MEMORANDUM

To: Хопь Л Вайсс
From: Nathan Delos Santos
Date: 9/22/2023
Re: Replicating Figures Produced In Paper "Windkessel Model Analysis In MATLAB"

Summary:
The academic paper this memo is trying to replicate the figures "Windkessel Model Analysis In MATLAB" has provided suspicious results and figures. Block diagrams were created using values provided in the paper's for the 2-, 3-, and 4- Element Windkessel Model, which outputs the pressure in the vessels modeled as voltage $u(t)$, for a given input current $i(t)$ representing the flow coming into the blood vessels. Like the figure provided, the graphs that the block diagrams produce show the pressure in the blood vessels $u(t)$ vs time t in seconds. The graph from the paper does not match any of the graphs from the created model.

Procedure:
To create the graphs, the block diagrams needed to be created first. The Windkessel models provide a set of differential equations for each n-element model, with $i(t)$ being the input, or forcing function, and $u(t)$ being the output variable. However, being differential equations, the output is mixed in with its own derivatives. To get a clear output $u(t)$ (which is solved numerically), the lowest derivative given must be integrated until the output function is returned. For example, the 2-element Windkessel model gives the equation:

$$i(t) \ = \ \frac{1}{R}u(t) \ + \ C\frac{du(t)}{dt} \qquad (1)$$

Where is $\frac{du(t)}{dt} = \frac{du(t)}{dt}$ the lowest derivative of $u(t)$. To get out $u(t)$, $\dot{u}(t)$ must be integrated once. (In the block diagram, $\dot{u}(t)$ is u_dot). $\frac{1}{s}$ is the integrator block.

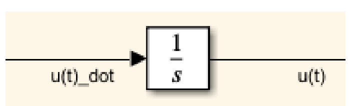

Figure 1: Integrating in the block diagram in simulink

Unfortunately, this is still meaningless, as the values of $u(t)$ are unknown since the values of$\dot{u}(t)$ are unknown. To see how $\dot{u}(t)$ is computed, the differential equation must be rearranged to solve in terms of $\dot{u}(t)$.

$$i(t) \ = \ \frac{1}{R}u(t) \ + \ C\frac{du(t)}{dt} \quad \Rightarrow \ \frac{du(t)}{dt} \ = \ \frac{1}{C}\left(i(t) \ - \ \frac{1}{R}u(t)\right) \qquad (1)$$

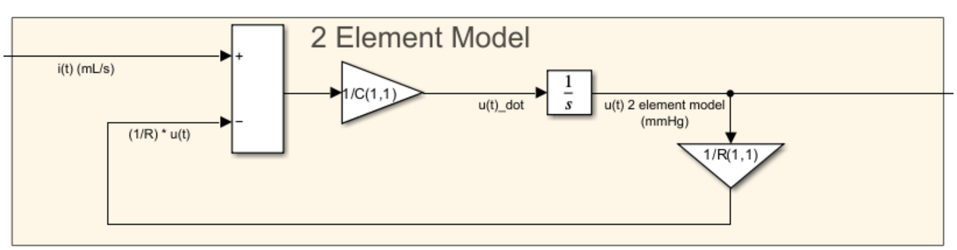

Figure 2: Seeing the feedback in a block diagram

It can be seen that $\frac{du(t)}{dt}$ is the difference of the input current minus the actual $u(t)$ with some gain involved. The feedback can be clearly seen in the block diagram. Similar processes were repeated for the 3-Element and 4-Element models, each one being fed the same forcing function, $i(t)$. The other n-element Windkessel Model Equations, and their block diagrams can be found in the appendix.

The input current, $i(t)$ was copied exactly as it was from the academic paper. It was given in wrong units of m/s, which the block diagrams corrected to mL/s. It was given that the piecewise function, where $T_s = 0.3s$, $T = 0.8s$, $I_0 = 500mL$:

$$i(t) = \begin{cases} I_0 \sin^2\left(\frac{\pi t}{T_s}\right) & 0 \le t \le T_s \\ 0 & T_s \le t \le T \end{cases} \tag{7}$$

