2$
  - $0 = A_1 \sin\phi_1 + A_2 \sin\phi_2$

    smash into eachother:

    $\cancel{0} = \cancel{2}A_1 \sin\phi_1$
  - $\frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right) = \omega_{n1} A_1 \cos\phi_1 - \omega_{n2} A_2 \cos\phi_2$
  - $\frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right) = \omega_{n1} A_1 \cos\phi_1 + \omega_{n2} A_2 \cos\phi_2$

    smash into eachother

    $\cancel{0} = \cancel{\omega_{n1}} A_1 \cos\phi_1$

$\Rightarrow$ CONT

(2 cont)

- finding A_1 & ϕ_1
  - $0 = A_1 \sin\phi_1$
  - $0 = A_1 \cos\phi_1$

  } $\sin\phi_1$ & $\cos\phi_1$ cant both be 0 at the same time so only way for this to be true: $\boxed{A_1 = 0}$, $\boxed{\phi_1 \text{ doesnt matter}}$

- finding A_2 & ϕ_2
  - plugging in $A_1 = 0$
  - $\left(\frac{1}{3}\right)\frac{-\sqrt{2}}{2} = \cancel{\omega_{n1} A_1 \cos\phi_1} - \omega_{n2} A_2 \cos\phi_2$
  - $0 = \cancel{\omega_{n1} A_1 \sin\phi_1} - \omega_{n2} A_2 \sin\phi_2$

  $\omega_{n2} A_2 = \frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{\cos\phi_2}\right)$

  $0 = \frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right)\frac{\sin\phi_2}{\cos\phi_2} \Rightarrow 0 = \tan\phi_2 \Rightarrow \boxed{\phi_2 = 0}$

- finding A_2

  ☆ $\omega_{n2} = 2$

  $\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right) = -\omega_{n2} A_2 \cos(0) \Rightarrow \boxed{\frac{-\sqrt{2}}{12} = A_2}$

- finally!

  $\vec{x}(t) = \cancel{A_1 \sin(\omega_{n1} t + \phi_1)\begin{bmatrix}1\\3\end{bmatrix}} + A_2 \sin(\omega_{n2} t + \phi_2)\begin{bmatrix}-1\\3\end{bmatrix}$

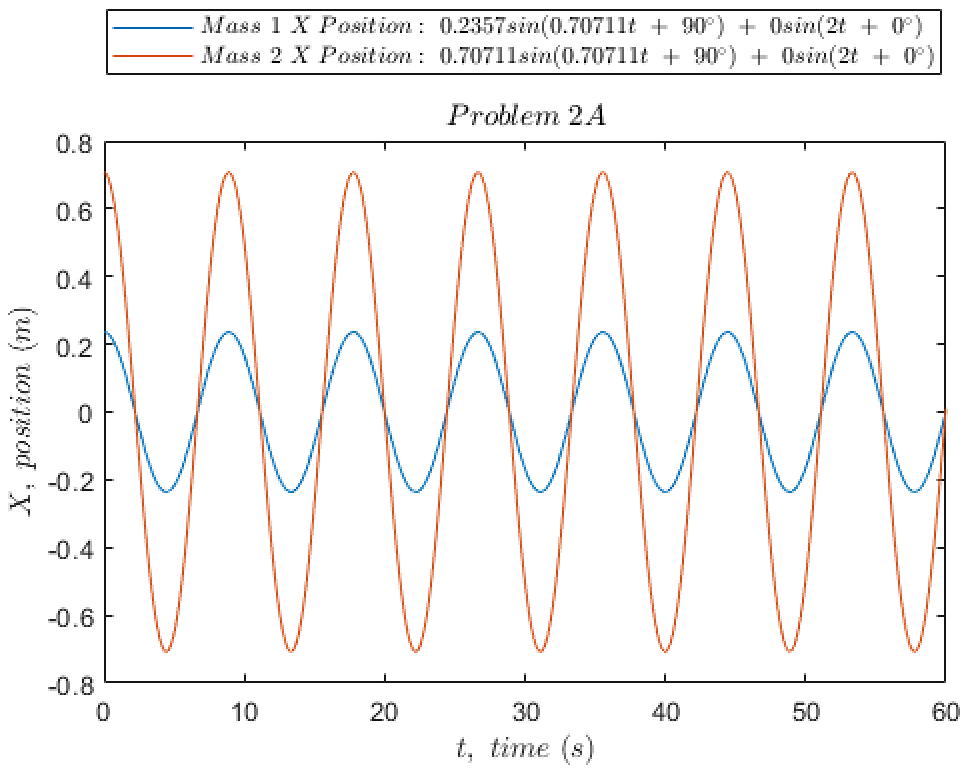
$$\boxed{\vec{x}(t) = \frac{-\sqrt{2}}{12}\sin(2t)\begin{bmatrix}-1\\3\end{bmatrix}}$$

