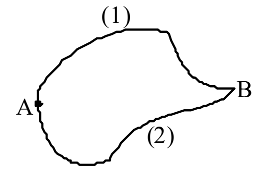
ONE DIMENSION MOTION

1. DISTANCE

Distance is the actual length of the path. It is the characteristic property of any path i.e. path is always associated when we consider distance between two positions. Distance between A and B while moving through path (A) may or may not be equal to the distance between A and B while moving through path (B).



(i) It is a scalar quantity

(ii) Dimension : $[M^0L^1T^0]$

(iii) Unit : In C.G.S. centimeter (cm), In M.K.S. (m)

2. DISPLACEMENT

Displacement of a particle is a position vector of its final position w.r.t. initial position.

Displacement = $\overrightarrow{AB} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

It is the characteristic property of any point i.e. depends only on final and initial positions.

To Determine The Position Vector :

Position vector of A w.r.t. $O = \overrightarrow{OA}$

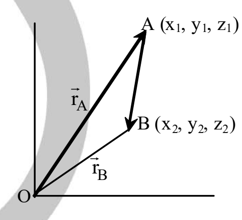
$\Rightarrow \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Position Vector of B w.r.t. O = $\overrightarrow{OB}$

$\Rightarrow \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

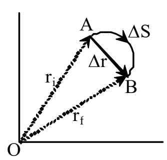
Position vector of B w.r.t. A;

$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$



3. COMPARATIVE STUDY OF DISTANCE & DISPLACEMENT

Distance is the actual path travelled by a moving body, while displacement is the change in the position.



In the above figure distance travelled is ΔS, while displacement is $\overrightarrow{\Delta r} = \vec{r}_f - \vec{r}_i$

3.1 Regarding distance and displacement it is worth noting that :

(A) Distance is scalar, while displacement is vector both having same dimensions [L] and same SI unit metre.

(B) The magnitude of displacement is equal to minimum possible distance so,
Distance $\geq$ |Displacement|

(C) For motion between two points displacement is single valued, while distance depends on actual path and so can have many values.

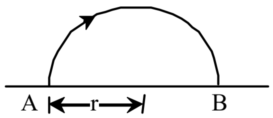
(D) For a moving particle distance can never decreases with time while displacement can. Decrease in displacement means body is moving towards the initial position.

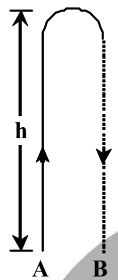
(e) For moving particle distance can never be negative or zero, while displacement can be. (Zero displacement means that body after motion has come back to initial position)
Distance > 0 but |Displacement| > = or < 0

(f) In general magnitude of displacement is not equal to distance. However it can be so if the motion is along a straight line without change in direction.

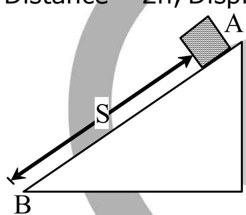
Comparative study of distance & displacement :

Distance = πr ,
|Displacement| = 2r
|Displacement| = s



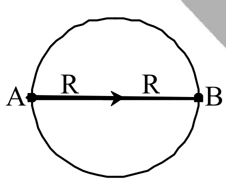


Distance = 2h, Displacement = 0



Distance = s, |Displacement| = s

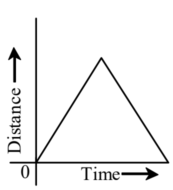
Note : Distance and Displacement, while moving in a circle from A to B and then from B to A.

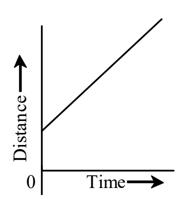


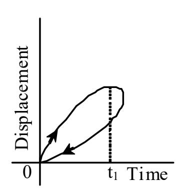
	Half Cycle	Full Cycle
Distance	πR	$2\pi R$
Displacement	2R	0
Direction	1. $A \to B$, When particle moves from A to B 2. $B \to A$, When particle moves from B to A	

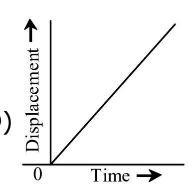
Example based on concept of distance and displacement

Ex.1 Which of the following graph(s) is / are not possible ?

(A) 

(B) 

(C) 

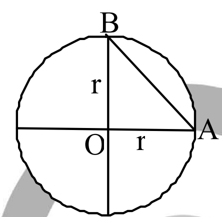
(D) 

Sol. The (A) graph shows that with increase in time distance first increases and then decreases. However, distance can never decrease with time so this graph is not physically possible. The graph (C) shows that at certain instant of time (t_1) body is present at two positions. Also it shows that time first increases then decreases. These conditions are not possible physically.
Hence correct answer is (A) and (C).

Ex.2 A body covered a distance of L m along a curved path of a quarter circle. The ratio of distance to displacement is-

(A) $\pi/2\sqrt{2}$ (B) $2\sqrt{2}/\pi$ (C) $\pi/\sqrt{2}$ (D) $\sqrt{2}/\pi$

Sol. Length of quarter circle path = L = $2\pi r/4$
$\therefore$ r = $2L/\pi$
Hence displacement AB = $\sqrt{r^2+r^2}=\sqrt{2}$ r.



From ΔOAB, magnitude of displacement = AB

$= \sqrt{2}\,r \;\therefore\; \frac{\text{Distance}}{\text{Displacement}} = \frac{2\pi r/4}{\sqrt{2}.r} = \pi/2\sqrt{2}$

Hence correct answer is (A).

Ex.3 An old man goes for morning walk on a semicircular track of radius 40 m ; if he starts from one end of the track and reaches to other end, the distance covered by the man and his displacement will respectively be-

(A) 126 m, 80 m (B) 80 m, 126 m (C) 80 m, 252 m (D) 252 m, 80 m

Sol. Distance covered by man = Length of the path = $\pi R = \pi \times 40$ = 126 m
Displacement of the man = The least distance between initial and final points = Diameter of semicircular path = 2R = 2 × 40 = 80 m
The direction of displacement will be from initial point to final point.
Hence correct answer is (A).

4. SPEED

It is the distance covered by the particle in one second.

(i) It is a scalar quantity
(ii) Unit : In M.K.S. m/s or km/sec.
In C.G.S. cm/sec
(iii) Dimension : $[M^0L^1T^{-1}]$

Types of speed :

(A) Instantaneous speed
(B) Average speed
(C) Uniform speed
(D) Non-uniform speed

(A) Instantaneous Speed :
It is the speed of a particle at particular instant. Instantaneous speed = $\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}$

(B) Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

(C) Uniform speed : If during the entire motion speed of the body remains same, the body is said to have uniform speed.

(D) Non-uniform speed : If speed changes, the body is said to have non-uniform speed.

5. VELOCITY

It is defined as rate of change of displacement.

(i) It is a vector quantity

(ii) Its direction is same as that of displacement

(iii) Unit and dimension : Same as that of speed

Types of Velocity :

(A) Instantaneous velocity

(B) Average velocity

(C) Uniform velocity

(D) Non-uniform velocity

(A) Instantaneous velocity : It is defined as the velocity at some particular instant.

Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(B) Average velocity :

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

(C) Uniform velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line reversing its direction.

(D) Non-uniform velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

6. COMPARATIVE STUDY OF INSTANTANEOUS SPEED AND INSTANTANEOUS VELOCITY

Instantaneous velocity or simply velocity is defined as rate of change of particle's position with time

$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ where the position $\vec{r}$ of a particle at any instant changes by $\Delta \vec{r}$ in a small time Δt

The magnitude of velocity is called speed i.e. speed = | velocity| i.e. v = $|\vec{v}|$

Note : In straight line motion there is no change in direction so $\vec{v}$ and v both have same meaning

Example Based on Relation between displacement and velocity in straight line motion

Ex.4 A particle moves along the x-axis in such a way that its x-coordinates varies with time as $x = 2 - 5t + 6t^2$. What will be its initial velocity ?

(A) 5 m/s (B) –5 m/s (C) 2 m/s (D) –2 m/s

Sol. Here displacement, $x = 2 - 5t + 6t^2$

The velocity at any instant t is given by

$v = \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2) \Rightarrow v = -5 + 12t$

This is the velocity at time t. Initially t = 0,

$\therefore v = -5$ m/s

Hence correct answer is (B).

Note : Hence speed will be $|\vec{v}| = 5$ m/s.

Ex.5 The displacement of a particle moving in one-dimensional direction under a force at time t is given by $t = \sqrt{x} + 3$, where x is in m and t in sec. The displacement of the particle, when its velocity is zero, will be-

(A) 0 (B) 3m (C) –3m (D) 2m

Sol. Given $t = \sqrt{x} + 3 \Rightarrow x = t^2 - 6t + 9 \Rightarrow \frac{dx}{dt} = 2t - 6$

$\Rightarrow$ Instantaneous velocity, $v = \frac{dx}{dt} = 2t - 6$

when v = 0, $2t - 6 = 0 \Rightarrow t = 3$ sec. Thus at t = 3 sec, $x = (t^2 - 6t + 9) = 0$

Hence correct answer is (A).

Note :

(A) Velocity is a vector while speed is a scalar having same units (m/s) and dimension $[LT^{-1}]$

(B) If during motion velocity remains constant throughout a given interval of time, the motion is said to be uniform and for uniform motion, $\vec{v}$ = constant = $\vec{v}_{av}$

However converse may or may not be true i.e. If $\vec{v} = \vec{v}_{av}$, the motion may or may not be uniform.

(C) If velocity is constant, speed (= | velocity |) will also be constant. However converse may or may not be true i.e. if speed = constant, velocity may or may not be constant as velocity has a direction in addition to magnitude which may or may not change. e.g. in case of uniform rectilinear motion. $\vec{v}$ = constant and so speed $|\vec{v}|$ = constant

while in case of uniform circular motion, v = constant but $\vec{v} \neq$ constant due to change in direction.

(D) Velocity can be positive or negative, as it is a vector but speed can never be negative as it is the magnitude of velocity

i.e. $v = |\vec{v}|$

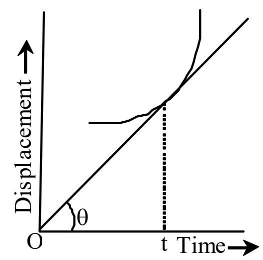
(5) If displacement is given as a function of time, the time derivative of displacement will give velocity and modulus of velocity gives speed.

e.g. $s = A_0 - A_1t + A_2t^2$, $v = \frac{ds}{dt} = -A_1 + 2A_2t$. So, initially (t = 0), velocity = $-A_1$,

while speed = $|-A_1| = A_1$

(6) As by definition, $v = \frac{ds}{dt}$, the slope of displacement versus time graph gives velocity.

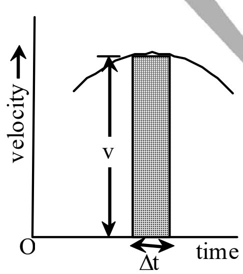
i.e. $v = \frac{ds}{dt} = \tan\theta$ = slope of s-t curve



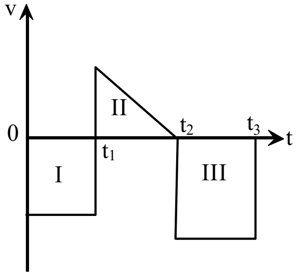
(7) As, $v = \frac{ds}{dt} \Rightarrow ds = vdt$

From figure vdt = dA. so, dA = ds

$\therefore s = \int dA = \int vdt$



Area under velocity versus time graph with proper algebraic sign gives displacement while without sign gives distance.



e.g. From the adjoining v-t graph. The distance travelled by body in time t_3 = Area I + Area II + Area III and the displacement of body = Area II – Area III – Area I

Example Based on Velocity and Speed

Ex.6 Can a body have uniform velocity but non-uniform speed ?

