en la parábola $= \dfrac{y^2}{4a}$

$$
\begin{aligned}
V &= 2\int_0^{2a} \pi \left[a^2 - \left(\frac{y^2}{4a}\right)^2\right] dy \\
V &= 2\pi \int_0^{2a} \left(a^2 - \frac{y^4}{16a^2}\right) dy \\
V &= 2\pi \left[a^2 y - \frac{y^5}{80a^2}\Big|_0^{2a}\right] \\
V &= 2\pi \left[\left(2a^3 - \frac{32a^5}{80a^2}\right) - (0-0)\right] \\
V &= 2\pi \left[2a^3 - \frac{2}{5}a^3\right] \\
V &= 2\pi \left(\frac{8}{5}a^3\right) \\
V &= \frac{16}{5}\pi a^3 u^3
\end{aligned}
$$

Ejemplo 3.6

Calcule el volumen del toro que se engendra cuando el área encerrada por la circunferencia $x^2 + y^2 = a^2$ gira alrededor de la recta $x = b$ con $b > a$.

Solución

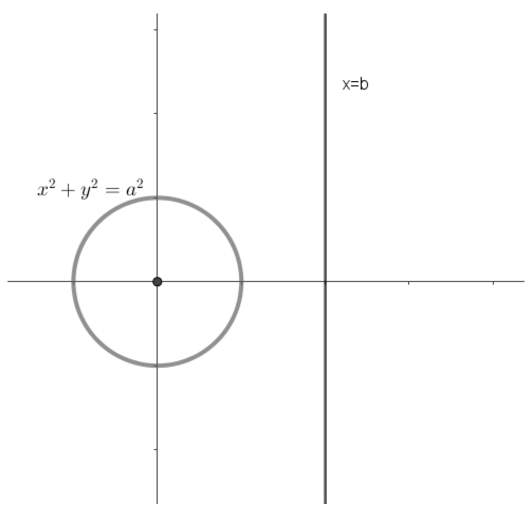

Figura 3.9: Ejemplo 2 a

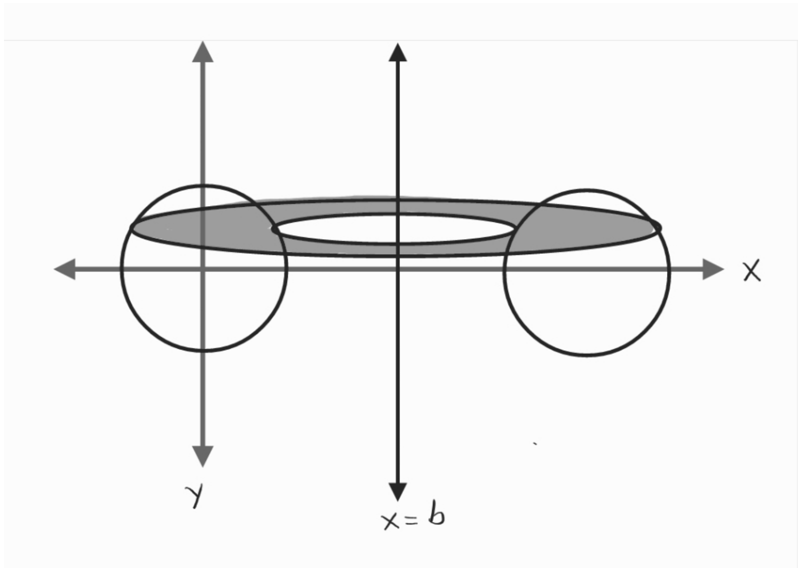

Figura 3.10: Ejemplo 2 b

$R = b + x \qquad r = b - x$

$$
\begin{aligned}
V &= 2\int_0^a \pi\left[(b+x)^2 - (b-x)^2\right] dy \\
V &= 2\int_0^a \pi\left[\left(b^2 + 2bx + x^2\right) - \left(b^2 - 2bx + x^2\right)\right] dy \\
V &= 2\pi\int_0^a \left(\cancel{b^2} + 2bx + \cancel{x^2} - \cancel{b^2} + 2bx - \cancel{x^2}\right) dy \\
V &= 2\pi\int_0^a 4bx \cdot dy \\
V &= 8\pi b\int_0^a x dy \therefore x^2 + y^2 = a^2
\end{aligned}
$$

