CAR SUSPENSION SYSTEM

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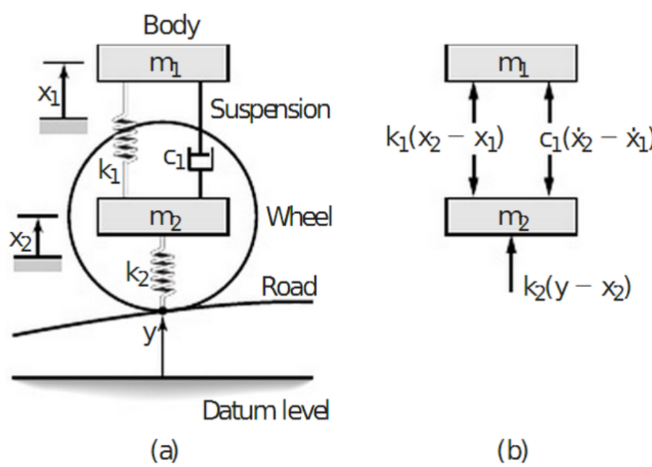
ABSTRACT

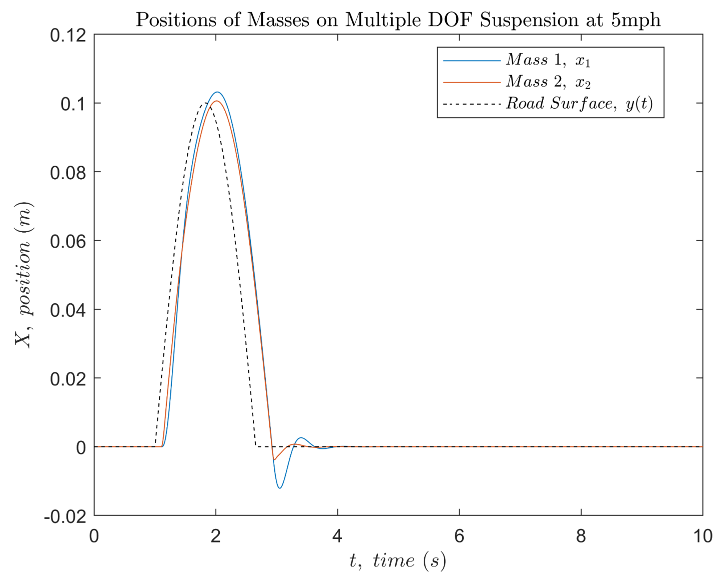
A car suspension analysis can be made simpler by only analyzing only one wheel and suspension (known as a quarter car model), and by lumping the springiness and dampening of components such as tires (known as lumped capacitance modeling). By only analyzing a quarter of a car, and analyzing the chassis and tire assembly as a separate mass, while modeling the suspension and tire itself as mass spring damper systems, it is possible to optimize or properly size the components of a suspension system, such as the proper spring stiffness or shock values.This analysis is complex, as this results is a system of differential equations with two equations. The addition of a damper also exacerbates the messiness of the analysis. However, using computing and state space models, it is possible to easily analyze the positions, velocities, and accelerations of the quarter chassis and tire when it rides over a bump or a pothole. The solutions are numerically integrated using MATLAB.

NOMENCLATURE

m_1	pipe diameter	k_1	suspension spring
m_2	standard pipe diameter	k_2	tire springiness
c	shock damper value	x_1	chassis position wrt its equilibrium
$y(t)$	road surface	x_2	tire assembly position wrt its equilibrium

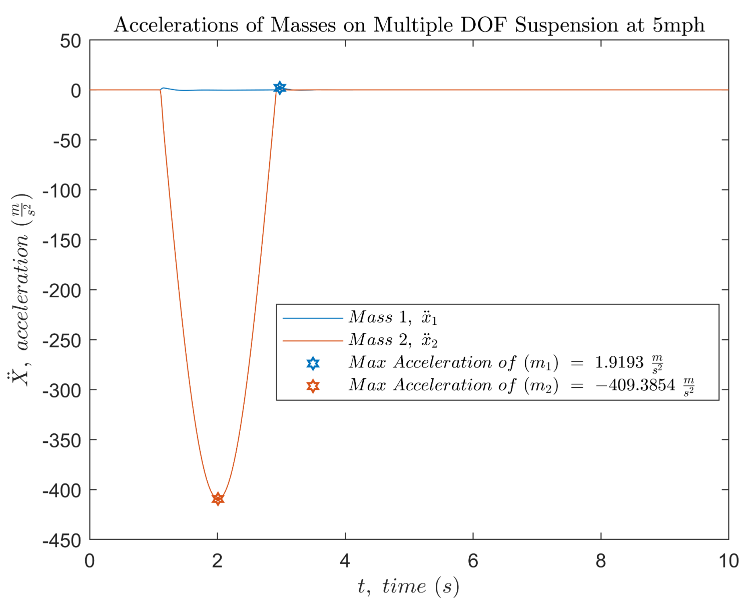
Because the masses are measured with respect to their spring equilibrium, the force of gravity has been taken into account for already, and is therefore not in the analysis of this system.