(A) Yes (B) No

(C) Depend on magnitude (D) Unpredictable

Sol. No.

Velocity = Speed + Direction

Hence correct answer is (B)

Ex.7 Can a body have uniform speed but non-uniform velocity ?

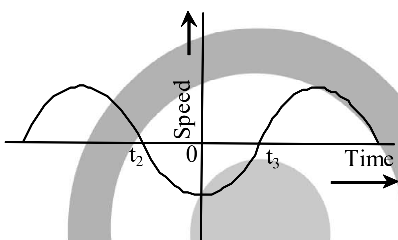
(A) Yes (B) No

(C) Depend on direction (D) Unpredictable

Sol. Yes, hence correct answer is (A).

e.g. Speed of a particle in circular path is constant but due to change in direction its velocity changes.

Ex.8 State whether the following graph can be seen in nature or not. Explain.



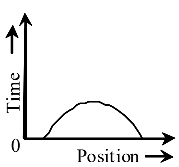
(A) Yes (B) No

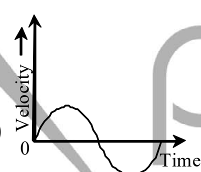
(C) Sometime (D) At a particular instant

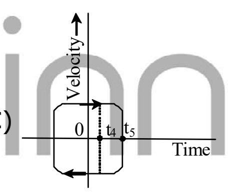
Sol. This graph shows that speed is negative for some interval of time (t_2 to t_3). Since speed can never be negative, so this graph is physically not possible.

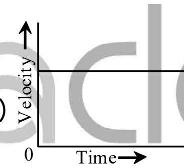
Hence correct answer is (B).

Ex.9 Out of the following graph(s), which is / are not possible ?

(A) 

(B) 

(C) 

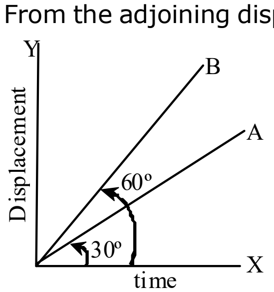
(D) 

Sol. The graph (A) shows that on increasing position (x), time first increases, then decreases, which is impossible. The graph (C) shows that at a given instant of time (t_4) particle has two velocities. Also it shows that at time (t_5) the acceleration is infinite (= slope of $\vec{v}$/t curve). Since both these conditions cannot be achieved practically, then these graphs are not possible.

Hence correct answer is (A) and (C).

Example Based on Calculation of Velocity by Displacement-Time Graph

Ex.10 From the adjoining displacement-time graph for two particles A & B the ratio of velocities $v_A : v_B$ will be-



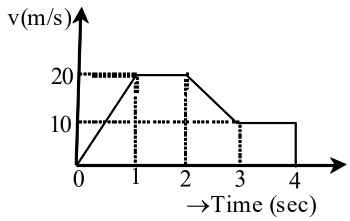
(A) 1 : 2 (B) 1 : $\sqrt{3}$ (C) $\sqrt{3}$: 1 (D) 1 : 3

Sol. The line having greater slope has greater velocity, hence the line making an angle 60º with time axis has greater velocity. Now, $\frac{v_A}{v_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

Hence correct answer is (D).

Example Based on Calculation of Distance By Velocity-Time Graph

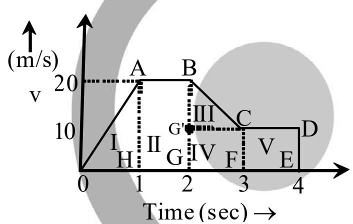
Ex.11 From the adjoining graph, the distance traversed by particle in 4 sec, is-



(A) 60 m (B) 25 m (C) 55 m (D) 30 m

Sol. The given graph can be drawn as shown in figure

Distance travelled = Area under v-t graph



= Area I + Area II + Area III + Area IV + Area V

= (1/2) (OH × AH) + HG × AH + 1/2 (G'C × BG')+ (GF × GG') + (EF × CF)

= (1/2) (1 × 20) + (1 × 20) + 1/2 (1 × 10) + (1 × 10) + (1 × 10) = 10 + 20 + 5 + 10 + 10 = 55 m

Hence correct answer is (C).

7. COMPARATIVE STUDY OF AVERAGE SPEED & AVERAGE VELOCITY

The average speed of a particle for a given interval of time is defined as the ratio of distance travelled to the time taken, while average velocity is defined as the ratio of displacement to time taken.

Average speed $= \frac{\text{Distance traveled}}{\text{Time taken}}$

i.e. $v_{av} = \frac{\Delta s}{\Delta t}$

Average velocity $= \frac{\text{Displacement}}{\text{Time taken}}$

i.e $\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$

Note :

(A) Average speed is a scalar, while average velocity is a vector both having same unit (m/s)

(B) Both have dimension $[M^0LT^{-1}]$

(C) For a given time interval average velocity is single valued, while average speed can have many values depending on path followed.

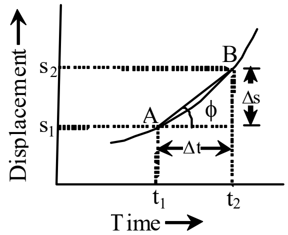
(D) If after motion body comes back to its initial position $\vec{v}_{av} = 0$ [as $\Delta \vec{r} = 0$], but $v_{av} > 0$ and finite (as $\Delta s > 0$)

(5) For a moving body average speed can never be negative or zero (unless $t \to \infty$), while average velocity can be i.e.

$v_{av} > 0$ while $\vec{v}_{av}$ > = or < 0

(6) In general average speed is not equal to magnitude of average velocity (as $\Delta s \neq |\Delta \vec{r}\,|$). However it can be so if the motion is along a straight line without change in direction (as $\Delta s = |\Delta \vec{r}\,|$).

(7) If a graph is plotted between distance (or displacement) and time, the slope of chord during a given time interval gives average speed (or velocity)



$v_{av} = \dfrac{\Delta s}{\Delta t}$ = tan ϕ = slope of chord

(8) If a particle travels distances L_1, L_2, L_3 etc at speeds v_1, v_2, v_3 etc. respectively, then

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{L_1 + L_2 + \ldots\ldots + L_n}{\frac{L_1}{v_1} + \frac{L_2}{v_2} + \ldots\ldots + \frac{L_n}{v_n}} = \frac{\Sigma L_i}{\Sigma \frac{L_i}{v_i}}$$

If $L_1 = L_2$ ------- $= L_n = L$ then $\dfrac{1}{v_{av}} = \dfrac{1}{n}\left[\dfrac{1}{v_1} + \dfrac{1}{v_2} + \ldots\ldots\right] = \dfrac{1}{n}\Sigma\dfrac{1}{v_i}$

Special Note :

If a particle moves a distance at speed v_1 and comes back with speed v_2, then $v_{av} = \dfrac{2v_1v_2}{v_1 + v_2}$ while $\vec{v}_{av} = 0$

[as displacement = 0]

(i) If a particle travels at speeds v_1, v_2 etc. for intervals t_1 , t_2 etc. respectively, then

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + \ldots\ldots}{t_1 + t_2 + \ldots\ldots} = \frac{\Sigma v_i t_i}{\Sigma t_i}$$

If $t_1 = t_2$ = ... $= t_n = t$ then $v_{av} = \dfrac{v_1 + v_2 + \ldots\ldots}{n} = \dfrac{1}{n}\Sigma v_i$

i.e. average speed is arithmetic mean of individual speeds.

Special Note :

If a particle moves for two equal time-intervals $v_{av} = \dfrac{v_1 + v_2}{2}$

Example Based on Average Speed and Average Velocity

Ex.12 A car travels first half distance between two places with a speed of 40 km/h and the rest half distance with a speed of 60 km/h. The average speed of the car will be-
(A) 100 km/hr (B) 50 km/hr (C) 48 km/hr (D) 200 km/hr

Sol. Let the total distance travelled be x. Time taken to travel first half distance

$t_1 = \dfrac{x/2}{40} = \dfrac{x}{80}$ hr

Time taken to travel the rest half distance

$t_2 = \dfrac{x/2}{60} = \dfrac{x}{120}$ hr

$\therefore$ Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{x}{(x/80) + (x/120)}$ = 48 km/hr

Hence correct answer is (C).

Ex.13 A table has its minute hand 4.0 cm long. The average velocity of the tip of the minute hand between 6.00 a.m. to 6.30 a.m. and 6.00 a.m. to 6.30 p.m. will respectively be- (in cm/s)

(A) 4.4×10^{-3}, 1.8×10^{-4} (B) 1.8×10^{-4}, 4.4×10^{-3}

(C) 8×10^{-3}, 4.4×10^{-3} (D) 4.4×10^{-3}, 8×10^{-4}

Sol. At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m. it is 180^0 away. Thus the straight line distance between the initial and final positions of the tip is equal to the diameter of the clock.

Displacement = 2 R = 2 × 4 cm = 8 cm

Time taken from 6 a. m. to 6.30 a.m. is 30 minutes = 1800 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8.0 \text{ cm}}{1800 \text{ s}} = 4.4 \times 10^{-3} \text{ cm/s}$$

Again time taken from 6 am to 6.30 p.m. = 12 hrs + 30 minutes = 45000 s

$$\therefore v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8}{45000} = 1.8 \times 10^{-4} \text{ cm/s}$$

Hence correct answer is (A).

Example Based on Comparative Study of Average Speed and Average Velocity

Ex.14 The average speed and average velocity during one complete cycle of radius R will respectively be- (T is the time to take one complete revolution)

(A) $\frac{\pi R}{T}$, 0 (B) $\frac{2\pi R}{T}, \frac{\pi R}{T}$ (C) $\frac{2\pi R}{T}, 0$ (D) $0, \frac{2\pi R}{T}$

Sol Average speed $v_{av} = \frac{2\pi R}{T}$ and

average velocity $\vec{v}_{av} = 0/T = 0$

Hence correct answer is (C).

Example Based on Formula of Average Speed

Ex.15 A boy covers a distance AB of 2 km with speed of 2.5 km/h, while going from A to B and comes back from B to A with speed 0.5 km/hr, his average speed will be-

(A) 1.5 km/hr (B) 0.83 km/hr (C) 1.2 km/hr (D) 3 km/hr

Sol. As boy goes from A to B and then comes back from B to A hence his average speed

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2} = \frac{2 \times 2.5 \times 0.5}{2.5 + 0.5} = \frac{2.5}{3} = 0.8 \text{ km/hr}$$

Hence correct answer is (B).

Example Based on Comparative Study of Average Speed and Average Velocity

Ex.16 Usually "average speed" means the ratio of total distance covered to the time elapsed. However some time the phrase " average speed" can mean the magnitude of the average velocity. Are the two same?

Sol. No, usually they have different meanings, as according to I-definition, $v_{av} = \frac{\text{distance}}{\text{time}}$, while according to II-definition $v_{av} = \frac{|\text{displacement}|}{\text{time}}$. Now as distance ≥ | displacement |, so $v_{av} \geq |\vec{v}_{av}|$

i.e. usually average speed is greater than the magnitude of average velocity

e.g. If a body returns to its starting point after some motion, then as distance travelled is finite while displacement is zero so $v_{av} > 0$ but $|\vec{v}_{av}| = 0$. However in case of motion along a straight-line without change in direction, as |displacement| = distance, the two definition will mean same.

8. ACCELERATION

It is defined as the rate of change of velocity.

(i) It is a vector quantity.

(ii) Its direction is same as that of change in velocity and not of the velocity (That is why acceleration in circular motion is towards the centre)

(iii) There are three ways possible in which change in velocity may occur

When only direction	When only magnitude changes	When both the direction and magnitude change
To change the direction net acceleration or net force should be perpendicular to direction of velocity **Example:** Uniform circular motion	In this case, net force or net acceleration should be parallel or anti-parallel to the direction of velocity. (straight line motion) **Example:** When ball is thrown up under gravity	In this case, net force or net acceleration has two components. One component is parallel or anti-parallel to velocity and another one is perpendicular to velocity **Example :** Projectile motion

Types of acceleration :

(A) Instantaneous acceleration :

It is defined as the acceleration of a body at some particular instant.