$$x^2 = a^2 - y^2$$

$$x^2 = \pm\sqrt{a^2 - y^2}$$

$$V = 8\pi b\int_0^a \sqrt{a^2 - y^2} dy$$

Sustitución Trigonométrica

$$y = a\sin\theta \quad \longrightarrow \quad dy = a\cos\theta d\theta$$

$$\sin\theta = \frac{y}{a} \quad \longrightarrow \quad \theta = \sin^{-1}\left(\frac{y}{a}\right)$$

$$y\Big|_0^a \qquad \theta\Big|_0^{\frac{\pi}{2}}$$

$$
\begin{aligned}
V &= 8\pi b \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2\theta} \cdot a\cos\theta d\theta \\
V &= 8\pi b \int_0^{\frac{\pi}{2}} a\cos\theta a\cos\theta d\theta \\
V &= 8\pi a^2 b \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \\
V &= 8\pi a^2 b \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(1+\cos\left(2\theta\right)\right) d\theta \\
V &= 4\pi a^2 b \int_0^{\frac{\pi}{2}} \left(1+\cos\left(2\theta\right)\right) d\theta \\
V &= 4\pi a^2 b \left[\int_0^{\frac{\pi}{2}} d\theta + \int_0^{\frac{\pi}{2}} \cos(2\theta) d\theta\right]
\end{aligned}
$$

$$h = 2\theta \quad ; \quad dh = 2d\theta$$

$$d\theta = \frac{dh}{2} \quad \theta\Big|_0^{\frac{\pi}{2}} \quad h\Big|_0^{\pi}$$

$$
\begin{aligned}
V &= 4\pi a^2 b \left[\int_0^{\frac{\pi}{2}} d\theta + \int_0^{\pi} \cos(h) \frac{dh}{2}\right] \\
V &= 4\pi a^2 b \left[\theta\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\sin(h)\Big|_0^{\pi}\right] \\
V &= 4\pi a^2 b \left[\left(\frac{\pi}{2} - 0\right) + \frac{1}{2}\cancelto{0}{(\sin(\pi) - \sin(0))}\right] \\
V &= 4\pi a^2 b \left(\frac{\pi}{2}\right) \\
V &= 2\pi^2 a^2 b u^3
\end{aligned}
$$

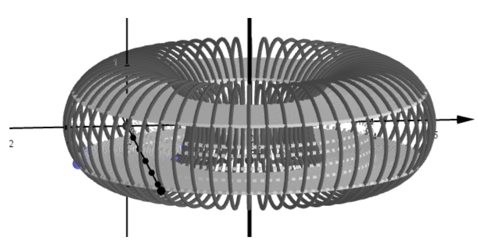

Figura 3.11: Ejemplo 2 c

$y\Big|_0^{60} \qquad A\triangle = \frac{bh}{2} \qquad b = 30ft \qquad h = 10ft$

$$\begin{aligned} V &= \int_0^{60} \frac{30ft10ft}{2} dy \\ V &= 150ft^2 \int_0^{60} dy \\ V &= 150ft^2 \left(y\Big|_0^{60} \right) \\ V &= 150ft^2 (60-0)\, ft \\ V &= 9000ft^3 \end{aligned}$$

Ejemplo 3.11

La aguja de la torre de una iglesia mide 30 pies de altura y tiene secciones cuadradas, cuyo lado varía linealmente desde 3 pies en la base hasta 6 pulgadas en lo alto. Calcular el volumen.