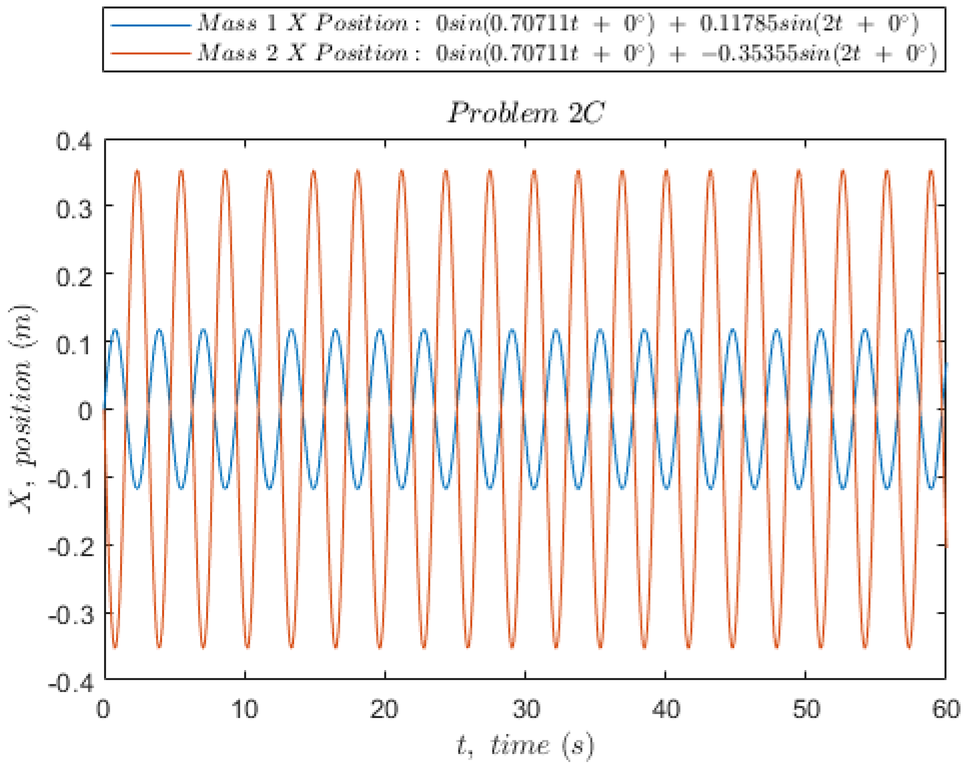
```
clc;
clear;
close all;

res = 1000;
t_max = 60;
t = linspace(0,t_max,res);

w_n_Array = [sqrt(2)/2,2]; %[w_n1, w_n2, ...]

u_vectors_Matrix = [[1,3];[-1,3]]; %[u_1 ; u_2, ...]
%u_1 = [1,3];
%u_2 = [-1,3];

%PHI IS IN DEGREES

%%Part A
A_Array = [sqrt(2)/6,0]; %[A_1,A_2, ...]
phi_Array = [90,0]; %[phi_1, phi_2, ...] in degrees
%NOTE: phi_2 doesn't matter for part A

X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,"$Problem \ 2A$");

%%Part C
A_Array = [0/2,-sqrt(2)/12]; %[A_1,A_2, ...]
phi_Array = [0,0]; %[phi_1, phi_2, ...] in degrees
%NOTE: phi_1 doesn't matter for part C

X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,"$Problem \ 2C$");

%%Coupled 3 DoF Test
A_Array = zeros(3);
w_n_Array = zeros(3);
u_vectors_Matrix = [];
A_Array = [1,-sqrt(2)/12,1]; %[A_1,A_2, ...]
phi_Array = [40,70,20]; %[phi_1, phi_2, ...] in degrees
%NOTE: phi_1 doesn't matter for part C
w_n_Array = [sqrt(5)/5,2,4]; %[w_n1, w_n2, ...]

u_vectors_Matrix = [[1,3,4];[-1,3,5];[5,4,6]]; %[u_1 ; u_2, ...]


X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,"$Coupled \ 3 \ DoF \ TEST$");

%%Coupled 4 DoF Test
A_Array = zeros(4);
w_n_Array = zeros(4);
u_vectors_Matrix = [];
A_Array = [6,-sqrt(2)/12,1,4]; %[A_1,A_2, ...]
```

