

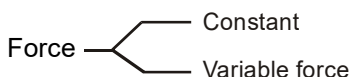
# WORK, POWER AND ENERGY



## 1. WORK

Whenever a force acting on a body displaces it, work is said to be done by the force.

Work done by a force is equal to scalar product of force applied and displacement of the body.



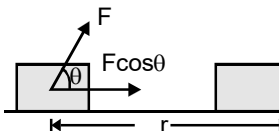
### 1.1 Work done by a constant force :

If the direction and magnitude of a force applying on a body is constant, the force is said to be constant. Work done by a constant force,

$W = \text{Force} \times \text{component of displacement along force} = \text{displacement} \times \text{component of force along displacement.}$

If a  $\vec{F}$  force is acting on a body at an angle  $\theta$  to the horizontal and the displacement  $\vec{r}$  is along the horizontal, the work done will, be  $W = (F \cos \theta) r$

$$= F (r \cos \theta)$$



In vector form,  $W = \vec{F} \cdot \vec{r}$

If  $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$  and  $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ , the work done will be,  $W = F_x \cdot x + F_y \cdot y + F_z \cdot z$

Note : The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

### 1.2 Work done by multiple forces:

If several forces act on a particle, then we can replace  $\vec{F}$  in equation  $W = \vec{F} \cdot \vec{S}$  by the net force  $\Sigma \vec{F}$  where

$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

$$\therefore W = [\Sigma \vec{F}] \cdot \vec{S} \quad \dots(i)$$

This gives the work done by the net force during a displacement  $\vec{S}$  of the particle.

We can rewrite equation (i) as :

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + \dots$$

$$\text{or } W = W_1 + W_2 + W_3 + \dots$$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

### 1.3 Work done by a variable force

If the force applying on a body is changing in its direction or magnitude or both, the force is said to be variable suppose a constant force causes displacement in a body from position  $P_1$  to position  $P_2$ . To calculate the work done by the force the path from  $P_1$  to  $P_2$  can be divided into infinitesimal element, each element is so

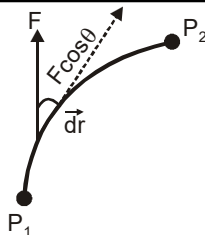
small that during displacement of body through it, the force is supposed to be constant. If  $d\vec{r}$  be small

displacement of body and  $\vec{F}$  be the force applying on the body, the work done by force is  $dW = \vec{F} \cdot d\vec{r} \dots\dots$

(i) The total work done in displacing body from  $P_1$  to  $P_2$  is given

$$\text{by, } \int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$

$$\text{or } W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$$



If  $\vec{r}_1$  and  $\vec{r}_2$  be the position vectors of the points  $P_1$  and  $P_2$  respectively, the total work done will be -

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

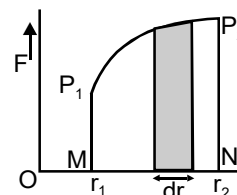
**Note :** When we consider a block attached to a spring, the force on the block is  $k$  times the elongation of the spring, where  $k$  is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

### 1.4 Calculation of work done from force displacement graph :

Suppose a body, whose initial position is  $r_1$ , is acted upon by a variable force  $\vec{F}$  and consequently the body acquires its final position  $r_2$ . From position  $r$  to  $r + dr$  or for small displacement  $dr$ , the work done will be  $Fdr$  whose value will be the area of the shaded strip of width  $dr$ .

The work done on the body in displacing it from position  $r_1$  to  $r_2$  will be equal to the sum of areas of all the such strips

$$\begin{aligned} \text{Thus, total work done, } W &= \sum_{r_1}^{r_2} dW \\ &= \sum_{r_1}^{r_2} F \cdot dr \\ &= \text{Area of } P_1P_2NM \end{aligned}$$



The area between the graph between force and displacement axis is equal to the work done.

**Note :** To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between  $F \cos \theta$  and  $r$ .

(i) Work is a scalar quantity

(ii) The dimensions of work :  $ML^2T^{-2}$

(iii) Unit of work : there are two types of unit of work

(a) Absolute unit : Joule (in M.K.S), Erg (in C.G.S.) (Note :  $10^7$  erg = 1 joule)

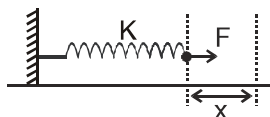
(b) Gravitational unit : Kilogram - metre (in M.K.S), Gram-cm (in C.G.S)

(Note : 1 kilogram metre = 9.8 joule =  $10^5$  gram cm)

### 1.5 Work done by spring force

#### Solved Examples

Ex.1



Initially spring is relaxed. A person starts pulling the spring by applying a variable force  $F$ . Find out the work done by  $F$  to stretch it slowly to a distance by  $x$ .

**Sol.**  $\int dW = \int \mathbf{F} \cdot d\mathbf{s} = \int_0^x Kx \, dx \quad \Rightarrow \quad W = \left( \frac{Kx^2}{2} \right)_0^x = \frac{Kx^2}{2}$

**Ex.2** In the above example

- (i) Where has the work gone ?
- (ii) Work done by spring on wall is zero. Why?
- (iii) Work done by spring force on man is \_\_\_\_\_.

**Sol.** (i) It is stored in the form of potential energy in spring.  
 (ii) Zero, as displacement is zero.

(iii)  $-\frac{1}{2}Kx^2$

**Ex.3** Find out work done by applied force to slowly extend the spring from  $x$  to  $2x$ .

**Sol.** Initially the spring is extended by  $x$

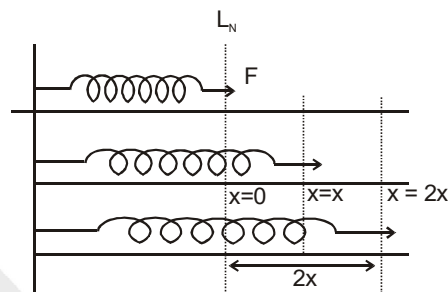
$$W = \vec{F} \cdot \vec{ds}$$

$$W = \int_x^{2x} Kx \, dx$$

$$W = \left[ \frac{Kx^2}{2} \right]_x^{2x} = \frac{3}{2}Kx^2$$

It can also found by difference of PE.

$$\text{i.e. } U_f = \frac{1}{2}K(2x)^2 = 2Kx^2 \quad \Rightarrow \quad U_i = \frac{1}{2}Kx^2 \quad \Rightarrow \quad U_f - U_i = \frac{3}{2}Kx^2$$



## 1.6 Work done by internal force

Sum of internal forces is zero. But it is not necessary that work done by internal force is zero. There must be some deformation or reformation between the system to do internal work. In case of a rigid body work done by internal force is zero.

## 1.7 Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero

(a) **Positive work** : As  $W = \vec{F} \cdot \vec{r} = F r \cos \theta$

$\therefore$  When  $\theta$  is acute ( $<90^\circ$ )  $\cos \theta$  is positive. Hence work done is positive.

(i) When a body falls freely under the action of gravity  $\theta = 0^\circ$ ,  $\cos \theta = +1$ , therefore work done by gravity on a body, falling freely is positive.

(b) **Negative work** : When  $\theta$  is obtuse ( $>90^\circ$ ),  $\cos \theta$  is negative. Hence work done is negative

(i) When a body is through up, its motion is opposed by gravity. The angle  $\theta$  between the gravitational force  $\vec{F}$  and displacement  $\vec{r}$  is  $180^\circ$ . As  $\cos \theta = -1$ , therefore, work done by gravity is negative.

(ii) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction.

Hence work done by frictional force is negative. **Note that work done by the applied force is not negative**

(iii) When a positive charge is moved closer to another positive charge, work done by electrostatic force of repulsion between the charges is negative.

(c) **Zero work** : When force  $\vec{F}$  or the displacement  $\vec{r}$  or both are zero, work done  $W$ , will be zero. Again when angle  $\theta$  between  $\vec{F}$  and  $\vec{r}$  is  $90^\circ$ , the work done will be zero.

- (i) When we fail to move a heavy stone, however hard we may try, work done by us is zero,  $\vec{r} = 0$   
 (ii) When a collie carrying some load on his head moves on horizontal platform,  $\theta = 90^\circ$ . Therefore, workdone by the collie is zero. This is because  $\theta = 90^\circ$   
 (iii) Tension in the string of simple pendulum is always perpendicular to displacement of the bob. Therefore, work done by tension is always zero.

Note : Another way of expressing negative or positive work is that when energy is transferred to the object work done is positive and when energy is transferred from object the work done is negative and hence the work which is a transfer of energy has same dimensions as energy.

### Solved Examples

**Ex.4** A position dependent force  $\vec{F} = 7 - 2x + 3x^2$  acts on a small body of mass 2kg and displaces it from  $x = 0$  to  $x = 5$  m. The work done in joule will be

**Sol.** 
$$W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2) dx = \left[ 7x - \frac{2x^2}{2} + \frac{3x^3}{3} \right]_0^5 = 135\text{J}$$

**Ex.5** A uniform chain of mass  $M$  and length  $L$  is lying on a frictionless table in such a way that its  $1/3$  part is hanging vertically down. The work done in pulling the chain up the table is

**Sol.** If length  $x$  of the chain is pulled up on the table, then the length of hanging part of the chain would be

$\left(\frac{L}{3} - x\right)$  and its weight would be  $\frac{M}{L}\left(\frac{L}{3} - x\right)g$ . If it is pulled up further by a distance  $dx$ , the work done in pulling up.

$$= \frac{M}{L}\left(\frac{L}{3} - x\right)g dx \quad \therefore \quad w = \int_0^{L/3} \frac{M}{L}\left(\frac{L}{3} - x\right)g dx = \frac{MgL}{18}$$

**Ex.6** The work done in pulling a body of mass 5 kg along an inclined plane (angle  $60^\circ$ ) with coefficient of friction 0.2 through 2m, will be

**Sol.** The minimum force with a body is to be pulled up along the inclined plane is  $mg (\sin\theta + \mu\cos\theta)$

Work done,  $W = \vec{F} \cdot \vec{d}$

$$= Fd \cos \theta^\circ$$

$$= mg (\sin\theta + \mu \cos\theta) \times d$$

$$= 5 \times 9.8 (\sin 60^\circ + 0.2 \cos 60^\circ) \times 2$$

$$= 98.08 \text{ J}$$



## 2. ENERGY

The energy of a body is defined as the capacity of doing work.

There are various form of energy

- (i) mechanical energy      (ii) chemical energy      (iii) electrical energy      (iv) magnetic energy  
 (v) nuclear energy      (vi) sound energy      (vii) light energy etc

Energy of system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

### Example

Electric energy	Motor →	Mechanical energy
Mechanical energy	Generator →	Electrical energy
Light energy	Photocell →	Electrical energy

Electrical energy	Heater →	Heat energy
Electrical energy	Radio →	Sound energy
Nuclear energy	Nuclear Reactor →	Electrical energy
Chemical energy	Cell →	Electrical energy
Electrical energy	Secondary Cell →	Chemical energy
Heat energy	Incandescence nt lamp →	Light

Energy is a scalar quantity

Unit : Its unit is same as that of work or torque.

In MKS : Joule, watt sec

In CGS : Erg

**Note** : 1 eV =  $1.6 \times 10^{-19}$  joule

1 KWh =  $36 \times 10^5$  joule

$10^7$  erg = 1 joule

Dimension  $[M^1L^2T^{-2}]$

According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other

Energy equivalent of mass m is,  $E = mc^2$

Where, m : mass of the particle

c : velocity of light

E : equivalent energy corresponding to mass m.

In mechanis we are concerned with mechanical energy only which is of two type

(a) kinetic energy (ii) potential energy

## 2.1 Kinetic energy

The energy possessed by a body by virtue of its motion is called kinetic energy

If a body of mass m is moving with velocity v, its kinetic energy

$$KE = \frac{1}{2} mv^2, \text{ for translatory motion}$$

$$KE = \frac{1}{2} I\omega^2, \text{ for rotational motion}$$

Kinetic energy is always positive

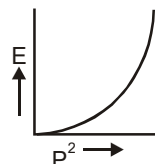
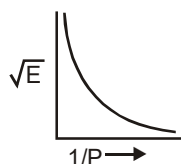
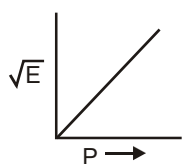
$$\text{If linear momentum of body is } p, KE = \frac{p^2}{2m} = \frac{1}{2} mv^2 - \text{ for translatory motion}$$

$$\text{If angular momentum of body is } J, KE = \frac{J^2}{2I} = \frac{1}{2} I\omega^2 - \text{ for rotational motion}$$

p or J  $\propto \sqrt{E}$

p : momentum

E : kinetic energy



The kinetic energy of a moving body is measured by the amount of work which has been done in bringing the body from the rest position to its present moving position or

The kinetic energy of a moving body is measured by the amount of work which the body can do against the external forces before it comes to rest.

If a body performs translatory and rotational motion simultaneously, its total kinetic energy =  $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

### 3. WORK-ENERGY THEOREM

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where,  $W_C$  is the work done by all the conservative forces.

$W_{NC}$  is the work done by all non-conservative forces.

$W_{PS}$  is the work done by all pseudo forces.

#### Modified Form of Work-Energy Theorem :

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$

### Solved Examples

**Ex.7** A body of mass  $m$  when released from rest from a height  $h$ , hits the ground with speed  $\sqrt{gh}$ .

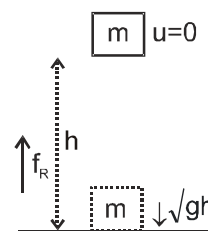
Find work done by resistive force.

**Sol.** Identify initial and final state and identify all forces.

$$W_g + W_{\text{air res.}} + W_{\text{int force}} = \Delta K$$

$$mgh + W_{\text{air res.}} + 0 = \frac{1}{2}m(\sqrt{gh})^2 - 0$$

$$\Rightarrow W_{\text{air res.}} = -\frac{mgh}{2}$$

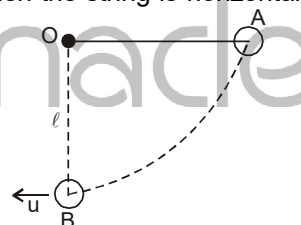


**Ex.8** The bob of a simple pendulum of length  $l$  is released when the string is horizontal. Find its speed at the bottom.