Solución

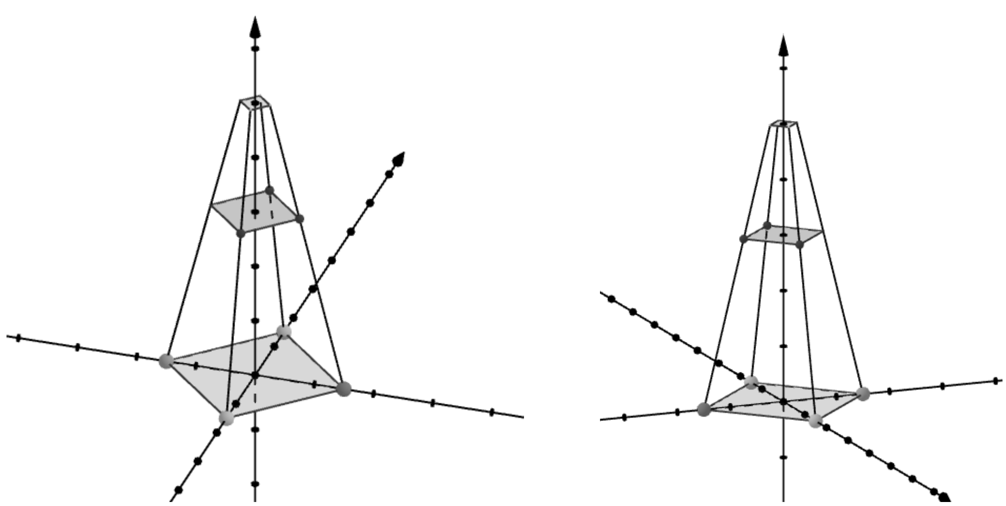

Figura 3.19: Ejemplo 4 a

$$1ft = 12in; \qquad \frac{1}{4}ft = 3in;$$

$\left(\frac{3}{2}, 0\right) \qquad \left(\frac{1}{4}, 30\right) \qquad m\,?$

$m\left(x - x'\right) = y - y' \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

$l = 2x \qquad V = \int_0^{30} (2x)^2 \, dy$

$\left(\frac{1}{4}, 30\right) \qquad \left(\frac{3}{2}, 0\right)$

$(x_1, y_1) \qquad (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 30}{\frac{3}{2} - \frac{1}{4}} = \frac{-30}{\frac{5}{4}} = \frac{-120}{5} = -24$$

$$m\left(x - x'\right) = y - y' \qquad \left(x' - y'\right) - \left(\frac{3}{2} - 0\right)$$

$$-24\left(x - \frac{3}{2}\right) = y - 0$$

$$x - \frac{3}{2} = -\frac{y}{24} \longrightarrow x = \frac{3}{2} - \frac{y}{24}$$

$$l = 2x = 2\left(\frac{3}{2} - \frac{y}{24}\right) = 3 - \frac{y}{12}$$

$$V = \int_0^{30} l^2 dy = \int_0^{30} \left(3 - \frac{y}{12}\right)^2 dy \approx 107{,}5 ft^3$$

3.5 LONGITUD DE UNA CURVA

LA LONGITUD DE UN ARCO AB de una curva es, por definición, el límite de la suma de las longitudesde las dis$_i$tintas cuerdas $AP_1, P_1P_2, \cdots, P_{n-1}B$, que unen los distintos puntos de l arco, cuando el número de estos crece indefinidamente, de manera que la longitud de cada una de las cuerdas tiende a cero.

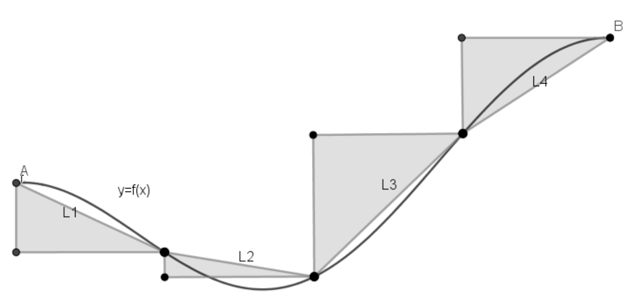

Figura 3.20: Longitud de un Arco

Sean $A(a,c)$ y $B(b,d)$ dos puntos de una curva $y = f(x)$, con $f(x)$ y su derivada, $f'(x)$, continua en el intervalo $a \leq x \leq b$; en estas condiciones, la longitud del arco AB viene dada

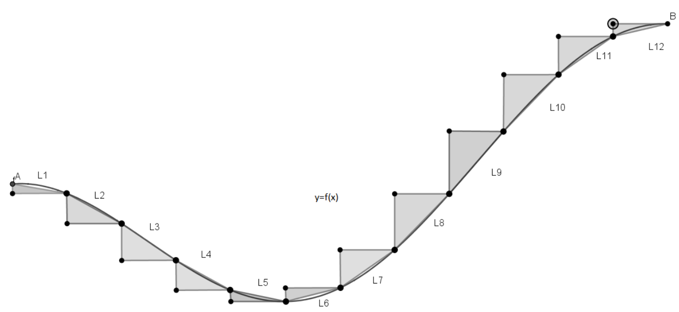

Figura 3.21: Longitud de un Arco

por

$$s = \int_{AB} ds = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Análogamente, si $A(a,c)$ y $B(b,d)$ son dos puntos de la curva $x = g(y)$ y su derivada con respecto a y continua en el intervalo $c \leq y \leq d$, la longitud del arco AB viene dada por

$$s = \int_{AB} ds = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

Si $A(u = u_1)$ y $B(u = u_2)$ son dos puntos de una curvas definida por las ecuaciones paramétricas $x = f(u)$, y $y = g(u)$ que cumplen las condiciones de continuidad, la longitud del arco AB viene dada por

$$s = \int_{AB} ds = \int_{u_1}^{u_2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

