Fully-Connected Neural Nets

In the previous homework you implemented a fully-connected two-layer neural network on CIFAR-10. The implementation was simple but not very modular since the loss and gradient were computed in a single monolithic function. This is manageable for a simple two-layer network, but would become impractical as we move to bigger models. Ideally we want to build networks using a more modular design so that we can implement different layer types in isolation and then snap them together into models with different architectures.

In this exercise we will implement fully-connected networks using a more modular approach. For each layer we will implement a `forward` and a `backward` function. The `forward` function will receive inputs, weights, and other parameters and will return both an output and a `cache` object storing data needed for the backward pass, like this:

```
def layer_forward(x, w):
  """ Receive inputs x and weights w """
  # Do some computations ...
  z = # ... some intermediate value
  # Do some more computations ...
  out = # the output

  cache = (x, w, z, out) # Values we need to compute gradients

  return out, cache
```

The backward pass will receive upstream derivatives and the `cache` object, and will return gradients with respect to the inputs and weights, like this:

```
def layer_backward(dout, cache):
  """
  Receive dout (derivative of loss with respect to outputs) and
cache,
  and compute derivative with respect to inputs.
  """
  # Unpack cache values
  x, w, z, out = cache

  # Use values in cache to compute derivatives
  dx = # Derivative of loss with respect to x
  dw = # Derivative of loss with respect to w
```

```
    return dx, dw
```

After implementing a bunch of layers this way, we will be able to easily combine them to build classifiers with different architectures.

In addition to implementing fully-connected networks of arbitrary depth, we will also explore different update rules for optimization, and introduce Batch/Layer Normalization as a tool to more efficiently optimize deep networks.

In [1]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'cs6353/assignments/assignment3/'
# FOLDERNAME = 'assignment3'
# assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
# import sys
# sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/MyDrive/MS/CS6353/assignment3/cs6353/datasets
!bash get_datasets.sh
%cd ../..

# Install requirements from colab_requirements.txt
# TODO: Please change your path below to the colab_requirements.txt file
# ! python -m pip install -r /content/drive/My\ Drive/$FOLDERNAME/requiremen
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, cal
l drive.mount("/content/drive", force_remount=True).
/content/drive/MyDrive/MS/CS6353/assignment3/cs6353/datasets
--2024-10-21 03:57:42--  https://www.cs.toronto.edu/~kriz/cifar-10-python.ta
r.gz
Resolving www.cs.toronto.edu (www.cs.toronto.edu)... 128.100.3.30
Connecting to www.cs.toronto.edu (www.cs.toronto.edu)|128.100.3.30|:443... c
onnected.
HTTP request sent, awaiting response... 200 OK
Length: 170498071 (163M) [application/x-gzip]
Saving to: ‘cifar-10-python.tar.gz’

cifar-10-python.tar 100%[===================>] 162.60M  25.0MB/s    in 6.6s

2024-10-21 03:57:50 (24.8 MB/s) - ‘cifar-10-python.tar.gz’ saved [170498071/
170498071]

cifar-10-batches-py/
cifar-10-batches-py/data_batch_4
cifar-10-batches-py/readme.html
cifar-10-batches-py/test_batch
cifar-10-batches-py/data_batch_3
cifar-10-batches-py/batches.meta
cifar-10-batches-py/data_batch_2
cifar-10-batches-py/data_batch_5
cifar-10-batches-py/data_batch_1
/content/drive/MyDrive/MS/CS6353/assignment3
```

In [2]:

```
# As usual, a bit of setup
from __future__ import print_function
import time
import numpy as np
import matplotlib.pyplot as plt
from cs6353.classifiers.fc_net import *
from cs6353.data_utils import get_CIFAR10_data
from cs6353.gradient_check import eval_numerical_gradient, eval_numerical_gr
from cs6353.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ip
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [3]:
```
# Load the (preprocessed) CIFAR10 data.
data = get_CIFAR10_data()
for k, v in list(data.items()):
  print(('%s: ' % k, v.shape))
```

```
('X_train: ', (49000, 3, 32, 32))
('y_train: ', (49000,))
('X_val: ', (1000, 3, 32, 32))
('y_val: ', (1000,))
('X_test: ', (1000, 3, 32, 32))
('y_test: ', (1000,))
```

Affine layer: foward

Open the file `cs6353/layers.py` and implement the `affine_forward` function.

Once you are done you can test your implementaion by running the following:

In [4]:
```
# Test the affine_forward function

num_inputs = 2
input_shape = (4, 5, 6)
output_dim = 3

input_size = num_inputs * np.prod(input_shape)
weight_size = output_dim * np.prod(input_shape)

x = np.linspace(-0.1, 0.5, num=input_size).reshape(num_inputs, *input_shape)
w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape), ou
b = np.linspace(-0.3, 0.1, num=output_dim)

out, _ = affine_forward(x, w, b)

correct_out = np.array([[ 1.49834967,  1.70660132,  1.91485297],
                        [ 3.25553199,  3.5141327,   3.77273342]])

# Compare your output with ours. The error should be around e-9 or less.
print('Testing affine_forward function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing affine_forward function:
difference:  9.769849468192957e-10
```

Affine layer: backward

Now implement the `affine_backward` function and test your implementation using numeric gradient checking.

In [5]:
```
# Test the affine_backward function
np.random.seed(231)
x = np.random.randn(10, 2, 3)
w = np.random.randn(6, 5)
b = np.random.randn(5)
dout = np.random.randn(10, 5)

dx_num = eval_numerical_gradient_array(lambda x: affine_forward(x, w, b)[0],
dw_num = eval_numerical_gradient_array(lambda w: affine_forward(x, w, b)[0],
db_num = eval_numerical_gradient_array(lambda b: affine_forward(x, w, b)[0],

_, cache = affine_forward(x, w, b)
dx, dw, db = affine_backward(dout, cache)

# The error should be around e-10 or less
print('Testing affine_backward function:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing affine_backward function:
dx error:  5.399100368651805e-11
dw error:  9.904211865398145e-11
db error:  2.4122867568119087e-11
```

ReLU activation: forward

Implement the forward pass for the ReLU activation function in the `relu_forward` function and test your implementation using the following:

In [6]:
```
# Test the relu_forward function

x = np.linspace(-0.5, 0.5, num=12).reshape(3, 4)

out, _ = relu_forward(x)
correct_out = np.array([[ 0.,          0.,          0.,          0.,
                        [ 0.,          0.,          0.04545455,  0.13636364,
                        [ 0.22727273,  0.31818182,  0.40909091,  0.5,

# Compare your output with ours. The error should be on the order of e-8
print('Testing relu_forward function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing relu_forward function:
difference:  4.999999798022158e-08
```

ReLU activation: backward

Now implement the backward pass for the ReLU activation function in the `relu_backward` function and test your implementation using numeric gradient checking:

In [7]:
```
np.random.seed(231)
x = np.random.randn(10, 10)
dout = np.random.randn(*x.shape)

dx_num = eval_numerical_gradient_array(lambda x: relu_forward(x)[0], x, dout

_, cache = relu_forward(x)
dx = relu_backward(dout, cache)

# The error should be on the order of e-12
print('Testing relu_backward function:')
print('dx error: ', rel_error(dx_num, dx))
```

```
Testing relu_backward function:
dx error:  3.2756349136310288e-12
```

Inline Question 1:

We've only asked you to implement ReLU, but there are a number of different activation functions that one could use in neural networks, each with its pros and cons. In particular, an issue commonly seen with activation functions is getting zero (or close to zero) gradient flow during backpropagation. Which of the following activation functions have this problem? If you consider these functions in the one dimensional case, what types of input would lead to this behaviour?