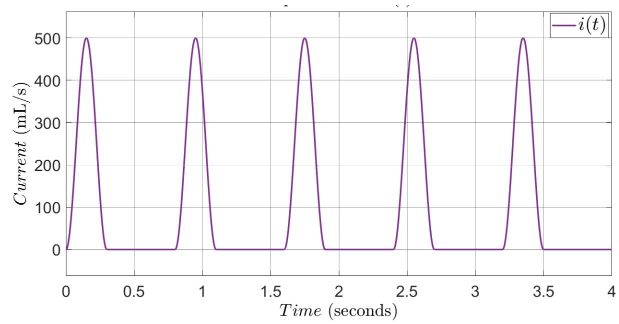

Figure 3: i(t) as a function of time, graphed as a repeated sequence

Results:

The academic paper in question gave three values for R, L, C, r for the 3-Element and 4-Element models. For the created block diagrams, each value was used, and their outputs were the voltage $u(t)$ for each system, vs time. The simulation ran for 4 seconds each time.

Tab. 1: *The values of normal human parameters of the Windkessel models*

	R [mmHg.s.cm^{-3}]	C [cm^3.mmHg^{-1}.s^2.cm^{-3}]	r [mmHg.s.cm^{-3}]	L [mmHg.s^2.cm^{-3}]
2 WM	1	1	-	-
3 WM	1 0.79 0.63	1 1.75 5.16	0.05 0.033 0.03	-
4 WM	1 0.79 0.63	1 1.22 2.53	0.05 0.056 0.045	0.005 0.0051 0.0054

Table 1: The table given from "Windkessel Analysis in MATLAB"

