2. RELATIONS & FUNCTIONS

SYNOPSIS

Relations:

Introduction: Much of mathematics can be built up from set theory this was a project which was carried out by philosophers, logicians, and mathematicians largely in the first half of the 20th century. Whitehead and Russell were among the pioneers, with their great work principia mathematical. Defining Mathematical notions on the basis if set theory does not add anything 'Mathematical', and is not of particular interest to the "working Mathematician", but it is of great interest for the foundations of mathematics, showing how little needs to be assumed as "primitive".

We illustrate some bits of that project here, with some basic set – theoretic definitions of ordered pairs, relations, and functions, along with some standard notions concerning relations and functions.

Ordered pairs and Cartesian products: As we see, there is no order imposed on the elements of a set. To describe functions and relations we will need the notion of an **ordered pair,** written (a,b), for example, in which a is considered the **first member (element)** and b is the **second member (element)** of the pair. So, in general, $(a,b) \neq (b,a)$ (whereas for a set, $\{a,b\} = \{b,a\}$)

Is there a way to define ordered pairs is terms of sets? You might think not, since sets are themselves unordered. But there are in fact various ways it can be done.

Here is one way to do it, usually considered the most conventional.

The ordered pair can be defined as follows.

Definition: (a, b) = {{a}, {a, b}}: How can we be sure that that definition does the job it's supposed to do? What's crucial is that for every ordered pair, there is indeed exactly one corresponding set of the form {{a}, {a, b}}, and two different ordered pairs always have two different corresponding sets. We want try to prove that that holds, but it does.

These would be nothing wrong with taking the notion of ordered pair as another primitive notion (i.e. undefined notion), alongside the notion of set. But mathematicians like seeing how far they can reduce the number of primitives (undefined notions), and it's an interesting discovery to see that the notion of order can be defined in terms of set theory.

Cartesian product: Suppose we have two sets A and B and we form ordered pairs by taking an element of A as the first member of the pair and an element of B as the second member. The Cartesian product of A and B, written $A \times B$, is the set consisting of all such pairs. The predicate notation defines it as:

$A \times B = \{(x, y) / x \in A \text{ and } y \in B\}$

Here are some examples of Cartesian products:

Eg - 1: If $A = \{a,b,c\}, B = \{1,2\}$, then

$A \times B = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

Note-1: What happens if either A or B is ϕ?

Suppose $A = \{a,b,c\}$ what is $A \times \phi$?

Eg - 2: $K=\{a,b,c,d\}$ and $L=\{1,2\}$, then

$K\times L\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2),(d,1),(d,2)\}$

$L\times K=\{(1,a),(1,b),(1,c),(1,d),(2,a),(2,b),(2,c),(2,d)\}$

$L\times L=\{(1,1),(1,2),(2,1),(2,2)\}$

Note 2: Why Cartesian products are called products

Note 3: Look at the cardinalities of the sets above, and see if you can figure out in general what the cardinality of the set $A\times B$ will be, given the cardinalities of sets A and B.

What about ordered triples? The definition of ordered pairs can be extended to ordered triples and in general to ordered n tuples for any natural n.

For example, ordered triples are usually defined as.

(a, b, c) = ((a, b), c)

And for three sets *A*, *B* and *C* the Cartesian product can be defined as

$A\times B\times C=((A\times B)\times C)$

In the case when *A* = *B* = C........a special notation is used: $A\times A=A^2, A\times A\times A=A^3$, etc., and we put $A^1=A$.

Illustration - 1: Let $A=\{a,b,c\}, B=\{d,e\}, C=\{e,f,h\}$ and $D=\{f,g,h\}$

(i) $A\times(B\cap C)$

(ii) $(A\times B)\cap(A\times C)$

(iii) $A\times(B\cup C)$

(iv) $(A\times B)\cup(A\times C)$

(v) $(A\times B)\cap(C\times D)$

(vi) $(A\cap C)\times(B\cap D)$

(vii) $(A\times B)\cap(C\times D)$

(viii) $(A\cup C)\times(B\cup D)$

What do you observe?

Solution: By the definition of intersection of two sets, $B\cap C=\{e\}$

Therefore $A\times(B\cap C)=\{(a,e),(b,e)(c,e)\}$

(ii) Now $A\times B=\{(a,d),(a,e),(b,d),(b,e),(c,d),(c,e)\}$

and $A\times C=\{(a,e),(a,f),(a,h),(b,e),(b,f),(b,h),(c,e),(c,f),(c,h)\}$

Therefore $(A\times B)\cap(A\times C)=\{(a,e),(b,e),(c,e)\}$

(iii) Now $(B\cup C)=\{d,e,f,h\}$, we have

$A\times(B\cup C)=\{(a,d),(a,e),(a,f),(a,h),(b,d),(b,e),(b,f),(b,h),(c,d),(c,e),(c,f),(c,h)\}$

(iv) using the sets $A\times B$ and $A\times C$ from part (ii) above, we obtain

$(A\times B)\cup(A\times C)=\{(a,d),(a,e),(a,f),(a,h),(b,d),(b,e),(b,f),(b,h),(c,d),(c,e),(c,f),(c,h)\}$

(v)using $(A\times B)$ from part (ii) above, and $C\times D=\{(e,f),(e,g),(e,h),(f,f),(f,g),(f,h),(h,f),(h,g),(h,h)\}$

Therefore $(A\times B)\cap(C\times D)=\phi$

(vi) Now by the definition of intersection of two sets, $A\cap C=\phi$ and $B\cap D=\phi$

Therefore $(A\cap C)\times(B\cap D)=\phi$

(vii) using $(A\times B)$ from part (ii) and $(C\times D)$ from part (vi) above, we obtain

$(A\times B)\cup(C\times D)=\{(a,d),(a,e),(b,d),(b,e),(c,d),(c,e),(e,f),(e,g),(e,h),$

$(f,f),(f,g),(f,h),(h,f),(h,g)(h,h)\}$

(viii) By the definition of union of two sets, $A\cup C=\{a,b,c,e,f,h\}$ and $(B\cup D)=\{d,e,f,g,h\}$

Therefore $(A\cup C)\times(B\cup D)=\{(a,d),(a,e),(a,f),(a,g),(a,h),(b,d),(b,e),(b,f),$

$(b,g),(b,h),(c,d),(c,e),(c,f),(c,g),(c,h),(e,d),(e,e),(e,f),(e,g),(e,h),(f,d),(f,e),(f,f),$

$(f,g),(f,h),(h,d),(h,e),(h,f),(h,g),(h,h)\}$

We conclude that (i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(iii) $(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$

(iv) $(A\times B)\cup(C\times D)\subseteq(A\cup C)\times(B\cup D)$

Theorem - 1: Suppose A and B are sets. Then $A\times B=B\times A$, if either $A=\phi, B=\phi$, or $A=B$

Illustration - 2: If $(a+5,3)=(8,b-6)$ find the values of a and b.

Solution: Since the ordered pairs are equal, so corresponding elements are equal.

Therefore $a+5=8$ and $3=b-6$

Solving we get $a=3$ and $b=9$

Illustration - 3: If $K=\{x,y,z\}, L=\{p,q,r\}$, form the sets $K\times L$ and $L\times P$. What do you observe?

Solution: By the definition of Cartesian product,

K x L = { (x, p), (x, q), (x, r), (y, p), (y, q), (y, r), (z, p), (z, q), (z, r)}

and L x K = { (p, x), (p, y), (p, z), (q, x), (q, y), (q, z), (r, x), (r, y), (r, z)}

Since by the definition of ordered pairs, the pair $(p,x)\neq(x,p)$, we conclude that $K\times L\neq L\times K$

Illustration - 4: If $A=\{a,b\}$ form the sets (i) $A\times A$ and (ii) $A\times A\times A$

Solution: (i) We have $A\times A=\{(a,a),(a,b),(b,a),(b,b)\}$

(ii) using the set $A\times A$ from part (i) above, we obtain $A\times A\times A=((A\times A)\times A)$

i.e., $\{(a,a,a),(a,a,b),(a,b,a),(a,b,b),(b,a,a),(b,a,b),(b,b,a),(b,b,b)\}$

Illustration - 5: If R is the set of all real numbers, what do the Cartesian products $R\times R$ and $R\times R\times R$ represents?

Solution: By the definition of Cartesian product, $R\times R=\{(x,y)/x,y\in R\}$

Which represents the coordinates of all the points in two dimensional space and the Cartesian product $R \times R \times R$ represents the set $R \times R \times R = \{(x, y, z) / x, y, z \in R\}$ which represents the coordinates of all the points in three dimensional space.

Illustration - 6: If $A \times B = \{(1,2),(3,4),(1,5)(3,5)(1,4)\}$ find A and B.

Solution: A is a set of first elements $= \{1,3\}$

B is a set of second elements $= \{2,4,5\}$

Relations: In natural language relation are a kind of links existing between two objects. Examples: 'Mother Of' neighbor of, 'parent of', 'father of', 'is older than', ' is an ancestor of ', 'is a subset of'. etc. These are binary relations. Formally we will define relations between elements of sets.

We may write Rxy or x Ry for " a related to b under the relation or by R. And when we formalize relations as sets of ordered pairs of elements, we will officially write $(a,b) \in R$.

If A and B are any sets and $R \subseteq A \times B$, we call R a **binary relation from** A **to** or a **binary relation between and .** A relation is called a relation in or on .

The set **dom** $R = \{a / (a,b) \in R \text{ for some b}\}$ is called the domain of the relation R and the set **range** $R = \{b / (a,b) \in R \text{ for some a}\}$ is called the range of the relation R. The whole set B is called co domain of the relation R.

Note: Range $R \subseteq$ Codomain of R.

Specifying Relations (Representation of a Relations)

1. **Roster form:** We can defined a relation by listing all ordered pairs

   Example: Let $A = B = \{1,2,3,4,5,6,7\}$ define the divisibility relation between A and B by listing all its elements.

   Solution: Observe that $|A| = |B| = 7, |A \times B| = 7 \times 7 = 49,$ so the relation divisibility consists of at most 49 pairs. There is some hope that we can list all the elements.

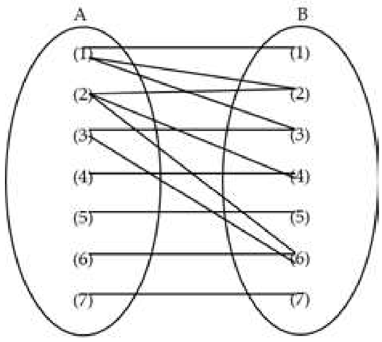
   For each possibility for the first component, we list all its multiples in the second component. This gives us a representation of the relation divisibility as the set $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),$ $(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6),(7,7),\}$

2. **Set Builder form:** we can also define a relation by rule form. In this form, the relation R is represented as $\{(a,b) / a \in a, b \in B, a......b\}$, the blank is to be replaced by the rule which associates a to b.

   the above example can be defined as $R = \{(a,b) / a \in A, b \in A, a|b\}$

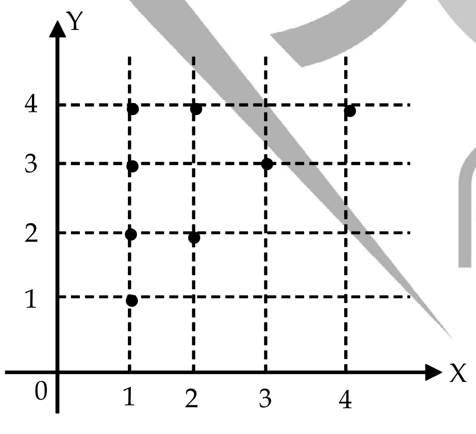
   Here the '|' means "exactly divisible by" symbol

3. **B i partite graph (Arrow diagram):** In this form, we list all elements of A one side and elements of B on the other side. If aRb, we connect the nodes corresponding to elements $a \in A$ and $b \in B$ with an edge.
The above example can be defined as follows:



bipartite graph

4. **By Lattice method:** In this form, the relation R is represented by drawing dots in the lattice or rectangular co-ordinate system for all ordered pairs which satisfy the given relation R.
Example: Let $A = \{1,2,3,4\}$ $B = \{1,2,3,4\}$ and R be the relation "is a divisor of" from A to B then
$R = \{(1,1),(1,2),(1,3),(1,4),(2,2),(2,4),(3,3),(4,4)\}$
This relation R from A to B can be represented by the lattice as shown in the given figure. The points marked by dots represent the ordered pairs satisfied by the given relation.



Operations on Relations: The complement of a relation $R \subseteq A \times B$ is defined as $R^c = (A \times B) - R$

Note: The complement of a relation is depends on what universe we are considering. A given relation may certainly be a subset of more than one Cartesian product, and its complement will differ according to what Cartesian product we are taking to be the relavant universe.

Eg: what is the complement of the relation $R=\{(a,d),(a,e),(b,c)\}$ on the universe $\{a,b\}\times\{c,d,e\}$?

Solution: Universe $U=\{(a,c),(a,d),(a,e),(b,c),(b,d),(b,e)\}$ and $R=\{(a,d),(a,e),(b,c)\}$

By the definition of complement of a set, $R^c=U-R=\{(a,c),(b,d),(b,e)\}$

Inverse of a relation $R\subseteq A\times B$: The inverse of a relation $R\subseteq A\times B$ is defined as the relation $R^{-1}\subseteq B\times A$, $R^{-1}=\{(b,a)/(a,b)\in R\}$.

Note: 1 If R is a relation from A into B, then $\left(R^{-1}\right)^{-1}=R$.

Eg: 1 Find the inverse of a relation R defined in the above example.

Solution: $R^{-1}=\{(d,a),(e,a),(c,d)\}$

Eg: 2 Let N be the set of natural numbers, $\{1,2,3,4,......\}$.

Let R be " is less than " on N (i.e. on $N\times N$) find R.

Solution: $R=\{(a,b)/a,b\in N; a<b\}$

Total number for relations: Let A and B be two sets having p and q elements respectively. Then $|A\times B|=|A|\times|B|$ or $n(A\times B)=n(A)\times n(B)$ $\therefore |A\times B|=pq$

Each subset associated with Cartesian product of A and B is a relation from A to B, therefore the total number of relations is 2^{pq}

Eg: Let $A=\{a,b,c\}, B=\{3,4\}$. Find number of relations from A to B

Solution: As we know each subset of a Cartesian product of A and B is a relation from A to B

Here $|A|=3;|B|=2$ $\quad\therefore |A\times B|=3\times 2=6$

Hence total number of relations is 2^6

Composite Relation: Suppose R is a relation from A to B and S is a relation from B to C. Then the composition of S and R is the relation S o R from A to C defined as follows

S o R $=\{(a,c)\in A\times C/(a,b)\in R,(b,c)\in S \text{ for some } b\in B\}$

Notice that we have assumed that the second coordinates of pairs in R and the first coordinate of pairs in S both come from the same set, B. If these sets were not the same, the composition So R would be undefined.

Ex: If $A=\{1,2,3\}, B=\{a,b,c\}, C=\{x,y\}$, $f=\{(1,a),(1,b),(2,a),(2,c)\}$ $g=\{(a,x),(b,x),(b,y)\}$ find fog and gof

Solution: $gof=\{(1,x),(2,x),(1,y)\}$, fog cannot defined

Theorem : Suppose R is a relation from A to B, S is a relation from B to C, and T is a relation from C to D, Then:

1. $\left(R^{-1}\right)^{-1} = R.$
2. $DOM\left(R^{-1}\right) = Ran\left(R\right)$
3. $Ran\left(R^{-1}\right) = DOM\left(R\right)$
4. $T \circ \left(S \circ R\right) = \left(T \circ S\right) \circ R.$
5. $\left(S \circ R\right)^{-1} = R^{-1} o S^{-1}$

Note: Inverse Relation $\left(R^{-1}\right)$ and complement of R are both different relations whereas

$R^{-1} = \{(b,a)/(a,b) \in R\}$ and $R^c = \{(a,b)/(a,b) \in \cup$ and $(a,b) \notin R\}$

Here 'U' means universal relation as defined in the topic complement of a relation.

Types of Relations: There are many special types of relations that deserve closer attention. These include

1. Functions
2. Equivalence relations
3. Order relations

We will discuss Equivalence relations and order relations in more details in this chapter and about function in the next chapter.

Relations on a set A: We will come across man if relations where the domain and the co domain are equal. We now introduce some properties such relations can have.

Definition: A relation R on set A is reflexive if $(\forall a \in A) aRa.$ It is anti -reflective if $(\forall a \in A) > aRa.$

Here the symbol '$\neg$' is negation.

Eg: (i) Any set is a subset of itself, $So \subseteq$ is reflexive

(ii) $a < b$ is anti -reflexive. Since no number can be less than itself.

Definition: A relation R on set A is symmetric if $(\forall a,b \in A) aRb \Leftrightarrow bRa.$ It is anti-symmetric if $(\forall a,b \in A)(aRb \cap bRa) \Rightarrow (a = b).$

Eg: (i) The relations $\Leftrightarrow$ and $\equiv$ are symmetric. Every relation that can be characterized by " a and b have the same….." is symmetric

(ii) The relation $\subseteq$ is not symmetric, since, whenever $A \subseteq B$ and $A \neq B,$ we have $A \not\subseteq B.$ Also note that if $A \subseteq B$ and $B \subseteq A,$ then $A = B,$ so $\subseteq$ is actually Anti-symmetric.

Definition: A relation R on set A is transitive if $(\forall a,b,c \in A)$ $(aRb \cap bRc) \Rightarrow aRc.$

Eg: All the relations $\subseteq, <, \leq, 1, \Leftrightarrow$ and $\equiv$ are easily seen to be transitive.

Order relations:

Definition: Suppose R is a relation on a set A. Then R is called a partial order on A.

(Or just partial order if A is clear from content) if it is reflexive, transitive, and anti – symmetric. It is called a total order on A (or just a total order) if it is a partial order, and in addition it has the following property.

$\forall x \in A \quad \forall y \in A\left(xRy \cup yRx\right)$

Eg: $G = \{(x, y) \in R \times R \,/\, x \geq y\}$

(i) Clearly $x \geq y$ is reflexive

(ii) $x \geq y$ is anti - symmetric, since $x \geq y$ and $y \leq x \Rightarrow x = y$.

(iii) Let $x, y, z \in R \quad \forall x, y, z \quad x \geq y, y \geq z \Rightarrow x \geq z.$

$\therefore$ The G is partial order.

Note:

1. Let A be a finite set and $n(A) = n,$ then

(i) $n(A \times A) = n^2$

(ii) The number of relations on $A = 2^{n^2}$

(iii) The number of reflexive relations on $A = 2^{n^2 - n}$

(iv) The number of symmetric relations on $A = 2^{\frac{n^2+n}{2}}$

2. (i) Let A be a set. If $B_1, B_2, ...B_n$ are nonempty subsets of A such that $B_1 \cup B_2 \cup ..... \cup B_n = A,$ and $B_i \cap B_j = \phi$ for $i \neq j,$ them $\{B_1, B_2, ...B_n\}$ is called a partition of A.

(ii) If A is finite set, then the number of equivalence relations on A is equal to the number of partitions of A.

FUNCTIONS:

1. **Domain, Codomain and Range of a Function** Let $f: A \to B,$ then the set A is known as the domain of f and the set B is known as codomain of f. The set of all f images of elements of A is known as the range of f. Thus Domain of $f = \{a | a \in A, (a, f(a)) \in f\}$

   Range of $f = \{f(a) | a \in A, f(a) \in B\}$

   It should be noted that range is a subset of codomain. If only the rule of function is given than the domain of the function is the set of those real numbers, where function is defined. For a continuous function, the interval from minimum to maximum value of a function gives the range.