Instantaneous acceleration = $\lim\limits_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(B) Average acceleration :

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

(C) Uniform acceleration :

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line.

Example : Parabolic motion

(D) Non-uniform acceleration :

A body is said to have non-uniform acceleration, if magnitude or direction or both change during motion.

Note :

(i) Acceleration is a vector with dimensions $[LT^{-2}]$ and SI units (m/s^2)

(ii) If acceleration is zero, velocity will be constant and motion will be uniform.

(iii) However if acceleration is constant then acceleration is uniform but motion is non-uniform and if acceleration is not constant then both motion and acceleration are non-uniform.

(iv) If a force $\vec{F}$ acts on a particle of mass m then by Newton's II law $\vec{a} = \vec{F}/m$

(v) As by definition

$$\vec{v} = \frac{d\vec{s}}{dt} \quad \text{so,} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right) = \frac{d^2\vec{s}}{dt^2}$$

i.e. if $\vec{s}$ is given as a function of time, second time derivative of displacement gives acceleration.

(vi) If velocity is given as function of position then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} \Rightarrow a = v\frac{dv}{dx} \qquad \left[\text{as} \frac{dx}{dt} = v\right]$$

(vii) As acceleration $\vec{a} = \frac{d\vec{v}}{dt}$ the slope of velocity -time graph gives acceleration i.e.

$$\vec{a} = \frac{d\vec{v}}{dt} = \tan\theta$$

(viii) The slope of $\vec{a}$ -t curve, i.e. $\frac{d\vec{a}}{dt}$ is a measure of rate of non-uniformity of acceleration. However we do not define this physical quantity as it is not involved in basic laws or equation of motion.

(ix) Acceleration can be positive or negative. Positive acceleration means velocity is increasing with time while negative acceleration called retardation means velocity is decreasing with time.

Example Based on Relation Between Displacement & Acceleration

Ex.17 The displacement x of a particle along a straight line at time t is given by $x = a_0 - a_1t + a_2t^2$. The acceleration of the particle is-

(A) a_0 (B) a_1 (C) $2a_2$ (D) a_2

Sol. $x = a_0 - a_1t + a_2t^2 \Rightarrow \frac{dx}{dt} = -a_1 + 2a_2t \Rightarrow \frac{d^2x}{dt^2} = 2a_2$

Hence correct answer is (C).

Example Based on Relation Between Velocity & Displacement

Ex.18 If the displacement of a particle is proportional to the square of time, then-

(A) velocity is inversely proportional to t (B) velocity is proportional to t

(C) velocity is proportional to $\sqrt{t}$ (D) acceleration is constant

Sol. Given that $s \propto t^2 \Rightarrow s = kt^2$, where k is constant

$\therefore$ velocity $v = \frac{ds}{dt} = 2kt$, velocity varies with time

acceleration $a = \frac{dv}{dt} = 2k$ = constant.

Hence acceleration of particle is constant

Hence correct answer are (B) & (D).

Example Based on Relation between Acceleration and Displacement

Ex.19 The displacement is given by $x = 2t^2 + t + 5$, the acceleration at t = 5 sec will be-

(A) 8 m/s^2 (B) 12 m/s^2 (C) 15 m/s^2 (D) 4 m/s^2

Sol. Given, $x = 2t^2 + t + 5 \Rightarrow \frac{dx}{dt} = 4t + 1 \Rightarrow \frac{d^2x}{dt^2} = 4 \Rightarrow \left(\frac{d^2x}{dt^2}\right)_{t=5sec} = 4\ m/s^2$

Hence correct answer is (D).

Ex.20 A particle moves along the x-axis in such a way that its x-co-ordinate varies with time as $x = 2 - 5t + 6t^2$. The initial velocity and acceleration of particle will respectively be-

(A) –5 m/s, 12 m/s (B) 5 m/s, –12 m/s

(C) –5 m/s, –12 m/s (D) 5 m/s, 12 m/s

Sol. $x = 2 - 5t + 6t^2 \Rightarrow v = \frac{dx}{dt} = -5 + 12t$, initially $t = 0 \Rightarrow \therefore v = -5$ m/s, $a = \frac{d^2x}{dt^2} = 12\ m/s^2$

Hence correct answer is (A).

Ex.21 The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration of the particle will be equal to zero at time –

(A) $\frac{2a}{3b}$ (B) $\frac{a}{b}$ (C) $\frac{a}{3b}$ (D) 0

Sol. Given that $x = at^2 - bt^3$ $\therefore$ Velocity $v = \frac{dx}{dt} = 2at - 3bt^2$ and

acceleration $a = \frac{d}{dt}\left(\frac{dx}{dt}\right) \Rightarrow 0 = 2a - 6bt \Rightarrow t = \frac{2a}{6b} = \frac{a}{3b}$

Hence correct answer is (C).

Example Based on Average Acceleration

Ex.22 In the above example, the average acceleration of the particle in the interval t = 1 to t = 3 sec will be-
(A) 12 a – 2b (B) 2b – 12 a (C) 2a – 12b (D) 12b – 2a

Sol. In the light of above example, we have $\frac{dx}{dt} = 2at - 3bt^2$

Now velocity at t = 1 sec, $v_1 = \left(\frac{dx}{dt}\right)_{t=1} = 2a - 3b$ and that at t = 3 sec, $v_2 = \left(\frac{dx}{dt}\right)_{t=3} = 6a - 27b$

Thus average acceleration $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6a - 27b - 2a + 3b}{3 - 1} = \frac{4a - 24b}{2} = 2a - 12b$

Hence correct answer is (C).

Example Based on Velocity & Acceleration Relation

Ex.23 The velocity v of a moving particle varies with displacement as $x = \sqrt{v+1}$, the acceleration of the particle at x = 5 unit will be-
(A) $\sqrt{6}$ unit (B) 24 unit (C) 240 unit (D) 25 unit

Sol. $x = \sqrt{v+1} \Rightarrow x^2 = v + 1 \Rightarrow v = x^2 - 1 \Rightarrow \frac{dv}{dx} = 2x$. Now acceleration

$a = v\frac{dv}{dx}$ $(x^2 - 1).\ 2x$

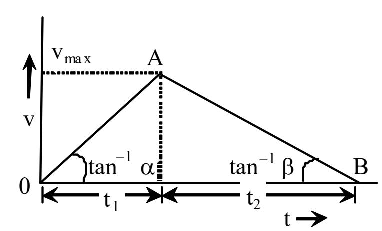
This is the acceleration at position x. Now at x = 5 unit, a = $(5^2 - 1)\ (2 \times 5) = 240$ unit
Hence correct answer is (C).

Example Based on Slope of Velocity-Time Graph

Ex.24 A car accelerates from rest at a constant rate α for sometime after which it decelerates at constant rate β to come to rest. If the total time elapsed is t sec. The maximum velocity of car will be-
(A) $\frac{\alpha\beta}{t(\alpha+\beta)}$ (B) $\frac{\alpha+\beta}{\alpha\beta}t$ (C) $\frac{\alpha\beta}{\alpha+\beta}t$ (D) $\frac{\alpha\beta}{(\alpha+\beta)}$

Sol. If the car accelerates for time t_1 and decelerates for time t_2, then according to given problems
$t = t_1 + t_2$ …. (A)
If v_{max} is the maximum velocity of the car, then from v/t curve, we have

$\alpha = \frac{v_{max}}{t_1}$, $\beta = \frac{v_{max}}{t_2}$



[as slope of v/t curve gives acceleration.]

so $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{t_1 + t_2}{v_{max}} \Rightarrow v_{max} = \frac{\alpha\beta}{(\alpha+\beta)}t$

[$\because t = t_1 + t_2$]
Hence correct answer is (C).
Special Note : In the above example the total distance travelled by car, s = 1/2 (area Δ OAB)

$s = (1/2)\ (v_{max})\ t = (1/2)\ \frac{\alpha\beta}{\alpha+\beta}t$ [as area of v/t graph gives total distance covered]

Example based on Acceleration and Displacement

Ex.25 If displacements of a particle varies with time t as s = $1/t^2$, then.
(A) The particle is moving with constant velocity
(B)The particle is moving with variable acceleration of decreasing order
(C)The particle is moving with constant retardation
(D)The particle has constant speed but variable velocity

Sol. $s = t^{-2}$, Velocity $v = \frac{ds}{dt} = -2/t^3$, acceleration $a = \frac{d^2s}{dt^2} = 6/t^4$
Hence correct answer is (B).

Example Based on Velocity-Acceleration Relation

Ex.26 The retardation of a moving particle, if the relation between time and position is $t = Ax^2 + Bx$ (where A and B are constant) will be-
(A) $2A\,(Ax + B)^{-3}$
(B) $2A\,(2Ax + B)^{-3}$
(C) $A/2\,(Ax + B)^{-3}$
(D) $A/2\,[2Ax + B]^{-3}$

Sol. As $t = Ax^2 + Bx \Rightarrow dt/dx = 2Ax + B$
$\Rightarrow v = (2Ax + B)^{-1}$ ... (A)
[as dx/dt = v], Now by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$$

$\Rightarrow a = (2Ax + B)^{-1} \frac{d}{dx}(2Ax + B)^{-1} = -2A\,(2Ax + B)^{-3}$
So retardation $= -a = 2A\,(2Ax + B)^{-3}$
Hence correct answer is (C).

Example Based on Concept Related to Speed & Acceleration

Ex.27 It is possible to be accelerating if you are travelling at constant speed ? Is it possible to round a curve, with zero acceleration ? With constant acceleration ? With variable acceleration ?
(A) No, yes, no, no
(B) Yes, no, yes, yes
(C) Yes, no, no, no
(D) No, no, yes, yes

Sol. If speed is constant, velocity may change due to change in direction and as acceleration is rate of change of velocity so acceleration may not be zero when speed is constant. Actually in uniform circular motion, speed = constant but acceleration $\neq 0$. For motion on a curve we at least have to change the direction of motion, so we will require a force and hence, acceleration i.e. it is not possible to round a curve with zero acceleration. However, in rounding a curve acceleration may be constant or variable. In case of projectile motion acceleration is constant ($=\vec{g}$) while in case of circular motion acceleration $\neq$ constant, either due to change in direction or both change in direction and magnitude.
Hence correct answer is (B).

Ex.28 What does $\frac{d|\vec{v}|}{dt}$ and $\left|\frac{d\vec{v}}{dt}\right|$ represent

Sol. $\frac{d|\vec{v}|}{dt}$ represents time rate of change of speed as $|\vec{v}| = v$, while $\left|\frac{d\vec{v}}{dt}\right|$ represents magnitude of acceleration.