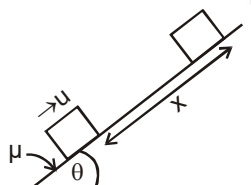
**Sol.**  $W_g + W_T = \Delta K$

$$mg\ell + 0 = \frac{1}{2}mu^2 - 0$$

$$u = \sqrt{2g\ell}$$



**Ex.9** A block is given a speed  $u$  up the inclined plane as shown.



Using work energy theorem find out  $x$  when the block stops moving.

**Sol.**  $W_g + W_f + W_N = \Delta K$

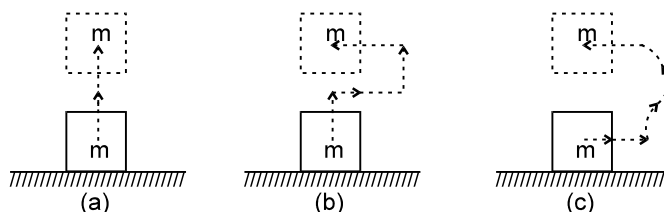
$$-mgx \sin \theta - \mu mgx \cos \theta + 0 = 0 - \frac{1}{2}mu^2$$

$$\Rightarrow x = \frac{u^2}{2g(\sin \theta + \mu \cos \theta)}$$



## 4. CONSERVATIVE FORCES

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



Consider a body of mass  $m$  being raised to a height  $h$  vertically upwards as shown in the above figure. The work done is  $mgh$ . Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result  $mgh$  once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal parts is zero. The work done along the vertical parts add up to  $mgh$ . Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

### Examples of Conservative forces :

- (i) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.
  - (ii) Elastic force in a stretched or compressed spring is a conservative force.
  - (iii) Electrostatic force between two electric charges is a conservative force.
  - (iv) Magnetic force between two magnetic poles is a conservative force.
- In fact, all fundamental forces of nature are conservative in nature.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrostatic forces are two important examples of central forces. Central forces are conservative forces.

### PROPERTIES OF CONSERVATIVE FORCES

- (i) **Work done by or against a conservative force depends only on the initial and final positions of the body.**
- (ii) Work done by or against a conservative force does not depend upon the nature of the path between initial and final positions of the body.

If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

- (iii) Work done by or against a conservative force in a round trip is zero.

If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

The concept of potential energy exists only in the case of conservative forces.

- (iv) The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work of a conservative force.

### NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

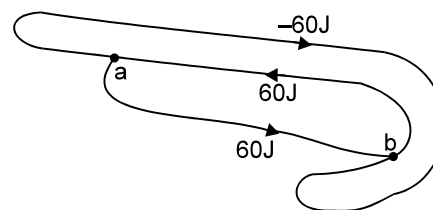
The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

S.No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done is not completely recoverable.

### Solved Examples

**Ex.10** The figure shows three paths connecting points a and b. A single force  $F$  does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force  $F$  conservative?



**Ans.** No

**Explanation :** For a conservative force, the work done in a round trip should be zero.

**Ex.11** Find the work done by a force  $\vec{F} = x\hat{i} + y\hat{j}$  acting on a particle to displace it from point A (0, 0) to B(2, 3).

**Sol.**  $dW = \vec{F} \cdot d\vec{s} = (x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$$W = \int_0^2 x dx + \int_0^3 y dy = \left[ \frac{x^2}{2} \right]_0^2 + \left[ \frac{y^2}{2} \right]_0^3 = \frac{13}{2} \text{ units}$$

### True or False

**Ex.12** In case of a non conservative force work done along two different paths will always be different.

**Ans.** False

**Ex.13.** In case of non conservative force work done along two different paths may be different.

**Ans.** True

**Ex.14** In case of non conservative force work done along all possible paths cannot be same.

**Ans.** True





## 5. POTENTIAL ENERGY

The energy which a body has by virtue of its position or configuration in a conservation force field

Potential energy is a relative quantity.

Potential energy is defined only for conservative force field.

Potential energy of a body at any position in a conservation force field is defined as the workdone by an external agent against the action of conservation force in order to shift it from reference point.

(PE = 0) to the present position or.

Potential energy of a body in a conservation force field is equal to the work done by the body in moving from its present position to reference position.

At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.

Relationship between conservative force field and potential energy (U)  $\vec{F} = -\nabla U = -\text{grad } (U)$

$$= -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} - \frac{\partial U}{\partial z} \hat{k}$$

### Solved Example

Ex. 15  $U = 3x^2$

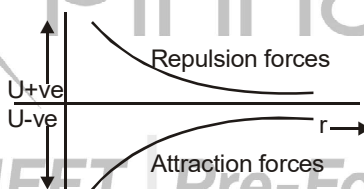
Sol.  $F_x = -\frac{dU}{dx}$

$$\Rightarrow \vec{F} = -6x\hat{i}$$

If force varies only with one dimension then  $F = -\frac{dU}{dx}$  or  $U = -\int_{x_1}^{x_2} F dx$

Potential energy may be positive or negative

- i) Potential energy is positive, if force field is repulsive in nature
- ii) Potential energy is negative, if force field is attractive in nature



If  $r \uparrow$  (separation between body and force centre),  $U \uparrow$ , force field is attractive or vice-versa.

If  $r \uparrow$ ,  $U \downarrow$ , force field is repulsive in nature.



## 6. POWER

(a) The time rate of doing work is called power

$$(b) \text{ Power} = \frac{dw}{dt} = \vec{F} \cdot \frac{d\vec{x}}{dt}$$

In translatory motion :  $\vec{P} = \vec{F} \cdot \vec{v}$

In rotational motion :  $P = \vec{\tau} \cdot \vec{\omega}$

(c) It is scalar quantity

(d) Unit :

In MKS - J/sec, watt

In CGS - erg/sec, (Note : 1KW =  $10^3$  watt, 1 HP = 746 watt)

(e) Dimension :  $[M^1L^2T^{-3}]$

Note : Power is the rate at which applied force transfers energy

(a) Power  $\bar{P} = \frac{W}{\Delta t}$  where W work is done in  $\Delta t$  time

(b) Instantaneous power  $P = \frac{dw}{dt}$ , it's value may change with time.

### Solved Examples

**Ex.16** A pump can lift 9000 kg coal per hour from a mine of depth 120 m. Assuming its efficiency to be 75% its power will be (in watts) -

**Sol.** Power =  $\frac{\text{work}}{\text{time}}$

$$\text{Efficiency of pump} = \frac{\text{output power}}{\text{input power}} ; \frac{3}{4} = \frac{\text{output power}}{P}$$

$$\therefore \text{Output power} = \frac{3P}{4}$$

$$\therefore \frac{3}{4}P = \frac{mgh}{t}$$

$$P = \frac{4}{3} \frac{mgh}{t} = \frac{4}{3} \times \frac{9000 \times 9.8 \times 120}{3600} = 3920 \text{ W}$$

**Ex.17** A person of mass 60 kg is capable of taking a 15 kg massive objects to a height of 10 m in 3 minutes. The efficiency of person is -

**Sol.** % Efficiency  $\eta = \frac{\text{work output}}{\text{work input}} \times 100 = \frac{mgh}{(m+M)gh} \times 100$

$$= \left( \frac{m}{m+M} \right) \times 100 = \left( \frac{15}{15+60} \right) \times 100 = 20\%$$

**Ex.18** An automobile of mass m accelerates from rest. If the engine supplies constant power p, the velocity at time t is given by

**Sol.** Given that power = Fv = p = constant

$$\text{or } m \frac{dv}{dt} v = p \quad [\text{as } F = ma = \frac{mdv}{dt}]$$

$$\text{or } \int v dv = \int \frac{p}{m} dt$$

$$\frac{v^2}{2} = \frac{p}{m}t + C_1$$

Now as initially, the body is at rest i.e.  $v = 0$  at  $t = 0$  so,  $C_1 = 0$

$$\therefore v = \sqrt{\frac{2pt}{m}}$$

**Ex.19** In the above problem, the position ( $s$ ) at time ( $t$ ) is given by

**Sol.** By definition  $v = \frac{ds}{dt}$  or  $\frac{ds}{dt} = \left(\frac{2pt}{m}\right)^{1/2}$  [From (1)]

$$\Rightarrow \int ds = \int \left(\frac{2pt}{m}\right)^{1/2} dt \Rightarrow s = \left(\frac{2p}{m}\right)^{1/2} \frac{2}{3} t^{3/2} + C_2$$

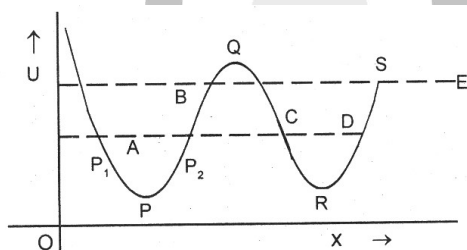
$$\text{Now as } t = 0, s = 0, \text{ so } C_2 = 0 \Rightarrow s = \left(\frac{8p}{9m}\right)^{1/2} t^{3/2}$$



## 7. POTENTIAL ENERGY CURVE

A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve

Using graph, we can predict the rate of motion of a particle at various positions.



Force on the particle is  $F_{(x)} = -\frac{dU}{dx}$

**Case : I** On increasing  $x$ , if  $U$  increase, force is in (–)ve  $x$ -direction i.e. attraction force.

**Case : II** On increasing  $x$ , if  $U$  decreases, force is in (+)ve  $x$ -direction i.e. repulsion force.

**Different positions of a particle**

**Position of equilibrium**

If net force acting on a body is zero, it is said to be in equilibrium for equilibrium

$\frac{dU}{dx} = 0$  Points P, Q, R and S are the states of equilibrium positions.

**Types of equilibrium :**

**Stable equilibrium** - When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in state equilibriums position.

Necessary conditions -  $\frac{dU}{dx} = 0$ ,  $\frac{d^2U}{dx^2} = +ve$

**Unstable equilibrium :** When a particle is displaced slightly from a position a position and force acting on it tries to displace the practice further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition :  $\frac{dU}{dx} = 0$  potential energy is max i.e.  $\frac{d^2U}{dx^2} = -ve$

**Neutral equilibrium :** In the neutral equilibrium potential energy is constant when a particle is displaced from its position it does not experiences any force to acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

## 8. MECHANICAL ENERGY :

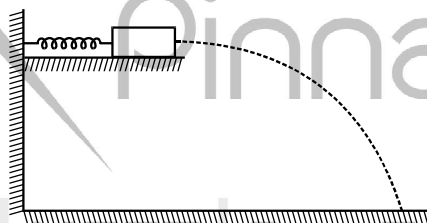
**Definition:** Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e.,  $E = K + U$

**Important Points for M.E.:**

1. It is a scalar quantity having dimensions  $[ML^2T^{-2}]$  and SI units joule.
2. It depends on frame of reference.
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if  $E = 0$  either both PE and KE are zero or PE may be negative and KE may be positive such that  $KE + PE = 0$ .
4. As mechanical energy  $E = K + U$ , i.e.,  $E - U = K$ . Now as K is always positive,  $E - U \geq 0$ , i.e., for existence of a particle in the field,  $E \geq U$ .
5. As mechanical energy  $E = K + U$  and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if  $K > |U|$  and E will be negative if  $K < |U|$  i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

### Solved Examples

**Ex.20** A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (figure). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring?



**Sol.** When block released, the block moves horizontally with speed V till it leaves the spring.

By energy conservation  $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$$v^2 = \frac{kx^2}{m}$$

$$\Rightarrow v = \sqrt{\frac{kx^2}{m}}$$

$$\text{Time of flight } t = \sqrt{\frac{2H}{g}}$$

So, horizontal distance travelled from the free end of the spring is  $V \times t$

$$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{100 \times (0.05)^2}{0.1}} \times \sqrt{\frac{2 \times 2}{10}} = 1 \text{ m}$$

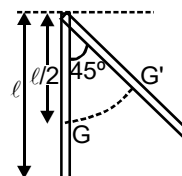
So, At a horizontal distance of 1 m from the free end of the spring.

**Ex.21** A meter scale of mass  $m$  initially vertical is displaced at  $45^\circ$  keeping the upper end fixed, the change in PE will be-

**Sol.** Work = change in PE = Force  $\times$  displacement

$$dU = mg \frac{\ell}{2} (1 - \cos \theta)$$

$$= mg \times \frac{1}{2} (1 - \cos 45^\circ) \quad (\because \ell = 1\text{m}) = \frac{mg}{2} \left(1 - \frac{1}{\sqrt{2}}\right)$$



**Ex.22** If the speed of a car increases 4 times, the stopping distance for this will increase by -

**Sol.** Work = Change in KE

$$\therefore FS = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

$$\frac{S'}{S} = \frac{v'^2}{v^2} \Rightarrow \frac{S'}{S} = 16 \Rightarrow S' = 16S$$

**Ex.23** If the potential energy function for a particle is  $U = a - \frac{b}{x} + \frac{c}{x^2}$  the force constant for oscillation will be.

**Sol.**  $U = a - \frac{b}{x} + \frac{c}{x^2}$  .....(1)

$$\therefore \frac{dU}{dx} = -\frac{b}{x^2} - \frac{2c}{x^3}$$
 .....(2)

$$\text{and } \frac{d^2U}{dx^2} = \frac{1}{x^3} \left(-2b + \frac{6c}{x}\right)$$
 .....(3)

for equilibrium

$$\frac{dU}{dx} = 0 \quad \therefore \quad x = \frac{2c}{b} \quad \therefore \quad \text{Substituting this value in (3)} \quad \frac{d^2U}{dx^2} = \left(\frac{b}{2c}\right)^3 \left[-2b + \frac{6c}{2c/b}\right] = \frac{b^4}{8c^3}$$

$$\text{as } \frac{d^2U}{dx^2} = K \quad \therefore \quad K = b^4/8c^3$$

### Miscellaneous Problem

**Problem 1 :** On passing through a wooden sheet a bullet loses 1/20 of initial velocity. The minimum number of sheets required to completely stop the bullet will be-

**Solution :** Use  $v^2 = u^2 + 2as$

for a sheet of thickness  $s$   $v = (19/20)u$

$$\left(\frac{19}{20}u\right)^2 = u^2 + 2as$$

$$2as = (361/400)u^2 - u^2 \quad a = -\left(\frac{39}{400}\right)\frac{u^2}{2s}$$

suppose for  $n$  sheet  $v = 0$   $\therefore 0^2 = u^2 + 2a(ns)$   $n = -\frac{u^2}{2as} = \frac{u^2}{2\left(\frac{39}{400}\right)\frac{u^2}{2s} \times s} \approx 11$

**Problem 2 :** The work done in taking out 2 lit of water using a bucket of mass 0.5 kg from a well of depth 6m will be-

**Solution :**  $W = mgh$

$$= (m_{\text{bucket}} + m_{\text{water}})gh [2 \text{ Lit water} = 2 \text{ kg water}]$$

$$= (0.5 + 2.00) \times 9.8 \times 6$$

$$= 15 \times 9.8 = 147 \text{ J}$$

**Problem 3 :** A body has velocity 200 m/s and its kinetic energy is 200 J. The mass of the body would be

**Solution :**  $\frac{1}{2}mv^2 = E$  or  $m = \left(\frac{2E}{v^2}\right) = \frac{2 \times 200}{(200)^2} = \frac{4 \times 10^2}{4 \times 10^4} = \frac{1}{100} \therefore m = 0.01 \text{ kg}$

**Problem 4 :** A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as  $x = 1/2t^2$  where  $x$  is in meter and  $t$  in sec. The work done by the force in first two seconds.