2. Important Type of Functions

   (i) Polynomial Function If a function f is defined by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$ where n is a non – negative integer and $a_0, a_1, a_2, ..., a_n$ are real numbers and $a_0 \neq 0,$ then f is called a polynomial function of degree n.

   Note (i) A polynomial of degree one with no constant term is called an odd linear function. i.e, $f(x) = ax, a \neq 0$

   There are two polynomial function, satisfying the relation; $f(x) \cdot f(1/x) = f(x) + f(1/x)$. They are

   (a) $f(x) = x^{n+1}$ and

   (b) $f(x) = 1 - x^n,$ where n is a positive integer.

   (ii) Algebraic function y is an algebraic function of $x,$ if it is a function that satisfies an algebraic equation of the form $P_0(x)y^n + P_1(x)y^{n-1} + ... + P_{n-1}(x)y + P_n(x) = 0$ where n is a positive integer and $P_0(x), P_1(x), ...$ are polynomials in x.

   eg, $y = |x|$ is an algebraic function, since it satisfies the equation $y^2 - x^2 = 0.$

   Note that all polynomial functions are algebraic but not the converse. A function that is not algebraic is called Transcendental Function.

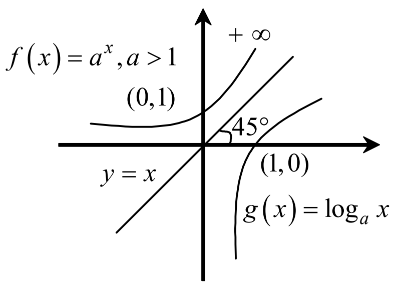
   (iii) Fractional Rational Function A ration function is a function of the form.

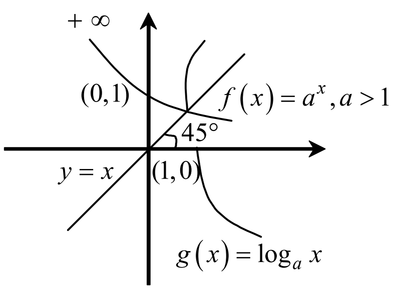
   $$y = f(x)\frac{g(x)}{h(x)},$$

   Where $g(x)$ and $h(x)$ are polynomials and $h(x) \neq 0.$

   (iv) Exponential Function A function $f(x) = a^x - e^{x/na}$ $(a > 0, a \neq 1, x \in R)$ is called an exponential function. The inverse of the exponential function is called the logarithmic function. i.e., $g(x) = \log_a X.$

   Note that $f(x)$ and $g(x)$ are inverse of each other and their graphs are as shown.



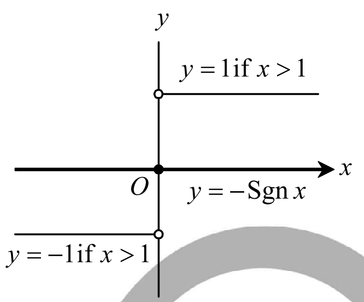


(v) Absolute Value Function A function $y = f(x) = |x|$ is called the absolute value function or Modulus function. it is defined as

$$y = |x| \begin{cases} x, \text{if } x \ge 0 \\ -x, \text{if } x < 0 \end{cases}$$

(vi) Signum Function A function $y = (x) = \operatorname{Sgn}(x)$ is defined as follows

$$y = f(x) \begin{cases} 1 & \text{for } \ x > 0 \\ 0 & \text{for } \ x = 0 \\ -1 & \text{for } \ x < 0 \end{cases}$$



It is also written as $\operatorname{Sgn} x = |x| / x;$

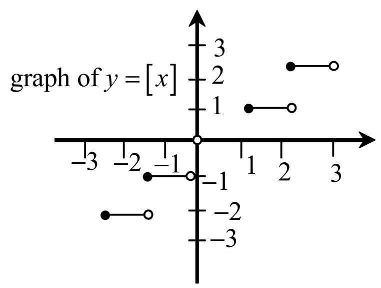
$$x \ne 0; f(0) = 0$$

(vii) Greatest Integer or Step Up Function The function $y = f(x) = [x]$ is called the greatest integers function where $[x]$ denotes the greatest integer less than or equal to $x.$ Note that for

$-1 \le x < 0 \ ; \ [x] = -1 \quad 0\text{€} \le \text{€}x < 1 \ ; \ [x] = 0$

$1 \le x < 2 \ ; \ [x] = 1 \quad 2 \le x < 3 \ ; \ [x] = 2$

and so on.
Properties of greatest integer function



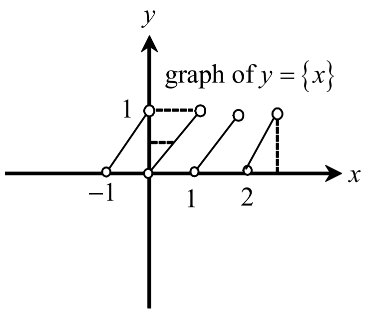
(a) $[x] \le x < [x]+1$ and $x-1 < [x] \le x, 0 \le x-[x] < 1$

(b) $[x+m]=[x]+m$ if m is an integer.

(c) $[x]+[y] \le [x+y] \le [x]+[y]+1$

(d) $[x]+[-x]=0$ if x is an integer $=-1$ otherwise.

(viii) Fractional Part Function It is defined as



eg. The fractional part of the no. 2.1 is $2.1-2=0.1$ and the fractional part of -3.7 is $0.3.$ The period of this function is 1 and graph of this function is an shown.

3. Domains and Ranges of Common Function:

Function $(y=f(x))$	**Domain** (i.e, values taken by x)	**Range** (i.e., values taken by $f(x)$)
A. Algebraic function:		
(i) $x^n, (n \in N)$	R = (set of real numbers)	R, if n is odd $R^+ \cup \{0\}$, if n is even
(ii) $\frac{1}{x^n}, (n \in N)$	R - {0}	R - {0}, if n is odd R⁺, if n is even
(iii) $x^{\ln}, (n \in N)$	R, if n is odd $R^+ \cup \{0\}$, if n is even $R-\{0\}$, if n is odd	R if n is odd $R^+ \cup \{0\}$, if n is even $R-\{0\}$, if n is odd
(iv) $\frac{1}{x^{1/n}}, (n \in N)$	R ⁺, if n is even	R ⁺, if n is even
B. Trigonometric Function:		
(i) $\sin x$	R	$[-1,+1]$
(ii) $\cos x$	R	$[-1,+1]$
(iii) $\tan x$	$R-(2k+1)\frac{\pi}{2}, k \in I$	R

(iv) $\sec x$	$R-(2k+1)\frac{\pi}{2}, k \in I$	$(-\infty,-1]\cup[1,\infty)$
		$(-\infty,-1]\cup[1,\infty)$
(v) $\operatorname{cosec} x$	$R-k\pi, k \in I$	R
(vi) $\cot x$	$R-k\pi, k \in I$	

C. Inverse Circular Functions: (refer after inverse in taught)

(i) $\sin^{-1} x$	$[-1,+1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
(ii) $\cos^{-1} x$	$[-1,+1]$	$[0,\pi]$
(iii) $\tan^{-1} x$	R	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
(iv) $\operatorname{cosec}^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
(v) $\sec^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
(vi) $\cot^{-1} x$	R	$(0,\pi)$

D. Exponential Function:

(i) e^x	R	R^+
(ii) $e^{1/x}$	R - {0}	R^+ - {1}
(iii) $a^x, a>0$	R	R^+
(iv) $a^{1/x}, a>0$	R - {0}	R^+ - {1}

E. Logarithmic Function:

(i) $\log_a x, (a>0)(a\neq 1)$	R^+	R^+
(ii) $\log_x a = \frac{1}{\log_a x}$, $(a>0)(a\neq 1)$	R^+ - (1)	R - {0}

F. Integral Part Functions:

(i) $[x]$	R	I
(ii) $\frac{1}{\{x\}}$	$R-[0,1)$	$\left\{\frac{1}{n}, \text{n} \in \text{I}-\{0\}\right\}$

G. Fractional Part Functions:

(i) $[x]$	R	[0, 1)
(ii) $\frac{1}{\{x\}}$	R - I	$(1,\infty)$

H. Modulus Functions:

(i) $\lvert x\rvert$	R	$R^{+}\cup\{0\}$
(ii) $\frac{1}{\lvert x\rvert}$	$R-\{0\}$	R $^{+}$

I. Signum Function

$\operatorname{Sgn}(x)=\frac{\lvert x\rvert}{x}, x\neq 0$ $=0, x=0$	R $^{+}$	{-1, 0, 1}

J. Constant Function

Say $f(x)=c$	R	{c}

4. **Equal or identical function** Two functions f and g are said to be equal if

   (i) The domain of $f=$ the domain of g.

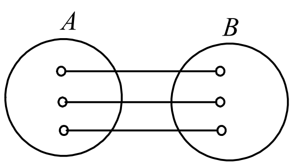
   (ii) The range of $f=$ the range domain of g.

   (iii) $f(x)=g(x)$, for every x belonging to their common domain. eg, $f(x)=\frac{1}{x}$ and $g(x)\frac{x}{x^2}$ are identical functions.

5. **Classification of Functions:**

   One – one function (Injective mapping) A function $f:A\rightarrow B$ is said to be one – one function or injective mapping if different elements of A have different f images in B. Thus for $x_1, x_2\in A$ and $f(x_1), f(x_2)\in B, f(x_1)=f(x_2), x_1=x_2$ or $x_1\neq x_2 \Leftrightarrow f(x_1)\neq f(x_2)$.

Diagrammatically an injective mapping can be shown as:



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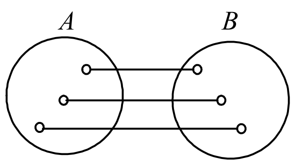
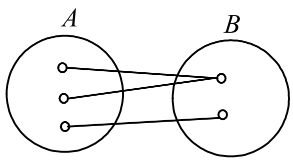


Note (i) Any function which is entirely increasing or decreasing in whole domain then $f(x)$ is one – one

(ii) If any line parallel to $x-$ axis cuts the graph of the function atmost at one point, then the function is one – one.

Many – one function A function $f: A \to B$ is said to be a many one function, if two or more elements of A have the same f image in B. Thus $f; A \to B$ is many one if for; $x_1, x_2 \in A, f(x_1) = f(x_2)$ but $x_1 \neq x_2$.
Diagrammatically a many one mapping can be shown as



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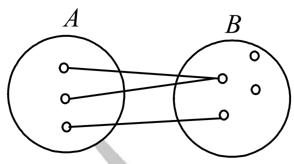
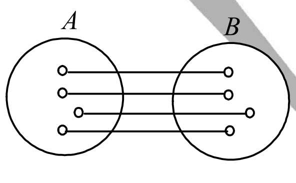


Note (i) Any continuous function which has at least one local maximum or local minimum, then $f(x)$ is many – one. In other words, if a line parallel to $x-$ axis cuts the graph of the function at least at two points, then f is many – one.

(ii) If a function is one – one, it cannot be many – one and vice – versa.

Onto function (Surjective mapping) If the function $f: A \to B$ is such that each element in B (codomain) is the f image of at least one element in A, then we say that f is a function of A ' 'onto" B. Thus $f: A \to B$ is surjective iff $\forall b \in B, \exists$ some $a \in A$ such that $f(a) = b$.

Diagrammatically surjective mapping can be shown as



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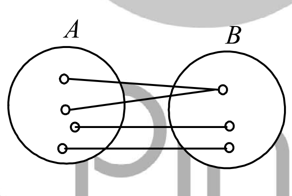
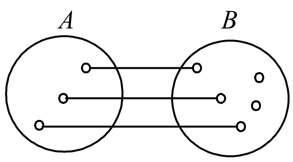
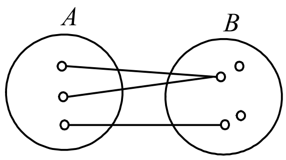


Note that If range = codomain, then $f(x)$ is onto.

Into function If $f: A \to B$ is such that there exists at least one element in codomain which is not the image of any element in domain, then $f(x)$ is onto.
Diagrammatically into function can be shown as



or

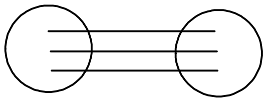


Note that If a function is onto, it cannot be into and vice – versa.

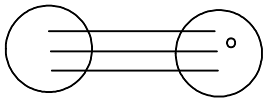
A polynomial of degree even will always be into.

Thus a function can be one of these four types.

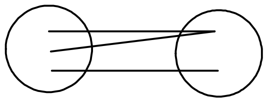
(a) one – one onto (injective and surjective).



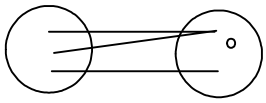
(b) one – one into (injective but not surjective)



(c) many – one ont (surjective but not injective)



(d) many – one into (neither surjective nor injective)



Note (i) If f is both injective and surjective, then it is called a bijective mapping.

The bijective functions are also named as invertible, non – singular or biuniform functions.

(ii) If a set A contains n distinct elements, then the number of different functions defined from $A \to A$ is n^n and out of it $n!$ are one one.

Identity function The function $f : A \to A$ defined by $f(x) = x, \forall x \in A$ is called the identity of A and is denotd by I_A. It is easy to observe that identity function is bijection.

Constant function A function $f : A \to B$ is said to be a constant function if every element of A has the same f image in B. Thus $f : A \to B; f(x) = c, \forall x \in A, c \in B$ is a constant function. note that the range of a constant function is a singleton and a constant function may be one – one or many – one, onto or into.

6. **Algebraic Operation on Functions** If f and g are real valued functions of x with domain set A, B respectively, then both f and g are defnined in $A \cap B$. Now we define $f + g, g - g, (f.g)$ and (f / g) as follows

(i) $(f \pm g)(x) = f(x) \pm g(x)$

(ii) $(f.g)(x) = f(x).g(x)$

domain in each case is $A \cap B$

(iii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ domain is $\{x | x \in A \cap B \text{ s.t.} g(x) \neq 0\}$.

7. **Composite of Uniformly and Non – uniformly Defined:**

Functions Let $f : A \to B$ and $g : B \to C$ be two factions. Then, the function $gof : A \to C$ defined by $(gof)(x)$

$= g(f(x)) \forall x \in A$ is called the composite of the two function f and g.

Diagramaticall $y \xrightarrow{x} f \xrightarrow{f(x)} g(f(x))$.

Thus the image of every $x \in A$ under the function gof is the $g-$ image of the $f-$ image of x.

Note that gof is defined only if $\forall x \in A, f(x)$ is an element of the domain of g so that we can take its $g-$ image. Hence, for the product gof of two functions f and g, the range of f must be a subset of the domain of g.

Properties of Composite Functions:

(i) The composite of functions is not commutative i.e., $gof \neq fog$.

(ii) $(f.g)(x) = f(x).g(x)$ (ii) $(f.g)(x) = f(x).g(x)$

(ii) The composite of functions is associative i.e., if f, g, h are three functions such that $fo(goh)$ and $(fog)oh$ are defined, then $fo(goh) = (for)oh$.

(iii) The composite of two bijections is a bijeciton i.e., if f and g are two bijections such that gof is defined, then gof is also a bijection.

8. **Homogeneous Functions** A function is said to be homogeneous with respect of any set of variables when each of its terms is of the same degree with respect to those variables. For example $5x^2 + 3y^2 - xy$ is homogeneous in x and y. Symbolically if, $f(tx, ty) = t^n.f(x, y)$, then $f(x, y)$ is homogeneous function of degree n.

9. **Bounded function** A function is said to be bounded if $|f(x)| \leq M$, where M is a finite quantity.

10. **Implicit and Explicit Function** A function defined by an equation not solved for the dependent variable is called an Implicit Function. For eg, the equation $x^3 + y^3 = 1$ defines y as an implicit function. If y has been expressed in terms of x alone then it called an Explicit Function.

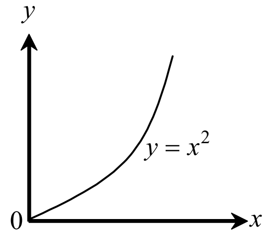
11. **Inverse of a Function** Let $f : A \to B$ be a one – one and onto function, then their exists a unique function $g : B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x, \forall x \in A$ and $y \in B$. Then g is said to be inverse of f.

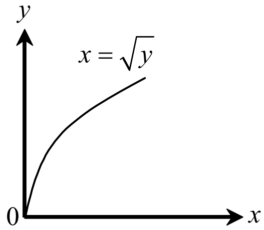
Thus $g = f^{-1} : B \to A = \{f(x), x)(x, f(x)) \in f\}$.

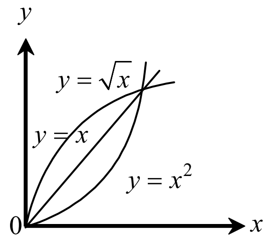
Properties of Inverse Function:

(i) The inverse of a bijection is unique.

(ii) If $f : A \to B$ is a bijection and $g : B \to A$ is the inverse of f, then $fog = I_B$ and $gof = I_A$, where I_A and I_B are identity functions on the sets A and B respectively. Note that the graphs of f and g are the mirror images of each other in the line $y = x$. As shown in the figure given below a point (x', y') corresponding to $y = x^2 (x \geq 0)$ changes to (y', x') corresponding to $y = +\sqrt{x}$, the changed form of $x = \sqrt{y}$.







(iii) The inverse of a bijection is also a bijection.

(iv) If f and g are two bijections $f : A \to B, g : B \to C$ then the inverse of gof exists and $(fog)^{-1} = f^{-1}og^{-1}$.

12. **Odd and Even Functions** If $f(-x) = f(x)$ for all x in the domain of $'f'$ then f is said to be an even function. eg, $f(x) = \cos x; g(x) = x^2 + 3.$ If $f(-x) = -f(x)$ for all x in the domain of $'f'$ then f is said to be an odd function. eg: f(x) = sinx,g(x)=x^3+x.

Note: (i) $f(x) - f(-x) = 0 \Rightarrow f(x)$ is even and $f(x) + f(-x) = 0 \Rightarrow f(x)$ is odd.

(ii) A function may neither be odd nor even.

(iii) Inverse of an even function is not defined.

(iv) Every even function is symmetric about the $y-$ axis and every odd function is symmetric about the origin.

(v) Every function can be expressed as the sum of an even and an odd function.

eg, $$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{\text{even}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{\text{odd}}$$

(vi) The only function which is defined on the entire number line and is even and odd at the same time is $f(x) = 0.$

(vii) If f and g both are even or both are odd then the function $f \cdot g,$ will be even but if any one of them is odd then $f \cdot g$ will be even but if any one of them is odd then $f \cdot g$ will be odd.

13. **Periodic function** A function $f(x)$ is called periodic if there exists a positive number $T(T > 0)$ called the period of the function such that $f(x+T) = f(x),$ for all values of x within the domain of $x.$

eg, The function $\sin x$ and $\cos x$ both are periodic over 2π and $\tan x$ is periodic over $\pi.$

Note (i) $f(T) = f(0) = f(-T),$ where $'T'$ is the period.

(ii) Inverse of a periodic function does not exist.

(iii) Every constant function is always periodic, with no fundamental period.

