Assignment

Subject: Mathematics

Topic: Differentiability

1. The function $f(x) = e^{-|x|}$ is

(a) continuous everywhere but not differentiable at x = 0
(b) continuous and differentiable everywher
(c) not continuous at x = 0 (d) none of these

2. Consider $f(x) = \begin{cases} \dfrac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

(a) f(x) is discontinuous everywhere (b) f(x) is continuous everywhere
(c) f'(x) exists in (–1, 1) (d) f'(x) exists in (–2, 2)

3. Let $f(x) = a + b|x| + c|x|^4$, where a, b and c are real constants. Then f(x) is differentiable at x = 0, if
(a) a = 0 (b) b = 0 (c) c = 0 (d) none of these

4. Exhaustive set of points where f(x) = cos | x | is differentiable, is

(a) $(-\infty, \infty) \sim \{0\}$ (b) $(-\infty, \infty) \sim \{1\}$ (c) $(-\infty, \infty) \sim [-1]$ (d) $(-\infty, \infty)$

5. If $f(x) = (x^2-4)|(x^3-6x^2+11x-6)| + \dfrac{x}{1+|x|}$, then the set of points at which the function $f(x)$ is not differentiable is

(a) {–2, 2, 1, 3} (b) {–2, 0, 3} (c) {– 2, 2, 0} (d) {1, 3}

6. Let $f: R \to R$ be any function. Define $g: R \to R$ by $g(x) = |f(x)|$ for all x. Then g is
(a) onto if f is onto (b) one one if f is one one
(c) continuous if f is continuous (d) differentiable if f is differentiable

7. **Statement 1** : $|x^3|$ is differentiable at x = 0
Statement 2 : if f(x) is differentiable at x = a then |f(x)| is also differentiable at x = a.

8. **Statement-1** : If f(x) = |x|. sin x is differentiable at x = 0.
Statement-2 : If f(x) is not differentiable and g(x) is differentiable at x = a, then f(x) . g(x) can still be differentiable at x = a.

9. Let $f(x) = (x^2-1)|x^2-3x+2| + \cos|x|$ then f(x) is non differentiable at :

(A) –1 (B) 0 (C) 1 (D) 2

10. Which of the following function is non differentiable at x = 0 is :

(A) $\cos|x| + |x|$ (B) $\cos|x| - |x|$ (C) $\sin|x| + |x|$ (D) $\sin|x| - |x|$

11. Let f(x) = ||x| – 1|, then points wher f(x), is not differentiable is/are :

(a) 0, ±1 (b) ±1 (c) 0 (d) 1

12. If the function $f(x) = |x^2 + a|x| + b|$ has exactly three points of non-derivability, then find a, b.
16. $b \le 0,\ a < 0$

13. If $f(x) = x|x|\ \forall\ x \in R$, then :

(a) f is discontinuous at x = 0 (b) f is derivable at x = 0 but $f'(0) \neq 0$
(c) f is derivable at x = 0 and $f'(0) \neq 1$ (d) none of these

14. Let $f(x) = [x] + |1-x|,\ x \in (-1, 3)$, where [.] denotes the greatest integer function. Total number of points where f(x) is non differentiable, is
(a) 5 (b) 2 (c) 3 (d) 4

15. $f(x) = [\sin x] + [\cos x],\ x \in [0, 2\pi]$. where [.] denotes the greatest integer function.
Total number of points where f(x) is non-differentiable is equal to
(a) 2 (b) 3 (c) 5 (d) 4

16. Let [.] represents the greatest integer function and $f(x) = [\tan^2 x]$, then
(a) $\lim_{x\to 0} f(x)$ does not exist (b) $f(x)$ is continuous at x = 0
(c) $f(x)$ is non-differentiable at x = 0 (d) $f'(0) = 1$

17. Let f(x) = cos x and g(x) = [x + 2], where [.] denotes the greatest integer function. The value of
$(gof)'\left(\frac{\pi}{2}\right)$ is
(a) 1 (b) 0 (c) –1 (d) does not exist

18. For a real number y, let [y] denote the greatest integer less than or equal to y. Then
$f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is
(a) discontinuous at some x (b) continuous at all x, but the derivative f'(x) does not exist for some x
(c) f'(x) exists for all x but second derivative f''(x) does not exist (d) f'(x) exists for all x

19. If [x] denotes the integral part of x and $f(x) = [n + p\sin x],\ 0 < x < \pi,\ \in I$ and p is a prime number, then the number, of points, where $f(x)$ is not differentiable is
(a) p – 1 (b) p (c) 2p – 1 (d) 2p + 1

20. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2)$, [.] denotes the greatest integer function, is continuous and differentiable in (4, 6), then
(a) $a \in [8, 64]$ (b) $a \in (0, 8]$ (c) $a \in [64, \infty)$ (d) none of these