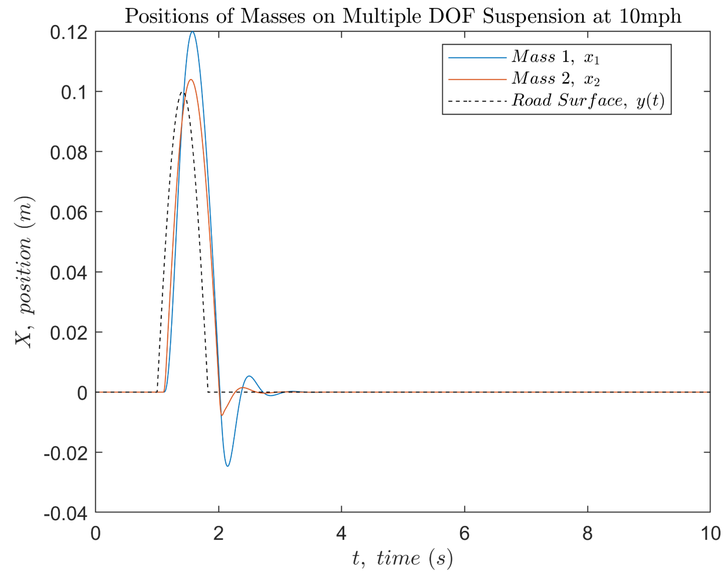
Positions of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) with respect to their equilibrium positions when the car goes 5mph over a 0.1m high bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 5 miles per hour, the displacement of both the tires and the quarter chassis isn't very noticeable. Even coming down, when the car slams and dips, the displacement isn't too severe.



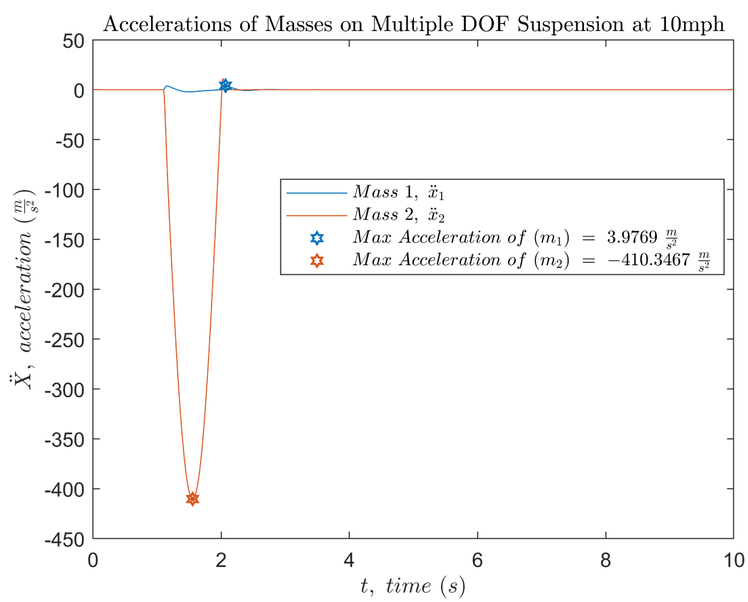
Accelerations of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) positions when the car goes 5mph when the car goes over a 0.1m high bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 5 miles per hour, the acceleration of the tire is extreme compared to that of the quarter chassis. This makes sense, since the tire assembly has significantly less mass, making it easier to accelerate. However, the chassis still accelerates at most at 1.9193 m/s^2, when the ISO 2631 standard states that passenger and car accelerations between 1.25 to 2.5 m/s^2 as very uncomfortable.



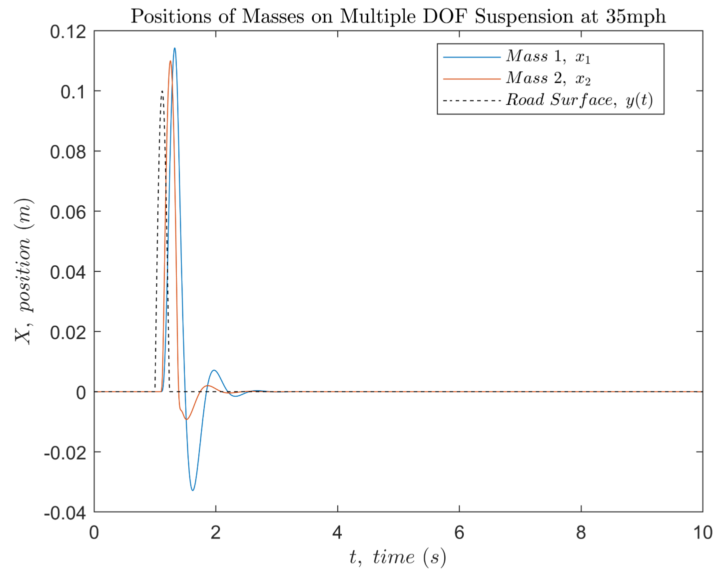
Positions of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) with respect to their equilibrium positions when the car goes 10mph over a 0.1m high, 3.7m long bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 10 miles per hour, the displacement of the quarter chassis is more pronounced as it rides off the bump, and when coming down, when the car slams and dips, the chassis slams deeper.