(iv) If $f(x)$ has a period T and $g(x)$ also has a period T then it does not mean that $f(x) + g(x)$ must have a period $T.$

eg, $f(x) = |\sin x| + |\cos x|.$

(v) If $f(x)$ has a period $p,$ then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has a period $p.$

(vi) If $f(x)$ has a period $T,$ then $f(ax+b)$ has a period $T/a(a>0).$

14. **General** If x, y are independent variables, then

(i) $f(xy)=f(x)+f(y) \Rightarrow f(x)=k\ln x$ or $f(x)=0$

(ii) $f(xy)=f(x)\cdot f(y) \Rightarrow f(x)=x^n, n \in R$

(iii) $f(x+y)=f(x)\cdot f(y) \Rightarrow f(x)=a^{kx}$

(iv) $f(x+y)=f(x)+f(y) \Rightarrow f(x)=kx,$ where k is a constant.

EXERCISE - I

WORK SHEET – I

1. **If $f(x)=\alpha x+\beta$ and f ={(1, 1), (2, 3), (3, 5), (4, 7)}, then the values of α, β are**

1) 2, -1 2) -2, 1 3) 3, -1 4) -2, -1

2. **If** $f(x)=\begin{cases} 3^{-x}-1, & \text{for } -1\le x<0 \\ \tan(x/2), & \text{for } 0\le x\le \pi \\ \dfrac{x}{x^2-2}, & \text{for } \pi\le x\le 6 \end{cases}$ **, then** $\sqrt{f(0)+f(\pi/6)+\frac{1}{5}f(5)-\frac{1}{23}}=$

1) 0 2) $\frac{27}{23}-\sqrt{3}$ 3) $\frac{27}{23}+\sqrt{3}$ 4) $\frac{\sqrt{3}-1}{\sqrt{2}}$

3. **If** $f(x)=\begin{cases} 2x-1, \text{ if } x>1 \\ x^2+1, \text{ if } -1\le x\le 1 \end{cases}$ **and if** $\frac{f(1)+f(3)+f(x)}{f(2)+f(-1)+f(1/2)}=\frac{32}{25}$**, then x =**

1) 1 2) 0 3) 4 4) –2

4. **If f and g are real functions defined by $f(x)=2x-1$ and $g(x)=x^2$, then**

1) $(3f-2g)(1)=1$ 2) $(fg)(2)=10$ 3) $g^3(2)=128$ 4) $\left(\frac{\sqrt{f}}{g}\right)(2)=\frac{\sqrt{3}}{2}$

5. **$f:N\to R$ such that $f(x)=\frac{2x-1}{2}$ and $g:Q\to R$ such that $g(x)=x+2$ be two functions. Then $(gof)\left(\frac{3}{2}\right)$ is equal to**

1) 3 2) $\frac{7}{2}$ 3) 1 4) not defined

6. **If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=2x+3$ and $g(x):x^2+7$, then the values of x for which $f(g(x))$ =25 are**

1) ± 1 2) ± 2 3) ± 3 4) ± 4

7. **If $g(x)=\sqrt[3]{x^2+11}$, $f(x)=\sqrt{x^3-2}$, then $fog(-4)$=**

1) 3 2) 4 3) 5 4) 6

8. **If $y=f(x)=\frac{5x+3}{4x-5}$ then $f(y)$ =**

1) $-x$ 2) x 3) $\frac{5x+3}{4x-5}$ 4) 2x

9. **If $f:[-6, 6]\to R$ is defined by $f(x)=x^2-3, \forall x\in R$, then $(fofof)(-1)+(fofof)(0)+(fofof)(1)$ =**

1) $f(4\sqrt{2})$ 2) $f(3\sqrt{2})$ 3) $f(2\sqrt{2})$ 4) $f(\sqrt{2})$

10. **If $f(x)=|x|, x \in R$, then**
 1) $f(x)=2f(x)$ 2) $f(x)=x$ 3) $f(x)=(f \times f)(x)$ 4) $f(x)=(fof)(x)$

11. **Which of the following is an odd function**
 1) $f(x)=\cos x$ 2) $f(x)=2-x^2$ 3) $f(x)=2^{x-x^4}$ 4) $f(x)=x^3-x$

12. **log (x – 3) is**
 1) is an even function 2) an odd function 3) neither even nor odd 4) cannot be determined

13. **$f(x)$ is an odd polynomial function. Then $\cos[f(x)]$ is**
 1) an even function 2) an odd function 3) neither even nor odd 4) periodic function

14. **$f(x)$ is an odd polynomial function. Then $f(f(x))$ is**
 1) an even function 2) an odd function 3) neither even nor odd 4) periodic function

15. **If f is an even function and g is an odd function, then $f\,g$ is .... function**
 1) even 2) odd 3) neither even nor odd 4) either even or odd

16. **If $f: R \to R$ defined by $f(x)=x^2+1$, then the set of all pre-images of $17=f^{-1}(17)$, the set of all pre-images of $-3=f^{-1}(-3)$ are respectively**
 1) ϕ, {4, –4} 2) $\{3,-3\}$,ϕ 3) $\{4,-4\}$,ϕ 4) $\{4,-4\},\{2,-2\}$

17. **If $f: R^+ \to R$ such that $f(x)=\log_2 x$, then $f^{-1}(x)=$**
 1) $\log_x 2$ 2) 2^x 3) 2^{-x} 4) doesn't exist

18. **Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$. Then inverse of f is**
 1) $g(y)=\frac{y-3}{4}$ 2) $g(y)=\frac{3y+4}{3}$ 3) $g(y)=4+\frac{y+3}{4}$ 4) $g(y)=\frac{y+3}{4}$

19. **If $f: R \to R$, $g: R \to R$ functions defined by $f(x)=3x-2$, $g(x)=x^2+1$, then $(gof^{-1})(2)=$**
 1) $\frac{25}{9}$ 2) $\frac{25}{3}$ 3) $\frac{16}{9}$ 4) $\frac{4}{3}$

20. **If the functions of f and g are defined by $f(x)=3-x$, $g(x)=2+3x$ for $x \in R$ respectively, then $g^{-1}(f^{-1}(5))=$**
 1) 1 2) $\frac{1}{3}$ 3) $-\frac{4}{3}$ 4) $\frac{1}{4}$

21. **If $f(x)=\frac{3x+2}{5x-3}$, then**
 1) $f^{-1}(x)=f(x)$ 2) $f^{-1}(x)=-f(x)$ 3) $f^{-1}(x)=-\frac{1}{19}f(x)$ 4) $(fof)(x)=-x$

22. **The domain of $\frac{10^x+10^{-x}}{10^x-10^{-x}}$ is**
 1) R 2) R - {0} 3) R - {1} 4) R^+

23. **The domain of sec $5x$ is**
 1) $R-\{n\pi : n \in N\}$ 2) $R-\left\{(2n+1)\frac{\pi}{2}; n \in Z\right\}$ 3) $R-\left\{\frac{n\pi}{5}; n \in Z\right\}$ 4) $R-\left\{(2n+1)\frac{\pi}{10}; n \in Z\right\}$

24. Domain of $\sqrt[3]{x}\,.\cot x$ is

1) R - {0} 2) R^+ 3) $R - \{x/x = n\pi, n \in Z\}$ 4) R

25. The domain of $\dfrac{x+1}{\sqrt{x^2-5x+6}}$ is

1) R – {2, 3} 2) $(3, \infty)$ 3) $(-\infty, \infty)$ 4) $(-\infty, 2) \cup (3, \infty)$

26. The domain of the function $f(x) = \log_3(18x - x^2 - 77)$ is

1) (7, 11) 2) (7, 10) 3) (8, 11) 4) (8, 10)

27. The domain of $\log_x 5$ is

1) $(0,\infty)$ 2) $R^+ - \{1\}$ 3) $R - \{1\}$ 4) $(0,5)$

28. If $f(x) = \left|\dfrac{x}{2}\right|$ is a real function, then $f: R \to B$ is a surjection then B equals

1) R 2) R^+ 3) R^- 4) $R^+ \cup \{0\}$

29. $A = \{x/x \in R, x \neq 0, -4 \leq x \leq 4\}$ and $f: A \to R$ is defined by $f(x) = \dfrac{|x|}{x}$ for $x \in A$. Then the range of f is

1) {1, –1} 2) $\{x : 0 \leq x \leq 4\}$ 3) {1} 4) $\{x : -4 \leq x \leq 0\}$

30. Domain, Range of $f(x) = |x| + |x+1|$ are respectively

1) R – {1}, $[0,\infty)$ 2) R – {–1}, $(0,\infty)$ 3) R, $(1,\infty)$ 4) R, $[1,\infty)$

31. f and h are from A into B where $A=\{a,b,c,d\}$, $B=\{s,t,u\}$ defined as $f(a)=t$, $f(b)=s$, $f(c)=s$, $f(d)=u$; $h(a)=s$, $h(b)=t$, $h(c)=s$, $h(a)=u$, $h(d)=u$. Which one of the following statement is true

1) f and h are functions
2) f is a function and h is not a function
3) f and h are not functions
4) f is not a function and h is a function

32. $A = \{1, 2, 5, 6, 11\}$, $B = \{2, -1, 1, 0, 11, 108\}$ and $f(x) = x^2 - x - 2$, then $f: A \to B$ is

1) function 2) one one 3) onto 4) not a function

33. The function $f: N \to N$ defined by $f(n)=2n+3$ is

1) surjective
2) injective
3) bijective
4) neither one-one nor onto

34. $f: N \to A$, where N is the set of natural numbers and A is the set of even natural numbers, defined by $f(x) = 2x$, then f is

1) injective only
2) surjective only
3) a bijection
4) neither one-one nor onto

35. Let Z denote the set of all integers and $f: Z \to Z$ defined by $f(x) = \begin{cases} x/2 & (x \text{ is even}) \\ 0 & (x \text{ is odd}) \end{cases}$. Then f is

1) onto but not one-one
2) one-one but onto
3) one-one and onto
4) neither one-one nor onto

36. If $f: [0,\infty) \to R$ defined by $f(x) = x^2$, then f is

1) a function 2) one one 3) onto 4) one one onto

37. Let X and Y be subsets of R, the set of all real numbers. The function $f : X \to Y$ defined by $f(x) = x^2$ for $x \in X$ is one-one but not onto if

1) $X=Y=R^+$ 2) $X=R$, $Y=R^+$ 3) $X=R^+$, $Y=R$ 4) $X=Y=R$

38. If $f : A \to A$ is defined by $f(x) = x^3$ where $A = \{x / -1 \le x \le 1\}$, then f is

1) only one-one 2) only onto 3) bijection 4) not a function

39. The function $\cos(3x - 4)$ defined from R to R is

1) injective 2) surjective
3) one-one onto 4) neither injective nor surjective

40. A is a finite set and B is a singleton set. Then $f : A \to B$ is

1) one one 2) onto 3) a bijection 4) an into function

41. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is

1) n^2 2) n^n 3) $2n$ 4) $n!$

42. The number of one one functions that can be defined from {1, 2, 3, 4, 5} into{a, b, c, d} is

1) 120 2) 24 3) 20 4) 0

WORK SHEET – II

1. If A is a set and $n(A) = 4$, then the number of equivalence relations on A is

(a) 16 (b) 15 (c) 32 (d) 31

2. If A is a finite set and $n(A) = 6$, then the number of relations on A which are symmetric but not reflexive is

(a) 2^{15} (b) 15×2^{15} (c) 31×2^{15} (d) 63×2^{15}

3. Let A be the set of all students in a class. Define the relation $R = \{(a,b) \in A \times A / a \text{ is a brother of b}\}$. Then R is _______ relation on A.

(a) reflexive only (b) transitive only
(c) symmetric and transitive (d) reflexive and symmetric

4. If A, B are two sets, $n(A) = 4$ and $n(B) = 6$ then the number of relations from A to B having domain A is

(a) 63 (b) 31 (c) 127 (d) 130

5. If a set A has n elements, then the number of relations defined on A is

(a) 2^{n^2} (b) $2^{n^2} - 1$ (c) 2^n (d) 2^{2n}

6. Let N denote the set of all natural number and R be the relation on $N \times N$ defined by $(a,b) R (c,d)$ if $ad(b+c) = bc(a+d)$. Then R is

(a) symmetric only (b) reflexive only (c) transitive only (d) equivalence relation

7. Let R be a relation defined by $R = \{(a,b) / a > b, (a,b) \in R\}$ then R is

(a) reflexive only (b) symmetric only
(c) Antisymmetric only (d) Antisymmetric and transitive.

8. Let R be a relation on the set N defined by $\{(x,y)/x,y\in N \text{ and } 2x+y=41\}$. Then R is
(a) reflexive only (b) Symmetric (c) transitive (d) not equivalence

9. For real numbers x and y we write $xRy \Leftrightarrow x-y+\sqrt{2}$ is an irrational number then the relation R is
(a) reflexive only (b) symmetric only (c) transitive only (d) equivalence only

10. If $A=\{1,2\}, B=\{2,3\}$ and $C=\{4\}$, then $A\times B\times C$ is
(a) $\{(1,2,4),(2,2,4),(1,3,4),(2,3,4)\}$ (b) $\{(1,2,4),(1,4,3),(2,3,4)\}$
(c) $\{(1,3,4),(2,3,4),(2,1,3),(2,2,4)\}$ (d) $\{(1,2,3),(2,3,4),(1,3,2)\}$

11. If $A=\{1,2,4\}, B=\{2,4,5\}, C=\{2,5\}$ then $(A-B)\times(B-C)$ is
(a) $\{(1,2),(1,5),(2,5)\}$ (b) $\{(1,4)\}$ (c) $(1,4)$ (d) $(2,5)$

12. Let A be non – void set of the children in a family. The relation x is a brother of y, then A is
(a) Reflexive (b) Symmetric (c) transitive (d) reflexive and symmetric

13. Let $R=\{(1,3),(2,2),(3,2)\}$ and $S=\{(2,1),(3,2),(2,3)\}$ be two relations on set $A=\{1,2,3\}$. Then $RoS=$
(a) $\{(1,3),(2,2),(3,2),(2,1),(2,3)\}$ (b) $\{(3,2),(1,3)\}$
(c) $\{(2,3),(3,2),(2,2)\}$ (d) $\{(2,3),(3,2)\}$

14. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $aRb \Leftrightarrow a\perp rb$ $a,b\in L$, then R is
(a) Reflexive (b) symmetric (c) transitive (d) reflexive and transitive

15. $'n|M'$ means that n is a factor of m, then the relation '|' is
(a) reflexive and symmetric (b) transitive and symmetric
(c) Reflexive, transitive and symmetric (d) reflexive, transitive, and not symmetric

16. Assume R and S are (non empty) relation in a set A. Which of the relation given below is false?
(a) If R, S are transitive, then $R\cap S$ is transitive (b) If R,S are transitive then $R\cup S$ is transitive
(c) If R,S are symmetric, then is symmetric (d) If R,S are reflexive, then is reflexive

17. Let $A=\{1,2,3\} B=\{a,b,c\}$ then $n(A\times B)=$ ________
(a) 6 (b) 9 (c) 30 (d) 64

18. $A=\{a,b,c,d\}, B=\{a,b,c\}$ then the number of relations from A to B is
(a) 32 (b) 6 (c) 64 (d) 72

19. Let A and B be two sets such that $A\times B=\{(a,1),(b,3),(a,3),(b,1),(a,2),(b,2)\}$. Then
(a) $A=\{1,2,3\}$ and $B=\{a,b\}$ (b) $A=\{a,b\}$ and $B=\{1,2,3\}$
(c) $A=\{1,2,3\}$ and $B=\{a,6\}$ (d) $A=\{a,6\}$ and $\{1,2,3\}$

20. Let A and B be two non – empty sets having n elements in common. Then, the number of elements common to $A\times B$ and $B\times A$ is
(a) $2n$ (b) n (c) n^2 (d) n^3

21. Let A be the set of first ten natural numbers and R be a relation on A defined by $(x, y) \in R \Leftrightarrow x+2y=10.$ then the domain of R is

(a) $\{1,2,3,.......10\}$ (b) $\{2,4,6,8\}$ (c) $\{1,2,3,4\}$ (d) $\{2,4,6,8,10\}$

22. If R is a relation on Z defined by $xRy \Leftrightarrow x$ divides y. Then R is

(a) reflexive and symmetric (b) reflexive and transitive

(c) symmetric and transitive (d) equivalence

23. If R be a relation defined on the set of real numbers by $aRb \Leftrightarrow 1+ab>0$. Then R is

(a) reflexive and symmetric (b) transitive

(c) symmetric only (d) equivalence

24. Let $A=\{1,2,3,4,5\}$ and a relation on it be $R=\{(x,y)/x,y\in A\}$ and $x+y=5$ then R is _______

(a) not reflexive, not symmetric but transitive (b) not reflexive, not transitive but symmetric

(c) not reflexive, not symmetric but tranitive (d) equivalence.

25. A relation R is defined from $\{2,3,4,5\}$ to $\{3,6,7,10\}$ by $xRy \Leftrightarrow$ is relatively prime to y. Then domain R is

(a) $\{2,3,5\}$ (b) $\{3,5\}$ (c) $\{2,3,4\}$ (d) $\{2,3,4,5\}$

26. Let w denote the set of words in the English dictionary. Define the relation R by $R=\{(x,y)\in w\times w/$ the words x and y have atleast one letter in common$\}$. Then R is

(a) reflexive, symmetric and not transitive (b) reflexive, symmetric and transitive

(c) reflexive, not symmetric and transitive (d) not reflexive, symmetric.