9. MOTION WITH UNIFORM ACCELERATION

Let $\vec{u}$ = Initial velocity (at t = 0), $\vec{v}$ = Velocity of the particle after time t
$\vec{a}$ = Acceleration (uniform), $\vec{s}$ = Displacement of the particle during time 't'

(A) Acceleration, $\vec{a} = \frac{\vec{v} - \vec{u}}{t}$ [Because of uniform acceleration, this acceleration is instantaneous as well average acceleration]. From above equation

$$\vec{v} = \vec{u} + \vec{a}t \quad ....(i)$$

(B) Displacement $\vec{s}$ = Average velocity × time,

$$\vec{s} = \frac{\vec{u} + \vec{v}}{2}t \quad ....(ii)$$

[This is very useful equation, when acceleration is not given]

(C) From (i) and (ii) $\vec{s} = \vec{u}\,t + (1/2)\,\vec{a}\,t^2$ ....(iii)
Again from (i) and (iii)

$$\vec{s} = \vec{v}t - (1/2)\vec{a}t^2$$

[Here negative sign does not indicate that retardation is occurring]

(D) From (i) and (ii) $\vec{v}^{\,2} = \vec{u}^{\,2} + 2\,\vec{a}.\vec{s}$ .....(iv)

$\vec{s}_n$ = displacement of particle in n^{th} second

$= \vec{s}_n - \vec{s}_{n-1} = \{\vec{u}(n) + (1/2)\vec{a}\,n^2\} - \{\vec{u}\,(n-1) + (1/2)\vec{a}\,(n-1)^2\}$

$\vec{s}_n = \vec{u} + 1/2\ \vec{a}\,(2n - 1)$

[This equation is dimensionally non balanced because we have substituted value of t = 1s and second is neglected that's why it seems to be unbalanced]

Equations (i), (iii) and (iv) one called 'equations of motion' and are very useful in solving the problems of motion along a straight line with constant acceleration.

Note :

(i) These equations can be applied only and only when acceleration is constant. In case of circular motion or simple harmonic motion as acceleration is not constant (due to change in direction or magnitude) so these equation can not be applied.

(ii) $\vec{v} = \vec{u} + \vec{a}t$ and $\vec{s} = \vec{u}\,t + (1/2)\ \vec{a}\,t^2$ are vector equation, while $\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s} \Rightarrow v^2 = u^2 + 2\vec{a}.\vec{s}$ is a scalar equation

(iii) If the velocity and acceleration are collinear, we conventionally take the direction of motion to be positive, so equation of motions becomes
$v = u + at$, $s = ut + (1/2)\,at^2$, $v^2 = u^2 + 2as$
If the velocity and acceleration are anti-parallel then,
$v = u - at$
$s = ut - (1/2)\,at^2$
$v^2 = u^2 - 2as$

Example Based on First Equation of Motion

Ex.29 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. Time taken to reach the velocity 7.5 m/s will be-
(A) 5 s (B) 2 s (C) 10 s (D) 15 s

Sol. We have $v = u + at$ or $7.5 = 2.5 + 0.50\,t \Rightarrow t = 10$ s
Hence correct answer is (C)

Ex.30 A car accelerates from rest at a constant rate α for sometime after which it decelerates at constant rate β to come to rest. If the total time elapsed is t sec. The maximum velocity will be-

(A) $\frac{\alpha\beta}{(\alpha+\beta)t}$ (B) $\frac{\alpha^2\beta}{(\alpha+\beta)}t$ (C) $\frac{\alpha\beta}{(\alpha+\beta)}t$ (D) $\frac{\alpha\beta^2 t}{(\alpha+\beta)}$

Sol. From " $v = u + at$" we have, for the car,
$v = 0 + \alpha\,t_1$ and $0 = v - \beta t_2$
[**Note :** velocity 'v' after time t_1 will be initial velocity for next motion with retardation β]

from these two equations, we get $v = \frac{\alpha\beta t}{(\alpha+\beta)}$

Hence correct answer is (C)

Example Based on Second Equation of Motion

Ex.31 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. The distance travelled by the particle in first two seconds will be-
(A) 4 m (B) 5m (C) 1m (D) 6 m

Sol. We have, $s = ut + (1/2)\,at^2 = (2.5)\,(B) + (1/2)\,(0.50)\,(B)^2 = 6$m
Since the particle does not return back, it is also the distance travelled.
Hence correct answer is (D)

Ex.32 A car accelerates from rest at a constant rate α for sometime after which it decelerates at constant rate β to come to rest. If the total time elapsed is t sec. What will be the total distance traveled ?

(A) $\frac{\alpha\beta t^2}{\alpha+\beta}$ (B) $\frac{\alpha+\beta}{\alpha\beta}t^2$ (C) (1/2) $\frac{\alpha\beta}{(\alpha+\beta)}t^2$ (D) $\frac{\alpha\beta t^2}{\alpha+\beta}2t^2$

Sol. From " $s = ut + (1/2)\,at^2$ ",
we have $s_1 = (1/2)\,\alpha t_1^2$, $s_2 = vt_2 - (1/2)\,\beta t_2^2$

Total distance = $s_1 + s_2 = (1/2)\,\frac{\alpha\beta t^2}{\alpha+\beta}$

[Putting value of s_1 and s_2 and $v = \frac{\alpha\beta t}{\alpha+\beta}$]

Hence correct answer is (C).

Ex.33 A passenger is standing 'd' m away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed v towards the bus. The minimum speed of the passenger so that he may catch the bus will be-

(A) 2ad (B) $\sqrt{ad}$ (C) $\sqrt{2ad}$ (D) ad

Sol. Let the passenger catch the bus after time t. From "$s = ut + (1/2)\,at^2$", the distance travelled by the bus
$s_1 = 0 + (1/2)\,at^2$ ... (A)
and the distance travelled by the passenger
$s_2 = ut + 0$ ... (B)
[**Note :** acceleration of passenger = 0]
Now the passenger will catch the bus if,
$d + s_1 = s_2$ ... (C)
In the light of eq. (A) & (B), eq. (C) gives

$(1/2)\,at^2 - ut + d = 0 \Rightarrow t = \frac{u \pm \sqrt{u^2 - 2ad}}{a}$

So the passenger will catch the bus if t is real i.e. $u^2 \geq 2ad \Rightarrow u \geq \sqrt{2ad}$

So, the minimum speed of passenger for catching the bus is $\sqrt{2ad}$

Hence correct answer is (C)

Ex.34 A body moving with uniform acceleration describes 4 m in 3rd second and 12 m in the 5th second. The distance described in next three second is-
(A) 100 m (B) 80 m (C) 60 m (D) 20 m

Sol. Let u is the initial velocity and a is the acceleration then
$S_n = u + (1/2)\,a(2n - 1)$ $\therefore$ $S_3 = u + (1/2)\,a(3 \times 2 - 1)$

$\Rightarrow 4 = u + \frac{5}{2}\,a$ ...(i)

similarly for 5th second
$S_5 = u + (1/2)\,a\,(2\times5-1) \Rightarrow 12 = u + (9/2)\,a$ ...(ii)
From (i) & (ii) $u = -6$ m/s and $a = 4$ m/s^2,
so, distance travelled in 5 sec,
From "$s = ut + 1/2\,at^2$" , $s = -6 \times 5 + (1/2) \times 4 \times 5^2 = 20$ m
Similarly distance travelled in 8 sec = $-6 \times 8 + (1/2)\,4 \times 8^2 = 80$ m
So distance travelled in next 3 sec = 80 – 20 = 60 m
Hence correct answer is (C)

Example Based on Third Equation of Motion

Ex.35 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. The distance covered in reaching the velocity 7.5 m/s will be-

(A) 25 m (B) 50 m (C) 75 m (D) 100 m

Sol. We have, $v^2 = u^2 + 2a\,x$ or $(7.5)^2 = (2.5)^2 + 2(0.50)\,x \Rightarrow x = 50$ m

Hence correct answer is (B)

Ex.36 A particle starts moving from position of rest under a constant acceleration. If it travels a distance x in t sec. The distance it will travel in next t sec will be-

(A) 2x (B) 3x (C) 4x (D) 5x

Sol. The velocity of particle after time t will be $v = u + at = 0 + at = at$

Now for next t sec, it will be the initial velocity,

From " $s = u't + (1/2)\,at^2$", we have

$\Rightarrow x' = (at)\,t + (1/2)\,at^2$ [Here u' = at]

$x' = 3/2\,at^2$ ...(A)

This is the distance travelled in next t sec

Also given that particle travels x distance in t sec. so again using "$s = ut + (1/2)\,at^2$"

We have, $x = \frac{1}{2}\,at^2$ ...(B)

From (A) & (B) , we have, $x' = 3x$

Hence correct answer is (B)

Ex.37 A truck and a car are brought to a hault by application of same breaking force. Which one will come to stop in shorter distance if they are moving with same (A) velocity (B) K.E. (C) momentum

(A) Both car, truck (B) Truck, car, car (C) Car, both, truck (D) Car, truck, truck

Sol. By breaking force the body is brought to rest so, v = 0 and a = (–F/m) (as it is retardation)

If s is the distance travelled in stopping (called stopping distance), from $v^2 = u^2 + 2as$

we have, $0 = u^2 - 2(F/m)s \Rightarrow s = \frac{mu^2}{2F}$, But KE = (1/2) mu^2 and also

$KE = \frac{p^2}{2m}$ ($\because$ p = mu), So $s = \frac{mu^2}{2F} = \frac{KE}{F} = \frac{p^2}{2mF}$.

From this it is clear that,

(A) If u is same, $s \propto \frac{mu^2}{2F} \Rightarrow s \propto m$

Now as mass of car is lesser than that of truck, so car will stop in shorter distance.

(B) If K.E. is same, $s \propto \frac{KE}{F}$

So both will stop after travelling same distance.

(C) If p is same, $s \propto \frac{p^2}{2mF} \Rightarrow s \propto \frac{1}{m}$

Now as mass of truck is more than that of car so truck will stop in a shorter distance.

Hence correct answer is (C)

Note : As $s = \frac{mu^2}{2F}$, so for a given body if breaking force remains unchanged. $s \propto u^2$

[as m is constant]

i.e. if the speed of a moving body is made n times the stopping distance will become n^2 times.

10. MOTION UNDER GRAVITY

Ideal Motion :

The most important example of motion in a straight line with constant acceleration is motion under gravity. In case of motion under gravity unless stated it is taken for granted that.

(i) The acceleration is constant, i.e.

$\vec{a} = \vec{g} = 9.8\ m/s^2$

and directed vertically downwards.

(ii) The motion is in vacuum i.e. viscous force or thrust of the medium has no effect on the motion. Now in the light of above assumptions, there are two possibilities.

10.1 Body Falling Freely Under Gravity :

Taking initial position as origin and direction of motion (i.e. downward direction) as positive, here we have u = 0 (as body starts from rest)

$a = +g$

(as acceleration is in the direction of motion)

So, if the body acquires velocity v after falling a distance h in time t, equations of motion viz

$v = u + at$

$s = ut + \left(\frac{1}{2}\right) at^2$

and $v^2 = u^2 + 2as$

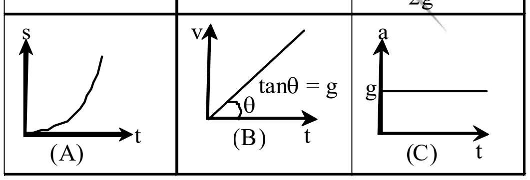
reduces to $v = gt$ ....(A)

$s = \left(\frac{1}{2}\right) gt^2$ ....(B)

$v^2 = 2gh$ ....(C)

These equations can be used to solve most of the problems of freely falling as

t is given	h is given	v is given
From eq. (1) & (2)	From eq. (2) & (3)	From eq. (3) & (1)
$v = gt$	$t = \sqrt{\frac{2h}{g}}$	$t = \frac{v}{g}$
and $s = \frac{1}{2} gt^2$	$v = \sqrt{2gh}$	$s = \frac{v^2}{2g}$



Note :

(A) If the body is dropped from a height H, as in time t, it has fallen a distance h from its initial position, the height of the body from the ground will be $h' = H - h$, with $h = 1/2\ gt^2$.

(B) As $h = (1/2)\ gt^2$ i.e. $h \propto t^2$, distance fallen in time t, 2t, 3t etc. will be in the ratio of $1^2 : 2^2 : 3^2 :$ ------- i.e. square of integers.