21. $f(x) = \frac{[x]+1}{\{x\}+1}$ for $f : [0, \frac{5}{2}) \to (\frac{1}{2}, 3]$ whre [.] represents greatest integer function and {.} represents fractional part of x, then which of the following is true.
(A) f(x) is injective discontinous function (B) f(x) is surjective non differentiable function
(C) $\min\left(\lim_{x\to 1-} f(x), \lim_{x\to 1+} f(x)\right) = f(1)$ (D) max (x values of point of discoutinuity) = f(1)

22. **Statement-1 :** f(x) = |[x] x| in $x \in [-1, 2]$ where [.] represents greatest integer functioin, is non differentiable at x = 2
Statement-2 : Discontinuous function is always non differntiable.

23. The function, $f(x) = [\,|x|\,] - |\,[x]\,|$ where [x] denotes greatest integer function
(A) is continuous for all positive integers
(B) is discontinuous for all non positive integers
(C) has finite number of elements in its range
(D) is such that its graph does not lie above the x – axis .

24. f(x) is function defined as $f(x) = x[x], -1 \le x \le 3$, where [ ] denotes greatest integer function then f is:
(A) continuous in its domain but non differentiable at finite number of points
(B) discontinuous and non differentiable at every point in its domain
(C) discontinuous at finite number of points but non differentiable at every point in its domain
(D) discontinuous and non differentiable at finite number of points

25. If $f(x)=[x.\sin \pi x]$, where [ ] denotes greatest integer function, then f(x) is :

(A) continuous at x=0 (B) continuous in $(-1,0)$

(C) differentiable at x =1 (D) differentiable in $(-1,1)$

26. In the following [x] denotes the greatest integer less than or equal to x.

Column I		Column II	
(i)	$x\,|x|$	(A)	continuous in (–1, 1)
(ii)	$\sqrt{|x|}$	(B)	differentiable in (–1, 1)
(iii)	$x+[x]$	(C)	strictly increasing in (–1, 1)
(iv)	$|x-1|+|x+1|$	(D)	not differentiable at least one point (–1, 1)

27. If f(x) = [x] $(\sin kx)^p$ is continous for real x, then

(A) k $\in$ {nπ, n$\in$I), p > 0 (B) k $\in$ (2nπ, n$\in$i} π > 0

(C) k $\in$ {nπ, n$\in$I}, p $\in$ R – {0} (D) k $\in$ {nπ, n$\in$I n $\neq$ 0}, p $\in$ R – {0}

28. If f(x) = $(x+[x^3+1])^{x^2+\sin x}$, find the derivative when $x\in(1,3/2)$ and indicate the points where it does not exists. (Where [.] denotes the greatest integer function).

29.

Column-I	Column-II
(A) Number of points of discontinuity of f(x) = $\tan^2 x - \sec^2 x$ in (0, 2π) is	(p) 4
(B) Number of points of which f(x) = sin–x + $\tan^{-1}x$ + $\cot^{-1}x$ is non-differeitable in (–1, 1) is	(q) 3
(C) Number of points of discontinuity of y = [sin x], x $\in$ [0, 2π] where [.] represents greatest integer function	(r) 2
(D) Number of points where y = $|(x-1)^3| + |(x-2)^5| + |x-3|$ is non-differentiable	(s) 1
	(t) 0

30. Let $f(x) = x^3$ sgn x for all x $\in$R, then :

(a) f is continuous but not derivable at 0 (b) f is derivable at 0

(c) L$f'(0) = -3$ (d) R$f'(0) = -3$

31. Given the function $f(x)=2x\sqrt{x^3-1}+5\sqrt{x}\sqrt{1-x^4}+7x^2\sqrt{x-1}+3x+2$ then :

(A) the function is continuous but not differentiable at $x=1$

(B) the function is discontinuous at $x=1$

(C) the function is both cont. & differentiable at $x=1$

(D) the range of f(x) is R^+.

32. If $f(x)=\sin^{-1}(\sin x)$; $x\in R$ then f is

(A) continuous and differentiable for all x

(B) continuous for all x but not differentiable for all $x=(2k+1)\frac{\pi}{2}$, $k\in I$

(C) neither continuous nor differentiable for $x=(2k-1)\frac{\pi}{2}$; $k\in I$

(D) neither continuous nor differentiable for $x\in R-[-1,1]$

33. Let f: R $\to$ R be any function and g(x) = $\frac{1}{f(x)}$. Then g is

(A) onto if it onto (B) one-one if f is one-one

(C) continuous is f is continous (D) differentiable if f is differentiable

34. If $f(x)$ is differentiable function and $(f(x) . g(x))$ is differentiable at $x = a$, then

(a) $g(x)$ must be differentiable at $x = a$ (b) If $g(x)$ is discontinuous, then $f(a) = 0$

