CIS 420/520 Automata Theory
Fall 2014

Assignment 1

due October 8, 2014

Please complete this first assignment on your own! Typically I will encourage you to work in groups on homework assignments. However, I will be using this first assignment to assess proof writing skills.

1. Design deterministic finite automata for each of the following sets:

   (a) the set of strings in $\{4, 8, 1\}^*$ containing the substring 481;

   (b) the set of strings in $\{a\}^*$ whose length is divisible by either 2 or 7;

   (c) the set of strings $x \in \{0, 1\}^*$ such that the number of 0's in x is even and the number of 1's in x is a multiple of three;

   (d) the set of strings over the alphabet $\{a, b\}$ containing at least three occurrences of three consecutive b's, overlapping permitted (e.g., the string *bbbbb* should be accepted);

2. Consider the following two deterministic finite automata: $M_1 = (\{1, 2\}, \{a, b\}, \delta_1, 1, \{2\})$ and $M_2 = (\{1, 2, 3\}, \{a, b\}, \delta_2, 1, \{3\})$.

δ_1	a	b
1	1	2
2	2	1

δ_2	a	b
1	2	3
2	3	1
3	1	2

   Use the *Cartesian product* construction described on page 46 of Sipser to produce DFA's for both the intersection and the union of the two sets defined by these automata.

3. For any string $w = w_1 w_2 \dots w_n$, the *reverse* of w, written w^R, is the string w in reverse order, $w_n \dots w_2 w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

4. Let B be a set of strings over a fixed finite alphabet. We say B is *transitive* if $BB \subseteq B$ and *reflexive* if $\epsilon \in B$. Prove that for any set of strings A, A^* is the smallest reflexive and transitive set containing A. That is, show that A^* is a reflexive transitive set containing A, and if B is any other reflexive and transitive set containing A, then $A^* \subseteq B$.