27. Let L denote the set of all straight lines in a plane. Let a relation R be defined on L by $xRy \Leftrightarrow x$ is parallel to y for $x, y \in L$. Then R is

(a) only symmetric (b) only transitive (c) anti symmetric (d) an equivalence relation

28. If $A=\{a,b,c,d\}$, then the relation $R=\{(a,b),(b,a)(a,a)\}$ on A is

(a) Symmetric and transitive (b) reflexive and transitive

(b) Symmetric only (d) transitive only

29. If R is a relation from the set {11, 12, 13} to the set {8, 10, 12} defined by R={(x,y)/y=x-3}. Then R^{-1} is

(a) $\{(8,11),(10,13)\}$ (b) $\{11,8\},(13,10)$ (c) $\{10,13\},(12,13)$ (d) $\{(11,8),(10,13),(12,15)\}$

30. Which of the following is not an equivalence relation on the set of integers?

(a) aRb if $a+b$ is an even integer (b) aRb if $a-b$ is an even integer

(c) aRb if $a<b$ (d) aRb if $a=b$(a) Reflexive

31. If A is a non – empty set then the relation $\subseteq$ (is a subset of) on the power set A is

(a) only reflexive relation (b) only symmetric relation

(c) an equivalence relation (d) not symmetric relation

32. If $n(A)=3$ and $n(B)=4,$ then the number of relations from A and B is

(a) 8 (b) 256 (c) 128 (d) 4096

33. If $n(A)=4,$ then the number of relations that can be defined on A is

(a) 2^4 (b) 2^8 (c) 2^{16} (d) 2^{32}

34. If $n(A)=3,$ then the number of reflexive relation that can be defined on A is

(a) 2^3 (b) 2^6 (c) 2^9 (d) 2^{27}

35. If $n(A)=4,$ then the number of symmetric relations that can be defined on A is

(a) 2^{10} (b) 2^4 (c) 2^8 (d) 2^{16}

36. If $n(A)=5,$ then the number of relations that are both reflexive and symmetric is

(a) 2^{10} (b) 2^{15} (c) 2^6 (d) 2^{20}

37. If $n(A)=4,$ then the number of relation on A that are not reflexive is

(a) 2^{16} (b) 2^{12} (c) 15×2^{12} (d) 17×2^{12}

38. If $n(A)=5,$ then the number of relation on A that are not symmetric is

(a) 2^{25} (b) 2^{15} (c) $2^{15}(255)$ (d) $2^{15}(1023)$

39. If A is a set, then any subset of $A\times A\times A$ is called a ternary relation on A. If $n(A)=4,$ then the number of ternary relations on A is

(a) 2^4 (b) 2^{16} (c) 2^{64} (d) 2^{256}

40. If $A=\{a,b\},B=\{c,d\},C=\{d,e\}$ then $\{(a,c),(a,d),(a,e),(b,c),(b,d),(b,e)\}$ is equal to

(a) $A\cap(B\cup C)$ (b) $A\cup(B\cap C)$ (c) $A\times(B\cup C)$ (d) $A\times(B\cap C)$

41. If R be a relation on a set A such that $R=R^{-1}$, then R is

(a) Reflexive (b) symmetric (c) Transitive (d) Cannot be defined

42. Let R be a reflexive relation on a set A and I be the identiry relation on A. then

(a) $R\subset I$ (b) $I\subset R$ (c) R=I (d) None

43. Let R and S be two equivalence relation a set A. then

(a) $R\cup S$ is *an* equivalence relation on A (b) $R\cap S$ is equivalence relation on A
(c) is equivalence relation on A (d) none of the above

44. Let R and S be two non-void relations on a set A which of the following statements is false?

(a) R and S are transitive $\Rightarrow R\cup S$ is transitive
(b) R and S are transitive is transitive
(c) R and S are Symmetric is Symmetric (d) R and S are reflexive is reflexive

45. If R is a relation $<$ from $A=\{1,2,3,4\}$ to $B=\{1,3,5\}$ i.e., $(a,b)\in R\Leftrightarrow a<b,$ then RoR^{-1} is

(a) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$ (b) $\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$

(c) $\{(3,3),(3,5),(5,3),(5,5)\}$ (d) $\{(3,3),(3,4),(4,5)\}$

46. Let R be a relation on the set N of natural numbers defined by $nRm\Leftrightarrow n$ is a factor of m. Then R is

(a) reflexive and Symmetric (b) transitive and Symmetric

(c) equivalence (d) reflexive, transitive, but not symmetric

47. For real numbers X and Y we write ${}_xR_y \Leftrightarrow x-y+\sqrt{2}$ is an irrational number. Then the relation R is

(a) reflexive (b) symmetric (c) transitive (d) None of these

48. If $R=\{(x,y)/x,y\in z \text{ and } x^2+y^2\le 4\}$ is a relation on z then domain of R is

(a) $\{0,1,2\}$ (b) $\{0,-1,-2\}$ (c) $\{-2,-1,0,1,2\}$ (d) ϕ

49. The minimum number of elements that must be added to the relation $R=\{(1,2),(2,3)\}$ on the set $\{1,2,3\}$ so that it is an equivalence relation is

(a) 4 (b) 7 (c) 6 (d) 5

50. Let R^+ be a relation defined in the set of real numbers by $aRb \Leftrightarrow 1+ab>0,$ then R is

(a) an equivalence relation (b) transitive relation

(c) Symmetric relation (d) anti – symmetric relation.

WORK SHEET - III

1. If $f(x)=\log x, g(x)=x^3,$ then $f(g(a))+f(g(b))$ =

1) $f(g(a)+g(ab))$ 2) $f(g(ab))$ 3) $g(f(ab))$ 4) $g(f(a)+f(b))$

2. If $f(x)=px+q$ and $g(x)=rx+s$, then,

1) $f(p)=g(q)$ 2) $f(q)=g(q)$ 3) $f(s)=g(q)$ 4) $f(r)=g(p)$

3. If $f(x)=\sin^2 x+\sin^2(x+\pi/3)+\cos x.\cos\left(x+\frac{\pi}{3}\right)$ and $g(5/4)=1$, then $(gof)(x)$ =

1) 1 2) 0 3) $\sin x$ 4) $\cos x$

4. If $f(x)=\frac{x}{\sqrt{1-x^2}}$, then $(fofof)(x)=$

1) $\frac{3x}{\sqrt{1-3x^2}}$ 2) $\frac{2x}{\sqrt{1-4x^2}}$ 3) $\frac{x}{\sqrt{1-3x^2}}$ 4) $\frac{4x}{\sqrt{1-4x^2}}$

5. Let $f(x)=\frac{x+1}{x-1}$, $fof(x)=f^2(x)$; $fofof(x)=f^3(x)$, $fofofof(x)=f^4(x)$..... then $f^{2008}_{(x)}$ =

1) x 2) $f(x)$ 3) 0 4) $\frac{1}{x}$

6. If $f(x)=\frac{\alpha x}{x+1}, x\ne -1$, then the value of α for which $f(f(x))=x$ is

1) $\sqrt{2}$ 2) $-\sqrt{2}$ 3) 1 4) –1

7. If $f(x)=(10-x^7)^{1/7}$, then $fof(x)$ =

1) 10 2) x 3) x^7 4) 10^7

8. If $f(x)$ is defined on [0, 1] as $f(x)=\begin{cases} x, & \text{if } x\in Q \\ 1-x, & \text{if } x\notin Q\end{cases}$ where $(fof)(x)=$

1) 1 2) x 3) $1-x$ 4) $1+x$

9. If $f(n)=(-1)^{n-1}(n-1), G(n)=n-f(n)$ for every $n\in N$ then (GOG) (n) =

1) n 2) n - 1 3) 1 4) 2

10. If $f(1)=1, f(n+1)=2f(n)+1,\ n\geq 1,$ then $f(n)$ is

1) 2^{n+1} 2) 2^{n} 3) $2^{n}-1$ 4) $2^{n-1}-1$

11. If $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x,$ then $f(x)=$

1) $1+2x^2$ 2) $2+x^2$ 3) $1+x$ 4) $2+x$

12. If $f:R\to R$ is defined by $f(x)=x^2-3x+2$, and $f(x^2-3x-2)=ax^4+bx^3+cx^2+dx+e$ then $a+b+c+d+e$ =

1) 1 2) 2 3) 30 4) 20

13. If $f:R\to R$ is defined by $f(x-1)=x^2+3x+2,$ then $f(x-2)=$

1) x^2+x 2) x^2-3x+2 3) x^2+2x 4) x^2-x

14. The function $y=f(x)$ satisfying the condition $f(x+1/x)=x^3+1/x^3$ is

1) $f(x)=x^2$ 2) $f(x)=x^2-2$ 3) $f(x)=x^2+2$ 4) $f(x)=x^3-3x$

15. If $f:N\to Z$ is defined by $f(n)=\begin{cases} 2, & \text{if } n=3k, & k\in Z \\ 10-n, & \text{if } n=3k+1, & k\in Z \\ 0, & \text{if } n=3k+2, & k\in Z \end{cases}$ then $\{n\in N: f(n)>2\}$ =

1) {3, 6, 4} 2) {1, 4, 7} 3) {4, 7} 4) {7}

16. If $f(x)=\cos(\log x),$ then $f(x^2).f(y^2)-\frac{1}{2}\left[f(x^2y^2)+f\left(\frac{x^2}{y^2}\right)\right]=$

1) -2 2) -1 3) $\frac{1}{2}$ 4) 0

17. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=k\,f\left(\frac{200x}{100+x^2}\right)$, then k =

1) 0.5 2) 0.6 3) 0.7 4) 0.8

18. $f(x)=\log\left(\frac{1+x}{1-x}\right)$ satisfies the equation $f(x_1)+f(x_2)=$

1) $f\left(\frac{x_1+x_2}{1+x_1x_2}\right)$ 2) $f\left(\frac{x_1+x_2}{1-x_1x_2}\right)$ 3) $f\left(\frac{x_1-x_2}{1+x_1x_2}\right)$ 4) $f(x_1 x_2)$

19. If $f(x)=|x-2|$ and $g(x)=f(f(x))$, then for $x>20$, $g(x)$ =

1) $-x$ 2) x 3) $x-4$ 4) $4-x$

20. If $f(x)=-|x|$, then $(fofof)(x)+(fofof)(-x)$ =

1) $-2f$ 2) $2|f|$ 3) $2f$ 4) $-2|f|$

21. If $f(x)=x$ and $g(x)=|x|$, then $f(x)+g(x)$ is equal to

1) 0 2) $2x$ 3) $2x$ if $x \geq 0$ 4) $2x$ if $x \geq 0$

22. If $f(x)=-x^2$ for $x<0$, $f(0)=0$, $f(x)=x^2$ for $x>0$, then on R, $f(x)$ is

1) $|x^2|$ 2) $-|x^2|$ 3) $-x|x|$ 4) $x|x|$

23. Suppose $f:[-2,2]\to R$ is defined by $f(x)=\begin{cases}-1, for -2\leq x\leq 0\\ x-1, for\, 0\leq x\leq 2,\end{cases}$ then $\{x\in[-2,2]: x\leq 0$ and $f(|x|)=x\}$ =

1) $\{-1\}$ 2) $\{0\}$ 3) $\left\{\frac{-1}{2}\right\}$ 4) ϕ

24. If for $x\in[0,\infty)$, $g[f(x)]=|\sin x|$ $f[g(x)]=(\sin\sqrt{x})^2$, then

1) $f(x)=\sin^2 x, g(x)=\sqrt{x}$ 2) $f(x)=\sin x, g(x)=|x|$
3) $f(x)=\sin x^2, g(x)=\sin\sqrt{x}$ 4) f, g cannot be determined

25. If $f(x)=\cos[e^2]x+\cos[-e^2]x$ where $[x]$ stands for greatest integer function, then

1) $f(\pi)=1$ 2) $f(2\pi)=1$ 3) $f(\pi/2)=1$ 4) $f(\pi/4)=1$

26. If $f: R\to R$ is defined by $f(x)=x-[x]-\frac{1}{2}$ for $x\in R$ where $[x]$ is the greatest integer not exeeding x, then $\left\{x\in R: f(x)=\frac{1}{2}\right\}$ =

1) Z, the set of all integers 2) N, the set of all natural numbers
3) $\varnothing$, the empty set 4) R

27. Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & if\ x<0\\ 0, & if\ x=0\\ 1, & if\ x>0\end{cases}$ then $\frac{f(g(2009))}{g(f(2009))}$ =

1) x 2) 1 3) $f(x)$ 4) $g(x)$

28. If $f: R\to R$ and $g: R\to R$ are defined by $f(x)=x-[x]$ and $g(x)=[x]\ \forall x\in R$, $f(g(x))$=

1) x 2) 0 3) $f(x)$ 4) $g(x)$

29. If $f(x)=[x], g(x)=x-[x]$ then which of the following functions is the zero function

1) $(f+g)(x)$ 2) $(fg)(x)$ 3) $(f-g)(x)$ 4) $(fog)(x)$

30. If $f: R\to R$ and $g: R\to R$ are given by $f(x)=|x|$ and $g(x)=[x]$ for each $x\in R$, then $\{x\in R: g(f(x))\leq f(g(x))\}$=

1) $z\cup(-\infty,0)$ 2) $(-\infty,0)$ 3) z 4) R

31. $f(x)=\begin{cases}[x] & if\ -3<x\leq -1\\ |x| & if\ -1<x<1\\ |[-x]| & if\ 1\leq x\leq 3\end{cases}$, then $\{x: f(x)\geq 0\}$ =

1) $(-1,3)$ 2) $[-1,3)$ 3) $(-1,3]$ 4) $[-1,3]$

32. If $f(x)$ is a polynomial in $x(>0)$ satisfying the equation $f(x)+f(1/x)=f(x).f(1/x)$ and $f(2)=9$, then $f(3)$ =

1) 26 2) 27 3) 28 4) 29

33. If $f(x)$ is a polynomial function such that $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ and $f(3)=-80$ then $f(x)-f\left(\frac{1}{x}\right)=$

1) $x^4+\frac{1}{x^4}$ 2) $x^4-\frac{1}{x^4}$ 3) $x^4-\frac{1}{x^4}$ 4) $-\frac{1}{x^4}-x^4$

34. If $f(x)$ is a function such that $f(x+y)=f(x)+f(y)$ and $f(1)=7$, then $\sum_{r=1}^{n} f(r)=$

1) $\frac{7n}{2}$ 2) $\frac{7(n+1)}{2}$ 3) $7n(n+1)$ 4) $\frac{7n(n+1)}{2}$

35. If $f(x)$ is a function such that $f(xy)=f(x)+f(y)$ and $f(2)=1$, then $f(x)$

1) x^2 2) 2^x 3) $\log_2 x$ 4) $\log_x 2$

36. $f:R\to R$ is given by $f(x)=\frac{a^x}{a^x+\sqrt{a}}$ $\forall$ $x\in R$, then $f\left(\frac{1}{1997}\right)+f\left(\frac{2}{1997}\right)+.....+f\left(\frac{1995}{1997}\right)+f\left(\frac{1996}{1997}\right)=$

1) 997 2) 998 3) 1997 4) 1998

37. A real valued function $f(x)$ satisfies the functional equation $f(x-y)=f(x)f(y)-f(a-x)f(a+y)$ where a is a given constant and $f(0)=1$, $f(2a-x)$ is equal to:

1) $f(-x)$ 2) $f(a)+f(a-x)$ 3) $f(x)$ 4) $-f(x)$

38. If $f(x)=\frac{(x-a)(x-b)}{x}$ and $\frac{f(x)}{(x-y)(x-z)}+\frac{f(y)}{(y-z)(y-x)}+\frac{f(z)}{(z-x)(z-y)}=\frac{K}{xyz}$, then $K=$

1) a 2) b 3) ab 4) $3ab$

39. If $2f(x)-3f\left(\frac{1}{x}\right)=x^2$, $x\neq 0$, then $f(2)=$

1) $-\frac{7}{4}$ 2) $\frac{5}{2}$ 3) -1 4) 2

40. If $f(x+y,x-y)=xy$, then the arithmetic mean of $f(x, y)$ and $f(y, -x)$ is

1) 0 2) x 3) y 4) 1

41. A : $f(x)=\log x^3$ and $g(x)=3\log x$ are equal functions

R : Two functions f and g are said to be equal if their domains are equal and $f(x)=g(x)\ \forall\ x$.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true but R is flase
4) A is false but R is true

42. Let $f(x)=px^2+qx^4+r$. Then for f to be an even function

1) p, q, r can be any real numbers
2) $p, q\in R$ and $r\in R^+$
3) $p, q\in R^+$ and $r\in R$
4) $p, q, r\in R^+$

43. If $f(x)=ax^5+bx^3+cx+d$ is an odd function, then d =

1) 0 2) 1 3) -1 4) any real number

44. $f(x)$ is an even polynomial function. Then sin $(f(x)-3x)$ is

1) an even function 2) an odd function 3) neither even nor odd 4) periodic function

45. **If $f(x)$ and $g(x)$ are be two functions with all real numbers as their domains, then $h(x)=\left[f(x)+f(-x)\right]\left[g(x)-g(-x)\right]$ is**

1) always an odd function 2) an odd function with both f and g are odd
3) an odd function when f is even and g is odd 4) always an even function

46. **If $f(x) + g(x) = e^{-x}$ where $f(x)$ is an even function and $g(x)$ is an odd function then $f(x)$=**

1) $\frac{e^{-x}}{2}$ 2) $\frac{e^{x}+e^{-x}}{2}$ 3) $\frac{e^{x}-e^{-x}}{2}$ 4) $\frac{e^{x}}{2}$

47. **A : The function $f(x) = \cos x$ is symmetric about the line $x = 0$**
R : Every even function is symmetric about y-axis

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true but R is flase 4) A is false but R is true

48. **A function whose graph is symmetrical about the y-axis is given by**

1) $f(x)=\cos[\log(x+\sqrt{x^2+1})]$ 2) $f(x)=\frac{\sec^4 x+\operatorname{cosec}^4 x}{x^3+x^4\cot x}$
3) $f(x+y)=f(x)+f(y)\ \forall\ x,y\in R$ 4) $f(x)=\frac{\sec^4 x-\operatorname{cosec}^4 x}{x^3-x^4\cot x}$

49. **The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then**

1) $f(x+2)=f(x-2)$ 2) $f(2+x)=f(2-x)$ 3) $f(x) = f(-x)$ 4) $f(x) = -f(-x)$

50. **If $f: (1, 2, 3, ...)\to\{0, \pm 1, \pm 2, ...\}$ is defined by $f(n) = \begin{cases}\frac{n}{2} \text{ if } n \text{ is even} \\ -\left(\frac{n-1}{2}\right) \text{ if } n \text{ is odd}\end{cases}$, then $f^{-1}(-100)$ is**

1) 100 2) 199 3) 201 4) 200

51. **If $f(x)=\left(4-(x-7)^3\right)^{\frac{1}{5}}$, then $f^{-1}(x)=$**

1) $(4+(7-x)^3)^{\frac{1}{5}}$ 2) $\sqrt[3]{4-x^5}+7$ 3) $\sqrt{4-x^{\frac{1}{5}}}-7$ 4) $\sqrt[3]{4-x}+7$

52. **If $f:[1,\infty)\to[2,\infty)$ is given by $f(x)=x+\frac{1}{x}$, then $f^{-1}(x)=$**

1) $\frac{x+\sqrt{x^2-4}}{2}$ 2) $\frac{x}{1+x^2}$ 3) $\frac{x-\sqrt{x^2-4}}{2}$ 4) $\frac{x-\sqrt{x^2-4}}{2}$

53. **If $f:[1\ ,\ \infty)\to[1,\ \infty)$ is defined by $f(x)=2^{x(x-1)}$ then $f^{-1}(x)=$**

1) $\frac{x(x-1)}{2}$ 2) $\frac{(1+\sqrt{1+4\log_2 x})}{2}$ 3) $\frac{(1-\sqrt{1+4\log_2 x})}{2}$ 4) $2x(x+1)$

54. **If $f:R\to R$ is defined by $f(x) = x - [x]$, then the inverse function $f^{-1}(x)$=**

1) $\frac{1}{x-[x]}$ 2) $[x]-x$ 3) $x+[x]$ 4) not defined

55. **Suppose $f(x)=(x+1)^2$ for $x\geq -1$. If $g(x)$ is the function whose grpah is the reflection of the graph of $f(x)$ w.r.t the line $y = x$, then $g(x)$ is equal to**

1) $-\sqrt{x}-1, x\geq 0$ 2) $\frac{1}{(x+1)^2}, x>-1$ 3) $\sqrt{x+1}, x\geq -1$ 4) $\sqrt{x}-1, x\geq 0$

56. **Let $f:[-1,\infty)\to R$ be given by $f(x)=(x+1)^2-1$, $x > -1$, then $f^{-1}(x)$**

1) $-1+\sqrt{x+1}$ 2) $-1-\sqrt{x+1}$
3) doesnot exists because f is not 1–1 4) doesnot exists because f is not onto

57. $\left\{x\in R:\frac{2x-1}{x^3+4x^2+3x}\in R\right\}=$

1) R – {0} 2) R – {0,1,3} 3) $R-\left\{0,-1,-3,\frac{1}{2}\right\}$ 4) R – {0,–1,–3}