Ejemplo 3.12

Calcule la longitud de la curva $y = \ln\left(1 - x^2\right)$ en $\frac{1}{4} \leq x \leq \frac{3}{4}$

Solución

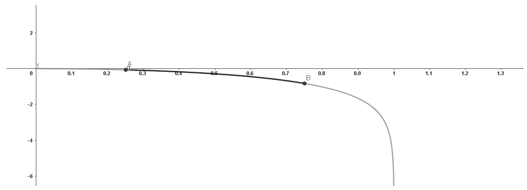

Figura 3.22: Ejemplo 1

$$
\begin{aligned}
L &= \int_a^b \sqrt{1+[y']^2}\ dx \\
y &= \ln\left(1-x^2\right) \\
y' &= \frac{1}{1-x^2}\left(1-x^2\right)' \\
y' &= \frac{-2x}{1-x^2} \\
1+\left(y'\right)^2 &= 1+\left(\frac{-2x}{1-x^2}\right)^2 \\
1+\left(y'\right)^2 &= 1+\frac{4x^2}{(1-x^2)^2} \\
1+\left(y'\right)^2 &= \frac{\left(1-x^2\right)^2+4x^2}{(1-x^2)^2} \\
1+\left(y'\right)^2 &= \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2} \\
1+\left(y'\right)^2 &= \frac{1+2x^2+x^4}{(1-x^2)^2} \\
1+\left(y'\right)^2 &= \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)^2} \\
\sqrt{1+(y')^2} &= \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}} \\
\sqrt{1+(y')^2} &= \frac{1+x^2}{1-x^2}
\end{aligned}
$$

$$
\begin{aligned}
L = \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1+x^2}{1-x^2}\right) dx &= \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{2}{1-x^2} - 1\right) dx \\
L &= 2\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{dx}{1-x^2} - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= 2\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{1dx}{(1+x)(1-x)} - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= 2\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{\left(\frac{(1+x)(1-x)}{2}\right)}{(1+x)(1-x)} dx - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= \frac{2}{2}\int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1\cancel{(1+x)}}{\cancel{(1+x)}(1-x)} + \frac{1\cancel{(1-x)}}{(1+x)\cancel{(1-x)}}\right) dx - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1}{1+x} + \frac{1}{1-x}\right) dx - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1}{1+x} - \frac{-1}{1-x}\right) dx - \int_{\frac{1}{4}}^{\frac{3}{4}} dx \\
L &= \left(\ln|1+x| - |1-x| \Bigg|_{\frac{1}{4}}^{\frac{3}{4}}\right) - x\Bigg|_{\frac{1}{4}}^{\frac{3}{4}} \\
L &= \ln\left|\frac{1+x}{1-x}\right|\Bigg|_{\frac{1}{4}}^{\frac{3}{4}} - \left(\frac{3}{4} - \frac{1}{4}\right) \\
L &= \ln\left(\frac{1+\frac{3}{4}}{1-\frac{3}{4}}\right) - \ln\left(\frac{1+\frac{1}{4}}{1-\frac{1}{4}}\right) - \frac{2}{4} \\
L &= \ln\left(\frac{\frac{7}{4}}{\frac{1}{4}}\right) - \ln\left(\frac{\frac{5}{4}}{\frac{3}{4}}\right) - \frac{1}{2} \\
L &= \ln(7) - \ln\left(\frac{5}{3}\right) - \frac{1}{2} \\
L &= \ln\left|\frac{7}{\frac{5}{3}}\right| - \frac{1}{2} \\
L &= \ln\left(\frac{21}{5}\right) - \frac{1}{2} \quad \text{unidades}
\end{aligned}
$$

Ejemplo 3.13

La posición de un punto en el instante t viene dada por $x = \frac{1}{2}t^2$, $y = \frac{1}{9}(6t+9)^{\frac{3}{2}}$. Hallar el espacio recorrido por el punto desde $t = 0$ hasta $t = 4$