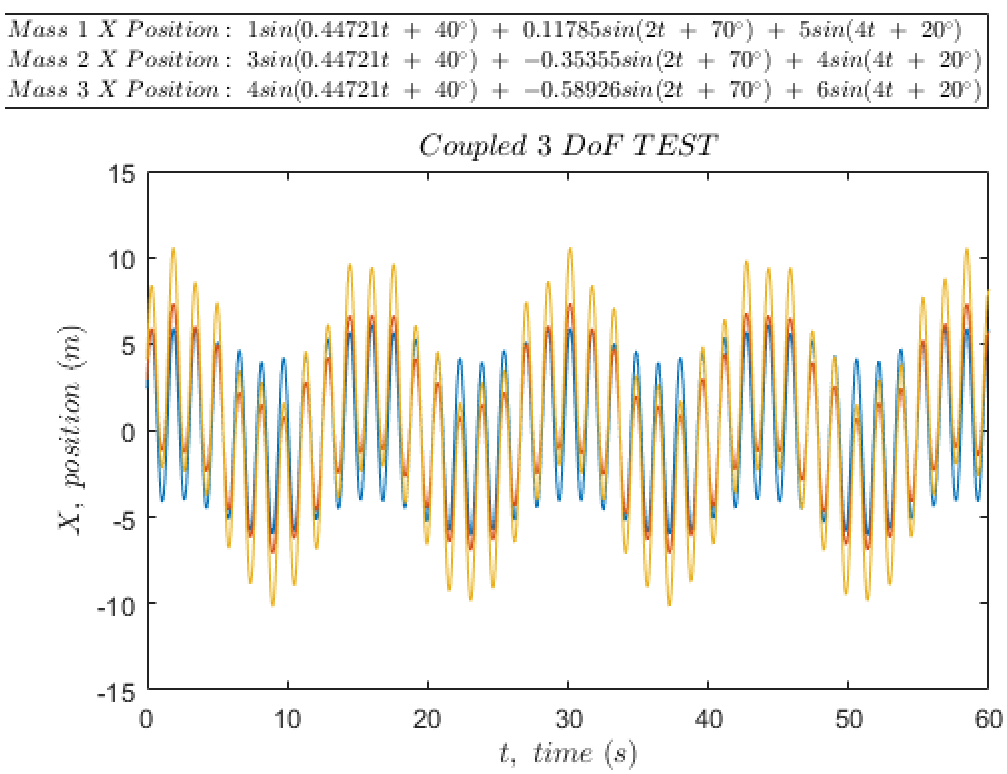
```
phi_Array = [40,70,20,10]; %[phi_1, phi_2, ...] in degrees
%NOTE: phi_1 doesn't matter for part C
w_n_Array = [sqrt(5)/5,2,4,5]; %[w_n1, w_n2, ...]

u_vectors_Matrix = [[1,3,4,1];[-1,3,5,8];[5,4,6,7];[1,2,3,4]]; %[u_1 ;
 u_2, ...]


X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,"$Coupled \ 4 \ DoF \ TEST$");

%%Coupled 5 DoF Test
A_Array = zeros(5);
w_n_Array = zeros(5);
u_vectors_Matrix = [];
A_Array = [1,6,-sqrt(2)/12,1,4]; %[A_1,A_2, ...]
phi_Array = [0,40,70,20,10]; %[phi_1, phi_2, ...] in degrees
%NOTE: phi_1 doesn't matter for part C
w_n_Array = [sqrt(5)/5,2,4,5,8]; %[w_n1, w_n2, ...]

u_vectors_Matrix = [[1,3,4,1,0];[-1,3,5,8,1/2];[5,4,6,7,sqrt(2)];[1,2,3,4,5];
[5,4,3,2,1]]; %[u_1 ; u_2, ...]


X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,"$Coupled \ 5 \ DoF \ TEST$");


%%Coupled n-DoF Test
DoF = 12;
% randMin = 0;
% randMax = 10;
A_Array = zeros(DoF);
w_n_Array = zeros(DoF);
u_vectors_Matrix = [];

for i = 1:DoF
    randMin = -10;
    randMax = 10;
    randomNum = ((randMax-randMin)*rand + randMin)/(randMax-randMin);
    A_Array(i) = randomNum;
    randMin = 0;
    randMax = 7;
    randomNum = (randMax-randMin)*rand + randMin;
    w_n_Array(i) = randomNum;
    for ii = 1:DoF
        randomNum = ((randMax-randMin)*rand + randMin)/(randMax-randMin);
        u_vectors_Matrix(i,ii) = randomNum;
    end
end

w_n_Array = sort(w_n_Array);

randMin = -360;
```

```
randMax = 360;

for i = 1:DoF
    randomNum = (randMax-randMin)*rand + randMin;
    phi_Array(i) = randomNum;
end

X_vals = coupledDoFPlotter(A_Array,
 phi_Array,w_n_Array,u_vectors_Matrix,t,strcat("$Coupled \ ", num2str(DoF), "
 \ DoF \ TEST$"));



function [coupledDoFValues] = coupledDoFPlotter(A_Values,
 phi_Values,natFreqs,u_Vectors,t_vals,titled)
    X = zeros([length(A_Values),length(t_vals)]);
    for i = 1:length(A_Values) %gets X mass1, X mass2, ... and selects which u
 vector to use
        %X(i,:) = X(i,:) + A_Values(1)*sin(natFreqs(1)*t_vals +
 phi_Values(1)*180/pi)*u_Vectors(1,i);
        for ii = 1:length(A_Values) %within X mass1, goes thru A_1, A2...,
 phi_1, phi_2..., selects
            %which value in the specified u vector to use
            X(i,:) = X(i,:) + A_Values(ii)*sin(natFreqs(ii)*t_vals +
 phi_Values(ii)*pi/180)*u_Vectors(ii,i);
        end
    end
    coupledDoFValues = X;
    figure;
    for i = 1:length(A_Values)
        plot(t_vals,X(i,:))
        hold on;
    end
    hold off;
    a = strcat("$Mass \ ", num2str(1)," \ X \ Position$");
    legendStuff = strings(1,length(A_Values)); %creates aray of empty string
    %legendStuff(1) = strcat("$Mass \ ", num2str(1)," \ X \ Position$");
    %legendStuff(2) = strcat("$Mass \ ", num2str(2)," \ X \ Position$");
    %legendStuff = [strcat("$Mass \ ",num2str(1)," \ X \ Position
$"),strcat("$Mass \ ", num2str(2)," \ X \ Position$")];
    for i = 1:length(A_Values)
        a = strcat("$Mass \ ", num2str(i)," \ X \ Position : \ ");
        b = "";
        for ii = 1:length(A_Values)
            if ii > 1
              b = strcat(b, " \ + \ ") ;
            end
            b = strcat(b,
 num2str(u_Vectors(ii,i)*A_Values(ii)),"sin(",num2str(natFreqs(ii)),"t \ + \
 ", num2str(phi_Values(ii)), "^{\circ}  )");
        end
        c = strcat(a,b," $");
        legendStuff(i) = c;
    end
```