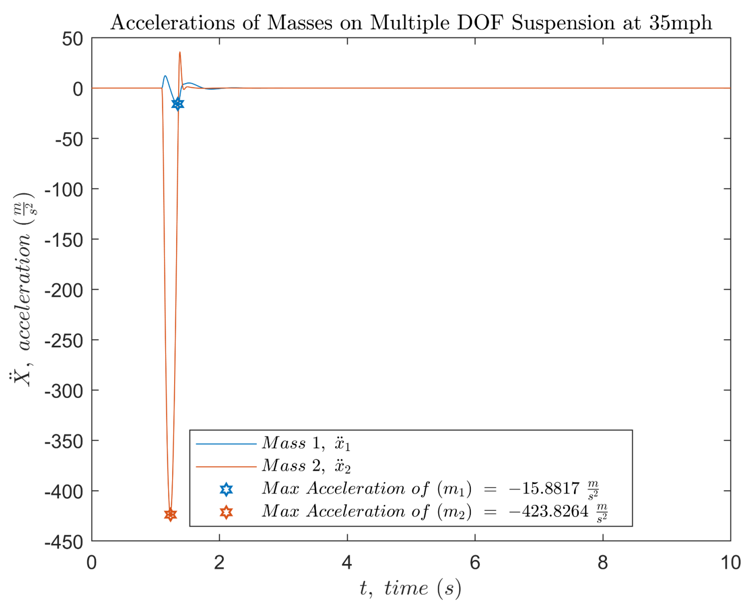
Accelerations of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) positions when the car goes 10mph when the car goes over a 0.1m high, 3.7m long bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 10 miles per hour, the acceleration of the tire is extreme compared to that of the quarter chassis. This makes sense, since the tire assembly has significantly less mass, making it easier to accelerate. However, the chassis still accelerates at most at 3.9769 m/s^2, when the ISO 2631 standard states that passenger and car accelerations between 1.25 to 2.5 m/s^2 as very uncomfortable, and anything beyond as extremely uncomfortable.



Positions of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) with respect to their equilibrium positions when the car goes 35mph over a 0.1m, 3.7m long high bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 35 miles per hour, the displacement of the quarter chassis is more severe as it rides off the bump, and when the car slams and dips, the chassis slams deeper.



Accelerations of Mass 1 (The Quarter Chassis) and Mass 2 (The Tire Assembly) positions when the car goes 35mph when the car goes over a 0.1m high bump.

It can be seen that when the car rides over a bump 0.1m tall (about 3.93701in) at 35 miles per hour, the acceleration of the tire is extreme compared to that of the quarter chassis. This makes sense, since the tire assembly has significantly less mass, making it easier to accelerate. However, the chassis still accelerates at most at -15.8817 m/s^2, when the ISO 2631 standard states that passenger and car accelerations between 1.25 to 2.5 m/s^2 as very uncomfortable, and anything beyond as extremely uncomfortable. This would be a very uncomfortable experience.

Speed bumps are typically 3 to 6 inches in height (76-90mm), and about 12 to 14 feet long (3.7-4.3m). Cars are typically instructed to drive over them at about 20mph. This is to force people to drive slower. The ISO 2631 Standard states that any accelerations between 1.25-2.5m/s^2, and anything beyond, as extremely uncomfortable for the driver.

C.2.3 Comfort reactions to vibration environments

Acceptable values of vibration magnitude for comfort in accordance with 8.2 depend on many factors which vary with each application. Therefore, a limit is not defined in this part of ISO 2631. The following values give approximate indications of likely reactions to various magnitudes of overall vibration total values in public transport.

However, as stated before, the reactions at various magnitudes depend on passenger expectations with regard to trip duration and the type of activities passengers expect to accomplish (e.g. reading, eating, writing, etc.) and many other factors (acoustic noise, temperature, etc.).

