11. SEQUENCE & SERIES

SYNOPSIS

A sequence is a function whose domain is the set N of natural numbers. A sequence whose range is subset of R is called a real sequence. If $a_1, a_2, a_3 .... a_n$ is a sequence, then the expression $a_1 + a_2 + ... + a_n$ is a series.

Progressions : It is not necessary that the terms of a sequence always follow a certain pattern (or) they are described by some explicit formula for the n^{th} term. Those sequences whose terms follow certain patterns are called progressions.

ARITHMETIC PROGRESSION (A..P) : A Sequence is called an arithmetic progression if the difference of a term and the previous term is always same,

i.e. $a_{n+1} - a_n$ = constant (= d) for all $n \in N$.

The constant difference, generally denoted by d is called the common difference.

General term of An A.P. : Let 'a' be the first term and 'd' be the common difference of an A.P. Then its nth term $t_n = a + (n - 1) d$.

SUM TO n TERMS OF AN A.P.

1. The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by

   $S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}[a + l]$ where l = last term = a + (n - 1)d

2. If the sum S_n of n terms of a sequence is given, then the n^{th} term a_n of the sequence can be determined by the formula $T_n = S_n - S_{n-1}$.

PROPERTIES OF A.P.

1. a, b, c are in A.P. $\Leftrightarrow$ 2b = a + c
2. If a constant term is added to (or) subtracted from each term of an A.P., then the resulting sequence is also an A.P. with same common difference. Let $a_1, a_2, a_3, ...$ are in A.P. Then $a_1 \pm k,\ a_2 \pm k,\ a_3 \pm k.......$ are also in A.P.
3. If each term of a given A.P. is multiplied (or) divided by a non zero constant k, then the resulting sequence is also an A.P. with common difference Kd or d/k. Where 'd' is the common difference of the given A.P.
4. In a finite A.P. the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of the first and last term.

   i.e. $a_1 + a_n = a_k + a_{n-(k-1)}$ $\forall$ k = 1, 2, 3, ... (n - 1) $a_2 + a_{n-1} = a_3 + a_{n-2} \equiv .... = a_1 + a_n$
5. A sequence is an A.P. iff its n^{th} term is a linear expression is n i.e. $T_n = An + B$, where A, B are constants. In such a case the coefficient of n in Tn is the c.d. of the A.P.
6. A sequence is an A.P. iff the sum of its first n terms is of the $An^2 + Bn$ where A, B are constants independent of n. In such a case the common difference is 2A. i.e. 2 times the coefficient of n^2.
7. If the terms of an A.P. are chosen at regular intervals, then they form an A.P.

Selection of terms in an A.P.

No. of terms	Terms	c . d
3	a - d, a, a + d	d
4	a - 3d, a - d, a + d, a + 3d	2d
5	a - 2d, a - d, a, a + d, a + 2d	d
6	a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d	2d

It should be noted that in case of an odd no. of terms, the middle term is a and the c.d. is d. While in case of an even no. of terms are a - d, a + d and c.d. is 2d.

Insertion of Arithmetic means :

In between two given quantities a and b we have to insert n quantities $A_1, A_2, \ldots A_n$. Such that $a, A_1, A_2, \ldots, A_n, b$ form an A.P. then we say that $A_1, A_2 \ldots A_n$ are arithmetic means between a and b.

Let $A_1, A_2, \ldots A_n$ be n arithmetic means between the quantities a and b, then $a, A_1, A_2, \ldots A_n, b$ is an A.P.

Let 'd' be the c.d. of the A.P., clearly it contains (n + 2) terms b = $(n + 2)^{th}$ term

=> $b = a + (n + 1)d \Rightarrow d = \dfrac{b-a}{n+1}$

$A_1 = a + d = \left(a + \dfrac{b-a}{n+1}\right)$ $\quad A_2 = a + 2d = a + \dfrac{2(b-a)}{n+1}$ ..... $A_n = a + nd = a + \dfrac{n(b-a)}{n+1}$

These are the required airthimatic means between a and b.

Let A be the A.M. of a and b then $A = \dfrac{a+b}{2}$

If $a_1, a_2 \ldots a_n$ are n numbers, then thier AM = $\dfrac{a_1 + a_2 + \ldots + a_n}{n}$

The sum of n AM's between two nos is n times the single A.M. between them.

i.e. $A_1 + A_2 + A_3 + \ldots + A_n = n\left(\dfrac{a+b}{2}\right) = n \times [\text{A.M. between a and b}]$

Geometric Progression : A sequence of non zero numbers is called a geometric progression (G.P) if the ratio of a term and the term preceding to it is always a constant quantity. The constant ratio is called the common ratio of the G.P.

The sequence $a_1, a_2, a_3 \ldots a_n \ldots$ is called a G.P.

If $\dfrac{a_{n+1}}{a_n}$ = constant for all $n \in N$ $\quad$ In a G.P. $T_n = a.\, r^{n-1}$.

The series is $a, ar, ar^2, ar^3, \ldots, ar^{n-1}, \ldots$

The n^{th} term from the end of a finite G.P. consisting of m terms is $a.\, r^{m-n}$ where a is the first term and r is the common ratio of the G.P.

The n^{th} term from the end of a G.P. with last term *l* and common ratio r is $\ell\left(\dfrac{\ell}{r}\right)^{n-1}$

Selection of terms in G.P.

No. of terms	**Terms**	**Common ratio**
3	$\dfrac{a}{r}, a, ar$	r
4	$\dfrac{a}{r^3}, \dfrac{a}{r}, ar, ar^3$	r^2
5	$\dfrac{a}{r^2}, \dfrac{a}{r}, ar, ar^2$	r

Sum of n terms of a G.P. :

$S_n = \dfrac{a(r^n - 1)}{r-1} (\text{if } r > 1)$ $\quad$ or $\quad$ $S_n = \dfrac{a.(1-r^n)}{1-r} \text{ if } (r < 1)$

If *l* is the last term of a G.P. Then $l = a.r^{n-1}$ and $S_n = \dfrac{a - \ell r}{\ell - r}$ or $S_n = \dfrac{\ell r - a}{r - 1}$

The sum of an infinite G.P. with first term a and common ratio r $(|r| < 1)$ is $S_\infty = \dfrac{a}{1-r}$

Properties of Geometric progressions :

1. If all the terms of a G.P. be multiplied or divided by the same non zero constant, then it remains a G.P. with the same common ratio.
2. The reciprocals of the terms of a given G.P. form a G.P.
3. If each term of a G.P. be raised to the same power, then the resulting sequence also forms a G.P. i.e., $a_1, a_2, \ldots a_n$ be

a G.P. with common ratio 'r' then a_1^k, a_2^k, a_3^k,.....a_n^k is a G.P. with common ratio r^k.

4. In a finite G.P. the product of the terms equidistant from the beginning and the end is always same and is equal to the produc t of the first and the last terms.

   i.e. $a_k\, a_{n-k+1} = a_1 r^{k-1}.\, a_n r^{n-k} = a_1^2.r^{n-1} = a_1 a_1 r^{n-1} = a_1 a_n \quad \forall\ k = 2, 3, ..., n-1$

5. a, b, c are in G.P. $\Leftrightarrow$ $b^2 = ac$

6. If the terms of a given G.P. are chosen at regular intervals, then the new sequence so formed also forms a G.P.

7. If $a_1, a_2, a_3, ... a_n$ is a G.P. of non zero, non - negative terms, then $\log a_1, \log a_2, ... \log a_n$ is an A.P. and vice - versa.

Insertion of geometric means between two given numbers :

Let a and b be two given numbers. If n numbers $G_1, G_2, ... G_n$ are inserted between a and b such that the sequence a, $G_1, G_2, ... G_n$ b is a G.P. Then the numbers $G_1, G_2 .... G_n$ are known as G.M's between a and b.

a, G, b are in G.P $\Leftrightarrow G^2 = ab \Leftrightarrow G = \sqrt{ab}$

Let $G_1, G_2 ...... G_n$ be n geometric means between a and b.

a, $G_1, G_2, G_3, ... G_n$, b is a G.P. consisting of (n + 2) terms. Let r be the common ratio of this G.P. then

$b = (n + 2)^{th}$ term $= a.r^{n+1}$ $\quad \Rightarrow r^{n+1} = \dfrac{b}{a} \quad r = \left(\dfrac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar = a.\left(\dfrac{b}{a}\right)^{\frac{1}{n+1}}$; $G_2 = ar^2 = a.\left(\dfrac{b}{a}\right)^{\frac{2}{n+1}}$ .... $G_n = ar^n = a.\left(\dfrac{b}{a}\right)^{\frac{n}{n+1}}$

$G_1 G_2 .... G_n = \left(\sqrt{ab}\right)^n = G^n$

i.e. If n geometric means are inserted between two quantities, then the product of n geo metric means is the nth power of the single geometric mean betwen the two quantities.

HARMONIC PROGRESSION :

A sequence $a_1, a_2 .... a_n$ of non zero numbers is called a H.P. if the sequence $\dfrac{1}{a_1}, \dfrac{1}{a_2}, ...\dfrac{1}{a_n}$ .... is an A.P. The n^{th} term of a H.P. is the reciprocal of the n^{th} term of the corresponding A.P. If $a_1 a_2 ..... a_n$ is a HP and the common difference of the corresponding AP is d.