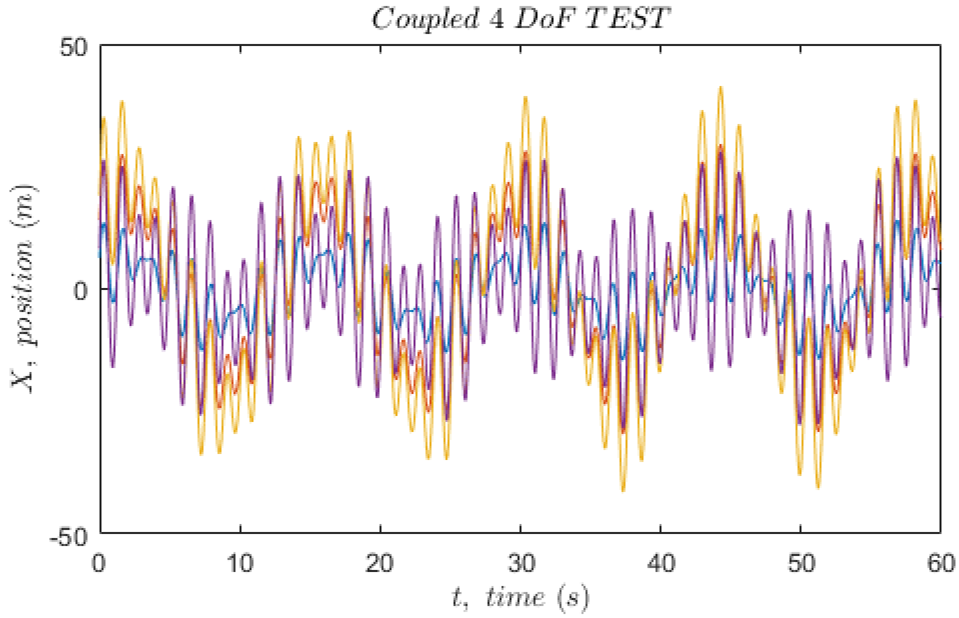
```
    lgnd= legend(legendStuff,'Location','northoutside');
    set(lgnd, 'Interpreter','latex')
    %lgnd.Location = 'best';
    title(titled,'Interpreter','latex')
    xlabel("$ t, \ time \ (s) $",'Interpreter','latex')
    ylabel("$X, \ position \ (m)$",'Interpreter','latex')
end

%%This is the model
% A_1 = sqrt(2)/2;
% A_2 = 0;
% phi_1 = 90;
% phi_2 = 0; %doesn't matter for part A
%
% u_1 = [1;3];
% u_2 = [-1;3];
%
% X= [;];
% X(1,:) = A_1*sin(w_n1*t + phi_1*180/pi)*u_1(1) + A_2*sin(w_n2*t + phi_2*180/
pi)*u_2(1);
% X(2,:) = A_1*sin(w_n1*t + phi_1*180/pi)*u_1(2) + A_2*sin(w_n2*t + phi_2*180/
pi)*u_2(2);
```