Less than 0,315 m/s²:	not uncomfortable
0,315 m/s² to 0,63 m/s²:	a little uncomfortable
0,5 m/s² to 1 m/s²:	fairly uncomfortable
0,8 m/s² to 1,6 m/s²:	uncomfortable
1,25 m/s² to 2,5 m/s²:	very uncomfortable
Greater than 2 m/s²:	extremely uncomfortable

While speed bumps are supposed to be uncomfortable to drive over at high speed (such as at 35 or 40mph), they are not supposed to be uncomfortable driving over at the recommended speed of 20 mph - especially not when going over at 5mph. When this model travels over the 0.1m high, 3.7m long bump at 5mph, the quarter chassis (in which the driver sits in) already accelerates (at max) at 1.9313 m/s^2, which is already listed as very uncomfortable.

$$\Sigma F = m_1 \ddot{x}_1$$

$$k_1(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = m_1 \ddot{x}_1$$

$$\ddot{x}_1 = \frac{k_1}{m_1} x_2 - \frac{k_1}{m_1} x_1 + \frac{c}{m_1} \dot{x}_2 - \frac{c}{m_1} \dot{x}_1$$

$$-k_1(x_2 - x_1) - c(\dot{x}_2 - \dot{x}_1) + k_2(y - x_2) = m_2 \ddot{x}_2$$

$$\ddot{x}_2 = \frac{1}{m_2}\left(k_1 x_1 - k_1 x_2 + c_1 \dot{x}_1 - c_1 \dot{x}_2 + k_2 y - k_2 x_2\right)$$

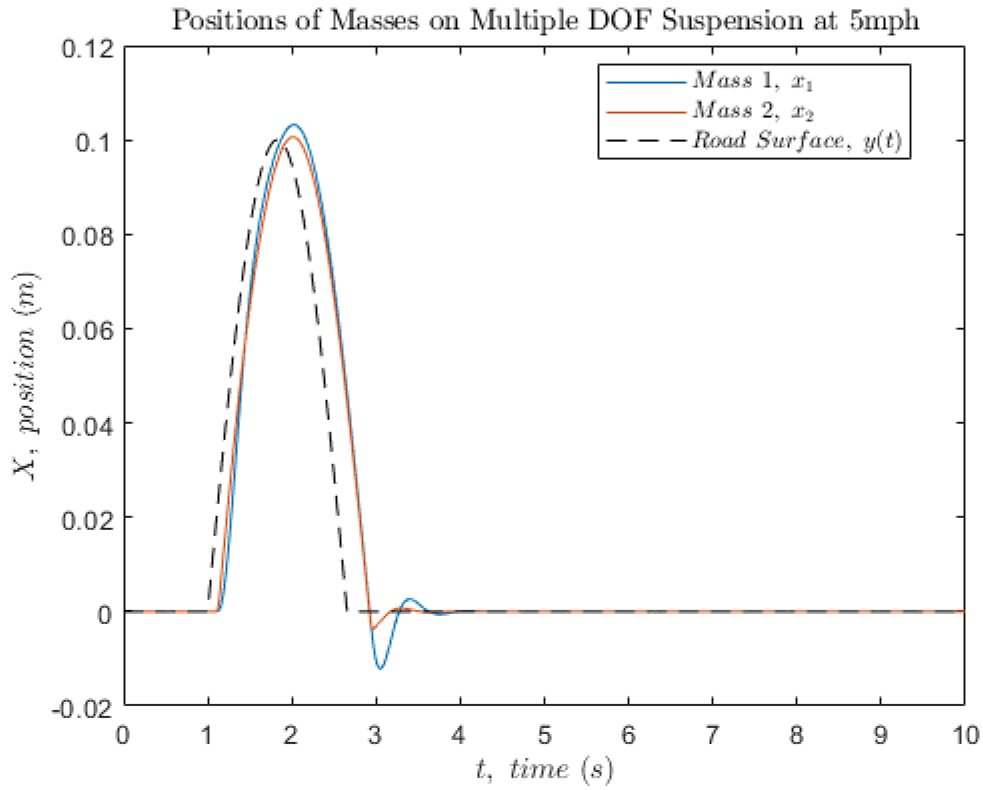
$$= \frac{1}{m_2}\left(k_1 x_1 - (k_1 + k_2) x_2 + c_1 \dot{x}_1 - c_1 \dot{x}_2 + k_2 y\right)$$

From these equations, it can be seen that the acceleration of the quarter chassis can be easily decreased by enlarging its mass, m1. However, doing this presents several other problems, namely, making it harder to accelerate the car, and making it more expensive to go at the same speed than what it takes to do so with its current mass. Increasing the stiffness of the suspension spring (k_1), the tire springiness (k2), or the shock dampening (c) doesn’t clearly show its effects on the acceleration of the quarter chassis, because sometimes, the add, while other times, they subtract from its value. However, because MATLAB was able to numerically integrate the values for x_1, x_1, x_1dot and x_2dot, these values may be optimized computationally.

