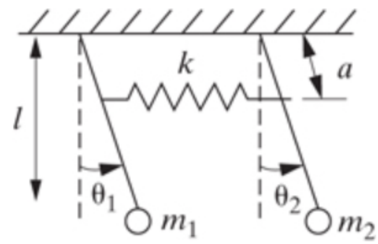
Homework #10

Due online: Thursday April 27 at 11pm

Turn in the following problems

1. Consider the system in the figure to the right which consists of two pendulums coupled by a spring where $k = 20N/m$, $\ell = 0.5m$, and $m_1 = m_2 = 10kg$, $a = 0.1m$ along the pendulum.



   (a) Find the equations of motion and write them in matrix form.

   Answer: $\begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix} \ddot{\boldsymbol{\theta}} + \begin{bmatrix} 49.25 & -0.2 \\ -0.2 & 49.25 \end{bmatrix} \boldsymbol{\theta} = \mathbf{0}$

   (a) Determine the natural frequencies for the system

   Answer: $\omega_1 = 4.429 \frac{rad}{s}$, $\omega_2 = 4.448 \frac{rad}{s}$

   (b) Determine the associated mode shapes for the system.

   Answer: $\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\boldsymbol{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (remember the vectors could be any multiple of the answers given)

2. Consider the example from class where the two-degree-of-freedom system was defined by

$$\begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

   and the solution was

$$\boldsymbol{x}(t) = A_1 \sin(\omega_1 t + \phi_1) \boldsymbol{u_1} + A_2 \sin(\omega_2 t + \phi_2) \boldsymbol{u_2}$$

   with $\omega_1 = \sqrt{2} \frac{rad}{s}$, $\omega_2 = 2 \frac{rad}{s}$ $\boldsymbol{u}_1 = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}$, and $\boldsymbol{u}_2 = \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix}$.

   (a) Calculate the response of the system to the initial conditions (i.e. determine A_1, A_2, ϕ_1, and ϕ_2 given the initial conditions)

$$\boldsymbol{x}(0) = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix} \qquad \dot{\boldsymbol{x}}(0) = \mathbf{0}$$

   Answer: $\boldsymbol{x}(t) = \begin{bmatrix} 0.236 \cos(1.414t) \\ 0.707 \cos(1.414t) \end{bmatrix}$

   (b) Plot the response of both masses found in part (a) on the same plot. Clearly label your plot and pick an appropriate time scale.

   (c) Calculate the response of the system to the initial conditions

$$\boldsymbol{x}(0) = \mathbf{0} \qquad \dot{\boldsymbol{x}}(0) = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{3} \\ -1 \end{bmatrix}$$

   Answer: $\boldsymbol{x}(t) = \begin{bmatrix} 0.118 \sin(2t) \\ -0.354 \sin(2t) \end{bmatrix}$

   (d) Plot the response of both masses found in part (c) on the same plot. Clearly label your plot and pick an appropriate time scale.