(c) $f(a) \neq 0$, then $g(x)$ must be differentiable (d) none of these

35. Let f be a differentiable function on the open interval (a,b). Which of the following statements must be true?

I. f is continuous on the closed interval [a,b]

II. f is bounded on the open interval (a,b)

III. If $a<a_1<b_1<b$, and $f(a_1)<0<f(b_1)$, then there is a number c such that $a_1<c<b_1$ and $f(c) = 0$

(a) I and II only (b) I and III only (c) II and III only (d) only III

36. S_1 : If f is continuous and g is discontinuous at x = a, then f(x). g(x) is discontinuous at x = a.

S_2 : $f(x) = \sqrt{2-x} + \sqrt{2-x}$ is not contitnuous at x = 2.

S_3 : $e^{-|x|}$ is differntiable at x = 0.

S_4 : If f(x) is differentiable every where, then $|f|^2$ is differentiable everywhere.

(A) TTFF (B) TTFT (C) FTFT (D) FFFT

37. S_1 : If $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} [f(x)]$ (where [ ] denotes greatest integer function) and f(x) is non constant continuous function then f(a) is an integer.

S_2 : cos |x| + |x| is differentiable at x = 0

S_3 : If a function has a tangent at x = a then it must be differentiable at x = a.

S_4 : if f(x) & g(x) both are discontinuous at any point, then there composition may be differentiable at that point.

(A) FTFT (B) TFFT (C) TFFF (D) FFFT

38. If $f(x) = \begin{cases} -x-\frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x-1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$, then

(a) $f(x)$ is continuous at $x = -\frac{\pi}{2}$ (b) $f(x)$ is not differentiable at $x = 0$

(c) $f(x)$ is differentiable at $x = 1$ (d) $f(x)$ is differentiable at $x = -\frac{3}{2}$

39. The function $f(x) = \begin{cases} |x-7|, x \ge 5 \\ \frac{x^3}{4} - \frac{7}{2}x + \frac{53}{4}, x < 5 \end{cases}$ is :

(a) continuous at x = 5 (b) continuous at x = 7

(c) dirrerentiable at x = 5 (d) differentiable at x = 7

40. If $f(x) = \begin{cases} x^2 & \text{if } x \le x_0 \\ ax+b & \text{if } x > x_0 \end{cases}$ derivable $\forall x \in R$ then the values of a and b are respectively

(A) $2x_0, -x_0^2$ (B) $-x_0, 2x_0^2$ (C) $-2x_0, -x_0^2$ (D) $2x_0^2, -x_0$

41. Let $f(x) = \begin{cases} \tan^{-1} x, |x| \ge 1 \\ \frac{x^2-1}{4}, |x| < 1 \end{cases}$, then domain of $f'(x)$ is

(a) $(-\infty, \infty) \sim \{1\}$ (b) $(-\infty, \infty) \sim \{-1\}$ (c) $(-\infty, \infty) \sim \{1,-1\}$ (d) $(-\infty, \infty) \sim \{-1,0,1\}$

42. The values of constants a and b so as to make the function $f(x)=\begin{cases}\frac{1}{|x|}, & |x|\ge 1\\ ax^2+b, & |x|<1\end{cases}$,

continuous as well as differentiable for all x, are

(a) $a=-\frac{1}{2}, b=\frac{3}{2}$ (b) $a=\frac{1}{2}, b=\frac{3}{2}$ (c) $a=-\frac{1}{2}, b=-\frac{3}{2}$ (d) none of these

43. If $f(x)=\begin{cases}\frac{\sin[x^2]\pi}{x^2-3x-18}+ax^3+b, & \text{for } 0\le x\le 1\\ 2\cos\pi x+\tan^{-1}x, & \text{for} 1<x\le 2\end{cases}$

differentiable function in [0,2], find a and b. (where[.] denotes the greatest integer function).

Comprehension # 1

Consider two function y = f(x) and y = g(x) defined as f(x) = $\begin{cases}ax^2+b & , 0\le x\le 1\\ 2bx+2b & , 1<x\le 3\\ (a-1)x+2a-3 & , 3<x\le 4\end{cases}$ and

g(x) = $\begin{cases}cx^2+d & , 0\le x\le 2\\ dx+3-c & , 2<x<3\\ x^2+b+1 & , 3\le x\le 4\end{cases}$

44. f(x) is continuous at x = 1 but not differentiable at x = 1, if

(A) a = 1, b = 0 (B) a = 1, b = 2 (C) a = 3, b = 1 (D) c = 1, d = 4

45. g(x) is continuous at x = 2, if

(A) c = 1, d = 2 (B) c = 2, d = 3 (C) c =1, d = –1 (D) c = 1, d = 4

46. If f is contnuous and differentiable at x = 3, then

(A) $a=-\frac{1}{3}, b=\frac{2}{3}$ (B) $a=\frac{2}{3}, b=-\frac{1}{3}$ (C) $a=\frac{1}{3}, b=-\frac{2}{3}$ (D) $a=2, b=\frac{1}{2}$