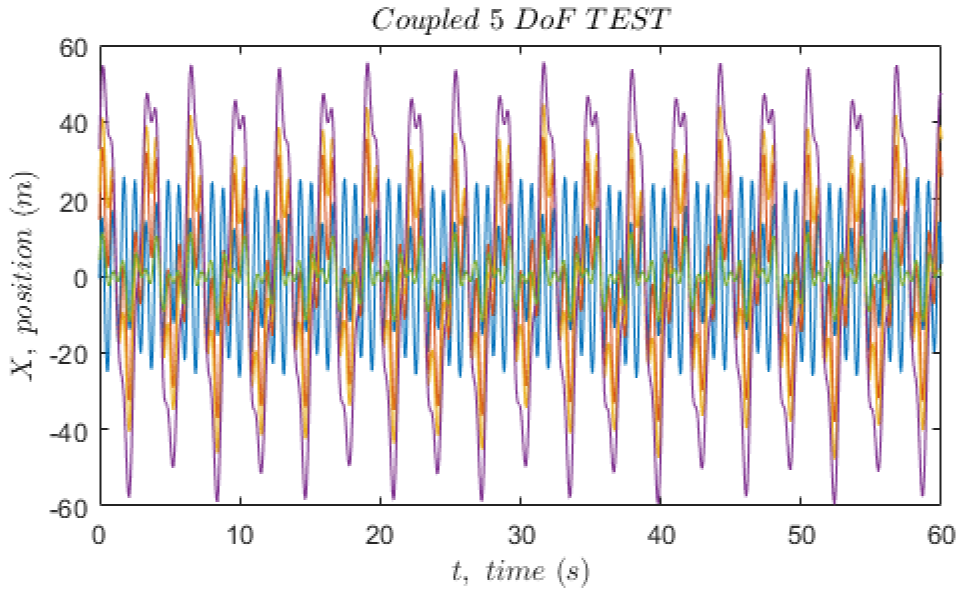

$Mass\ 1\ X\ Position:\ 0sin(0.70711t\ +\ 0^{\circ})\ +\ 0.11785sin(2t\ +\ 0^{\circ})$
$Mass\ 2\ X\ Position:\ 0sin(0.70711t\ +\ 0^{\circ})\ +\ -0.35355sin(2t\ +\ 0^{\circ})$
$Problem\ 2C$
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
$X,\ position\ (m)$
0
10
20
30
40
50
60
$t,\ time\ (s)$