**Solution :** Work done = change in kinetic energy

or  $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{2t}{2}\right)^2 = \frac{1}{2} \times 8 \times \left[\frac{2 \times 2}{2}\right]^2 = 16 \text{ Joules}$

**Problem 5 :** A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then body would reach upto a height of

**Solution :**  $\frac{1}{4}mgh = mgh'$

$$\therefore h' = \frac{h}{4} = \frac{1}{4} \times 20 = 5 \text{ cm}$$

**Problem 6 :** Potential energy function describing the interaction between two atoms of a diatomic molecule is

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

In stable equilibrium, the distance between them would be

**Solution :** In stable equilibrium potential energy is minimum. For minimum value of  $U(x)$

$$\frac{d}{dx}[U(x)] = 0$$

$$\text{or } \frac{d}{dx} \left( \frac{a}{x^{12}} - \frac{b}{x^6} \right) = 0$$

$$\text{or } \frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$$

$$\text{or } \frac{6}{x^{13}} (-2a + bx^6) = 0$$

$$\text{or } bx^6 - 2a = 0$$

$$\therefore x = \left( \frac{2a}{b} \right)^{1/6}$$

**Problem 7 :** Two electrons are at a distance of  $1 \times 10^{-12} \text{m}$  from each other. Potential energy (in eV) of this system would be

**Solution :** Potential energy of the system

$$U = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{1 \times 10^{-12}} = 23.04 \times 10^{-17} \text{ Joule}$$

$$= \frac{23.04 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} = 1.44 \times 10^3 \text{ eV}$$

**Problem 8 :** Potential energy function  $U(r)$  corresponding to the central force  $F = \frac{K}{r^2}$  would be

**Solution :** Central force is conservative. Therefore

$$\vec{F}(r) = -\frac{dU}{dr} \hat{r} \quad \text{or} \quad dU = -\vec{F}(r) \cdot d\vec{r} = -F(r) dr$$

$$\therefore U = \int dU = -\int F(r) dr = -\int \frac{K}{r^2} dr = -K \int \frac{1}{r^2} dr = Kr^{-1} + C$$

If at  $r = \infty$ ,  $U = 0$ , then  $C = 0$

$$U = Kr^{-1} = \frac{K}{r}$$

**Problem 9 :** The stopping distance for a vehicle of mass  $M$  moving with speed  $v$  along level road, will be ( $\mu$  is the coefficient of friction between tyres and the road)

**Solution :** When the vehical of mass  $m$  is moving with velocity  $v$ , the kinetic energy of the where  $K = 1/2 mv^2$  and if  $S$  is the stopping distance, work done by the firction

$$W = FS \cos \theta = m MgS \cos 180^\circ = -m MgS$$

So by Work-Energy theorem,  $W = DK = K_f - k_i$

$$\Rightarrow -\mu MgS = 0 - 1/2 Mv^2 \quad \Rightarrow \quad S = \frac{v^2}{2\mu g}$$

**Problem 10 :** A particle of mass  $m$  is moving in a horizontal circle of radius  $r$ , under a centripetal force equal to  $(-k/r^2)$ , where  $k$  is constant. The total energy of the particle is

**Solution :** As the particle is moving in a circle, so

$$\frac{mv^2}{r} = \frac{k}{r^2} \quad \text{Now} \quad K.E = \frac{1}{2} mv^2 = \frac{k}{2r}$$

$$\text{Now as } F = -\frac{dU}{dr} \quad \text{P.E, } U = -\int_{\infty}^r F dr$$

$$= \int_{\infty}^r \left( \frac{k}{r^2} \right) dr = -\frac{k}{r}$$

So total energy = U + K.E.

$$= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$$

Negative energy means that particle is in bound state.

**Problem 11 :** The work done by a person in carrying a box of mass 10 kg. through a vertical height of 10 m is 4900J. The mass of the person is

**Solution :** Let the mass of the person is m .

Work done, W = P.E at height h above the earth surface.

$$= (M + m) gh$$

$$\text{or } 4900 = (M + 10) 9.8 \times 10$$

$$\text{or } M = 40 \text{ kg}$$

**Problem 12 :** A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. The work done in keeping it vertical with one of its ends touching the ground, will be - .

**Solution :** As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length = l/2

$$\text{Work done } w = mgh = mg \frac{\ell}{2} = 20 \times 9.8 \times \frac{4}{2} = 392 \text{ J}$$

**Problem 13 :** A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save

**Solution :** In first case,  $W_1 = \frac{1}{2} m(v_1)^2 + mgh$

$$= \frac{1}{2} m(12)^2 + m \times 10 \times 12$$

$$= 72 m + 120 m$$

and in second case,  $W_2 = mgh$

$$= 120 m$$

$$\text{The percentage of energy saved} = \frac{192m - 120m}{192m} \times 100 = 38\%$$

\*\*\*\*\*



# **KEY CONCEPT**



## **Work done by Constant Force :**

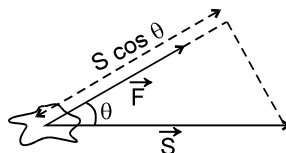
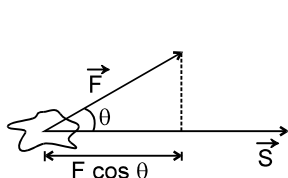
$$\mathbf{W} = \vec{F} \cdot \vec{S}$$

Since work is the dot product of two vectors therefore it is a scalar quantity.

$$W = FS \cos \theta$$

or

$$W = (F \cos \theta)S$$



## **Work Done by Multiple Forces :**

If several forces act on a particle, then we can replace  $\vec{F}$  in equation  $W = \vec{F} \cdot \vec{S}$  by the net force  $\Sigma \vec{F}$  where

$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

$$W = [\Sigma \vec{F}] \cdot \vec{S} \quad \dots(i)$$

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + \dots$$

or  $W = W_1 + W_2 + W_3 + \dots$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.



## **Dimensions of Work :**

$$[\text{Work}] = [\text{Force}] [\text{Distance}] = [MLT^{-2}] [L] = [ML^2T^{-2}]$$

Work has one dimension in mass, two dimensions in length and '-2' dimensions in time,



## **Work in Terms of Rectangular Components :**

In terms of rectangular components,  $\vec{F}$  and  $\vec{S}$  may be written as :

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \quad \text{and} \quad \vec{S} = S_x \hat{i} + S_y \hat{j} + S_z \hat{k}$$

$$\vec{F} \cdot \vec{S} = F_x S_x + F_y S_y + F_z S_z$$



## **Work Done by a Variable Force :**

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant

$$dW = \vec{F} \cdot d\vec{s}$$

The total work done will be sum of infinitely small work

$$W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (\vec{F} \cos \theta) d\vec{s}$$

In terms of rectangular components,

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$W_{A \rightarrow B} = \int_{x_A}^{x_B} F_x dx + \int_{x_A}^{x_B} F_y dy + \int_{x_A}^{x_B} F_z dz$$

**Relation Between Momentum and Kinetic Energy :**

**Important Points for K.E.**

1. As mass  $m$  and  $v^2$  ( $\vec{v} \cdot \vec{v}$ ) are always positive, kinetic energy is always positive scalar i.e, kinetic energy can never be negative.
2. The kinetic energy depends on the frame of reference,

$$K = \frac{p^2}{2m} \quad \text{and} \quad P = \sqrt{2mK} \quad ; \quad P = \text{linear momentum}$$

**Potential Energy :**

In case of conservative force

$$\int_{U_1}^{U_2} dU = - \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$\text{i.e.,} \quad U_2 - U_1 = - \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = -W, \quad \text{where } W \text{ is work done by conservative forces}$$

Whenever and wherever possible, we take the reference point at  $\infty$  and assume potential energy to be zero there, i.e., If we take  $r_1 = \infty$  and  $U_1 = 0$  then

$$U = - \int_{\infty}^r \vec{F} \cdot d\vec{r} = -W$$

**Types of Potential Energy :**

**(a) Elastic Potential Energy:**

$$U = \frac{1}{2}ky^2$$

where  $k$  is force constant and 'y' is the stretch or compression. Elastic potential energy is always positive.

**(b) Electric Potential Energy:** It is the energy associated with charged particles that interact via electric force. For two point charges  $q_1$  and  $q_2$  separated by a distance 'r',

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

As charge can be positive or negative, therefore electric potential energy can also be positive or negative.

**(c) Gravitational Potential Energy:** It is due to gravitational force. For two particles of masses  $m_1$  and  $m_2$  separated by a distance 'r', it is given by:

$$U = -G \frac{m_1m_2}{r}$$

which for a body of mass 'm' at height 'h' relative to surface of the earth reduces to  $U = mgh$   
Gravitational potential energy can be positive or negative.

**Mechanical Energy :**

**Definition:** Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e.,

$$E = K + U$$

**Conservative Forces :**

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.

$$F = - \frac{\partial U}{\partial r}$$



### Conservative Force & Potential Energy :

We know that  $F = -\frac{\partial U}{\partial r}$



### Types of Equilibrium :

- (a) **Stable equilibrium** : When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions :  $-\frac{dU}{dx} = 0$ , and  $\frac{d^2U}{dx^2} = +ve$  Potential energy is minimum.

- (b) **Unstable Equilibrium** : When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition :  $-\frac{dU}{dx} = 0$  potential energy is maximum i.e.  $\frac{d^2U}{dx^2} = -ve$

- (c) **Neutral equilibrium** : In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.



### Work-Energy Theorem :

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where,  $W_C$  is the work done by all the conservative forces.

$W_{NC}$  is the work done by all non-conservative forces.

$W_{PS}$  is the work done by all psuedo forces.



### Modified Form of Work-Energy Theorem :

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$



### Power :

**Power is defined as the time rate of doing work.**

The average power ( $\bar{P}$  or  $p_{av}$ ) delivered by an agent is given by

$$\bar{P} \text{ or } p_{av} = \frac{W}{t}$$

where  $W$  is the amount of work done in time  $t$ .

$$P = \frac{\vec{F} \cdot d\vec{S}}{dt} = \vec{F} \cdot \frac{d\vec{S}}{dt} = \vec{F} \cdot \vec{v}$$

By definition of dot product,

$$P = Fv \cos \theta$$

where  $\theta$  is the smaller angle between  $\vec{F}$  and  $\vec{v}$ .

This  $P$  is called as instantaneous power if  $dt$  is very small.

**EXERCISE – 1**  
**BUILDING A FOUNDATION**

**SECTION-A WORK DONE BY CONSTANT FORCE**

- A-1.** A man pushes wall and fails to displace it. He does  
 (a) Negative work (b) Positive but not maximum work  
 (c) No work at all (d) Maximum work
- A-2.** A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is  
 (a)  $0^\circ$  (b)  $30^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
- A-3.** A particle moves from position  $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$  to position  $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$  under the action of force  $4\hat{i} + \hat{j} + 3\hat{k}$  N. The work done by this force will be  
 (a) 100 J (b) 50 J (c) 200 J (d) 75 J
- A-4.** A 50 kg man with 20 kg load on his head climbs up 20 steps of 0.25 m height each. The work done by the man on the block during climbing is  
 (a) 5 J (b) 350 J (c) 1000 J (d) 3540 J
- A-5.** A body which is constrained for move along Y-direction is acted upon by a force  $\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$  N. The work done by this force in displacing the body by 10m along Y-axis is-  
 (a) 105 (b) 150 (c) 250 (d) 100
- A-6.** Find the work done in moving a 50 kg. block through a horizontal distance of 10 m. by applying a force of 100 N which makes an angle of  $60^\circ$  with the horizontal-  
 (a) 50 Joule (b) 50 ergs (c) 500 Joule (d) 500 ergs
- A-7.** A body is constrained to move in y-direction. If is subjected to a force  $(-2\hat{i} + 15\hat{j} + 6\hat{k})$  N. The work done by this force in moving the body through a distance 10 m is :  
 (a) 100 J (b) 120 J (c) 400 J (d) 150 J
- A-8.** If no work is done by a force on an object, then which of the alternative is correct ?  
 (a) the force is always perpendicular to its velocity  
 (b) the force is always perpendicular to its acceleration  
 (c) the force moves the object  
 (d) none of these
- A-9.** A point charge Q is moved along a circular path around another fixed point charge . The work done is zero :  
 (a) only if Q return to its starting point  
 (b) only if the two charges have the same magnitude  
 (c) only if the two charges have the same magnitude and opposite signs  
 (D) in all the above cases
- A-10.** The momentum of a car is increased by 10% and then decreased by 10%. In the process the kinetic energy of car :  
 (a) increases by 2% (B) decreases by 2% (c) increases by 1% (d) decreases by 1%

**A-11.** The work done by the frictional force in drawing a circle of radius  $r$  by a pencil of negligible mass with a normal pressing force  $N$  (coefficient of friction  $\mu_k$ ) is :

- (a)  $4\pi r^2 \mu_k N$       (b)  $-2\pi r^2 \mu_k N$       (c)  $-3\pi r^2 \mu_k N$       (d)  $-2\pi r \mu_k N$

