Ramanujan Numbers

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Риниваса Рамануджан – индийский математик, славившийся своей интуицией в области чисел. Когда английский математик Годфри Харди навестил его однажды в больнице, он обмолвился, что номером такси, на котором он приехал, было 17291729, такое скучное и заурядное число. На что Рамануджан ответил: "Нет, нет! Это очень интересное число. Это наименьшее число, выражаемое как сумма двух кубов двумя разными способами". Другими словами:

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

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```r
# Create an hash table of cubes from 1 to 100
numbers <- 1:100
cubes <- numbers ^ 3

# The possible pairs of numbers
pairs <- combn(numbers, 2)

# Now sum the cubes of the combinations
# This takes every couple and sums the values of the cubes
# with the appropriate index
sums <- apply(pairs, 2, function(x) {sum(cubes[x])})
```

numbers will be: 1, 2, 3, 4, ..., 98, 99, 100

cubes will be: 1, 8, 27, 64, ..., 941192, 970299, 1000000

pairs will contain:

```
     [,1] [,2] [,3] [,4] [,5] ... [,4949] [,4950]
[1,]   1    1    1    1    1  ...    98      99
[2,]   2    3    4    5    6  ...   100     100
```

```r
which(sums == 1729) # the ids of the couples summing to 1729
```

```
## [1]  11 765
```

```r
pairs[,11]
```

```
## [1]  1 12
```

```r
pairs[,765]
```

```
## [1]  9 10
```

Now, let's find which are the couples with the same sums.

table(sums) gives us a neat summary like

9 28 35 65 72 91 ... 1674 1729 1736
    1 1 1 1 1 1 .... <lots of 1s here> ... 1 2 1

So let's just find which elements of table(sums) are == 2

```
doubles <- which(table(sums) == 2)

taxi.numbers <- as.integer(names(doubles))
```

And finally (to be read two-by-two), the corresponding integer pairs

```
pairs[,doubles]
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
## [1,]    1    1    3    4    5    5    5    6    7     8    11    11    12    13
## [2,]   53   99   35   17   35   88   98   54   94    12    29    32    83    75
##      [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25] [,26]
## [1,]    14    15    16    17    17    20    23    23    23    26    27    29
## [2,]    71    18    60    69    71    45    35    64    87    53    92    90
##      [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37] [,38]
## [1,]    30    33    33    34    34    36    38    43    44    50    52    55
## [2,]    45    93    98    38    71    59    51    67    68    81   100    86
##      [,39] [,40] [,41] [,42] [,43] [,44] [,45]
## [1,]    55    58    60    62    63    64    65
## [2,]    95    69    87    95    82    97    78
```

So:

1,12 and 9,10 give 1729 2,16 and 9,15 give 4104 2,24 and 18,20 give 13832 and so on!