

COMPARING QUANTITIES

PROFIT AND LOSS

In our daily routine, we have to buy some articles from various shops. The shopkeepers purchase these articles either from wholesalers or directly from the manufacturers by paying a certain price. Generally, the shopkeeper sells his articles at a different price. These prices and difference in these prices are given special names such as cost price, selling price, profit, loss etc.

Cost Price

The price for which an article is purchased is called the cost price and abbreviated as C.P.

Selling Price

The price for which an article is sold is called the selling price and abbreviated as S.P.

Profit

If selling price is more than cost price, then the difference between selling price and the cost price is called the profit.

$$\therefore \text{Profit} = \text{Selling Price} - \text{Cost Price}$$

Loss

If selling price is less than cost price, then the difference between the selling price and cost price is called loss.

$$\therefore \text{Loss} = \text{Cost Price} - \text{Selling Price}$$

Overheads

Usually, a merchant has to spend some money on freight or transport, labour or maintenance of the purchased articles. These extra expenditures are called overheads. The overheads are an essential part of cost price.

$$\therefore \text{Cost Price} = (\text{Payment made while purchasing the articles}) + \text{overhead charges}$$

SOME USEFUL FORMULAE TO FIND THE ABOVE DEFINED TERMS

1. Profit or Gain (S.P. > C.P.)

- | | |
|--|---|
| 1. Profit = S.P. - C.P. | 2. S.P. = Profit + C.P. |
| 3. C.P. = S.P. - Profit | 4. Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$ |
| 5. Profit = $\frac{\text{C.P.} \times \text{Profit}\%}{100}$ | 6. S.P. = C.P. $\left(\frac{100 + \text{Profit}\%}{100} \right)$ |
| 7. C.P. = $\frac{100 \times \text{S.P.}}{(100 + \text{Profit}\%)}$ | |

2. Loss (S.P. < C.P.)

- | | |
|--|---|
| 1. Loss = C.P. - S.P. | 2. S.P. = C.P. - Loss |
| 3. C.P. = Loss + S.P. | 4. Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$ |
| 5. Loss = $\frac{\text{C.P.} \times \text{Loss}\%}{100}$ | 6. S.P. = C.P. $\left(\frac{100 - \text{Loss}\%}{100} \right)$ |
| 7. C.P. = $\frac{100 \times \text{S.P.}}{(100 - \text{Loss}\%)}$ | |

SIMPLE INTEREST

- (i) Simple Interest = $\frac{\text{Principal} \times \text{Rate of interest} \times \text{Time}}{100}$
- (ii) Amount = Principal + Simple Interest.

COMPOUND INTEREST

When we borrow money from a financial agency (bank, financial agency or individual), it is called the lender.

The borrowed money is called the principal.

We have to pay some additional money together with the borrowed money for a certain time period, for the benefit of using his or her money. The additional money that we pay is called the interest.

If the principal remains the same for the whole loan period (or time), then the interest is called the simple interest.

The interest together with the principal is called the amount.

If the principal does not remain the same for the whole loan period due to addition of (compounding of) interest to the principal after a certain interval of time to form the new principal, then the interest so obtained is called the compound interest.

To understand compound interest, we consider the following example -

"A man lends Rs 5,000 to a finance company at 10% per annum. What interest does he get after one year ? What will be the amount then ? At the end of the year, if he decides to deposit the whole sum (amount after one year) for another year, what interest does he get at the end of the second year ?"

Interest after one year

$$= \text{Rs } \frac{5000 \times 1 \times 10}{100} = \text{Rs } 500$$

$$\begin{aligned} \therefore \text{Amount after one year} \\ &= \text{Rs } 5000 + \text{Rs } 500 \\ &= \text{Rs } 5500. \end{aligned}$$

When the deposit is Rs 5,500 in the company for one more year, the amount of Rs 5,500 due at the end of first year becomes the principal for the second year.

\therefore Interest at the end of the second year

$$= \text{Rs } \frac{5000 \times 1 \times 10}{100} = \text{Rs } 550$$

Thus, the interest for two years is

$$\text{Rs } 500 + \text{Rs } 550 = \text{Rs } 1050.$$

What do we notice ?

We notice that the interest for the second year is more than that for the first year.

Why, it is so ?

It is clear that in the second year, interest has been calculated on Rs 5500, which is equal to Rs 5,000 (Principal at the beginning) + Rs 500 (Interest for the first year). So, for the second year, interest on the interest has also been calculated. Interest calculated in this manner is known as compound interest.

COMPUTATION OF COMPOUND INTEREST BY USING FORMULAE

Formula : 1

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P \left(1 + \frac{R}{100} \right)^n$$

$$\text{C.I.} = A - P = P \left\{ \left(1 + \frac{R}{100} \right)^n - 1 \right\}$$

Formula : 2

Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually, then the amount A and the compound interest C.I. at the end of n years are given by

$$A = P \left(1 + \frac{R}{100k} \right)^{nk}$$

and, C.I. = A - P

$$= P \left\{ \left(1 + \frac{R}{100k} \right)^{nk} - 1 \right\} \text{ respectively.}$$

Here, interest is payable k times in a year.

Particular Cases :

Case 1 : When the interest is compound half- yearly or semi-annually.

In this case, k = 2

$$\therefore A = P \left(1 + \frac{R}{2 \times 100} \right)^{2n} \quad \text{and} \quad \text{C.I.} = P \left[\left(1 + \frac{R}{2 \times 100} \right)^{2n} - 1 \right]$$

Case 2 : When interest is compounded quarterly.