$T_n = \dfrac{1}{a + (n-1)d}$ where $a = \dfrac{1}{a_1}$

Insertion of n HM's & between two given numbers.

Let a, b be two given numbers, If n numbers, $H_1, H_2 .... H_n$ are inserted between a and b such that the sequence a, $H_1, H_2 ..... H_n$, b be is an H.P. Then $H_1, H_2 .... H_n$ are called n harmonic means between a and b.

Now, a, $H_1, H_2, ... H_n$, b are in H.P. $\dfrac{1}{a}, \dfrac{1}{H_1}, \dfrac{1}{H_2}, ...\dfrac{1}{H_n}, \dfrac{1}{b}$ are in A.P.

Let D be the common difference of this A.P. Then $D = \dfrac{a-b}{(n+1)ab}$

$$\therefore \frac{1}{H_1} = \frac{1}{a} + D, \frac{1}{H_2} = \frac{1}{a} + 2D, ...; \frac{1}{H_n} = \frac{1}{a} + nD$$

a, H, b are in A.P. $\Leftrightarrow$ H = $\dfrac{2ab}{a+b}$

If $a_1, a_2, a_3 \ldots\ldots a_n$ are n non zero numbers, then the harmonic mean (H) of these numbers

$$\frac{1}{H} = \frac{\frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots + \frac{1}{a_n}}{n}$$

Properties of A.M., G.M., H.M, between two given numbers.

Let A, G, H be the A.M., G.M., H.M., between two numbers a and b then

$$A = \frac{a+b}{2};\ G = \sqrt{ab};\ H = \frac{2ab}{a+b}$$

1. $A \geq G \geq H$
2. A, G, H are in G.P. i.e. $G^2 = A.H.$
3. The equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$
4. If A, G, H are A.M., G.M., H.M. between three numbers a,b,c then the equation having a, b, c as its roots is $x^3 - 3Ax^2 + \dfrac{3G^3}{H}x - G^3 = 0$
5. $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M., G.M., H.M between a and b for n = 1, $\dfrac{1}{2}$, 0

A.G.P.

If a, a + d, a + 2d, a + 3d, ... is an A.P. and b, br, br^2, is a G.P. then ab, (a + d) br, (a + 2d) br^2,..... is an arthmetrico geometric progression. The general form of an A.G.P. is a, (a + d)r, (a + 2d)r^2, (a + 3d)r^3,......

In an A.G.P. $T_n = \left[(a + (n-1)d\right].r^{n-1}$

In an A.G.P. $S_n = \begin{cases} \dfrac{a}{1-r} + dr\dfrac{(1-r^{n-1})}{(1-r)^2} - \dfrac{[a+(n-1)d]r^n}{1-r} (r \neq 1) \\ \dfrac{n}{2}[2a+(n-1)d] \left(\text{when } r = 1\right) \end{cases}$

In an infinite A.G.P. $S_\infty = \dfrac{a}{(1-r)} + \dfrac{dr}{(1-r)^2} \left(\text{when } |r| < 1\right)$

Some special sequences :

1. $1 + 2 + 3 + \ldots\ldots + n = \sum_{k=1}^{n} (k) = \dfrac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \sum_{k=1}^{n} k^3 = \dfrac{n^2(n+1)^2}{4}$

EXERCISE - I

WORK SHEET- I

Problems on A.P.

1. If $\frac{3+5+7+....+n \text{ terms}}{5+8+11+.....+10 \text{ terms}}=7$ then the value of n is

1) 35 2) 36 3) 37 4) 40

2. If $\frac{1+3+5+......n \text{ terms}}{1+2+3+.........n \text{ terms}}=\frac{12}{7}$ then n =

1) 6 2) 4 3) 7 4) 5

3. If the first term of an A.P. is 2 and the sum of first five terms is equal to one fourth of the sum of the next five terms, then the sum of the first 30 terms is

1) 2550 2) 3000 3) - 2550 4) - 3000

4. The first, second and last terms of an A.P. are α, β, γ respectively then the sum of first n terms is

1) $\beta+\gamma-2\alpha$ 2) $\frac{\beta+\gamma-2\alpha}{\beta-\alpha}$ 3) $\frac{\beta+\gamma+2\alpha}{\beta+\alpha}$ 4) $\frac{(\alpha+\gamma)(\beta+\gamma-2\alpha)}{2(\beta-\alpha)}$

5. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$, then the ratio of the m^{th} and n^{th} terms is

1) 2m - 1 : 2n - 1 2) m - 1 : n - 1 3) 2m + 1 : 2n + 1 4) m + 1 : n + 1

6. If s_n be the sum of n terms of an A.P. and $\frac{s_{pn}}{s_n}$ is independent of n then the common difference is

1) a 2) 2a 3) 3a 4) 4a

7. If S_n denotes the sum of first n terms of an A.P. and $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ =

1) 10 2) 8 3) 7 4) 6

8. If $s_1, s_2, s_3......s_q$ are the sums of n terms of q A.P.'s whose first terms are 1, 2, 3 ....q and common differences are 1,3,5, ... (2q–1), respectively then $s_1+s_2+s_3+...+s_q=$

1) $\frac{1}{2}nq(nq+1)$ 2) $\frac{1}{2}nq(nq-1)$ 3) nq 4) $\frac{nq}{2}$

9. There are n A.P's whose common differences are 1, 2, 3 ... n respectively the first term of each being unity. Then sum of their n th terms is

1) $n(n+1)^2$ 2) $n^2(n+1)$ 3) $\frac{1}{2}n(n^2+1)$ 4) $\frac{1}{2}n^2(n+1)$

10. If the sum of 5 terms of an A.P. is same as the sum of its 11 terms then sum of 16 terms is

1) 0 2) 16 3) –16 4) 32

11. The nth term of an A.P is p and the sum of the first n terms is s. The first term is

1) $\frac{2p+sn}{n}$ 2) $\frac{2p-sn}{n}$ 3) $\frac{2s+pn}{n}$ 4) $\frac{2s-pn}{n}$

12. Let $a_1, a_2, a_3.....$ be terms of an A.P. If $\frac{a_1+a_2+....a_p}{a_1+a_2+....a_q}=\frac{p^2}{q^2}$ $(p \neq q)$ then $\frac{a_6}{a_{21}}=$

1) $\frac{7}{2}$ 2) $\frac{2}{7}$ 3) $\frac{11}{41}$ 4) $\frac{41}{11}$

13. Sum of the series $(x-y)^2+x^2+y^2+(x+y)^2+....+[(x+y)^2+6xy]$ is

1) $2(x^2+y^2+3xy)$ 2) $3(x^2+y^2+3xy)$ 3) $6(x^2+y^2+3xy)$ 4) $4(x^2+y^2+3xy)$

14. If x, y, z are real numbers satisfying the equation $25(9x^2+y^2)+9z^2-15(5xy+yz+3zx)=0$ then x, y, z are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

15. If x, y, z are real numbers satsifying the equation $16(25x^2+4y^2)+25z^2-160xy-40yz-100zx=0$ then x, y, z are in

1) A.P 2) G.P 3) H.P 4) A.G.P

16. If p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term of A.P. is $\frac{1}{p}$ then sum of the first pq terms is

1) $\frac{p+q}{pq}$ 2) $\frac{pq}{p+q}$ 3) $\frac{1}{2}(pq-1)$ 4) $\frac{1}{2}(pq+1)$

17. If $a_1, a_2, a_3,....... a_n$ are in A.P. where $a_i > 0$, $\forall i$ then $\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+........+$

$\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{k}{\sqrt{a_1}+\sqrt{a_n}}$ then k =

1) 1 - n 2) n - 1 3) n 4) n + 1

18. The sum to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+....$ is

1) $\sqrt{2n+1}$ 2) $\left(\frac{1}{2}\right)\sqrt{2n+1}$ 3) $\sqrt{2n+1}-1$ 4) $\frac{1}{2}\left\{\sqrt{2n+1}-1\right\}$

19. If $a_1, a_2, a_3,....... a_n$ be an A.P. of non-zero terms then $\frac{1}{a_1 a_2}+\frac{1}{a_2 a_3}+........+\frac{1}{a_{n-1} a_n}=$

1) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ 2) $\frac{n-1}{a_1+a_n}$ 3) $\frac{n-1}{a_1.a_n}$ 4) $\frac{1-n}{a_1.a_n}$

20. If $a_1, a_2, a_3,....... a_n$ be an A.P. with common difference d, then

$\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + ... + \sec a_{n-1} \sec a_n =$

1) $\frac{Tan d}{\sin a_1 \sin a_2}$ 2) $\frac{Tan\, a_n - Tan\, a_1}{\sin d}$ 3) $\frac{n-1}{Tan a_1\, Tan a_2}$ 4) $\frac{Tan\ a_1 - Tan\ a_n}{\sin\ d}$

21. If $a_1, a_2, a_3......... a_n$ are in A.P. with common difference d then sin d [cosec a_1 cosec a_2 + cosec a_2 cosec a_3 + cosec a_3 cosec a_4 +......n terms] =

1) cot a_1–cot a_{n+1} 2) tan a_1–tan a_{n+1} 3) sec a_1–sec a_{n+1} 4) cosec a_1–cosec a_{n+1}