1. Sigmoid
2. ReLU
3. Leaky ReLU

Answer:

Sigmoid and ReLu would still have this problem. Sigmoid's gradient would have vanishing gradient for input close to 0 and relu's gradient is either 0 or 1. In one dimensional case, one example sigmoid would have vanishing gradient would be if the input is [-1e5, 1e5] and whereas relu any negative input, i.e [-1, -2, -3]

Leaky ReLU doesn't have this problem because it considers small negative slope when we have negative values, i.e., if x < 0 then alpha else x, where alpha can be equal to 0.01. Hence, no vanishing gradient

"Sandwich" layers

There are some common patterns of layers that are frequently used in neural nets. For example, affine layers are frequently followed by a ReLU nonlinearity. To make these common patterns easy, we define several convenience layers in the file `cs6353/layer_utils.py`.

For now take a look at the `affine_relu_forward` and `affine_relu_backward` functions, and run the following to numerically gradient check the backward pass:

In [8]:
```
from cs6353.layer_utils import affine_relu_forward, affine_relu_backward
np.random.seed(231)
x = np.random.randn(2, 3, 4)
w = np.random.randn(12, 10)
b = np.random.randn(10)
dout = np.random.randn(2, 10)

out, cache = affine_relu_forward(x, w, b)
dx, dw, db = affine_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w, b
dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w, b
db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w, b

# Relative error should be around e-10 or less
print('Testing affine_relu_forward and affine_relu_backward:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing affine_relu_forward and affine_relu_backward:
dx error:  2.299579177309368e-11
dw error:  8.162011105764925e-11
db error:  7.826724021458994e-12
```

Loss layers: Softmax and SVM

You implemented these loss functions in the last assignment, so we'll give them to you for free here. You should still make sure you understand how they work by looking at the implementations in `cs6353/layers.py`.

You can make sure that the implementations are correct by running the following:

In [9]:
```
np.random.seed(231)
num_classes, num_inputs = 10, 50
x = 0.001 * np.random.randn(num_inputs, num_classes)
y = np.random.randint(num_classes, size=num_inputs)

dx_num = eval_numerical_gradient(lambda x: svm_loss(x, y)[0], x, verbose=Fal
loss, dx = svm_loss(x, y)

# Test svm_loss function. Loss should be around 9 and dx error should be arc
print('Testing svm_loss:')
print('loss: ', loss)
print('dx error: ', rel_error(dx_num, dx))

dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x, verbose
loss, dx = softmax_loss(x, y)

# Test softmax_loss function. Loss should be close to 2.3 and dx error shoul
print('\nTesting softmax_loss:')
print('loss: ', loss)
print('dx error: ', rel_error(dx_num, dx))
```

```
Testing svm_loss:
loss:  8.999602749096233
dx error:  1.4021566006651672e-09

Testing softmax_loss:
loss:  2.302545844500738
dx error:  9.384673161989355e-09
```

Two-layer network

In the previous assignment you implemented a two-layer neural network in a single monolithic class. Now that you have implemented modular versions of the necessary layers, you will reimplement the two layer network using these modular implementations.

Open the file `cs6353/classifiers/fc_net.py` and complete the implementation of the `TwoLayerNet` class. This class will serve as a model for the other networks you will implement in this assignment, so read through it to make sure you understand the API. You can run the cell below to test your implementation.

In [10]:
```
np.random.seed(231)
N, D, H, C = 3, 5, 50, 7
X = np.random.randn(N, D)
y = np.random.randint(C, size=N)

std = 1e-3
model = TwoLayerNet(input_dim=D, hidden_dim=H, num_classes=C, weight_scale=s

print('Testing initialization ... ')
W1_std = abs(model.params['W1'].std() - std)
b1 = model.params['b1']
W2_std = abs(model.params['W2'].std() - std)
b2 = model.params['b2']
assert W1_std < std / 10, 'First layer weights do not seem right'
assert np.all(b1 == 0), 'First layer biases do not seem right'
assert W2_std < std / 10, 'Second layer weights do not seem right'
assert np.all(b2 == 0), 'Second layer biases do not seem right'

print('Testing test-time forward pass ... ')
model.params['W1'] = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
model.params['b1'] = np.linspace(-0.1, 0.9, num=H)
model.params['W2'] = np.linspace(-0.3, 0.4, num=H*C).reshape(H, C)
model.params['b2'] = np.linspace(-0.9, 0.1, num=C)
X = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
scores = model.loss(X)
correct_scores = np.asarray(
  [[11.53165108,  12.2917344,   13.05181771,  13.81190102,  14.57198434, 15.
   [12.05769098,  12.74614105,  13.43459113,  14.1230412,   14.81149128, 15.
   [12.58373087,  13.20054771,  13.81736455,  14.43418138,  15.05099822, 15.
scores_diff = np.abs(scores - correct_scores).sum()
assert scores_diff < 1e-6, 'Problem with test-time forward pass'

print('Testing training loss (no regularization)')
y = np.asarray([0, 5, 1])
loss, grads = model.loss(X, y)
correct_loss = 3.4702243556
```

```
assert abs(loss - correct_loss) < 1e-10, 'Problem with training-time loss'

model.reg = 1.0
loss, grads = model.loss(X, y)
correct_loss = 26.5948426952
assert abs(loss - correct_loss) < 1e-10, 'Problem with regularization loss'

# Errors should be around e-7 or less
for reg in [0.0, 0.7]:
  print('Running numeric gradient check with reg = ', reg)
  model.reg = reg
  loss, grads = model.loss(X, y)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False)
    print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name]
```

```
Testing initialization ...
Testing test-time forward pass ...
Testing training loss (no regularization)
Running numeric gradient check with reg =  0.0
W1 relative error: 1.83e-08
W2 relative error: 3.12e-10
b1 relative error: 9.83e-09
b2 relative error: 4.33e-10
Running numeric gradient check with reg =  0.7
W1 relative error: 2.53e-07
W2 relative error: 2.85e-08
b1 relative error: 1.56e-08
b2 relative error: 7.76e-10
```

