

## Statistical physics 2, homework 3

### Monte Carlo simulation of the 2D Ising model

Write and document a Monte Carlo simulation code for the classical 2D Ising model on a square lattice. Please follow the steps below:

1. Use a square lattice of size  $L \times L = 10 \times 10$  with periodic boundary conditions.
2. The Hamiltonian is the following:

$$\mathcal{H} = -K \sum_{i,j} \sigma_i \sigma_j - h \sum_i \sigma_i$$

Set the parameters as  $K = 1$ ,  $k_B T = 5$ .

3. Assign each lattice  $j$  a random spin  $\sigma_j = \pm 1$ .
4. A Monte Carlo time step consists of  $L \times L$  elementary steps in which a spin is chosen randomly and is flipped with the Metropolis probabilities.
5. Measure the Monte Carlo correlation time  $\tau$  from the exponential decay of the average magnetization starting from the ferromagnetic “all spins up” configuration.
6. Modify the external field  $h$  in the range  $h \in [0, 10]$  and plot the  $M(h)$  curve.
7. Measure the susceptibility using  $\chi = \partial M / \partial h|_{h=0}$ . [Hint: Use small fields.]
8. Verify the susceptibility from the fluctuations of the magnetization,  $\chi = (\langle M^2 \rangle - \langle M \rangle^2) / (k_B T)$ .