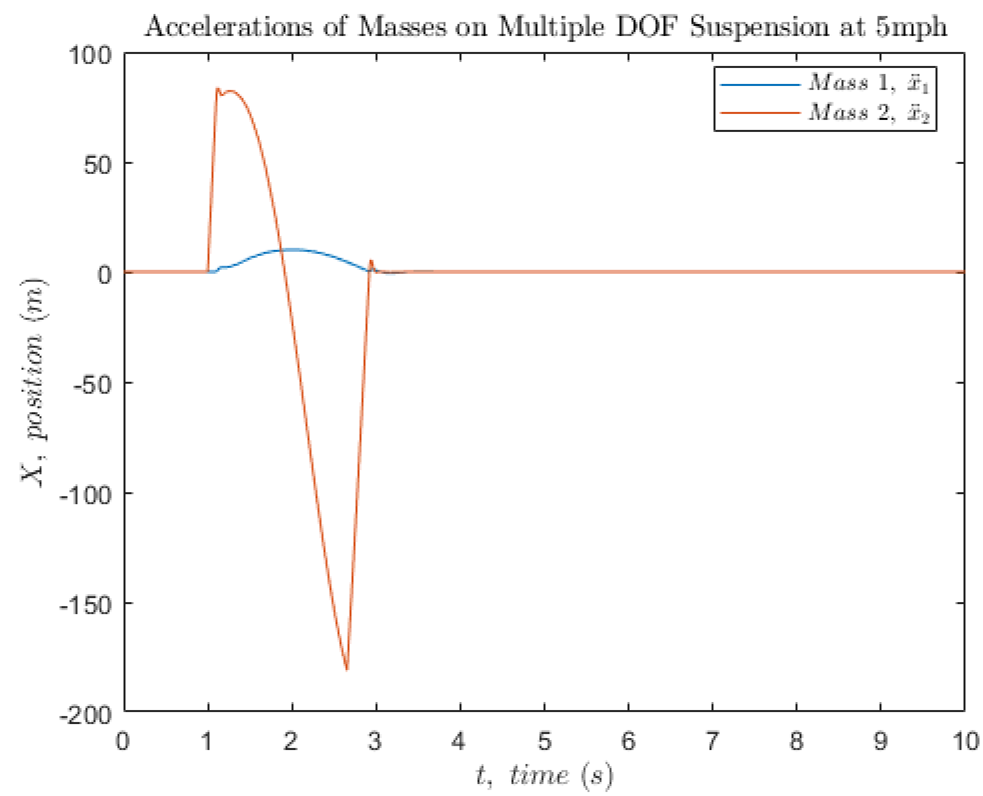
[1]ISO 2631-1 Mechanical vibration and shock-Evaluation of human exposure to whole body vibration
[2] Margaret Parkhill, P.Eng., Rudolph Sooklall, M.A.Sc, Geni Bahar, P.Eng, ”Updated Guidelines for the Design and Application of Speed Humps”

```
clear ; clc ; close all ;

m = [320,44];
k = [3.2*10^4,1.8*10^5];
c = [3430];

v = [5,10,35] * 5280*12*0.0254/3600;

h = 0.1;
L = 3.7;

resolution = 1500;
tStart =0; % time starts at 0 seconds
tEnd = 10; % time ends at this amount of time
% specify the time vector
time = linspace(tStart, tEnd, resolution);
yTime = linspace(tStart,tEnd+1,resolution);
% initial conditions
x_0 =[0,0]; % in m
x_dot_0 =[0,0]; % in m / s
IC =[ x_0(1) , x_0(2) , x_dot_0(1), x_dot_0(2) ]';


for i = 1:length(v)
    d = 1*v(i);
    y = zeros([1,length(time)]);
    [ ~, idxStart] = min(abs(time - d/v(i) ));
    [ ~, idxEnd] = min(abs(time - (d+L)/v(i) ));
    y(idxStart:idxEnd) = h*sin( (pi*v(i)/L) * (time(idxStart:idxEnd) - d/
v(i) ));

    % using the ode solver
    [t , z]= ode45 ( @ (t , z )
 multipleDOF_Car_Suspension_Function(t,z,y,yTime,m,c,k) , time , IC );
    %only passing time into function, and not just ode45, so that it can
    %interpolate the input y for any time t
    figure;
    plot(t , z(: ,1)) %plots all of z_1, aka, all of x_1
    hold on
    plot(t , z(: ,2)) %plots all of z_2, aka, all of x_2
    hold on
    plot(t,y,'k--')
    legendStuff = ["$Mass \ 1, \ x_{1}$","$Mass \ 2, \ x_{2}$","$Road \
 Surface, \ y(t)$"];
    lgnd= legend(legendStuff);
    set(lgnd, 'Interpreter','latex')
    lgnd.Location = 'best';
    title( strcat("Positions of Masses on Multiple DOF Suspension at ",
 num2str(v(i)/(5280*12*0.0254/3600)) ,"mph"),'Interpreter','latex')
    xlabel("$ t, \ time \ (s) $",'Interpreter','latex')
    ylabel("$X, \ position \ (m)$",'Interpreter','latex')
    hold off;
```