58. **The domain of $\frac{1}{\sqrt{x-x^2}}+\sqrt{3x-1-2x^2}$ is**

1) $\left[\frac{1}{2},1\right]$ 2) $\left[\frac{1}{2},1\right)$ 3) $\left(\frac{1}{2},1\right]$ 4) $\left(\frac{1}{2},1\right)$

59. **Domain of the real valued function $\sqrt{25-9x^2}+\sqrt{x^2+x+1}$ is**

1) $\left(-\frac{5}{3},\frac{5}{3}\right)$ 2) $\left[-\frac{5}{3},\frac{5}{3}\right]$ 3) $(-\infty,-5/3)\cup[5/3,\infty]$ 4) $\left(-\infty,-\frac{5}{3}\right)\cup\left(\frac{5}{3},\infty\right)$

60. **The domain of $f(x)=\sqrt{\frac{x-3}{x+3}}+\sqrt{\frac{2-x}{2+x}}$ is**

1) $(-\infty,2)\cup(3,\infty)$ 2) (2, 3) 3) R 4) ϕ

61. **The domain of $f(x)=\frac{1}{\sqrt{(x-1)(x-2)(x-3)}}$ is**

1) $(-\infty,1)\cup(3,\infty)$ 2) $(1, 2)\cup(3, \infty)$ 3) $(-\infty,2)$ 4) R

62. **The domain of $\sqrt{\frac{2x+1}{2x^3+3x^2+x}}\sqrt{2}$ is**

1) $(-\infty,-1)$ 2) $(0,\infty)$ 3) $(-\infty,-1)\cup(0,\infty)$ 4) R

63. **The domain of the function $f(x)=\sqrt[3]{\frac{2x-1}{x^2-10x-11}}$ is**

1) $(-\infty,0)$ 2) $(0,\infty)$
3) $(-\infty,-1)\cup(-1,11)\cup(11,\infty)$ 4) $(-\infty,\infty)$

64. A is the domain of $f(x)=\dfrac{1}{\sqrt{|x|-x}}$ and B is the domain of $g(x)=\sqrt{1-|x|}$ then $A\cap B=$

1) (-1, 0) 2) [-1, 0) 3) (-1, 1) 4) [-1, 1]

65. The domain of $f(x)=\sqrt{x^2-4|x|+3}$ is

1) $(-\infty,3]$ 2) $[1,\infty)$ 3) $(-\infty,-3]\cup[-1,\infty)$ 4) $(-\infty,-3]\cup[-1,1]\cup[3,\infty)$

66. The domain of the function $\sqrt{\dfrac{1}{|\cos x|}}$ is

1) $R-\{\pi/2\}$ 2) $R-\{\pi/2\ ,\ 3\pi/2\}$ 3) $R-\{x/x=2n\pi+\pi/2\ ,\ n\in I\}$ 4) $R-\{x=n\pi+\pi/2,\ n\in I\}$

67. Domain of $\sqrt{x^2-[x]^2}$ is

1) $(-\infty,\ 0]$ 2) $[0,\ \infty)$ 3) $R^+\cup Z$ 4) $R-\{0\}$

68. The domain of $x^{\left(\frac{1}{\log x}\right)}$ is

1) $(0,\ \infty)$ 2) $(1,\ \infty)$ 3) $(0,\ 1)\cup(1,\ \infty)$ 4) $[1,\ \infty)$

69. The domain of $f(x)=\log\left(\dfrac{x-5}{x^2-10x+24}\right)-\sqrt[3]{x+5}$ is

1) (4, 5) 2) $(6,\ \infty)$ 3) $(4,5)\cup(6,\infty)$ 4) $(4,5]\cup(6,\infty)$

70. Domain of $1/\log|x|$ is

1) R - {0, 1, -1} 2) R - {0} 3) R - {-1, 1} 4) R

71. The domain of $\log\left(\dfrac{\sqrt{4-x^2}}{1+x}\right)$ is

1) $(-2,2)$ 2) $(-1,\infty)$ 3) $[-1,2]$ 4) $(-1,2)$

72. The domain of $\sqrt{\log_a x}\,(a>1)$ is

1) $(0,1)$ 2) [0, 1] 3) $[1,\ \infty)$ 4) $(1,\ \infty)$

73. The domain of the function $\sqrt{\log_{10}\left(\dfrac{5x-x^2}{4}\right)}$ is

1) $[1,\ 4]$ 2) $[-1,\ 4]$ 3) $[0,\ 5]$ 4) $[-1,\ 5]$

74. The domain of the function $f(x)=\log\left(\sqrt{\log_{0.2} x}\right)$ is

1) (0, 1) 2) (0, 1] 3) $[1,\ \infty)$ 4) $(1,\ \infty)$

75. The domain of the function $\sqrt{\log \frac{1}{|\sin x|}}$ is

1) R - {0}
2) $R-\{0,\pi\}$
3) $R-\{x : x = n\pi / n \in I\}$
4) $R-\{x : x = 2n\pi / n \in I\}$

76. The domain of the function log $(\sin^2 x)$ is

1) $[0,\ 2\pi]$
2) $[-\pi,\ \pi]$
3) R
4) $R-\{n\pi : n \in z\}$

77. The domain of $f(x) = \log_{10}\left(\sqrt{x-4}+\sqrt{6-x}\right)$ is

1) [4 , 6]
2) $(-\infty,\ 6)$
3) $(2,3)$
4) $(-\infty,\ 4)$

78. Domain of $\log_e|\log_e x|$ is

1) $(1,\ \infty)$
2) $(e,\ \infty)\cup(-\infty,\ 0)$
3) $(0,\ \infty)$
4) $(0,\ 1)\cup(1,\ \infty)$

79. The domain of function $\log_{10}\log_{10}\log_{10}\log_{10}\log_{10}^{x}$ is

1) $(10^4,\ \infty)$
2) $(10^{10},\ \infty)$
3) $(10^{10^{10}},\ \infty)$
4) $(10^{100},\ \infty)$

80. The functions $f(x)=\log(x-1)-\log(x-2)$ and $g(x) = \log\left(\frac{x-1}{x-2}\right)$ are identical on

1) [1, 2]
2) $[2,\ \infty)$
3) $(2,\ \infty)$
4) R

81. The domain of the function $f(x) = [x]\sin\frac{\pi}{[x+1]}$, where [ ] denote greatest integer function is

1) $R-\{-1\}$
2) $(-\infty,\ -1)\cup[0,\ \infty)$
3) $(-\infty,\ -1]\cup[0,\ \infty)$
4) $(-\infty,\ -1)\cup(0,\ \infty)$

82. If A is the set of real values of x such that $e^{(1/x)-1} < 1$, then A =

1) $(-\infty,\ 0)\cup(1,\ \infty)$
2) $(-\infty,\ 0)$
3) $(1,\ \infty)$
4) (0, 1)

83. If $e^x + e^{f(x)} = e$, then domain of $f(x)$ is

1) $(-\infty,\ 0]$
2) [0, 1]
3) $(-\infty,\ 1)$
4) $(1,\ \infty)$

84. If $f : R^+ \to B$ such that $f(x) = x^2 - 4x + 5$ is a bijection, B =

1) R
2) $[0,\ \infty)$
3) $[1,\ \infty)$
4) $[5,\ \infty)$

85. If $f(x) = x^2 - 4x + 5$ then the range of f is

1) $[4,\ \infty)$
2) $(4,\ \infty)$
3) $(5,\ \infty)$
4) R

86. The range of $x^2 + 4y^2 + 9z^2 - 6yz - 3xz - 2xy$ is

1) ϕ
2) R
3) $[0,\ \infty)$
4) $(-\infty,\ 0)$

87. If $a^2 + b^2 + c^2 = 2$ then the range of $ab+bc+ca$ is

1) $[-1/2, 1]$
2) $[-1/2,\ \infty)$
3) [-1, 2]
4) $[1,\ \infty)$

88. The range of $f(x)=\dfrac{x^2}{1+x^2}$ is

1) [0, 1) 2) (0, 1) 3) $(0, \infty)$ 4) $(0,2)$

89. The range of the function $f(x)=\dfrac{x^2}{x^4+1}$ is

1) $\left(0,\frac{1}{2}\right)$ 2) $\left[0,\frac{1}{2}\right]$ 3) $[0, \infty)$ 4) $[0,2]$

90. The range of the function $f(x)=\dfrac{1-\tan x}{1+\tan x}$ is

1) R - {0} 2) R 3) R - {-1} 4) R - {1}

91. The range of $f(x)=\dfrac{\sin\pi[x^2-1]}{x^4+1}$ is

1) R 2) [-1, 1] 3) {0, 1} 4) {0}

92. If $f : R\rightarrow R$ and $g : R\rightarrow R$ are defined by $f(x)=|x|$ and $g(x)=[x-3]\ \forall\ x\in R$, then $\left\{g(f(x): -\frac{8}{5}<x<\frac{8}{5}\right\}$ =

1) {0, 1} 2) {1, 2} 3) {-3, -2} 4) {2, 3}

93. Range of [sin$x$] is
1) [-1, 1] 2) {-1, 1} 3) {-1, 0, 1} 4) (0, 1)

94. The range of $f(x)$ = [tan x] is
1) R 2) Z 3) {1, 3} 4) N

95. The range of the function $f(x)=\cos[x]$ where $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$ is

1) {-1, 1, 0} 2) {cos 1, cos 2} 3) {cos 1, cos 2, 1} 4) {0, 1}

96. The range of [x] – x is A and x-[x] is B then $A\cap B$ =
1) {0} 2) (-1, 1) 3) (0, 1) 4) ϕ

97. If $f: R\rightarrow R$ is defined by $f(x)$ = [2x]–2[x] for $x\in R$, where [x] is the greatest integer not exceeding x, then the range of f is
1) $\{x\in R : 0\le x\le 1\}$ 2) {0, 1} 3) $\{x\in R : x\ge 0\}$ 4) $\{x\in R : x\le 0\}$

98. The range of sin $\log\left[\dfrac{\sqrt{4-x^2}}{(1-x)}\right]$ is

1) [–1, 1] 2) (–2, 1) 3) (–2, –1) 4) [0 , 1]

99. Let $f(x)=\sin x$ and $g(x)=\log|x|$. If the ranges of composite functions fog and gof are R_1 and R_2 repsectively, then

1) $R_1=\{u:-1\le u\le 1\},\ R_2=\{\upsilon:-\infty<\upsilon<0\}$
2) $R_1=\{u:-\infty<u\le 0\},\ R_2=\{\upsilon:-1\le\upsilon\le 1\}$
3) $R_1=\{u:-1<u<1\},\ R_2=\{\upsilon:-\infty<\upsilon<0\}$
4) $R_1=\{u:-1\le u\le 1\},\ R_2=\{\upsilon:-\infty<\upsilon\le 0\}$

100. **If f from [–1, 1] into [–1, 1] defined by $f(x) = 3x - 5$ then f is**

1) not a function 2) a function 3) one one 4) onto

101. **If $f : D \to R$ be the function with domain $D = \left\{x : -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$ and $f(x) = 3+4x$, R being the set of all real, then which one of the following statement is correct ?**

1) f is not one-one but onto on R
2) f is one-one but not onto on R
3) f is one-one as well as onto on R
4) f is neither one-one nor onto on R

102. **If $f : R \to R$ defined by $f(x) = \begin{cases} 2x+5, if\ x > 0 \\ 3x-2, if\ x \le 0 \end{cases}$ then f is**

1) a function 2) one one 3) onto 4) one one onto

103. **A function 'f' from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, \text{ when n is odd} \\ -\frac{n}{2}, \text{ when n is even} \end{cases}$**

1) one-one but not onto
2) onto but not one-one
3) one-one and onto both
4) Neither one-one nor onto

104. **If $f : R \to R$ is defined by $f(x) = x + \sqrt{x^2}$, then f is**

1) an injection 2) onto 3) a bijection 4) Only function

105. **$A = \{x : -1 \le x \le 1\}$. $f : A \to A$ defined by $f(x) = x|x|$. Then f is**

1) a bijection
2) an injection but not surjection
3) a surjection but not an injection
4) neither an injection nor a surjection

106. **If $f(x) = |x-1| + |x-2| + |x-3|$, f:[2,3] $\to R$ is**

1) one-one onto function
2) an onto function only
3) an identity function
4) an into function only

107. **$y = f(x) = \frac{x}{1+|x|}, x \in R, y \in R$ is**

1) one-one and onto
2) onto but not one-one
3) one-one but not onto
4) neither one-one

108. **$f : R \to R$ is a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then f is**

1) one-one and onto
2) one-one but not onto
3) onto but not one-one
4) neither one-one nor onto

109. **If $f : R \to R$ defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, then f is**

1) one-one but not onto
2) not one-one but onto
3) one-one and onto
4) neither one-one nor onto

110. If $f:[0,1]\to[-1,3]$ defined by $f(x)=x^2+x+1$, then f is

1) a function 2) one one 3) onto 4) one one onto

111. If $f:R\to R$ defined by $f(x)=x^2-2x-3$, then f is

1) a function 2) one one 3) onto 4) one one onto

112. If $f:R\to(0,1]$ defined by $f(x)=\dfrac{1}{x^2+1}$, then f is

1) not one-one 2) not onto
3) not one-one but onto 4) one-one but not onto

113. If $f:R\to R$ is defined by $f(x)=\dfrac{x^2-4}{x^2+1}$, then $f(x)$ is

1) one-one and not onto 2) one-one and onto
3) not one-one but onto 4) neither one-one nor onto

114. Which of the following functions is not injective ?

1) $f(x)=|x+1|,\ x\in[-1,\ \infty)$ 2) $g(x)=x+\dfrac{1}{x},\ \ x\in(0,\infty)$
3) $h(x)=x^2+4x-5,\ x\in(0,\ \infty)$ 4) $k(x)=e^{-x},\ x\in[0,\ \infty)$

115. The function $f:R\to R$ defined by $f(x)=x-[x],\ \forall x\in R$ is

1) one-one 2) onto
3) Both one-one and onto 4) neither one-one nor onto

116. $f:R^+\to R$ defined by $f(x)=2^x, x\in(0,1),\ f(x)=3^x, x\in[1,\infty)$ is

1) onto 2) one-one
3) neither one-one nor onto 4) one one onto

117. $f:R^+\to R$ defined by $f(x)=\log_e x, x\in(0,1),\ \ f(x)=2\log_e x, x\in[1,\infty)$ is

1) onto 2) one-one 3) not one-one 4) a bijection

118. Statement I : $f:A\to B$ is one - one and $g:B\to C$ is a one-one function, then $gof:A\to C$ is one - one
Statement II : If $f:A\to B$, $g:B\to A$ are two functions such that $gof=I_A$ and $fog=I_B$, then $f=g^{-1}$.
Statement III : $f(x)=\sec^2 x-\tan^2 x$, $g(x)=\operatorname{cosec}^2 x-\cot^2 x$, then $f=g$
Which of the above statement/s is/are true.

1) only III 2) both I & III 3) both I & II 4) I, II, III

119. If $f(x)=\dfrac{e^x+e^{-x}}{2}$, then the inverse function of $f(x)$ is

1) $\log_e(x+\sqrt{x^2+1})$ 2) $\log_e\sqrt{x^2+1}$ 3) $\log_e(x+\sqrt{x^2-1})$ 4) $\log_e(x-\sqrt{x^2-1})$

120. If $f(x)=\dfrac{10^x-10^{-x}}{10^x+10^{-x}}$ then $f^{-1}(x)=$

1) $\log_{10}(2-x)$ 2) $\dfrac{1}{2}\log_{10}\dfrac{1+x}{1-x}$ 3) $\dfrac{1}{2}\log_{10}(2x-1)$ 4) $\dfrac{1}{4}\log_{10}\dfrac{2x}{2-x}$

121. Let $f:\left[\frac{1}{2},1\right]\to[-1,1]$ is given by $f(x)=4x^3-3x$ then $f^{-1}(x)$ is given by

1) $\cos\left[\frac{1}{3}\cos^{-1}x\right]$ 2) $3\cos\left(\sin^{-1}x\right)$ 3) $3\sin^{-1}(\cos x)$ 4) $\sin\left(\frac{1}{3}\cos^{-1}x\right)$

122. The domain of the function $f(x)=\dfrac{\tan 2x}{6\cos x+2\sin 2x}$ is

1) $R-\left\{(2n+1)\frac{\pi}{2}:\ n\in Z\right\}$ 2) $R-\left\{(2n+1)\frac{\pi}{4}:\ n\in Z\right\}$

3) $R-\left\{(2n+1)\frac{\pi}{2}:\ n\in Z\right\}\cup\left\{(2n+1)\frac{\pi}{4}:\ n\in Z\right\}$ 4) R

123. The domain of $f(x)=\dfrac{1}{|\sin x|+\sin x}$ is

1) R 2) $\underset{n\in Z}{U}((2n+1)\pi,\ 2(n+1)\pi)$

3) $\underset{n\in Z}{U}(2n\pi,\ (2n+1)\pi)$ 4) ϕ

124. The domain of the function $f(x)=\sqrt{\operatorname{cosec} x-1}$ is

1) $((2n-1)\pi,\ 2n\pi)$ 2) $(2n\pi,\ (2n+1)\pi)$ 3) $\left(2n\pi,\ 2n\pi+\frac{\pi}{2}\right]$ 4) ϕ

125. If $f(x)=|\sin x|$ has an inverse if its domain is

1) $[0,\ \pi]$ 2) $\left[0,\frac{\pi}{2}\right]$ 3) $[-\pi/4\ ,\ \pi/4]$ 4) $[-\pi/2,\ \pi/2]$

126. Let $f(x)=\sin x+\cos x$, $g(x)=x^2-1$. Then $g(f(x))$ is invertible for $x\in$

1) $\left[-\frac{\pi}{2},0\right]$ 2) $\left[-\frac{\pi}{2},\pi\right]$ 3) $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ 4) $\left[0,\frac{\pi}{2}\right]$

127. Domain of $\sin^{-1}5x$ is

1) $\left(-\frac{1}{5},\frac{1}{5}\right)$ 2) $\left[-\frac{1}{5},\frac{1}{5}\right]$ 3) $[-1,1]$ 4) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

128. The domain of the function $f(x)=\dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

1) [2, 3] 2) [2, 3) 3) [1, 2] 4) [1, 2)

129. The domain of $\sqrt{1-3x}+\cos^{-1}\dfrac{3x-1}{2}$ is

1) $(-\infty,-1)$ 2) $(0,\infty)$ 3) $(-\infty,\ -1)\cup(0,\ \infty)$ 4) $\left[-\frac{1}{3},\frac{1}{3}\right]$

130. The domain of $\cos^{-1}\sqrt{3x}$ is :

1) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ 2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ 3) $\left(0, \frac{1}{3}\right)$ 4) $\left[0, \frac{1}{3}\right]$

131. The domain of the function $f(x) = \sqrt{\sin^{-1} x}$ is

1) [0, 1] 2) [-1, 1] 3) $(-\infty, \infty)$ 4) (0, 1)

132. Domain of $\sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is :

1) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ 2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ 3) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ 4) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

133. The domain of $\sin^{-1}\frac{3x-1}{2} + \sqrt{\cos(\sin x)}$ is

1) [-1, 1] 2) [-1/3, 0] 3) (0, 1] 4) [-1/3, 1]

134. The domain of the function $f(x) = \sqrt{\sin^{-1}(\log_2 x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right) + \sqrt{\cos(\sin x)}$ is

1) {1} 2) {-1, 1} 3) $\{x : 1 \le x \le 2\}$ 4) Not defined for any real x.