(C) The distance fallen in n^{th} sec.,

$h_n - h_{n-1} = (1/2)\ g(n)^2 - (1/2)\ g(n-1)^2$

$= (1/2)\ g(2n-1)$

So distance fallen in I^{st}, II^{nd}, III^{rd} sec will be in the ratio 1 : 3 : 5 i.e. odd integers only.

10.2 Body is projected vertically up :

Taking initial position as origin and direction of motion (i.e. vertically up) as positive, here we have v = 0 [as at the highest point, velocity = 0], a = – g [as acceleration is downwards while motion upwards]. So, if the body is projected with velocity u and reaches the highest point at a distance h above the ground in time t, the equations of motion viz

$v = u + at$

$s = ut + \left(\frac{1}{2}\right) at^2$

and $v^2 = u^2 + 2as$ reduces to

$0 = u - gt$

$h = ut - 1/2\ gt^2$

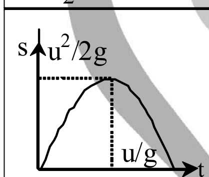
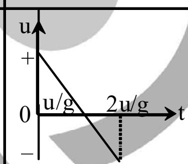
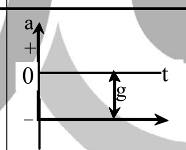
and $0 = u^2 - 2gh$

or $u = gt$ ....(A)

$h = 1/2\ gt^2$ ....(B)

($\because$ $u = gt$), $u^2 = 2\ gh$ ....(C)

These equations can be used to solve most of the problems of bodies projected vertically up as, if

If is given	If h is given	is u given
From eq. (1) & (2)	From eq. (2) & (3)	From eq. (3) & (1)
$u = gt$	$t = \sqrt{2h/g}$	$t = u/g$
$h = \frac{1}{2}gt^2$	$u = \sqrt{2hg}$	$h = u^2/2g$
 (A)	 (B)	 (C)

Discussion :

From cases (10.1) and (10.2) it is clear that :

(A) In case of motion under gravity for a given body, mass, acceleration and mechanical energy remains constant while speed, velocity, momentum, kinetic energy and potential energy changes.

(B) The motion is independent to the mass of the body as in any equation of motion mass is not involved. This is why a heavy and light body when released from same height reaches the ground simultaneously and with same velocity.

i.e. $t = \sqrt{2h/g}$ and $v = \sqrt{2gh}$

However, momentum, kinetic energy or potential energy depends on the mass of the body (all $\propto$ mass)

(C) As from case (B) time taken to reach a height h, $t_U = \sqrt{2h/g}$

And from case (A) time taken to fall down through a distance h,

$t_D = \sqrt{2h/g}$

so, $t_U = t_D = \sqrt{2h/g}$

So in case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance.

(D) If a body projected vertically up reaches a height h then from case (B), $u = \sqrt{2gh}$ and if a body falls freely through a height h from case (A), $v = \sqrt{2gh}$

So in case of motion under gravity the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection

Example Based on Second equation of motion under gravity

Ex.38 From the top of a building a ball is dropped, while another is thrown horizontally at the same time. Which ball will strike the ground first ?

(A) The ball projected horizontally (B) The ball projected vertically
(C) Both at the same time (D) It depends upon mass of the balls

Sol. Both the balls will reach the ground simultaneously as horizontal velocity does not effect the vertical motion, $t_1 = t_2 = \sqrt{(2h/g)}$ [from "h = 1/2 gt^2", as u = 0]

Note : However for the ball dropped vertically, $v_1 = \sqrt{(2gh)}$, while for the ball projected horizontally :

horizontal velocity $(v_H)_2$ = u and vertical velocity $(v_v)_2 = \sqrt{(2gh)}$, so that $v_2 = \sqrt{(u^2 + 2gh)}$

i.e. on hitting the ground speed of horizontally projected ball will be more than the ball dropped vertically
Hence correct answer is (C)

Ex.39 A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies 2 sec after the release of the second body ? If g = 9.8 m/s^2.

(A) 2.45 m (B) 24.5 m (C) 4.9 m (D) 9.8 m

Sol According to given problem 2^{nd} body falls for 2 s so that $h_2 = (1/2)\ g\ (B)^2$ ... (A)
While I^{st} has fallen for 2 + 1 = 3 s, so $h_1 = (1/2)\ (C)^2\ g$ ... (B)
$\therefore$ Separation between two bodies 2 s after the release of II^{nd} body
$d = h_1 - h_2 = (1/2)\ g(3^2 - 2^2) = 4.9 \times 5 = 24.5$ m
Hence correct answer is (B)

Ex.40 If a body travels half its total path in the last second of its fall from rest. The time and height of its fall, will respectively be- (g = 9.8 m/s^2)

(A) 0.59 s, 57 m (B) 3.41 s, 57 m (C) 5.9 s, 5.7 m (D) 5.9 s, 34.1 m

Sol If the body falls a height h in time t, from 2^{nd} equation of motion we have
$h = 1/2\ gt^2$ ....(A)
[u = 0 as body starts from rest]
Now the distance fallen in (t – 1) s will be
$h = 1/2\ g(t-1)^2$ ...(B)
So from eq. (A) & (B) distance fallen in the last second
$h - h' = (1/2)\ gt^2 - (1/2)\ g\ (t-1)^2$,
$h - h' = (1/2)\ g\ (2t - 1)$
But according to given problem as
$(h - h') = h/2$
i.e. $(1/2)\ h = (1/2)g\ (2t - 1)$
or $(1/2)\ gt^2 = g(2t - 1)$
[as from eq. (A) $h = (1/2)\ gt^2$]
or $t^2 - 4t + 2 = 0$
or $t = [4 \pm \sqrt{(4^2 - 4 \times 2)}\ ]/2$
or $t = 2 \pm \sqrt{2}$ or t = 0.59 or 3.41 s
0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground is lesser than one sec while according to the given problem time of motion must be greater than 1 s.
So t = 3.41 s &
$h = (1/2) \times (9.8) \times (3.41)^2 = 57$ m
Hence correct answer is (B)

Example Based on Concept of Projectile Motion Under Gravity

Ex.41 Statement given below is true or false ? Give reason in brief. "Two balls of different masses are thrown vertically upwards with the same speed. They reach through the point of projection in their downward motion with the same speed".

(A) True (B) False
(C) Depend upon conditions (D) None of these

Sol. The statement is true as motion under gravity is independent of mass of the body and as body comes back to the point of projection with same speed, so
$v_1 = u_1$ and $v_2 = u_2$, Here $u_1 = u_2 = u$ (given)
so, $v_1 = v_2 = u$
Hence correct answer is (A)

Example Based on Third Equation of Motion Under Gravity

Ex.42 A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throws another stone straight down with the same initial speed and from the same position. Find the ratio of the speed the stones would have attained when they hit the ground at the base of the cliff.
(A) $\sqrt{2}$: 1 (B) 1 : $\sqrt{2}$ (C) 1 : 1 (D) 1 : 2

Sol. As the stone thrown vertically up will come back to the point of projection with same speed, both the stones will move downward with same initial velocity, so both will hit the ground with velocity
$v^2 = u^2 + 2gh$ i.e., $v = \sqrt{(u^2 + 2gh)}$
So, the ratio of speeds attained when they hit the ground is 1 :1
Hence correct answer is (C)

Note : However the stone projected up will take (2u/g) time more to reach the ground than the stone projected downwards.

Ex.43 A juggler throws balls into air. He throws one, when ever the previous one is at its highest point. How high do the balls rise if he throws n balls such each sec, accelerating due to gravity is-
(A) g/n^2 (B) $g/2n^2$ (C) 2n/g (D) $2n^2/g$

Sol. A juggler is throwing n balls each second and 2nd when the first is at its highest point, so time taken by one ball to reach the highest point t = (1/n) sec and as at highest point v = 0,
From 1st equation of motion
$0 = u - (g)(1/n)$, i.e. $u = (g/n)$ .....(A)
Now from 3 rd equation of motion
i.e. $v^2 = u^2 + 2as$, $0 = u^2 - 2gh$
i.e. $h = (u^2/2g)$
$h = \frac{g}{2n^2}\left[\text{From Eq.(1)} u = \frac{g}{n}\right]$
Hence correct answer is (B)

Example Based on First and Second Equations of MOtion Under Gravity

Ex.44 A pebble is thrown vertically upwards from bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is 9.8 m/s^2 . The height of the bridge and velocity with which the pebble strike the water will respectively be-
(A) 4.9 m, 1.47 m/s (B) 9.8 m, 14.7 m/s
(C) 49 m, 1.47 m/s (D) 1.47 m, 4.9 m/s

Sol. Taking the point of projection as origin and downward direction as positive. By 2nd equation of motion, i.e. $s = ut + (1/2)at^2$, We have,
$h = -4.9 \times 2 + (1/2)\, 9.8 \times 2^2 = 9.8$
(u is taken to be negative as it is upwards)
From 1st equation of motion i.e. $v = u + at$, $v = -4.9 + 9.8 \times 2 = 14.7$ m/s
Hence correct answer is (B)

Example Based on First and Third Equation of Motion

Ex.45 A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up. (A) the maximum height reached. (B) After how much time from then will the maximum height be reached (Take g = 10 m/s^2)
(A) 36 km, 1 min (B) 6 km, 1 min (C) 36 km, 1 sec (D) 36 km, 1 sec

Sol. (A) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards is 10 m/s^2 will be

$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m ....(A)

And velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s ....(B)

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity oppose its motion. So, it will go to a height h_2 till its velocity becomes zero that

$0 = (600)^2 - 2gh_2 \quad \Rightarrow h_2 = 18000$ m

[as g = 10 m/s^2] ....(C)

So from eq. (A) and (C) the maximum height reached by the rocket from the ground.

$H = h_1 + h_2 = 18 + 18 = 36$ km

(B) As after burning of fuel the initial velocity from Eq. (B) is 600 m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it to reach the maximum height (for which v = 0)

$0 = 600 - gt$, i.e.t = 60 s

after finishing of fuel, the rocket goes up for 60 sec i.e., 1 minute more.

Hence correct answer is (A)

Example Based on All Three Equations of Motion Under Gravity

Ex.46 A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 m/s^2

(A) How long does it take to reach its highest point?

(B) How high does it rise above the point of projection?

(C) How long will it take for the ball to reach a point 10 m above the point of projection?

(A) 2 s, 20 m, 3.41 s (B) 5 s, 20 m, 3.41 s

(C) 2 s, 10 m, .59 s (D) 2 s, 5 m, .59 s

Sol. As here motion is vertically upwards a = – g and v = 0

(A) From 1st equation of motion

i.e., $v = u + at$

$\Rightarrow 0 = 20 - 10\,t$, i.e., t = 2s

(B) From 3rd equation of motion

i.e., $v^2 = u^2 + 2as$

$\Rightarrow 0 = (20)^2 - 2 \times 10 \times h$, i.e., h = 20 m

(C) From 2nd equation of motion,

i.e., $s = ut + (1/2)\,at^2$

$\Rightarrow 10 = 20t - (1/2) \times 10 \times t^2$

$t^2 - 4t + 2 = 0$, i.e. $t = 2 \pm \sqrt{2}$

i.e. t = 0.59 s or 3.41 s

i.e. there are two such times, as the ball passes twice through h = 10 m once when going up and once when coming down.

Hence correct answer is (A)

11. MOTION WITH VARIABLE ACCELERATION

There are only two equations in this type of motion.