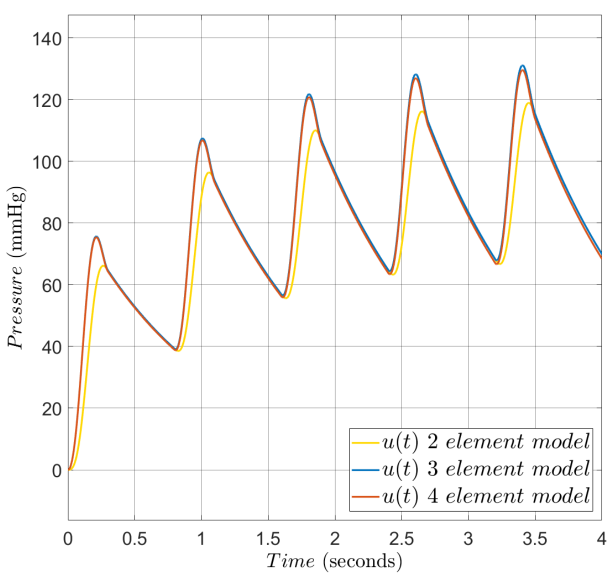

Figure 4: Windkessel Models vs Time, 1st Set Of Values

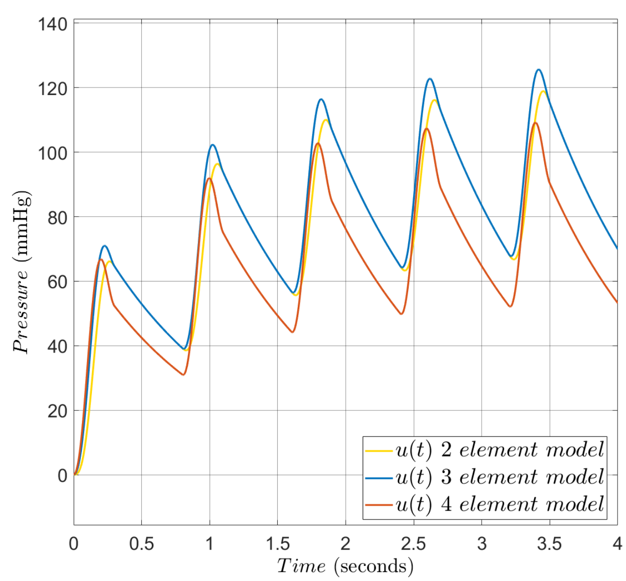

Figure 5: Windkessel Models vs Time, 2nd Set Of Values

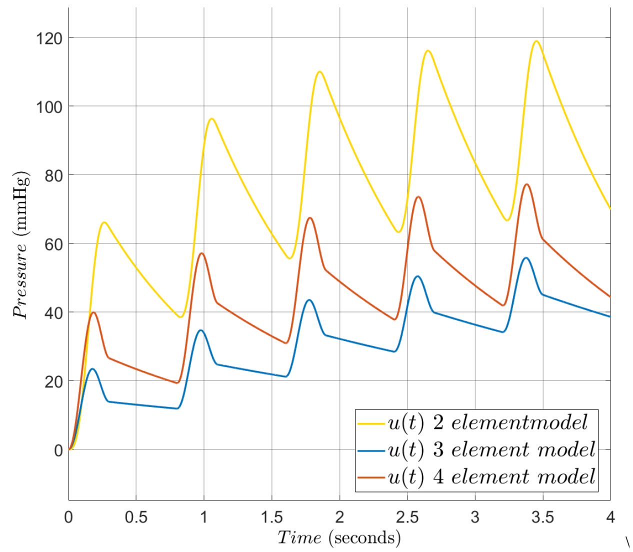

Figure 6: Windkessel Models vs Time, 3rd Set Of Values

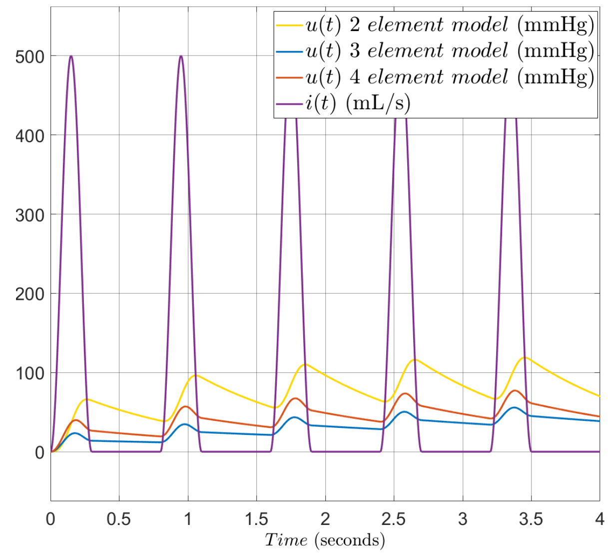

Figure 7: Windkessel Models vs Time, 3rd Set Viewed With Input Current, $i(t)$

It can be clearly seen that for all three models, and all three sets of values, that the rise and fall in "voltages" (pressures) correspond to the spikes and falls in current. When the current hit a peak (systole), it spurred the voltages to peak as well, with some delay. When the current flatlined to zero (diastol), the voltages gradually dropped down. Within the 4 second simulation time, the piecewise current was able to cycle five times. It can be seen that the voltages rise a little higher each cycle, as the model probably has not yet entered steady state within the given runtime. It can also be observed that the 1st set of values provides the n-element models with the closest matching graphs, and that the 3rd set of values provides the greatest discrepancy between them.

Conclusion:

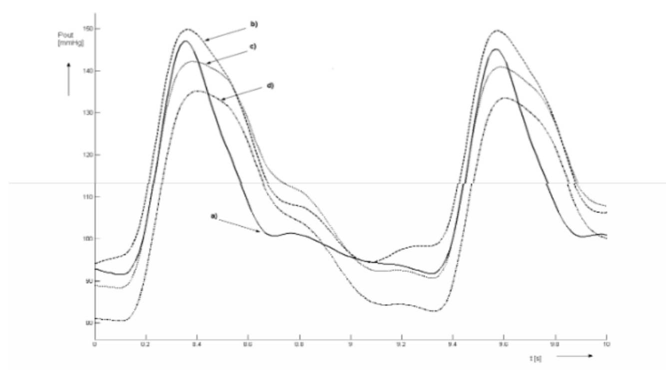

Fig. 3: *Arterial pressure for three WM (a – measured pressure (solid line), b – 4WM (dashed line), c – 3WM (dot line), d – 2WM (dot-and-dash line))*

Figure 8: Windkessel Graphs from "Windkessel Analysis in MATLAB"

The graph from the academic paper only has two major humps during its 10 second run, unlike the graphs from the block diagrams, which had five humps within 4 seconds, following the pulse of the input current, $i(t)$. Although the graphs from this memo do not reach steady state within 4 seconds, it is clear that the peaks should coincide with the peaks of the current. The fluctuations during the dips also do not make sense. The models from the block diagrams do not have a wavy shape during diastole, unlike the one from the paper. In addition, it is hard to read the values on both the time and voltage (pressure) axes. This suggests that either the memo's block diagrams are incorrect, or that the graph from the paper in question is incorrect, or used the wrong values not provided in its own table. The paper never specified which value set it used for the graph, nor did it show any block diagrams or how it got its graphs. These pieces of information are required for a complete comparison of methodology and results.