$Mass\ 1\ X\ Position:\ 1sin(0.44721t\ +\ 40^{\circ})\ +\ 0.11785sin(2t\ +\ 70^{\circ})\ +\ 5sin(4t\ +\ 20^{\circ})$
$Mass\ 2\ X\ Position:\ 3sin(0.44721t\ +\ 40^{\circ})\ +\ -0.35355sin(2t\ +\ 70^{\circ})\ +\ 4sin(4t\ +\ 20^{\circ})$
$Mass\ 3\ X\ Position:\ 4sin(0.44721t\ +\ 40^{\circ})\ +\ -0.58926sin(2t\ +\ 70^{\circ})\ +\ 6sin(4t\ +\ 20^{\circ})$
$Coupled\ 3\ DoF\ TEST$
15
10
5
0
-5
-10
-15
$X,\ position\ (m)$
0
10
20
30
40
50
60
$t,\ time\ (s)$

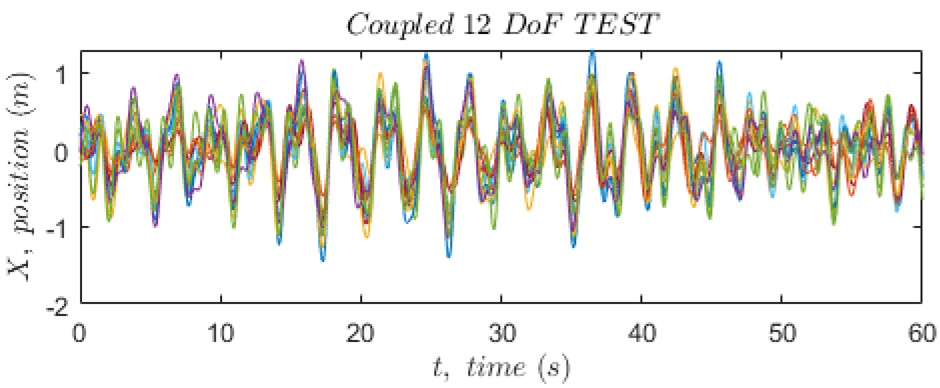
$6sin(0.44721t + 40^\circ) + 0.11785sin(2t + 70^\circ) + 5sin(4t + 20^\circ) + 4sin(5t + 10^\circ)$
$18sin(0.44721t + 40^\circ) + -0.35355sin(2t + 70^\circ) + 4sin(4t + 20^\circ) + 8sin(5t + 10^\circ)$
$24sin(0.44721t + 40^\circ) + -0.58926sin(2t + 70^\circ) + 6sin(4t + 20^\circ) + 12sin(5t + 10^\circ)$
$6sin(0.44721t + 40^\circ) + -0.94281sin(2t + 70^\circ) + 7sin(4t + 20^\circ) + 16sin(5t + 10^\circ)$



$^\circ) + -6sin(2t + 40^\circ) + -0.58926sin(4t + 70^\circ) + 1sin(5t + 20^\circ) + 20sin(8t + 10^\circ)$
$^\circ) + 18sin(2t + 40^\circ) + -0.4714sin(4t + 70^\circ) + 2sin(5t + 20^\circ) + 16sin(8t + 10^\circ)$
$^\circ) + 30sin(2t + 40^\circ) + -0.70711sin(4t + 70^\circ) + 3sin(5t + 20^\circ) + 12sin(8t + 10^\circ)$
$^\circ) + 48sin(2t + 40^\circ) + -0.82496sin(4t + 70^\circ) + 4sin(5t + 20^\circ) + 8sin(8t + 10^\circ)$
$^\circ) + 3sin(2t + 40^\circ) + -0.16667sin(4t + 70^\circ) + 5sin(5t + 20^\circ) + 4sin(8t + 10^\circ)$



$$\ldots 7.8296^\circ) \;+\; -0.27154sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.12395sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 7.8296^\circ) \;+\; -0.26239sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.06594sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 8296^\circ) \;+\; -0.019751sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.16298sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots .8296^\circ) \;+\; -0.1304sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.11299sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 96^\circ) \;+\; -0.17208sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.31673sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 8296^\circ) \;+\; -0.13613sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.30632sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 296^\circ) \;+\; -0.21454sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.017533sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 96^\circ) \;+\; -0.2051sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.24558sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 8296^\circ) \;+\; -0.095365sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.089571sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 8296^\circ) \;+\; -0.14098sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.14073sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 67.8296^\circ) \;+\; -0.0050583sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.18235sin(5.7228t \;+\; -236.7928^\circ)$$
$$\ldots 7.8296^\circ) \;+\; -0.32141sin(4.1093t \;+\; 359.3379^\circ) \;+\; -0.31377sin(5.7228t \;+\; -236.7928^\circ)$$



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