Solver

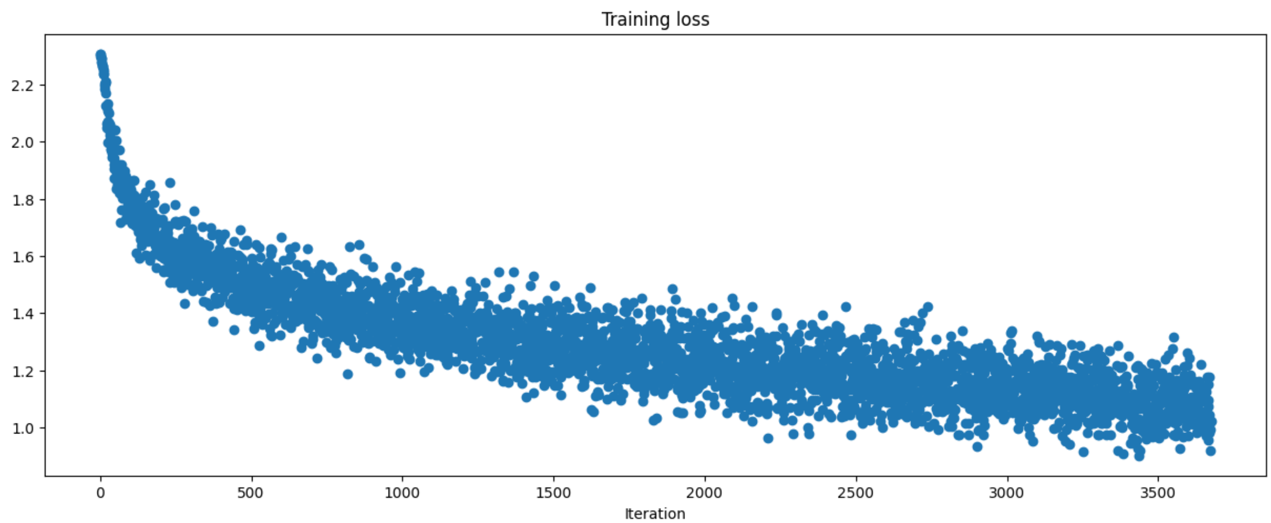
In the previous assignment, the logic for training models was coupled to the models themselves. Following a more modular design, for this assignment we have split the logic for training models into a separate class.

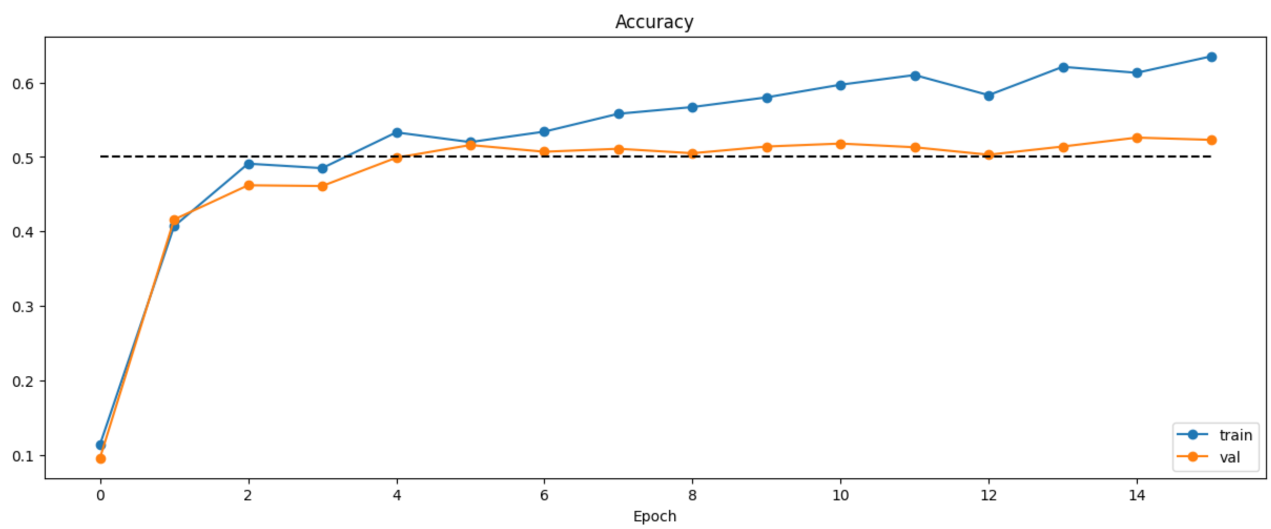
Open the file `cs6353/solver.py` and read through it to familiarize yourself with the API. After doing so, use a `Solver` instance to train a `TwoLayerNet` that achieves at least `50%` accuracy on the validation set.

In [11]:
```
model = TwoLayerNet()
solver = None

##############################################################################
# TODO: Use a Solver instance to train a TwoLayerNet that achieves at least
# 50% accuracy on the validation set.
##############################################################################
solver_data = {'X_train': data['X_train'],
               'y_train': data['y_train'],
               'X_val': data['X_val'],
               'y_val': data['y_val']}

# model = TwoLayerNet(input_dim=solver_data['X_train'].shape[1:], hidden_dim
solver = Solver(model, solver_data,
                update_rule='sgd',
                optim_config={'learning_rate': 1e-3,},
                lr_decay = 0.95,
                num_epochs=15,
                batch_size=200,
                print_every=100)

solver.train()
##############################################################################
#                             END OF YOUR CODE
##############################################################################
```

```
(Iteration 1 / 3675) loss: 2.305965
(Epoch 0 / 15) train acc: 0.114000; val_acc: 0.096000
(Iteration 101 / 3675) loss: 1.856329
(Iteration 201 / 3675) loss: 1.598619
(Epoch 1 / 15) train acc: 0.407000; val_acc: 0.416000
(Iteration 301 / 3675) loss: 1.548536
(Iteration 401 / 3675) loss: 1.661861
(Epoch 2 / 15) train acc: 0.491000; val_acc: 0.462000
(Iteration 501 / 3675) loss: 1.452303
(Iteration 601 / 3675) loss: 1.665413
(Iteration 701 / 3675) loss: 1.295910
(Epoch 3 / 15) train acc: 0.485000; val_acc: 0.461000
(Iteration 801 / 3675) loss: 1.393038
(Iteration 901 / 3675) loss: 1.395341
(Epoch 4 / 15) train acc: 0.533000; val_acc: 0.499000
(Iteration 1001 / 3675) loss: 1.449278
(Iteration 1101 / 3675) loss: 1.276145
(Iteration 1201 / 3675) loss: 1.207275
(Epoch 5 / 15) train acc: 0.520000; val_acc: 0.516000
(Iteration 1301 / 3675) loss: 1.339874
(Iteration 1401 / 3675) loss: 1.430487
(Epoch 6 / 15) train acc: 0.534000; val_acc: 0.507000
(Iteration 1501 / 3675) loss: 1.364778
(Iteration 1601 / 3675) loss: 1.167146
(Iteration 1701 / 3675) loss: 1.362156
(Epoch 7 / 15) train acc: 0.558000; val_acc: 0.511000
(Iteration 1801 / 3675) loss: 1.225039
(Iteration 1901 / 3675) loss: 1.310197
(Epoch 8 / 15) train acc: 0.567000; val_acc: 0.505000
(Iteration 2001 / 3675) loss: 1.271719
(Iteration 2101 / 3675) loss: 1.120800
(Iteration 2201 / 3675) loss: 1.279429
(Epoch 9 / 15) train acc: 0.580000; val_acc: 0.514000
(Iteration 2301 / 3675) loss: 1.229667
(Iteration 2401 / 3675) loss: 1.104043
(Epoch 10 / 15) train acc: 0.597000; val_acc: 0.518000
(Iteration 2501 / 3675) loss: 1.286715
(Iteration 2601 / 3675) loss: 1.143557
(Epoch 11 / 15) train acc: 0.610000; val_acc: 0.513000
(Iteration 2701 / 3675) loss: 1.048077
(Iteration 2801 / 3675) loss: 1.065507
(Iteration 2901 / 3675) loss: 1.130374
(Epoch 12 / 15) train acc: 0.583000; val_acc: 0.503000
(Iteration 3001 / 3675) loss: 1.146657
(Iteration 3101 / 3675) loss: 1.197794
(Epoch 13 / 15) train acc: 0.621000; val_acc: 0.514000
(Iteration 3201 / 3675) loss: 1.045194
(Iteration 3301 / 3675) loss: 0.974399
(Iteration 3401 / 3675) loss: 1.064486
(Epoch 14 / 15) train acc: 0.613000; val_acc: 0.526000
(Iteration 3501 / 3675) loss: 1.043400
(Iteration 3601 / 3675) loss: 1.065562
(Epoch 15 / 15) train acc: 0.635000; val_acc: 0.523000
```