22. If S_1, S_2, S_3 be the sum of n, 2n, 3n terms respectively of an A.P. Then

1) $S_3 = S_1 + S_2$ 2) $S_3 = 2(S_1 + S_2)$ 3) $S_3 = 3(S_2 - S_1)$ 4) $S_3 = 3(S_1 + S_2)$

23. The sums of n terms of three A.Ps are s_1, s_2, s_3 respectively. The first term of each is unity and common differences are 1, 2 and 3 respectively then $\frac{s_1 + s_3}{s_2} =$

1) 1 2) 2 3) 3 4) 4

24. If S_1, S_2, S_3 be the sum of 10, 20, 30 terms respectively of an A.P. Then

1) $S_3 = S_1+S_2$ 2) $S_3 = 2(S_1+S_2)$ 3) $S_3 = 3(S_1+S_2)$ 4) $S_3 = 3(S_2-S_1)$

25. If the sides of a right angled triangle are in A.P. then sines of acute angles are

1) $\frac{3}{5}, \frac{4}{5}$ 2) $\frac{5}{13}, \frac{12}{13}$ 3) $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$ 4) $\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}+1}{2}}$

26. If $1, \frac{1}{2}\log_3^{\left(3^{1-x}+2\right)}, \log_3^{\left(4.3^x-1\right)}$ are in A.P. then x equals

1) $\log_3^4$ 2) $1-\log_3^4$ 3) $1-\log_4^3$ 4) $\log_4^3$

27. If $\frac{a+b}{1-ab}$, b and $\frac{(b+c)}{1-bc}$ are in A.P. then c, abc, a are in

1) A.P. 2) G.P. 3) H.P 4) A.G.P

28. If a, b, c are in A.P then $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

29. If p, q, r are in A.P the $p^2(q+r), \quad q^2(r+p), r^2(p+q)$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

30. After inserting n A.M.'s between 2 and 38, the sum of the resulting progressions is 200. The value of n is

1) 7 2) 8 3) 9 4) 10

31. If n A.M.'s are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1:3 then the value of n is

1) 11 2) 12 3) 13 4) 14

32. If the sum of m A.M.'s between two positive numbers is α and sum of n A.M.'s between the same numbers is β then $\frac{\alpha}{\beta} =$

1) $\sqrt{\frac{m}{n}}$ 2) $\frac{m^2}{n^2}$ 3) $\frac{m}{n}$ 4) mn

33. If the A.M. between m th and n th term of an A.P. be equal to A.M. between p th and q th terms of the A.P then

1) m + n = p + q 2) m + q = p + n 3) m + p = n + q 4) m + n + p + q =0

34. Given that n AM's are inserted between two sets of numbers a,2b and 2a,b where $a, b \in R$ suppose further that the n^{th} mean between these sets of numbers is same then the ratio a : b =

1) 1: n 2) n : 1 3) n + 1 : 1 4) n + 2 : n - 1

Problems on G.P.

35. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.........\infty\right)} =$

1) 4 2) log 4 3) log 2 4) log 3

36. If 4th, 7th, 10th terms of a G.P. are p, q, r respectively then

1) $p^2 = q^2+r^2$ 2) $q^2 = pr$ 3) $p^2 = qr$ 4) pqr +pq+1=0

37. If x, y, z are in pth, qth, rth terms respectively of an A.P and also G.P then $x^{y-z}.y^{z-x}.z^{x-y} =$

1) x y z 2) 0 3) 1 4) - x y z

38. If $x^{18} = y^{21} = z^{28}$ then $3\log_y^x,\ 3\log_z^y,\ 7\log_x^z$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P

39. A G.P. consists of an even number of terms. If the sum of all the terms is five times the sum of those terms occuping the odd places, then common ratio is

1) 2 2) 3 3) 4 4) 5

40. In a G.P. of positive terms for a fixed n, the n^{th} term equal to the sum of the next two terms. Then the common ratio of the G.P. is

1) 2 sin 18^0 2) sin 18^0 3) cos 18^0 4) 2 cos 18^0

41. If s_n represents the sum of n terms of G.P. whose first term and common ratio are a and r respectively, then $s_1+s_2+s_3+....+s_n =$

1) $\frac{an}{1-r}-\frac{ar(1-r^n)}{(1-r)^2}$ 2) $\frac{ar(1-r^n)}{(1-r)^2}$ 3) $\frac{an}{1-r}-\frac{ar(1-r^{2n})}{(1-r)^2(1+r)}$ 4) $\frac{ar(1-r^{2n})}{(1+r)(1-r)^2}$

42. Let a_n be the n^{th} term of G.P. If $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, then the common ratio of the G.P. is

1) $\frac{\sqrt{\alpha}}{\beta}$ 2) $\frac{\sqrt{\beta}}{\alpha}$ 3) $\frac{\alpha}{\beta}$ 4) $\frac{\beta}{\alpha}$

43. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} (ab)^n$ where a<1, b<1 then

1) xy+xz = yz+x 2) xyz = x+y+z 3) xy+yz = xz+y 4) yz+zx = xy+z

44. The sum of an infinite G.P is 2. If the sum of their squares is 4/3 then the third term is

1) 1/2 2) 1 3) 1/4 4) 1/8

45. A square is drawn by joining the mid points of the given square a third square in the same way and this process continues indefinitely. If a side of the first square is 16 cm, then the sum of the areas of all the squares

1) 128 sq. cm 2) 256 sq. cm 3) 512 sq. cm 4) 1024 sq. cm

46. The sum to n terms of the series .5+.55+.555+...... is

1) $\frac{5n}{9}-\frac{5}{81}\left(1-\frac{1}{10^n}\right)$ 2) $\frac{5n}{9}+\frac{5}{81}\left(1-\frac{1}{10^n}\right)$ 3) $\frac{5n}{9}+\frac{5}{81}\left(1+\frac{1}{10^n}\right)$ 4) $\frac{5n}{9}-\frac{5}{81}\left(\frac{1}{10^n}-1\right)$

47. 0.7+0.77+0.777+....... n terms =

1) $\frac{7n}{9}-\frac{7}{81}\left(1-\frac{1}{10^n}\right)$ 2) $\frac{7\left(10^n-1\right)}{9}-\frac{7n}{8}$ 3) $\frac{10\left(10^n-1\right)}{81}-\frac{7n}{9}$ 4) $\frac{7n}{9}-\frac{7}{81}\left(\frac{1}{10^n+1}\right)$

48. One of the five geometric means between $\frac{1}{3}$ and 243 is

1) 79 2) 80 3) 81 4) 82

49. If x,y,z are three geometric means between 6,54 then z =

1) $9\sqrt{3}$ 2) 18 3) $18\sqrt{3}$ 4) 27

50. Let x be arthimetic mean and y,z be two gemetric means between any two positive number then the value of $\frac{y^3+z^3}{xyz}=$

1) 2 2) 3 3) 1/2 4) 3/2

51. If a, b, c are in A.P. and b-a, c-b, a are in G.P. then a:b:c =

1) 2:3:4 2) 1:2:4 3) 1:2:3 4) 1:3:5

52. If A and G are the A.M. and G.M. respectively between two numbers then the numbers are

1) $A\pm\sqrt{G^2+A^2}$ 2) $A\pm\sqrt{A^2-G^2}$ 3) $A\pm\sqrt{A^2+2G^2}$ 4) $G\pm\sqrt{A^2-G^2}$

53. If A and G are A.M and G.M between two positive numbers a and b are connected by the relation A+G=a–b then the numbers are in the ratio

1) 1 : 3 2) 1 : 6 3) 1 : 9 4) 1 : 12

54. If one G.M. 'G' and two arithmetic means p and q be inserted between any two given number then G^2 =

1) $(2p-q)(2q-p)$ 2) $(2p-q)(q-2p)$ 3) $\frac{2p+q}{2q+p}$ 4) $\frac{2p+q}{q+2p}$

55. If the A.M. and G. M between two numbers are in the ration m : n then the numbers are in the ratio

1) $m+\sqrt{n^2-m^2}:m-\sqrt{n^2-m^2}$ 2) $m+\sqrt{m^2+n^2}:m-\sqrt{m^2+n^2}$

3) $m+\sqrt{m^2-n^2}:m-\sqrt{m^2-n^2}$ 4) $n+\sqrt{m^2-n^2}:n-\sqrt{m^2-n^2}$

56. If the G.M. of two non-zero positive numbers is to their A.M is 12:13 then numbers are in the ratio

1) 5:8 2) 4:9 3) 6:11 4) 3:8

57. α,β,γ are the geometic means between ca, ab ; ab, bc; bc, ca respectively and a, b, c are in A.P then $\alpha^2,\beta^2,\gamma^2$ are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

Problems on H.P.