In this case, k = 4

$$\therefore A = P \left(1 + \frac{R}{4 \times 100} \right)^{4n} \quad \text{and} \quad \text{C.I.} = P \left[\left(1 + \frac{R}{4 \times 100} \right)^{4n} - 1 \right]$$

PERCENTAGE

The word percent is an abbreviation of the Latin phrase / word 'per centum' which means *per hundred or hundredths*.

Thus, the word 'percentage' literally means *per hundred or every hundred*'. Therefore whenever we calculate something as a part of 100 that 'part is numerically termed as percentage'.

Symbol : In short 'percentage is written as P.C. and symbolically it is denoted as %.

Conversion of percentage into fraction : To express a% as a fraction, divide a by 100 i.e., a% =

$$\frac{a}{100}$$

Ex. We have, 35% = $\frac{35}{100}$

Thus, a fraction with its denominator 100 is equal to that percent as is the numerator.

$$\text{So, } 8\% = \frac{8}{100}, 12\% = \frac{12}{100}, 60\% = \frac{60}{100} = \frac{3}{5} = 3 : 5$$

Conversion of a fraction into percentage : To convert a fraction into a percent, we multiply the fraction by 100 and put the percent sign %.

Ex. Thus $\frac{4}{5} = \left(\frac{4}{5} \times 100 \right) \% = 80\%$, $\frac{9}{20} = \left(\frac{9}{20} \times 100 \right) \% = 45\%$

Conversion of a percentage into decimal form : To convert a given percent in decimal form, we express it as a fraction with denominator as 100 and then the fraction is written in decimal form.

Ex. $65\% = \frac{65}{100} = 0.65$, $7.4\% = \frac{7.4}{100} = 0.074$

Conversion of a decimal fraction into percentage : In order to convert a given decimal into a percent, we move the decimal point on the right side by two digits and put the percent sign %

Ex. $0.122 = 12.2\%$, $0.275 = 27.5\%$, $0.037 = 3.7\%$

TO FIND THE VALUE OF A CERTAIN PERCENT OF ANY QUANTITY

To find the value of a certain percent of a given quantity we first convert the percent in to a fraction and then multiply the result with the given quantity.

i.e.
$$P\% \text{ of } x = \frac{P}{100} \times x$$

Ex.1 Find 12% of Rs. 1200.

Sol. 12% of Rs.1200 = Rs. $\frac{12}{100} \times 1200 = 144$

Ex.2 What is 45% of 500?

Sol. 45% of 500 = $500 \times \frac{45}{100} = 225$

SUCCESSIVE CHANGES IN PERCENTAGE

If a quantity x is increased or decreased successively by A%, B%, C% then the final value of x will be

$$= x \left(1 \pm \frac{A}{100} \right) \left(1 \pm \frac{B}{100} \right) \left(1 \pm \frac{C}{100} \right)$$

(+) → increase, (-) → Decrease

PROFIT-LOSS AND DISCOUNT

1. **Profit-loss :**

(A) **Cost price (C.P.) :** The price at which an article is purchased is called its cost price (C.P.)

(B) **Selling price (S.P.) :** The price at which an article is sold is known as its selling price.

(C) **Gain-profit :** When S.P. > C.P. then there is a gain.

$$\text{Gain or profit} = \text{S.P.} - \text{C.P.}$$

(D) **Loss :** when S.P. < C.P. then there is a loss.

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

Remark : The gain (profit) or loss is always reckoned on the cost price.

SOME IMPORTANT FORMULAE

1.
$$\text{Gain}\% = \left(\frac{\text{Gain}}{\text{C.P.}} \times 100 \right)\% \quad \text{or} \quad \text{Gain} = \frac{\text{C.P.} \times \text{Gain}\%}{100}$$

2.
$$\text{Loss}\% = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100 \right)\% \quad \text{or} \quad \text{Loss} = \frac{\text{C.P.} \times \text{Loss}\%}{100}$$

3. To find C.P. when S.P. and gain% or loss% are given.

(A)
$$\text{C.P.} = \frac{100}{100 + \text{Gain}\%} \times \text{S.P.}$$

(B)
$$\text{C.P.} = \frac{100}{100 - \text{Loss}\%} \times \text{S.P.}$$

4. To find S.P. when C.P. and gain% or loss% are given.

(A)
$$\text{S.P.} = \frac{100 + \text{Gain}\%}{100} \times \text{C.P.}$$

(B)
$$\text{S.P.} = \frac{100 - \text{Loss}\%}{100} \times \text{C.P.}$$

OVERHEAD

Sometimes, after purchasing an article, we have to pay some more money for things like transportation, labour charges, repairing charges, local taxes, etc. These extra expenses are called overhead. For calculating the total cost price, we add overhead to the purchase price.

The real cost price = Total investment = (Payment made for buying goods + overhead charges)

Note :

(i) If the S.P. of two objects are equal and one of them is sold at $x\%$ gain and other is at $x\%$ loss than

$$\text{there is always a loss of} = \left[\frac{\text{Common loss or gain}\%}{10} \right]^2 = \left(\frac{x}{10} \right)^2 \%$$

(ii) If the C.P. of two objects are equal and one of them is sold at $x\%$ gain and other is at $x\%$ loss than there is no loss – no profit.

DISCOUNT

- 1. Marked Price (M.P.) or List Price (L.P.) :** The price written on the article or tagged with the articles is called the Marked Price (M.P.) or List Price (L.P.).
- 2. Discount :** The deduction made on the marked price is called discount. Discount is generally given as a certain percent of the marked price. It is always calculated on the marked price or list price.
- 3. Selling Price/Net Price :** The difference between the marked price and discount is called the selling price or net price of the article.