Solución

$x = \frac{1}{2}t^2; \quad y = \frac{1}{9}(6t+9)^{\frac{3}{2}}; \quad 0 \leq t \leq 4$

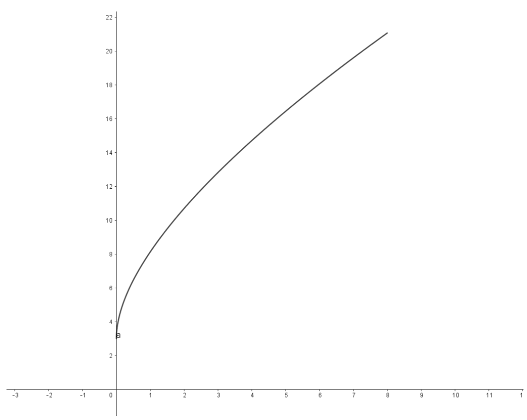

Figura 3.23: Ejemplo 2

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

$$
\begin{aligned}
x = \frac{1}{2}t^2 \quad &\longrightarrow \quad \frac{dx}{dt} = \frac{1}{2}2t = t \\
y = \frac{1}{9}(6t+9)^{\frac{3}{2}} \quad &\longrightarrow \quad \frac{dy}{dt} = \frac{1}{9}\frac{3}{2}(6t+9)^{\frac{1}{2}}6 \\
& \qquad\;\; \frac{dy}{dt} = \sqrt{6t+9}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= t^2 + \left(\sqrt{6t+9}\right)^2 \\
\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= t + 6t + 9 \\
\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (t+3)^2 \\
\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= \sqrt{(t+3)^2} \\
\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= t+3 \\
L = \int_0^4 (t+3)\,dt &= \left(\frac{t^2}{2} + 3t\right)\Bigg|_0^4 \\
&= \left(\frac{4^2}{2} + 3(4)\right) - \cancelto{0}{\left(\frac{0^2}{2} + 3(0)\right)} \\
&= 8 + 12 \\
&= 20 \ \text{ unidades}
\end{aligned}
$$

Ejemplo 3.14

$x = e^t \cos t; \qquad y = e^t \sin t; \qquad 0 \leq t \leq 4; \qquad L = ?$

Solución

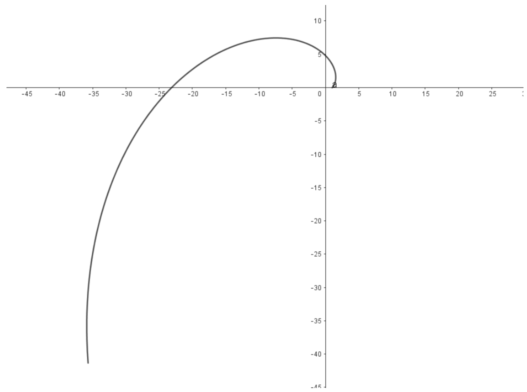

Figura 3.24

$$
\begin{aligned}
x = e^t \cos t \longrightarrow \frac{dx}{dt} &= -e^t \sin t + e^t \cos t \\
\frac{dx}{dt} &= e^t (\cos t - \sin t) \\
\left(\frac{dx}{dt}\right)^2 &= \left[e^t (\cos t - \sin t)\right]^2 = e^{2t} (\cos t - \sin t)^2 \\
\left(\frac{dx}{dt}\right)^2 &= e^{2t} \left(\cos^2 t - 2\cos t \sin t + \sin^2 t\right) \\
\left(\frac{dx}{dt}\right)^2 &= e^{2t} (1 - \sin(2t))
\end{aligned}
$$

$$
\begin{aligned}
y &= e^t \sin t \\
\frac{dy}{dt} = e^t \cos t + e^t \sin t &= e^t (\cos t + \sin t) \\
\left(\frac{dy}{dt}\right)^2 &= \left[e^t (\cos t + \sin t)\right]^2 \\
\left(\frac{dy}{dt}\right)^2 &= e^{2t} (\cos t + \sin t)^2 \\
\left(\frac{dy}{dt}\right)^2 &= e^{2t} \left(\cos^2 t + 2\cos t \sin t + \sin^2 t\right) \\
\left(\frac{dy}{dt}\right)^2 &= e^{2t} (1 + \sin(2t))
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 &= e^{2t}\left(1-\sin(2t)\right)+e^{2t}\left(1+\sin(2t)\right)\\
\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 &= e^{2t}\left(1-\cancel{\sin(2t)}+1+\cancel{\sin(2t)}\right)\\
\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2 &= 2e^{2t}\\
\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} &= \sqrt{2e^{2t}}\\
\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} &= \sqrt{2}\ \ e^{t}
\end{aligned}
$$

$$
\begin{aligned}
L &= \int_0^4 \sqrt{2}\ \ e^t dt\\
L &= \sqrt{2}\left[e^t\Big|_0^4\right]\\
L &= \sqrt{2}\left(e^4-e^0\right)\\
L &= \sqrt{2}\left(e^4-1\right)
\end{aligned}
$$