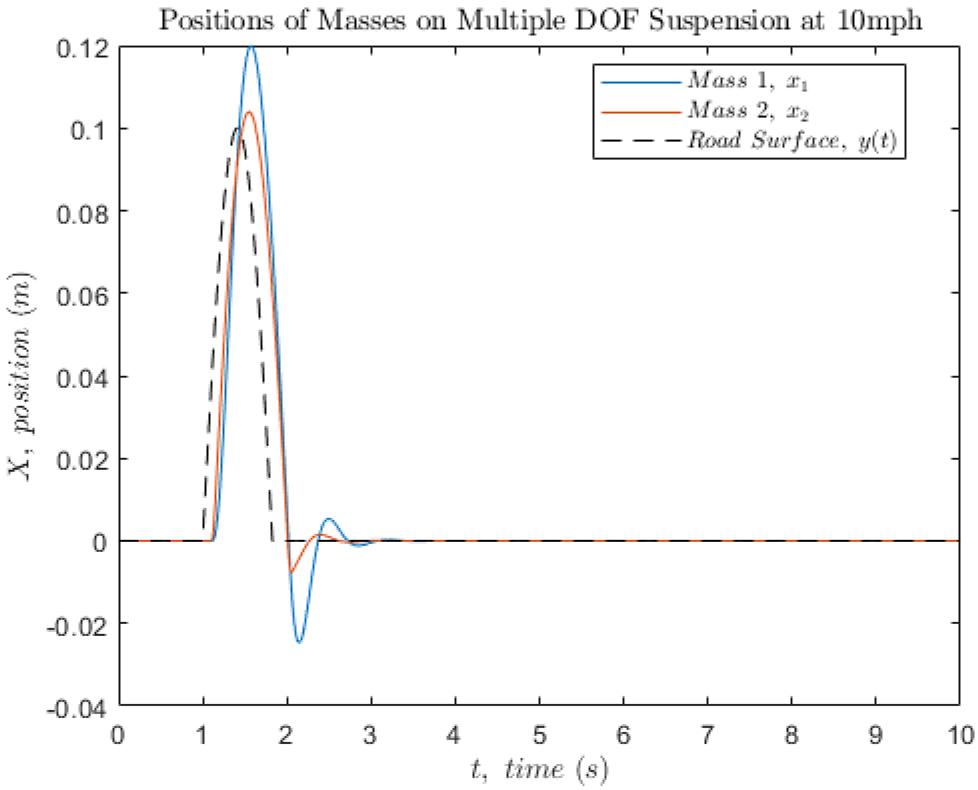
```
    plot1 = strcat("Positions of Masses on Multiple DOF Suspension at ",
 num2str(v(i)/(5280*12*0.0254/3600)) ,"mph");
    print('-r600','-dpng',plot1);

    figure;
    plot(t , (1/m(1))*( c(1)*z(:,4)+k(1)*z(:,2)-c(1)*z(:,3)-
k(1)*z(1) ) ) %plots all of dot_dot_z_1, check formula on state space work
    hold on
    %(1/m(2))*( k(2)*y'+c(1)*z(:,3)+k(1)*z(:,1)-c(1)*z(:,4)-
(k(1)+k(2))*z(:,2) )
    %(1/m(2))*( -k(2)*(z(:,2)-y') + k(1)*(z(:,1)-z(:,2)) + c(1)*(z(:,3)-
z(:,4)) )
    plot(t , (1/m(2))*( k(2)*y'+c(1)*z(:,3)+k(1)*z(:,1)-c(1)*z(:,4)-
(k(1)+k(2))*z(:,2) )  ) %plots all of dot_dot_z_2, check formula on state
 space work
    legendStuff = ["$Mass \ 1, \ \ddot{x}_{1}$","$Mass \ 2, \ \ddot{x}_{2}$"];
    lgnd= legend(legendStuff);
    set(lgnd, 'Interpreter','latex')
    lgnd.Location = 'best';
    title( strcat("Accelerations of Masses on Multiple DOF Suspension at ",
 num2str(v(i)/(5280*12*0.0254/3600)) ,"mph"),'Interpreter','latex')
    xlabel("$ t, \ time \ (s) $",'Interpreter','latex')
    ylabel("$X, \ position \ (m)$",'Interpreter','latex')
    hold off;

    plot1 = strcat("Accelerations of Masses on Multiple DOF Suspension at ",
 num2str(v(i)/(5280*12*0.0254/3600)) ,"mph");
    print('-r600','-dpng',plot1);
end
```