In [12]:
```
# Run this cell to visualize training loss and train / val accuracy

plt.subplot(2, 1, 1)
plt.title('Training loss')
plt.plot(solver.loss_history, 'o')
plt.xlabel('Iteration')

plt.subplot(2, 1, 2)
plt.title('Accuracy')
plt.plot(solver.train_acc_history, '-o', label='train')
plt.plot(solver.val_acc_history, '-o', label='val')
plt.plot([0.5] * len(solver.val_acc_history), 'k--')
plt.xlabel('Epoch')
plt.legend(loc='lower right')
plt.gcf().set_size_inches(15, 12)
plt.show()
```





Multilayer network

Next you will implement a fully-connected network with an arbitrary number of hidden layers.

Read through the `FullyConnectedNet` class in the file `cs6353/classifiers/fc_net.py`.

Implement the initialization, the forward pass, and the backward pass. For the moment don't worry about implementing batch/layer normalization; we will add those features soon.

Initial loss and gradient check

As a sanity check, run the following to check the initial loss and to gradient check the network both with and without regularization. Do the initial losses seem reasonable?

For gradient checking, you should expect to see errors around 1e-7 or less.

In [13]:
```
np.random.seed(231)
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for reg in [0, 3.14]:
  print('Running check with reg = ', reg)
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            reg=reg, weight_scale=5e-2, dtype=np.float64)

  loss, grads = model.loss(X, y)
  print('Initial loss: ', loss)

  # Most of the errors should be on the order of e-7 or smaller.
  # NOTE: It is fine however to see an error for W2 on the order of e-5
  # for the check when reg = 0.0
  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False,
    print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name]
```

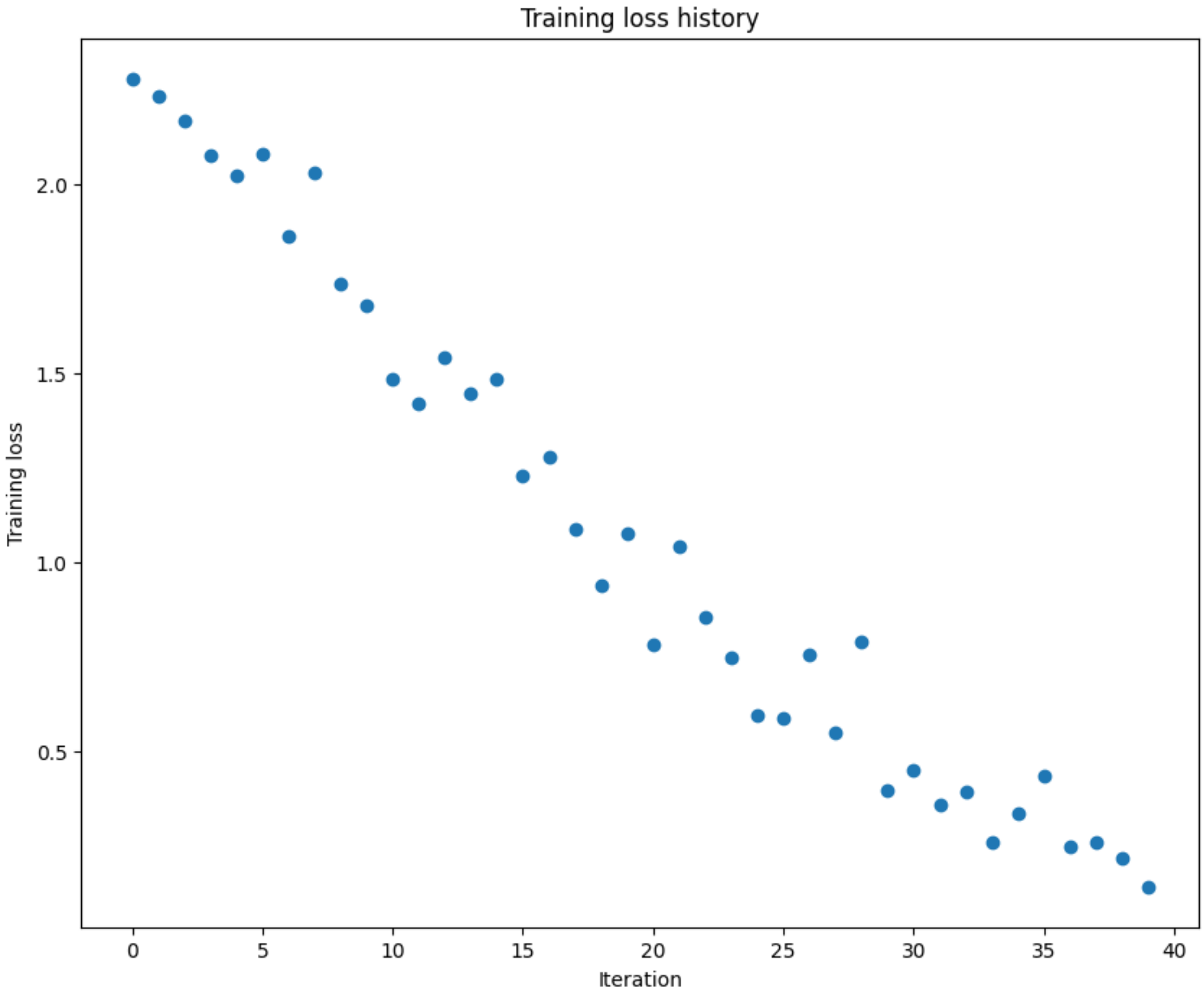
```
Running check with reg =  0
Initial loss:  2.3004790897684924
W1 relative error: 1.48e-07
W2 relative error: 2.21e-05
W3 relative error: 3.53e-07
b1 relative error: 5.38e-09
b2 relative error: 2.09e-09
b3 relative error: 5.80e-11
Running check with reg =  3.14
Initial loss:  7.052114776533016
W1 relative error: 7.36e-09
W2 relative error: 6.87e-08
W3 relative error: 3.48e-08
b1 relative error: 1.48e-08
b2 relative error: 1.72e-09
b3 relative error: 1.80e-10
```

As another sanity check, make sure you can overfit a small dataset of 50 images. First we will try a three-layer network with 100 units in each hidden layer. In the following cell, tweak the learning rate and initialization scale to overfit and achieve 100% training accuracy within 20 epochs.