Note : Discount is given on the marked price only.

$$\text{S.P.} = \text{M.P.} - \text{Discount}$$

$$\text{Discount} = \text{Marked price (M.P.)} - \text{Selling price (S.P.)}$$

$$\text{Discount \%} = \frac{\text{Discount}}{\text{M.P.}} \times 100 \quad \text{or} \quad \text{Discount} = \frac{\text{M.P.} \times \text{Discount \%}}{100}$$

$$\text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount \%}} \quad \text{or} \quad \text{S.P.} = \frac{\text{M.P.} \times (100 - \text{Discount \%})}{100}$$

$$\text{Discount \%} = \frac{\text{M.P.} - \text{S.P.}}{\text{M.P.}} \times 100$$

Successive discounts : Two successive discounts of $x\%$ and $y\%$ allowed on an item are equivalent to

$$\text{a single discount of} \left(x + y - \frac{xy}{100} \right) \%$$

Note : This discount is always less than the sum of individual discounts.

VALUE ADDED TAX (VAT) / SALES TAX (S.T.)

Sales tax (S.T.) is a Tax. which is paid by us when we buy items.

Remark : S.T. is charged by the government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. So **S.T., is always calculated on selling price** of an item and is added to the value of the bill. So, the prices include the tax known as value added tax (VAT).

Now a days value added tax (VAT) has been included as a tax.

$$\text{Calculation of sales tax : Sales tax} = \text{Selling price} \times \frac{\text{Rate of Sales tax}}{100}$$

$$\text{Rate of sales tax} = \frac{\text{Sales tax}}{\text{S.P.}} \times 100$$

$$\text{Price including Sales tax} = \text{Selling price} + \text{Sales tax.}$$

SIMPLE INTEREST & COMPOUND INTEREST

- 1. Simple Interest (S.I.) :** The interest charged on the initial principal is called simple interest. If P is the principal. R is the rate of interest per annum and T is the time in years, then the simple interest is given by

$$S.I. = \frac{P \times R \times T}{100} \quad \text{where } P = \text{Principal, } R = \text{Rate, } T = \text{Time, } SI = \text{Simple interest}$$

- 2. Compound Interest (C.I.) :** The interest charged every year on the amount of last year is called compound interest. When the interest is not paid in the specified period and the interest is accrued to the principal for the calculation of future interest such an interest is known as compound interest.
Compound interest = Amount of the last time period principal of first time period $CI = A - P$.

$$\text{Where, } A = P \left[1 + \frac{R}{100} \right]^n$$

Where P = Principal, R = Rate of interest, n = Time.

\therefore Compound interest (C.I.) = $A - P$

C.I. at the end of certain specified period is equal to the difference between the amount at the end of the period and the original principal.

$$\text{Hence, } C.I. = P \left[\left(1 + \frac{R}{100} \right)^n - 1 \right]$$

(A) Principle (P) : The money borrowed or lent out is called principal. $P = A - I$

(B) Interest (I) : The additional money paid by the borrower is called the interest. $I = A - P$

(C) Amount (A) : The total money (interest + Principal) paid by the borrower is called the amount.

$$A = P + I$$

(D) Rate of interest (R) : If the borrower paid Rs. x on Rs. 100 for 1 year, then the rate of interest is x percent per annum.

(E) Time (T) : The period for which the sum is borrowed is called the time.

(F) Conversion period : The time period after which the interest is added each time to form a new principal is called conversion period. It may be one year, six months or three months i.e., annually, semi-annually or quarterly etc.

Type-I : On finding the compound interest and the amount when interest is compounded annually :

Ex.3 Find the compound interest on Rs.10,000 for 2 years at rate of 8% per annum.

Sol. $P = 10,000$, $R = 8\%$, $T = 2$ years

$$A = P \left(1 + \frac{R}{100} \right)^n = 10000 \left(1 + \frac{8}{100} \right)^2 = 10000 \times \frac{108}{100} \times \frac{108}{100} = 11664$$

$$C.I. = A - P = 11664 - 10000 = \text{Rs. } 1664$$

Type-II : Computation of compound interest when interest is compounded Annually but time being a fraction:

Ex.4 Find the compound interest on Rs. 1500 for $2\frac{1}{2}$ years at 10% per annum, when the interest is payable annually.

Sol. $P = \text{Rs. } 1500$, $T = 2\frac{1}{2}$ years, $R = 10\%$

Interest is paid annually.

$$A = P \left(1 + \frac{R}{100}\right)^n = 1500 \left(1 + \frac{10}{100}\right)^{2\frac{1}{2}} = 1500 \left(1 + \frac{10}{100}\right)^2 \left(1 + \frac{10}{100}\right)^{\frac{1}{2}}$$

$$1500 \times \frac{110}{100} \times \frac{110}{100} \times \frac{105}{100} = \text{Rs. } 1905.75$$

[\therefore Rate of interest is 10% annually hence on Rs. 100 only Rs. 5 will be charged]

Compound interest = Amount – Principal = 1905.75 – 1500 = Rs. 405.75

Ex.5 Find the amount of Rs. 2000 for 12% per annum compounded annually for 1 year and 4 months.