### **SECTION-B WORK DONE BY VARIABLE FORCE**

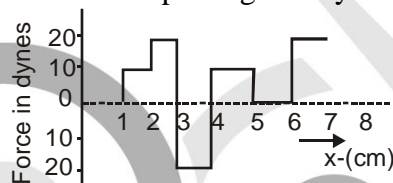
**B-1.** A force acting on a particle varies with the displacement  $x$  as  $F = ax - bx^2$ . Where  $a = 1 \text{ N/m}$  and  $b = 1 \text{ N/m}^2$ . The work done by this force for the first one meter ( $F$  is in newtons,  $x$  is in meters) is :

- (a)  $\frac{1}{6} \text{ J}$       (b)  $\frac{2}{6} \text{ J}$       (c)  $\frac{3}{6} \text{ J}$       (d) None of these

**B-2.** A position dependent force  $F = 7 - 2x + 3x^2$  newton acts on a small body of mass  $2 \text{ kg}$  and displaces it from  $x = 0$  to  $x = 5 \text{ m}$ . The work done in joules is

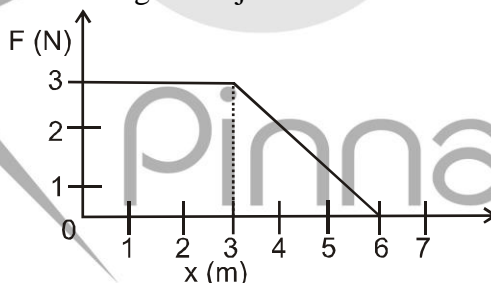
- (a) 70      (b) 270      (c) 35      (d) 135

**B-3.** The relationship between force and position is shown in the figure given (in one dimensional case). The work done by the force in displacing a body from  $x = 1 \text{ cm}$  to  $x = 5 \text{ cm}$  is –



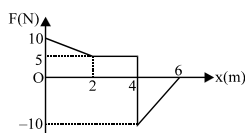
- (a) 20 ergs      (b) 60 ergs      (c) 70 ergs      (d) 700 ergs

**B-4.** A force  $F$  acting on an object varies with distance  $x$  as shown here. The force is in  $\text{N}$  and  $x$  is in  $\text{m}$ . The work done by the force in moving the object from  $x = 0$  to  $x = 6 \text{ m}$  is



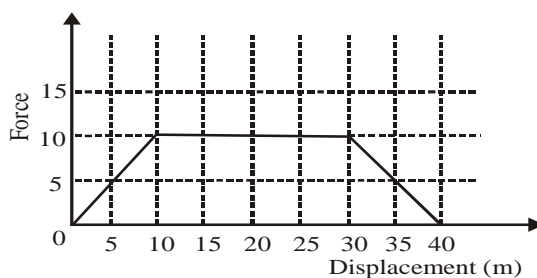
- (a) 18.0 J      (b) 13.5 J      (c) 4.5 J      (d) 9.0 J

**B-5.** A particle is acted upon by a force (in newtons) which varies with position  $X$  in meters as shown in the figure :



- (a) Work done by force  $F$  between  $X = 0$  to  $X = 2 \text{ m}$  is  $+20 \text{ J}$   
 (b) Work done by force  $F$  between  $X = 4 \text{ m}$  to  $X = 6 \text{ m}$  is  $-15 \text{ J}$   
 (c) Work done by force  $F$  between  $X = 4 \text{ m}$  to  $X = 6 \text{ m}$  is  $+10 \text{ J}$   
 (D) Change in KE between  $X = 0$  to  $X = 6 \text{ m}$  is  $15 \text{ J}$ .

**B-6.** Adjacent figure shows the force-displacement graph of a moving body, the work done by this force in displacing the body from  $x = 0$  to  $x = 35 \text{ m}$  is equal to :

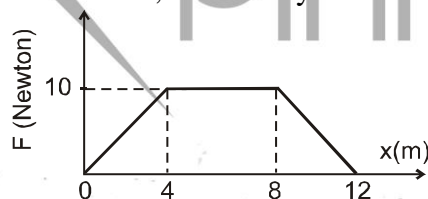


- (a) 50 J                      (b) 25 J                      (c) 287.5 J                      (d) 200 J

- B-7.** A particle moves under the effect of a force  $F = kx^2$  from  $x = 0$  to  $x = 4$  the work done by force is :  
 (a)  $\frac{8k}{3}$                       (b)  $\frac{32k}{3}$                       (c)  $\frac{64k}{3}$                       (d)  $\frac{128k}{3}$
- B-8.** Force acting on a particle moving in a straight line varies with the velocity of the particle as  $F = KV$ . Where  $K$  is constant. The work done by this force in time  $t$  is proportional to:  
 (a)  $KVt$                       (b)  $K^2V^2t^2$                       (c)  $K^2Vt$                       (d)  $KV^2t$
- B-9.** A force  $F = (15 + 0.50x)$  N acts on a particle in  $x$ -direction. The work done by this force during displacement from  $x = 0$  to  $x = 2.0$  m is :  
 (a) 3 J                      (b) 15 J                      (c) 31 J                      (d) 40 J

### **SECTION-C WORK ENERGY THEOREM**

- C-1.** The kinetic energy of a body of mass 2 kg and momentum of 2 Ns is  
 (a) 1 J                      (b) 2J                      (c) 3 J                      (d) 4 J
- C-2.** A particle of mass  $m$  at rest is acted upon by a force  $F$  for a time  $t$ . Its kinetic energy after an interval  $t$  is :  
 (a)  $\frac{F^2t^2}{m}$                       (b)  $\frac{F^2t^2}{2m}$                       (c)  $\frac{F^2t^2}{3m}$                       (d)  $\frac{Ft}{2m}$
- C-3.** A particle of mass 0.1 kg is subjected to a force which varies with distance as shown in figure. If it starts its journey from rest at  $x = 0$ , its velocity at  $x = 12$  m is



- (a) 0 m/s                      (b)  $20\sqrt{2}$  m/s                      (c)  $20\sqrt{3}$  m/s                      (d) 40 m/s

- C-4.** The total work done on a particle is equal to the change in its kinetic energy  
 (a) always  
 (b) only if the forces acting on it are conservative  
 (c) only if gravitational force alone acts on it  
 (d) only if elastic force alone acts on it.
- C-5.** When a man walks on a horizontal surface with constant velocity, work done by  
 (a) friction is zero                      (b) contact force is non zero  
 (c) gravity is non zero                      (d) None of these

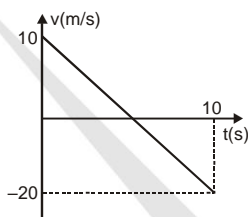
**C-6.** Given that the displacement of the body in metre is a function of time as follows :  $x = 2t^4 + 5$ . The mass of the body is 2 kg. What is the increase in its kinetic energy one second after the start of motion-

- (a) 8 J                      (b) 16 J                      (c) 32 J                      (d) 64 J

**C-7.** A body moves from rest with constant acceleration and acquires a velocity  $v$  in time  $t$ . The work done on the body in time  $T$  is proportional to:

- (a)  $\frac{v}{t}T$                       (b)  $\frac{v^2}{t^2}T$                       (c)  $\frac{v^2}{t^2}T^2$                       (d)  $\frac{v^2T^2}{t}$

**C-8.** Velocity-time graph of a particle moving in a straight line is as shown in figure. Mass of the particle is 2 kg. Work done by all the forces acting on the particle in time interval between  $t = 0$  to  $t = 10$  s is :



- (a) 300 J                      (b) -300 J                      (c) 400 J                      (d) -400 J

**C-9.** The momentum of a body of mass 5 kg is 500 kg m/s. The K.E. is :

- (a)  $3 \times 10^6$  J                      (b)  $10^4$  J                      (c)  $2.5 \times 10^4$  J                      (d)  $8 \times 10^4$  J

**C-10.** A body of mass 4 kg initially at rest is subjected to a force 16 N. The kinetic energy acquired by the body at the end of 10 seconds is :

- (a) 1600 J                      (b) 3200 J                      (c) 800 J                      (d) 1000 J

**C-11.** While catching a cricket ball of mass 200 g moving with a velocity of  $20 \text{ ms}^{-1}$ , Mr. Dhoni draws his hands backwards through 20 cm. The work done in catching the ball is :

- (a) 20 J                      (b) 50 J                      (c) 40 J                      (d) 60 J

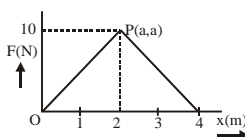
**C-12.** The ratio of work done by the internal forces of a car in order to change its speed from 0 to  $v$ , and from  $v$  to  $2v$  is (Assume that the car moves on a horizontal road) :

- (a) 1                      (b)  $1/2$                       (c)  $1/3$                       (d)  $1/4$

**C-13.** The work done by the external forces on a system equals the change in :

- (A) total energy                      (b) kinetic energy  
(c) potential energy                      (d) none of these

**C-14.** The graph between the net resistive force  $F$  acting on a body (moving in one direction along a straight line) and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be

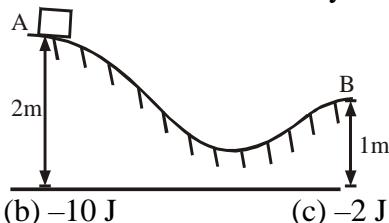


- (a) 50 J                      (b) 40 J                      (c) 20 J                      (d) 30 J

**C-15.** A particle of mass 0.5 kg travels in a straight line with velocity  $v = ax^{3/2}$  where  $a = 5 \text{ m}^{-1/2}\text{s}^{-1}$ . The work done by the net force during its displacement from  $x = 0$  to  $x = 2\text{m}$  is :

- (a) 40 J                      (b) 50 J                      (c) 60 J                      (d) 80 J

**C-16.** A block of 1 kg is placed at A of a rough track shown in the figure. If slightly pushed towards right, it stops at point B of the track. The work done by the frictional force is ( $g = 10 \text{ m/s}^2$ ) :



- (a) -5 J                      (b) -10 J                      (c) -2 J                      (d) -8 J

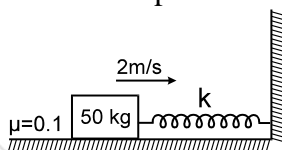
### **SECTION-D MECHANICAL ENERGY CONSERVATION**

- D-1.** The negative of the work done by the conservative internal forces on a system equals the change in  
(a) total energy                      (b) kinetic energy                      (c) potential energy                      (d) none of these
- D-2.** A spring when stretched by 2 mm its potential energy becomes 4 J. If it is stretched by 10 mm, its potential energy is equal to  
(a) 4 J                      (b) 54 J                      (c) 415 J                      (d) 100 J
- D-3.** A boy is sitting on a swing at a maximum height of 5m above the ground. When the swing passes through the mean position which is 2m above the ground its velocity is approximately-  
(a) 7.6 m/sec                      (b) 9.8 m/sec                      (c) 6.26 m/sec                      (d) None of these
- D-4.** A force of 10N acts on a body of 2kg mass for a distance of 1m. The kinetic energy received by the body is-  
(a) 20 J                      (b) 10 J                      (c) 5 J                      (d) 2.5 J
- D-5.** When a body of mass  $m$  is suspended from a spiral spring of natural length  $L$ , the spring gets stretched through a distance  $h$ . The potential energy of the stretched spring is-  
(a)  $\frac{mgh^2}{2}$                       (b)  $mgh$                       (c)  $\frac{1}{2}mgh$                       (d)  $\frac{1}{2}mg(L + h)$
- D-6.** If a spring extends by  $x$  on loading, then the energy stored by the spring is- ( $T$  is the tension in the spring and  $K$  is force constant)  
(a)  $\frac{2x}{t^2}$                       (b)  $\frac{T^2}{2K}$                       (c)  $\frac{2K}{T^2}$                       (d)  $\frac{T^2}{2x}$
- D-7.** The elastic potential energy of a spring-  
(a) Increases only when it is stretched.                      (b) Decreases only when it is stretched.  
(c) Decreases only when it is compressed.                      (d) Increases whether stretched or compressed.
- D-8.** The amount of work done in stretching a spring from a stretched length of 10 cm to a stretched length of 20 cm is-  
(a) Equal to the work done in stretching it from 20 cm to 30 cm.  
(b) Less than the work done in stretching it from 20 cm to 30 cm.  
(c) More than the work done in stretching it from 20 cm to 30 cm.  
(d) Equal to the work done in stretching it from 0 to 10 cm.



- D-9.** A block of mass 2 kg. collides with a horizontal weightless spring fixed at one end and of force constant 2 N/m. The block compresses the spring by 4 meter from the rest position. The speed of the block at the instant of collision is-
- (a) 16 m/sec. (b) 2 m/sec. (c) 4 m/sec. (d) 8 m/sec.

- D-10.** A block of mass 50 kg is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is 0.1. The block strikes a light spring of stiffness  $k = 100$  N/m with a velocity 2m/s. The maximum compression of the spring is :



- (a) 1 m (b) 2 m (c) 3 m (d) 4 m

- D-11.** A stone projected vertically up with a velocity  $u$  reaches a maximum height  $h$ . When it is at a height of  $3h/4$  from the ground, the ratio of KE and PE at that point is : (consider PE = 0 at the point of projection)

- (a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

- D-12.** A body is dropped from a certain height. When it loses  $U$  amount of its energy it acquires a velocity ' $v$ '. The mass of the body is :

- (a)  $2U/v^2$  (b)  $2v/U^2$  (c)  $2v/U$  (d)  $U^2/2v$

- D-13.** Mechanical Energy is conserved under :

- (a) Conservative system of forces (b) dissipative forces  
(c) (a) and (b) (d) none of these

- D-14.** A book is lifted from the floor and is kept on an almirah. One person says that P.E of the block is 20J and the other says it is 30J. Then

- (a) one of them is wrong  
(b) both are wrong  
(c) It depends on whether the work done in lifting the book and keeping it on the almirah is positive or negative  
(d) Both can be correct. PE depends on the choice of reference and potential energy assigned to the reference.

- D-15.** The mechanical energy of a projectile in its path :

- (a) remains constant  
(b) decreases throughout in its path  
(c) first decreases, then becomes zero then increases again  
(d) first increases and then decreases

- D-16.** There are two identical massless springs A and B of spring constants  $K_A$  and  $K_B$  respectively and  $K_A > K_B$ . Then :

- (a) If they are compressed to same distance, then work done on A > work done on B  
(b) If they are compressed by same force work done on A < work done on B  
(c) If they are compressed by same force work done on A > work done on B  
(d) Both A and B are correct.