135.The domain of $f(x) = \frac{\operatorname{cosec}^{-1} x}{[x]}$ is

1) [-1, 1] 2) R - [-1, 1] 3) R - (-1, 1) 4) R – (0, 1)

136. The domain of the function $\sin^{-1}\left(\frac{x}{2} - 1\right) + \log\sqrt{x - [x]}$ is

1) (0, 4) 2) R - I 3) {1, 2, 3} 4) (0, 4) - {1, 2, 3}

137. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ is defined, is

1) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ 2) $\left[0, \frac{\pi}{2}\right)$ 3) $[0, \pi]$ 4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

138. The domain of $\log(\tan^{-1} x)$ is

1) R 2) R^+ 3) $[0, \infty)$ 4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

139. The domain of $f(x) = \log_2 \ \log_3 \ \log_{\frac{4}{\pi}} (\tan^{-1} x)^{-1}$ is

1) (-1, 1) 2) (0, 1) 3) $\left(\frac{4}{\pi}, \infty\right)$ 4) R

140. The domain of $\frac{\tanh^{-1}(2x-3)}{\sqrt{4-x^2}}$ is

1) (-2, 2) 2) (1, 2) 3) [1, 2] 4) [0, 2]

141. If $f: R \to S$, defined by $f(x)=\sin x-\sqrt{3}\cos x+1$, is onto, then the interval of S is

1) [0, 3] 2) [-1, 1] 3) [0, 1] 4) [-1, 3]

142. The range of the function $f(x)=[\sin x+\cos x]$ (where $[x]$ denotes the greatest integer function) is

1) [-2, 1] 2) {-2, -1, 0, 1} 3) {-1, 1} 4){-2, -1, 1}

143. Let $f:(-1,1)\to B$, be a function defined by $f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then f is both one-one and onto when B is the interval

1) $\left(0, \frac{\pi}{2}\right)$ 2) $\left[0, \frac{\pi}{2}\right)$ 3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

144. The range of the function $f(x)=\tan^{-1}[x], -\frac{\pi}{4}\le x\le\frac{\pi}{4}$ where [.] denotes the greatest integer function

1) $\left\{-\frac{\pi}{4}, 0\right\}$ 2) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ 3) $\left\{\pm\frac{\pi}{4}\right\}$ 4) $\{-1, 0\}$

WORK SHEET - IV

1. If $f(x)+2f(1-x)=x^2+2 \ \forall\ x\in R$, then $f(x)$ is given by

1) $\frac{(x-2)^2}{3}$ 2) x^2-2 3) 1 4) x^2+2

2. $f: N\to R$ is given by $f(1)=1$ and $f(1)+2f(2)+3f(3)+\dots\dots\dots+nf(n)=n(n+1)f(n)$, for $n\ge 2$, then $f(1994)$ =

1) $\frac{1}{1994}$ 2) 1994 3) $\frac{1}{3988}$ 4) 3998

3. A single formula that gives $f(x)$ for all $x>0$, where $f(x)=\begin{cases}2+x, & 0\le x<2\\ 3x-2, & x\ge 2\end{cases}$ is

1) $f(x)=|x-2|+2x$ 2) $f(x)=|2x-1|+x$ 3) $f(x)=|3x-1|-3$ 4) $f(x)=|3x-2|+1$

4. If $f(n+1)=\frac{2f(n)+1}{2}$, $n=1, 2\dots$ and $f(1)=2$, then $f(101)$ =

1) 52 2) 49 3) 48 4) 51

5. The value of natural number 'a' for which $\sum_{k=1}^{n} f(a+k)=16(2^n-1)$, where the function satisfies the relation $f(x+y)=f(x)\cdot f(y)$ for all natural numbers x, y and further $f(1)=2$ is

1) 3 2) 4 3) 2 4) 1

6. Let $f:[-100\pi, 1000\pi]\to[-1,1]$ be defined by $f(\theta)=\sin^2\theta$. Then the number of values of $\theta\in[-100\pi, 1000\pi]$ for which $f(\theta)=0$ is

1) 1100 2) 1110 3) 1000 4) 1101

7. For a real number x, $[x]$ denotes the integral part of x. The value of

$$\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{100}\right]+\left[\frac{1}{2}+\frac{2}{100}\right]+\left[\frac{1}{2}+\frac{3}{100}\right]+.......+\left[\frac{1}{2}+\frac{99}{100}\right]=$$

1) 49 2) 50 3) 48 4) 51

8. If f is an even function defined on the interval [–5, 5], then the real values of x satisfying the equation $f(x)=f\left(\frac{x+1}{x+2}\right)$ are

1) $\frac{-1\pm\sqrt{5}}{2},\frac{-3\pm\sqrt{5}}{2}$ 2) $\frac{-3\pm\sqrt{5}}{2},\frac{-4\pm\sqrt{5}}{2}$ 3) $\frac{-2\pm\sqrt{5}}{2},\frac{-4\pm\sqrt{5}}{2}$ 4) $\frac{-4\pm\sqrt{5}}{2},\frac{-1\pm\sqrt{5}}{2}$

9. Let f be an injective map with domain $\{x, y, z\}$ and range {1, 2, 4} such that exactly one of the following statements is correct and the remaining are false: $f(x)=1$, $f(y)\neq 1$, $f(z)\neq 2$. The value of $f^{-1}(1)$ is

1) y 2) x 3) z 4) 0

10. Let $f:R\to R$ be given by $f(x)=(x+1)^2-1, x\geq -1$. Then the set of values of x for which $f(x)=f^{-1}(x)$ is given by

1) {0} 2) {-1, 0} 3) {-1} 4) {0, 1}

11. The domain of $\sqrt{\frac{1-|x|}{2-|x|}}$ is

1) $[-1,1]\cup(-\infty,-2)\cup(2,\infty)$ 2) $(-\infty,\infty)-[-2,2]$

3) $(-\infty,\infty)-[-1,1]$ 4) R

12. The domain of the function $f(x)=\log_{10}\sin(x-3)+\sqrt{16-x^2}$ is

1) (3, 4] 2) (–4, 4) 3) $(3,\pi+3)$ 4) (1, –1)

13. The domain of the function $f(x)=\log_x \cos x$ is

1) $\left(0,\frac{\pi}{2}\right)-\{1\}$ 2) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{1\}$ 3) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ 4) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

14. If a function f satisfies the condition $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}(x\neq 0)$ then domain of $f(x)$ is :

1) (–2, 2) 2) $(-\infty, 2)$ 3) $(2, \infty)$ 4) R –{0}

15. If $f(x) = [x]$ where $[x]$ denotes the greatest integer not exceeding x and $g(x)=\cos(\pi x)$, then the range of the function gof is

1) {0} 2) {-1, 1} 3){-1, 0, 1} 4) $\{x:-1\leq x\leq 1\}$

16. If domain of $|x|+x-[x]$ is (0, 3), then its range is

1) [0, 3) 2) [0, 4) 3) [0, 3] 4) (0, 4)

17. If $f(x)=\frac{x^2+1}{[x]}$, ([.] denotes the greatest integer function), $1\le x<4$, then

1) range of f is $\left[2,\frac{17}{3}\right)$
2) f is monotonically increasing in [1, 4]
3) the maximum value of $f(x)$ is $\frac{17}{3}$
4) the maximum value of $f(x)$ is $\frac{17}{4}$

18. If $f:R\to R$ where $f(x)=ax+\cos x$, if f is bijective, then

1) $a\in R$
2) $a\in R^+$
3) $a\in R^-$
4) $a\in R-(-1,1)$

19. If $f:R-\{1,2\}\to R-\{1,4\}$ defined by $f(x)=\frac{x^2-4}{x^2-3x+2}$ is

1) one-one
2) onto
3) bijective
4) neither one-one nor onto

20. The function $f:R\to R$ defined by $f(x)=4^x+4^{|x|}$ is

1) one-one and into
2) many one and into
3) one-one and onto
4) many one and onto

21. The function $f:(-\infty,-1)\to(0,e^5]$ defined by $f(x)=e^{x^3-3x+2}$ is

1) Many one and onto
2) Many one and into
3) One - one and onto
4) One - one and into

WORK SHEET - V

1. The graph of the equation $y+|y|-x-|x|=0$ is represented by

1) the x-axis
2) the bisector line of the first quadrant
3) a pair of lines bisecting all the quadrants
4) all points of the fourth quadrant

2. If $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$, where [x] is the greatest integer function,then

1) $f\left(\frac{\pi}{2}\right)=-1$
2) $f(\pi)=1$
3) $f(-\pi)=-1$
4) $f\left(\frac{\pi}{4}\right)=2$

3. If x satisfies | x-1 | + | x-2 | + | x-3 | $\ge$6, then

1) $0\le x\le 4$
2) $x\le -2$ or $x\ge 4$
3) $x\le 0$ or $x\ge 4$
4) $x\in R$

4. If y=3[x]+1=2[x-3]+5, then

1) [x]= -1
2) y= 5
3) [x-y] = 2
4) [x+y] = -7

5. Let g(x) be a function defined on [-1, 1] so that the area of the equilateral triangle with two of its vertices at (0, 0) and (x, g(x)) is $\frac{\sqrt{3}}{4}$. The function g(x) is equal to

1) $\sqrt{1-x^2}$
2) $-\sqrt{1+x^2}$
3) $\frac{1}{2}\sqrt{1-x^2}$
4) $\frac{\sqrt{3}}{8}\sqrt{1-x^2}$

6. Let f(x) = Ax^2 + Bx + C, where A, B, C are real numbers. If f(x) is an integer whenever x is an integer, then

1) A is an integer
2) B is an integer
3) C is an non-integer
4) A + B is an integer

7. If $f(x)=\left(\dfrac{x-1}{x+1}\right)$, **then which of the following statement(s) is/are correct**

1) $f\left(\dfrac{1}{x}\right)=f(x)$ 2) $f\left(\dfrac{1}{x}\right)=-f(x)$ 3) $f\left(-\dfrac{1}{x}\right)=\dfrac{1}{f(x)}$ 4) $f\left(-\dfrac{1}{x}\right)=-\dfrac{2}{f(x)}$

8. **If f(x+2y, x-2y)=xy, then f(x,y) equals**

1) $\dfrac{x^2-y^2}{8}$ 2) $\dfrac{x^2-y^2}{4}$ 3) $\dfrac{x^2+y^2}{4}$ 4) $\dfrac{x^2-y^2}{2}$

9. **If** $f(x)=ax+b$ *and* $g(x)=cx+d$, **then** $f(g(x))=g(f(x))\Rightarrow$

1) $f(a)=g(c)$ 2) $f(b)=g(b)$ 3) $f(d)=g(b)$ 4) $ad-b=bc+d$

10. **Let f(x)** $=\dfrac{\alpha x}{x+1}, x\neq -1$ **. Then for what value of** α **is f(f(x)) =x**

1) $\sqrt{2}$ 2) $-\sqrt{2}$ 3) 1 4) –1

11. **Let f be the greatest integer function and g be the modulous functions, then**

1) $(gof-fog)\left(-\dfrac{5}{3}\right)=1$ 2) $(f+2g)(-1)=1$

3) $(gof-fog)\left(\dfrac{5}{3}\right)=0$ 4) $(f+2g)(1)=1$

12. **The function** $\dfrac{e^{2x}-1}{e^{2x}+1}$ **is**

1) symmetric about y axis 2) symmetric in opposite quadrants
3) odd 4) even

13. $f(x)=\cos^2 x+\cos^2\left(\dfrac{\pi}{3}+x\right)-\cos x\cos\left(\dfrac{\pi}{3}+x\right)$ **is**

1) an odd function 2) an even function 3) a periodic function 4) f(0) = f(1)

14. **Which of the following functions are even?**

1) $f(x)=x\left(\dfrac{a^x+1}{a^x-1}\right)$ 2) $g(x)=\ln(x+\sqrt{(x^2+a^2)})$

3) $h(x)=\sqrt[3]{(1-x)^2}+\sqrt[3]{(1+x)^2}$ 4) $p(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ 1, & \text{if } x \text{ is irrational}\end{cases}$

15. **f(x) =** $\dfrac{\cos x}{\left[\dfrac{x}{\pi}\right]+\dfrac{1}{2}}$ **, where x is not an integral multiple of** π **and [x] denote the greatest integer funtion,**

is

1) and odd function 2) an even function
3) neither odd nor even 4) symmetric in opposite quadrants

16. **Let f : [-10, 10] $\rightarrow$ R, where f(x)=sin x+[x^2/a] be an odd function. Then set of values of parameter 'a' is/are:**

1) (-10, 10)-{0} 2) (1000, ∞) 3) [100, ∞) 4) (100, ∞)

Passage - I :

For $x \neq 0,1$, *define*

$$f_1(x) = x,\ f_2(x) = 1/x,\ f_3(x) = 1-x,$$

$$f_4(x) == 1/(1-x),\ f_5(x) = (x-1)/x,\ f_6(x) = x/(x-1)$$

This family of functions is closed under composition that is, the composition of any two of these functions is again one of these.

17. **Let F be a function such that f_1 0 F = f_4. Then F is equal to**

1) f_1 2) f_2 3) f_3 4) f_4

18. **Let G be a function such that G 0 f_3 = f_6. Then G is equal to**

1) f_5 2) f_4 3) f_3 4) f_2

19. **Let H be a function such that f_4 0 M = f_5. Then H is equal to**

1) f_6 2) f_4 3) f_5 4) f_3

Passage - II :

The function f(x) = mx satisfies f(x + y) = f(x) + f(y) and f(x) = a^x satisfies f(x + y) = f(x) + f(y) and f(x) = a^x satisfies f(x + y) = f(x) f(y).

From the given functional relations, we can determine several things about the functions. At times the function can be determined uniquely from the functional equation.

20. **If f(x + y) = f(x) + f(y) for all x, y then f(4) is equal to**

1) f(1) 2) 4f(1) 3) 2f(1) 4) 0

21. **If f(x + y) = f(x) + f(y) for all x, y and f(1) = 1 then f(- 9/8) is euqal to**

1) 9/8 2) 8/9 3) - 9/8 4) 1

22. **If f(x) + f(y) =** $f\left(x\sqrt{1-x^2} + y\sqrt{1-x^2}\right)$ **then**

1) $f(4x^2+3x)+3f(x)=0$ 2) $f(3x-4x^3)+3f(x)=0$

3) $f(4x^3+3x)-3f(x)=0$ 4) $f(4x^3-3x)+3f(x)=0$

23. **Column - 1 gives the functions and Column - 2 gives the nature of function**

Column - I	**Column - II**
1) $\dfrac{x}{e^x+1}$	p) Even
2) $\dfrac{x}{2} - \dfrac{x}{e^x+1}$	q) Odd
3) $\dfrac{\sqrt{x^2+1}+x-1}{\sqrt{x^2+1}+x-1}$	r) Both even and odd

4) In $(x^4+x^2+1)-2\ln(x^2+x+1)$ s) Neither even nor odd

24. **Let** $f(x)=\dfrac{x^2-6x+5}{x^2-5x+6}$

Column - I	Column - II
1) If - 1 < x < 1, then f(x) satisfies	p) $0<f(x)<1$
2) If 1 < x < 2, then f(x) satisfies	q) $f(x)<0$
3) If 3 < x < 5, then f(x) satisfies	r) $f(x)>0$
4) If x > 5, then f(x) satifies	s) $f(x)<1$

25. **If f(x) is a polynomial of least degree such that f(r) = 1/r, r = 1, 2, 3,.....9, then 10 f(10)**

26. **If f(x) is a polynomial such that**

$f(x)f(y)=f(x)+f(y)+f(xy)-2\forall\ x,y\ and\ f(2)=5,\ then\ f(4)-10=$

27. **If** $f(x)=1+x^{1/3}\ and\ g(f(x))=3-x^{1/3}+x,\ then\ g(5)-60=$

28. **If** $2f(xy)=(f(x))^y+(f(y))^x\forall\ x,y\ and\ f(1)=2,$ **then** $\left[\sum_{n=1}^{9}\dfrac{f(n)}{2^{10}}\right]$

29. **The number of roots of the equation** $|x|+|x-1|+|x+1|=1$ **is**

30. **If f(x) is a polynomial such that** $f(x)f\left(\dfrac{1}{x}\right)=f(x)+f\left(\dfrac{1}{x}\right),\forall\ x\neq 0,\ and\ f(-2)=33,$ **then f(1) =**

WORK SHEET - VI

1. **If f(x) = sinx + cos ax is a periodic function then a cannot be**

1) π 2) 3 3) 2 4) $\dfrac{1}{2}$

2. **The possible values of 'a' for which the function** $f(x)=e^{x-[x]}+\cos ax$ (where[.] **denotes the greatest integer function) is periodic with finite fundamental period is**

1) 2 2) 2π 3) 3 4) 1

3. **Period of** $f(x)=\sum_{r=1}^{n}\left(S\text{in}\dfrac{x}{2^{r-1}}+Tan\dfrac{x}{2^r}\right)$ **is**

1) $2^n\pi$ 2) 2π 3) 4π 4) 2^{n+2}

4. **If f(x)+f(x+4)=f(x+2)+f(x+6)** $\forall x\in R$ **, and f(5)=10, then** $\sum_{r=1}^{100}f(5+8r)$ **equal to**

1) 1000 2) 100 3) 10000 4) none of these

5. **Cosider y=f(x), a real valued function with domain as all real numbers. It is given that graph of the function is symmetrical about the lines x = a and x = b, where a<b.**
1) f is periodic 2) f is non periodic 3) f is one-one 4) f is onto

6. **If domain of f is D_1 and domain of g is D_2, then domain of f + g is**
1) D_1/D_2 2) $(D_1 \cup D_2)$ 3) $(D_2 - D_1)$ 4) $D_1 \cap D_2$

7. **Let** $f(x)=\dfrac{5\sqrt{\sin x}}{1+\sqrt[3]{\sin x}}$. **If D is the domain of f, then D contains**
1) $(0,\pi)$ 2) $(-2\pi,\pi)$ 3) $(\pi,3\pi)$ 4) $(4\pi,6\pi)$

8. **If domain for y=f(x) is [-3, 2], then domain of g(x)=f{|[x]|}.**
1) (-2, 3) 2) [-2, 3) 3) [-2, 3] 4) (-2, 3]

9. **Let** $f: R \to R$ **be a function defined as f(x) = 4^x - 2^x + 1. Then**
1) f(x) is one-one 2) f(x) is bijective 3) f(x) > 2 for all x 4) range of f(x) is $\left[\frac{3}{4},\infty\right)$

10. **If S is the set of all real numbers x for which** $\dfrac{2x-1}{2x^3+3x^2+x}>0$, **and P is the subset of S, then P can be**
1) $\left(-\frac{3}{2},-\frac{1}{4}\right)$ 2) $\left(\frac{1}{2},0\right)$ 3) $\left(\frac{1}{2},3\right)$ 4) $(0,\infty)$

11. **Let f(x) = ln|x| and g(x) = sinx. If A is the range of f(g(x)) and B is the range of g(f(x)), then**
1) $A\cup B=(-\infty,1)$ 2) $A\cup B=(-\infty,\infty)$ 3) $A\cap B=[-1,0]$ 4) $A\cap B=[0,1]$

12. **Which of the following functions is not injective?**
1) $f(x)=|x+1|, x\in[-1,0]$ 2) $f(x)=x+1/x, x\in(0,\infty)$
3) $f(x)=x^2+4x-5$ 4) $f(x)=e^{-x}, x\in[0,\infty)$

13. **Which of the following functions are not identical?**
1) $f(x)=\dfrac{x}{x^2}$ and $g(x)=\dfrac{1}{x}$ 2) $f(x)=\dfrac{x^2}{x}$ and $g(x)=x$
3) $f(x)=\ln x^4$ and $g(x)=4\ln x$
4) $f(x)=\ln\{(x-1)(x-2)\}$ and $g(x)=\ln(x-2)+\ln(x-3)$

14. **If** $f: R\to R$ **defined as f(x) =** $\dfrac{\sin([x]\pi)}{x^2+x+1}$, **where** $[\cdot]$ **is the greatest integer less than or equal to x, then**
1) f is one–one 2) f is many –one 3) f is in to 4) f is on to

15. **The entire graphs of the equation** $y=x^2+kx-x+9$ **is strictly above the x-axis if and only if**
1) k<7 2) -5<k<7 3) K>-5 4) none of these.