47. If f(x) = $\begin{cases}x^2\{e^{1/x}\} & x\ne 0\\ k & x=0\end{cases}$ is continous at x = 0, then

({} denotes fractional part function)

(A) It is differnentiable at x = 0 (B) k = 1

(C) continous but not differentiable at x = 0 (D) continous every where in its domain

48. Let the function f, g and h be defined as follows :

$$f(x)=\begin{cases}x\sin\left(\frac{1}{x}\right) & \text{for } -1\le x\le 1 \text{ and } x\ne 0\\ 0 & \text{for } \quad x=0\end{cases}$$

$$g(x)=\begin{cases}x^2\sin\left(\frac{1}{x}\right) & \text{for } -1\le x\le 1 \text{ and } x\ne 0\\ 0 & \text{for } \quad x=0\end{cases}$$

$$h(x)=|x|^3 \qquad \text{for} -1\le x\le 1$$

Which of these functions are differentiable at x = 0?

(A) f and g only (B) f and h only (C) g and h only (D) none

49. Let $f(x)=\begin{cases} x\sin\frac{1}{x}\sin\left(\dfrac{1}{x\sin\frac{1}{x}}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$, check continuity & differentiability at $x=0$

50. Let f(x) = $\begin{cases} |x|^p \sin\frac{1}{x}+x\,|\tan x|^q & , x\neq 0 \\ 0 & , x=0 \end{cases}$ be differentiable at x = 0, then find the least possible value of [p + q], (where [.] represents greatest integer function).

51. Let $f(x)=\begin{cases} g(x).\cos\frac{1}{x} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ where g(x) is an even function differentiable at x = 0, passing through the origin. Then f' (0)

(a) is equal to 1 (b) is equal to 0 (c) is equal to 2 (d) does ont exist

52. The function $f(x)=\begin{cases} 2x+1 & , x\in Q \\ x^2-2x+5 & , x\notin Q \end{cases}$ is

(A) continuous no where
(B) differentiable no where
(C) continuous but not differentiable exactly at one point
(D) differentiable and continuous only at one point and discontinuous elsewhere

53. Let $f(x)=\begin{cases} \int_0^x \{5+|1-t|\}\,dt & \text{if } x>2 \\ 5x+1 & \text{if } x\leq 2 \end{cases}$ then

(a) f(x) is not continuous at x = 2
(b) f(x) is continuous but not differentiable at x = 2
(c) f(x) is differentiable everywhere
(d) the right derivative of f(x) at x = 2 does not exist.

54. Let $f(x)=\begin{cases} 3-x^2 & , -1\leq x<2 \\ 2x-4 & , 2\leq x\leq 4 \end{cases}$,then find fof & check its continuity & differentiability.

55. If $\phi(x)=\begin{cases} |x|, & x\leq 1 \\ 2-x, & x>1 \end{cases}$, then discuss the continuity and differentiability of the function $\phi\{\phi(x)\}$. also give a rough sketch of the function.

56. If $f(x)=\begin{cases} 4, & -3<x<-1 \\ 5+x, & -1\leq x<0 \\ 5-x, & 0\leq x<2 \\ x^2+x-3, & 2\leq x<x \end{cases}$, then $f(|x|)$ is

(a) differentiable but not continuous in (–3, 3)
(b) continuous but not differentiable in (–3, 3)
(c) continuous as well as differentiable in (–3, 3)
(d) neither continuous nor differentiable in (–3, 3)

57. If $f(x)=\begin{cases}3, & x<0\\ 2x+1, & x\geq 0\end{cases}$, then

(a) both f(x) and f (| x |) are differentiable at x = 0
(b) f (x) is differentiable but f(| x |) is not differentiable at x = 0
(c) f (| x |) is differentiable but f(x) is not differentiable at x = 0
(d) both f(x) and f(| x |) are not differentiable at x = 0

58. Let *f*(x) be defined in the interval [–2,2] such that $f(x)=\begin{cases}-1, & -2\leq x\leq 0\\ x-1, & 0<x\leq 2\end{cases}$ and $g(x)=f(|x|)+|f(x)|$.

Test the differentiability of g(x) in [–2,2].