- D-17.** A particle is released from the top of an incline of height ' $h$ '

- (a) its KE at the bottom of the incline depends on angle of incline  
(b) its KE at the bottom will depend on angle of incline only if the incline is not smooth

- (c) its KE at bottom of the incline will not depend on angle of incline irrespective of whether incline is smooth or rough  
 (d) Cannot say unless mass is given.

**D-18.** When a body of mass  $M$  slides down an inclined plane of inclination  $\theta$ , having coefficient of friction  $\mu$  through a distance  $s$ , the work done against friction is :

- (a)  $\mu(Mg\cos\theta)s$  (b)  $\mu(Mg\sin\theta)s$   
 (c)  $Mg(\mu\cos\theta - \sin\theta)s$  (d) None of these

### **SECTION-E POWER**

**E-1.** If a motor boat moving with a velocity 3 m/s. If a force 500 N due to flow of water, then power of boat is-

- (a) 150 kW (b) 15 kW (c) 1.5 kW (d) 150 W

**E-2.** An electric motor creates a tension of 4500 N in a hoisting cable and reels it in at the rate of 2 ms<sup>-1</sup>. What is the power of the electric motor-

- (a) 15 kW (b) 9 kW (c) 225 kW (d) 9000 HP

**E-3.** A weight lifter lifts a weight 300 kg. from ground to a height of 2 m. in 3 sec. Average power developed by him-

- (a) 2210 watt (b) 8820 watt (c) zero watt (d) 1960 watt

**E-4.** If a force  $F$  is applied on a body and it moves with a velocity  $v$  the power will be-

- (a)  $Fv$  (b)  $F/v$  (c)  $F/v^2$  (d)  $Fv^2$

**E-5.** A man is riding on a cycle with velocity 7.2 km/hr up a hill having a slope 1 in 20. The total mass of the man and cycle is 100 kg. The power of the man is-

- (a) 98 W (b) 49 W (c) 196 W (d) 147 W

**E-6.** A body of mass  $m$  kg initially at rest attains a velocity of  $v$  m/sec in time  $t$  under the action of a constant force  $F$ . The power supplied to the mass is-

- (a)  $mv/t$  (b)  $mv^2/t$  (c)  $Fv$  (d)  $Fv/2$

**E-7.** The average power required to lift a 100 kg mass through a height of 50 metres in approximately 50 seconds would be

- (a) 50 J/s (b) 5000 J/s (c) 100 J/s (d) 980 J/s

**E-8.** A particle moves with a velocity  $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}$  m/s under the influence of a constant force  $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}$  N. The instantaneous power applied to the particle is :

- (a) 200 J/s (b) 40 J/s (c) 140 J (d) 170 J/s

**E-9.** A horse pulls a wagon with a force of 360 N at an angle of 60° with the horizontal at a speed of 10 Km/hr. The power of the horse is :

- (a) 1000 W (b) 2000 W (c) 500 W (d) 750 W.

**E-10.** A car of mass 1000 kg accelerates uniformly from rest to velocity of 54 km/h in 5 second. The average power of the engine during this period (Neglect friction ) is :

- (a) 20000 W (b) 22500 W (c) 3000 W (d) 5000 W

**E-11.** An electric motor crates a tension of 4500 N in hoisting cable and runs at the rate of 2 m/s. What is the power of electric motor ?

- (a) 9W (b) 9 kW  
(c) 225 W (d) 9000 H.P.

**E-12.** A machine gun is firing 60 bullets for minute with a velocity of  $700 \text{ ms}^{-1}$ . If each bullet has a mass of 50 g, the power developed by the gun is :

- (a) 20000W (b) 10000 W (c) 12250 W (d) 12000 W

**E-13.** A force  $\vec{F} = (3\hat{i} + 4\hat{j})$  N acts on a 2 kg movable object that moves from an initial position  $\vec{d}_i = (-3\hat{i} - 2\hat{j})$  m to final position  $\vec{d}_f = (5\hat{i} + 4\hat{j})$  in 6 sec. The average power delivered by the force during the interval is equal to :

- (a) 8 watt (b) 50/6 watt (c) 15 watt (d) 50/3 watt

**E-14.** A particle moves with a velocity  $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$  m/s under the influence of a constant force  $\vec{F} = 3\hat{i} + 15\hat{j} + 5\hat{k}$  N. The instantaneous power applied to the particle is :

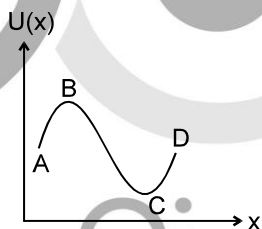
- (a) 200 J/s (b) 40 J/s (c) 0 J/s (d) 170 J/s

### **SECTION-F COMSERVATIVE AND NONCONSERVATIVE FORCES AND EQUILIBRIUM**

**F-1.** The potential energy of a particle in a field is  $U = \frac{a}{r^2} - \frac{b}{r}$ , where a and b are constant. The value of r in terms of a and b where force on the particle is zero will be :

- (a)  $\frac{a}{b}$  (b)  $\frac{b}{a}$  (c)  $\frac{2a}{b}$  (d)  $\frac{2b}{a}$

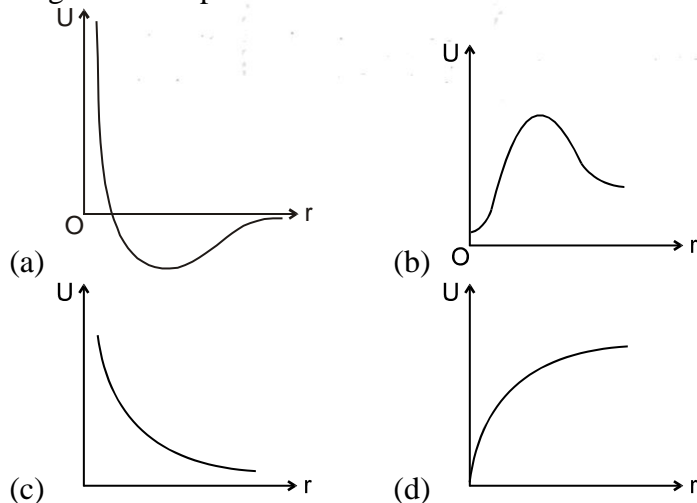
**F-2.** The potential energy of a particle varies with distance x as shown in the graph.



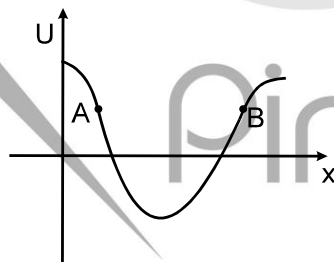
The force acting on the particle is zero at

- (a) C (b) B (c) B and C (d) A and D.

**F-3.** The diagrams represent the potential energy U of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature.



- F-4.** The potential energy for a force field is given by  $U(x, y) = \sin(x + y)$ . Magnitude of the force acting on the particle of mass  $m$  at  $(0, \frac{\pi}{4})$  is  
 (a) 1 (b)  $\sqrt{2}$  (c)  $\frac{1}{\sqrt{2}}$  (d) 0
- F-5.** A particle is moving in a potential region given by  $U = K(x^2 + y^2 + z^2)$ . The force acting on the particle is given by-  
 (a)  $-2K(x\hat{i} + y\hat{j} + z\hat{k})$  (b)  $K(x\hat{i} + y\hat{j} + z\hat{k})$   
 (c)  $\frac{K}{2}(x\hat{i} + y\hat{j} + z\hat{k})$  (d)  $K(x^2\hat{i} + y^2\hat{j} + z^2\hat{k})$
- F-6.** The potential energy of a particle varies with  $x$  according to the relation  $U(x) = x^2 - 4x$ . The point  $x = 2$  is a point of :  
 (a) stable equilibrium (b) unstable equilibrium  
 (c) neutral equilibrium (d) none of above
- F-7.** The potential energy function associated with the force  $\vec{F} = 4xy\hat{i} + 2x^2\hat{j}$  is :  
 (a)  $U = -2x^2y$  (b)  $U = -2x^2y + \text{constant}$   
 (c)  $U = 2x^2y + \text{constant}$  (d) not defined
- F-8.** One of the forces acting on a particle is conservative then which of the following statement(s) are true about this conservative force  
 (a) Its work is non zero when the particle moves exactly once around any closed path.  
 (b) Its work equals the change in the kinetic energy of the particle  
 (c) Then that particular force must be constant.  
 (d) Its work depends on the end points of the motion, not on the path between.
- F-9.** Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.



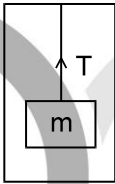
- (a) Positive, Positive (b) Positive, Negative  
 (c) Negative, Positive (d) Negative, Negative
- F-10.** A conservative force  $\vec{F}$  is acting on a body and body moves from Point A to B and then B to A then work done by the force is :  
 (a)  $W < 0$  (b)  $W > 0$  (c)  $W = 0$  (d)  $W \neq 0$
- F-11.** A simple pendulum has a string of length  $l$  and bob of mass  $m$ . When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a circular path about the point of suspension. The tension in the string at the lowest position of the bob is :  
 (a)  $3mg$  (b)  $4mg$  (c)  $5mg$  (d)  $6mg$

**SECTION-G VERTICAL CIRCULAR MOTION**

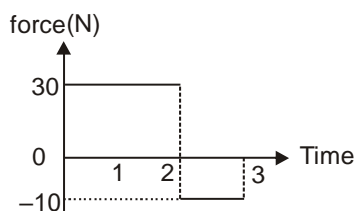
- G-1.** The tension in the string revolving in a vertical circle with a mass  $m$  at the end when it is at the lowest position.  
 (a)  $\frac{mv^2}{r}$  (b)  $\frac{mv^2}{r} - mg$  (c)  $\frac{mv^2}{r} + mg$  (d)  $mg$
- G-2.** A motorcycle is going on an overbridge of radius  $R$ . The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it :  
 (a) increase (b) decreases  
 (c) remains constant (d) first increases then decreases.
- G-3.** In a circus, stuntman rides a motorbike in a circular track of radius  $R$  in the vertical plane. The minimum speed at highest point of track will be :  
 (a)  $\sqrt{2gR}$  (b)  $2gR$  (c)  $\sqrt{3gR}$  (d)  $\sqrt{gR}$
- G-4.** A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles  $30^\circ$  and  $60^\circ$  from vertical (lowest positions) are  $T_1$  and  $T_2$  respectively. Then  
 (a)  $T_1 = T_2$  (b)  $T_2 > T_1$   
 (c)  $T_1 > T_2$  (d) Tension in the string always remains the same
- G-5.** A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break.  
 (a) When the mass is at the height point of the circle  
 (b) When the mass is at the lowest point of the circle  
 (c) When the wire is horizontal  
 (d) At an angle of  $\cos^{-1}(1/3)$  from the upward vertical
- G-6.** A motorcycle is going on an overbridge of radius  $R$ . The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it.  
 (a) Increases (b) Decreases (c) Remains the same (d) Fluctuates
- G-7.** A hollow sphere has radius 6.4 m. Minimum velocity required by a motor cyclist at bottom to complete the circle will be.  
 (a) 17.7 m/s (b) 10.2 m/s (c) 12.4 m/s (d) 16.0 m/s
- G-8.** A body of mass 100 g is rotating in a circular path of radius  $r$  with constant velocity. The work done in one complete revolution is.  
 (a) 100 rJ (b)  $(r/100)$  J (c)  $(100/r)$  J (d) Zero
- G-9.** A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 gm is tied to it and revolved in a circular path of radius 4 m in a vertical plane. If  $g = 10 \text{ ms}^{-2}$ , then the maximum angular velocity of the stone will be.  
 (a) 4 radians/sec (b) 16 radians/sec (c)  $\sqrt{21}$  radians/sec (d) 2 radians/sec
- G-10.** A small disc is on the top of a hemisphere of radius  $R$ . What is the smallest horizontal velocity  $v$  that should be given to the disc for it to leave the hemisphere and not slide down it ? [There is no friction]  
 (a)  $v = \sqrt{2gR}$  (b)  $v = \sqrt{gR}$  (c)  $v = \frac{g}{R}$  (d)  $v = \sqrt{g^2R}$

- G-11.** A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical.  
 (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
- G-12.** A cane filled with water is revolved in a vertical circle of radius 4 meter and the water just does not fall down. The time period of revolution will be-  
 (a) 1 sec (b) 10 sec (c) 8 sec (d) 4 sec

**EXERCISE – II**  
**READY FOR CHALLENGES**

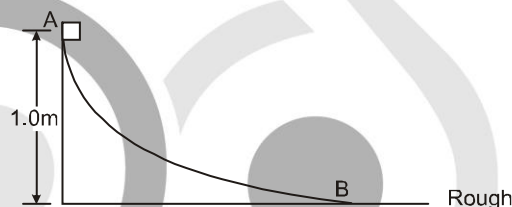
- A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is  
 (a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 1 : 3 : 5 (d) 1 : 5 : 3
- A block of mass  $m$  is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration  $a$ . The work done by tension on the block during  $t$  seconds is ( $u = 0$ ) :  

 (a)  $\frac{m}{2}(g + a)at^2$  (b)  $\frac{m}{2}(g - a)at^2$  (c)  $\frac{m}{2}gat^2$  (d) 0
- The work done by the frictional force on a surface in drawing a circle of radius  $r$  on the surface by a pencil of negligible mass with a normal pressing force  $N$  (coefficient of friction  $\mu_k$ ) is :  
 (a)  $4\pi r^2 \mu_k N$  (b)  $-2\pi r^2 \mu_k N$  (c)  $-2\pi r \mu_k N$  (d) zero
- A force  $F = 5 + 2x$  acts on a body in  $x$ -direction where  $x$  is in metres and  $F$  in newton. Find the work done in displacing the body from  $x = 0$  to  $x = 2$  m.  
 (a) 14 J (b) 18 J (c) 16 J (d) none of these
- A particle moves under the effect of a force  $F = Cx$  from  $x = 0$  to  $x = x_1$ . The work done in the process is  
 (a)  $Cx_1^2$  (b)  $\frac{1}{2}Cx_1^2$  (c)  $Cx_1$  (d) Zero
- A particle of mass  $m$  is moving with speed  $u$ . It is stopped by a force  $F$  in distance  $x$ . If the stopping force is  $4F$  then :  
 (a) work done by stopping force in second case will be same as that in first case.  
 (b) work done by stopping force in second case will be 2 times of that in first case.  
 (c) work done by stopping force in second case will be  $1/2$  of that in first case.  
 (d) work done by stopping force in second case will be  $1/4$  of that in first case.
- Starting at rest, a 10 kg object is acted upon by only one force as indicated in figure. Then the total work done by the force is



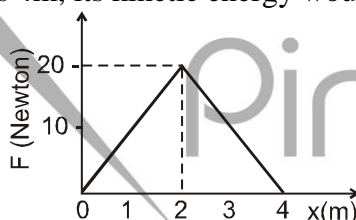


- (a) 90 J                      (b) 125 J                      (c) 245 J                      (d) 490 J
8. The kinetic energy of a particle continuously increases with time  
 (a) the resultant force on the particle must be parallel to the velocity at all instants.  
 (b) the resultant force on the particle must be at an angle greater than  $90^\circ$  with the velocity all the time  
 (c) its height above the ground level must continuously decrease  
 (d) the magnitude of its linear momentum is increasing continuously

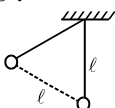
9. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is :



- (a) 5.0 m                      (b) 10.0 m                      (c) 15.0 m                      (d) 20.0 m
10. The graph between the resistive force  $F$  acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, its kinetic energy would be



- (a) 50 J                      (b) 40 J                      (c) 20 J                      (d) 10 J
11. A bob hangs from a rigid support by an inextensible string of length  $l$ . If it is displaced through a distance  $l$  (from the lowest position) keeping the string straight & then released. The speed of the bob at the lowest position is :



- (a)  $\sqrt{gl}$                       (b)  $\sqrt{3gl}$                       (c)  $\sqrt{2gl}$                       (d)  $\sqrt{5gl}$
12. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground ?