References:

[1]N. Westerhof, J.-W. Lankhaar, and B. E. Westerhof, “The arterial Windkessel,” *Medical & Biological Engineering & Computing*, vol. 47, no. 2, pp. 131–141, Jun. 2008, doi: https://doi.org/10.1007/s11517-008-0359-2.

[2]M. Hlaváč, “Windkessel Model Analysis in MATLAB,” 2004.

Appendix:

MATLAB Code To Generate $i(t)$:

```
clc;
clear;
%close all;
I_0 = 500;
t_s = 0.3;
t_end_cyc = 0.8;
resolution = 1000;
sampleTime = [t_end_cyc/resolution , 0]; %period,offset
t = linspace(0,t_end_cyc,resolution);
%t_flat = linspace(t_s,t_end_cyc,resolution);
t_humplast_index = max(find(t <= t_s));
i = zeros(1,resolution);
i(1:t_humplast_index) = I_0 * sin( pi* t(1:t_humplast_index) / t_s ) .^2;
i(t_humplast_index + 1 : resolution) = 0;
i_of_t = [t',i'];
R = [1,NaN,NaN;1,0.79,0.63;1,0.79,0.63];
C = [1,NaN,NaN;1,1.75,5.16;1,1.22,2.53];
r = [NaN,NaN,NaN;0.05,0.033,0.03;0.05,0.056,0.045];
L = [NaN,NaN,NaN;NaN,NaN,NaN;0.005,0.0051,0.0054];
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Windkessel Equations For 2-, 3-, 4- Element models:

- 2-Element:

$$i(t) = \frac{1}{R}u(t) + C\frac{du(t)}{dt} \Rightarrow \frac{du(t)}{dt} = \frac{1}{C}\left(i(t) - \frac{1}{R}u(t)\right) \quad (1)$$

- 3-Element:

$$i(t) = \frac{1}{R}u_c(t) + C\frac{du_C(t)}{dt} \Rightarrow \frac{du_c(t)}{dt} = \frac{1}{C}\left(i(t) - \frac{1}{R}u_c(t)\right), \quad (2)$$

$$u(t) = r \cdot i(t) + u_c(t) \quad (3)$$

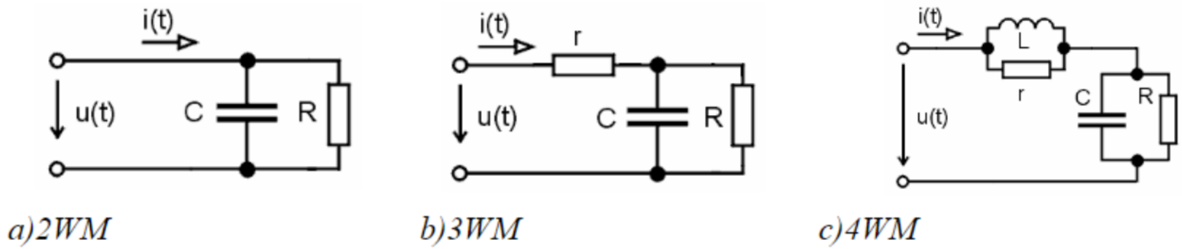
- 4-Element:

$$i(t) = \frac{1}{R}u_c(t) + C\frac{du_C(t)}{dt} \Rightarrow \frac{du_c(t)}{dt} = \frac{1}{C}\left(i(t) - \frac{1}{R}u_c(t)\right), \quad (2)$$

$$i(t) = i_L(t) + \frac{L}{R}\frac{di_L(t)}{dt}, \quad (4)$$

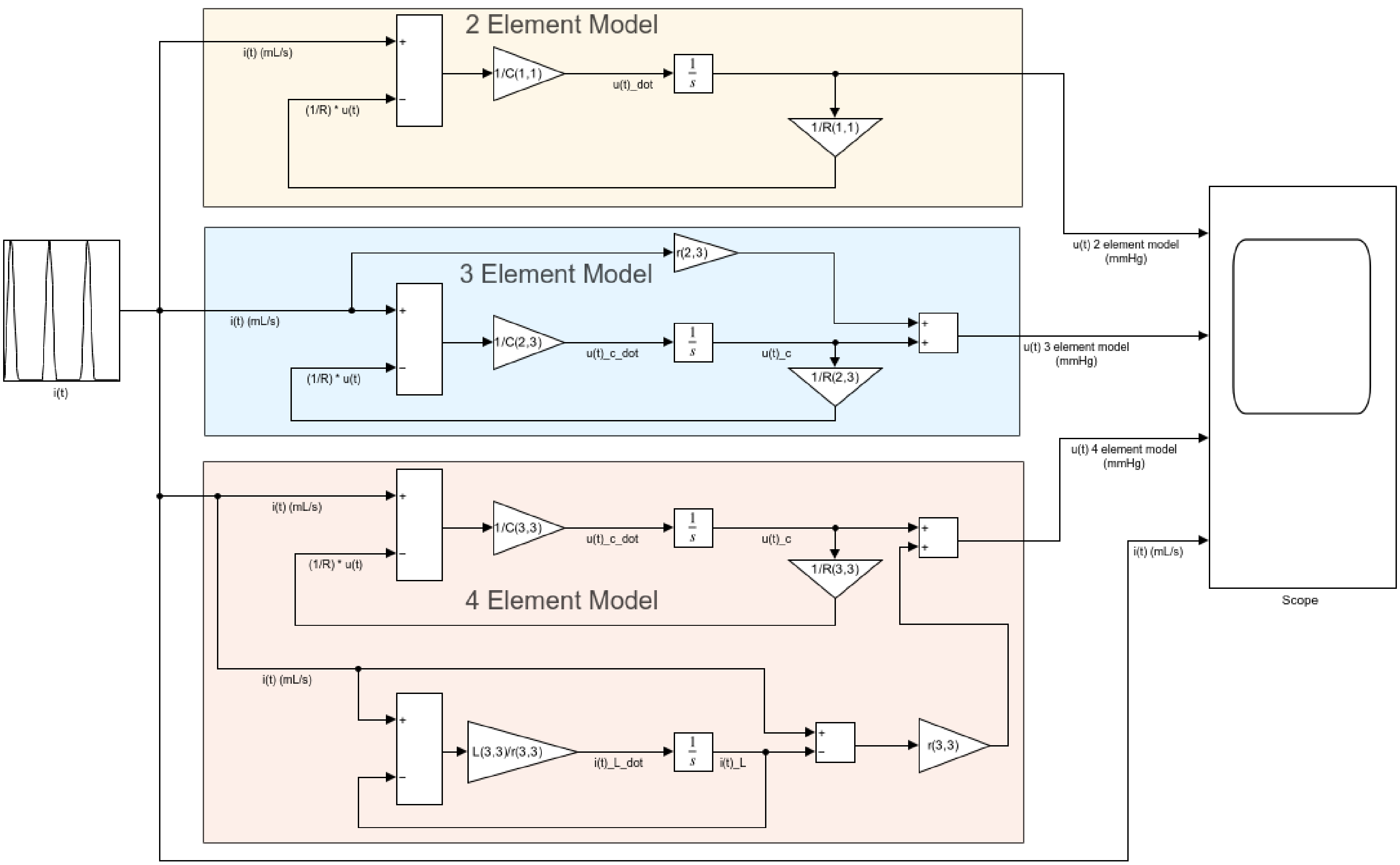
$$u(t) = u_c(t) + r\left(i(t) - i_L(t)\right) \quad (5)$$

Windkessel Circuit Models:



$Figure\ 9$: $Windkessel\ Circuit\ Models\ from\ "Windkessel\ Analysis\ in\ MATLAB"$

Windkessel Block Diagrams For 2-, 3-, 4- Element Models:



$Figure\ 10$: $2 - Element,\ 3 - Element,\ 4 - Element\ Windkessel\ block\ diagrams, with\ all\ given\ the\ same\ forcing\ function,\ i(t),\ and\ outputted\ to\ a\ scope$