Sol. Principal = Rs. 2000,

Time = 1 year 4 months = $1 + \frac{4}{12} = 1\frac{1}{3}$ years, Rate = 12%

$$\text{Amount} = \text{Principal} \left(1 + \frac{\text{rate}}{100}\right)^{\text{Time}} = 2000 \times \left(1 + \frac{12}{100}\right)^{1\frac{1}{3}} = 2000 \left(1 + \frac{12}{100}\right)^1 \left(1 + \frac{12}{100}\right)^{\frac{1}{3}}$$

$$= 2000 \times \frac{112}{100} \times \frac{104}{100} = \text{Rs. } 2329.60$$

Type-III : Computation of compound interest when the interest is compounded half-yearly

$$= A = P \left(1 + \frac{\frac{1}{2}R}{100}\right)^{2n} \text{ or } A = P \left(1 + \frac{R}{200}\right)^{2n}, \text{ Rate} = \left(\frac{R}{2}\right)\% \text{ half yearly and Time} = 2n \text{ half yearly}$$

Ex.6 Kapil deposited Rs. 1600 in a bank on 1st January 2005. Find the amount in his bank account on 1st January, 2006 if the bank pays interest at 8% per annum and the interest is calculated every year on 30th June and 31st December.

Sol. P = 1600, T = 1 years, Rate = 8%

Interest is calculated every year on 30th June and 31st December, the interest is payable half yearly.

Time = 1 year = 2 half years

Rate = 8% per annum = 4% half yearly

$$\text{Amount} = 1600 \times \left(1 + \frac{4}{100}\right)^2 = 1600 \times \frac{104}{100} \times \frac{104}{100} = \text{Rs. } 1730.56$$

Amount in Kapil's account on 1st Jan. 2006 = Rs. 1730.56

Type-IV : Computation of compound interest, when interest is compounded quarterly :

Rate = $\frac{R}{4}\%$ per quarterly and Time = 4n quarters

$$A = P \left(1 + \frac{\frac{1}{4}R}{100}\right)^{4n} \text{ or } A = P \left(1 + \frac{R}{400}\right)^{4n}$$

Ex.7 Find the compound interest on Rs. 1000 for 6 months at the rate of 16% per annum when the interest is payable quarterly.

Sol. P = 1000, T = 6 Month = 2 quarters, n = 2 years, R = 16% per annum = 4% quarterly.

$$\text{C.I.} = \left(1 + \frac{R}{100}\right)^n - p = 1000 \left(1 + \frac{4}{100}\right)^2 - 1000$$

$$= \text{Rs. } (1000 \times \frac{104}{100} \times \frac{104}{100} - 1000) = \text{Rs. } (1081.60 - 1000) = \text{Rs. } 81.60$$

Ex.8 Ramesh deposited R. 7500 in a bank which pays him 12% interest per annum compounded quarterly. What is the amount which he receives after 9 months?

Sol. Here, P = Rs. 7500, R = 12% per annum and T/n = 9 months = $\frac{9}{12}$ year = $\frac{3}{4}$ years.

$$\begin{aligned} \therefore \text{Amount after 9 months} &= P \left(1 + \frac{R}{400}\right)^{4n} \\ &= \text{Rs. } 7500 \times \left(1 + \frac{12}{100}\right)^{4 \times \frac{3}{4}} = \text{Rs. } 7500 \times \left(1 + \frac{3}{100}\right)^3 \\ &= \text{Rs. } 7500 \times \left(\frac{103}{100}\right)^3 = \text{Rs. } 7500 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100} = \text{Rs. } 8195.45 \end{aligned}$$

Type-V : Inverse problems on compound interest :

Ex.9 What sum of money will amount Rs. 3645 in 3 years at $12\frac{1}{2}\%$ per annum compound interest.

Sol. Suppose the sum of money = Rs. 100, Rate = $12\frac{1}{2}\%$ p.a., Time = 3 years

$$\begin{aligned} \therefore \text{Amount} &= \text{Rs. } 100 \left(1 + \frac{12\frac{1}{2}}{100}\right)^3 = \text{Rs. } 100 \left(1 + \frac{25}{200}\right)^3 = \text{Rs. } 100 \left(1 + \frac{1}{8}\right)^3 = \text{Rs. } 100 \left(\frac{9}{8}\right)^3 \\ &= \text{Rs. } 100 \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \text{Rs. } \frac{18225}{128} \end{aligned}$$

If the amount is Rs. $\frac{18225}{128}$ then the principal = Rs. 100

If the amount is Re 1 then the principal = Rs. $\frac{100 \times 128}{18225}$

If the amount is Rs. 3645 then the principal = Rs. $\frac{100 \times 128}{18225} \times 3645 = \text{Rs. } 2560$

Ex.10 The difference between the compound interest and simple interest on a certain sum of money at 10% per annum for 2 years is Rs. 500. Find the sum when the interest is compounded annually.

Sol. Let the sum be Rs. 100.

Computation of compound interest :

We have, Principal = Rs.100, R = 10% per annum and n = 2 years.

$$\text{Amount} = \text{Rs. } \left[100 \times \left(1 + \frac{10}{100}\right)^2\right] = \text{Rs. } \left[100 \times \left(\frac{11}{10}\right)^2\right] = \text{Rs. } 121$$

$$\therefore \text{C.I.} = \text{Rs. } 121 - \text{Rs. } 100 = \text{Rs. } 21$$

Computation of simple interest :

We have principal = Rs. 100, R = 10% and Time = 2 years.

$$\therefore \text{S.I.} = \text{Rs. } \left(\frac{100 \times 10 \times 2}{100}\right) = \text{Rs. } 20$$

Thus, difference in C.I. and S.I. = Rs. (21 - 20) = Rs. 1

Now,

If difference between C.I. and S.I. is Rs. 1, sum = Rs. 100

If difference between C.I. and S.I. is Rs. 500, sum = Rs. (100 × 500) = Rs. 50000.