- (a) At a horizontal distance of 1 m from the end of the track.  
 (b) At a horizontal distance of 2 m from the end of the track.  
 (c) At a horizontal distance of 3 m from the end of the track.  
 (d) Insufficient information
13. A body of mass 1 kg is thrown upwards with a velocity  $20 \text{ ms}^{-1}$ . It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? ( $g = 10 \text{ ms}^{-2}$ )  
 (a) 20 J (b) 30 J (c) 40 J (d) 10 J
14. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time  $t$  is proportional to :  
 (a)  $t^{\frac{3}{4}}$  (b)  $t^{\frac{3}{2}}$  (c)  $t^{\frac{1}{4}}$  (d)  $t^{\frac{1}{2}}$
15. A body of mass  $m$  accelerates uniformly from rest to  $v_1$  in time  $t_1$ . The instantaneous power delivered to the body as a function of time  $t$  is :  
 (a)  $\frac{mv_1 t}{t_1}$  (b)  $\frac{mv_1^2 t}{t_1^2}$  (c)  $\frac{mv_1 t}{t_1^2}$  (d)  $\frac{mv_1^2 t}{t_1}$
16. At time  $t = 0$  s a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time ' $t$ ', the net force acting on it must be proportional to :  
 (a) constant (b)  $t$  (c)  $\frac{1}{\sqrt{t}}$  (d)  $\sqrt{t}$
17. When a rubber-band is stretched by a distance  $x$ , it exerts a restoring force of magnitude  $F = ax + bx^2$  where  $a$  and  $b$  are constants. The work done in stretching the unstretched rubber-band by  $L$  is :  
 (a)  $aL^2 + bL^3$  (b)  $\frac{1}{2}(aL^2 + bL^3)$  (c)  $\frac{aL^2}{2} + \frac{bL^3}{3}$  (d)  $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$
18. A particle is dropped from a height  $h$ . A constant horizontal velocity is given to the particle. Taking  $g$  to be constant every where, kinetic energy  $E$  of the particle with respect to time  $t$  is correctly shown in  
  
 (a) (b) (c) (d)
19. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement  $x$  is proportional to  
 (a)  $x^2$  (b)  $e^x$  (c)  $x$  (d)  $\log_e x$
20. A block of mass  $m$  is attached to two unstretched springs of spring constants  $k_1$  and  $k_2$  as shown in figure. The block is displaced towards right through a distance  $x$  and is released. Find the speed of the block as it passes through the mean position shown.

(a)  $\sqrt{\frac{k_1 + k_2}{m}} x$

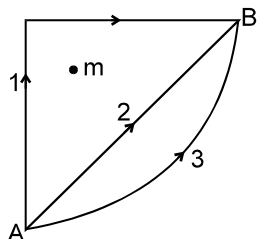
(b)  $\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}} x$

(c)  $\sqrt{\frac{k_1^2 k_2^2}{m(k_1^2 + k_2^2)}} x$

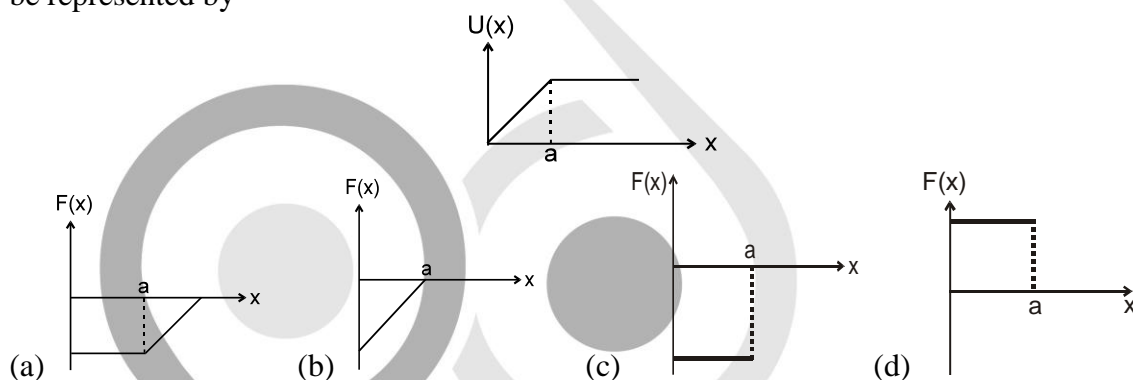
(d)  $\sqrt{\frac{k_1^3 k_2^3}{m(k_1^3 + k_2^3)}} x$



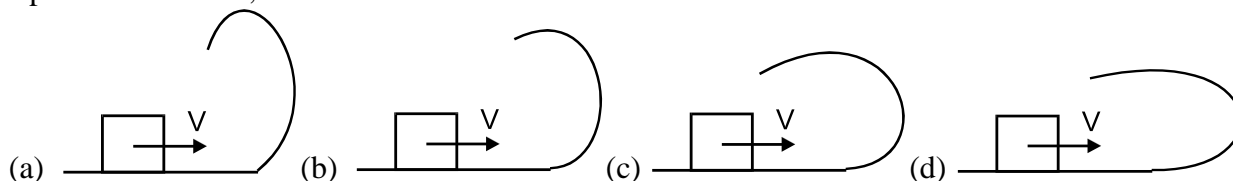
21. If  $W_1$ ,  $W_2$  and  $W_3$  represent the work done in moving a particle from A to B along three different paths 1, 2, 3 respectively (as shown) in the gravitational field of a point mass  $m$ , find the correct relation between  $W_1$ ,  $W_2$  and  $W_3$



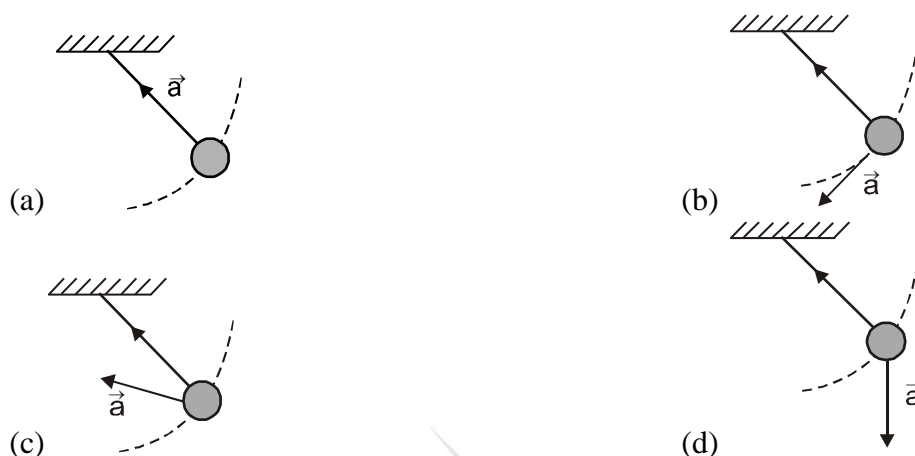
- (a)  $W_1 > W_2 > W_3$  (b)  $W_1 = W_2 = W_3$   
(c)  $W_1 < W_2 < W_3$  (d)  $W_2 > W_1 > W_3$
22. The potential energy of a system is represented in the first figure, the force acting on the system will be represented by



23. A stone tied to a string of length  $L$  is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed  $u$ . The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is  
(a)  $\sqrt{u^2 - 2gl}$  (b)  $\sqrt{2gl}$  (c)  $\sqrt{u^2 - gl}$  (d)  $\sqrt{2(u^2 - gl)}$
24. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance  $L$  from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration,  $\alpha$ . If the coefficient of friction between the rod and the bead is  $\mu$ , and gravity is neglected, then the time after which the bead starts slipping is—  
(a)  $\sqrt{\frac{\mu}{\alpha}}$  (b)  $\frac{\mu}{\sqrt{\alpha}}$  (c)  $\frac{1}{\sqrt{\mu\alpha}}$  (d) Infinitesimal
25. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in —

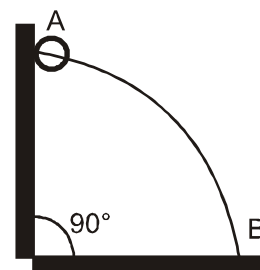


26. A simple pendulum is oscillating without damping. When the displacements of the bob is less than maximum, its acceleration vector is correctly shown in



27. A wire, which passes through the hole is a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

- (a) always radially outwards  
(b) always radially inwards  
(c) radially outwards initially and radially inwards later  
(d) radially inwards initially and radially outwards later.



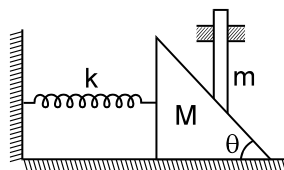
28. A particle of mass  $m$  begins to slide down a fixed smooth sphere from the top. What is its tangential acceleration when it breaks off the sphere ?

- (a)  $\frac{2g}{3}$  (b)  $\frac{\sqrt{5}g}{3}$  (c)  $g$  (d)  $\frac{g}{3}$

29. A spring of spring constant  $k$  placed horizontally on a rough horizontal surface is compressed against a block of mass  $m$  placed on the surface so as to store maximum energy in the spring. If the coefficient of friction between the block and the surface is  $\mu$ , the potential energy stored the spring is :

- (a)  $\frac{\mu^2 m^2 g^2}{k}$  (b)  $\frac{2\mu m^2 g^2}{k}$  (c)  $\frac{\mu^2 m^2 g^2}{2k}$  (d)  $\frac{3\mu^2 m g^2}{k}$

30. A wedge of mass  $M$  fitted with a spring of stiffness ' $k$ ' is kept on a smooth horizontal surface. A rod of mass  $m$  is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is :



- (a)  $\frac{mg^2 \tan^2 \theta}{2k}$  (b)  $\frac{m^2 g \tan^2 \theta}{2k}$  (c)  $\frac{m^2 g^2 \tan^2 \theta}{2k}$  (d)  $\frac{m^2 g^2 \tan^2 \theta}{k}$

31. If a particle under the action of a force  $\vec{F}$  has potential energy  $U$  then in equilibrium :

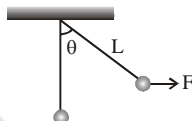
- (a)  $\vec{F} = 0$  and  $U = 0$  (b)  $\vec{F} = 0$  but  $U \neq 0$

- (c) Both 'a' and 'b' are correct (d) Neither 'a' nor 'b' are correct

**32.** A body dropped by a height  $H$  reaches the ground with a speed of  $\sqrt{1.5gH}$ . The work done by the air friction is :

- (a)  $+0.25 mgH$  (b)  $1.5 mgH$  (c)  $-0.25 mgH$  (d) zero

**33.** An object of mass  $m$  is tied to a string of length  $l$  and a variable horizontal force is applied on it which is initially zero and gradually increases until the string makes an angle  $\theta$  with the vertical. Work done by the force  $F$  is :



- (a)  $mg l(1 - \sin\theta)$  (b)  $mg l$  (c)  $mg l(1 - \cos\theta)$  (d)  $mg l(1 + \cos\theta)$

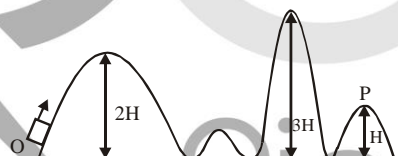
**34.** A 15 gm ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is : ( $g = 10 \text{ m/s}^2$ )

- (a) 6.0 m (b) 12.0 m (c) 10.0 m (d) 8.0 m

**35.** A small block of mass  $m$  is kept on a rough inclined surface of inclination  $\theta$  fixed in a elevator. The elevator goes up with a uniform velocity  $v$  and the block does not slide on the wedge. The work done by the force of friction on the block in time  $t$  will be :

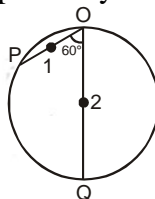
- (a) zero (b)  $mgvt \cos^2\theta$  (c)  $mgvt \sin^2\theta$  (d)  $mgvt \sin\theta$

**36.** On a vertical smooth curved track shown in the diagram, a particle is projected from the point O such that it just manages to reach the point P. The speed of the particle required at O is :



- (a) zero (b)  $\sqrt{2gH}$  (c)  $\sqrt{6gH}$  (d)  $\sqrt{3gH}$

**37.** Two particles 1 and 2 are allowed to descend on two frictionless chords OP and OQ. The ratio of the speeds of the particles 1 and 2 respectively when they reach on the circumference is :