58. If $a_1, a_2, ...a_n$ are in H.P. then $a_1.a_2+a_2.a_3+a_3.a_4+......+a_{n-1}.a_n =$

1) $(n-1)\, a_1a_n$ 2) $n\, a_1a_n$ 3) $(n+1)\, a_1a_n$ 4) $(n+2)\, a_1a_n$

59. Three numbers form a H.P. the sum of the numbers is 11 and the sum of their reciprocals is 1. Then one among those numbers is

1) 3 2) 4 3) $\frac{1}{6}$ 4) $\frac{1}{2}$

60. If a, b, c are in H.P. and a^2, b^2, c^2 are in H.P. then

1) a= b = c 2) 2b= 3a+ c 3) $b^2 = \frac{ac}{8}$ 4) 2c = 3b+ a

61. If a, b, c are in A.P., p, q, r are in H.P. and ap, bq, cr are in G.P. then $\frac{p}{r}+\frac{r}{p}=$

1) $\frac{a}{c}-\frac{c}{a}$ 2) $\frac{a}{c}+\frac{c}{a}$ 3) $\frac{b}{q}+\frac{q}{b}$ 4) $\frac{b}{q}-\frac{q}{b}$

62. If a, b, c are in A.P. a, mb, c are in G.P. then a, m^2b, c are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

63. If $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$ and p, q, r, be in A.P. then x, y, z are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

64. If $H_1, H_2, H_3....H_n$ be n harmonic means between a and b then $\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}=$

1) a 2) b 3) n 4) 2 n

65. H_1, H_2 are 2 H.M's between a,b then $\frac{H_1+H_2}{H_1H_2}=$

1) $\frac{a.b}{a+b}$ 2) $\frac{a+b}{ab}$ 3) $\frac{a-b}{ab}$ 4) $\frac{ab}{a-b}$

66. If 2(y-a) is the H.M between y - x and y - z then $x-a, y-a, z-a$ are in

1) A.P 2) G.P 3) H.P 4) A.G.P

67. The G.M. of two numbers is 6. Their A.M. 'A' and H.M. H satisfy the equation 90A+5H = 918 then

1) A=10,A = 4 2) A= $\frac{1}{5}$, A=10 3) A = 5, A = 10 4) A = $\frac{1}{5}$, A = 5

Problems on A.G.P.

68. The sum i–2–3i+4+........ up to 100 terms =

1) 50(1–i) 2) 25i 3) 25(1+i) 4) 100 (1–i)

69. If $|a|<1$ and $|b|<1$, then the sum of the series $1 + (1+a)b + (1+a+a^2)b^2 + (1+a+a^2+a^3)b^3 ...$ is

1) $\dfrac{1}{(1-a)(1-b)}$ 2) $\dfrac{1}{(1-a)(1-ab)}$ 3) $\dfrac{1}{(1-b)(1-ab)}$ 4) $\dfrac{1}{(1-a)(1-b)(1-ab)}$

70. The sum of n terms of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ...$ is

1) $-\dfrac{n(n+1)}{2}$ 2) $\dfrac{n(n+1)}{2}$ 3) $-(n+1)n$ 4) $\dfrac{n(n+1)(2n+1)}{6}$

Miscelleneous

71. If $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+.........\infty = \dfrac{\pi^2}{6}$ then $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+...... =$

1) $\dfrac{\pi^2}{8}$ 2) $\dfrac{\pi^2}{12}$ 3) $\dfrac{\pi^2}{3}$ 4) $\dfrac{\pi^2}{2}$

72. The sum of first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+.....$ is $\dfrac{n(n+1)^2}{2}$ when n is even, when n is odd the sum is (AIEEE - 2004)

1) $\dfrac{n(n+1)^2}{2}$ 2) $\dfrac{n^2(n+1)}{2}$ 3) $\dfrac{n(n+1)^2}{4}$ 4) $\dfrac{n^2(n+1)^2}{4}$

73. If (1+3+5+.........+p)+(1+3+5+......+q) = (1+3+5+......+r) where each set of parenthesis contains the sum of consecutive integers, then the smallest possible value of p + q + r (p>6) is

1) 12 2) 21 3) 45 4) 54

74. Suppose a,b,c are in A.P and a^2, b^2, c^2 are in G.P. If a < b < c and a+b+c = 3/2. Then the value of a is

1) $\dfrac{1}{2\sqrt{2}}$ 2) $\dfrac{1}{2\sqrt{3}}$ 3) $\dfrac{1}{2}-\dfrac{1}{\sqrt{3}}$ 4) $\dfrac{1}{2}-\dfrac{1}{\sqrt{2}}$

75. If $a_1, a_2, ...a_n$ are in H.P then $\dfrac{a_1}{a_2+a_3+.....+a_n}, \dfrac{a_2}{a_1+a_3+.....+a_n},$ $\dfrac{a_3}{a_1+a_2+a_4.....+a_n}.....\dfrac{a_n}{a_1+a_2+a_3+....a_{n-1}}$

1) A.P 2) G.P 3) H.P. 4) A.G.P.

76. If $\dfrac{a+be^x}{a-be^x}=\dfrac{b+ce^x}{b-ce^x}=\dfrac{c+de^x}{c-de^x}$ then a, b, c, d are in

1) A. P 2) G. P 3) H. P 4) A.G.P

77. The n^{th} terms of two series 3 + 10 + 17 + ... and 63 + 65 + 67 + ... are equal, then the value of n is

1) 9 2) 13 3) 19 4) 21

78. If the sum of n terms of the series 2, 5, 8, 11, ... is 60100, then the number of terms is

1) 100 2) 200 3) 150 4) 250

79. If the ratio between the sum of first n terms of two A.P's is 7n+1 : 4n+27 then the ratio of 11^{th} terms is

1) 4:3 2) 3:4 3) 78:61 4) 148:111

80. If the sum of 8 terms of an A.P. is equal to the sum of 5 terms of the A.P. then the sum of 13 terms is

1) –13 2) 13 3) 0 4) 40

81. If $a_1, a_2, a_3, a_4,$........ are in A.P. such that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=450$ then $a_1+a_8+a_{17}+a_{24}=$

1) 100 2) 150 3) 300 4) 450

WORK SHEET - II

1. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. then

1) a, b, c are in A.P. 2) a^2, b^2, c^2 are in A.P.

3) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. 4) $\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$ are in A.P.

2. If three non - zero numbers x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y, tan^{-1} z$ are in A.P. then

1) $x^2 = yz$ 2) $x^2 + z^2 = y^2$ 3) x = y = z 4) $x^2 + y^2 = z^2$

3. The numbers $3^{2\sin 2x-1}, 14, 3^{4-2\sin 2x}$ form first three terms of an A.P. then x =

1) $\frac{n\pi}{2}, n \in z$ 2) $\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}, n\in z$ 3) $\frac{n\pi}{2}+(-1)^n\frac{\pi}{12}, n\in z$ 4) $\frac{n\pi}{2}+(-1)^n\frac{\pi}{6}, n\in z$

4. If a, b, c, d, e, f are in A.P. then e - c =

1) 2 (c - a) 2) 2 (d - c) 3) 2 (f - d) 4) d - c

5. An employee gets Rs 300 per month in his 11^{th} year of service and got Rs 495 per month in his 24^{th} year of service. If his monthly salary is in A.P his initial salary and his incriment are

1) Rs 200,10 2) Rs 300,Rs10 3) Rs150, Rs 20 4) Rs 150,Rs 15

6. The interior angles of a polygon are in A.P. The smallest angle is 120^0 and the common difference is 5^0. Then the number of sides of the polygon is

1) 6 2) 7 3) 8 4) 9

7. Sum of all odd integers between 2 and 100 that are divisible by 3 is

1) 864 2) 867 3) 870 4) 873

8. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

1) 3050 2) 3150 3) 3250 4) 2550

9. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b then the value of n is

1) 0 2) 1 3) – 1 4) 1/2

10. n A.M.'s are inserted between 2 and 100 then sum of n A.M.'s is

1) 50 n 2) 51 n 3) 50 4) 51

11. n A.M's are inserted between n and 3n +2 then 10th A.M. is

1) n + 19 2) n + 20 3) n + 21 4) n + 22

12. Let t_r be the rth term of an A.P. whose first term is 'a' and common difference is d. If for some positive integers,

m, n $(m \neq n)$ $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a - d =

1) 0 2) 1 3) $\frac{1}{mn}$ 4) $\frac{1}{m}+\frac{1}{n}$

13. In a G.P. if the first term is 3, n^{th} term is 96 and the sum of n terms is 189, then the number of terms is

1) 5 2) 6 3) 8 4) 9

14. Sum of n terms of the series 6 + 66 + 666 ... is

1) $\frac{1}{3}(10^n - 1)$ 2) $\frac{2}{27}(10^{n+1} - 9n - 10)$ 3) $\frac{4}{27}(10^{n+1} - 9n - 10)$ 4) $\frac{8}{3}(10^n - 1)$

15. If x, 2x + 2, 3x + 3 are in G.P. then the fourth term is

1) 27 2) $-\frac{27}{2}$ 3) $\frac{27}{2}$ 4) - 27

16. If the p^{th} term of a G.P. is α and qth term is β then nth term is

1) $\left(\frac{\alpha^{n-p}}{\beta^{n-q}}\right)^{\frac{1}{p-q}}$ 2) $\left(\frac{\alpha^{n-q}}{\beta^{n-p}}\right)^{\frac{1}{p-q}}$ 3) $\left(\frac{\alpha^{n-q}}{\beta^{n-p}}\right)^{\frac{1}{q-p}}$ 4) $\left(\frac{\alpha^{n-p}}{\beta^{n-q}}\right)^{\frac{1}{q-p}}$