(A) $v = \frac{dx}{dt}$ (B) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Example Based on Velocity-Displacement Relation

Ex.47 The displacement of particle is zero at t = 0 and at t = t it is x. It starts moving in the x direction with velocity, which varies as $v = k\sqrt{x}$, where k is constant. The velocity -

(A) varies with time (B) independent to time

(C) inversely proportional to time (D) nothing can be said

Sol. $v = k\sqrt{x} \Rightarrow \frac{dx}{dt} = k\sqrt{x} \Rightarrow \int \frac{dx}{\sqrt{x}} = \int k\,dt \Rightarrow \frac{x^{+1/2}}{1/2} = kt + c$

Given that, at t = 0, x = 0 $\therefore$ c = 0

Now, $2x^{1/2} = kt \Rightarrow \sqrt{x} = (1/2)\,kt$,

Now, $v = k\,(1/2\,kt) = k^2t/2$

Thus velocity varies with time. Hence correct answer is (A)

Example Based on Velocity-Acceleration Relation

Ex.48 The acceleration of a particle is given as $a = 3x^2$. At t = 0, v = 0, x = 0. The velocity at t = 2 sec will be-

(A) 0.05 m/s (B) 0.5 m/s (C) 5 m/s (D) 50 m/s

Sol. $a = 3x^2 \Rightarrow v\dfrac{dv}{dx} = 3x^2$

$\Rightarrow vdv = 3x^2\,dx \Rightarrow \dfrac{v^2}{2} = 3\dfrac{x^2}{3} + c$

at t = 0, v = 0, x = 0

$\therefore$ c = 0 Now, $\dfrac{v^2}{2} = x^3$

$v^2 = 2x^3 \Rightarrow v = \sqrt{2}\,x^{3/2}$ ... (A)

$\Rightarrow \dfrac{dx}{dt} = \sqrt{2}\,x^{3/2}$

Remember, when a is function of x.

use $a = v\dfrac{dv}{dx}$

when a is function of t, $a = \dfrac{dv}{dt}$, $dx = \sqrt{2}\,x^{3/2}\,dt \Rightarrow \dfrac{dx}{x^{3/2}} = \sqrt{2}\,t + c'$, at t = 0, x = 0, v = 0

$\therefore$ $c' = 0$

Now $\dfrac{-2}{\sqrt{x}} = \sqrt{2}\,t \Rightarrow 4 = 2xt^2$

$\Rightarrow x = \dfrac{2}{t^2}$ ...(B)

From (A) and (B) $v = \sqrt{2}\left(\dfrac{2}{t^2}\right)^{3/2}$, at t = 2 sec $\Rightarrow$ v = 1/2 m/sec

Hence correct answer is (B)

Example Based on Velocity-Acceleration and Displacement-Velocity Relation

Ex.49 The acceleration of a particle is given by $a = 3t$ and at t = 0, v = 0, x = 0. The velocity and displacement at t = 2 sec will be-

(A) 6 m/s, 4 m (B) 4 m/s, 6 m (C) 3 m/s, 2 m (D) 2 m/s, 3 m

Sol. $a = 3t \Rightarrow \dfrac{dv}{dt} = 3t \Rightarrow \int dv = \int 3t\,dt \Rightarrow v = \dfrac{3t^2}{2} + c$

Substituting the initial conditions, At t = 0, v = 0 and x = 0

$\therefore$ c = 0 Hence, $v = \dfrac{3t^2}{2}$,

Velocity at t = 2 sec is $\dfrac{3\times 2^2}{2} = 6$ m/s

Also, $\dfrac{dx}{dt} = \dfrac{3t^2}{3} \Rightarrow \int dx = \dfrac{3}{2}\int t^2 dt \Rightarrow x = \dfrac{3}{2}\dfrac{t^3}{3} + c'$

at t = 0, x = 0 $\therefore c' = 0$, $\therefore x = \dfrac{t^3}{2}$,

Now displacement at t = 2 sec is $\dfrac{2^3}{2} = 4$ m

Hence correct answer is (A)

Note : Prohibit the use of definite integral to avoid blunders as constant may change from the given initial conditions.

12. RELATIVE – VELOCITY

(i) There is nothing in absolute rest or absolute motion.

(ii) Motion is a combined property of the object under study and the observer.

Example :

(i) A book placed on the table in a room is at rest, if it is viewed from the room but it is in motion, if it is viewed from the moon (another frame of reference). The moon is moving w.r.t. the book and the book w.r.t. the moon.

(ii) A robber enters a train moving at great speed with respect to the ground, brings out his pistol and says " Don't move, stand still".
The passengers stand still. The passengers are at rest with respect to the robber but are moving with respect to the rail track.

(iii) Relative motion means, the motion of a body with respect to another. Now if $\vec{V}_A$ and $\vec{V}_B$ are velocities of two bodies relative to earth, the velocity of B relative to A will be given by

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Note :

(A) If two bodies are moving along the same line in same direction with velocities V_A and V_B relative to earth, the velocity of B relative to A will be given by $V_{BA} = V_B - V_A$.
If it is positive the direction of V_{BA} is that of B and if negative the direction of V_{BA} is opposite to that of B.

(B) However, if the bodies are moving towards or away from each other, as direction of V_A and V_B are opposite, velocity of B relative to A will have magnitude $V_{BA} = V_B - (-V_A)$
$= V_B + V_A$ and directed towards A or away from A respectively.

(C) In dealing the motion of two bodies relative to each other $\vec{V}_{rel.}$ is the difference of velocities of two bodies, if they are moving in same direction and is the sum of two velocities if they are moving in opposite direction.

(D) If a man can swim relative to water with velocity $\vec{V}$ and water is flowing relative to ground with velocity $\vec{V}_R$, velocity of man relative to ground $\vec{V}_m$ will be

$\vec{V} = \vec{V}_m - \vec{V}_R$ i.e. $\vec{V}_m = \vec{V} + \vec{V}_R$

So if the swimming is in the direction of flow of water $V_m = V + V_R$
And if the swimming is opposite to the flow of water $V_m = V - V_R$

(5) If a boy is running with velocity $\vec{V}_{rel.}$ on a train moving with velocity $\vec{V}$ relative to the ground.
The velocity of boy relative to ground, $\vec{v}$ will be given by $\vec{V}_{rel.} = \vec{v} - \vec{V} \Rightarrow \vec{v} = \vec{V}_{rel.} + \vec{V}$
So, if the boy is running on the train in the direction of motion of train $v = V_{rel.} + V$
And if the boy is running on the train in a direction opposite to the motion of train
$v = V_{rel.} - V$

Special Note :

In case of motion of a body A on a moving body B, the velocity of A relative to ground is the sum of two velocities if A is moving on B in the same direction and is equal to difference of two velocities if they are moving in opposite direction.

Example Based on Relative Speed

Ex.50 Two trains along the same straight rails moving with constant speed 60 km/hr and 30km/hr respectively towards each other. If at time t = 0, the distance between them is 90 km, the time when they collide is-
(A) 1hr (B) 2 hr (C) 3 hr (D) 4 hr

Sol. The relative velocity
$v_{rel.} = 60 - (-30) = 90$ km/hr
Distance between the train
$S_{rel.} = 90$ km
$\therefore$ Time when they collide
$= \frac{S_{rel.}}{v_{rel.}} = \frac{90}{90} = 1$ hr
Hence correct answer is (a)

Example Based on Relative Velocity

Ex.51 Two cars are moving in the same direction with the same speed 30 km/hr. They are separated by a distance of 5 km, the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes, will be –

(A) 40 km/hr (B) 45 km/hr (C) 30 km/hr (D) 0 km/hr

Sol. As the two cars (say A and B) are moving with same velocity, the relative velocity of one (say B) with respect to the other A,

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ = v – v = 0

So the relative separation between them (= 5 km) always remains the same.

Now if the velocity of car (say C) moving in opposite direction to A and B, is $\vec{v}_C$ relative to ground then the velocity of car C relative to A and B will be $\vec{v}_{rel.} = \vec{v}_C - \vec{v}$

But as $\vec{v}$ is opposite to v_C ,

$v_{rel} = v_C - (-30) = (v_C + 30)$ km/hr

So, the time taken by it to cross the cars A and B is

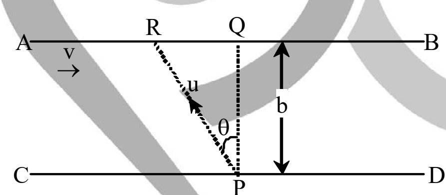
$t = \frac{d}{v_{rel.}}$

$\Rightarrow \frac{4}{60} = \frac{5}{v_C + 30} \Rightarrow v_C = 45$ km/hr

Hence correct answer is (B)

Some useful hints for solving problems regarding the motion of object across a river :

AB and CD $\rightarrow$ two banks of river, v $\rightarrow$ velocity of river, b $\rightarrow$ width of river



A swimmer wants to cross the river starting from a point P to reach a point directly opposite to P on the bank CD in a given time t, then

(A) $t = \frac{b}{u\cos\theta} \Rightarrow t = \frac{b}{\sqrt{u^2 - v^2}}$

(B) Resultant velocity of swimmer

$V = \sqrt{u^2 - v^2}$

(C) The distance travelled by swimmer

s = b

(D) For crossing the river in **minimum time**

$\theta = 0$, so $t_{min} = \frac{b}{u}$

(5) distance covered in the direction of flow = v = b

$s = \left(\frac{b}{u}\right) v = b\left(\frac{v}{u}\right)$

Example Based on Comparison of Motion in River Under Different Conditions

Ex.52 A stream boat goes across a lake and comes back (A) On a quite day when the water is still and (B) On a rough day when there is uniform current so as to help the journey onward and to impede the journey back. If the speed of the launch on both days was same, in which case it will complete the journey in lesser time-

(A) case (A) (B) case (B)

(C) same in both (D) Nothing can be predicted

Sol. If the breadth of the lake is L and velocity of boat is V. Time in going and coming back on a quite day

$t_Q = \frac{L}{V} + \frac{L}{V} = \frac{2L}{V}$ ....(A)

Now if v is the velocity of air-current then time taken in going across the lake,

$t_1 = \frac{L}{V+v}$

[as current helps the motion]
and time taken in coming back

$t_2 = \frac{L}{V+v}$

[as current opposes the motion]

$t_R = t_1 + t_2 = \frac{2L}{V[1-(v/V)^2]}$ ....(B)

From eq. (A) & (B)

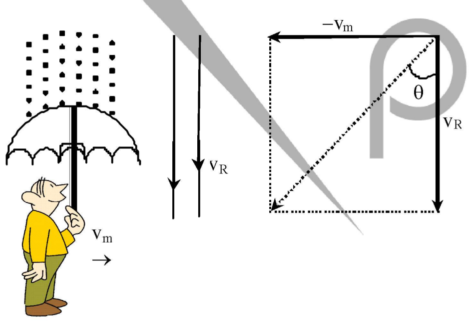
$\frac{t_R}{t_Q} = \frac{1}{[1-(v/V)^2]} > 1$

[as $1 - \frac{v^2}{V^2} < 1$] i.e. $t_R > t_Q$

i.e. time taken to complete the journey on quite day is lesser than that on rough day
Hence correct answer is (B)

Note: It is common-misconception that on a rough day in time decreases due to helping currents will be equal to increase in time due to opposition and so the time of journey on rough and quite day will be same.

If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_m$, the velocity of rain relative to observer :



$\vec{v}_{RM} = \vec{v}_R - \vec{v}_m$

which by law of vector addition has magnitude

$v_{RM} = \sqrt{v_R^2 + v_M^2}$

and direction $\theta = \tan^{-1}\left(\frac{v_M}{v_R}\right)$ with vertical.

Example Based on Calculation of Relative speed of rain

Ex.53 A man standing on a road holds his umbrella at 30º with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that rain drops are hitting his head vertically, the speed of raindrop with respect to the road will be-

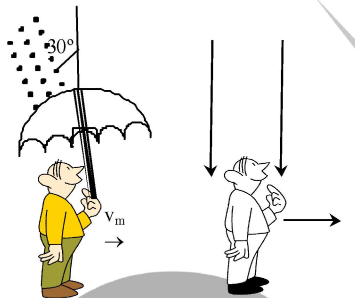
(A) 10 km/h (B) 20 km/h (C) 30 km/h (D) 40 km/h

Sol. When the man is at rest w.r.t. the ground, the rain comes to him at an angle 30º with the vertical. This is the direction of the velocity of rain drops with respect to the ground.