Type-VI : Computation of compound interest, when interest is compounded annually but rates being different for different years :

Respectively then amount is given by $A = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \dots$

Ex.11 Ram Singh buys a refrigerator for Rs. 4000 on credit. The rate of interest for the first year is 5% and of the second year is 15%. How much will it cost him if he pays the amount after two years?

Sol. Here, $P = 4000$, $R_1 = 5\%$, $R_2 = 15\%$

$$\begin{aligned} \therefore \text{Amount after 2 years} &= P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \\ &= \text{Rs. } 4000 \times \left(1 + \frac{5}{100}\right) \left(1 + \frac{15}{100}\right) \\ &= \text{Rs. } 4000 \times \frac{105}{100} \times \frac{115}{100} \\ &= \text{Rs. } 4000 \times \frac{21}{20} \times \frac{23}{20} = \text{Rs. } 4830 \end{aligned}$$

Thus, the refrigerator will cost Rs. 4830 to Ram Singh.

GROWTH AND DEPRECIATION

Growth Rate : The growth rate for a certain period is the ratio of the change in that period and the magnitude just before this period, which is expressed as percent.

P = Final value after n years.

$$\text{Then rate of growth} = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100\%$$

$$\text{or } R = \frac{P - P_0}{P_0} \times 100\%$$

Ex. 12 The numbers of viewers of a magic show on Monday was 3000. On Tuesday it was 3600 and on Wednesday, the number was 2700. Find the growth rate for Tuesday and Wednesday.

Sol. \therefore Number of viewers on Monday = 3000
Number of viewers on Tuesday = 3600
Number of viewers on Wednesday = 2700

Increase in the number of viewers on Tuesday as compared to Monday = $3600 - 3000 = 600$

The number of viewers on Monday = 3000

$$\therefore \text{Rate of growth for Tuesday} = \frac{600}{3000} \times 100 = 20\%$$

By Formula : $P_0 = 3000$, $P = 3600$

$$\text{Growth rate for Tuesday} = \frac{P - P_0}{P_0} \times 100\%$$

$$= \frac{3600 - 3000}{3000} \times 100$$

$$= \frac{600}{3000} \times 100 = 20\%$$

$$\text{Growth rate for Wednesday} = \frac{2700 - 3600}{3600} \times 100 = \frac{-900}{3600} \times 100 = -25\%$$

The negative growth rate is called the rate of depreciation.

FORMULAE FOR POPULATION GROWTH**FORMULA-I :**

Let P be the population of a city or a town at the beginning of a certain year and the population grows at a constant rate of R% per annum, then

$$\text{Population after } n \text{ years} = P_0 \left[1 + \frac{R}{100} \right]^n$$

FORMULA-II :

Let P be the population of a city or a town at the beginning of a certain year. If the population grows at the rate of $R_1\%$ during first year and $R_2\%$ during second year, then

$$\text{Population after 2 years} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right)$$

FORMULA-III :

Let P be the population of a city or a town at the beginning of a certain year. If the population decreases at the rate of R% per annum, then

$$\text{Population after } n \text{ years} = P \left(1 - \frac{R}{100} \right)^n$$

Ex.13 The population of a town 2 years ago was 62500. Due to migration to cities, it decreases every year at the rate of 4% per annum. Find its present population.

Sol.

We have,

Population two years ago = 62500

Rate of decrease of population = 4% per annum.

$$\begin{aligned} \therefore \text{Present population} &= 62500 \times \left(1 - \frac{4}{100} \right)^2 \\ &= 62500 \times \left(1 - \frac{1}{25} \right)^2 = 62500 \times \left(\frac{24}{25} \right)^2 = 62500 \times \frac{24}{25} \times \frac{24}{25} = 57600. \end{aligned}$$

Hence, present population = 57600.

DEPRECIATION

The relative decrease in the value of a machine over a period of time is called its depreciation.

(i) If P_0 is the value of an article a certain time and R% per annum is the rate of depreciation, then the

value P_n at the end of n years is given by $P_n = P_0 \left(1 - \frac{R}{100} \right)^n$

(ii) If P_0 is the value of an article a certain time and the rate of depreciation is $R_1\%$ for first n_1 years, $R_2\%$ for next n_2 years and so on and $R_k\%$ for the last n_k years then the value at the end of $n_1 + n_2 + \dots + n_k$

is given by $P = P_0 \left(1 - \frac{R_1}{100} \right)^{n_1} \times \left(1 - \frac{R_2}{100} \right)^{n_2} \dots \left(1 - \frac{R_k}{100} \right)^{n_k}$

Ex.14 A new car costs Rs 360000. Its price depreciates at the rate of 10% a year during the first two years and at the rate of 20% a year thereafter. What will be the price of the car after 3 years?

Sol.

We have,

Cost of the car = Rs. 360000

Rate of depreciation in first two years = 10% per annum.

Rate of depreciation in the third year = 20%

\therefore Price of the car after 3 years

$$= \text{Rs. } \left\{ 360000 \times \left(1 - \frac{10}{100} \right) \times \left(1 - \frac{10}{100} \right) \times \left(1 - \frac{20}{100} \right) \right\}$$

$$= \text{Rs. } \left\{ 360000 \times \left(1 - \frac{1}{10} \right) \times \left(1 - \frac{1}{10} \right) \times \left(1 - \frac{1}{5} \right) \right\}$$

$$= \text{Rs. } \left\{ 360000 \times \frac{9}{10} \times \frac{9}{10} \times \frac{4}{5} \right\} = \text{Rs. } 233280$$

Hence, the price of the car after 3 years = Rs. 233280.