17. If $A = 1 + r^a + r^{2a} + r^{3a} + ...\infty$ and $B = 1 + r^b + r^{2b} + ...\infty$ then $\frac{a}{b} =$

1) $\log_{(1-B)}^{(1-A)}$ 2) $\log_{\left(\frac{B-1}{B}\right)}^{\left(\frac{A-1}{A}\right)}$ 3) $\log_B^A$ 4) $\log_A^B$

18. If $x = 1 + a + a^2 + ...\infty$, $y = 1 + b + b^2 + ...\infty$ where a and b are proper fractions, then $1 + ab + a^2b^2 + ...\infty =$

1) $\frac{xy}{x+y-1}$ 2) $\frac{x+y}{x-y}$ 3) $\frac{x^2+y^2}{x-y}$ 4) $\frac{xy}{x-y-1}$

19. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the G.M. between a and b, then the value of n is

1) 0 2) 1 3) $\frac{1}{2}$ 4) - 1

20. If the sum of an infinitely decreasing G.P. is 3, and the sum of the squares of its terms is $\frac{9}{2}$, then the sum of the cubes of the terms is

1) $\frac{105}{13}$ 2) $\frac{108}{13}$ 3) $\frac{729}{8}$ 4) $\frac{108}{9}$

21. The sides a, b, c of ΔABC are in G.P, and loga - log2b, log2b - log3c, log3c - loga are in A.P., then ΔABC is

1) acute angled 2) obtuse angled 3) right angled 4) triangle does not exist

22. If 8 G.M.'s are inserted between 2 and 3 then the product of the 8 G.M.'s is

1) 6 2) 36 3) 216 4) 1296

23. If the A .M of the roots of a quadratic in x is 3 and G.M is $2\sqrt{2}$, then the quadratic is

1) $x^2 - 3x + 8 = 0$ 2) $x^2 - 6x + 2\sqrt{2} = 0$ 3) $x^2 - 6x + 8 = 0$ 4) $x^2 - 3x + 2\sqrt{2} = 0$

24. In a geometrical progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals to

1) $\sqrt{5}$ 2) $\frac{\sqrt{5}-1}{2}$ 3) $\frac{1-\sqrt{5}}{2}$ 4) $\frac{\sqrt{5}}{2}$

25. If $x=\sum_{n=0}^{\infty}a^n,\ y=\sum_{n=0}^{\infty}b^n,\ z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P. and $|a|<1,|b|<1,|c|<1$ then x, y, z are in

1) A.P. 2) G. P 3) H.P 4) A.G.P

26. The least value of n for which $1+2+2^2+....n$ terms is greater than 100 is

1) 7 2) 8 3) 9 4) 10

27. The sum to infinity of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+....=$

1) 1/5 2) 7/24 3) 5/48 4) 3/16

28. If p^{th}, q^{th}, r^{th} terms of a G.P are a,b,c then $\sum(q-r)\log a=$

1) 0 2) 1 3) pqr 4) abc

29. If g_1, g_2, g_3 are three geometric means between two positive numbers a,b then $g_1g_3=$

1) g_2 2) g_2^2 3) $2g_2$ 4) $2g_2^2$

30. If 3rd term of an H.P. is 7 and 7^{th} term of H.P. is 3 then 10^{th} term is

1) $\frac{10}{21}$ 2) $\frac{21}{10}$ 3) $\frac{10}{7}$ 4) $\frac{3}{7}$

31. If a, b, c are in H.P then $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)=$

1) $\frac{2}{bc}+\frac{1}{b^2}$ 2) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}+\frac{1}{a^2}\right)$ 3) $\frac{3}{b^2}-\frac{2}{ab}$ 4) $\frac{1}{b}+\frac{1}{a}$

32. If the first two terms of a H.P. are $\frac{2}{5}$ and $\frac{12}{13}$ respectively then the largest term is

1) 2nd term 2) 6th term 3) 4th term 4) 5th term

33. If x, y, z are in G.P. and x+3, y+3, z+3 are in H.P. then

1) y=2 2) y=3 3) y=1 4) y=0

34. The A.M and H.M between two numbers are 27 and 12 respectively then G.M is

1) 18 2) 16 3) 20 4) 25

35. Two A.M.'s A_1 and A_2, two G.M.'s G_1, G_2 and two H.M.'s H_1, H_2 are inserted between any two non-zero positive numbers then $\frac{1}{H_1}+\frac{1}{H_2}=$

1) $\frac{1}{A_1}+\frac{1}{A_2}$ 2) $\frac{1}{G_1}+\frac{1}{G_2}$ 3) $\frac{G_1G_2}{A_1+A_2}$ 4) $\frac{A_1+A_2}{G_1G_2}$

36. The harmonic mean of two numbers is 4. Their arithmetic mean is A and geometric mean is G. If G satisifies $2A+G^2=27$, the numbers are

1) 1,13 2) 9,12 3) 3,6 4) 4,8

37. The value of $2^{1/4}.4^{1/8}.8^{1/16}........\infty$ is

1) 1 2) 2 3) 3 4) 4

38. Sum to infinity of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+.....=$

1) $\frac{16}{35}$ 2) $\frac{35}{16}$ 3) $\frac{1}{8}$ 4) $\frac{35}{8}$

39. If the sum of $1 + 4x + 7x^2 + 10x^3 + ... \infty$ (| x | < 1) is $\frac{35}{16}$ then x =

1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{5}$ 4) $\frac{1}{6}$

40. The sum of the series $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+...\infty$ is

1) $\frac{2}{9}$ 2) $\frac{-4}{9}$ 3) $\frac{4}{9}$ 4) $\frac{-2}{9}$

41. The sum of n terms of the series $1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+.....$ is

1) n^2 2) n(n+1) 3) $n\left(1+\frac{1}{n}\right)^2$ 4) $(n+1)^2$

42. If three positive real numbers a,b,c are in A.P such that abc = 4. Then the minimum possible value of b is

1) $2^{3/2}$ 2) $2^{2/3}$ 3) $2^{1/3}$ 4) $2^{5/2}$

43. Sum of the series $S=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+\frac{1}{4}(1+2+3+4)+...$ upto 20 terms is

1) 110 2) 111 3) 115 4) 116

44. If p,q,r are three positive real numbers then the value of $(p+q)(q+r)(r+p)$ is

1) > 8pqr 2) <8pqr 3) 8pqr 4) 8(p+q+r)

45. If $I_n=\int_0^{\pi/4}\tan^n x dx$ then I_2+I_4, I_3+I_5, I_4+I_6, I_5+I_7 ... are in

1) A.P 2) G.P 3) H.P 4) A.G.P

EXERCISE - I / ANSWERS

WORK SHEET - I

1) 1	2) 1	3) 3	4) 4	5) 1	6) 2	7) 4	8) 1	9) 3	10) 1
11) 4	12) 3	13) 3	14) 1	15) 1	16) 4	17) 2	18) 4	19) 3	20) 2
21) 1	22) 3	23) 2	24) 4	25) 1	26) 2	27) 1	28) 1	29) 1	30) 2
31) 1	32) 3	33) 1	34) 2	35) 1	36) 2	37) 3	38) 1	39) 4	40) 1
41) 1	42) 3	43) 4	44) 3	45) 3	46) 1	47) 1	48) 3	49) 3	50) 3
51) 3	52) 2	53) 3	54) 1	55) 3	56) 2	57) 1	58) 1	59) 1	60) 1
61) 2	62) 3	63) 3	64) 4	65) 2	66) 2	67) 2	68) 1	69) 3	70) 1
71) 1	72) 2	73) 2	74) 4	75) 3	76) 2	77) 2	78) 2	79) 4	80) 3
81) 3									

WORK SHEET - II

1) 2	2) 3	3) 3	4) 2	5) 4	6) 4	7) 2	8) 1	9) 2	10) 2
11) 2	12) 1	13) 2	14) 2	15) 2	16) 2	17) 2	18) 1	19) 3	20) 2
21) 2	22) 4	23) 3	24) 2	25) 3	26) 1	27) 4	28) 1	29) 2	30) 2
31) 3	32) 1	33) 2	34) 1	35) 4	36) 3	37) 2	38) 2	39) 3	40) 1
41) 1	42) 2	43) 3	44) 1	45) 3					

EXERCISE- II

WORK SHEET (HW) - I

(Single and More than one answer type)

1. If three positive real numbers, a, b, c are in A.P. such that $abc=4$, then the minimum value of b is

1) $2^{1/3}$ 2) $2^{2/3}$ 3) $2^{1/2}$ 4) $2^{3/2}$

2. The maximum sum of the series $20+19\frac{1}{3}+18\frac{2}{3}+$............ is

1) 310 2) 300 3) 320 4) 340

3. The largest term common to the sequences 1, 11, 21, 31, ... to 100 terms and 31, 36, 41, 46 ..... to 100 terms is

1) 381 2) 471 3) 281 4) 521

4. The number of terms common between the series 1 + 2 + 4 + 8 +........+ to 100 terms and 1+ 4 + 7+10+ .... to 100 terms is

1) 6 2) 4 3) 5 4) 10

5. Consider an A.p. $a_1, a_2, a_3,$ .... such that $a_3+a_5+a_8=11$ and $a_4+a_2=-2$, then the value of $a_1+a_6+a_7$ is

1) –8 2) 5 3) 7 4) 9

6. For an increasing A.P. $a_1, a_2, ...a_n$ if $a_1+a_3+a_5=-12$ and $a_1a_3a_5=80$, then which of the following is/are true?

