TEST INFORMATION

DATE : 13.05.2015 **PART TEST (PT-05)**

Syllabus : Differential Equation, Complex Number, Permutation & Combination, Probability

DATE : 17.05.2015 **MAJOR TEST (MT)**

Syllabus : Full Syllabus

REVISION DPP OF

CONIC SECTION, TANGENTS & NORMALS

Total Marks : 153 **Max. Time : 113.5 min.**

Single choice Objective (–1 negative marking) Q. 1 to 5	**(3 marks 2.5 min.)**	**[15, 12.5]**
Multiple choice objective (–1 negative marking) Q. 6 to 30	**(4 marks, 3 min.)**	**[100, 75]**
Comprehension (MCQ)(–1 negative marking) Q.31 to 32	**(4 marks 3 min.)**	**[8, 6]**
Comprehension (SCQ) (–1 negative marking) Q.33 to 34	**(3 marks 2.5 min.)**	**[6, 5]**
Single digit type Questions (no negative marking) Q. 35 to 40	**(4 marks 2.5 min.)**	**[24, 15]**

1. Variable ellipses are drawn with $x = -4$ as a directrix and origin as corresponding foci. The locus of extremities of minor axes of these ellipses is

(A) $y^2 = 4x$ (B) $y^2 = 2x$

(C) $y^2 = x$ (D) $x^2 = 4y$

2. An endless inextensible string of length 15m passes around two pins, A & B which are 5m apart. This string is always kept tight and a small ring, R, of negligible dimensions, inserted in this string is made to move in a path keeping all segments RA, AB, RB tight (as mentioned earlier). The ring traces a path, given by conic C, then

(A) Conic C is an ellipse with eccentricity 1/2

(B) Conic C is an hyperbola with eccentricity 2

(C) Conic C is an ellipse with eccentricity 2/3

(D) Conic C is a hyperbola with eccentricity 3/2

3. Let PQ and RS be 2 perpendicular focal chords of a rectangular hyperbola, which are not parallel to its axes, then

(A) PQ = RS (B) $PQ^2 + (RS)^2 = (\text{latus rectum})^2$

(C) PQ + RS = (latus rectum) (D) None of these

4. If a variable line has intercepts of e_1 and e_2 on the co-ordinate axes, where $\frac{e_1}{2}$ & $\frac{e_2}{2}$ are the eccentricities of a hyperbola and its conjugate, then the line always touches a fixed circle $x^2 + y^2 = r^2$, then 'r' is

(A) 1 (B) 2 (C) 3 (D) 4

5. P is a point on the circle $x^2 + y^2 = 1$. Line OP, where O is origin, and x = 1 meet at Q. L_1 is a line parallel to x-axis drawn from Q. A line is drawn parallel to x axis from P meeting x = 1 at R. OR meets L_1 at S. Then locus of S is

(A) Circle (B) Parabola
(C) Ellipse (D) Hyperbola

6. Tangent to the curve $y = x^3 - 3x^2 + 2x + 1$ at $P(\alpha, \beta)$ does not meet the curve at any point other than P. Then identify the correct statement(s)

(A) There is only one such tangent
(B) There are two such tangents
(C) $\alpha + \beta = 2$
(D) Equation of normal at P can be y = x

7. Consider the curve $x^n + y^n = a^n (a > 0)$. Tangent at any arbitrary point $P(x_1, y_1)$ of the curve meets x-axis at A and y-axis at B.$(x_1y_1 \neq 0)$, then

(A) OA + OB = constant $\Rightarrow n = \frac{1}{2}$ (O is origin)

(B) AB = constant $\Rightarrow n = \frac{2}{3}$

(C) Mid-point of AB remain same $\forall (x_1, y_1) \Rightarrow n = 1$

(D) Slope of AB $= -\frac{x_1}{y_1} \Rightarrow n = 2$

8. The curve $y^2 = x^3 + 1$ touches a circle whose centre is (4, 0). Then abscissa of point of contact of these curves can be

(A) –1 (B) –2 (C) 4/3 (D) 1/3

9. Let $P(\alpha, 0)$ & $Q(0, \beta)$ be two-points on x-axis and y-axis respectively. Tangents from P touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $M_1 (x_1, y_1)$ & $M_2 (x_2, y_2)$, similarly tangents from Q to this hyperbola touches it at $M_3 (x_3, y_3)$ and $M_4 (x_4, y_4)$, then (given $\alpha, \beta \neq 0$)

(A) $x_1 = x_2$ & $y_1 + y_2 = 0$ (B) $x_1 + x_2 = 0$, $y_1 = y_2$
(C) $x_3 + x_4 = 0$, $y_3 = y_4$ (D) $x_3 + x_4$, $y_3 + y_4 = 0$

10. Circles are drawn with OA & OB as diameters, where A & B are points of parabola $y^2 = 4x$. These circles meet at P (other than O). m_1 and m_2 are slope of tangents at A & B respectively and m is slope of chord AB, then (given $m_1 + m_2 \neq 0$, A, B are points other than origin and 'O' is origin)

(A) A, P, B are collinear points
(B) m is harmonic mean of m_1 and m_2
(C) m is arithmetic mean of m_1 and m_2
(D) OP is perpendicular to AB

11. Tangents are drawn to hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$. ('b' being parameter) from A(0, 4). The locus of point of contact of these tangent is a conic C, then

(A) Eccentricity of conic C is 1
(B) (0, 3) is focus of C
(C) Eccentricity of conic C is 1/2
(D) (0, 5) is focus of C

12. Major and minor axis of an ellipse are 8 and 6 respectively. Initially it touches positive x and y axis and line joining the two focii is parallel to x-axis. It then rotates in anti-clockwise sense, always touching both the positive co-ordinate axes, and the rotation stops when the line joining their focii is vertical for the first time. C is centre of ellipse and O is origin, then

(A) Locus of C is the complete portion of $x^2 + y^2 = 25$ lying in Ist quadrant
(B) Locus of C is part of the circle $x^2 + y^2 = 100$
(C) Total distance covered by C is $5\tan^{-1}\left(\frac{7}{24}\right)$
(D) Initial and final positions of C lies on the curve xy = 12

13. From centre O, of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, two perpendicular rays are drawn meeting the ellipse at P & Q, N is the foot of perpendicular from O to PQ, then

(A) $\frac{1}{OP^2} + \frac{1}{OQ^2} = \frac{25}{144}$
(B) $\frac{1}{OP^2} - \frac{1}{OQ^2} = \frac{25}{144}$
(C) ON = $\frac{12}{5}$
(D) ON = $\frac{6}{5}$

14. Two distinct tangents are drawn to parabola $y^2 = 4x$ from P(h, k) then (given $h \neq 0$).

(A) If slopes of both the tangents are positive then hk > 0
(B) if h < 0, slopes of the two tangents are of different signs
(C) If product of slopes of tangents is negative and hk > 0, then sum of slopes is positive
(D) If product of slopes of tangents is negative and hk > 0, the sum of slopes is negative

15. Tangents are drawn to the curve y = $\frac{3x+1}{x-2}$ .These tangents meet x = 2 and y = 3 at P & Q respectively if point R is (2, 3) then

(A) Area of triangle PQR is 7 square units

(B) Area of triangle PQR is 14 square units

(C) Locus of circumcentre of triangle PQR is (y – 3)(x –2) = 1

(D) Locus of circumcentre of triangle PQR is (y – 3)(x –2) = 7

16. y = x is tangent to an ellipse whose foci are (1, 0) and (3, 0) then

(A) Major axis of ellipse is = $\sqrt{6}$

(B) Major axis of ellipse is = $\sqrt{10}$

(C) $\left(\frac{3}{4},\frac{3}{4}\right)$ is the point of contact of this ellipse and this tangent

(D) $\left(\frac{1}{2},\frac{1}{2}\right)$ is the point of contact of this ellipse and this tangent

17. Two perpendicular tangents to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}$ = 1 are such that the auxiliary circle intercepts chord of length ℓ_1 & ℓ_2 on these tangents, then

(A) These tangents intersect on the circle $x^2 + y^2 = 49$

(B) These tangents intersect on the circle $x^2 + y^2 = 25$

(C) $\ell_1^2 + \ell_2^2 = 28$

(D) $\ell_1^2 + \ell_2^2 = 25$

18. Focus & vertex of a parabola are A(5, 2) and B(8, 6) respectively. P & Q are two points on the parabola such that the tangents meet at T(11, 10). Then

(A) P & Q are mirror images of each other in the line 4x – 3y = 14

(B) Area of quadrilateral formed by tangent & normal at P & Q is 400 sq. units

(C) Area of quadrilateral formed by tangents & normal at P & Q is 200 sq. units

(D) P & Q are extremities of latus rectum of this parabola

19. Chord joining A(θ_1) & B(θ_2) is reflected by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1, at B. if AB is a focal chord and the reflected ray meets the ellipse again at C(θ_3), then (given $\theta_1, \theta_2 \neq \frac{n\pi}{2}$, $n \in Z$)

(where e equal to eccentricity of ellipse)

(A) $e = \left|\frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}\right|$

(B) $e = \left|\frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right|$

(C) $\tan\frac{\theta_1}{2}\tan\frac{\theta_3}{2} = \cot^2\left(\frac{\theta_2}{2}\right)$

(D) $\tan\frac{\theta_1}{2}\tan\frac{\theta_3}{2} = \tan^2\left(\frac{\theta_2}{2}\right)$

20. Let set S consists of all the points (x, y) satisfying $16x^2 + 25y^2 \le 400$. For points in S let maximum and minimum value of $\frac{y-4}{x-9}$ be M and m respectively, then

(A) M = 1 (B) $M = \frac{65}{7}$ (C) m = 1 (D) $m = \frac{7}{65}$

21. Consider the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where x, y are real variables and a, b, c , f, g, h are real constants. Let $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, and curve S be the locus of point of intersection of perpendicular tangents of the above curve.

(A) If $\Delta \neq 0$ and $h^2 = ab$, then S is a straight line

(B) If $\Delta \neq 0$, h = 0, $a = b \neq 0$ then S is a circle of radius $\sqrt{2\left(g^2 + f^2 - c\right)}$

(C) If $\Delta = 0$, a + b = 0, then S is a point only

(D) IF $\Delta = 0$, a + b = 0 then S is a pair of straight lines.

22. The ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ has a double contact with a circle at the extremity of latus rectum. The point of contact lying in first and fourth quadrant.

(A) Centre of circle is (0, 0) (B) Centre of circle is $\left(\frac{1}{4}, 0\right)$

(C) Radius of circle is $\frac{3\sqrt{5}}{4}$ (D) Radius of circle is $\frac{3\sqrt{5}}{2}$

23. Normal at point $P(x_1, y_1)$, not lying on x-axis, to the hyperbola $x^2 - y^2 = a^2$ meets x-axis at A and y-axis at B. If O is origin then

(A) Circumcentre of triangle OAB is P.

(B) Slope of OP + slope of AB = 0

(C) Slope of OP = slope of AB

(D) Locus of centroid of triangle OAB is a rectangular hyperbola

24. Tangents at $A(a\cos\theta_1, b\sin\theta_1)$ & $B(a\cos\theta_2, b\sin\theta_2)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are perpendicular and their point of intersection is $T(x_1, y_1)$. Normal at A & B meet at point N(h, k). Then

(A) $(a^2 + b^2)\cos^2\left(\frac{\theta_1 - \theta_2}{2}\right) = a^2\cos^2\left(\frac{\theta_1 + \theta_2}{2}\right) + b^2\sin^2\left(\frac{\theta_1 + \theta_2}{2}\right)$

(B) Origin, N and T are vertices of a right angle triangle

(C) $\cos^2\left(\frac{\theta_1 - \theta_2}{2}\right) = \frac{a^2 + b^2}{(a+b)^2}$

(D) Origin, N and T are collinear points

25. A & B two points on the curve $xy = a^2$. Let N be the mid-point of AB. The line through A and B meets. x-axis at P and y-axis at Q, then

(A) N bisects PQ
(B) ON is perpendicular to AB (where O is origin)
(C) AP = BQ
(D) AQ = BP

26. Two parabolas have same focus and same axis, but in opposite directions. Let P & Q be the point of intersection of these two parabolas, then

(A) PQ is latus rectum of at least one of these parabolas.
(B) PQ is a double ordinate of these parabolas
(C) These two parabolas meet orthogonally
(D) Tangent at P(or Q) to these parabola are equally inclined to both the axis

27. Let $A(x_1, y_1)$, $x_1 \neq 0$, be a point of the curve $y^2 = x^3$. Tangent at A meets the curve again at $B(x_2, y_2)$. M and N are foot of perpendicular drawn to x-axis from point A & B respectively. T is the point where tangent at A meets x-axis, then

(A) $y_1y_2 > 0$
(B) $y_1y_2 < 0$
(C) Area if triangle AMT = 8(Area of triangle BNT)
(D) Area of triangle AMT = 64(Area of triangle BNT)

28. Let, S, be a conic whose centre is M(p, q). Locus of middle points of chords of this conic, which passes through a fixed point $N(\alpha, \beta)$ is

(A) Another conic which has a centre
(B) Another conic with same focus
(C) Another conic with centre as $\left(\frac{\alpha+p}{2}, \frac{\beta+q}{2}\right)$
(D) Another conic with centre as $\left(\frac{\alpha-p}{2}, \frac{\beta-q}{2}\right)$

29. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+11)} = 1$, where f(x) is a strictly decreasing positive function, then

(A) the set of values of k for which the major axis of the ellipse is x-axis is (–3, 2)
(B) the set of values of k for which the major axis of the ellipse is y-axis is $(-\infty, 2)$
(C) the set of values of k for which the major axis of the ellipse is y-axis is $(-\infty, -3) \cup (2, \infty)$
(D) the set of values of k for which the major axis of the ellipse is x-axis is $(-3, \infty)$

30. Two concentric ellipses are such that the foci of one lie on the other and the length of their major-axes are equal. If e_1 & e_2 be their eccentricities, then

(A) the quadrilateral formed by joining their foci is a parallelogram
(B) the angle between their axes is given by $\cos\theta = \sqrt{\frac{1}{e_1^2} + \frac{1}{e_2^2} - \frac{1}{e_1^2 e_2^2}}$
(C) their axes are perpendicular if $e_1 = \sqrt{1-e_2^2}$
(D) None of these

Comprehension (Q. 31 to 32)

Consider the circle, S, with equation $x^2 + y^2 + 2gx + 2fy + c = 0$. This circle meets the parabola $y^2 = 4ax$ at $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$. Also let x-intercept of the circle, S, be X_L.

31. Identify the correct identity (identities)

(A) $y_1 + y_2 + y_3 + y_4 = 0$ (B) $x_1 + x_2 + x_3 + x_4 = -(8a + 4g)$

(C) $y_1y_2y_3y_4 = a^2c$ (D) $y_1y_2y_3y_4 = 16a^2c$

32. If A, B, C are co-normal points and $X_L = 2\sqrt{9^2 + f^2 - c}$, then

(A) $x_4 = 0$ (B) $x_1x_2x_3 = 0$

(C) Circle, touches parabola (D) $(y_1 + y_2)(y_2 + y_3)(y_3 + y_1) = 0$

Comprehension : (Q.33 to 34)

Let P, Q, R be three distinct points on the circle $x^2 + y^2 = 25$. L, M, N are points on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. PL, QM, NR are perpendicular to x-axis, with each segment not intersecting the x-axis. Further none of these points lie on coordinate axes and P,Q,R have been so chosen that area of triangle PQR is maximum.

33. Area of triangle LMN is (in square units)

(A) $45\sqrt{3}$ (B) $\frac{75\sqrt{3}}{4}$

(C) $25\sqrt{3}$ (D) $15\sqrt{3}$

34. Normals to the ellipse at L, M and N are

(A) Concurrent at a point.

(B) such that they all pass through origin

(C) sides of an equilateral triangle with non-zero area.

(D) such that two of them are necessarily perpendicular.

35. AB is focal chord of a parabola. Let D and C be foot of perpendicular from A & B on it's directrix respectively. If CD= 6 and area of trapezium ABCD is 24 square units, then find length of chord AB.

36. A circle is drawn whose centre is on x-axis and it touches y-axis. If no part of the circle lies ouside the parabola $y^2 = 8x$, then maximum possible radius of the circle is

37. Let P be a point on the curve $y = \ell n\left(1+\sqrt{1-x^2}\right) - \ell nx - \sqrt{1-x^2}$, Tangent at point P meet y-axis at Q, then find the length of segment PQ.

38. Normal at the point P to the parabola $y^2 = 4x$, intersects the circle with SP as diameter at Q also. If PQ = 2 units (given that point S is focus of the given parabola), then find the abscissa of point P.

39. Parabola, P_1 has focus at S(2, 2) and y-axis is it's directrix. Parabola, P_2 is confocal with P_1 and it's directrix is x-axis. Let $Q(x_1, y_1)$ and $R(x_2, y_2)$ be real points of intersection of parabolas P_1 and P_2.
If the ratio $\frac{RS}{QS} = a + b\sqrt{b}$ find (a + b) (given $x_2 > x_1$ and $a, b \in N$)

40. From point P $\left(-\frac{5}{4}, 2\right)$ variable straight lines are drawn to meet the curve $y = 2\sqrt{x}$ at A & B. Q is a point on this line such that $PA.PB = (PQ)^2$, then locus of point Q is the line ax + y = b, where (a + b) is equal to

ANSWER KEY

DPP # 10

REVISION DPP OF PERMUTATION & COMBINATION AND PROBABILITY

1. (A) **2.** (D) **3.** (D) **4.** (D) **5.** (A) **6.** (C) **7.** (A)
8. (B) **9.** (B) **10.** (B) **11.** (D) **12.** (B) **13.** (C) **14.** (C)
15. (C) **16.** (A) **17.** (A) **18.** (C) **19.** (A,B,C,) **20.** (B,C)
21. (A,B,D) **22.** (B,D) **23.** (B,C,D) **24.** (A,C) **25.** (A,B,C) **26.** (B,C) **27.** (A,B,C)
28. (A,C) **29.** (A,B,C,D) **30.** (B,C,D) **31.** (A,B,C,D) **32.** (A,B)
33. (C,D) **34.** (D) **35.** (A) **36.** (B) **37.** 4 **38.** 5 **39.** 16
40. 175

ANSWER KEY

DPP # 11

REVISION DPP OF CONIC SECTION, TANGENTS & NORMALS

1. (A) **2.** (A) **3.** (A) **4.** (B) **5.** (D) **6.** (A,C,D)
7. (A,B,C,D) **8.** (A,C) **9.** (A,C) **10.** (A,B,D) **11.** (A,B)
12. (C,D) **13.** (A,C) **14.** (A,B,C) **15.** (B,D) **16.** (B,C) **17.** (B,C)
18. (A,C,D) **19.** (A,C) **20.** (A,D) **21.** (A,C) **22.** (B,C) **23.** (A,B,D)
24. (A,D) **25.** (A,C,D) **26.** (B,C) **27.** (B,D) **28.** (A,C) **29.** (A,C)
30. (A,B,C) **31.** (A,B,D) **32.** (A,B,C,D) **33.** (D) **34.** (A)
35. 8 **36.** 4 **37.** 1 **38.** 3 **39.** 5 **40.** 5

! ! BEST OF LUCK FOR JEE-ADVANCED 2015 ! !