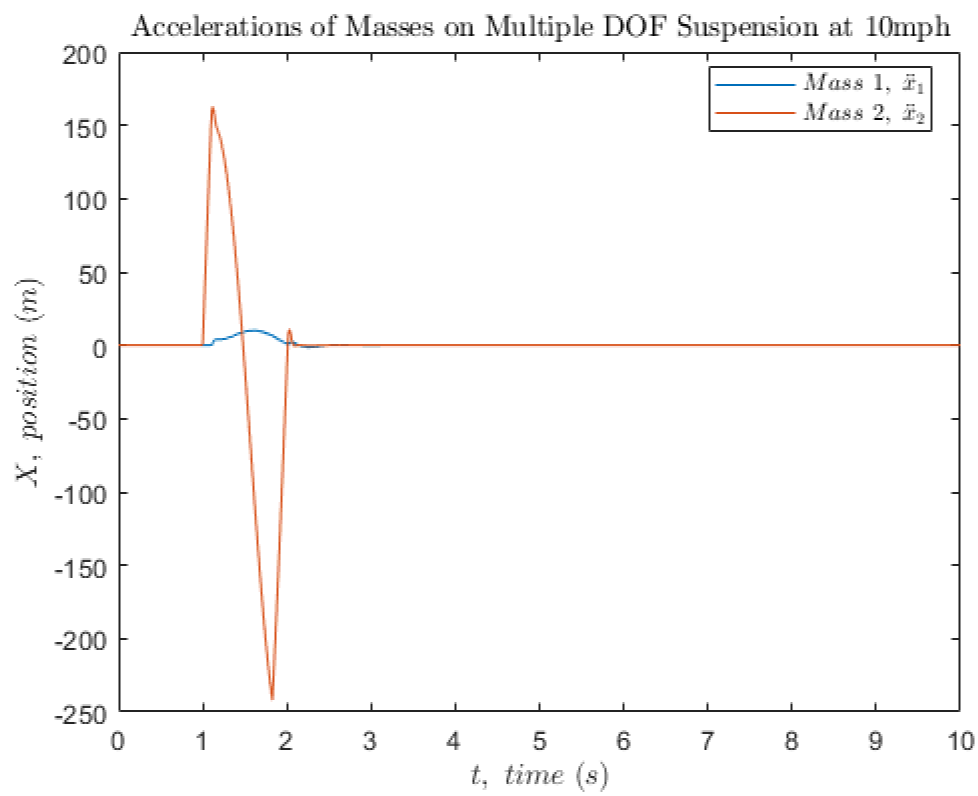
Positions of Masses on Multiple DOF Suspension at 5mph
Mass 1, x_1
Mass 2, x_2
Road Surface, $y(t)$
X, position (m)
t, time (s)



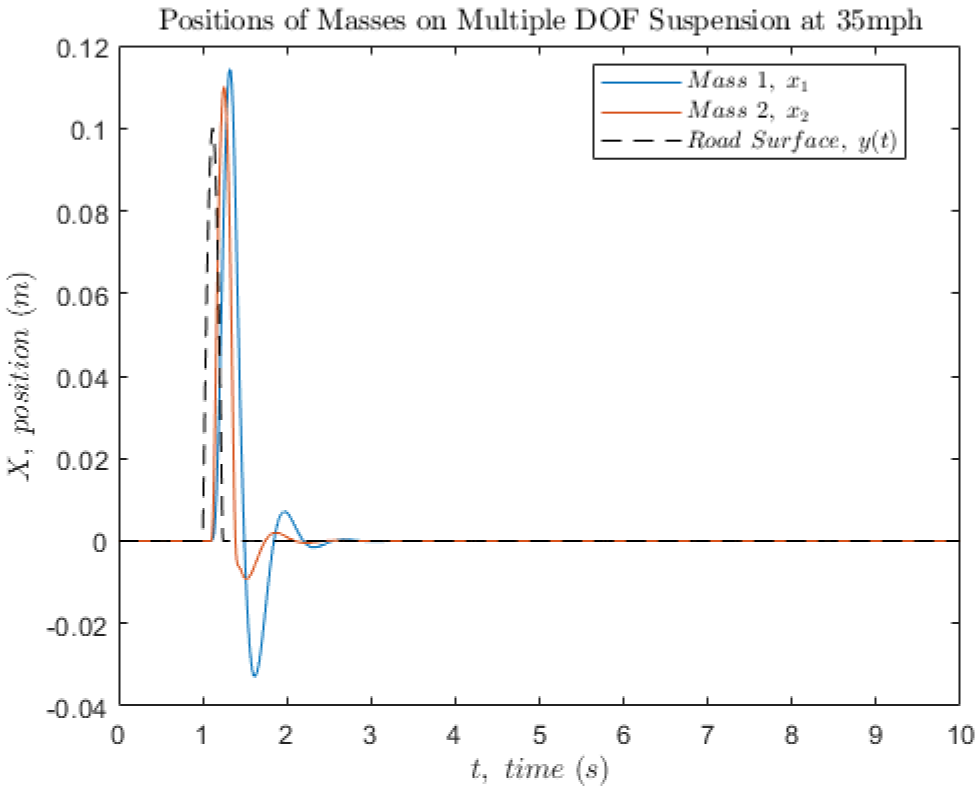
Accelerations of Masses on Multiple DOF Suspension at 5mph
Mass 1, $\ddot{x}_1$
Mass 2, $\ddot{x}_2$
X, position (m)
t, time (s)