Here $\vec{v}_{r,g}$ = velocity of rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground.

and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man,



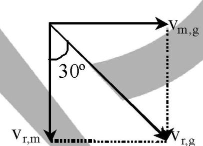
We have $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$ ....(A)

Taking horizontal components eq. (A) gives

$v_{r,g} \sin 30º = v_{m,g} = 10$ km/h

or $v_{r,g} = \dfrac{10}{\sin 30º} = 20$ km/h

Hence correct answer is (B)



Ex.54 In the above example, the speed of raindrops w.r.t. the moving man, will be-

(A) $10/\sqrt{3}$ km/h (B) 5 km/h (C) $10\sqrt{3}$ km/h (D) $5/\sqrt{3}$ km/h

Sol. Taking vertical components eq. (A) gives

$v_{r,g} \cos 30º = v_{r,m}$ or $v_{r,m} = 20\,\dfrac{\sqrt{3}}{2} = 10\sqrt{3}$ km/hr

Hence correct answer is (C)

Special Note :

If speeds are comparable to the velocity of light c, according to theory of relativity, velocity of B relative to A (When both are moving along the same line in opposite directions) is given by

$v_{BA} = \dfrac{v_B + v_A}{\left[1 + \dfrac{v_A v_B}{c^2}\right]}$, from this it is clear that

if v_A or v_B is equal to c , $v_{BA} = \dfrac{v + c}{1 + v/c} = c$

i.e. speed of light is independent of relative motion between source and observer, the basic postulate of special theory of relativity

NOTE : 'Effect of Medium on Motion Under Gravity' IS NOT IN SYLLABUS :

POINTS TO REMEMBER

1. If body starts from rest or falls freely or is dropped then, u = 0

2. If the body is thrown upwards then it will rise until its vertical velocity becomes zero. Maximum height attained is $h = u^2/2g$.

3. If air resistance is negligible then the time of the rise is equal to time of fall and each is equal to $t = u/g$.

4. The body returns to the starting point with the same speed with which it was thrown.

5. The straight line inclined to time axis in x-t graph represents constant velocity.

6. In x-t graph the straight line inclined to time axis at an angle greater than 90º shows negative velocity.

7. No line in x – t graph can be perpendicular to time axis because it will represent infinite velocity.

8. If the x – t graph is a curve whose slope decreases continuously with time, then the velocity of the body goes on decreasing continuously and the motion of the body is retarded.

9. If the v – t graph is a straight line parallel to time axis, then the acceleration of the body is zero.

10. If the graph is a straight line inclined to time axis with positive slope, then that body is moving with constant acceleration.

11. If v – t graph is a straight line inclined to time axis with negative slope, then the body is retarded.

12. Velocity and acceleration of a body need not be zero simultaneously.

13. A body in equilibrium has zero acceleration only. All other quantities need not be zero.

14. If a body travels with a uniform acceleration a_1 for a time interval t_1 and with uniform acceleration a_2 for a time interval t_2, then the average acceleration $a = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$

15. For a body moving with uniform acceleration, the average velocity $(u + v)/2$, where u is the initial velocity and v is the final velocity.

16. The distance travelled by the body in successive second in the ratio 1 : 3 : 5 : 7 ....etc.

17. When the body is starting from rest, the distances travelled by the body in the first second, first two seconds, first three seconds, etc. are in the ratio of 1 : 4 : 9 : 16 : 25 ....etc.

18. When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.

19. If the v – t graph is a curve whose slope decreases with time then the acceleration goes on decreasing.

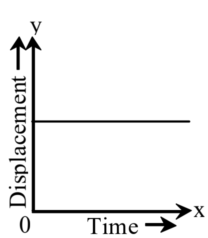
20. If the v-t graph is a curve whose slope increases with time then the acceleration of the body goes on increasing.

21. The v-t graph normal to time axis is not a practical possibility because it means that the acceleration of the body is infinite.

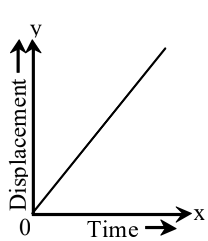
Various Graphs Related to Motion :

A. Displacement-Time Graph :

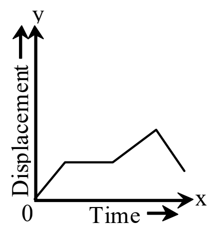
(A) For a stationary body



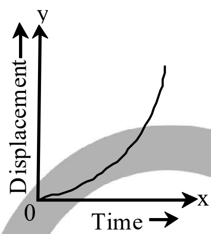
(B) For a body moving with constant velocity



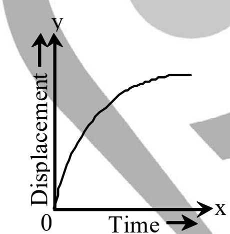
(C) For a body moving with non-uniform velocity



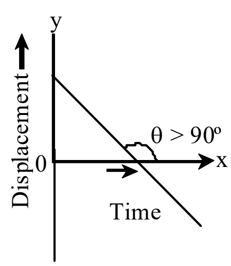
(D) For a body with accelerated motion



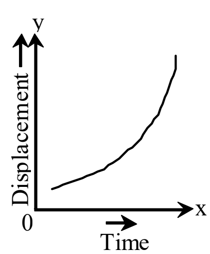
(5) For a body with decelerated motion



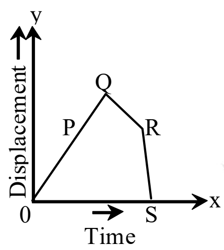
(6) For a body which returns towards the point of reference



(7) For a body whose velocity constantly changes

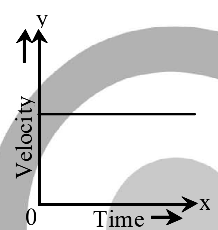


(8) For a body whose velocity changes after certain interval of time

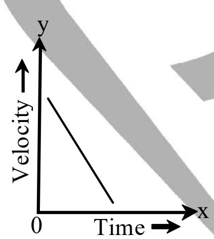


B. Velocity-Time Graph:

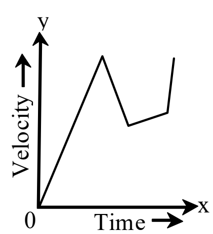
(A) For the body having constant velocity or zero acceleration



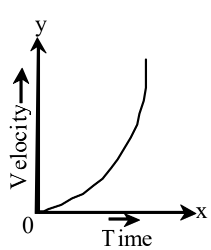
(B) When the body is moving with constant retardation and its initial velocity is not zero



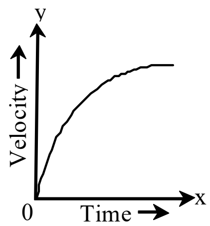
(C) When body moves with non-uniform acceleration and its initial velocity is zero.



(D) When the body is accelerated and its initial velocity is zero

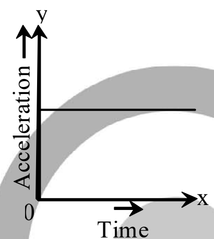


(5) When the body is decelerated.

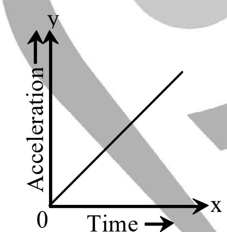


C. Acceleration-Time Graph :

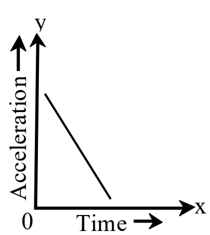
(A) When acceleration is constant



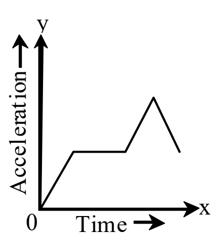
(B) When acceleration is increasing and is positive



(C) When acceleration is decreasing and is negative



(d) When initial acceleration is zero and rate of change of acceleration is non-uniform



SOLVED EXAMPLE

Ex.1 The velocity acquired by a body moving with uniform acceleration is 20 meter/second in first 2 seconds and 40 m/sec in first 4 sec. The initial velocity will be -
(A) 0 m/sec (B) 40 m/sec (C) 20 m/sec (D) None

Sol. Acceleration = $\frac{\text{Change in velocity}}{\text{Time}} = \frac{40-20}{4-2}$ = 10 m/sec^2
From the relation, v = u + at
$\Rightarrow$ 20 = u + 10 × 2 $\Rightarrow$ u = 0 m/sec
Hence correct answer is (A).

Ex.2 A body moves along the sides AB, BC and CD of a square of side 10 meter with velocity of constant magnitude 3 meter/sec. Its average velocity will be-
(A) 3 m/sec (B) 0.87 m/sec (C) 1.33 m/sec (D) None

Sol. Average velocity of the body = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{AD}{(AB+BC+CD)/v} = \frac{10}{30/3}$ = 1 m/sec
Hence correct answer is (D)

Ex.3 A body covers half the distance with a velocity 10 m/s and remaining half with a velocity 15 m/s along a straight line. The average velocity will be-
(A) 12 m/s (B) 10 m/s (C) 5 m/s (D) 12.5 m/s

Sol. Average velocity v = $\frac{2v_1v_2}{v_1+v_2} \Rightarrow \frac{2\times10\times15}{10+5}$ = 12 m/sec
Hence correct answer is (A)

Ex.4 A point travelling along a straight line traverse one third the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. The mean velocity of the point averaged over the whole time of motion will be-

(A) $\frac{v_0(v_1+v_2)}{3(v_1+v_2+v_3)}$ (B) $\frac{3v_0(v_1+v_2)}{v_1+v_2+v_3}$ (C) $\frac{v_0(v_1+v_2)}{v_1+v_2+4v_3}$ (D) $\frac{3v_0(v_1+v_2)}{v_1+v_2+4v_0}$

Sol. Let s be the total distance . Let (s/3) distance be covered in time t_1 while the remaining distance (2s/3) in time t_2 second.

Now $\left(\frac{s}{3}\right) = v_0 t_1$ or $t_1 = \frac{s}{3v_0}$ ....(A)

and $\left(\frac{2s}{3}\right) = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right)$

or $t_2 = \frac{4s}{3(v_1+v_2)}$ ....(B)

Average velocity = $\frac{s}{t_1+t_2} = \frac{s}{\frac{s}{3v_0}+\frac{4s}{(v_1+v_2)}} = \left.\frac{3v_0(v_1+v_2)}{v_1+v_2+4v_0}\right]$

Hence correct answer is (D)

Ex.5 A particle moves along the x-axis in such a way that its x-coordinate varies with time according to the equation x = 2 – 5t + 6t^2. The initial velocity and acceleration will respectively be-
(A) 5 m/s, 12 m/s^2 (B) –12 m/s, –5 m/s^2
(C) 12, – 5 m/s^2 (D) –5 m/s, 12 m/s^2

Sol. $x = 2 - 5t + 6t^2$,

$\because v = \frac{dx}{dt} = -5 + 6 \times 2 \times t$

Initial velocity at t = 0, v = – 5 m/sec

$\because a = \frac{d^2x}{dt^2} = 12\ m/sec^2$

Hence correct answer is (D)

Ex.6 The position of a body with respect to time is given by $x = 4t^3 - 6t^2 + 20t + 12$. Acceleration at t = 0 will be-

(A) –12 units (B) 12 units (C) 24 units (D) –24 units

Sol. $\because a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$\because \frac{dx}{dt} = 12t^2 - 12t + 20$

$\Rightarrow \frac{d^2x}{dt^2} = 24t - 12$

when t = 0, a = 0 – 12 = –12 units

Hence correct answer is (A).