16. **The value of the parameter** α, **for which the function f(x)=1+**$\alpha x, \alpha\neq 0$ **is the inverse of itself, is**
1) -2 2) -1 3) 1 4) 2

Passage - I :

We know that any real number x can be expressed as following $x=[\alpha]+\{x\}$, where [x] is an integer and $0 \le \{x\} < 1$. We define $[\alpha]$ as the greatest integer less than or equal to x or integral part of x and [x] as the fractional part of x. Suppose for any real number x, we write $x=(x)-(x)$, where (x) is integer and $0 \le (x) < 1$. We define (x) as the least integer greater than (or) equal to x. For example (3.26) = 4(-14, 4) = -14 (5) = 5 elearly, if $x \in I$ then (x) = [x]. If $x \notin I$, then (x) = $[\alpha]+1$. We can also define that $x \in (n, n+1) \Rightarrow (x) = n+1$, where $n \in I$

17. **The domain of defination of the function** $f(x)=\dfrac{1}{\sqrt{x-(x)}}$ **is**

1) I 2) R - I 3) $(0,\infty)$ 4) ϕ

18. **The range of the function** $f(x)=\dfrac{1}{\sqrt{(x)-[x]}}$ **is**

1) ϕ 2) {1} 3) $\left\{\dfrac{1}{\sqrt{n}}, n \in N\right\}$ 4) $(1,\infty)$

19. **The solution set of the equation** $(x)^2=[x]^2+2x$ **contains**

1) no integer 2) exactly one integer
3) exactly two integers 4) infinite integers

Passage - II :

Let $f(x)=\log_e\left(x+\sqrt{x^2+1}\right)$**, domain of f is where f(x) is defined for real values of x. If f is bijective then $f^{-1}(x)$ exists**

20. **f is defined on**

1) $(0, \infty)$ 2) $(-\infty, \infty)$ 3) [0, e] 4) $(-\infty, 0)$

21. **$f^{-1}(x)$ is defined on**

1) $(0, \infty)$ 2) $(-\infty, \infty)$ 3) [0, e] 4) $(-\infty, 0)$

22. **The inverse of f is positive on**

1) $(0, \infty)$ 2) $(-\infty, \infty)$ 3) [0, e] 4) $(-\infty, 0)$

23. Column - 1 gives functions and column 2 the nature of the functions

Column - I	**Column - II**
1) $f:[0,\infty)\to[0,\infty), f(x)=\dfrac{x}{1+x}$	p) one - one onto
2) $f:R-\{0\}\to R, f(x)=x-\dfrac{1}{x}$	q) one - one but not onto
3) $f:R-\{0\}\to R, f(x)=x+\dfrac{1}{x}$	r) onto but not. one - one
4) $f:R\to R, f(x)=2x+\sin x$	s) neither one - one nor onto

EXERCISE - I / ANSWERS

WORK SHEET -I

1) 1	2) 4	3) 2	4) 1	5) 4	6) 2	7) 3	8) 2	9) 1	10) 4
11) 4	12) 3	13) 1	14) 2	15) 2	16) 3	17) 2	18) 1	19) 1	20) 3
21) 1	22) 2	23) 4	24) 3	25) 4	26) 1	27) 2	28) 4	29) 1	30) 4
31) 2	32) 4	33) 2	34) 3	35) 1	36) 2	37) 3	38) 3	39) 4	40) 2
41) 2	42) 4								

WORK SHEET - II

1) 2	2) 4	3) 2	4) 1	5) 3	6) 4	7) 4	8) 4	9) 1	10) 1
11) 2	12) 3	13) 3	14) 2	15) 4	16) 2	17) 1	18) 3	19) 2	20) 3
21) 2	22) 2	23) 1	24) 2	25) 4	26) 1	27) 4	28) 3	29) 1	30) 3
31) 4	32) 4	33) 3	34) 2	35) 1	36) 1	37) 3	38) 4	39) 4	40) 3
41) 2	42) 2	43) 2	44) 1	45) 3	46) 4	47) 1	48) 3	49) 2	50) 1

WORK SHEET - II

1) 2	2) 3	3) 1	4) 3	5) 1	6) 4	7) 2	8) 2	9) 3	10) 3
11) 2	12) 3	13) 1	14) 4	15) 2	16) 4	17) 1	18) 1	19) 3	20) 3
21) 3	22) 4	23) 3	24) 1	25) 3	26) 3	27) 2	28) 2	29) 4	30) 4
31) 3	32) 3	33) 3	34) 4	35) 3	36) 2	37) 4	38) 3	39) 1	40) 1
41) 1	42) 1	43) 1	44) 3	45) 1	46) 2	47) 1	48) 1	49) 2	50) 3
51) 2	52) 1	53) 2	54) 4	55) 4	56) 4	57) 4	58) 2	59) 2	60) 4
61) 2	62) 3	63) 3	64) 2	65) 4	66) 4	67) 3	68) 3	69) 3	70) 1
71) 4	72) 3	73) 1	74) 1	75) 3	76) 4	77) 1	78) 4	79) 3	80) 3
81) 2	82) 1	83) 3	84) 3	85) 1	86) 3	87) 3	88) 1	89) 2	90) 3
91) 4	92) 3	93) 3	94) 2	95) 3	96) 1	97) 2	98) 1	99) 4	100) 1
101) 2	102) 2	103) 3	104) 4	105) 1	106) 3	107) 3	108) 4	109) 4	110) 2
111) 1	112) 3	113) 4	114) 2	115) 4	116) 2	117) 4	118) 3	119) 3	120) 2
121) 1	122) 3	123) 3	124) 2	125) 2	126) 3	127) 2	128) 2	129) 4	130) 4
131) 1	132) 1	133) 4	134) 1	135) 3	136) 4	137) 2	138) 2	139) 2	140) 2
141) 4	142) 2	143) 4	144) 1						

WORK SHEET - III

1) 1	2) 3	3) 1	4) 1	5) 1	6) 4	7) 2	8) 1	9) 1	10) 2
11) 1	12) 1	13) 1	14) 4	15) 2	16) 4	17) 1	18) 4	19) 3	20) 2
21) 4									

WORK SHEET - VI

1) 2	2) 1	3) 3	4) 4	5) 1	6) 4	7) 2	8) 1	9) 3	10) 4
11) 1,2,3	12) 2,3	13) 2,3,4	14) 1,2,4	15) 1,4	16) 2,4	17) 4	18) 1	19) 2	20) 2

21) 3 22) 4 23) 1 - s,2 - p,3 - q,4 - q. 24) 1 - p,r,s,2 - q,s,3 - q,s,4 - p,r,s 25) 2

26) 7 27) 3 28) 0 29) 0 30) 0

WORK SHEET - V

1) 1	2) 2	3) 1	4) 1	5) 1	6) 4	7) 1	8) 2	9) 4	10) 3
11) 1,3	12) 2,3	13) 2,3,4	14) 2,3	15) 2	16) 2	17) 4	18) 2	19) 2	20) 2

21) 2 22) 1 23) 1 - q,2 - r,3 - s,4 - p.

EXERCISE - II

WORK SHEET (HW) - I

1. **If** $y=|x-2|-|x+1|$, **then**

1) $for\ x<-2, y=3$
2) $for\ x>3, y=3$
3) $for\ 0\le x\le 1, y=-2x+1$
4) $for\ 1\le x\le 2,\ y=-2x+1$

2. **Number of solutions of equation |2x-1| = 3[x]+2{x} for x is.**

1) 2
2) 3
3) 0
4) 1

3. **Solution of** $x\in R$, $\left|x^2+6x+8\right|=\left|x^2+4x+5\right|+\left|2x+3\right|$ **is .**

1) $\left[-\frac{3}{2},\infty\right)$
2) $\left(-\frac{3}{2},\infty\right)$
3) $\left(-\infty,-\frac{3}{2}\right)$
4) $(-\infty,\infty)$

4. **If [x]= the greatest integer less than or equal to x, and (x)=the least integer greater than or equal to x, and** $[x]^2+(x)^2>25$, **then x belongs to**

1) $(-\infty,-4]\cup[4,\infty)$
2) $(-\infty,-1/4]\cup[2,\infty)$
3) $(-\infty,-4]\cup[3,\infty)$
4) $(-\infty,-5]\cup[3,\infty)$

5. **The number of solutions of |[x]-2x| =4, where [x] is the greatest integer** $\le$**x.**

1) 2
2) 3
3) 4
4) infinite

6. **If {x} and[x] represent fractional and integral part of x, then find the value of** $[x]+\sum_{r=1}^{2011}\frac{\{x+r\}}{2011}$.

1) 2000
2) x
3) {x}
4) 2x

7. **Let f(x) be a linear function which maps [-1, 1] onto [0, 2], then f(x) can be**

1) x + 1
2) - x - 1
3) - x + 1
4) x - 1

8. **If** $y=f(x)=\frac{x+2}{x-1}$, **then**

1) $x=f(y)$
2) $f(1)=3$
3) y increases with x for x < 1
4) f is rational function of x

9. **Let f be a function defined by** $f(x)=\frac{x-5}{x-3},\ x\ne 3,2$ **$f^k(x)$ denote the composition of f with it self taken k times i.e.** $f^3(x)=f\left(f(f(x))\right)$ **then**

1) $f^{2012}(2009)=2009$
2) $f^{2009}(2010)=\frac{2005}{2007}$
3) $f^{2009}(2009)=\frac{1002}{1003}$
4) $f^{2012}(2012)=2012$

10. **ABCD is a square of side a. A line parallel to the diagonal BD at a distance x from the vertex A cuts the two adjacent sides. The area of the segment to the square with A at the vertex, as a function of x**

1) $f(x)=x^2, o\le x\le \frac{a}{\sqrt{2}}$ 2) $f(x)=2\sqrt{2}\,ax-x^2-a^2, \frac{a}{\sqrt{2}}<x\le\sqrt{2}\,a$

3) $f(x)=2\sqrt{2}\,ax-x^2-a^2, 0\le x\le \frac{a}{\sqrt{2}}$ 4) $f(x)=x^2, \frac{a}{\sqrt{2}}<x\le\sqrt{2}\,a$

11. For all real values of u and v, $2f(u)\cos v=f(u+v)+f(u-v)$ then which of the following is true for all $x\in R$

1) $f(x)+f(-x)=$ 2 a cos x, a is constant 2) $f(\pi-x)+f(x)=0$

3) $f(\pi-x)+f(x)=2b\sin x, b$ is constant 4) $f(x)=a\cos x+b\sin x, a, b$ are constants

12. If $f(x+y+1)=\left(\sqrt{f(x)}+\sqrt{f(y)}\right)^2$ and f (0) = 1, $\forall\ x,y\in R$ then

1) $f\left(\frac{1}{2}\right)=\frac{9}{4}$ 2) $f(2)=9$ 3) $f(-1)=0$ 4) $f(2)=\frac{9}{4}$

13. Which of the following functions satisfy $f\left(\frac{x_1+x_2}{2}\right)\le\frac{f(x_1)+f(x_2)}{2}$

1) $Sinx, 0<x<\frac{\pi}{2}$ 2) $e^x, 0<x<\infty$ 3) $Tan\,x, 0<x<\frac{\pi}{2}$ 4) $\log x, 0<x<\infty$

14. Let $f:R\to R$ such that f(x f(x) + f(y)) = $(f(x))^2$ + y, for all $x,y\in R$ then

1) f (x) = x 2) f (x) = –x 3) f(x) = x^2+1 4) f (x) = x+1

15. If f(x) = $\frac{(x-3)(x+2)(x+5)}{(x+1)(x-7)}$.

1) f(x) > 0 $\forall$ x $\in$ (-5, -2) $\cup$ (-1, 3) $\cup$(7, ∞) 2) f(x) < 0 $\forall$ x $\in$ ($-\infty$, -5) $\cup$ (-2, -1) $\cup$ (3, 7).

3) f(x) > 0 $\forall$ x $\in$ (-5, -2) $\cup$ (-1, 3) $\cup$(9, ∞) 4) f(x) < 0 $\forall$ x $\in$ ($-\infty$, -5) $\cup$ (-2, -1) $\cup$ (3, 4).

16. If f(x) $=\begin{cases}x^2-4x+3, & x<3\\ x-4, & x\ge 3\end{cases}$, $g(x)=\begin{cases}x-3 & x<4\\ x^2+2x+2, & x\ge 4.\end{cases}$

1) $\frac{f(x)}{g(x)}=\begin{cases}x-1, & x<3\\ \frac{x-4}{x-3}, & 3<x<4.\\ \frac{x-4}{x^2+2x+2}, & x\ge 4\end{cases}$ 2) $\frac{f(x)}{g(x)}=\begin{cases}x+1, & x<3\\ \frac{x+4}{x+3}, & 3<x<4.\\ \frac{x-4}{x^2+2x+2}, & x\ge 4\end{cases}$

3) $\frac{f(x)}{g(x)}=\begin{cases}x-1, & x<3\\ \frac{x-4}{x+3}, & 3<x<4.\\ \frac{x+4}{x^2+2x+2}, & x\ge 4\end{cases}$ 4) $\frac{f(x)}{g(x)}=\begin{cases}x-1, & x<3\\ \frac{x-4}{x-3}, & 3<x<4.\\ \frac{x+4}{x^2+2x+2}, & x\ge 4\end{cases}$

17. **If** $f(x)=\begin{cases} x^2, & \text{for } x\ge 0 \\ x, & \text{for } x<0 \end{cases}$**, then fof(x) is given by**

1) x^2 for $x\ge 0$ 2) x^4 for $x\ge 0$ 3) x for $x<0$ 4) $-x^2$ for $x<0$

18. **Let** $f(x)=\begin{cases} 2+x & if \quad x\ge 0 \\ -x & if \quad x<0 \end{cases}$ **and g(x) = fof(x). Then**

1) g(x) is an even function
2) range of g(x) = $[2,\infty)$
3) for every $x\in R\, g(x)$ has exactly two values
4) $g(x)>f(x)\forall x\in R$

19. **Let** $f:R\to R$ **be given by f (x) =3 + 4x and** $g(x)=(f\ of\ of\ o----n\ times)$ **(x) then**

1) g (x) = $3+4^n x$
2) g(x) = $4^n (x+1)-1$
3) $g^{-1}(x)=\dfrac{x-3}{4^n}$
4) $g^{-1}(x)=4^{-n}(x+1)-1$

20. **If f(x) be defined on [-2,2] and is given by f(x)=**$\begin{cases} -1, & -2\le x\le 0 \\ x-1, & 0<x\le 2 \end{cases}$

1) $f(|x|)=\begin{cases} -1, & x=0 \\ -x-1, & -2\le x<0 \\ x-1 & 0\le x<1 \end{cases}$

2) $|f(x)|=\begin{cases} 1, & -2\le x\le 0 \\ -x+1, & 0<x<1 \\ x-1 & 1\le x\le 2 \end{cases}$

3) $f(|x|)=\begin{cases} 1, & x=0 \\ -x-1, & -2\le x<0 \\ x-1 & 0\le x<1 \end{cases}$

4) $|f(x)|=\begin{cases} -1, & -2\le x\le 0 \\ -x+1, & 0<x<1 \\ x-1 & 1\le x\le 2 \end{cases}$

21. **The function** $\log_e(x^3+\sqrt{x^6+1})$ **is of the following type(s)**

1) symmetric about y axis
2) symmetric in opposite quadrants
3) odd
4) even

22. **Let** $f(x)=\begin{cases} 0, & \text{for} \quad x=0 \\ x^2\sin\left(\dfrac{\pi}{x}\right), & \text{for} \quad -1<x<1, (x\ne 0) \\ x|x| & \text{for} \quad x\ge 1 \text{ or } x\le -1 \end{cases}$**, then**

1) f(x) is an odd function
2) f(x) is an even function
3) f(x) is neither odd nor even
4) f'(x) is an even function

23. $f(x)=\sin\alpha+\cos\alpha-1$**, where** $\alpha=\sin^{-1}\sqrt{\{x\}}$, $\{\cdot\}$ **is the fractional part of x, then f (x) is**

1) an even function 2) an odd functions 3) a periodic function 4) zero, $x\in z$

24. **f (x) is a real valued function satisfying f (x+y) + f (x-y)=2 f(x) f(y) for all $x, y \in R$ then**

1) f (x) is an even function
2) f (x) is even if f (0) = 1
3) f (x) is odd if f (0) = 0
4) f (x) is even if f (0) = 0

25. **If f (x) is an even function and g (x) is an odd function and satisfies the relation** $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$ **then**

1) $f(x) = 0, \forall x \in R$
2) f (2009) =0
3) f is constant function
4) g (x) = 0, $\forall x \in R$

26. **If f(x)=** $\begin{cases} x^3 + x^2 & \text{for } 0 \le x \le 2 \\ x+2 & \text{for } 2 < x \le 4 \end{cases}$**. then odd extension of f(x) is**

1) $\begin{cases} -x+2, & \text{for } -4 \le x < -2 \\ -x^3 + x^2 & \text{for } -2 \le x \le 0 \end{cases}$
2) $\begin{cases} x-2, & \text{for } -4 \le x < -2 \\ x^3 - x^2 & \text{for } -2 \le x \le 0 \end{cases}$
3) $\begin{cases} x+2, & \text{for } -4 \le x < -2 \\ x^3 + x^2 & \text{for } -2 \le x \le 0 \end{cases}$
4) $\begin{cases} x+1, & \text{for } -4 \le x < -2 \\ x^3 + x & \text{for } -2 \le x \le 0 \end{cases}$

27. **If f(x)** $= \begin{cases} x|x| & , \quad x \le -1 \\ [1+x]+[1-x] & , \quad -1 < x < 1 \\ -x|x| & , \quad x \ge 1 \end{cases}$

1) and odd function
2) an even function
3) neither odd nor even
4) can't be determined

28. **Let f be real valued function defined for all real numbers x such that for some fixed a > 0,**
f(x + 1) = $\frac{1}{2} + \sqrt{f(x) - \{f(x)\}^2}$ **for all x, then**

1) range of f(x) is $\left[\frac{1}{2}, 1\right]$
2) f(x) is many - one
3) f(x) is periodic
4) f(x) is odd

29. **Which of the following function is periodic**

1) $\text{Sgn}(e^{-x})$
2) $\sin x + |\sin x|$
3) min(sin x, |x|)
4) $\left[x + \frac{1}{2}\right] + \left[x - \frac{1}{2}\right] + 2[-x]$

([x] denotes the greatest integer function)