SOLVED EXAMPLES

Ex.1 Anshul purchased 100 oranges at the rate of Rs 2 per orange. He sold 60% of the oranges at the rate of Rs 2.50 per orange and the remaining oranges at the rate of Rs 2 per orange. Find his profit percent.

Sol. S.P. of 100 oranges
 $= \text{Rs } 2 \times 100 = \text{Rs } 200$

$$60\% \text{ of } 100 \text{ oranges} = \frac{60}{100} \times 100 \text{ oranges} = 60 \text{ oranges}$$

Now S.P. of 60 oranges

$$= \text{Rs } 2.50 \times 60 = \text{Rs } 150$$

and S.P. of the remaining $(100 - 60)$, i.e., 40 oranges

$$= \text{Rs } 2 \times 40 = \text{Rs } 80$$

\therefore S.P. of all the 100 oranges

$$= \text{Rs } 150 + \text{Rs } 80 = \text{Rs } 230$$

Therefore, profit = S.P. - C.P.

$$= \text{Rs } (230 - 200) = \text{Rs } 30$$

Hence, Profit percent

$$= \frac{30}{200} \times 100 = 15\%$$

Thus, Anshul's profit is 15%.

Ex.2 By selling 144 eggs, Anuj lost the S.P. of 6 eggs. Find his loss percent.

Sol. Let S.P. of 1 egg = Re 1

\therefore S.P. of 144 eggs

$$= \text{Rs } 144 \times 1 = \text{Rs } 144$$

and, Loss = S.P. of 6 eggs

$$= \text{Re } 1 \times 6 = \text{Rs } 6$$

\therefore C.P. of 144 eggs = S.P. + Loss

$$= \text{Rs } 144 + \text{Rs } 6 = \text{Rs } 150$$

Therefore, loss %

$$= \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{6}{150} \times 100 = 4$$

Thus, Anuj's loss is 4%.

Ex.3 Mahender bought two cows at Rs 20,000 each. He sold one cow at 15% gain. But he had to sell the second cow at a loss. If he had suffered a loss of Rs 1,800 on the whole dealing, find the selling price of the second cow.

Sol. Total C.P. of the two cows

$$= 2 \times \text{Rs } 20000 = \text{Rs } 40000$$

Loss = Rs 1800

$$\therefore \text{Total S.P.} = \text{Rs } 40000 - \text{Rs } 1800 = \text{Rs } 38200$$

... (i)

Now, S.P. of the first cow at 15% profit

$$= \text{C.P.} \left(\frac{100 + \text{Profit}\%}{100} \right)$$

$$= \text{Rs } 20000 \times \frac{(100+15)}{100}$$

$$= \text{Rs } 20000 \times \frac{115}{100}$$

$$= \text{Rs } 200 \times 115 = \text{Rs } 23000$$

.. (ii)

\therefore S.P. of the second cow

$$= \text{Rs } 38200 - \text{Rs } 23000$$

[From (i) and (ii)] = Rs 15200

Thus, the selling price of the second cow is Rs 15,200.

Ex.4 A man buys 60 pens at Rs 10 per pen and sells 40 pens at Rs 12 per pen and remaining 20 pens at Rs 9 per pen. Find his gain or loss percent.

Sol. Cost of 60 pens = Rs 10 × 60 = Rs 600

S.P. of 40 pens = Rs 12 × 40 = Rs 480

S.P. of 20 pens = Rs 9 × 20 = Rs 180

⇒ Total S.P.

$$= \text{Rs } 480 + \text{Rs } 180 = \text{Rs } 660$$

Since, S.P. > C.P.

$$\therefore \text{Profit} = \text{Rs } 660 - \text{Rs } 600 = \text{Rs } 60$$

$$\therefore \text{Profit percent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{60}{600} \times 100 = 10\%.$$

Ex.5 By selling an air-cooler for Rs 6,800, Mr. Avinash lost 15%. For what price should he sell it to get a profit of 10% ?

Sol. This sum will be solved in two parts. In 1st part, we find the C.P. and in 2nd part, we find the required S.P.

Part I :

S.P. of the air cooler = Rs 6800

Loss = 15% i.e., for every Rs 100 he is losing Rs 15.

∴ If C.P. is Rs 100, then S.P.

$$\text{Rs } 100 - \text{Rs } 15 = \text{Rs } 85$$

∴ If S.P. is Rs 85, then C.P. = Rs 100

If S.P. is Re 1, then C.P.

$$= \text{Rs } \frac{100}{85}$$

If S.P. is Rs 6800, then C.P.

$$= \text{Rs } \frac{100}{85} \times 6800 = \text{Rs } 100 \times 80$$

$$= \text{Rs } 8000.$$

Part II :

C.P. = Rs 8000 Profit = 10%

∴ Profit = 10% of Rs 8000

$$= \frac{10}{100} \text{Rs } 8000 = \text{Rs } 800$$

∴ S.P. = C.P. + Profit

$$= \text{Rs } 8000 + \text{Rs } 800 = \text{Rs } 8800.$$

Hence, the air-cooler should be sold for Rs 8,800 in order to make a profit of 10%.

Ex.6 A man sold two scooters for Rs 18000 each. On one, he gained 20% and on the other, he lost 20%. Find his total loss or gain.