Ex.7 A body travels 200 cm in the first two seconds and 220 cm in the next four second. The velocity at the end of the seventh second from the start will be-

(A) 10 cm/s (B) 5 cm/s (C) 15 cm/s (D) 20 cm/s

Sol. Let 'u' and 'a' be the initial velocity and acceleration of the body respectively. For first two second (t = 2 sec), the distance covered is 200 cm.

Now using, $s = ut + \frac{1}{2}at^2$

we have, $200 = u(B) + (1/2)a(B)^2$ .....(A)

After four seconds of this journey i.e., after a time t = 6 sec the distance covered is 200 cm + 220 cm = 420 cm.

Hence $420 = u(6) + (1/2)a(6)^2$ .....(B)

Solving equations (A) and (B),

we get, u = 115 cm/sec and a = – 15 cm/sec^2

Now velocity after 7 seconds = u + a t = 115 + (–15) 7 = 10 cm/sec

Hence correct answer is (A)

Ex.8 An α particle travels along the inside of straight hollow tube, 2.0 metre long, of a particle accelerator. Under uniform acceleration, how long is the particle in the tube if it enters at a speed of 1000 m/s and leaves at 9000 m/s -

(A) 4×10^{-4} sec (B) 2×10^{-7} sec

(C) 40×10^{-4} sec (D) 20×10^{-7} sec

Sol. Let a be the uniform acceleration of α-particle. According to the given problem s = 2.0 m, v = 9000 m/sec and u = 1000 m/sec

Using the formula $v^2 = u^2 + 2as$

we have, $a = \frac{v^2 - u^2}{2s} = \frac{(9000)^2 - (1000)^2}{2 \times (2.0)} = \frac{8 \times 10^7}{4.0} = 2.0 \times 10^7\ m/sec^2$

Let the particle remains in the tube for t sec.

Then $v = u + at \Rightarrow t = \frac{v - u}{a} \Rightarrow \frac{9000 - 1000}{2 \times 10^{-7}} = 4 \times 10^{-4}$ sec

Hence correct answer is (A)

Ex.9 A truck starts from rest with an acceleration of 1.5 m/s^2 while a car 150 m behind starts from rest with an acceleration of 2 m/s^2. How long will it take before both the truck and car side by side, and how much distance is travelled by each ?
(A) 2.45 s, 500 m (truck), 650 m (car) (B) 5 s, 450 m (truck), 600 m (car)
(C) 24.5 s, 450 m (truck), 600 m (car) (D) 5.3 s, 500 m (truck), 650 m (car)

Sol. Let x be the distance travelled by the truck when truck and car are side by side. The distance travelled by the car will be (x + 150) as the car is 150 metre behind the truck. Applying the formula
$s = ut + (1/2)\,a\,t^2$,
we have, $x = 1/2 \times (1.5)\,t^2$ ....(A)
and $(x + 150) = (1/2) \times (B)\,t^2$ ....(B)
Here t is the common time.
From equations (A) and (B)
we have, $\frac{x+150}{x} = \frac{2}{1.5}$
Solving we get, x = 450 m (truck)
and x + 150 = 600 m (car)
Substituting the value of x in eq. (A), we get
$450 = 1/2\,(1.5)\,t^2$
$\therefore\ t = \sqrt{\frac{450\times 2}{1.5}} = \sqrt{600}$ = 24.5 sec.
Hence correct answer is (C)

Ex.10 Two car travelling towards each other on a straight road at velocity 10 m/sec and 12 m/sec respectively. When they are 150 m apart, both drivers apply their brakes and each car decelerates at 2 m/sec^2 until it stops. How far apart will they be when they have both come to a stop?
(A) 8.9 m (B) 89 m (C) 809 m (D) 890 m

Sol. Let x_1 and x_2 be the distance travelled by the car before they stop under deceleration. Using the formula $v^2 = u^2 + 2a\,s$,
we have, $0 = (10)^2 - 2 \times 2 \times x_1$
and $0 = (12)^2 - 2 \times 2 \times x_2$
Solving we get,
x_1 = 25 metre and x_2 = 36 metre,
Total distance covered by the two cars
$x_1 + x_2 = 25 + 36 = 61$ metre
Distance between the two cars when they stop
= 150 – 61 = 89 metre
Hence correct answer is (B)

Ex.11 The driver of a train travelling at 115 km/hour sees on the same track 100 m in front of him a slow train travelling in same direction at 25 km/hr. The least retardation that must be applied to the faster train to avoid a collision will be-
(A) 3.125 m/s^2 (B) 31.25 m/s^2
(C) 312.5 m/s^2 (D) 0.3125 m/s^2

Sol. The velocity of faster train with respect to slow train = (115 – 25) = 90 km/hr.
The distance between two trains = 100 m.
If the collision is to be avoided, the relative speed should become zero till distance 100 m is covered.
Using the formula $v^2 = u^2 + 2\,a\,s$, we have
$0 = \left(90\times\frac{5}{18}\right)^2 + 2\,a \times 100$
($\because$ 90 km/h = $\frac{90\times 10^3}{60\times 60}$ = $90 \times \frac{5}{18}$ m/sec)
$a = -\frac{1}{200}\left(90\times\frac{5}{18}\right)^2$ m/sec^2 = – 3.125 m/sec^2
$\therefore$ Retardation = – a = 3.125 m/s^2
Hence correct answer is (A)

Ex.12 A car is moving with a velocity of 20 m/sec. The driver sees a stationary truck at a distance of 100 m ahead. After some reaction time Δt he applies the brakes, produces a retardation of 4 m/s^2. The maximum reaction time to avoid collision will be –

(A) 5 sec (B) 2.5 sec (C) 4 sec (D) 10 sec

Sol. Car covers a distance s before coming to rest using relation as $v^2 = u^2 + 2as$

$\Rightarrow s = \frac{20\times 20}{4\times 2} = 50$ m

To avoid the clash the remaining distance 100 – 50 = 50 m must be covered by the car with uniform velocity 20 m/s during the reaction time Δt.

Hence $\frac{50}{\Delta t} = 20$ or $\Delta t = \frac{50}{20} = 2.5$ sec

Hence correct answer is (B)

Ex.13 A balloon going upward with a velocity of 12 m/sec is at a height of 65 m from the earth at any instant. Exactly at this instant a packet drops from it. How much time will the packet take in reaching the earth? (g = 10 m/sec^2)

(A) 7.5 sec (B) 10 sec (C) 5 sec (D) None

Sol. $h = -ut + 1/2\, gt^2 \Rightarrow 65 = -12t + 5t^2$

$\Rightarrow 5t^2 - 12t - 65 - 0 \Rightarrow t = 5$ sec

Hence correct answer is (C)

Ex.14 A body is falling from a height 'h'. It takes t_1 sec to reach the ground, the time taken to reach the half of the height will be-

(A) $\sqrt{2}\ t_1$ (B) $\frac{t_1}{2}$ (C) $\frac{t_1}{\sqrt{2}}$ (D) $2t_1$

Sol. Time of fall to ground = $t_1 = \sqrt{\frac{2h}{g}}$.

Let the time taken to cover h/2 is t_2,

then $t_2 = \sqrt{\frac{2h}{2g}} = \sqrt{\frac{h}{g}} = \frac{t_1}{\sqrt{2}}$

Hence correct answer is (C).

Ex.15 A body thrown up with a velocity reaches a maximum height of 100 m. Another body with double the mass thrown up with double the initial velocity will reach a maximum height of-

(A) 400 m (B) 200 m (c) 100 m (D) 250 m

Sol. The maximum height reached by a vertically projected body is given by $S = u^2/2g$,

i.e. $S \propto u^2$ as 2g is constant,

$= \frac{S_1}{S_2} = \frac{u_1^2}{u_2^2} \Rightarrow \frac{100}{S_2} = \frac{u^2}{(2u)^2} = \frac{1}{4} \Rightarrow S_2 = 400$ m

Hence correct answer is (A)

Ex.16 A ball dropped from the top of a building takes 0.5 sec to clear the window of 4.9 m height. What is the height of building above the window?

(A) 2.75 m (B) 5.0 m (C) 5.5 m (D) 4.9 m

Sol. As the ball clears the window of height 4.9 m. Now from,

$S = ut + 1/2\, gt^2$,

We get, $4.9 = u \times (1/2) + (1/2)9.8 \times (1/2)^2$

$\Rightarrow \frac{u}{2} = 4.9 - \frac{4.9}{4} = 4.9 \times \frac{3}{4} \Rightarrow u = \frac{4.9\times 3}{2}$ m/sec

If h is the height of the building above the window. For the flight from top of the building.

$h = \frac{u^2}{2g} = \frac{4.9\times 3\times 4.9\times 3}{2\times 9.8\times 2\times 2} = 2.75$ m

Hence correct answer is (A).

Ex.17 A ball is thrown straight upward with an initial speed of 12 m/sec. After 1 sec velocity & displacement will respectively be-

(A) 2 m/sec, 7 m (B) 7m, 2 m/sec (C) 2.20 m/s, 7.10 m (D) None

Sol. $v = u - gt = 12 - 9.8 \times 1 \Rightarrow v = 12 - 9.8 = 2.20$ m/s $\Rightarrow h = ut - (1/2)\, gt^2$

$= 12 \times 1 - (1/2) \times 9.8\, (A)^2 \Rightarrow h = 12 - 4.9 = 7.10$ m.

Hence correct answer is (C)

Ex.18 A ball is thrown from ground vertically upward, reaches the roof of a house 100 meters high. At the moment this ball was thrown vertically upward, another ball is dropped from rest vertically downward from the roof of the house. At which height from the ground do the balls pass each other and after what time?

(A) $t = \frac{100}{\sqrt{1960}}$ sec, h = 25 m (B) $t = \frac{100}{\sqrt{1690}}$ sec; h = 25 m

(C) $t = \frac{200}{\sqrt{1690}}$, h = 75 m (D) $\frac{100}{\sqrt{1960}}$ sec; h = 75 m

Sol. Let the two balls meet at a height h from the top of the house or at a height (100 – h) from the bottom. Let the balls meet after a time 't' Then for the ball moving down,

$h = (1/2)\, gt^2$ ...(A)

For the ball moving up,

$100 - h = ut - (1/2)\, gt^2$ ...(B)

Since the up going ball reaches to a height of 100 m, its initial velocity u is

$0 = u^2 - 2 \times 9.8 \times 100 \Rightarrow u = \sqrt{1960}$ m/sec

From (A) & (B), $100 - (1/2)\, gt^2 = ut - 1/2\, gt^2$

$\Rightarrow t = 100/u$ ...(C)

Put (C) in (A)

$\Rightarrow h = (1/2) \times \frac{9.8 \times 100 \times 100}{1960} = 25$ m

Thus the height of their meeting point from ground is 100 – h = 75 m and they will meet after

$t = \frac{100}{\sqrt{1960}}$ sec

Hence correct answer is (D)

Ex.19 A motor boat covers the distance between two spots on the river in $t_1 = 8$ hr and $t_2 = 12$ hr downstream and upstream respectively. The time required for the boat to cover this distance in still water will be-

(A) 6.9 hr (B) 9.6 hr (C) 69 sec (D) 96 sec

Sol. Let s be the distance between that two spots. Also assume that the velocity of the motor boat in still water is v and the velocity of flow of water is u.

Then , for downward journey,

$s/t_1 = v + u$ ...(A)

For upward journey,

$s/t_2 = v - u$ ...(B)

Adding eq. (A) to (B),

$s/t_1 + s/t_2 = 2v$

or $t = \frac{s}{v} = \frac{2t_1t_2}{(t_1 + t_2)} = \frac{2 \times 8 \times 12}{(8 + 12)} = 9.6$ hr

Hence correct answer is (B)