30. **Which of the following functions are periodic?**

1) $f(x) = \sin x + |\sin x|$
2) $g(x) = \frac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\text{cosec} x)}$
3) $h(x) = \max(\sin x, \cos x)$
4) $p(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + 10,$

31. **If the function f(x) satisfies f(10+x) = f(10-x) and f(20-x) = –f(20+x) then f(x) is**

1) an odd function
2) an even function
3) a periodic functions
4) a periodic odd function

32. **If f(x)+f(x+a)+f(x+2a)+.........+f(x+na)=constant; $\forall x \in \mathbb{R}$ and a>0 and f(x) is periodic, then period of f(x), is**

1) (n+1)a
2) $e^{(n+1)a}$
3) na
4) e^{na}

33. **Let f(x,y)be a periodic function, satisfying the condition f(x,y) = f(2x+2y, 2y-2x) $\forall x, y \in \mathbb{R}$ and let g(x) be a function defined as g(x) =f(2^x,0).**

1) g(x) is periodic
2) period of g(x) is 12
3) period of g(x) is 2
4) period of g(x) is 1

34. **If f be a real value function which satisfies** $f\left(x+\frac{3}{2}\right)+f(x)=f(x+1)+f\left(x+\frac{1}{2}\right)$ **and |f(x)| ≤ 2.** $\forall x \in \mathbb{R}$, **then**

1) f is periodic
2) period of f(x) is 1
3) period of f(x) is 2
4) period of f(x) is 3

35. **If f(x) be a function such that,** $f(x-1)+f(x+1)=\sqrt{3}f(x), \forall x \in R$

1) f is periodic
2) period of f(x) is 12
3) period of f(x) is 2
4) period of f(x) is 3

36. **Let** $f(x)=\sin\frac{\pi}{x}$ and $D=\{x : f(x)>0\}$, **then D contains**

1) $\left(\frac{1}{3},\frac{1}{2}\right)$
2) $\left(\frac{1}{5},\frac{1}{4}\right)$
3) $\left(-1,-\frac{1}{2}\right)$
4) $\left(-\pi,-\frac{1}{2}\right)$

37. **Domain of** $f(x)=\sin^{-1}[2-4x^2]$ **is ([.] denotes the greatest integer function)**

1) $\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]\sim\{0\}$
2) $\left[-\frac{\sqrt{3}}{2},1\right)$
3) $\left[-\frac{\sqrt{3}}{2},0\right)\cup\left(0,\frac{\sqrt{3}}{2}\right]$
4) $\left[-\frac{\sqrt{3}}{2},8\right]$

38. **If** $e^x+e^{f(x)}=e$, **then for f(x)**

1) domain = $(-\infty,1)$
2) range = $(-\infty,1)$
3) domain = $(-\infty,0]$
4) range = $(-\infty,1]$

39. **Let f(x) =** $\sec^{-1}[1+\cos^2 x]$, **where [.] denotes the greatest integer function, then**

1) the domain of f is R
2) the domain of f is [1, 2]
3) the range of f is [1, 2]
4) the range of f is $\{\sec^{-1}1, \sec^{-1}2\}$

40. **Let** $f(x)=\frac{x}{1+x^2}$ *and* $g(x)=\frac{e^{-x}}{1+[x]}$, **where** $[\cdot]$ **is the greatest integer less than or equal to x then**

1) Domain (f + g) = R– [–2, 0)
2) Domain (f–g) = R –[–1, 0)
3) Range f $\cap$ Range $g=\left[-2,\frac{1}{2}\right]$
4) Range g = R–{0}

41. Let $f(x)=\left\{\left[\sqrt{n^2+1}\right]-\left[\sqrt{n^2+x}\right]\right\}^{1/2}$, where $[\cdot]$ is the greatest integer function and $n \in N$ then the value of x for which f (x) is defined.

1) $\sqrt{k}$, where k =0, 1,2, --------n
2) $[-n^2, 2n+1)$
3) $[0, 2n+1)$
4) none of these

42. If the domain of f(x) be (-1,2), then

1) domain of f(sinx) will be$(-\infty,\infty)$
2) domain of f(log x) will be $\left(\frac{1}{e}, e^2\right)$
3) domain of f([x]) will be (0,2)
4) none of these

43. The domain of the function $f(x)=\sqrt{\cos(\sin x)}+\sqrt{\log_x\{x\}}$ where {.} indicate fractional part function

1) $[1, \pi)$
2) $(0,2\pi)-[1,\pi)$
3) $\left(0,\frac{\pi}{2}\right)-\{1\}$
4) (0,1)

44. Let $f(x)=\frac{x-[x]}{1+x-[x]}, x \in R,$ then f(R) cannot contain, where [x] is the greatest integer less than or equal to x.

1) 1
2) $\frac{3}{4}$
3) $\frac{1}{4}$
4) $-\frac{1}{2}$

45. Let $f(x)=\log_{100x}\left(\frac{2\log_{10}x+2}{-x}\right)$ and g(x) = {x}, where {x} denotes the fractional part of x. If the function fog(x) exists then the range of g(x) contains

1) $\left(0,\frac{1}{100}\right)$
2) $\left(\frac{1}{100},\frac{1}{10}\right)$
3) $\left(\frac{1}{10},1\right)$
4) $(1,\infty)$

46. If $f(x)=\frac{x}{x^2+1}$ and $f(A)=\left\{y:-\frac{1}{2}\le y<0\right\}$, then set A is

1) [-1, 0)
2) $(-\infty,-1]$
3) $(-\infty,0)$
4) $(-\infty,\infty)$

47. If the functions f (x) = $Sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $g(x)=Cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ are identical then

1) Domain [–1, 1]
2) Range $\left[0,\frac{\pi}{2}\right]$
3) Domain [0, 1]
4) Range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

48. If a,b,c,d,e are positive real numbers, such that $a+b+c+d+e=8$ and $a^2+b^2+c^2+d^2+e^2=16$, the range of e.

1) $\left(0,\frac{16}{5}\right)$
2) $\left(0,\frac{5}{16}\right)$
3) $\left[0,\frac{5}{16}\right]$
4) $\left[0,\frac{16}{5}\right]$

49. The range of f(x) $=\cot^{-1}(2x-x^2)$

1) $\left[\frac{\pi}{2},\pi\right)$ 2) $\left[\frac{\pi}{4},\pi\right)$ 3) $\left(\frac{\pi}{4},\pi\right)$ 4) $\left[\frac{3\pi}{4},\pi\right)$

WORK SHEET (HW) - II

Passage - I :

Consider the functions $f(x)=\begin{cases} x+1, & x £ 1 \\ 2x+1, & 1<x £ 2 \end{cases}$ **and** $g(x)=\begin{cases} x^2, & -1 £ x<2 \\ x+2, & 2 £ x £ 3 \end{cases}$

1. **Domain of function f(g(x)) is**

1) $\left[0,\sqrt{2}\right]$ 2) $\left[-1,2\right]$ 3) $\left[-1,\sqrt{2}\right]$ 4) None of these

2. **The range of the function f(g(x)) is**

1) $\left[1,5\right]$ 2) $\left[2,3\right]$ 3) $\left[1,2\right]\cup\left(3,5\right]$ 4) None of these

3. **The number of the roots of the equation f(g(x)) = 2 is**

1) 1 2) 2 3) 4 4) None of these

Passage - II :

Mr. x is a teacher of mathematics. His students want to know the ages of his son's S_1 and S_2. He told that their ages are 'a' and 'b' respectively such that f(x+y)-ax=f(x)+by² $\forall x, y \in R$ after some time students said that information is insufficient, please give more information. Teacher says that f(1)=8 and f(2)=32.

4. **The age of S_1 & S_2 will be respectively**

1) 4, 16 2) 8, 16 3) 16, 8 4) 32, 8

5. **The funcion f(x) is**

1) even 2) odd
3) neither even nor odd 4) periodic as well as odd

6. **The function** $f:R\to R$**, then function will be**

1) one one onto 2) one one into 3) many one onto 4) many one into

WORK SHEET (HW) - III

1. **Let [a, b] be the range of f(x)**

f(x)	**b - a**
1) $\frac{1}{p^2}((\cos^{-1}x)^2-(\sin^{-1}x)^2)$	p) $\frac{27}{32}$
2) $\frac{1}{p^2}((\cos^{-1}x)^2+(\sin^{-1}x)^2)$	q) 1
3) $\frac{1}{p^3}((\cos^{-1}x)^3-(\sin^{-1}x)^3)$	r) 3/4

4) $\frac{1}{p^3}((\cos^{-1}x)^3+(\sin^{-1}x)^3)$ s) 5/4

2. **Let** $f(x)=\cos^{-1}\frac{x-3}{2}+\log_5(6-x)$ **and** $g(x)=\frac{1}{\sqrt{\sin x}}+\sqrt[3]{\sin x}$

Column - I	Column - II
1) f is defined on	p) [1, 3]
2) g is defined on	q) [1, 2]
3) f is continuous on	r) [1/2, 4]
4) g is continuous on	s) [2, 5]

3. **Let** $f(x)=\log\frac{1+x}{1-x}$ *and* $g(x)=\frac{3x+x^3}{1+3x^2}$, **then**

Column - I	Column - II
1) $f\,o\,g\frac{1-e}{1+e}$	p) 3
2) $g\,o\,f\left(\frac{e-1}{e+1}\right)$	q) - 3
3) $f\,o\,g(0)$	r) 0
4) $f\,o\,g\left(\frac{e-1}{e+1}\right)$	s) 1

4. **Let** $f:R\to[-1,1]$ **is defined by** $f(x)=\sin(2x+1)$. **If domain is restricted to**

Column - I	Column - II
1) $\left[-\frac{3\pi}{4}-\frac{1}{2},\ -\frac{\pi}{2}-\frac{1}{2}\right]$	p) f is one-one and onto
2) $[-3\pi/4-1/2,-1/2]$	q) f is one-one but not onto
3) $[\pi/4-1/2,3\pi/4-1/2]$	r) f is onto out not one-one
4) $\left[-\frac{3\pi}{4}-\frac{1}{2},\frac{3\pi}{2}-\frac{1}{2}\right]\cup\left[\frac{5\pi}{4}-\frac{1}{2},\frac{3\pi}{2}-\frac{1}{2}\right]$	s) f is neither one-one nor onto

5. **Column-I** | **Column-II**

Column-I	Column-II
1) The number of values of 'x' satisfying $e^x \ln x=1$ is/are	p) 0
2) The number of real solution of the equation $x^{2\log_x(x+3)}=16$	q) 1
3) The number of roots of the equation $x+2\tan x=\frac{\pi}{2}$ in the interval $[0, 2\pi]$ is	r) 2
4) The perid of the function sin $3\pi t+\sin 4\pi t$ is	s) 3

WORK SHEET (HW) - IV

1. **If $f(x)=\frac{1-x}{1+x}, x>0$, then the least value of $f\{f(x)\}+f\{f(1/x)\}$ is.**

2. **If f(x) is a polynomial such that f(x) f(y) = f(xy) - f(x) - f(y) $\forall x, y$ and f(2) = 7, then -f(-2) =**

3. **If $f(x+2)-5f(x+1)+6f(x)=0, f(0)=0, f(1)=1,$ then the least positive prime factor of f(2008) is.**

4. **The number of positive integer solutions of the equation $\left[\frac{x}{9}\right]=\left[\frac{x}{11}\right]$ is l then $\left[\frac{l}{20}\right]$**

5. **The number of real numbers x such that 4{x} = x + [x] is**

6. **Let $f: R \to R$ such that f(x-f(y))=f(f(y))+xf(y)+f(x)-1 $\forall x,y \in R$. Then the value of |(f(16))|-125.**

7. **Let f be a function from the set of positive integers to the set of real number such that**

(i) f(1)=1 **ii)** $\sum_{r=1}^{n} r\,f(r)=n(n+1)f(n), \forall n \geq 2$ **then the value of 4022 f(2011).**

8. **Let f(x) be a function such that f(x-1)+f(x+1)=$\sqrt{2}$ f(x) $\forall \in R$. If f(2)=7 then the value of**

$$\sum_{r=0}^{17}\left[f(2+8r)-7\right].$$

9. **If f² (x).** $f\left(\frac{1-x}{1+x}\right)=x^3\left[x \neq -1,1 \ \& \ f(x) \neq 0\right]$ **then find |[f(-2)]| (where [.] is the g.i.f).**

10. **If 'f' is polynomial such that** $f\left(\frac{1-x}{1+x}\right).f\left(\frac{1+x}{1-x}\right)=f\left(\frac{1-x}{1+x}\right)+f\left(\frac{1+x}{1-x}\right)$ **where ($x \neq 0, \pm 1$) and f(3)= 28, then find the value of** $\left(\sum_{n=1}^{10}(f(n)-1)\right)$.

ADDITIONAL EXERCISE

1. **The equation** $e^x - ax - b = 0$ **has**

1) one real root if $a \le 0$
2) one real root if b > 0, $a \le 0$
3) two real roots if a > 0, $a \ln a \ge a - b$
4) no real root if a > 0, a log a < a - b

2. **If** $f'(x) = \dfrac{1-2\sin^2 x}{f(x)}, (f(x) \ge 0, \forall x \in R$ **and f(0) = 1) then f(x) is a periodic function with the period**

1) π
2) 2π
3) $\pi/2$
4) none of these

3. **For real** $x, f(x) = \dfrac{(x-a)(x-b)}{x-c}$ **assumes all real values if**

1) a > b > c
2) a < b < c
3) a > c > b
4) a < c < b

4. **If [x] denotes the greatest integer less than or equal to x, the extreme values of the function** $f(x) = [1+\sin x] + [1+\sin 2x] + [1+\sin 3x] + ... + [1+\sin nx], n \in I^+, x \in (0,\pi)$ **are**

1) n - 1
2) n
3) n + 1
4) n + 2

5. **If f(x) and g (x) are two functions such that f (x+y) = f (x) g(y) + g(x) f(y) then**

$$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$$ **is independent of**

1) α
2) β
3) γ
4) θ

Passage - I :

Let the function f(x) be defined on the set A (domain) and have a range B. If for each $y \in B$, **there exists a single value of x such that f(x) = y(one-one correspondence), then this correspondence defines a certain function x = g(y) is said inverse with respect to the given function.** $y = f(x), g(y) = x = f^{-1}(y)$. **The sufficient condition for the existance of an inverse function is a strictly monotonic of the original function y = f(x), if the function increasing or decreases of the original function y = f(x), then the inverse function x = g(y),** $f^{-1}(x)$ **also increases or decreases. If the inverse function is writeen in the form of y = f(x) then the graph of the inverse function will be symmetirc to that of the function y = f(x) with respect to the bisector of the first and third quadrants.**

6. **If f and g, two continuous functions** $f(x) = x^2 - x + 1, x > \dfrac{1}{2}, g(x) = \dfrac{1}{2} + \sqrt{x - \dfrac{3}{4}}, x > \dfrac{3}{4}$ **are mutually inverse, then**

1) x = {1 }
2) $x = \left[\dfrac{3}{4}, 1\right]$
3) $[1, \infty[$
4) $\left[\dfrac{3}{4}, \infty\right[$

7. If $f(x)=x^4-2x^2+3,$ then which one is in correct?

1) f(x) attains minimum value at $x=\pm 1$
2) f(x) attains maximum value at $x=3\ in\ [-1,1]$
3) f(x) increases $\forall x\geq 1$
4) Range of the functions is $[1,\infty[$.

8. If $f(x)=\dfrac{2x}{1+x^2}$, then which one of the following is incorrect?

1) f(x) is an odd function
2) Graph is symmetric about origin
3) Maximum value of the function f(x) is 1
4) Range of the function is]-1, 1]

Passage - II :

If $y=f(x)$, the set of all values of x for which function is defined, is said domain of the function and the set of values of y for which $x=f^{-1}(y)$, is defined, is said range of the function and the function is written as $f:A\rightarrow B$, then A is said domain and B is said co-domain.

9. If $f(x)={}^{g(x)}P_{h(x)}$, where $g(x)=19x-9-2x^2\ and\ h(x)=\sqrt{x-4}$, then the domain of the function f(x) is.

1) [4, 8]
2) [4, 9]
3) {4,5,6,7,8}
4) {4,5,8}

10. The range of f(x) is

1) {36, 105}
2) {1,36,105}
3) {35,36,108}
4) None of these

11. If $f:(0,\pi)\rightarrow R$, defined by $f(x)=\sum_{k=1}^{n}[1+\sin kx]$, where [.] denotes the greatest integral part of x, then the range of f(x) is.

1) [1, 2]
2) [0, 2]
3) [n - 1, n + 1]
4) [n, n + 1]

12. Consider the local maximum and local minimum of $f(x)=\dfrac{(x-a)(x-b)}{(x-3)(x-6)}$

Column - I	Column - II
1) a = 1, b = 2	p) no maximum, no minimum
2) a = 2, b = 7	q) one maximum, no miniuum
3) a = 4, b = 5	r) one minimum, no maximum
4) a = 2, b = 4	s) one maximum, one minimum

13. Consider the functions $f_1(x)=\sin^{-1}\sin x, f_2(x)=\sin^{-1}\cos x, f_3(x)=\cos^{-1}\sin x, f_4(x)=\cos^{-1}\cos x.$

Column I gives the values of the derivatives $f_1'(x), f_2'(x), f_3'(x), f_4'(x)$ in that order.

Column - I	Column - II
1) 3	p) -1, -1, 1, 1
2) 4	q) -1, 1, 1, -1
3) 5	r) 1, 1, -1, -1
4) 7	s) 1, -1, -1, 1

EXERCISE - II / ANSWERS

WORKS HEET (HW) - I

1) 1,3,4 2) 4 3) 1 4) 1 5) 3 6) 2 7) 1,3 8) 1 9) 1,2,3,4

10) 1,2 11) 1,2,3,4 12) 1,2,3 13) 2,3 14) 1,2 15) 1,2 16) 1 17) 2,3 18) 2,4 19) 2,4

20) 1,2 21) 2,3 22) 1,4 23) 1,3,4 24) 1,2,3,4 25) 1,2,3 26) 2 27) 2 28) 1,2,3 29) 1,2,3,4

30) 1,2,3,4 31) 1,3,4 32) 1 33) 1,2 34) 1,2 35) 1,2 36) 1,2,3 37) 1,3 38) 1,2 39) 1,4

40) 2,4 41) 1,2,3 42) 1,2,3 43) 4 44) 1,2,4 45) 1,2 46) 1,2,3 47) 2,3 48) 4 49) 2

WORKS HEET (HW) - II

1) 3 2) 3 3) 2 4) 3 5) 1 6) 4

WORKS HEET (HW) - III

1) 1 - q,2 - q,3 - s,4 - r. 2) 1- pqs, 2 -pq, 3 - pqs,4 - pq. 3) 1 - q,2 - s,3 - r,4 - p.

4) 1 - q,2 - r,3 - p,4 - s. 5) 1 - q,2 - p,3 - s,4 - r.

WORKS HEET (HW) - IV

1) 2 2) 9 3) 5 4) 1 5) 2 6) 2 7) 1 8) 0 9) 2 10) 5

ADDITIONAL EXERCISE

1) 2,3,4 2) 1,2 3) 3,4 4) 2,3 5) 1,2,3,4 6) 4 7) 1 8) 1

9) 4 10) 1 11) 4 12) 1 - s,2 - r,3 - q,4 - p. 13) 1 - p,2 - q,3 - r,4 - s.