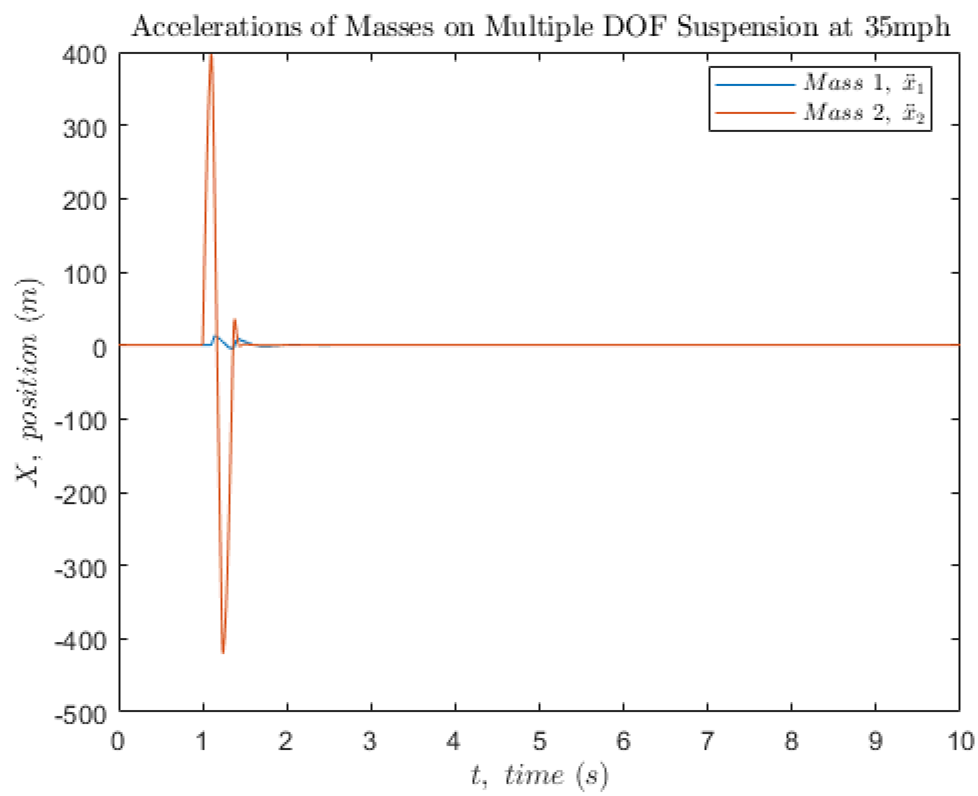
Positions of Masses on Multiple DOF Suspension at 10mph
Mass 1, x_1
Mass 2, x_2
Road Surface, $y(t)$
X, position (m)
t, time (s)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0 1 2 3 4 5 6 7 8 9 10



Accelerations of Masses on Multiple DOF Suspension at 10mph
Mass 1, $\ddot{x}_1$
Mass 2, $\ddot{x}_2$
X, position (m)
t, time (s)
200
150
100
50
0
-50
-100
-150
-200
-250
0 1 2 3 4 5 6 7 8 9 10



Positions of Masses on Multiple DOF Suspension at 35mph
Mass 1, x_1
Mass 2, x_2
Road Surface, $y(t)$
X, position (m)
t, time (s)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0 1 2 3 4 5 6 7 8 9 10



Accelerations of Masses on Multiple DOF Suspension at 35mph
Mass 1, $\ddot{x}_1$
Mass 2, $\ddot{x}_2$
X, position (m)
t, time (s)
400
300
200
100
0
-100
-200
-300
-400
-500
0 1 2 3 4 5 6 7 8 9 10

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```
function [z_dot] = multipleDOF_Car_Suspension_Function(t,z,y,time,m,c,k)


%t isn't actually an array. The function gets passed one individual value
 %of t at a time. check it out with
t;
length(t);

% y = 0;
% if t >= d/v && t <= (d+L)/v
%    y = h*sin( (pi*v/L) * (t - d/v));
% end

y = interp1(time,y,t); %values of y at whatever t must be interpolated,
%since there might not be a given y value for that value of t

z_dot(1) = z(3);
z_dot(2) = z(4);
z_dot(3) = (1/m(1)) * ( c(1)*z(4) + k(1)*z(2) - c(1)*z(3) - k(1)*z(1) );
z_dot(4) = (1/m(2)) * ( k(2)*y + c(1)*z(3) + k(1)*z(1) - c(1)*z(4) -
 (k(1)+k(2))*z(2) );

z_dot = z_dot';
end
```

```
Not enough input arguments.

Error in multipleDOF_Car_Suspension_Function (line 6)
t;
```

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