- (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c) 1 (d)  $\frac{1}{2\sqrt{2}}$

**38.** A stone projected up with a velocity  $u$  reaches a maximum height  $h$ . When it is at a height of  $3h/4$  from the ground, the ratio of KE and PE at that point is : (consider PE=0 at the point of projectory)

- (a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

**39.** An ideal spring with spring constant  $k$  is hung from the ceiling and a block of mass  $M$  is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is :

- (a)  $\frac{Mg}{2k}$  (b)  $\frac{Mg}{k}$  (c)  $\frac{2Mg}{k}$  (d)  $\frac{4Mg}{k}$

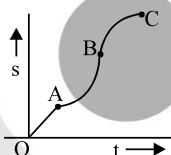
40. A man pulls a bucket of water from a well of depth  $H$ . If the mass of the rope and that of the bucket full of water are  $m$  and  $M$  respectively, then the work done by the man is :  
 (a)  $(m + M)gh$  (b)  $\left(\frac{m}{2} + M\right)gh$  (c)  $\left(\frac{m+M}{2}\right)gh$  (d)  $\left(\frac{M}{2} + m\right)gh$
41. A rod of length 1 m and mass 0.5 kg hinged at one end, is initially hanging vertically. The other end is now slowly raised until it makes an angle  $60^\circ$  with the vertical. The required work is ( $g = 9.8 \text{ m/s}^2$ ) :  
 (a) 1.522 J (b) 1.225 J (c) 2.125 J (d) 3.125 J
42. A car of mass  $m$  is driven with acceleration 'a' along a straight line road against external resistive force  $R$ . When the velocity of the car is  $V$ , the rate at which the engine of the car is doing work will be :  
 (a)  $RV$  (b)  $maV$  (c)  $(R + m)V$  (d)  $(ma + R)V$
43. A force  $\vec{F} = 3\hat{i} + 5\hat{j}$  acts on a body and its position varies as  $\vec{S} = (2t^2\hat{i} - 5\hat{j})$ . Work done by this force in initial 2s is :  
 (a) 24 J (b) 32 J (c) zero (d) can't be obtained
44. A particle is moved from  $(0, 0)$  to  $(a, a)$  under a force  $\vec{F} = (3\hat{i} + 4\hat{j})$  from two paths. Path 1 is OP and path 2 is OQP. Let  $W_1$  and  $W_2$  be the work done by this force in these two paths. Then find the ratio  $W_1/W_2$ .
- 
- (a) 1 (b)  $3/4$  (c)  $4/3$  (d)  $1/2$
45. A force at any point is inversely proportional to the square of the distance from fixed origin O and is directed along the tangential direction at that point as shown in figure. Then work done along a closed circular path of radius  $R$ , which encloses the origin is :
- 
- (a) zero (b)  $\frac{2\pi A}{R}$  (c)  $\frac{2\pi A}{R^2}$  (d)  $-\frac{2\pi A}{R^2}$
46. Force acting on a particle is  $(2\hat{i} + 3\hat{j})$  N. Work done by this force is zero, when a particle is moved on the line  $3y + kx = 5$ . Here value of  $k$  is :  
 (a) 2 (b) 4 (c) 6 (d) 8
47. The relation between the displacement  $x$  and the time  $t$  for a body of mass 2kg moving under the action of a force is given by  $x = t^3/3$ . The work done by the body in first 2 seconds is :  
 (a) 10 J (b) 16 J (c) 24 J (d) 5 J
48. An object is displaced from a point A  $(0,0,0)$  to B  $(1\text{m}, 1\text{m}, 1\text{m})$  under a force  $F = (y\hat{i} + x\hat{j})$  N. The work done by this force in this process is :  
 (a) 0.5 J (b) 2 J (c) 1 J (d) 3 J
49. Potential energy function describing the interaction between two atoms of a diatomic molecule is

$$U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$

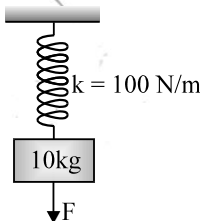
Force acting between them will be zero when the distance between them would be :

- (a)  $\left(\frac{2a}{b}\right)^{1/6}$  (b)  $\left(\frac{b}{2a}\right)^{1/6}$  (c)  $\left(\frac{a}{b}\right)^{1/6}$  (d)  $\left(\frac{b}{a}\right)^{1/6}$

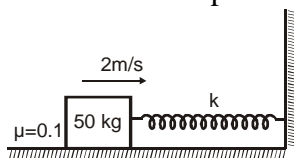
50. The potential energy of a particle varies with position  $x$  according to the relation  $U(x) = 2x^4 - 27x$  the point  $x = 3/2$  is point of :  
 (a) Unstable equilibrium (b) Stable equilibrium  
 (c) neutral equilibrium (d) none of these
51. A particle is moving in a force field given by potential  $U = -\lambda(x + y + z)$  from the point (1, 1, 1) to (2, 3, 4). The work done in the process is :  
 (a)  $3\lambda$  (b)  $1.5\lambda$  (c)  $6\lambda$  (d)  $12\lambda$
52. A particle moving in a horizontal circle of radius  $r$  has centripetal force  $F = -k/r^2$ , where  $k$  is a constant. The kinetic energy of the particle is :  
 (a)  $k/r$  (b)  $k^2/r^2$  (c)  $2k/r$  (d)  $k/2r$
53. The kinetic energy of a particle  $K$  moving along a circle of radius  $R$  depends on the distance covered 'S' as  $K = aS^2$ . The force acting on the particle is :  
 (a)  $2a \frac{S^2}{R}$  (b)  $2aS(1 + S^2/R^2)^{1/2}$  (c)  $2aS$  (d)  $2s R^2/S^2$
54. Displacement time graph of a particle moving in a straight line is as shown in figure. Select correct alternative



- (a) Work done by all forces in region OA is positive  
 (b) Work done by all forces in region AB is negative  
 (c) Work done by all forces in region BC is positive  
 (d) Work done by all forces in region AB is positive
55. A vertical spring of force constant  $100 \text{ N/m}$  is attached with a hanging mass of  $10 \text{ kg}$ . Now an external force is applied on the mass so that the spring is stretched by additional  $2 \text{ m}$ . The work done by the force  $F$  is ( $g = 10 \text{ m/s}^2$ ) :

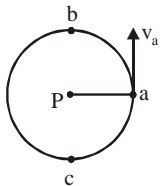


- (a)  $200 \text{ J}$  (b)  $400 \text{ J}$  (c)  $450 \text{ J}$  (d)  $600 \text{ J}$
56. A block of mass  $50 \text{ kg}$  is projected horizontally on a rough horizontal floor. The coefficient of friction between the block and the floor is  $0.1$ . The block strikes a light spring of stiffness  $k=100 \text{ N/m}$  with a velocity  $2 \text{ m/s}$ . The maximum compression of the spring is :

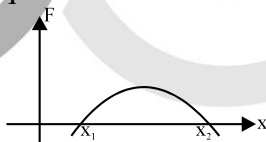


- (a)  $1 \text{ m}$  (b)  $2 \text{ m}$  (c)  $3 \text{ m}$  (d)  $4 \text{ m}$

57. If a particle is tied to a light inextensible string of length  $L$  fixed at  $P$  and particle is projected at  $a$  with velocity  $v_a = \sqrt{4gL}$  as shown. The ratio of tension at  $c$  and  $b$  is :



- (a) 5 : 1                      (b) 7 : 1                      (c) 6 : 1                      (d) 1 : 4
58. A block of mass  $m$  is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and plane is  $\mu$ . The power delivered by the external agent at a time  $t$  from the beginning is equal to :
- (a)  $ma^2t$                       (b)  $\mu mgat$                       (c)  $\mu m(a+\mu g)gt$                       (d)  $m(a + \mu g)at$
59. A pump motor is used to lift water to a height  $4m$  and to deliver water of  $0.12 \text{ m}^3/\text{minute}$  with a velocity of  $10 \text{ m/s}$ . The power of pump is [density of water =  $10^3 \text{ kg/m}^3$ ,  $g = 10 \text{ m/s}^2$ ] :
- (a)  $100 \text{ W}$                       (b)  $180 \text{ W}$                       (c)  $200 \text{ W}$                       (d)  $280 \text{ W}$
60. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time  $t$  is proportional to-
- (a)  $t^{1/2}$                       (b)  $t^{3/4}$                       (c)  $t^{3/2}$                       (d)  $t^2$
61. The force acting on a body moving along  $x$ -axis varies with the position of the particle as shown in figure. The body is in stable equilibrium at :

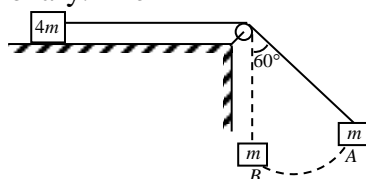


- (a)  $x = x_1$                       (b)  $x = x_2$                       (c) both  $x_1$  and  $x_2$                       (d) neither  $x_1$  nor  $x_2$
62. Force acting on a particle moving in a straight line varies with the velocity of the particle as  $F = KV$ . Where  $K$  is constant. The work done by this force in time  $t$  is proportional to:
- (a)  $KVt$                       (b)  $K^2V^2t^2$                       (c)  $K^2Vt$                       (d)  $KV^2t$

### EXERCISE – III CROSSING THE HURDLES

#### MORE THAN ONE CORRECT

1. In the system shown in the figure, the mass  $m$  moves in a circular arc of angular amplitude  $60^\circ$ . The mass  $4m$  remains stationary. Then



- (a) The minimum value of coefficient of friction between the mass  $4m$  and the surface is  $0.5$ .



- (b) The work done by gravitational force on the block  $m$  is positive when it moves from A to B.  
 (c) The power delivered by the tension when  $m$  moves from A to B is zero.  
 (d) The kinetic energy of  $m$  in position B equals the work done by gravitational force on the block when it moves from position A to B.

2. A block is suspended by an ideal spring of force constant  $k$ . The block is pulled down by applying a constant force  $F$  and maximum displacement of block from its initial mean position is  $x_0$ . Then

- (a) Increase in energy stored in spring is  $kx_0^2$  (b)  $x_0 = \frac{2F}{2k}$   
 (c)  $x_0 = \frac{2F}{k}$  (d) Work done by applied force is  $Fx_0$ .

3. Consider two observers moving with respect to each other at a speed  $v$  along a straight line. They observe a block of mass  $m$  moving a distance  $l$  m on a rough surface. The following quantities will be different as observed by the two observers

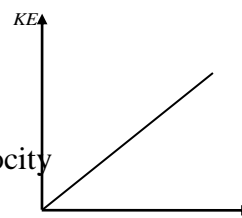
- (a) Kinetic energy of the block at time  $t$  (b) Work done by friction  
 (c) total work done on the block (d) Acceleration of the block

4. A block of mass  $M$  is hanging over a smooth light pulley through a light string. The other end of the string is pulled by a constant force  $F$ . The kinetic energy of the block increases by 20 J in 1 s.

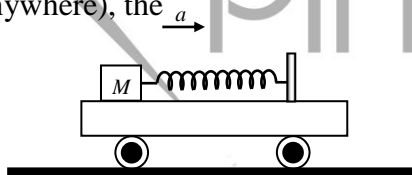
- (a) The total work done on the block in 1 s is 20 J.  
 (b) The tension in the string is  $F$   
 (c) The work done by the tension on the block is 20 J in 1 s.  
 (d) The work done by the force of gravity is  $-20$  J in 1 s.

5. The kinetic energy of body moving along a straight line varies with time as shown in the figure. The force acting on the body is

- (a) Directly proportional to  $\sqrt{t}$  (b) Inversely proportional to  $\sqrt{t}$   
 (c) Directly proportional to velocity (d) Inversely proportional to velocity



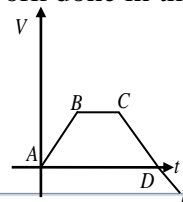
6. A block of mass  $M$  is attached with a spring of spring constant  $K$ . The whole arrangement is placed on a vehicle as shown in the figure. If the vehicle starts moving towards right with an acceleration  $a$  (there is no friction anywhere), the



- (a) Maximum elongation in the spring is  $\frac{Ma}{K}$   
 (b) Maximum elongation in the spring is  $\frac{2Ma}{K}$   
 (c) Maximum compression in the spring is  $\frac{2Ma}{K}$   
 (d) Maximum compression in the spring is zero.

7. The velocity - time graph of a particle is shown in figure. The work done in the interval

- (a) AB is positive  
 (b) BC is zero  
 (c) CD is negative  
 (d) DE is positive



8. A heavy stone is thrown from a cliff of height  $h$  in a given direction. The speed with which it hits the ground
  - (a) Must depend on the speed of projection
  - (b) Must be larger than the speed of projection
  - (c) Must be independent of the speed of projection
  - (d) Must be smaller than the speed of projection
9. A Particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
  - (a) Its velocity is constant
  - (b) Its acceleration is constant
  - (c) Its kinetic energy is constant
  - (d) it move in a circular path.
10. No work is done on an object by a force if
  - (a) The force is always perpendicular to its velocity
  - (b) The force is always perpendicular to its acceleration
  - (c) The object is stationary
  - (d) The object moves in such a way that point of application of force remains fixed.
11. The potential energy  $U$  for a force field  $\vec{F}$  is such that  $U = -Kxy$ , where  $K$  is a constant. Then
  - (a)  $\vec{F} = ky\hat{i} + kx\hat{j}$
  - (b)  $\vec{F} = Kx\hat{i} + Ky\hat{j}$
  - (c)  $\vec{F}$  is a conservative force
  - (d)  $\vec{F}$  is a non-conservative force
12. A force  $F = kx^3$  is acting on a block moving along  $x$ -axis. Here,  $k$  is a positive constant. Work done by this force is
  - (a) Positive if displacing the block from  $x = 3$  to  $x = 1$ .
  - (b) Positive in displacing the block from  $x = -1$  to  $x = -3$ .
  - (c) Negative in displacing the block from  $x = 3$  to  $x = 1$ .
  - (d) Negative in displacing the block from  $x = -1$  to  $x = -3$ .