1) $a_1=-10$ 2) $a_2=-1$ 3) $a_3=-4$ 4) $a_5=-2$

7. Let $a_1+a_2+a_3$ .... be terms of an A.P. If $\frac{a_1+a_2+.....a_p}{a_1+a_2+......a_q}=\frac{p^2}{q^2}$, $p\neq q$ then $\frac{a_6}{a_{21}}$ equals

1) 41/11 2) 7/2 3) 2/7 4) 11/41

8. If $a_1, a_2, a_3,$ ......a_{2n+1} are in A.P, then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+.....+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to

1) $\frac{n(n+1)}{2}\times\frac{a_2-a_1}{a_{n+1}}$ 2) $\frac{n(n+1)}{2}$ 3) $(n+1)(a_2-a_1)$ 4) $n(a_2-a_1)$

9. If $(r)_n$ denotes the number rrr ... (n digits), where r = 1,2,3, ..., 9 and $a=(6)_n, b=(8)_n, c=(4)_{2n}$, then

1) $a^2+b+c=0$ 2) $a^2+b-c=0$ 3) $a^2+b-2c=0$ 4) $a^2+b-9c=0$

10. Let A = 111 ......1 (12 digits), B = 333.....3 (6 digits) and C = 111 ....1 (6 digits), then $\frac{A-B^2}{C}$ divides

1) 6 2) 10 3) 16 4) 24

11. Let $t_n=\underbrace{1.1....1}_{n\,times}$, then

1) t_{92} is not prime 2) t_{951} is not prime 3) t_{480} is not prime 4) t_{91} is not prime

12. If a, b, c are digits, then the rational number represented by 0. $cababab$ ... is

1) $\frac{99c+ab}{990}$ 2) $\frac{99c+10a+b}{99}$ 3) $\frac{99c+10a+b}{990}$ 4) $\frac{99c-10a+b}{990}$

13. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is

1) $2-\sqrt{3}$ 2) $2+\sqrt{3}$ 3) $\sqrt{3}-2$ 4) $3+\sqrt{2}$

14. If S denotes the sum to infinity and S_n the sum of n terms of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$.........., such that $S-S_n<\frac{1}{1000}$, then the least valu of n is

1) 8 2) 9 3) 10 4) 11

15. Let a_n be the nth term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n}=\alpha$ and $\sum_{n=1}^{100} a_{2n-1}=\beta$, such that $\alpha\neq\beta$, then the common ratio is

1) $\frac{\alpha}{\beta}$ 2) $\frac{\beta}{\alpha}$ 3) $\sqrt{\frac{\alpha}{\beta}}$ 4) $\sqrt{\frac{\beta}{\alpha}}$

16. a, b, c are three distinct real numbers in a G.P. If $a + b + c = xb$, then

1) $x\leq -1$ or $x\geq 3$ 2) $x\leq -1$ or $x\geq 2$ 3) $x\leq -3$ or $x\geq 1$ 4) $x\leq -1$ or $x\geq -5$

17. If two geometric means g_1 and g_2 and one arithmetic mean A be inserted between two numbers, then $\frac{g_1^2}{g_2}+\frac{g_2^2}{g_1}$ =

1) $4A$ 2) $3A$ 3) $2A$ 4) A

18. The sum of infinite terms of a decreasing G.P. is equal to the greatest value of the function $f(x)=x^3+3x-9$ in the interval [–2, 3] and the difference between the first two terms is $f^1(0)$. The common ratio of the G.P. is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $-\frac{2}{3}$

19. Concentric circles of radii 1, 2, 3 .... 100cm are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. Then, the total area of the green regions in sq. cm is equal to

1) 1000π 2) 5050π 3) 4950π 4) 5151π

20. If $x, 2y, 3z$ are in A.P., where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is

1) 3 2) $\frac{1}{3}$ 3) 2 4) $\frac{1}{2}$

21. In a sequence of 21 terms, the first 11 terms are in A.P. with common difference 2 and the last 11 terms are in G.P. with common ratio 2. If the middle term the A.P. is equal to the middle term of the G.P., then the middle term of the entire sequence is

1) –10/31 2) 10/31 3) 32/31 4) –31/32

22. Let there be a_1, a_2, a_3..............a_n terms in G.P. whose common ratio is r. Let S_K denote the sum of first k terms of this G.P. and $S_{m-1}S_m=k\sum_{i<j}^{m}a_ia_j$, then k is

1) $\frac{r+1}{r-1}$ 2) $\frac{r}{r-1}$ 3) $\frac{r}{r+1}$ 4) $\frac{1}{r+1}$

23. If $p(x)=\frac{1+x^2+x^4+\ldots+x^{2n-2}}{1+x+x^2+\ldots+x^{n-1}}$ is a polynomial in x, then n can be

1) 5 2) 10 3) 20 4) 17

24. If a, b, c, d are in G.P. while $a-2, b-7, c-9, d-5$ are in A.P., then $a + b + c + d$ is divisible by
1) 2 2) 3 3) 5 4) 7

25. For the series, $S = 1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}(1+2+3+4)^2+....$
1) 7^{th} term is 16
2) 7^{th} term is 18
3) sum of first 10 terms is $\frac{505}{4}$
4) sum of first 10 terms is $\frac{405}{4}$

26. If three successive terms of a G.P. with common ratio r(r > 1) form the sides of a ΔABC and [r] denotes greates integer function, then $[r] + [-r] =$
1) 0 2) 1 3) –1 4) –1/2

27. The solution of the equations $\log x + \log x^{1/2} + \log x^{1/4} + ... = y$ and $\frac{1+3+5+....+(2y-1)}{4+7+10+....+(3y+1)} = \frac{20}{7\log x}$ is
1) $x = 10^5, 10^{-5/7}$ 2) $y = 10, -\frac{10}{7}$ 3) $x = 10, -\frac{10}{7}$ 4) $y = 10^5, 10^{-5/7}$

28. If $H_1, H_2, ... H_{20}$ harmonic means between 2 and 3, then $\frac{H_1+2}{H_1-2}+\frac{H_{20}+3}{H_{20}-2} =$
1) 20 2) 21 3) 40 4) 38

29. Let a_1, a_2, a_3, a_4 and a_5 be such that a_1, a_2 and a_3 are in A.P., a_2, a_3 and a_4 are in G.P,. and a_3, a_4 and a_5 are in H.P. Then $\log_e a_1, \log_e a_3$ and $\log_e a_5$ are in
1) G.P. 2) A.P. 3) H.P. 4) none of these

30. The 15^{th} term of the series $2\frac{1}{2}+1\frac{7}{13}+1\frac{1}{9}+\frac{20}{23}+.........$ is
1) $\frac{5}{12}$ 2) $\frac{10}{21}$ 3) $\frac{10}{23}$ 4) $\frac{11}{23}$

31. If x, y, z are real and $4x^2+9y^2+16z^2-6xy-12yz-8zx=0$, then x, y, z are in
1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

32. If $x^2+9y^2+25z^2 = xyz\left(\frac{15}{x}+\frac{5}{y}+\frac{3}{z}\right)$, then
1) $x, y,$ and z are in H.P.
2) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.
3) x, y, z are in G.P.
4) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in G.P.

33. Given that x+y+z=15 where a, x, y, z, b are in A.P. and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$ when a, x, y, z, b are in H.P. Then
1) G.M. of a and b is 3
2) one possible value of $a + 2b$ is 11
3) A.M. of a and b is 6
4) greatest value of a–b is 8

34. If $A_1, A_2; G_1, G_2;$ and H_1, H_2 are two arithmetic, geometric and harmonic means respectively, between two quantities a and b, then ab is equal to

1) A_1H_2 2) A_2H_1 3) G_1G_2 4) none of these

35. There are two numbers a and b whose product is 192 and the quotient of A.M. by H.M. of their greatest common divisor and least common multiple is $\frac{169}{48}$. The smaller of a and b is

1) 2 2) 4 3) 6 4) 12

36. Suppose a, $b > 0$ and x_1, x_2, x_3 ($x_1 > x_2 > x_3$) are roots of $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$ and $x_1 - x_2 - x_3 = c$, then a, b, c are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

37. If 0.272727 ..., x and 0.727272 .... are in H.P., then x must be

1) rational 2) integer 3) irrational 4) Natural number

38. If a_1, a_2, a_3, a_4 are in H.P., then $\frac{1}{a_1a_4}\sum_{r=1}^{3} a_r a_{r+1}$ is a root of

1) $x^2 + 2x + 15 = 0$ 2) $x^2 + 2x - 15 = 0$ 3) $x^2 - 6x - 8 = 0$ 4) $x^2 - 9x + 20 = 0$

39. If $H_n = 1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$, then the value of $1+\frac{3}{2}+\frac{5}{3}+...+\frac{2n-1}{n}$ is

1) $n - H_n$ 2) $2n - H_n$ 3) $(n-1) - H_n$ 4) $n - 2H_n$

40. If S_1, S_2 and S_3 denote the sums up to $n > 1$ terms of three sequences in A.P. whose first are unity and common differences are in H.P. then n =

