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DPP: 6
Subject: Mathematics
Topic: Limits

1. $\lim\limits_{x\to 0}(1+ax)^{b/x}$

2. $f(x) = \left(\frac{x}{2+x}\right)^{2x}$, then

(A) $\lim\limits_{x\to\infty} f(x) = -4$ (B) $\lim\limits_{x\to\infty} f(x) = 2$ (C) $\lim\limits_{x\to\infty} f(x) = e^{-4}$ (D) $\lim\limits_{x\to\infty} f(x) = \frac{1}{9}$

3. For $x \in R, \lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^{x} =$

(A) e (B) e^{-1} (C) e^{-5} (D) e^{5}

4. $\lim\limits_{x\to\infty}\left(\frac{x+1}{x+5}\right)^{2x}$

5. $\lim\limits_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$

6. $\lim\limits_{x\to\infty}\left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}}$

7. $\lim\limits_{n\to\infty}\left(1+\sin\frac{a}{n}\right)^{n}$

8. $\lim\limits_{x\to 0}(1+x^2+\sin x)^{3/\tan x}$

9. $\lim\limits_{x\to 1}(1+\sin \pi x)^{\cot \pi x} =$

(A) 1/e (B) 2/e (C) 1 (D) 0

10. $\lim\limits_{x\to 1}(\log_2 2x)^{\log_x 5}$

11. $\lim\limits_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$

12. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}} =$

(A) e (B) e^{2} (C) e^{3} (D) 1

13. $\lim\limits_{x\to\infty}\left\{\sin\left(\frac{1}{x}\right)+\cos\left(\frac{1}{x}\right)\right\}^{x}$

14. $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

15. $\lim\limits_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}} =$

(A) $e^{\tan a}$ (B) $e^{\sin a}$ (C) $e^{\cot a}$ (D) N.O.T

16. $\lim\limits_{x\to 0}\left(\frac{5}{2+\sqrt{9+x}}\right)^{\frac{1}{\sin x}}$

17. $\lim\limits_{x\to 0}\left\{\frac{p^x+q^x+r^x}{3}\right\}^{2/x}$ where p, q, r > 0

18. $\lim\limits_{x\to 0}\left(\frac{1^x+2^x+3^x+\ldots+n^x}{n}\right)^{1/x}$

19. $\lim\limits_{n\to\infty} n^{-n^2}\left[(n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)\ldots\ldots\left(n+\frac{1}{2^{n-1}}\right)\right]^{n}$.

20. Let $f(x) = \frac{\tan x}{x}$, then prove that $\lim\limits_{x\to 0}\left([f(x)] + x^2\right)^{\frac{1}{\{f(x)\}}} = e^3$.

(where [.] and {.} denotes greatest integer and fractional part function respectively)

21. If $\lim\limits_{x\to 1}\left(1 + ax + bx^2\right)^{\frac{c}{x-1}} = e^3$, then find condition on a, b and c.

22. If $\lim\limits_{x\to 0}(\cos x + a\sin bx)^{\frac{1}{x}} = e^2$ then values of a&b can be :

(A) $a = 1, b = 2$ (B) $a = 2, b = 1$ (C) $a = 2\sqrt{2}, b = \frac{1}{\sqrt{2}}$ (D) $a = -2, b = -1$

23. Find the polynomial f(x) of degree 6, which satisfies $\lim\limits_{x\to 0}\left(1 + \frac{f(x)}{x^3}\right)^{\frac{1}{x}} = e^2$

24. $\lim\limits_{n\to\infty}\sin^{2n} x =$

(A) $1 \text{ when } x \neq (2k+1)\frac{\pi}{2}, k \in I$ (B) $0 \text{ when } x \neq (2k+1)\frac{\pi}{2}, k \in I$

(C) $1 \text{ when } x = (2k+1)\frac{\pi}{2}, k \in I$ (D) $0 \text{ when } x = (2k+1)\frac{\pi}{2}, k \in I$

25. $\lim\limits_{n\to\infty}\frac{\left(x^{2n+2} - \cos x\right)}{x^{2n} + 1} =$

(A) $-\cos x, \text{ for } -1 < x < 1$ (B) $\left(\frac{1-\cos 1}{2}\right) \text{ for } x = \pm 1$

(C) $x^2 \text{ for } |x| > 1$ (D) N.O.T

26. Set of all values of x such that $\lim\limits_{n\to\infty}\frac{1}{1 + \left(\frac{4\tan^{-1} 2x}{\pi}\right)^n}$ is non-zero and finite number, where $n \in N$, is

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $(-1, 1)$ (D) $\left[-\frac{1}{2}, 0\right]$

27. Find $\lim\limits_{n\to\infty}\frac{x}{1 + (2\sin x)^{2n}}$

28. $\lim\limits_{m\to\infty}\lim\limits_{n\to\infty}(\cos m!\pi x)^{2n} =$

(A) 1 when $x \in$ irrational (B) 1 when $x \in$ rational

(C) 0 when $x \in$ rational (D) 0when $x \in$ irrational

Answer Key

1. e^{ab}	**2.C**	**3.C**	**4.** e^{-8}	**7.** e^x	**8.** e^3	**9.A**
10. $e^{\log_2 5}$	**11.** $e^{2/\pi}$	**12.B**	**13.** 1	**14.** 1	**15.C**	**16.** $e^{-1/30}$
17. $(pqr)^{2/3}$	**18.** $(n!)^{1/x}$	**19.** e^2	**21.** a + b = 0, bc = 3.		**22. ABCD**	**23.** 1/2
24.BC	**25.ABC**	**26.A**	**27.** x when $\lvert\sin x\rvert < 1/2$; x/2 $\lvert\sin x\rvert = 1/2$; 0 when $\lvert\sin x\rvert \geq 1/2$			
28.BD						