### **COMPREHENSION TYPE QUESTION**

#### **PASSAGE I**

A small sphere of mass  $m$  suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released ( $\tan^{-1} \sqrt{2} = 54.7^\circ$ )

13. The total acceleration of the sphere as a function of angle  $\theta$  with the vertical is
  - (a)  $g\sqrt{1+\cos^2 \theta}$
  - (b)  $g\sqrt{1+3\cos^2 \theta}$
  - (c)  $g \cos \theta$
  - (d)  $g \sin \theta$
14. Then tension in the string as a function of angle  $\theta$  with the vertical is.
  - (a)  $3 mg \cos \theta$
  - (b)  $mg \cos \theta$
  - (c)  $mg\sqrt{1+\cos^2 \theta}$
  - (d)  $mg\sqrt{1+3\cos^2 \theta}$
15. The tension in the thread at the moment the vertical component of the sphere's velocity is maximum is



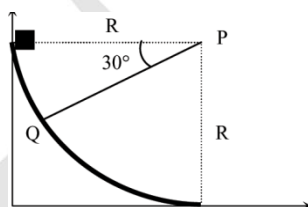
- (a)  $mg$  (b)  $\frac{mg}{\sqrt{2}}$  (c)  $\sqrt{3}mg$  (d)  $\frac{mg}{2}$

16. The angle  $\theta$  made by the thread with the vertical at the moment when total acceleration vector of the particle is directed horizontally is.

- (a)  $54^\circ$  (b)  $53^\circ$  (c)  $54.7^\circ$  (d)  $53.7^\circ$

### PASSAGE II

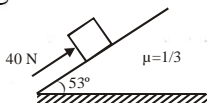
A small block of mass 1 kg is released from rest at the top of a rough track. The track is circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, below, is 150 J. (Take the acceleration due to gravity,  $g = 10 \text{ m/s}^2$ ).



17. The speed of the block when it reaches the point Q is  
(a)  $5 \text{ ms}^{-1}$  (b)  $10 \text{ ms}^{-1}$  (c)  $10\sqrt{3} \text{ ms}^{-1}$  (d)  $20 \text{ ms}^{-1}$
18. The magnitude of the normal reaction that acts on the block at the point Q is  
(a) 7.5 N (b) 8.6 N (c) 11.5 N (d) 22.5 N

### PASSAGE III

19. A block of mass 10 kg is acted upon by a force of 40 N parallel to the inclined plane as shown. Find the work done by applied force during first 10 seconds of its motion. Initially block was at rest.



- (a) -4000 J (b) 4000 J (c) -8000 J (d) 800 J
20. In the previous problem, find work done by frictional force on the block .  
(a) -3000 J (b) -2000 J (c) -4000 J (d) -6000 J
21. In the previous problem, work done by the gravity is .  
(a) -8000 J (b) -4000 J (c) 8000 J (d) 4000 J

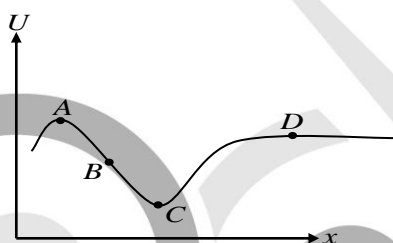
**PASSAGE IV**

22. The displacement  $x$  in metre of a particle of mass  $m$  kg moving in one dimension under the action of a force is related to the time  $t$  in seconds by the equation  $x = \sqrt{t} + 3$ . Then the displacement of the particle when its velocity is zero is :  
 (a) 3m (b) zero (c) 6 m (d) None of these
23. In the previous problem, the work done in first six seconds is :  
 (a) 18 mJ (b) zero (c) 9/2 mJ (d) 36 mJ

**MATRIX MATCH TYPE**

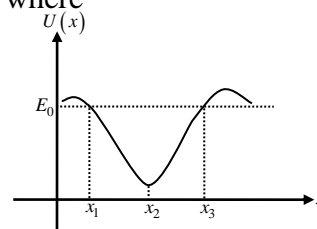
**Note :** Each statement in Column I has one or more than one match in Column II.

24. Potential energy of conservative field versus  $x$  graph is as shown in the figure, where  $x$  is the displacement in the direction of force. Four points A, B, C, and D are marked in the graph. Match the column I with column II.



Column – I	Column – II
A. At point A	(P) $\sum F_{\text{net}} = 0$
B. At point B	(Q) Potential energy is maximum.
C. At point C	(R) Potential energy minimum.
D. At point D	(S) $\sum F_{\text{net}} \neq 0$
	(T). Potential energy is constant.

25. The potential energy of a particle moving along  $x$ -axis is shown in figure, where  $E_0$  is the total mechanical energy of the particle.



Column – I	Column – II
A. Force on the particle is towards +ve $x$ -axis if	(P) $x_1 < x < x_2$
B. Particle cannot be found where	(Q) $x_2 < x < x_3$
C. Force is towards the equilibrium position if	(R) $x_1 > x > x_3$
D. Force is towards the -ve $x$ -axis if	(S) $x_3 > x_2 > x$

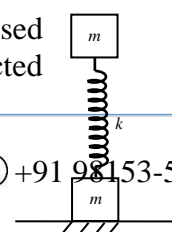
(T).  $x_1 < x < x_3$

26. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and  $U_0$  are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I		Column II	
(A)	$U_1(x) = \frac{U_0}{2} \left[ 1 - \left( \frac{x}{a} \right)^2 \right]^2$	(P)	The force acting on the particle is zero at $x = a$ .
(B)	$U_2(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2$	(Q)	The force acting on the particle is zero at $x = 0$ .
(C)	$U_3(x) = \frac{U_0}{2} \left( \frac{x}{a} \right)^2 \exp \left[ - \left( \frac{x}{a} \right)^2 \right]$	(R)	The force acting on the particle is zero at $x = -a$ .
(D)	$U_4(x) = \frac{U_0}{2} \left[ \frac{x}{a} - \frac{1}{3} \left( \frac{x}{a} \right)^3 \right]$	(S)	The particle experiences an attractive force towards $x = 0$ in the region $ x  < a$ .
		(T)	The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$ .

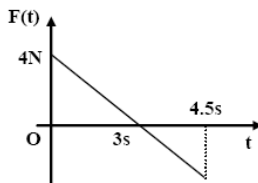
### EXERCISE – IV

- A pump motor is used to lift water to a height 4m and to deliver water of  $0.12 \text{ m}^3/\text{minute}$  with a velocity of 10 m/s. The power of pump is [density of water =  $10^3 \text{ kg/m}^3$ ,  $g = 10 \text{ m/s}^2$ ]:
- Power supplied to a particle of mass 2 kg varies with time as  $\frac{3t^2}{2}$  watt. Here t is in second. If velocity of particle at  $t = 0$  is  $v = 0$ . The velocity of particle at time  $t = 2 \text{ s}$  will be :  
(a) 1 m/s                      (b) 4 m/s                      (c) 2 m/s                      (d)  $2\sqrt{2} \text{ m/s}$
- A particle slides down from the top outside smooth surface of a fixed sphere of radius  $a = 10 \text{ m}$ . Find the initial horizontal velocity (in m/sec) to be imparted to the particle at the top, if it leaves the surface at a point whose vertical height above the centre of sphere is  $\frac{3a}{4}$ . ( $g = 10 \text{ m/s}^2$ )
- A horse pulls a wagon of 3075 kg from rest against a constant resistance of 90 N. The pull exerted initially is 600 N and it decreases uniformly with the distance covered to 400 N at a distance of 15 m from starts. Find the velocity (in m/sec) of wagon at this point.
- A system consists of two identical slabs each of mass m linked by compressed weightless spring of stiffness k as shown in figure. The slabs are also connected

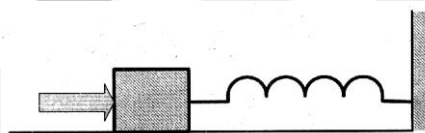


by a thread, which is burnt at a certain moment. Find the lower value of  $\Delta \ell$  the initial compression of spring (in meters); the lower slab will bounce up after the thread is burned through. (Take  $\frac{mg}{k} = 1$ )

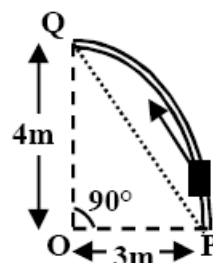
6. A block of mass 2 kg is free to move along the x-axis. It is at rest and from  $t = 0$  onwards it is subjected to a time-dependent force  $F(t)$  in the x direction. The force  $F(t)$  varies with  $t$  as shown in the figure. The kinetic energy of the block after 4.5 seconds is



7. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is  $V = N/10$ . Then  $N$  is



8. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in m/s) of the particle is zero, the speed (in m/s) after 5 s is
9. Consider an elliptically shaped rail PQ in the vertical plane with  $OP = 3$  m and  $OQ = 4$  m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is  $(n \times 10)$  Joules. The value of  $n$  is (take acceleration due to gravity =  $10 \text{ ms}^{-2}$ )



10. A uniform chain of length  $\ell = 1\text{m}$  and mass  $m = 9\text{kg}$  overhangs a smooth table with its two third part lying on the table. Find the kinetic energy of the chain as it completely slips off the table.
11. A small body is placed on the top of a smooth hemisphere of radius  $R = 60\text{m}$ . When the sphere is given a uniform horizontal acceleration  $a_0$  the body starts sliding down. (a) Find the velocity of body relative to sphere at the instant of losing contact. (b) Find the angle  $\phi$  between radius vector drawn to the body from centre of sphere at the time of losing contact if  $a_0 = g$ . ( $g = 10 \text{ m/s}^2$ )

12. A heavy particle hanging from a fixed point by a light inextensible string of length  $L = 30m$  is projected horizontally with speed  $\sqrt{gL}$ . find the speed of the particle at the instant of the motion when the tension in the string equals the weight of the particle. ( $g = 10m/s^2$ )

## ANSWERS

### EXERCISE – I BUILDING A FOUNDATION

#### SECTION-A

- |          |           |           |          |
|----------|-----------|-----------|----------|
| A-1. (c) | A-2. (c)  | A-3. (a)  | A-4. (c) |
| A-5. (b) | A-6. (c)  | A-7. (d)  | A-8. (a) |
| A-9. (d) | A-10. (d) | A-11. (d) |          |

#### SECTION-B

- |          |          |          |          |
|----------|----------|----------|----------|
| B-1. (a) | B-2. (d) | B-3. (a) | B-4. (b) |
| B-5. (d) | B-6. (c) | B-7. (c) | B-8. (d) |

#### SECTION-C

- |           |           |           |           |
|-----------|-----------|-----------|-----------|
| C-1. (a)  | C-2. (b)  | C-3. (d)  | C-4. (a)  |
| C-5. (a)  | C-6. (d)  | C-7. (c)  | C-8. (a)  |
| C-9. (c)  | C-10. (b) | C-11. (c) | C-12. (c) |
| C-13. (b) | C-14. (d) | C-15. (b) | C-16. (b) |

#### SECTION-D

- |           |           |           |           |
|-----------|-----------|-----------|-----------|
| D-1. (c)  | D-2. (d)  | D-3. (a)  | D-4. (b)  |
| D-5. (c)  | D-6. (b)  | D-7. (d)  | D-8. (b)  |
| D-9. (c)  | D-10. (a) | D-11. (c) | D-12. (a) |
| D-13. (a) | D-14. (d) | D-15. (a) | D-16. (d) |
| D-17. (b) | D-18. (a) |           |           |

#### SECTION-E

- |           |           |           |           |
|-----------|-----------|-----------|-----------|
| E-1. (c)  | E-2. (b)  | E-3. (d)  | E-4. (a)  |
| E-5. (a)  | E-6. (d)  | E-7. (d)  | E-8. (c)  |
| E-9. (c)  | E-10. (b) | E-11. (b) | E-12. (c) |
| E-13. (a) | E-14. (c) |           |           |

#### SECTION-F

- |          |          |          |          |
|----------|----------|----------|----------|
| F-1. (c) | F-2. (c) | F-3. (a) | F-4. (a) |
| F-5. (a) | F-6. (a) | F-7. (b) | F-8. (d) |

**F-9. (b)**
**F-10. (c)**
**F-11. (d)**
**SECTION-G**
**G-1. (c)**
**G-2. (a)**
**G-3. (d)**
**G-4. (c)**
**G-5. (b)**
**G-6. (a)**
**G-7. (a)**
**G-8. (d)**
**G-9. (a)**
**G-10. (b)**
**G-11. (d)**
**G-12. (d)**

<b>EXERCISE – II</b> <b>READY FOR CHALLENGES</b>
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1. (c)	2. (a)	3. (d)	4. (a)	5. (b)
6. (a)	7. (b)	8. (d)	9. (a)	10. (d)
11. (a)	12. (a)	13. (a)	14. (b)	15. (b)
16. (c)	17. (c)	18. (a)	19. (a)	20. (a)
21. (b)	22. (c)	23. (d)	24. (a)	25. (a)
26. (c)	27. (d)	28. (b)	29. (c)	30. (c)
31. (C)	32. (c)	33. (c)	34. (c)	35. (c)
36. (c)	37. (b)	38. (c)	39. (c)	40. (b)
41. (b)	42. (d)	43. (a)	44. (a)	45. (b)
46. (a)	47. (b)	48. (c)	49. (a)	50. (b)
51. (c)	52. (d)	53. (b)	54. (d)	55. (a)
56. (a)	57. (b)	58. (d)	59. (b)	60. (b)
61. (c)	62. (d)			

<b>EXERCISE – III</b> <b>CROSSING THE HURDLES</b>
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**MORE THAN ONE CORRECT**

1. (a,b,c,d)	2. (c,d)	3. (a,b,c)	4. (a,b)	5. (b,d)
6. (b,d)	7. (a,b,c,d)	8. (a,b)	9. (c)	10. (a,c)
11. (a,c)	12. (b,c)	13. (b)	14. (a)	15. (c)
16. (c)	17. (b)	18. (a)	19. (a)	20. (b)

21. (c)	22.(b)	23. (b)
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**MATCH THE COLUMN**

24. (A-PQ, B-S, C-PR, D-PT)

25. (A-P, B-R, C-T, D-Q)

26. (A-PQRT, B-QS, C-PQRS, D-PRT)

**EXERCISE – IV**

1. 180W
2. 2m/s
3. 5
4. 2
5. 3
6. 5.06J
7. 4
8. 5
9. 5
10. 10m/s
11. 170cm/s
12. 336cm/s