3.6 ÁREA DE UNA SUPERFICIE DE REVOLUCIÓN

Sean $A(a,c)$ y $B(b,d)$ son dos puntos en una curva $y=f(x)$ con $f(x)$ y $f'(x)$ funciones continuas definidas en un $[a,b]$

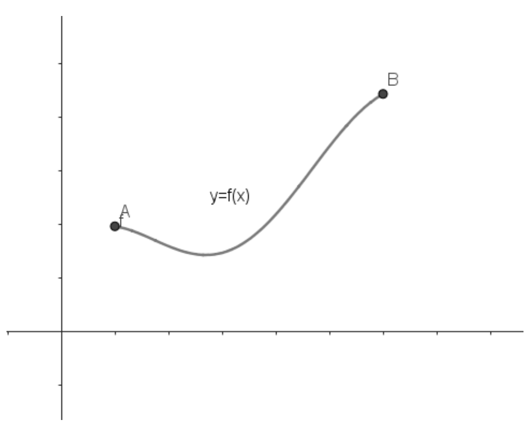

Figura 3.25: Superficie de Revolución

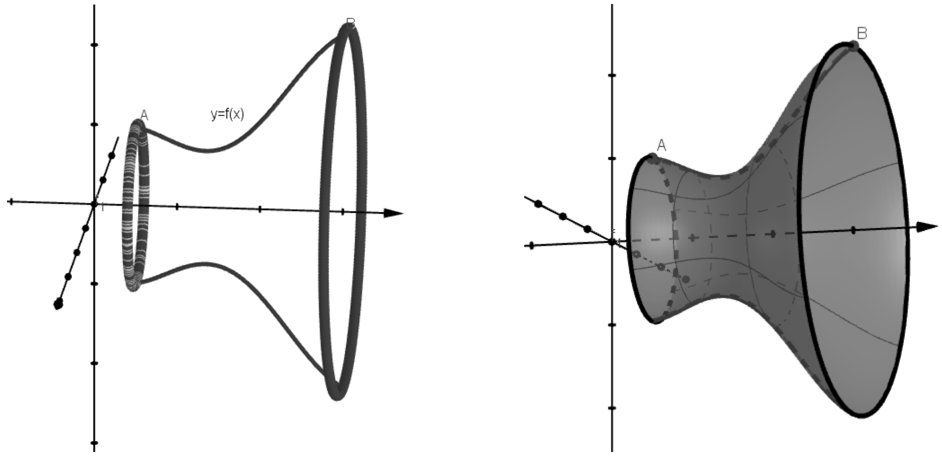

Figura 3.26: Superficie de Revolución

Si hacemos cortes perpendiculares al eje de revolución obtendremos circunferencias $L_0 = 2\pi r$

Si hacemos una partición del $[a,b]$ en n-subintervalos de igual longitud $\Delta x = \dfrac{b-a}{n}$

$$\begin{aligned}
A_s &= l_1 + l_2 + l_3 + \cdots + l_n \\
A_s &= \sum_{i=1}^{n} l_i = l_i L \\
A_s &\approx 2\pi f(x) L \\
A_s &= \int_a^b 2\pi f(x)\sqrt{1+(y')^2}\, dx \quad \circlearrowleft x
\end{aligned}$$