Sol. S.P. of the first scooter = Rs 18000

Gain = 20%

$$\text{Therefore, C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit}\%)} = \text{Rs } \frac{100 \times 18000}{(100 + 20)}$$

$$= \text{Rs } \frac{100 \times 18000}{120} = \text{Rs } 100 \times 150 = \text{Rs } 15000$$

... (i)

S.P. of the second scooter = Rs 18000

Loss = 20%

$$\text{Therefore, C.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{Loss}\%)}$$

$$= \text{Rs } \frac{100 \times 18000}{(100 - 20)}$$

$$= \frac{100 \times 18000}{80} = \text{Rs } 100 \times \frac{225}{80} = \text{Rs } 22500 \quad \dots \text{ (ii)}$$

Now, total C.P.

$$= \text{Rs } 15000 + \text{Rs } 22500$$

[From (i) and (ii)]

$$= \text{Rs } 37500$$

and, total S.P.

$$= 2 \times \text{Rs } 18000 = \text{Rs } 36000$$

Hence, loss = C.P. - S.P.

$$= \text{Rs } 37500 - \text{Rs } 36000$$

$$= \text{Rs } 1,500.$$

Ex.7 The cost price of 10 tables is equal to the selling price of 8 tables. Find the loss or profit percent.

Sol. Let the C.P. of each table = Rs 100

$$\therefore \text{C.P. of 10 tables} = \text{Rs } 1000$$

$$\therefore \text{S.P. of 8 tables} = \text{Rs } 1000$$

$$\text{So, S.P. of 1 table} = \text{Rs } \frac{1000^{125}}{8} = \text{Rs } 125$$

$$\therefore \text{Profit on 1 table}$$

$$= \text{Rs } 125 - \text{Rs } 100 = \text{Rs } 25$$

$$\text{or Profit percent} = \frac{25}{100} \times 100 = 25\%.$$

Ex.8 Marked price of a pen is Rs 20. It is sold at a discount of 15%. Find the discount allowed on the pen and its selling price.

Sol. Marked Price of the pen = Rs 20

Rate of discount = 15%

\therefore Discount allowed

$$= 15\% \text{ of Rs } 20 = \frac{15}{100} \times \text{Rs } 20 = \text{Rs } 3$$

Therefore, selling price of the pen = Rs 20 - Rs 3 = Rs 17.

Ex.9 A chain with marked price Rs 1,200 was sold to a customer for Rs 1,000. Find the rate of discount allowed on the chain.

Sol. Marked Price = Rs 1200

Selling Price = Rs 1000

Discount = Rs 1200 - Rs 1000 = Rs 200

$$\text{Rate of discount} = \frac{\text{Discount}}{\text{M.P.}} \times 100\%$$

$$= \frac{200}{1200} \times 100\% = 16.66\%$$

Ex.10 A shopkeeper offers 15% season discount to the customers and still makes a profit of 19%. What is the cost price for the shopkeeper on a saree marked at Rs 2,240 ?

Sol. M.P. = Rs 2240

Rate of discount = 15%

Discount allowed

$$= \text{Rs } \frac{15}{100} \times 2240 = \text{Rs } 336$$

Thus, S.P. of the saree

$$= \text{Rs } (2240 - 336) = \text{Rs } 1904$$

Now, profit % of the shopkeeper = 19%

$$\text{Therefore, C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit}\%)}$$

$$= \text{Rs } \frac{100 \times 1904}{(100 + 19)} = \text{Rs } \frac{100 \times 1904}{119}$$

$$= \text{Rs } 100 \times 16 = \text{Rs } 1600$$

Thus, the cost price of the saree is Rs 1,600.

Ex.11 A Jacket was sold for Rs 680 after allowing a discount of 15% on the marked price. Find the marked price of the Jacket.

Sol. Let M.P. be Rs x.

\therefore Discount = 15% on Rs x

$$= \text{Rs } \frac{15}{100} \times x = \text{Rs } \frac{3x}{20}$$

$$\therefore \text{S.P.} = \text{Rs } \left(x - \frac{3x}{20} \right) = \text{Rs } \left(\frac{20x - 3x}{20} \right)$$

$$= \text{Rs } \frac{17x}{20}$$

According to the given condition,

$$\frac{17x}{20} = 680 \text{ or } x = \frac{680 \times 20}{17} = \text{Rs } 800$$

Thus, marked price of the Jacket is Rs 800.

Ex.12 Rakesh purchased a cycle for Rs 660 including sales tax. If the rate of sales tax is 10%, find the selling price of cycle.

Sol. Let the selling price be Rs x.

Sales tax = 10% of x

$$= \frac{10}{100} \times x = \frac{x}{10}$$

\therefore Amount to be paid for the cycle

$$= x + \frac{x}{10} = \frac{11x}{10}$$

$$\text{Now, } \frac{11x}{10} = 660 \text{ (given)}$$

$$\text{Therefore, } x = \frac{660 \times 10}{11} = 600$$

Hence, the selling price of the cycle is Rs 600.

Ex.13 In a school out of 600 students, $12\frac{1}{2}\%$ were absent. Find the number of students absent.

Sol. \therefore Total number of students = 600

\therefore Absent students $12\frac{1}{2}\%$ i.e., $\frac{25}{2}\%$

\therefore Number of absent students = $600 \times 12\frac{1}{2}\%$

$$\therefore = 600 \times \frac{25}{2} \times \frac{1}{100} = 75$$

Ex.14 Malvika gets 98 marks in her exams. This amounts to 56% of the total marks. What are the maximum marks.

Sol. Let the maximum marks be x. Then,
56% of x = 98

$$\Rightarrow \frac{56}{100} \times x = 98$$

$$\Rightarrow x = 98 \times \frac{100}{56}$$

$$\Rightarrow x = 175$$

Hence, the maximum marks are 175.

Ex.16 A football team won 10 games from the total they played. This was 40% of the total. How many games were played in all?

Sol. Let total games played be x

$$\text{So, } 40\% \text{ of } x = 10 \text{ or } x \times \frac{40}{100} = 10$$

$$\text{or } x = \frac{10 \times 100}{40} = 25$$

Thus, total games were 25.

Ex.17 What percentage of 364 is 273?

$$\text{Sol. } \text{Percentage} = \frac{273}{364} \times 100 = 75\%$$

Ex.18 What percentage of 75 is 125?

$$\text{Sol. } \text{Percentage} = \frac{125}{75} \times 100 = 166.66\%$$

Ex.19 The balance in the savings bank account of Sunil was Rs. 7500. He withdrew Rs. 1500 from his account. Find what percent of the amount did Sunil withdraw?

Sol. \therefore Out of 7500 he withdrew Rs. 1500

$$\therefore \text{ Out of 100 he withdrew Rs. } \frac{1500}{7500} \times 100 = 20$$

Therefore Sunil withdrew 20% of his balance from the bank.

Ex.20 Find the number of which is 30% more than 600.

$$\text{Sol. } 30\% \text{ more of } 600 = 600 \times \left(\frac{100+30}{100} \right) = 600 \times \left(\frac{130}{100} \right) = 780$$

Ex.21 Find the number of which is 10% less than 350.

$$\text{Sol. } 10\% \text{ less of } 350 = 350 \times \left(\frac{100-10}{100} \right) = 350 \times \left(\frac{90}{100} \right) = 315$$

Ex.22 The price of a scooter which was Rs. 34000 last year increased by 20% this year. What is the price now?

Sol. Let the original price be Rs. 100, Increased in price = Rs. 20

$$\therefore \text{Increased price} = \text{Rs. } 100 + \text{Rs. } 20 = \text{Rs. } 120$$

If original price is Rs. 100, increased price

$$= \text{Rs. } 120$$

If original price is Rs. 34000, increased price

$$= \text{Rs. } \frac{120}{100} \times 34000 = \text{Rs. } 40800.$$

Ex.23 The result of annual examination of class IX was 65%. If 195 students passed find the total number of students appeared in the examination.

Sol. Suppose the number of students appeared is x.

According, number of students passed is 65%

$$\text{i.e. } x \times \frac{65}{100} = 195$$

$$x = 195 \times \frac{100}{65} = 300$$

Ex.24 'A' has 30% more money than 'B', 'B' has 10% more money than 'C'. Find.

(i) What percent more 'A' has as compared to 'C'?

(ii) If 'C' has Rs. 400 what does 'A' have.

Sol. (i) 'A' is compared to 'B' and 'B' is compared to 'C' therefore we shall start from 'C'.

Suppose 'C' has Rs. 100 then 'B' will have Rs. 110

Again 'A' has 30% more than 'B' therefore if 'B' has Rs. 100 then 'A' will have Rs. 130.

If 'B' has Rs. 110 then 'A' will have

$$\frac{130}{100} \times 110 = 143 \text{ Rs.}$$

Therefore 'A' has more than 'C' = $143 - 100 = 43 \text{ Rs.} = 43\%$

(ii) If 'C' has Rs. 100 then 'A' has Rs. 143

If 'C' has Rs. 400 then 'A' has

$$\text{Rs. } \frac{143}{100} \times 400 = \text{Rs. } 572$$

Ex.25 Mohan's salary is 25% more than that of Ram's Salary, find how much less salary does Ram get in comparison to Mohan.

Sol. Ram's salary is less than that of Mohan's by $\frac{100 \times 25}{100 + 25} \% = \frac{100 \times 25}{125} \% = 20\%$

Ex.26 The salary of Raman is 25% less than that of Aman's salary. By what percentage Aman salary more that of Raman's?

Sol. Aman's salary will be more than Raman's by $\frac{100 \times 25}{100 - 25} \% = \frac{100 \times 25}{75} \% = \frac{100}{3} = 33\frac{1}{3} \%$

Ex.27 If the price of rice increased by 20%. By what percent the consumption should be decreased by the consumer, if

- (i) the expenditure on rice remains unchanged.
- (ii) the expenditure on rice is increased by 12%.

Sol. Suppose a person purchases 100 kg. of rice for Rs. 100.

When the price increases by 20% he will purchase 100 kg rice for Rs. 120.

- (i) If the expenses remains same i.e. Rs. 100, then

$$\text{the rice purchased for Rs. 100} = \frac{100}{120} \times 100 = \frac{250}{3} \text{ kgs.}$$

$$\therefore \text{The decline in consumption} = 100 - \frac{250}{3} = \frac{50}{3} \text{ kgs.} = 16\frac{2}{3} \%$$

- (ii) If the expenses increases by 12% i.e. Rs. 112.

\therefore Rs. 120 will be spent for purchase of 100 kg. rice

\therefore Rs. 112 will be spent for purchase

$$= \frac{100}{120} \times 112 = \frac{280}{3} \text{ kgs.}$$

\therefore The decline in the consumption

$$= 100 - \frac{280}{3} = \frac{20}{3} \text{ kgs.} = 6\frac{2}{3} \%$$