In [20]:

```
# TODO: Use a three-layer Net to overfit 50 training examples by
# tweaking just the learning rate and initialization scale.

num_train = 50
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

weight_scale = 1e-2
learning_rate = 5e-3
model = FullyConnectedNet([100, 100],
              weight_scale=weight_scale, dtype=np.float64)
solver = Solver(model, small_data,
                print_every=10, num_epochs=20, batch_size=25,
                update_rule='sgd',
                optim_config={
                  'learning_rate': learning_rate,
                }
         )
solver.train()

plt.plot(solver.loss_history, 'o')
plt.title('Training loss history')
plt.xlabel('Iteration')
plt.ylabel('Training loss')
plt.show()
```

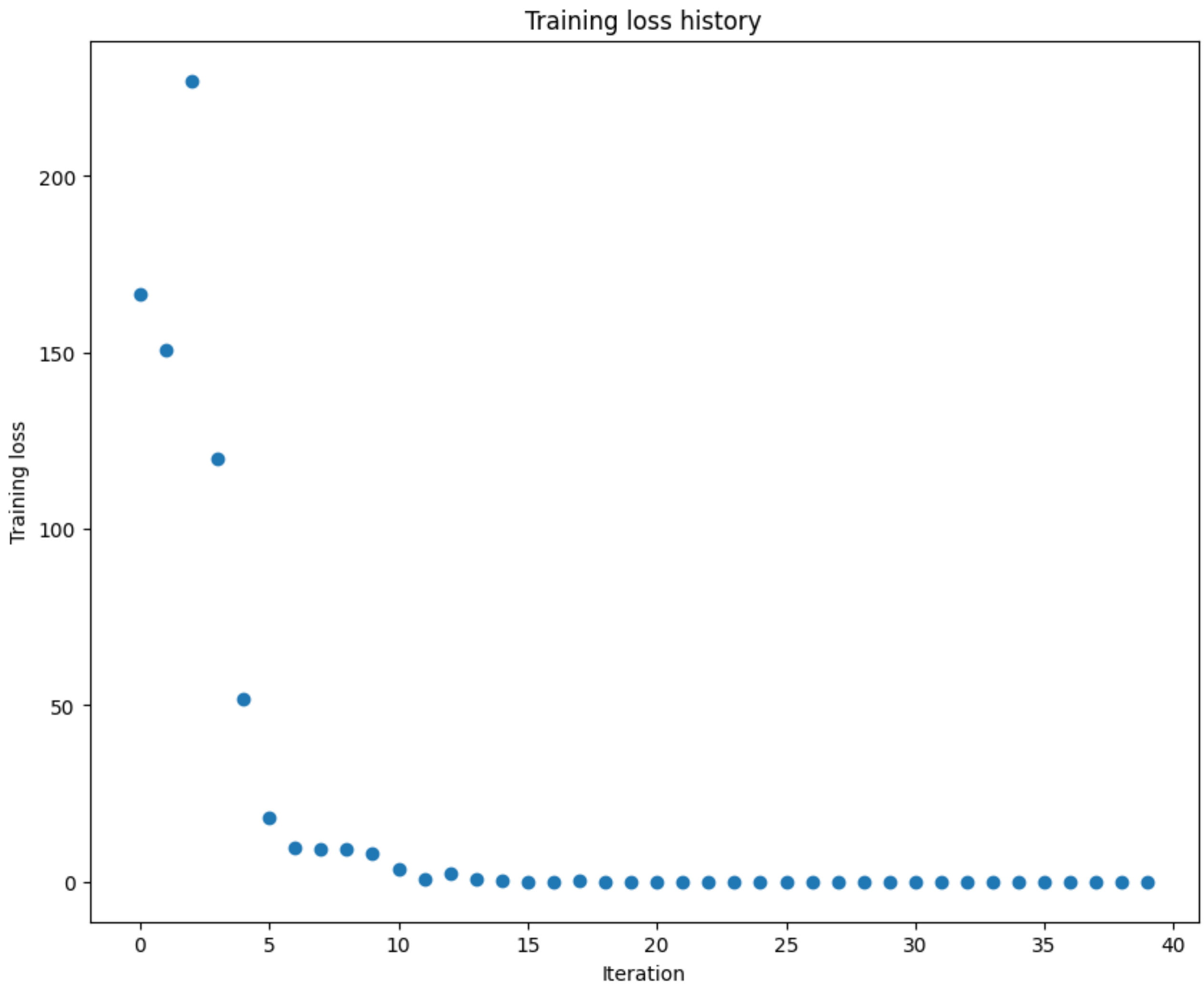
```
(Iteration 1 / 40) loss: 2.280581
(Epoch 0 / 20) train acc: 0.260000; val_acc: 0.119000
(Epoch 1 / 20) train acc: 0.280000; val_acc: 0.154000
(Epoch 2 / 20) train acc: 0.420000; val_acc: 0.146000
(Epoch 3 / 20) train acc: 0.480000; val_acc: 0.133000
(Epoch 4 / 20) train acc: 0.500000; val_acc: 0.173000
(Epoch 5 / 20) train acc: 0.520000; val_acc: 0.182000
(Iteration 11 / 40) loss: 1.486073
(Epoch 6 / 20) train acc: 0.560000; val_acc: 0.183000
(Epoch 7 / 20) train acc: 0.660000; val_acc: 0.199000
(Epoch 8 / 20) train acc: 0.700000; val_acc: 0.175000
(Epoch 9 / 20) train acc: 0.720000; val_acc: 0.170000
(Epoch 10 / 20) train acc: 0.820000; val_acc: 0.199000
(Iteration 21 / 40) loss: 0.782952
(Epoch 11 / 20) train acc: 0.820000; val_acc: 0.178000
(Epoch 12 / 20) train acc: 0.900000; val_acc: 0.203000
(Epoch 13 / 20) train acc: 0.820000; val_acc: 0.182000
(Epoch 14 / 20) train acc: 0.900000; val_acc: 0.197000
(Epoch 15 / 20) train acc: 0.940000; val_acc: 0.202000
(Iteration 31 / 40) loss: 0.449988
(Epoch 16 / 20) train acc: 0.980000; val_acc: 0.205000
(Epoch 17 / 20) train acc: 0.980000; val_acc: 0.216000
(Epoch 18 / 20) train acc: 1.000000; val_acc: 0.198000
(Epoch 19 / 20) train acc: 0.980000; val_acc: 0.192000
(Epoch 20 / 20) train acc: 1.000000; val_acc: 0.202000
```



Now try to use a five-layer network with 100 units on each layer to overfit 50 training examples. Again you will have to adjust the learning rate and weight initialization, but you should be able to achieve 100% training accuracy within 20 epochs.