1) $\frac{2S_3S_1+S_1S_2+S_2S_3}{S_1-2S_2+S_3}$ 2) $\frac{2S_3S_1-S_1S_2-S_2S_3}{S_1+2S_2+S_3}$ 3) $\frac{2S_3S_1-S_1S_2-S_2S_3}{S_1-2S_2+S_3}$ 4) $\frac{S_3S_1-S_1S_2-S_2S_3}{S_1-2S_2+S_3}$

41. The sum of the infinite A.G.P. 3, 4, 4....... is

1) 27 2) 30 3) 24 4) 25

42. The sum of the infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+$.................. is

1) 2 2) 3 3) 4 4) 6

43. The sum to n terms of the series $1+5\left(\frac{4n+1}{4n-3}\right)+9\left(\frac{4n+1}{4n-3}\right)^2+13\left(\frac{4n+1}{4n-3}\right)^3+$..... is

1) n (4n–3) 2) n(4n + 3) 3) (n + 1) (4n) 4) (n +1) (4n –3)

44. Sum to 10 terms of the series $1 + 2(1.1) + 3(1.1)^2 + 4(1.1)^3 +$ ...... is

1) 85.12 2) 92.5 3) 96.75 4) 100

45. The sum of the series $1 + 2.2 + 3.2^2 + 4.2^3 + 5.2^4 + .. + 100.2^{99}$ is

1) $99.2^{100}+ 1$ 2) 100.2^{100} 3) 99.2^{100} 4) $99.2^{100} + 1$

46. The sum to 50 terms of the series $1+2\left(1+\frac{1}{50}\right)+3\left(1+\frac{1}{50}\right)^2+$................ is

1) 2500 2) 2550 3) 2450 4) 2650

47. Let $S=\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+\ldots$ up to ∞. Then S is equal to

1) 40/9 2) 38/81 3) 36/171 4) $\frac{18}{171}$

48. The sum of series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots\infty$ is

1) 7/16 2) 5/16 3) 105/64 4) 35/16

49. If t_n denotes the n^{th} term of the series 2 + 3 + 6 + 11 + 18 +... then t_{50} is

1) 49^2-1 2) 49^2 3) 50^2+1 4) 49^2+2

50. The coefficient of n in the n^{th} term of the sequence 1, 2, 5, 10, 17, 26 ....

1) –2 2) –1 3) 3 4) 2

51. The sum of 1 + 3 + 7 + 15 + 31+... to 100 terms is

1) $2^{100}-102$ 2) $2^{99}-101$ 3) $2^{101}-102$ 4) $2^{101}-19$

52. Sum to n terms of the series 2 +5 + 14 + 41 + ... is

1) $\frac{n}{2}+\frac{1}{4}(3^n-1)$ 2) $\frac{n}{2}+\frac{3}{4}(3^n-1)$ 3) $\frac{n}{2}+\frac{1}{2}(3^n-1)$ 4) $\frac{n}{2}+\frac{1}{3}(3^n-1)$

53. The sum to 50 terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots$ is

1) $\frac{100}{17}$ 2) $\frac{150}{17}$ 3) $\frac{200}{51}$ 4) $\frac{50}{17}$

54. If $t_n=\frac{1}{4}(n+2)(n+3)$ for n = 1, 2, 3, ..., then $\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\ldots\ldots+\frac{1}{t_{2003}}=$

1) $\frac{4006}{3006}$ 2) $\frac{4003}{3007}$ 3) $\frac{4006}{3008}$ 4) $\frac{4006}{3009}$

55. $\sum_{r=1}^{50}\left(\frac{1}{49+r}-\frac{1}{2r(2r-1)}\right)=$

a) $\frac{1}{10}$ 2) $\frac{1}{100}$ 3) $\frac{1}{5}$ 4) $\frac{1}{500}$

56. $\sum_{r=1}^{10}\frac{r}{1+r^2+r^4}=$

1) $\frac{33}{111}$ 2) $\frac{1}{111}$ 3) $\frac{5}{111}$ 4) $\frac{55}{111}$

57. If $S_n=\sum_{r=1}^{n}t_r=\frac{1}{6}n(2n^2+9n+13)$, then $S_n=\sum_{r=1}^{n}\sqrt{t_r}$, equals

1) $\frac{1}{2}n(n+1)$ 2) $\frac{1}{2}n(n+2)$ 3) $\frac{1}{2}n(n+3)$ 4) $\frac{1}{2}n(n+5)$

58. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{1\times3\times5\times7\times9\times.....\times(2r+1)}$ is equal to

1) $\frac{1}{3}$ 2) $\frac{3}{2}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

59. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\ to\ \infty=\frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......$ equals

1) $\pi^2/8$ 2) $\pi^2/12$ 3) $\pi^2/3$ 4) $\pi^2/2$

60. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....$ up to $\infty=\frac{\pi^4}{90}$, then the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.....$ up to ∞ is

1) $\frac{\pi^4}{45}$ 2) $\frac{\pi^4}{96}$ 3) $\frac{\pi^4}{124}$ 4) $\frac{\pi^4}{85}$

61. The largest term of the sequence $\frac{1}{503},\frac{4}{524},\frac{9}{581},\frac{16}{692},.....$ is

1) $\frac{16}{692}$ 2) $\frac{4}{524}$ 3) $\frac{49}{1529}$ 4) $\frac{49}{1592}$

62. If $\langle a_n\rangle$ and $\langle b_n\rangle$ be two sequences given by $a_n=(x)^{1/2^n}+(y)^{1/2^n}$ and $b_n=(x)^{1/2^n}-(y)^{1/2^n}$ for all $n\in N$. Then $a_1a_2a_3\ .....\ a_n$ is equal to

1) $x-y$ 2) $\frac{x+y}{b_n}$ 3) $\frac{x-y}{b_n}$ 4) $\frac{xy}{b_n}$

63 If $a+b+c=3$ and $a>0$, $b>0$, $c>0$, then the greatest value $a^2b^3c^2$ is

1) $\frac{3^{10}.2^4}{7^7}$ 2) $\frac{3^9.2^4}{7^7}$ 3) $\frac{3^8.2^4}{7^7}$ 4) $\frac{3^{10}.2^2}{7^7}$

64. In the sequence 1, 2, 2, 3, 3, 3,........where n occurs n times, then the 2007th term is divisible by

1) 3 2) 5 3) 7 4) 11

65. Consider the sequence 1, 22, 4444, 88888888, ... Then 1025th term will be

1) 2^9 2) 2^{11} 3) 2^{10} 4) 2^{12}

66. The value of $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{\substack{k=0\\(i\neq j\neq k)}}^{\infty}\frac{1}{3^i3^j3^k}$

1) $\frac{71}{208}$ 2) $\frac{81}{218}$ 3) $\frac{81}{208}$ 4) $\frac{8}{211}$

67. If $S_n=1^2-2^2+3^2-4^2+5^2-6^2+.....$, then

1) $S_{40}=-820$ 2) $S_{2n}>S_{2n+2}$ 3) $S_{51}=1275$ 4) $S_{2n+1}>S_{2n-1}$

WORK SHEET (HW) – II

Comprehension Type Questions

Passage – I

Consider three distinct real numbers *a, b, c* in a G.P. with $a^2 + b^2 + c^2 = t^2$ and $a + b + c = \alpha t$. Sum of the common ratio and its reciprocal is denoted by S .

1. Complete set of α^2 is

1) $\left(\frac{1}{3},3\right)$ 2) $\left[\frac{1}{3},3\right]$ c) $\left(\frac{1}{3},3\right)-\{1\}$ 4) $\left(-\infty,\frac{1}{3}\right)\cup(3,\infty)$

2. Complete set of S is

1) (–2, 2) 2) $(-\infty,2)\cup(2,\infty)$ 3) (–1, 1) 4) $(-\infty,-1)\cup(1,\infty)$

3. If *a, b* and *c* also represent the sides of a triangle, then the complete set of α^2 is

1) $\left(\frac{1}{3},3\right)$ 2) (2, 3) 3) $\left[\frac{1}{3},2\right]$ 4) $\left(\frac{\sqrt{5}+3}{2},3\right)$

Passage – II

Consider the sequence in the form of groups(1), (2, 2), (3, 3, 3), (4, 4, 4, 4,), (5, 5, 5, 5, 5), ...

4. The 2000th term of the sequence is

1) 3 2) 9 3) 7 4) 63

5. The sum of first 2000 terms is

1) 84336 2) 96324 3) 78466 4) 78664

6. The sum of the remaining terms in the group after 2000th term in which 2000th term lies is

1) 1088 2) 1008 3) 1040 4) 1060

Passage – III

**Let $T_1, T_2, T_3, \ldots\ldots\ldots\ldots T_n$ be the terms of a sequence and let $(T_2 - T_1) = T'_1$, $(T_3 - T_2) = T'_2$,...........
$(T_n - T_{n-1}) = T'_{n-1}$.**

Case – I:

If $T'_1, T'_2, \ldots\ldots\ldots T'_{n-1}$ are in A.P., then T_n is quadratic in 'n'. If $T'_1 - T'_2, T'_2 - T'_3, \ldots\ldots\ldots$ are in A.P., then T_n is cubic in *n*.