59. Let $f(x)=\begin{cases}x-1, & -1\leq x<0\\ x^2, & 0\leq x\leq 1\end{cases}$ and g(x) = sin x

further let $h(x)=f(|g(x)|)+|f(g(x))|$. Then,

(a) $h(x)$ is continuous for $x\in[-1,1]$
(b) $h(x)$ is differentiable for $x\in[-1,1]$
(c) $h(x)$ is differentiable for $x-\{0\}$
(d) $h(x)$ is differentiable for $x\in(-1,1)-\{0\}$

60. Construct the graph of the function

$g(x)=f(x+l)+f(x-l)$, where $f(x)=\begin{cases}k\left\{1-\frac{|x|}{l}\right\}, & \text{for}|x|\leq l\\ 0, & \text{for}|x|>l\end{cases}$

Also discus the continuity and differentiability of the function g (x).

61. If $\sin^{-1}x+|y|=2y$ then y as a function of x is :

(a) defined for $-1\leq x\leq 1$
(b) continuous at x = 0
(c) differentiable for all x.
(d) such that $\frac{dy}{dx}=\frac{1}{3\sqrt{1-x^2}}$ for x < 0

62. If $f(x)=\min\{|x|,|x-2|,2-|x-1|\}$, then check the continuity & differentiability of f(x).

63. If f(x) = maximum $\left(\cos x,\frac{1}{2},\{\sin x\}\right)$, $0\leq x\leq 2\pi$, where { . } represents fractional part function, then number of points of which f(x) is continous but not differentiable, is

(A) 1 (B) 2 (C) 3 (D) 4

64. Number of points where the functioin f(x) = max (|tan x|, cos |x| is non differentiable in the interval $(-\pi,\pi)$ is

(A) 4 (B) 6 (C) 3 (D) 2

65. The function f(x) = maximum $\left\{\sqrt{x(2-x)},2-x\right\}$ is non-differebtiable at x equal to :

(A) 1 (B) 0, 2 (C) 0, 1 (D) 1, 2

66. If both f(x) & g(x) are differentiable functions at $x = x_0$, then the function defined as, h(x) = Maximum {f(x), g(x)} :

(A) is always differentiable at $x=x_0$
(B) is differentiable at $x=x_0$ provided $f(x_0)\neq g(x_0)$
(C) is never differentiable at $x=x_0$
(D) cannot be differentiable at $x=x_0$ if $f(x_0)=g(x_0)$

67. Let $f(x)=x-x^2$ and $g(x)=\begin{cases}\max f(t), 0\leq t\leq x, 0\leq x\leq 1,\\ \sin\pi x, x>1\end{cases}$

check the differentiability of $g(x)$ in $[0,\infty]$

68. Let $f(x) = \sin x$ $g(x) = \begin{cases} \max.\{f(t),\ 0 \le t \le x & \text{for } 0 \le x \le \pi\} \\ \dfrac{1-\cos x}{2}, & \text{for } x > \pi \end{cases}$. Then, g(x) is :

(a) differentiable for all $x \in R$ (b) differentiable for all $x \in R - \{\pi\}$
(c) differentiable for all $x \in (0, \infty)$ (d) differentiable for all $x \in (0, \infty) - \{\pi\}$

69. Let $f(x) = x^3 - x^2 + x + 1$ and
$g(x) = \max\ \{f(t), 0 \le t \le x\},\ 0 \le x \le 1,$
$3 - x, \quad 1 < x \le 2.$
Discuss the continuity and differentiability of g(x) in the interval [0,2].

70. Let $f(x) = 1 + 4x - x^2, \forall\ x \in R$
$g(x) = \max.\ \{f(t); x \le t \le (x + 1);\ 0 \le x < 3\}$
$= \min.\ \{(x + 3); 3 \le x \le 5\}$
Verify continuity of g(x) for all $x \in [0,5]$

71. $f(x) = x^2 - 2|x|,\ \ g(x) = \begin{cases} \min & \{f(t): -2 \le 1 \le x\},\ x \in [-2, 0) \\ \max & \{f(t): 0 \le t \le x\},\ x \in [0, 3] \end{cases}$. Then g(x) is differentiable in

(a) $[-2, 3] \sim \{-1, 0, 2\}$ (b) $[-2, 3] \sim \{-1, 2\}$
(c) $[-2, 3] \sim \{0, 2\}$ (d) $[-2, 3] \sim \{-1, 0\}$

72. A function y = f(x) is defined parametrically as $y = t^2 + t|t|,\ x = 2t - |t|,\ t \in R,$
Then at $x = 0,\ \ f(x)$ is

(a) continuous but non-differentiable (b) differentiable
(c) discontinuous (d) none of these

73. If $f(x) = \begin{cases} \dfrac{x\ .\ \ell n(\cos x)}{\ell n\left(1 + x^2\right)} & x \ne 0 \\ 0 & x = 0 \end{cases}$ then :

(A) f is continuous at $x = 0$ (B) f is continuous at x = 0 but not differentiable at x=0
(C) f is differentiable at $x = 0$ (D) f is not continuous at $x = 0$.

74. The function $f(x) = \begin{cases} \sin\dfrac{\pi x}{2}, & x < 1 \\ [2x - 3]x, & x \ge 1 \end{cases}$, where [.] denotes the greatest integer function, is

(a) Continuous and differentiable at x = 1 (b) continuous but not differentiable at x = 1
(c) Discontinuous at x =1 (d) None of these

75. If $f(x) = [x].\sin(\pi x)$,then left hand derivative of f(x) at $x = k, k \in I$ is:

(A) $(-1)^k (k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k (k-1)k\pi$ (D) $(-1)^{k-1}(k-1)k\pi$

76. Consider the following statements :
S_1 : If f'(a+) and f'(a$^-$) exist finitely at a point then f is continuous at x = a.
S_2 : The function f(x) = 3 tan 5x – 7 is differentiable at all points in its domain
S_3 : The existence of $\lim\limits_{x \to c}$ (f(x) + g(x) does not imply of existance of $\lim\limits_{x \to 0}$ f(x) and $\lim\limits_{x \to c}$ g(x).
S_4 : If f(x) < g(x) then f'(x) < g'(x).
(A) TTTF (B) FFTF (C) TFTF (D) TFTT

77. Let $f(x)=\left(\dfrac{2\left(\sin x-\sin^3 x\right)+\left|\sin x-\sin^3 x\right|}{2\left(\sin x-\sin^3 x\right)-\left|\sin x-\sin^3 x\right|}\right), x\in(0,\pi)-\left\{\dfrac{\pi}{2}\right\}, f\left(\dfrac{\pi}{2}\right)=3$,

where [ ] denotes the greatest integer function then:

(A) f is continuous and differentiable at $x=\dfrac{\pi}{2}$

(B) f is continuous but nondifferentiable at $x=\dfrac{\pi}{2}$

(C) f is neither continuous nor differentiable at $x=\dfrac{\pi}{2}$

(D)N.O.T

78. Let $f(x)=\begin{cases}\dfrac{x\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)}{\left(e^{\frac{1}{x}}+e^{-\frac{1}{x}}\right)} & , x\neq 0\\ 0 & , x=0\end{cases}$, check the differentiability of f(x) at x = 0.

Comprehension # 2

Left hand derivative and Right hand derivative of a function f(x) at a point x = a are defined as

$f'(a^-)\ \lim\limits_{h\to 0^+}\dfrac{f(a)-f(a-h)}{h}=\lim\limits_{h\to 0^-}\dfrac{f(a+h)-f(a)}{h}$ and

$f(a^+)=\lim\limits_{h\to 0^+}\dfrac{f(a+h)-f(a)}{h}=\lim\limits_{h\to 0^-}\dfrac{f(a)-f(a-h)}{h}=\lim\limits_{x\to a^+}\dfrac{f(a)-f(x)}{a-x}$ respectively.

Lef f be a twice differentiable function.

79. If f is odd, which of the following is Left hand derivative of f at x = – a

(A) $\lim\limits_{h\to 0^-}\dfrac{f(a-h)-f(a)}{-h}$

(B) $\lim\limits_{h\to 0^-}\dfrac{f(a-h)-f(a)}{h}$

(C) $\lim\limits_{h\to 0^+}\dfrac{f(a)+f(a-h)}{-h}$

(D) $\lim\limits_{h\to 0^-}\dfrac{f(-a)-f(-a-h)}{-h}$

80. If f is even which of the following is Right and hand derivative of f' at x = a.

(A) $\lim\limits_{h\to 0^-}\dfrac{f'(a)+f'(-a+h)}{h}$

(B) $\lim\limits_{h\to 0^+}\dfrac{f'(a)+f'(-a-h)}{h}$

(C) $\lim\limits_{h\to 0^-}\dfrac{-f'(-a)+f'(-a-h)}{-h}$

(D) $\lim\limits_{h\to 0^+}\dfrac{f'(a)+f'(-a+h)}{-h}$

81. The statement $\lim\limits_{h\to 0}\dfrac{f(-x)-f(-x-h)}{h}=\lim\limits_{h\to 0}\dfrac{f(x)-f(x-h)}{-h}$ implies that

(A) f is odd

(B) f is even

(C) f is neither odd nor even

(D) nothing can be concluded

Comprehension #

Let a function of defined as $f(x)=\begin{cases}[x] & , -2\le x\le -\dfrac{1}{2}\\ 2x^2-1 & , -\dfrac{1}{2}<x\le 2\end{cases}$, where [.] denotes greatest integer function.

Answer the follwoing questions by using the above information.

82. The number of points of discontinuity of f(x) is

(A) 1 (B) 2 (C) 3 (D) N.O.T.

83. The function f(x – 1) is discontinuous at the points

(A) $-1, -\frac{1}{2}$ (B) $-\frac{1}{2}, 1$ (C) $0, \frac{1}{2}$ (D) 0,1

84. Number of points where |f(x)| is not differentiable is

(A) 1 (B) 2 (C) 3 (D) 4

85. **Column-I** **Column-II**

(A) Number of points where the function

$$f(x) = \begin{cases} 1+\left[\cos\frac{\pi x}{2}\right] & 1 < x \le 2 \\ 1-\{x\} & 0 \le x < 1 \\ |\sin \pi x| & -1 \le x < 0 \end{cases}$$ and f(1) = 0 is continuos but non-differentiable (p) 0

(B) $f(x) = \begin{cases} x^2 e^{1/x} & , x \ne 0 \\ 0 & , x = 0 \end{cases}$ then $f'(0^-)$ = (q) 1

(C) The number of points at which $g(x) = \dfrac{1}{1+\frac{2}{f(x)}}$ is not differentiable where $f(x) = \dfrac{1}{1+\frac{1}{x}}$, is (r) 2

(D) Number of points where tangent does not exist for the curve $y = \text{sgn}(x^2 - 1)$ (s) 3

(t) 4

86. Let $f(x) = \begin{cases} \left|x - \frac{1}{2}\right| & ;0 \le x < 1 \\ x[x] & ;1 \le x < 2 \end{cases}$, where [ ] denotes the greatest integer function.check the diffentiability of f(x).

87. $f(x)=\begin{cases} \left[|x|\left[\frac{1}{|x|}\right]\right], & |x| \ne \frac{1}{n}, n \in N, \\ 0, & |x| = \frac{1}{n} \end{cases}$ then; (where[.] denotes greatest integral function).

(a) f is differentiable everywhere (b) f is continuous everywhere

(c) f is periodic (d) f is not an odd function

88. If $|f(x_1)-f(x_2)|\le(x_1-x_2)^2$, for all $x_1, x_2 \in R$. Find the equation of tangent to the curve y = f(x) at the point (1, 2).

89. Given a differentiable function $f(x)$ defined for all real x, and is such that $f(x+h) - f(x) \le 6h^2$ for all real h and x. Show that $f(x)$ is constant.

90. Let f(x) is a function which satisfies $f(x)=\{|x|, \quad -1\le x\le 1$ and $f(x+2)=f(x)$. If $g(x)=\lim\limits_{n\to\infty}(f(x))^n$, then draw the graphs of f(x) and g(x) and discuss their continuity and differentiability.

91. If $f(x)=\begin{cases} x-[x], & x\notin I \\ 1, & x\in I\end{cases}$, where I is integer and [.] represents the greatest integer function and

$g(x)=\lim\limits_{n\to\infty}\dfrac{\{f(x)\}^{2n}-1}{\{f(x)\}^{2n}+1}$, then

(a) Draw graphs of f (2x), g(x) and g {g(x)} and discuss their continuity.
(b) Find the domain and range of these functions.
(c) Are these function periodic ? If yes, find their periods.

92. Let $f(x)$ be a function defined on (-a, a) with a > 0. Assume that $f(x)$ is continuous at x = 0 and $\lim\limits_{x\to 0}\dfrac{f(x)-f(kx)}{x}=\alpha$, where $k \in (0, 1)$ then compute $f'(0^+)$ and $f'(0^-)$, and comment upon the differentiability of f at x = 0.

93. **Statement-1 :** If $f : R \to R$ is a continuous function such that $f(x) = f(3x)\ \forall\ x \in R$, then f is constant function.

Statement-2 : If f is continuous at $x = \lim\limits_{x\to a} g(x)$, then $\lim\limits_{x\to a} f(g(x)) = f\left(\lim\limits_{x\to a} g(x)\right)$

94. If function $f:[-2a,2a]\to R$ is an odd function satisfies $f(x)=f(2a-x)$ in [a,2a] if left hand derivative at x = a is zero, then find left hand derivative at x = -a.

95. Let f(x) be a function satisfying the condition f (-x) = f(x) for all real x. If f'(0) exists, then its value is

96. Let $\alpha \in R$. Prove that a function f: $R \to R$ is differentiable at α if and only if there is a function $g: R \to R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x-\alpha)$ for all $x \in R$.

97. Let f(x) be non constants twice differentiable function on $(-\infty,\infty)$, such that $f(x)=f(1-x)$ & $f`(1/4)=0$ then :

(A) f``(x) vanishes at least twice on [0,1]
(B) $f`(1/2)=0$
(C) $\int_{-1/2}^{1/2} f(x+1/2).\sin x\,dx = 0$
(D) $\int_{0}^{1/2} f(t).e^{\sin \pi t}\,dt = \int_{1/2}^{1} f(1-t).e^{\sin \pi t}\,dt$

98. If the function $f(x)\forall x,y\in R$ satisfies the conditions

(A) $f(x+y)=f(x)+f(y)$
(B) $f(x)=1+x.g(x)$ *where* $\lim\limits_{x\to 0} g(x)=1$

Then prove that f`(x) exists and f`(x) = f(x).

99. Let f(x) be a derivable function at x = 0 & $f\left(\dfrac{x+y}{k}\right)=\dfrac{f(x)+f(y)}{k}$ $(k \in R, k\ne 0, 2)$. Show that f(x) is either a zero or an odd linear function.

100. A function f(x) satisfies the relation $f(x + y) = f(x) + f(y) + xy(x + y)$ $\forall$, x, y $\in$ R. If $f'(0) = -1$, then

(A) f(x) is a polynomial function (B) f(x) is an exponential function

(C) f(x) is twice differentiable for all $x \in R$ (D) $f'(3) = 8$

101. A function $f : R \to R$ where R is a set of real numbers satisfies the equation $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)+f(0)}{3}$ for all x, y $\in$ R. If the function is differentiable at x = 0 then show that it is differentiable for all x in R.

102. If $f(x+y) = f(x)f(y)$ for all x, y $\in$R, and $f(x) = 1 + g(x)G(x)$, where $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} G(x)$ exists. Prove that f(x) is continuous for all x $\in$R.

103. Let f be a function satisfying $f(x+y) = f(x)f(y) - \sqrt{4 - f(y)}$ for all x, y $\in$R, and $\lim_{x\to 0} f(x) = 4$. Discuss the continuity of f(x).

104. If $f(x + y) = f(x) . f(y)$ for x, y $\in$ R & f(x) is differentiable everywhere then find f(x).

105. Let f(x) be a differentiable function which satisfies the equation $f(xy) = f(x) + f(y)$ for all $x > 0, y > 0$ then $f'(x)$ is equal to

(A) $\frac{f'(1)}{x}$ (B) $\frac{1}{x}$ (C) $f'(1)$ (D) $f'(1).(\ln x)$

106. (a) Let $\frac{f(x+y)-f(x)}{2} = \frac{f(y)-a}{2} + xy$ for all real x and y. If f(x) is differentiable and $f'(0)$ exists for all real permissible values of 'a' and is equal to $\sqrt{5a-1-a^2}$. Prove that f(x) is positive for all real x.

(b) If $f(x - y) = f(x) . g(y) - f(y) . g(x)$ and $g(x - y) = g(x) . g(y) + f(x) . f(y)$ for all x, y $\in$ R. If right hand derivative at x = 0 exists for $f(x)$. Find derivative of $g(x)$ at x = 0.

107. A function f is definded in [–1, 1] as $f'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$; $x \neq 0$; $f(0) = 0$; $f(1/\pi) = 0$.
Discuss the continuity and derivability of f an x = 0.

108. Let f be a function that in every where differentiable and that has the following properties:

(i) $f(x+h) = \frac{f(x)+f(h)}{f(-x)+f(-h)}$ for all real x and h. (ii) $f(x) > 0$ for all real x.

(iii) $f'(0) = -1$ (iv) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).f(h)$

use the definition of derivative to find $f(x)$ in terms of $f(x)$

Answer Key

1.a	**2.b**	**3.b**	**4.d**	**5.d**	**6.c**	**7.c**	**8.a**	**9.d**	**10.d**	**11.a**	**12.** $b \leq 0, a < 0$
13.d	**14.c**	**15.c**	**16.d**	**17.d**	**18.d**	**19.c**	**20.c**	**21.abd**		**22.a**	**23.abcd**
24.d											
25.abd		**26.** (i)-A, B, C (ii)-A, D (iii)-C, D (iv)-A, B						**27.a**		**29. A-R, B-T, C-R, D-Q**	
30.b	**31.b**	**32.b**	**33.b**	**36.c**	**37.b**	**38.abcd**		**39.abc**	**40.a**	**41.c**	**42.a**
44.c	**45.a**										
46.d	**47.a**	**48.c**	**49.** Continuous but not differentiable					**50.1**	**52.d**	**53.b**	**56.b**
57.c	**63.d**										
64.a	**65.d**	**66.b**	**71.c**	**72.b**	**73.ac**	**74.c**	**75.a**	**76.a**	**77.a**	**79.a**	**80.a**
81.b	**82.b**										
83.c	**84.c**	**85. A-Q, B-P, C-S, D-Q**				**88.** $y - 2 = 0$		**92.** $f'(0)\frac{\alpha}{1-k}$			**93.a**
94. 0	**95.** 0										
97.abcd		**100.ac**		**105.a**		**108.** $f'(x) = -f(x)$					