In [15]:
```
# TODO: Use a five-layer Net to overfit 50 training examples by
# tweaking just the learning rate and initialization scale.

num_train = 50
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

learning_rate = 2e-3
weight_scale = 1e-1
model = FullyConnectedNet([100, 100, 100, 100],
                weight_scale=weight_scale, dtype=np.float64)
solver = Solver(model, small_data,
                print_every=10, num_epochs=20, batch_size=25,
                update_rule='sgd',
                optim_config={
                  'learning_rate': learning_rate,
                }
         )
solver.train()

plt.plot(solver.loss_history, 'o')
plt.title('Training loss history')
plt.xlabel('Iteration')
plt.ylabel('Training loss')
plt.show()
```

```
(Iteration 1 / 40) loss: 166.501707
(Epoch 0 / 20) train acc: 0.100000; val_acc: 0.107000
(Epoch 1 / 20) train acc: 0.320000; val_acc: 0.101000
(Epoch 2 / 20) train acc: 0.160000; val_acc: 0.122000
(Epoch 3 / 20) train acc: 0.380000; val_acc: 0.106000
(Epoch 4 / 20) train acc: 0.520000; val_acc: 0.111000
(Epoch 5 / 20) train acc: 0.760000; val_acc: 0.113000
(Iteration 11 / 40) loss: 3.343141
(Epoch 6 / 20) train acc: 0.840000; val_acc: 0.122000
(Epoch 7 / 20) train acc: 0.920000; val_acc: 0.113000
(Epoch 8 / 20) train acc: 0.940000; val_acc: 0.125000
(Epoch 9 / 20) train acc: 0.960000; val_acc: 0.125000
(Epoch 10 / 20) train acc: 0.980000; val_acc: 0.121000
(Iteration 21 / 40) loss: 0.039138
(Epoch 11 / 20) train acc: 0.980000; val_acc: 0.123000
(Epoch 12 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 13 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 14 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 15 / 20) train acc: 1.000000; val_acc: 0.121000
(Iteration 31 / 40) loss: 0.000644
(Epoch 16 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 17 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 18 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 19 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 20 / 20) train acc: 1.000000; val_acc: 0.121000
```



Inline Question 2:

Did you notice anything about the comparative difficulty of training the three-layer net vs training the five layer net? In particular, based on your experience, which network seemed more sensitive to the initialization scale? Why do you think that is the case?

Answer:

When comparing the difficulty of training these two networks, the five-layer network showed greater sensitivity to the initialization scale. I believe this is because of the increased depth of the network. In deeper networks, setting a small scale can cause the gradients to vanish due to small products, while a large scale can lead to gradient explosion from large products—issues that occur more frequently than in shallower networks. Therefore, finding the appropriate scale for the five-layer network is more challenging compared to the three-layer network, though it can be achieved through random search.

Update rules

So far we have used vanilla stochastic gradient descent (SGD) as our update rule. More sophisticated update rules can make it easier to train deep networks. We will implement a few of the most commonly used update rules and compare them to vanilla SGD.

SGD+Momentum

Stochastic gradient descent with momentum is a widely used update rule that tends to make deep networks converge faster than vanilla stochastic gradient descent.

Open the file `cs6353/optim.py` and read the documentation at the top of the file to make sure you understand the API. Implement the SGD+momentum update rule in the function `sgd_momentum` and run the following to check your implementation. You should see errors less than e-8.

In [16]:
```
from cs6353.optim import sgd_momentum

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
v = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-3, 'velocity': v}
next_w, _ = sgd_momentum(w, dw, config=config)

expected_next_w = np.asarray([
  [ 0.1406,      0.20738947,  0.27417895,  0.34096842,  0.40775789],
  [ 0.47454737,  0.54133684,  0.60812632,  0.67491579,  0.74170526],
  [ 0.80849474,  0.87528421,  0.94207368,  1.00886316,  1.07565263],
  [ 1.14244211,  1.20923158,  1.27602105,  1.34281053,  1.4096    ]])
expected_velocity = np.asarray([
  [ 0.5406,      0.55475789,  0.56891579, 0.58307368,  0.59723158],
  [ 0.61138947,  0.62554737,  0.63970526,  0.65386316,  0.66802105],
  [ 0.68217895,  0.69633684,  0.71049474,  0.72465263,  0.73881053],
  [ 0.75296842,  0.76712632,  0.78128421,  0.79544211,  0.8096    ]])

# Should see relative errors around e-8 or less
print('next_w error: ', rel_error(next_w, expected_next_w))
print('velocity error: ', rel_error(expected_velocity, config['velocity']))
```

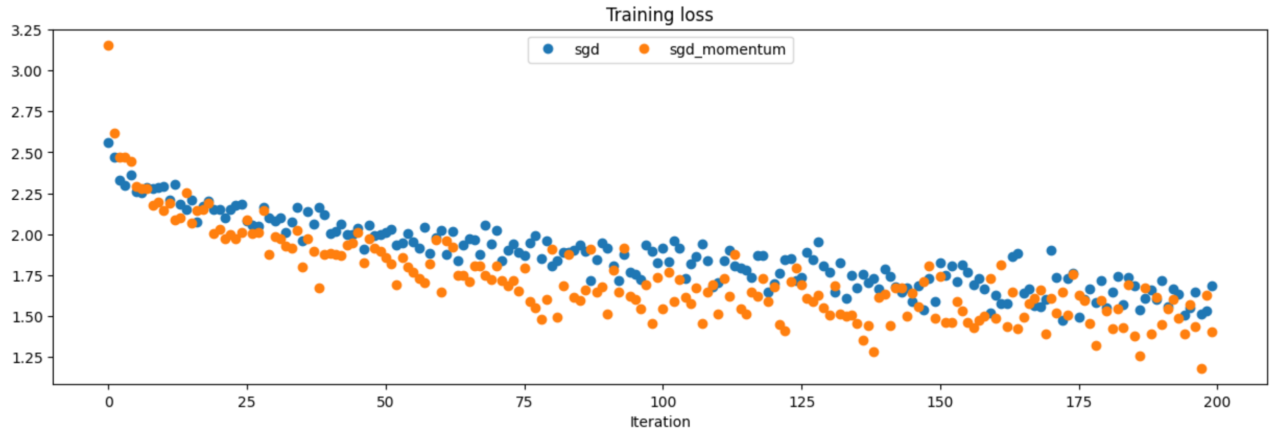
```
next_w error:  8.882347033505819e-09
velocity error:  4.269287743278663e-09
```