Case – II:

If $T'_1, T'_2, \ldots\ldots\ldots T'_{n-1}$ are not in A.P., but in G.P., then $T_n = ar^n + b$, where *r* is the common ratio of the G.P. $T'_1, T'_2, T'_3, \ldots\ldots$ and $a, b \in R$. Again,

if $T'_1, T'_2 \ldots\ldots T'_{n-1}$ are not in G.P. but $T'_2 - T'_1, T'_3 - T'_2, \ldots\ldots\ldots$, are in G.P., then T_n is of the form ar^n+bn+c $a, b, c \in R$.

7. The sum of 20 terms of the series 3 + 7 + 14 + 24 + 37 + ... is

1) 4010 2) 3860 3) 4240 4) 3680

8. The 100th term of the series 3 + 8 + 22 + 72 + 226 + 1036 + ... is divisible by 2^n, then maximum value of *n* is

1) 4 2) 2 3) 3 4) 5

9. For the series 2 + 12 + 36 + 80 + 150 + 252 + ..., the value of $\lim_{n\to\infty}\frac{T_n}{n^3}$ is (where T_n is nth term)

1) 2 2) 1/2 3) 1 4) 1/3

Passage – IV

Let $n \in N$. The A.M., G.M., H.M. and R.M.S. (root–mean square) of the *n* numbers n+1, n+2, n+3, ..., n+n are A_n, G_n, H_n, R_n respectively. Then

10. $\lim_{n\to\infty} \frac{A_n}{n} =$

1) 1 2) $\frac{3}{2}$ 3) 2 4) $\frac{1}{2}$

11. $\lim_{n\to\infty} \frac{G_n}{n} =$

1) $\frac{1}{e}$ 2) $\frac{2}{e}$ 3) $\frac{3}{e}$ 4) $\frac{4}{e}$

12. $\lim_{n\to\infty} \frac{H_n}{n} =$

1) ln2 2) $\frac{1}{\ln 2}$ 3) 1 4) e

13. $\lim_{n\to\infty} \frac{R_n}{n}$

1) $\sqrt{3}$ 2) $\sqrt{\frac{5}{3}}$ 3) $\sqrt{\frac{7}{3}}$ 4) 3

Passage – V

Let $(x+1)(x+2)(x+3)\,.....\,(x+n) = x^n + A_1x^{n-1} + A_2x^{n-2} + A_3x^{n-3} + ... + A_n$

14. $A_1 + A_n$

1) $\frac{n}{2}+n!$ 2) $\frac{n+1}{2}+n!$ 3) $\frac{n(n+1)}{2}+n!$ 4) (n + 1)!

15. $A_2 =$

1) $\frac{(n+1)n(n+1)}{12}$ 2) $\frac{n(n+1)(3n+1)}{12}$ 3) $\frac{(n+1)(3n+1)}{24}$ 4) $\frac{(n+1)n(n+1)(3n+2)}{24}$

WORK SHEET (HW) – III

(Matching Type Questions)

1. **COLUMN – I** | **COLUMN – II**

1) If *a, b, c* are in G.P., then $\log_a 10, \log_b 10, \log_c 10$ are in — p) A.P.

2) If $\frac{a+be^x}{a-be^x} = \frac{b+ce^x}{b-ce^x} = \frac{c+de^x}{c-de^x}$, then *a, b, c, d* are in — q) H.P.

3) If *a,b,c* are in A.P., *a,x, b* are in G.P. and *b,y,c* are in G.P., then x^2, b^2, y^2 are in — r) G.P.

4) If *x,y,z* are in G.P., $a^x = b^y = c^z$, then log *a*, log *b*, log *c* are in — 4) A.G.P.

2. **COLUMN – I** | **COLUMN – II**

1) If $\sum n = 210$, then $\sum n^2$ is divisible by the greatest prime number which is greater than — p. 16

2) Between 4 and 2916 is inserted odd number (2n + 1) G.M.'s. Then the(n + 1)th G.M. is divisible by greatest odd integer which is less than — q) 10

3) In a certain progression, three consecutive erms are 40, 30, 24, 20. Then the integral part of the next term of the progression is more than — r) 34

4) $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+....$ to $\infty = \frac{a}{b}$, where H.C.F. (a, 2) = 1, — s) 30

then a – b is less than

3. **COLUMN – I** | **COLUMN – II**

COLUMN – I	COLUMN – II
1) $1^2 - 2^2 + 3^2$............ to 21 terms	p) 680
2) $1^3 - 2^3 + 3^3 - 4^3 + $..... to 15 terms	q) 2556
3) $1^2 + 3^2 + 5^3 + $... to 8 terms	r) 1856
4) $1^3 + 3^3 + 5^3 + $... to 6 terms	s) 231

4. **COLUMN – I** | **COLUMN – II**

COLUMN – I	COLUMN – II
1) If the first term of an infinite G.P. is 1 and each term is twice the sum of the suceeding terms, then the common ratio is	p) $\frac{2}{9}$
B) Sum to infinity of the series $\frac{2}{3} - \frac{5}{6} + \frac{2}{3} = \frac{11}{24} + \dots$ is	q) $\frac{3}{2}$
C) $\lim\limits_{n\to\infty} (1 + 3^{-1})(1 + 3^{-2})(1 + 3^{-4})(1 + 3^{-8}) \dots (1 + 3^{-2^n}) =$	r) 1
D) If $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$, then $a + b + c + d + e =$	s) $\frac{1}{3}$

5. **COLUMN – I** | **COLUMN – II**

COLUMN – I	COLUMN – II
A) If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in	p) A.P.
B) If $2(y - 1)$ is the H.M. between $y - x$, $y - z$ then $x - a$, $y - a$, $z - a$ are in	q) G.P.
C) If three numbers are in H.P., then the numbers obtained by subtracting half of the middle number from each of them are in	r) H.P.
D) If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in	s) A.G.P.

WORK SHEET (HW) – IV

(Integer type Questions)

1. In the series 3, 7, 11, 15, . and 2, 5, 8, ... each continued to 100 terms, and number of terms that are identical is 5λ then λ is
2. If p, q, r are in A.P. and x, y, z are in G.P., then $x^{q-r}.\, y^{r-p}.z^{p-q} =$.
3. If $\sum_{j=1}^{21} a_j = 693$, where $a_1, a_2, \dots, a_{21}$ are in A.P. and $\sum_{i=0}^{10} a_{2i+1} = 5k + l$ then $\frac{k+l}{11} =$
4. Let $p, q, r \in R^+$ and $27\, pqr \geq (p + q + r)^3$ and $3p + 4q + 5r = 12$ then $p^3 + q^4 + r^5 =$ .....
5. If n arithmetic means $a_1, a_2, \dots, a_n$ are inserted between 50 and 200 and n harmonic means $h_1, h_2, \dots h_n$ are inserted between the same two numbers, then $a_2\, h_{n-1}$ is equal to K then $\frac{K-9000}{500} =$
6. If a, b, c are three positive unequal numbers in H.P. then $a^5 + c^5 > kb^5$, where $k =$
7. If 9 A.M.s and again 9 H.M.s are inserted between 2 and 3 and if A is any $A.M.$ and H, the corresponding H.M., then $A + \frac{6}{H} =$ .....
8. If pth, qth and rth terms of both an A.P. and a G.P. be respectively a, b and c, then $a^b.b^c.c^a. - a^c.b^a.c^b =$
9. If x = 111.....1 (20 digits), y = 333 ....3 (10 digits) and z = 222 ....2 (10 digits), then $\frac{x^2 - y^2}{z} =$
10. If a, b, c are positive, and $[(1+1)\,(1+2)\,(1+c)]^7 > 7^k\, a^4 b^4 c^4$ then k is

EXERCISE - II / ANSWERS

WORK SHEET (HW) - I

1) 1	2) 1	3) 4	4) 3	5) 3	6) 134	7) 4	8) 1	9) 2	10) 3
11) 1234	12) 3	13) 4	14) 4	15) 1	16) 1	17) 3	18) 2	19) 2	20) 2
21) 1	22) 3	23) 1	24) 2	25) 13	26) 3	27) 12	28) 3	29) 2	30) 1
31) 3	32) 12	33) 124	34) 123	35) 2	36) 3	37) 1	38) 2	39) 2	40) 3
41) 1	42) 2	43) 1	44) 4	45) 4	46) 1	47) 2	48) 4	49) 4	50) 13
51) 3	52) 2	53) 1	54) 4	55) 2	56) 4	57) 3	58) 3	59) 1	60) 2
61) 3	62) 3	63) 1	64) 1	65) 3	66) 3	67) 123			

WORK SHEET (HW) - II

1) 3	2) 3	3) 4	4) 4	5) 1	6) 2	7) 3	8) 3	9) 3	10) 2
11) 4	12) 2	13) 3	14) 3	15) 4					

WORK SHEET (HW) - III

1) 1–q, 2–r, 3–p, 4–r
2) 1–p,q,r, s ; 2–r,s; 3–p,q; 4–r, s
3) 1–s, 2–r, 3–p, 4–q
4) 1 – s; 2 – p; 3 – q; 4 – r
5) 1 – q; 2 – q; 3 – q; 4 – p

WORK SHEET (HW) - IV

1) 5	2) 1	3) 9	4) 3	5) 2	6) 2	7) 5	8) 0	9) 1	10) 7