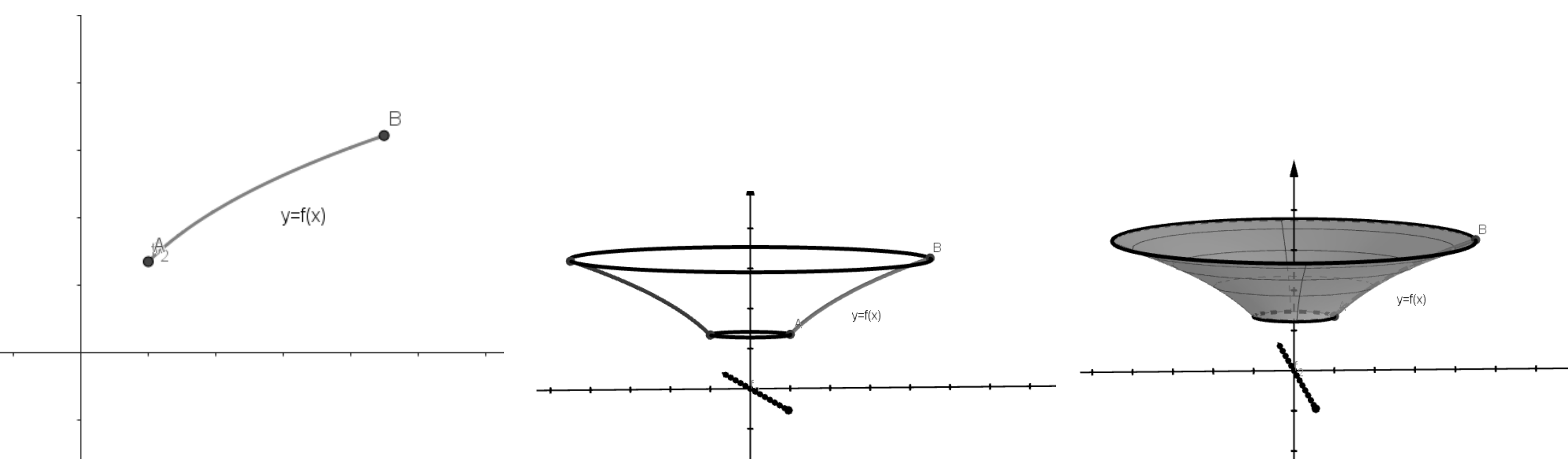

Figura 3.27: Cortes Perpendiculares

$$L_0 = 2\pi x; \qquad x\Big|_a^b; \qquad A_s = 2\pi x L$$

$$A_s = \int_a^b 2\pi x \sqrt{1 + (y')^2}\, dx \quad \circlearrowleft y$$

3.7 Coordenadas Paramétricas

$x = f(t); \qquad t_1 \leq t \leq t_2$

$y = g(t)$

- $\circlearrowleft x \qquad \text{radio} = y = g(t)$

$$A_s = \int_{t_1}^{t_2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\; dt$$

- $\circlearrowleft y \qquad \text{radio} = x = f(t)$

$$A_s = \int_{t_1}^{t_2} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\; dt$$

Ejemplo 3.15

Calcule el área superficial generada por la rotación del arco dado por:

$y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ desde $x = 1$ hasta $x = 2$; eje y

Solución

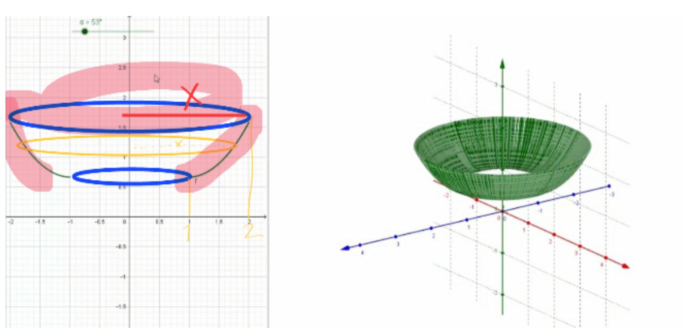

Figura 3.28: Ejemplo 1

Radio de cada circunferenxia es x ; $\qquad 1 \leq x \leq 2$

$$A_s = \int_1^2 2\pi x\sqrt{1+(y')^2}dx$$

$$\begin{aligned}
y = \frac{x^3}{6} + \frac{1}{2x} &= \frac{x^3}{6} + \frac{1}{2}x^{-1} \\
y' = \frac{3}{6}x^2 - \frac{1}{2}x^{-2} &= \frac{1}{2}x^2 - \frac{1}{2}x^{-2} \\
(y')^2 + 1 &= \left[\frac{1}{2}\left(x^2 - x^{-2}\right)\right]^2 + 1 \\
(y')^2 + 1 &= \frac{1}{4}\left(x^2 - x^{-2}\right)^2 + 1 \\
(y')^2 + 1 &= \frac{x^4 - 2 + x^{-4}}{4} + \frac{4}{4} \\
(y')^2 + 1 &= \frac{x^4 + 2 + x^{-4}}{4} = \frac{\left(x^2 + x^{-2}\right)^2}{4} \\
\sqrt{1+(y')^2} &= \sqrt{\frac{(x^2+x^{-2})^2}{4}} = \frac{x^2 + x^{-2}}{2}
\end{aligned}$$

$$
\begin{aligned}
A_s &= \int_1^2 2\pi x\sqrt{1+(y')^2}dx \\
A_s &= \cancel{2}\pi\int_1^2 x\left(\frac{x^2+x^{-2}}{\cancel{2}}\right)dx \\
A_s &= \pi\int_1^2 \left(x^3+x^{-1}\right)dx = \pi\int_1^2\left(x^3+\frac{1}{x}\right)dx \\
A_s &= \pi\left[\left(\frac{x^4}{4}+\ln|x|\right)\Big|_1^2\right] \\
A_s &= \pi\left[\left(\frac{2^4}{4}+\ln|2|\right)-\left(\frac{1^4}{4}+\cancelto{0}{\ln|1|}\right)\right] \\
A_s &= \pi\left[4+\ln|2|-\frac{1}{4}\right] \\
A_s &= \pi\left(\frac{15}{4}+\ln|2|\right)u^2
\end{aligned}
$$

Ejemplo 3.16

$x=e^t\cos t \qquad y=e^t\sin t \qquad 0\le t\le\frac{\pi}{2} \qquad$ ↻ eje x

Solución
redio $= y$

$$A_s=\int_0^{\frac{\pi}{2}} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt$$

- $x=e^t\cos t$
  $\dfrac{dx}{dt}=-e^t\sin t+e^t\cos t=e^t\left(\cos t-\sin t\right)$
- $y=e^t\sin t$
  $\dfrac{dy}{dt}=e^t\cos t+e^t\sin t=e^t\left(\cos t+\sin t\right)$
- $\left(\dfrac{dx}{dt}\right)^2=\left[e^t\left(\cos t-\sin t\right)\right]^2$
  $\left(\dfrac{dx}{dt}\right)^2=e^{2t}\left(\cos t-\sin t\right)^2$

$$\left(\frac{dx}{dt}\right)^2 = e^{2t}\left(\cos^2 t - 2\sin t\cos t + \sin^2 t\right)$$
$$\left(\frac{dx}{dt}\right)^2 = e^{2t}\left(1 - 2\sin t\cos t\right)$$

- $$\left(\frac{dy}{dt}\right)^2 = \left[e^{t}\left(\cos t + \sin t\right)\right]^2$$
$$\left(\frac{dy}{dt}\right)^2 = e^{2t}\left(\cos t + \sin t\right)^2$$
$$\left(\frac{dy}{dt}\right)^2 = e^{2t}\left(\cos^2 t + 2\cos t\sin t + \sin^2 t\right)$$
$$\left(\frac{dy}{dt}\right)^2 = e^{2t}\left(1 + 2\cos t\sin t\right)$$

- $$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{2t}\left(1 - 2\sin t\cos t\right) + e^{2t}\left(1 + 2\cos t\sin t\right)$$
$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{2t}\left[1 - \cancel{2\sin t\cos t} + 1 + \cancel{2\cos t\sin t}\right]$$
$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{2t}\left(2\right)$$
$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{2e^{2t}} = \sqrt{2}\ \ e^{t}$$

$$A_s = \int_0^{\frac{\pi}{2}} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$
$$A_s = 2\pi\int_0^{\frac{\pi}{2}} e^{t}\sin t\sqrt{2}\ \ e^{t}\,dt$$
$$A_s = 2\sqrt{2}\pi\int_0^{\frac{\pi}{2}} e^{2t}\sin t\ \ dt \qquad \text{por partes } \checkmark$$
$$A_s = 2\sqrt{2}\pi\left[\left(\frac{2\sin(t) - \cos(t)}{5}\right)e^{2t}\Big|_0^{\frac{\pi}{2}}\right]$$
$$A_s = \frac{2\sqrt{2}\pi}{5}\left[\left(2-0\right)e^{\pi} - \left(0-1\right)e^{0}\right]$$
$$A_s = \frac{2\sqrt{2}\pi}{5}\left[2e^{\pi} + 1\right]u^2$$