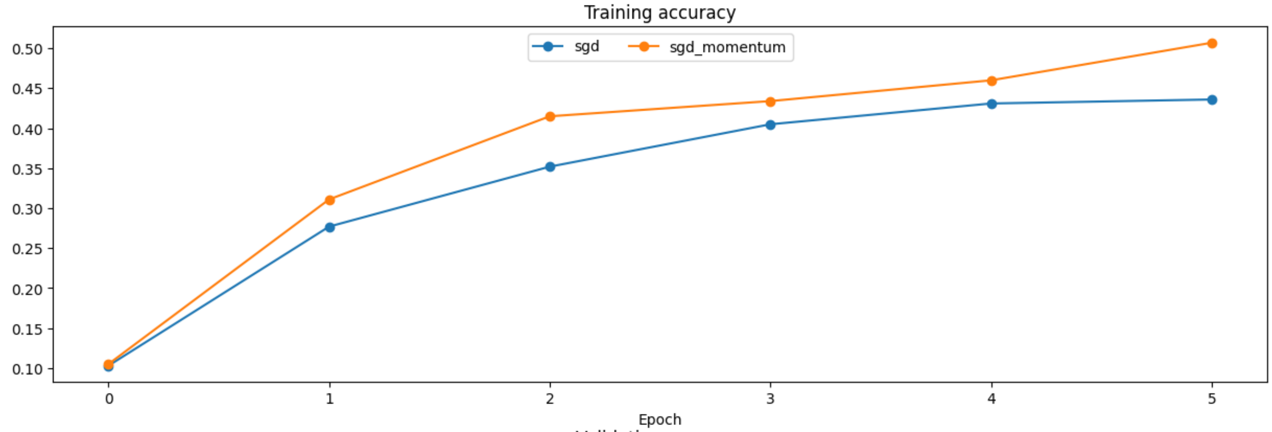
Once you have done so, run the following to train a six-layer network with both SGD and SGD+momentum. You should see the SGD+momentum update rule converge faster.

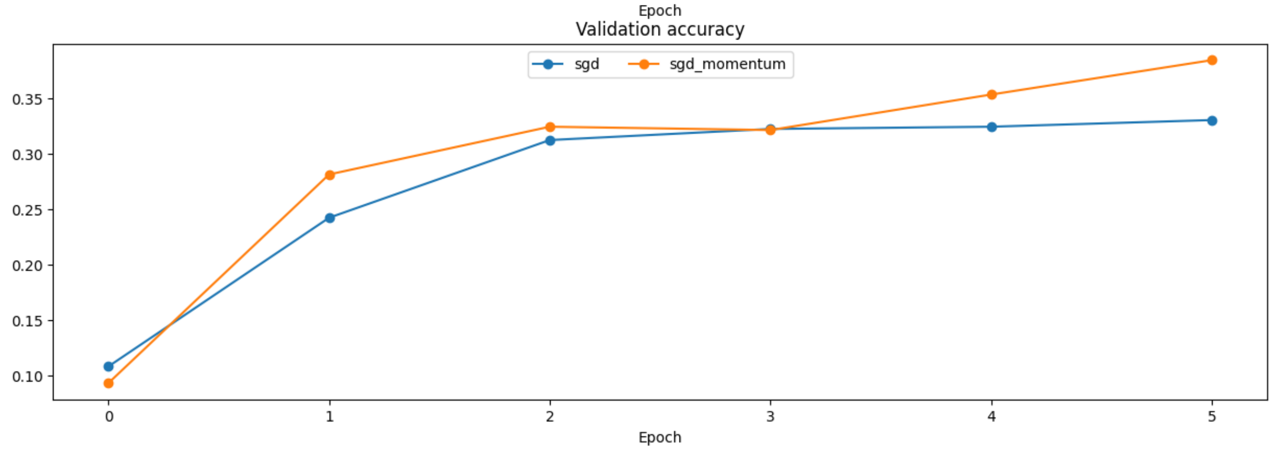
In [17]:
```
num_train = 4000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

solvers = {}

for update_rule in ['sgd', 'sgd_momentum']:
  print('running with ', update_rule)
  model = FullyConnectedNet([100, 100, 100, 100, 100], weight_scale=5e-2)

  solver = Solver(model, small_data,
                  num_epochs=5, batch_size=100,
                  update_rule=update_rule,
                  optim_config={
                    'learning_rate': 1e-2,
                  },
                  verbose=True)
  solvers[update_rule] = solver
  solver.train()
  print()

plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')

plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')

plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')

for update_rule, solver in list(solvers.items()):
  plt.subplot(3, 1, 1)
  plt.plot(solver.loss_history, 'o', label=update_rule)

  plt.subplot(3, 1, 2)
  plt.plot(solver.train_acc_history, '-o', label=update_rule)

  plt.subplot(3, 1, 3)
  plt.plot(solver.val_acc_history, '-o', label=update_rule)

for i in [1, 2, 3]:
  plt.subplot(3, 1, i)
  plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```

```
running with  sgd
(Iteration 1 / 200) loss: 2.559978
(Epoch 0 / 5) train acc: 0.103000; val_acc: 0.108000
(Iteration 11 / 200) loss: 2.291086
(Iteration 21 / 200) loss: 2.153591
(Iteration 31 / 200) loss: 2.082693
(Epoch 1 / 5) train acc: 0.277000; val_acc: 0.242000
(Iteration 41 / 200) loss: 2.004171
(Iteration 51 / 200) loss: 2.010409
(Iteration 61 / 200) loss: 2.023753
(Iteration 71 / 200) loss: 2.026621
(Epoch 2 / 5) train acc: 0.352000; val_acc: 0.312000
(Iteration 81 / 200) loss: 1.807163
(Iteration 91 / 200) loss: 1.914256
(Iteration 101 / 200) loss: 1.917176
(Iteration 111 / 200) loss: 1.706193
(Epoch 3 / 5) train acc: 0.405000; val_acc: 0.322000
(Iteration 121 / 200) loss: 1.697994
(Iteration 131 / 200) loss: 1.768837
(Iteration 141 / 200) loss: 1.784967
(Iteration 151 / 200) loss: 1.823291
(Epoch 4 / 5) train acc: 0.431000; val_acc: 0.324000
(Iteration 161 / 200) loss: 1.626499
(Iteration 171 / 200) loss: 1.901366
(Iteration 181 / 200) loss: 1.550534
(Iteration 191 / 200) loss: 1.716921
(Epoch 5 / 5) train acc: 0.436000; val_acc: 0.330000

running with  sgd_momentum
(Iteration 1 / 200) loss: 3.153778
(Epoch 0 / 5) train acc: 0.105000; val_acc: 0.093000
(Iteration 11 / 200) loss: 2.145874
(Iteration 21 / 200) loss: 2.032563
(Iteration 31 / 200) loss: 1.985848
(Epoch 1 / 5) train acc: 0.311000; val_acc: 0.281000
(Iteration 41 / 200) loss: 1.882353
(Iteration 51 / 200) loss: 1.855372
(Iteration 61 / 200) loss: 1.649133
(Iteration 71 / 200) loss: 1.806432
(Epoch 2 / 5) train acc: 0.415000; val_acc: 0.324000
(Iteration 81 / 200) loss: 1.907840
(Iteration 91 / 200) loss: 1.510681
(Iteration 101 / 200) loss: 1.546872
(Iteration 111 / 200) loss: 1.512047
(Epoch 3 / 5) train acc: 0.434000; val_acc: 0.321000
(Iteration 121 / 200) loss: 1.677301
(Iteration 131 / 200) loss: 1.504686
(Iteration 141 / 200) loss: 1.633253
(Iteration 151 / 200) loss: 1.745081
(Epoch 4 / 5) train acc: 0.460000; val_acc: 0.353000
(Iteration 161 / 200) loss: 1.485411
(Iteration 171 / 200) loss: 1.610416
(Iteration 181 / 200) loss: 1.528331
```

```
(Iteration 191 / 200) loss: 1.449159
(Epoch 5 / 5) train acc: 0.507000; val_acc: 0.384000
```







Train a good model!

Train the best fully-connected model that you can on CIFAR-10, storing your best model in the `best_model` variable. We require you to get at least 50% accuracy on the validation set using a fully-connected net.

If you are careful it should be possible to get accuracies above 55%, but we don't require it for this part and won't assign extra credit for doing so. Later in the assignment we will ask you to train the best convolutional network that you can on CIFAR-10, and we would prefer that you spend your effort working on convolutional nets rather than fully-connected nets.

You might find it useful to complete the `BatchNormalization.ipynb` notebook before completing this part, since those techniques can help you train powerful models.

In [65]:
```
best_model = None
################################################################################
# TODO: Train the best FullyConnectedNet that you can on CIFAR-10. You might
# find batch/layer normalization useful. Store your best model in  #
# the best_model variable.
################################################################################
from itertools import product
num_train = 50
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

learning_rates = [0.12894736842105264]
# regularization_strengths = [6.233333333333334e-05]

# learning_rates = np.linspace(1e-2, 1e-1, num=20)
regularization_strengths = np.linspace(6.2e-5, 6.3e-5, num=5)
update_rules = ['sgd', 'sgd_momentum']
weight_scale = [5e-2]
momentum = [0.8, 0.9]

# best validation accuracy: 0.227 with lr 0.12894736842105264, reg 6.2333333

hyperparameters = list(product(*[learning_rates, regularization_strengths, u

best_val = 0
best_lr = 0
best_reg = 0
```

```
best_ur = ''
best_m = 0

for lr, reg, ur, m, w in hyperparameters:
    model = FullyConnectedNet([100, 100, 100, 100, 100],
                    weight_scale=w, reg=reg, dtype=np.float64, normalization
    solver = Solver(model, small_data,
                    print_every=0, num_epochs=50, batch_size=10,
                    update_rule=ur,
                    optim_config={
                    'learning_rate': lr,
                    'momentum': m
                    },
                    verbose=False
            )
    solver.train()

    if solver.best_val_acc > best_val:
        best_val = solver.best_val_acc
        best_lr = lr
        best_reg = reg
        best_ur = ur
        best_m = m
        best_w = w
        best_model = model
        print(f'** New best! best validation accuracy: {best_val} with lr {b


print(f'best validation accuracy: {best_val} with lr {best_lr}, reg {best_re
################################################################################
#                                END OF YOUR CODE
################################################################################
```

```
** New best! best validation accuracy: 0.204 with lr 0.12894736842105264, re
g 6.2e-05, weight scale 0.05 and best update rule sgd and best momentum 0.8
** New best! best validation accuracy: 0.208 with lr 0.12894736842105264, re
g 6.2e-05, weight scale 0.05 and best update rule sgd and best momentum 0.9
** New best! best validation accuracy: 0.214 with lr 0.12894736842105264, re
g 6.225e-05, weight scale 0.05 and best update rule sgd_momentum and best mo
mentum 0.9
** New best! best validation accuracy: 0.217 with lr 0.12894736842105264, re
g 6.275000000000001e-05, weight scale 0.05 and best update rule sgd_momentum
and best momentum 0.9
best validation accuracy: 0.217 with lr 0.12894736842105264, reg 6.275000000
000001e-05, and weight scale 0.05 and best update rule sgd_momentum and best
momentum 0.9
```

best validation accuracy: 0.177 with lr 0.05736152510448681 and weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.204 with lr 0.12689610031679222, reg 1e-05, weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.219 with lr 0.2782559402207124, reg 1e-05, weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.189 with lr 0.1291549665014884, reg 1.2915496650148827e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.189 with lr 0.12, reg 6e-05, and weight scale 0.05 and best update rule sgd_momentum and best momentum 0.9

best validation accuracy: 0.205 with lr 0.12897435897435897, reg 7.000000000000001e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.211 with lr 0.12897435897435897, reg 6.111111111111111e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.22 with lr 0.12894736842105264, reg 6.222222222222222e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.9

best validation accuracy: 0.217 with lr 0.12894736842105264, reg 6e-05, and weight scale 0.05 and best update rule sgd_momentum and best momentum 0.9

best validation accuracy: 0.221 with lr 0.12894736842105264, reg 6.111111111111111e-05, and weight scale 0.05 and best update rule sgd_momentum and best momentum 0.9

best validation accuracy: 0.226 with lr 0.12894736842105264, reg 6e-05, and weight scale 0.05 and best update rule sgd_momentum and best momentum 0.9

best validation accuracy: 0.211 with lr 0.12894736842105264, reg 6e-05, and weight scale 0.05 and best update rule sgd_momentum and best momentum 0.9

best validation accuracy: 0.227 with lr 0.12894736842105264, reg 6.233333333333334e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.9

** New best! best validation accuracy: 0.215 with lr 0.004641588833612777, reg 0.00021544346900318823, weight scale 0.02 and best update rule adam and best momentum 0.99

best validation accuracy: 0.212 with lr 0.12894736842105264, reg 6.2e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.5

** New best! best validation accuracy: 0.226 with lr 0.12894736842105264, reg 6.1e-05, weight scale 0.05 and best update rule sgd and best momentum 0.5

best validation accuracy: 0.216 with lr 0.12894736842105264, reg 6e-05, and weight scale 0.05 and best update rule sgd and best momentum 0.99

Best parameters: {'learning_rate': 0.04814179393818249, 'reg': 6.881218080170343e-05, 'weight_scale': 0.02, 'update_rule': 'adam', 'momentum': 0.8} Best validation accuracy: 0.216

Best parameters: {'learning_rate': 0.04046327613077516, 'reg': 6.322120254343489e-05, 'weight_scale': 0.02, 'update_rule': 'sgd_momentum', 'momentum': 0.9} Best validation accuracy: 0.216

Test your model!

Run your best model on the validation and test sets. You should achieve above 50% accuracy on the validation set.

In [66]:
```
y_test_pred = np.argmax(best_model.loss(data['X_test']), axis=1)
y_val_pred = np.argmax(best_model.loss(data['X_val']), axis=1)
print('Validation set accuracy: ', (y_val_pred == data['y_val']).mean())
print('Test set accuracy: ', (y_test_pred == data['y_test']).mean())
```

```
Validation set accuracy:  0.208
Test set accuracy:  0.199
```

In [ ]: