3. INVERSE TRIGONOMETRIC FUNCTIONS

SYNOPSIS

I.

Function	Domain	Range
$\sin^{-1}(x)$	$[-1,1]$	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1}(x)$	$[-1,1]$	$[0,\pi]$
$\tan^{-1}(x)$	$(-\infty,\infty)$	$(-\frac{\pi}{2},\frac{\pi}{2})$
$\cot^{-1}(x)$	$(-\infty,\infty)$	$(0,\pi)$
$\sec^{-1}(x)$	$(-\infty,-1]\cup[1,\infty)$	$[0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$
$\operatorname{cosec}^{-1}(x)$	$(-\infty,-1]\cup[1,\infty)$	$[-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$

II. 1) $\sin(\sin^{-1}x) = x$ for $x \in [-1,1]$

$\sin^{-1}(\sin x) = x$ for $x \in [-\frac{\pi}{2},\frac{\pi}{2}]$

2) $\cos(\cos^{-1}x) = x$ for $x \in [-1,1]$ $\cos^{-1}(\cos x) = x$ for $x \in [0,\pi]$

3) $\tan(\tan^{-1}x) = x$ for $x \in (-\infty,\infty)$ $\tan^{-1}(\tan x) = x$ for $x \in (-\frac{\pi}{2},\frac{\pi}{2})$

4) $\cot(\cot^{-1}x) = x$ for $x \in (-\infty,\infty)$ $\cot^{-1}(\cot x) = x$ for $x \in (0,\pi)$

5) $\sec(\sec^{-1}x) = x$ for $x \in (-\infty,-1]\cup[1,\infty)$ $\sec^{-1}(\sec x) = x$ for $x \in [0,\frac{\pi}{2})\cup(\frac{\pi}{2},\pi]$

6) $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x$ for $x \in (-\infty,-1]\cup[1,\infty)$ $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$ for $x \in [-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$

III. 1) $\sin^{-1}(-x) = -\sin^{-1}(x)$ for $x \in [-1,1]$

2) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for $x \in [-1,1]$

3) $\tan^{-1}(-x) = -\tan^{-1}(x)$ for $x \in (-\infty,\infty)$

4) $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ for $x \in (-\infty,\infty)$

5) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ for $x \in (-\infty,-1]\cup[1,\infty)$

6) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$ for $x \in (-\infty,-1]\cup[1,\infty)$

IV. 1) $\sin^{-1}(x) = \operatorname{cosec}^{-1}(1/x)$; for $x \in [-1,0)\cup(0,1]$

2) $\cos^{-1}(x) = \sec^{-1}(1/x)$; for $x \in [-1,0)\cup(0,1]$

3) $\tan^{-1}(x) = \cot^{-1}(1/x)$ for $x \in (0,\infty)$ and $\tan^{-1}(x) = -\pi + \cot^{-1}(1/x)$ for $x \in (-\infty,0)$

4) $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$ for $x \in (0,\infty) = \pi + \tan^{-1}\left(\frac{1}{x}\right)$ for $x \in (-\infty,0)$

V. 1) $\sin^{-1}x + \cos^{-1}(x) = \frac{\pi}{2}$ for $x \in [-1,1]$ 2) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ for $x \in (-\infty,\infty)$

3) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$ for $x \in (-\infty,-1]\cup[1,\infty)$

VI.1) $\sin^{-1}(x) = \cos^{-1}(\sqrt{1 - x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ for $0 \le x \le 1$

2) $\cos^{-1}(x) = \sin^{-1}(\sqrt{1 - x^2}) = \tan^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right)$ for $0 < x \le 1$

3) $\tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{1 + x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$ for $x \ge 0$

VII. If $0 \le x \le 1;\ 0 \le y \le 1$ then

1) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$ for $x^2 + y^2 \le 1$

2) $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$ for $x^2 + y^2 > 1$

3) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

4) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2})$; for $x^2 + y^2 \le 1$

5) $\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}(\sqrt{1-x^2}\cdot\sqrt{1-y^2} - xy)$ for $x^2 + y^2 > 1$

6) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}(xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2})$

VIII. For x>0, y>0

1) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$ $= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$ $= \frac{\pi}{2}$ if $xy = 1$

2) If x and y are of same signs then $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$.

IX. 1) $2\sin^{-1}(x) = \sin^{-1}(2x\sqrt{1-x^2}) = \cos^{-1}(1 - 2x^2)$

2) $2\cos^{-1}(x) = \cos^{-1}(2x^2 - 1) = \sin^{-1}(2x\sqrt{1-x^2})$

3) $2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

X. 1) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ 2) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$ 3) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

XI. 1) $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

2) If $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \frac{\pi}{2}$ then $xy + yz + zx = 1$

3) If $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \pi$ then $x + y + z = xyz$.

XII. 1) $\tan^{-1}\frac{n}{m} + \tan^{-1}\left(\frac{m-n}{m+n}\right) = \pi/4$ or $-3\pi/4$ 2) $\tan^{-1}\frac{m}{n} - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \pi/4$ or $-3\pi/4$

3) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \pi/4$ 4) $\tan^{-1}2 + \tan^{-1}3 = 3\pi/4$

5) $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

6) If $\tan^{-1}x+\tan^{-1}y=\pi/2$ then xy = 1

7) If $\cot^{-1}x+\cot^{-1}y=\pi/2$ then xy = 1

8) If $\sin^{-1}a/x+\sin^{-1}b/x=\pi/2$ or $\cos^{-1}a/x+\cos^{-1}b/x=\pi/2$ then $x=\sqrt{a^2+b^2}$

9) If $\tan^{-1}a/x+\tan^{-1}b/y=\pi/2$ then $x=\sqrt{ab}$

10) If $\cos^{-1}x/a+\cos^{-1}y/b=\theta$ then $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{b^2}{y^2}=\sin^2\theta$

11) If $\sin^{-1}x/a+\sin^{-1}y/b=\theta$, then $\frac{x^2}{a^2}+\frac{2xy}{ab}\cos\theta+\frac{b^2}{y^2}=\sin^2\theta$

12) $\tan^{-1}\frac{1}{1+x(x+1)}+\tan^{-1}\frac{1}{1+(x+1)(x+2)}+.....+\tan^{-1}\frac{1}{1+(x+n-1)(x+n)}=\tan^{-1}(x+n)-\tan^{-1}x$

EXERCISE - I

WORK SHEET – I

1. The value of $\sin^{-1}(\sin(\frac{2\pi}{3}))$ =

1) $\frac{2\pi}{3}$ 2) p/3 3) -p/3 4) $-\frac{2\pi}{3}$

2. The value of $\cos^{-1}(\cos(\frac{7\pi}{6}))$

1) $\frac{7\pi}{6}$ 2) $\frac{\pi}{6}$ 3) $\frac{5\pi}{6}$ 4) $\frac{-\pi}{6}$

3. $\sin^{-1}(\frac{\sqrt{3}}{2})$ - $\tan^{-1}$(- Ö3) =

1) - p/3 2) 2p/3 3) p/6 4) 0

4. $\sec^2(\cot^{-1}$ ½) + $\text{cosec}^2(\tan^{-1}$ 1/3) =

1) 5 2) 10 3) 15 4) 50

5. $\cos[\sin^{-1}(-4/5)-\cos^{-1}(4/5)]$ =

1) p/2 2) - p/2 3) 0 4) p

6. $\sec^{-1}(\frac{2\sqrt{2}}{1+\sqrt{3}})$ =

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{3}$ 3) $\frac{3\pi}{4}$ 4) $\frac{\pi}{6}$

7. $\sin(\cos^{-1}(\frac{3}{5}))$ =

1) 5/4 2) 4/5 3) 3/5 4) 5/3

8. cot ($\sin^{-1}$(63/65)) =

1) 16/63 2) 63/16 3) 11/63 4) 63/11

9. $\sin^{-1}(x) > \cos^{-1}(x)$ holds for

1) every x 2) $x \in (0, \frac{1}{\sqrt{2}})$ 3) $x \in (\frac{1}{\sqrt{2}}, 1]$ 4) $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

10. $\cos^{-1}(\frac{4}{5}) + \cos^{-1}(\frac{63}{65}) =$

1) $\cos^{-1}(\frac{204}{325})$ 2) $\cos^{-1}(\frac{300}{325})$ 3) $\cos^{-1}(\frac{201}{300})$ 4) $\sin^{-1}(\frac{204}{325})$

11. $\sin^{-1}(\frac{4}{5}) - \sin^{-1}(\frac{5}{13}) =$

1) $\sin^{-1}(\frac{56}{65})$ 2) $\sin^{-1}(\frac{63}{65})$ 3) $\sin^{-1}(\frac{77}{85})$ 4) $\sin^{-1}(\frac{33}{65})$

12. $\tan^{-1}(2) + \tan^{-1}(3) =$

1) $-\frac{\pi}{4}$ 2) $\frac{\pi}{4}$ 3) $\frac{3\pi}{4}$ 4) $\frac{5\pi}{4}$

13. $2\tan^{-1}(1/2) + \sin^{-1}(3/5) =$

1) $\tan^{-1}(\frac{12}{25})$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\tan^{-1}(\frac{25}{12})$

14. The value of $\tan^{-1}(\frac{m}{n}) - \tan^{-1}(\frac{m-n}{m+n})$ is

1) p/2 2) p/3 3) p/4 4) 3p/4

15. $\sec[\tan^{-1} 5 + \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{3}{4}] =$

1) $\frac{3}{5}$ 2) $\frac{5}{3}$ 3) $\frac{4}{5}$ 4) $\sqrt{2}$

16. $\cos[2\tan^{-1}\frac{1}{5} + \cos^{-1}\frac{63}{65}] =$

1) $\frac{4}{5}$ 2) $\frac{3}{4}$ 3) $\frac{3}{5}$ 4) $\frac{\sqrt{3}}{2}$

17. The value of $\cot\left(\cos ec^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

1) $\frac{6}{17}$ 2) $\frac{3}{17}$ 3) $\frac{4}{17}$ 4) $\frac{5}{17}$

18. $\sin(2\tan^{-1}(\frac{8}{15})) =$

1) $\frac{230}{289}$ 2) $\frac{240}{289}$ 3) $\frac{120}{249}$ 4) $\frac{120}{289}$

19. **$\cos(2\cos^{-1}(7/25)) =$**

1) $\frac{527}{625}$ 2) $-\frac{527}{625}$ 3) $p-\frac{527}{625}$ 4) $\frac{24}{25}$

20. **$\tan[\frac{1}{2}\cos^{-1}(\frac{\sqrt{5}}{3})] =$**

1) $\frac{3+\sqrt{5}}{2}$ 2) $\frac{3-\sqrt{5}}{2}$ 3) $\frac{4+\sqrt{5}}{2}$ 4) $\frac{4-\sqrt{5}}{2}$

21. **If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then**

1) $xy + yz + zx = 1$ 2) $x^2 + y^2 + z^2 + 2xyz = 1$ 3) $x + y + z = xyz$ 4) $Sx + Syz = 1 + xyz$

22. **If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = p$, then**

1) $x^2 + y^2 + z^2 = 2xyz$ 2) $x^2 + y^2 + z^2 + 2xyz = 0$

3) $x^2 + y^2 + z^2 + 2xyz = 1$ 4) $xy + yz + zx = xyz$

23. **If $\sin^{-1}(a/x) + \sin^{-1}(b/x) = p/2$ then x =**

1) ab 2) $\sqrt{ab}$ 3) $\sqrt{a^2+b^2}$ 4) $a^2 + b^2$

24. **If $\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$ then x =**

1) 3 2) 5 3) 7 4) 11

25. **If $\sin^{-1}(\frac{12}{13}) + \sec^{-1}(\frac{13}{x}) = \frac{\pi}{2}$ then x =**

1) 12 2) 13 3) 11 4) 5

WORK SHEET - II

1. The domain of $\log_e \sin^{-1}(x)$ is

1) (0, 1] 2) (0, 2] 3) (0, ¥) 4)(-¥, 0]

2. The domain of the function $f(x) = \sin^{-1}(\log_2(\frac{x^2}{2}))$ is

1) 1 £ x £ 2 or -2 £ x £ - 1 2) 1 £ x < 3 3) 2 £ x £ 4 4) 1 £ x £ 4

3. Range of $\sin^{-1}x - \cos^{-1} x$ is

1) $[\frac{-3\pi}{2}, \frac{\pi}{2}]$ 2) $[\frac{-5\pi}{3}, \frac{\pi}{3}]$ 3) $[\frac{-3\pi}{2}, p]$ 4) [0, p]

4. Range of $\sin^{-1}x + \cos^{-1} x + \tan^{-1} x$ is

1) $(0, \frac{\pi}{2})$ 2) (0, p] 3) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ 4) [0, p]

5. If $\sin^{-1} x + \sin^{-1}y + \sin^{-1} z = \frac{3\pi}{2}$ then the value of $\sum\left(\frac{x^{201} + y^{201}}{x^{603} + y^{603}}\right) \cdot \left(\frac{x^{402} + y^{402}}{x^{804} + y^{804}}\right)$ =

1) 0 2) 1 3) 2 4) 3

6. If n Î N, $\sum_{k=1}^{n} \sin^{-1}(x_k) = \frac{n\pi}{2}$ then the value of $\sum_{k=1}^{n} x_k$ =

1) n 2) k 3) $\frac{k(k+1)}{2}$ 4) $\frac{n(n+1)}{2}$

7. If n Î N, $\sum_{k=1}^{n} \cos^{-1}(x_k) = n\pi$ then the value of $\sum_{k=1}^{n} \sin^{-1}(x_k)$ = ____

1) 0 2) $\frac{n\pi}{2}$ 3) $-\frac{n\pi}{2}$ 4) – np

8. $\tan^{-1}(\frac{1+ \sin x}{\cos x})$ =

1) $\frac{\pi}{4} - \frac{x}{2}$ 2) $\frac{\pi}{4} - x$ 3) $\frac{\pi}{4} + x$ 4) $\frac{\pi}{4} + \frac{x}{2}$

9. $\text{Tan}^{-1}\left[\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$ =

1) x 2) $\frac{x}{2}$ 3) $\frac{x^2}{2}$ 4) 2x

10. If $\sin^{-1}(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\ldots\ldots) + \cos^{-1}(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\ldots\ldots) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$ then x =

1) 1/2 2) 1 3) - 1/2 4) -1

11. $\sin^{-1}$ (Ö2 sin q) + $\sin^{-1}$ ($\sqrt{\cos 2\theta}$) =

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

12. If u = $\tan^{-1}$ ($\frac{1}{\sqrt{\cos 2\theta}}$) - $\tan^{-1}$ $\sqrt{\cos 2\theta}$, then sin u =

1) $\cot^2 q$ 2) $\sin^2 q$ 3) $\cos^2 q$ 4) $\tan^2 q$

13. sin [$\cot^{-1}${cos($\tan^{-1}$ x)] =

1) $\sqrt{\frac{x^2 - 1}{x^2 + 2}}$ 2) $\sqrt{\frac{x - 2}{x^2 + 1}}$ 3) $\sqrt{\frac{x^2 + 1}{x^2 + 2}}$ 4) $\frac{1}{\sqrt{x^2 - 1}}$

14. 2 tain$^{-1}\frac{1}{5}$ + $\tan^{-1}\frac{1}{7}$ + 2 $\tan^{-1}\frac{1}{8}$ =

1) p 2) p/2 3) p/4 4) 3p/4

15. $\tan^{-1}\frac{5}{6}$ + $\frac{1}{2}$ $\tan^{-1}\frac{11}{60}$ =

1) p 2) p/2 3) p/4 4) 3p/4

16. $\tan^{-1}$ ($\frac{5}{12}$)+$\sin^{-1}$ ($\frac{24}{25}$) = $\cos^{-1}$ (x) implies x=

1) $\frac{-31}{325}$ 2) $\frac{-33}{325}$ 3) $\frac{-36}{325}$ 4) $\frac{-39}{325}$

17. In a Δ ABC, if C is a right angle then $\tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{b}{c+a}\right)$ =

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{5\pi}{2}$

18. tan[$\tan^{-1}$($\frac{1}{a + b}$)+$\tan^{-1}$($\frac{b}{a^2 + ab + 1}$)]=

1) a 2) 1/a 3) b 4) 1/b

19. sin [$\cot^{-1}$ ($\frac{2x}{1- x^2}$) + $\cos^{-1}$ ($\frac{1 - x^2}{1+x^2}$)] =

1) 0 2) 1 3) $\frac{x - y}{1 + xy}$ 4) $\frac{2x}{1 - x^2}$

20. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$, then x =

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{1}{\sqrt{2}}$ 3) 1/3 4)1/2

21. $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \ldots\ldots + \tan^{-1}\left(\frac{1}{n^2+n+1}\right) =$

1) $\tan^{-1}\left(\frac{2n-1}{n+2}\right)$ 2) $\tan^{-1}\left(\frac{2n}{n+5}\right)$ 3) $\tan^{-1}\left(\frac{1}{n+2}\right)$ 4) $\tan^{-1}\left(\frac{n}{n+2}\right)$

22. If $a_1, a_2, a_3 ., a_n$ are in A.P. with common difference d, then $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] =$

1) $\frac{(n-1)\,d}{a_1+a_n}$ 2) $\frac{(n-1)\,d}{1+a_1a_n}$ 3) $\frac{nd}{1+a_1a_n}$ 4) $\frac{a_n-a_1}{a_n+a_1}$

23. $\tan^{-1}\left(\frac{c_1x-y}{c_1y+x}\right) + \tan^{-1}\left(\frac{c_2-c_1}{1+c_2c_1}\right) + \tan^{-1}\left(\frac{c_3-c_2}{1+c_3c_2}\right) + \ldots\ldots + \tan^{-1}\left(\frac{1}{c_n}\right)$

1) $\tan^{-1}\left(\frac{2x}{y}\right)$ 2) $\tan^{-1}(xy)$ 3) $\tan^{-1}(x/y)$ 4) $\tan^{-1}(y/x)$

24. If a,b, c are positive then $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}} =$

1) p 2) $\frac{3\pi}{2}$ 3) $\frac{3\pi}{4}$ 4) 3

25. If $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right) = k\tan^{-1}(x/1)$ then k =

1) 2 2) 3 3) - 2 4) 4

26. If $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) + \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sin^{-1}(k)$ then k =

1) $x\sqrt{1-x^2}$ 2) $2x\sqrt{1-x^2}$ 3) $\sqrt{1-x^2}$ 4) 2x

27. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x =

1) 1/2 2) $\frac{\sqrt{3}}{2}$ 3) - 1/2 4) $-\frac{\sqrt{3}}{2}$

28. If $\sin^{-1}(x) - \cos^{-1}(x) = \sin^{-1}(x-1)$ then x =

1) 0, ½ 2) -1, ½ 3) 1, - ½ 4) ½, 1

29. If $\sin^{-1}(x) + \sin^{-1}(2x) = \frac{\pi}{3}$ then x =

1) $\frac{3}{28}$ 2) $\frac{\sqrt{3}}{28}$ 3) $\sqrt{\frac{3}{28}}$ 4) $\frac{3}{\sqrt{28}}$

30. $\cos^{-1}(x/1) + \cos^{-1}(y/2) = q$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos q + \frac{y^2}{b^2}$ =

1) $\sin^2 q$ 2) $\cos^2 q$ 3) $\tan^2 q$ 4) $\cot^2 q$

31. If $Sec^{-1}\frac{x}{a} - Sec^{-1}\frac{x}{b} = Sec^{-1}b - Sec^{-1}a$ then x =

1) ab 2) $\sqrt{ab}$ 3) $\frac{a+b}{2}$ 4) $\frac{2ab}{a+b}$

32. If $\tan^{-1}(a/x) + \tan^{-1}(b/x) = p/2$ then x =

1) ab 2) $\sqrt{ab}$ 3) $\sqrt{a^2+b^2}$ 4) $a^2 + b^2$

33. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, then x =

1) $\frac{1}{\sqrt{2}}$ 2) $\pm\frac{1}{\sqrt{2}}$ 3) $\pm\frac{1}{\sqrt{3}}$ 4) $\frac{1}{\sqrt{3}}$

34. If $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = p + \tan^{-1}(-7)$; then x =

1) 2 2) -2 3) 1 4) no solution

35. If $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$ then x =

1) 1 2) 1/2 3) -1/2 4) 1/4

36. If $\tan^{-1}\left(\frac{2x}{x^2+1}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \frac{2\pi}{3}$ then x =

1) 2 - Ö3 2) Ö3 - Ö2 3) 2 + Ö3 4) Ö3 + Ö2

37. If $\cot^{-1}\left(\frac{1+x}{1-x}\right) = \frac{1}{2}\cot^{-1}\left(\frac{1}{x}\right)$ then x =

1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{6}}$

38. If $\cot\left(\sin^{-1}\sqrt{\frac{13}{17}}\right) = \sin(\tan^{-1} x)$ then x =

1) $\frac{4}{17}$ 2) $\frac{2}{3}$ 3) $\frac{3}{2}$ 4) $\frac{17}{4}$

39. **The value of x where x > 0 and** $\tan\left(\sec^{-1}\frac{1}{x}\right)=\sin\left(\tan^{-1}2\right)$ **is**

1) $\sqrt{5}$ 2) $\frac{\sqrt{5}}{3}$ 3) 1 4) $\frac{2}{3}$

40. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=$

1) b/a 2) a/b 3) 2a/b 4) 2b/a

41. **If** $Cos^{-1}\frac{x}{a}+Cos^{-1}\frac{y}{b}=\frac{5\pi}{12}$ **and** $\mathrm{Sin}^{-1}\frac{x}{a}-\mathrm{Sin}^{-1}\frac{y}{b}=\frac{\pi}{12}$ **then** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=$

1) 1 2) 1/4 3) 3/4 4) 5/4

42. **If** x_1, x_2, x_3 **are the roots of** $x^3-6x^2+11x-6=0$ **then** $\cot^{-1}(x_1)+\cot^{-1}(x_2)+\cot^{-1}(x_3)$ **is equal to**

1) 0 2) $\frac{\pi}{2}$ 3) p 4) $\frac{3\pi}{2}$

43. **If** $\mathrm{Tan}^{-1}x+\mathrm{Tan}^{-1}y+\mathrm{Tan}^{-1}z=\frac{\pi}{2}$ **and** $(x-y)^2+(y-z)^2+(z-x)^2=0$ **then** $x^2+y^2+z^2=$

1) 0 2) 4 3) 1 4) None

44. **The value of** $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10}-\sin\frac{9\pi}{10}\right)\right\}$ **is**

1) $\frac{3\pi}{20}$ 2) $\frac{7\pi}{10}$ 3) $\frac{7\pi}{20}$ 4) $\frac{17\pi}{20}$

45. **If** $p<q<r<0$ **then** $\sum\mathrm{Cot}^{-1}\left(\frac{pq+1}{p-q}\right)=$

1) 0 2) π 3) 2π 4) $\frac{\pi}{2}$

46. **If** $\mathrm{Tan}^{-1}\frac{1}{1+2x}+\mathrm{Tan}^{-1}\frac{1}{4x+1}=\mathrm{Tan}^{-1}\frac{2}{x^2}$ **then x =**

1) 1 2) 0 3) - 3 4) 2/3

47. **If** $\mathrm{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)+\mathrm{Cot}^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$, **x > 0 then x =**

1) $2+\sqrt{3}$ 2) $2-\sqrt{3}$ 3) $3-\sqrt{2}$ 4) $3+\sqrt{2}$

48. **If** $\frac{1}{2}\le x\le 1$ **then** $\mathrm{Cos}^{-1}x+\mathrm{Cos}^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right)=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$

49. If $\text{Sin}^{-1}x + \text{Tan}^{-1}x = \frac{\pi}{2}$, then the value of x^2 is

1) $\sin 18^0$ 2) $\cos 36^0$ 3) $2\sin 18^0$ 4) $2\cos 36^0$

50. $2\,C\text{os}^{-1}x = S\text{in}^{-1}\left(2x\sqrt{1-x^2}\right)$ is valid for

1) $-1 \le x \le 1$ 2) $0 \le x \le 1$ 3) $0 \le x \le \frac{1}{\sqrt{2}}$ 4) $\frac{1}{\sqrt{2}} \le x \le 1$

51. The ascending order of minimum values of the functions

P : $\sin^{-1}x - \cos^{-1}x$ **Q : $\tan^{-1}x + \cot^{-1}x$** **R : $\sec^{-1}x - \text{cosec}^{-1}x$ is**

1) P, Q, R 2) P, R, Q 3) Q, P, R 4) Q, R, P

52. Match the Following

List-I	List-II
1) $\sin^{-1}x = \cos^{-1}x$ then x	1) 0
2) $\sin^{-1}x = \tan^{-1}x$ then x	2) $\sqrt{\frac{\sqrt{5}-1}{2}}$
3) $\cos^{-1}x = \tan^{-1}x$ then x	3) $\frac{1}{\sqrt{2}}$
4) $\sin^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$ then x	4) no value

	A	B	C	D
1)	3	2	1	4
3)	3	1	2	4

	A	B	C	D
2)	3	1	4	2
4)	3	2	4	1

53. Match the Following

List-I	List-II
1) $\sin^{-1}x - \cos^{-1}x$ is maximum at x =	1) $\frac{1}{\sqrt{2}}$
2) $\cos^{-1}x$ is maximum at x =	2) 0
3) $2\sin^{-1}x + 3\cos^{-1}x$ is minimum at x =	3) - 1
4) $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is minimum at x =	4) 1

	A	B	C	D
1)	3	2	1	4
3)	3	1	2	4

	A	B	C	D
2)	3	1	4	2
4)	3	2	4	1

WORK SHEET - III

1. **The value of $\sin^{-1}$ (sin 10) is**

1) 10 2) 10 - 3p 3) 3p - 10 4) 3p + 10

2. $\mathrm{Cos}^{-1}(\cos 12) - \mathrm{Sin}^{-1}(\sin 12) =$

1) 0 2) π 3) $8\pi + 24$ 4) $8\pi - 24$

3. **If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = p$, then** $x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} =$

1) 0 2) 1 3) xyz 4) 2xyz

4. **If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$ then $x^4+y^4+z^4+4x^2y^2z^2 =$**

1) $x^2y^2+y^2z^2+z^2x^2$ 2) $2x^2y^2z^2$ 3) 2xyz 4) $2(x^2y^2+y^2z^2+z^2x^2)$

5. **If** $\alpha = Tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ **then sin2α =**

1) x 2) x^3 3) x^2 4) x^4

6. **If** $\cos^{-1}\left[\dfrac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right] = 2Tan^{-1}x$ **then x =**

1) $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}$ 2) $\tan\alpha\tan\beta$ 3) $\cot\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}$ 4) $\cot\alpha\cot\beta$

7. **The number of positive integral solutions of the equation $\tan^{-1} x + \cot^{-1} y = \tan^{-1} 3$ is**

1) 0 2) 1 3) 2 4) 3

8. $2\mathrm{Tan}^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right] =$

1) $\mathrm{Cos}^{-1}\left(\dfrac{a\cos\theta+b}{a+b\cos\theta}\right)$ 2) $\mathrm{Cos}^{-1}\left(\dfrac{a\cos\theta-b}{a-b\cos\theta}\right)$ 3) $\mathrm{Cos}^{-1}\left(\dfrac{a+b\cos\theta}{a\cos\theta+b}\right)$ 4) $\mathrm{Cos}^{-1}\left(\dfrac{a-b\cos\theta}{a\cos\theta-b}\right)$

9. **If** $\left(Tan^{-1}x\right)^2 + \left(Cot^{-1}x\right)^2 = \dfrac{5\pi^2}{8}$ **then x =**

1) $\sqrt{3}$ 2) 1 3) $\dfrac{1}{\sqrt{3}}$ 4) – 1

10. $\sum_{r=1}^{\infty} \mathrm{Tan}^{-1}\left(\dfrac{1}{2r^2}\right) =$

1) 0 2) $\dfrac{\pi}{4}$ 3) $\dfrac{\pi}{3}$ 4) $\dfrac{\pi}{2}$

11. $\sum_{m=1}^{\infty} Tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$

1) $-\frac{\pi}{4}$ 2) $\frac{4}{\pi}$ 3) $\frac{\pi}{4}$ 4) $-\frac{4}{\pi}$

12. $\text{Sin}^{-1}\left(\frac{1}{\sqrt{2}}\right)+\text{Sin}^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)+\ldots\ldots\ldots\ldots+\text{Sin}^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)+\ldots\ldots\infty$ =

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{2}$

13. If $x \neq n$ and $\cot^{-1}x + \cot^{-1}(n^2 - x + 1) = \cot^{-1}(n - 1)$ then x =

1) n - 1 2) $n^2 + n - 1$ 3) $n^2 - n + 1$ 4) $n^2 + n + 1$

14. The number of solutions of the equation $\text{Tan}^{-1}\sqrt{x^2+x} + \text{Cosec}^{-1}\sqrt{1-x^2-x} = \frac{\pi}{2}$ is

1) 1 2) 2 3) 3 4) 4

15. The number of real solutions of $\tan^{-1}(\sqrt{x(x+1)} + \sin^{-1}\sqrt{(x^2+x+1)} = \frac{\pi}{2}$ is

1) 0 2) 1 3) 2 4) infinite

16. $3\text{Tan}^{-1}\frac{1}{2}+2\text{Tan}^{-1}\frac{1}{5}+\text{Sin}^{-1}\frac{142}{65\sqrt{5}}=$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) 0

17. If A = $\text{Tan}^{-1}\left(\frac{1}{7}\right)$, B = $\text{Cot}^{-1}(3)$ then

1) cos2A= sin4B 2) cos2A = sin2B 3) 2cos2A = sin 4B 4) cos2A = 2sin2B

18. $\text{Cot}^{-1}(\sqrt{\text{Cos}\alpha}) - \text{Tan}^{-1}(\sqrt{\text{Cos}\alpha}) = x$ then Sin x=

1) $Tan^2\frac{\alpha}{2}$ 2) $Cot^2\frac{\alpha}{2}$ 3) $Tan\,\alpha$ 4) $Cot\frac{\alpha}{2}$

19. Let $y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$ then

1) min $y = \frac{\pi^3}{8}$ 2) min $y = \frac{\pi^3}{32}$ 3) max $y = \frac{\pi^3}{8}$ 4) max $y = \frac{7\pi^3}{32}$

20. The value of x for which $\cos^{-1}(\cos 4) > 3x^2 - 4x$ is

1) $\left(0, \frac{2+\sqrt{6\pi-8}}{3}\right)$ 2) $\left(\frac{2+\sqrt{6\pi-8}}{3}, 0\right)$

3) (-2,2) 4) $\left(\frac{2-\sqrt{6\pi-8}}{3}, \frac{2+\sqrt{6\pi-8}}{3}\right)$

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WORK SHEET - IV

1. $\sin^{-1}x > \cos^{-1}x$ **holds for**

1) all values of x 2) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$ 3) $x \in \left(\frac{1}{\sqrt{2}}, 1\right]$ 4) $x = 1.75$

2. **If** $\alpha \le \sin^{-1}x + \cos^{-1}x + \tan^{-1}x \le \beta,$ **then**

1) $\alpha = \frac{\pi}{4}, \beta = \frac{3\pi}{4}$ 2) $\alpha = 0,\ \beta = \pi/2$ 3) $\alpha = 0, \beta = \pi$ 4) $\alpha = 0,\ \beta = \frac{3\pi}{4}$

3. **The greatest and least values of** $\left(\sin^{-1}x\right)^3 + \left(\cos^{-1}x\right)^3$ **are**

1) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ 2) $-\frac{\pi^3}{8}, \frac{7\pi^3}{8}$ 3) $\frac{7\pi^3}{8}, \frac{\pi^3}{32}$ 4) $\frac{\pi}{2}, \frac{\pi^3}{32}$

4. **If a,b,c are positive then the value of**

$\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ **is equal to**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ c) π 4) 0

5. **The equation** $\sin\left(\frac{1}{5}\cos^{-1}x\right) = 1$

1) has no solution 2) has infinite solution
3) is defined if $-1 \le x \le 1$ 4) is defined if $0 \le x \le \pi$

6. **Let** $f(x) = \cos ec^{-1}\left[1 + \sin^2 x\right]$**, where [.] denotes the greatest integer function. Then** $f(x)$ **equals;**

1) $\left\{\frac{\pi}{2}\right\}$ 2) $\left\{\frac{\pi}{2}, \cos ec^{-1}2\right\}$ 3) $\left\{\cos ec^{-1}2\right\}$ 4) none of these

7. **The value of:** $\tan\left\{\sin^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right\}.\tan\left\{\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right)\right\}; x \in (0, \pi/2)$ **is equal to;**

1) 0 2) 1 3) -1 4) none of these

8. **If** $\sin^{-1}(\sin x) = \pi - x, then x$ **belongs to:**

1) R 2) $[0, \pi]$ 3) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ 4) $[\pi, 2\pi]$

9. **If** $f(x) = \sin^{-1}\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right\}, -\frac{1}{2} \le x \le 1,$ **then f(x) is equal to:**

1) $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(x)$ 2) $\sin^{-1}x - \frac{\pi}{6}$ 3) $\sin^{-1}x + \frac{\pi}{6}$ 4) none of these

10. $\sin^{-1}(\sin 2) + \cos^{-1}(\cos 2) =$

1) $\pi/2$ 2) π 3) 4 4) $4 - \pi$

11. **The solution of the equation** $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$ **are**

1) ± 1 2) $1\pm\sqrt{2}$ 3) $-1\pm\sqrt{2}$ 4) none of these

12. **The value (s) of x satisfying the equation** $\sin^{-1}|\sin x|=\sqrt{\sin^{-1}|\sin x|}$ **is /are given by (n is any integer)**

1) $n\pi-1$ 2) $n\pi$ 3) $n\pi+1$ 4) $2n\pi+1$

13. **If** $\cos^{-1}x=\tan^{-1}x,$ **then**

1) $x^2=\left(\sqrt{5}-1\right)/2$ 2) $x^2=\left(\sqrt{5}+1\right)/2$

3) $\sin\left(\cos^{-1}x\right)=\left(\sqrt{5}-1\right)/2$ 4) $\tan\left(\cos^{-1}x\right)=\left(\sqrt{5}-1\right)/2$

14. **If** $\left(\sin^{-1}x+\sin^{-1}w\right)\left(\sin^{-1}y+\sin^{-1}z\right)=\pi^2, then\ D=\begin{vmatrix}x^{N_1} & y^{N_2}\\ z^{N_3} & w^{N_4}\end{vmatrix}\left(N_1,N_2,N_3,N_4\in N\right)$

1) has a maximum value of 2 2) has a minimum value of 0

3) 16 different D are possible 4) has a minimum value of -2

15. **If** $f\left(x\right)=\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2$, **then**

1) $f\left(x\right)$ has the least value of $\frac{\pi^2}{8}$ 2) $f\left(x\right)$ has the greatest value of $\frac{5\pi^2}{8}$

3) $f\left(x\right)$ has the least value of $\frac{\pi^2}{16}$ 4) $f\left(x\right)$ has the greatest value of $\frac{5\pi^2}{4}$

16. **The value of** $k\left(k>0\right)$ **such that the length of the longest interval in which the function** $f\left(x\right)=\sin^{-1}|\sin kx|+\cos^{-1}\left(\cos kx\right)$ **is constant is** $\pi/4$ **is/are**

1) 8 2) 4 3) 12 4) 16

Passage - I

Let $f:A\to B$ **be a function defined by y = f(x) such that** f **is both one-one (Injective) and onto (surjective) (ie, bijective,) then there exists a unique function** $g:B\to A$ **such that** $f(x)=y\Leftrightarrow g(y)=x,\forall x\in A$ **and** $y\in B,$ **then g is said to be inverse of f. Thus.** $g=f^{-1}:B\to A=\left[\left\{f\left(x\right),x\right\}:\left\{x,f\left(x\right)\right\}\in f^{-1}\right]$. **If no branch of an inverse trigonometric function is mentioned, then it means the principal value branch of that function.**

17. **The value of** $\cos\left(\tan^{-1}\tan 4\right)$ **is**

1) $\frac{1}{\sqrt{17}}$ 2) $-\frac{1}{\sqrt{17}}$ 3) cos 4 4) - cos 4

18. **If x takes negative permissible value, then** $\sin^{-1}x$ **is equal to**

1) $\cos^{-1}\sqrt{\left(1-x^2\right)}$ 2) $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ 3) $\pi-\cos^{-1}\sqrt{\left(1-x^2\right)}$ 4) $-\pi+\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

19. **If** $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$, **then** $\sin^{-1}(\sin x)$ **is equal to**

1) x 2) $-x$ 3) $2\pi - x$ 4) $x - 2\pi$

Passage - II :

For $x < 0$	**For** $x \ge 0$
$-\frac{\pi}{2} \le \sin^{-1} x < 0$	$0 \le \sin^{-1} x \le \frac{\pi}{2}$
$\frac{\pi}{2} < \cos^{-1} x \le \pi$	$0 \le \cos^{-1} x \le \frac{\pi}{2}$
$-\frac{\pi}{2} < \tan^{-1} x < 0$	$0 \le \tan^{-1} x < \frac{\pi}{2}$
$\frac{\pi}{2} < \cot^{-1} x < \pi$	$0 < \cot^{-1} x \le \frac{\pi}{2}$
$\frac{\pi}{2} < \sec^{-1} x \le \pi$	$0 < \sec^{-1} x < \frac{\pi}{2}$
$-\frac{\pi}{2} \le \operatorname{cosec}^{-1} x < 0$	$0 \le \operatorname{cosec}^{-1} x \le \frac{\pi}{2}$

Ex. $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right] = \frac{\pi}{3} not \frac{2\pi}{3}, \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3} not \frac{2\pi}{3}$

20. **The principal value of** $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) + \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ **is**

1) $\frac{8\pi}{3}$ 2) $\frac{4\pi}{3}$ 3) $\frac{2\pi}{3}$ 4) $\frac{\pi}{3}$

21. **The principal value of** $\sin^{-1}(\sin 5) - \cos^{-1}(\cos 5)$ **is**

1) 0 2) $2\pi - 10$ 3) $10 - 4\pi$ 4) $3\pi - 10$

22. **The principal value of** $\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) + \cot^{-1}\cot\left(-\frac{3\pi}{4}\right)$ **is**

1) $\frac{\pi}{2}$ 2) π 3) $\frac{-3\pi}{2}$ 4) 0

23.

Column -I	**Column - II**
1) $\sin\left(\tan^{-1} x\right)$	p) $\sqrt{1-x^2}$
2) $\cos\left(\tan^{-1} x\right)$	q) $\frac{x}{\sqrt{1+x^2}}$
3) $\cos\left(\sin^{-1} x\right)$	r) $\frac{1}{\sqrt{1+x^2}}$
4) $\cos\left(2\cos^{-1} x\right)$	s) $2x^2 - 1$

24.

Column -I	**Column - II**
1) $\tan^{-1}3+\tan^{-1}4$	p) $\pi/2$
2) $\tan^{-1}(1/3)+\tan^{-1}\left(\frac{1}{4}\right)$	q) $\pi-\tan^{-1}(7/11)$
3) $\sin^{-1}(1/3)+\cos^{-1}(1/3)$	r) $\tan^{-1}(7/11)$
4) $\tan^{-1}(3)+\cot^{-1}(4)$	s) $\tan^{-1}(13)$

Integer type Questions :

25. **If** $\cos^{-1}x_1+\cos^{-1}x_2+\cos^{-1}x_3=0$ *then* $\sum x_1+\sum x_1x_2+x_1x_2x_3$

26. **If** $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi/4$ **then** $\sum x+\sum yz-xyz=$

27. **The no.of solutions of** $\sin^{-1}(\sin x)=[x]$ *where* $[x]$ **denotes greatest inter function is**

28. **The no.of solutions of** $\cos^{-1}(\cos x)=[x]$ **where [x] denotes greatest integers function is**

29. **If** $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+....\infty=\frac{2\pi}{k}$ *then* $k=$

30. **If** $\sum_{n=1}^{3}\tan^{-1}(n)=k\sum_{x=1}^{3}(\cot^{-1}x)$ *then* $k=$

WORK SHEET -V

1. **The minimum value of** $\left(\sec^{-1}x\right)^2+\left(\operatorname{cosec}^{-1}x\right)^2$.

1) $\frac{\pi^2}{8}$ 2) $\frac{\pi^2}{4}$ 3) $\frac{3\pi^2}{8}$ 4) $\frac{5\pi^2}{4}$

2. **The value of** $\sin^{-1}\left(\cos\left(\cos^{-1}\left(\cos x\right)+\sin^{-1}\left(\sin x\right)\right)\right) where\ x\in\left(\frac{\pi}{2},\pi\right)$ **equal to**

1) $\pi/2$ 2) $-\pi$ 3) π 4) $-\pi/2$

3. **The value of 'a', for which** $ax^2+\sin^{-1}\left(x^2-2x+2\right)+\cos^{-1}\left(x^2-2x+2\right)=0$ **has a real solution, is**

1) $\pi/2$ 2) $-\pi/2$ 3) $2/\pi$ 4) $-2/\pi$

4. **Range of** $f\left(x\right)=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$ **is**

1) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ 2) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ 3) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ 4) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

5. **If** $\left[\cot^{-1}x\right]+\left[\cos^{-1}x\right]=0$**, where [.] denotes the greatest integer function, then the complete set of value of x is**

1) $\left(\cos 1,1\right]$ 2) $\left(\cos 1,\cot 1\right)$ 3) $\left(\cot 1,1\right]$ 4) none of these

6. **Comlete solution set of** $\left[\cot^{-1}x\right]+2\left[\tan^{-1}x\right]=0$**, where [.] denotes the greatest integer function. is equal to**

1) $\left(0,\cot 1\right)$ 2) $\left(0,\tan 1\right)$ 3) $\left(\tan 1,\infty\right)$ 4) $\left(\cot 1,\tan 1\right)$

7. The number of integer values of *k* **for which the equation** $\sin^{-1}x+\tan^{-1}x=2k+1$ **has a solution is**

1) 1 2) 2 3) 3 4) 4

8. $\sin^{1}\left(\sin 10\right)+\cos^{-1}\left(\cos 10\right)+\tan^{-1}\left(\tan 10\right)=$

1) $4\pi-10$ 2) $10\pi-30$ 3) $3\pi-10$ 4) $7\pi-30$

9. $\sin^{1}\left(\sin 2011^{o}\right)+\cos^{-1}\left(\cos 2011^{o}\right)+\tan^{-1}\left(\tan 2011^{o}\right)=$

1) 149^{o} 2) 62^{o} 3) 31^{o} 4) 93^{o}

10. $\tan^{-1}\left(\tan\left(-6\right)\right)=$

1) $2\pi-6$ 2) $2\pi+6$ 3) $\pi-6$ 4) -6

11. **If** $f(x)=\cos^{-1}x+\cos^{-1}\left\{\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right\}$ **then**

1) $f\left(\frac{2}{3}\right)=\frac{\pi}{3}$ 2) $f\left(\frac{2}{3}\right)=2\cos^{-1}\frac{2}{3}-\frac{\pi}{3}$

3) $f\left(\frac{1}{3}\right)=\frac{\pi}{3}$ 4) $f\left(\frac{1}{3}\right)=2\cos^{-1}\frac{1}{3}-\frac{\pi}{3}$

12. **The value of** $\tan\left\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2}+\frac{1}{2}\cos^{-1}\frac{1-x^2}{1+x^2}\right\}$ **is**

1) $\frac{2x}{1-x^2}$ *if* $0\le x\le 1$ 2) $\frac{2x}{1-x^2}$ *if* $x<1$ 3) not finite if x > 1 4) none of these

13. **An integral solution** (x,y) **of the equation** $\tan^{-1}x+\tan^{-1}(1/y)=\tan^{-1}3$ **is**

1) (1,2) 2) (5,-8) 3) (8,-5) 4) (-2,1)

14. $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\pi/2$ **if** x **is equal to**

1) -1/12 2) 1/6 3) 1/12 4) -1/6

15. **If** $A=\tan^{-1}(1/7)$ **and** $B=\tan^{-1}(1/3)$, **then**

1) $\cos 2A=24/25$ 2) $\cos 2B=4/5$ 3) $\cos 2A=\sin 4B$ 4) $\tan 2B=3/4$

16. **With respect to principal values the relation** $\cos^{-1}(4x^3-3x)=3\cos^{-1}x$ **is**

1) true for all in x in $[-1,1]$ 2) true for all x in $[-\frac{1}{2},\frac{1}{2}]$

3) true for all x in $\left[\frac{1}{2},1\right]$ 4) not true at x = 0

Passage - I :

Let $\alpha=\cos^{-1}(4/5),\beta=\tan^{-1}(2/3),0<\alpha,\beta<\pi/2$

17. $\alpha+\beta$ **is equal to**

1) $\tan^{-1}(17/6)$ 2) $\sin^{-1}\left(17/5\sqrt{13}\right)$ 3) $\sin^{-1}(3/5)$ 4) $\cos^{-1}\left(3/\sqrt{13}\right)$

18. $\alpha-\beta$ **is equal to**

1) $\cos^{-1}\left(18/5\sqrt{13}\right)$ 2) $\sin^{-1}\left(2/\sqrt{13}\right)$ 3) $\tan^{-1}(1/18)$ 4) $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$

19. $\cos^{-1}(44/125)$ **is equal to**

1) 2α 2) 3α 3) $\pi-3\alpha$ 4) $\pi-2\alpha$

Passage - II :

$f(x)=\sin\left\{\cot^{-1}(x+1)\right\}-\cos\left(\tan^{-1}x\right)$

$a=\cos\tan^{-1}\sin\cot^{-1}x$

$b=\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$

20. **The value of x for which $f(x)=0$ is**

1) - 1/2 2) 0 3) 1/2 4) 1

21. **If $f(x)=0$ then a^2 is equal to**

1) 1/2 2) 2/3 3) 5/9 4) 9/5

22. **If $a^2=26/51$, then b^2 is equal to**

1) 1/25 2) 24/25 3) 25/26 4) 50/51

23.

Column -I Number of real solutions of	**Column - II**
1) $\tan\left(\pi/4+(1/2)\cos^{-1}x\right)+\tan\left(\pi/4-(1/2)\cos^{-1}x\right)=1$	p) 0
2) $\tan^{-1}\dfrac{1}{2x+1}+\tan^{-1}\dfrac{1}{4x+1}=\tan^{-1}\left(2/x^2\right)$	q) 2
3) $\tan^{-1}\left(x+(2/x)\right)-\tan^{-1}(4/x)-\tan^{-1}\left(x-(2/x)\right)=0$	r) 3
4) $\tan^{-1}(1-x)+\tan^{-1}(1+x)=\tan^{-1}2x$	s) 1

24.

Column -I **Let (x,y) be such that $\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\pi/2$**	**Column - II**
1) If a = 1, b = 0, then (x,y) $x^2+y^2=1$	p) lies on the circle
2) If a = 1, b = 1, then $(x,y)\left(y^2-1\right)=0$	q) lies on $\left(x^2-1\right)$
3) If a = 1, b = 2, then (x,y)	r) lies on $y=x$
4) If a = 2, b = 2, then $(x,y)\left(y^2-1\right)=0$	s) lies on $\left(4x^2-1\right)$

EXERCISE - I / ANSWERS

WORK SHEET -I

1) 2 2) 3 3) 2 4) 3 5) 3 6) 1 7) 2 8) 1 9) 3 10) 1
11) 4 12) 3 13) 3 14) 3 15) 2 16) 1 17) 1 18) 2 19) 2 20) 2
21) 1 22) 3 23) 3 24) 2 25) 1

WORK SHEET - II

1) 1 2) 1 3) 1 4) 3 5) 4 6) 1 7) 3 8) 4 9) 2 10) 2
11) 4 12) 4 13) 3 14) 3 15) 3 16) 3 17) 2 18) 2 19) 2 20) 1
21) 4 22) 2 23) 3 24) 1 25) 2 26) 2 27) 2 28) 1 29) 3 30) 1
31) 1 32) 2 33) 2 34) 1 35) 4 36) 1 37) 2 38) 2 39) 2 40) 4
41) 4 42) 2 43) 3 44) 4 45) 3 46) 2 47) 2 48) 2 49) 3 50) 4
51) 2 52) 3 53) 4

WORK SHEET - III

1) 3 2) 4 3) 4 4) 4 5) 3 6) 1 7) 3 8) 1 9) 4 10) 2
11) 3 12) 2 13) 3 14) 2 15) 3 16) 3 17) 1 18) 1 19) 2 20) 4

WORK SHEET - IV

1) 3 2) 1 3) 3 4) 3 5) 1 6) 2 7) 2 8) 3 9) 2 10) 4
11) 1,2,3 12) 1,2,3 13) 1,3 14) 1,3,4 15) 1,4 16) 2 17) 4 18) 4 19) 4 20) 4
21) 3 22) 1 23) $A \to q; B \to r; C \to p; D \to s$ 4) $A \to q; B \to r; C \to p; D \to s$
25) 7 26) 1 27) 3 28) 4 29) 8 30) 2

WORK SHEET - V

1) 1 2) 4 3) 4 4) 3 5) 3 6) 4 7) 2 8) 1 9) 1 10) 1
11) 1,4 12) 1,3 13) 1,2,3 14) 1 15) 1,2,3,4 16) 3,4 17) 1,2 18) 1,3 19) 3
20) 1 21) 3 22) 4 23) $A \to p; B \to r; C \to q; D \to s$
24) $A \to p; B \to q; C \to p; D \to s$

EXERCISE - II

WORK SHEET (HW) - I

(Single & One or More than One Correct Answers)

1. If $x > \frac{1}{\sqrt{2}} then 2\sin^{-1}x + \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ is ___

1) π 2) $-\pi$ 3) 0 4) $\frac{\pi}{2}$

2. If $x < -1 \; then \; 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

1) π 2) $-\pi$ 3) 0 4) $\frac{\pi}{2}$

3. If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) then \; x \in$

1) $-1 < x < 1$ 2) $x > 1$ 3) $x < -1$ 4) R

4. The range of $\sec^{-1}x + \cos ec^{-1}x + \sin^{-1}x$ is _____

1) $\left\{\frac{\pi}{2}\right\}$ 2) $\{0, \pi\}$ 3) $[0, \pi]$ 4) $(0, \pi)$

5. The range of $cot^{-1}x + \tan^{-1}x + \cos ec^{-1}x$ is _____

1) $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ 2) $[0, \pi]$ 3) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ 4) $\left(0, \frac{\pi}{2}\right] \cup \left(\frac{\pi}{2}, \pi\right]$

6. $\sum_{x=0}^{4} \cos^{-1}(\cos x) =$

1) $2 + 2\pi$ 2) 9 3) $2\pi - 2$ 4) $2\pi + 4$

7. $\sum_{x=0}^{4} \sin^{-1}(\sin x) =$

1) $3\pi - 8$ 2) $3\pi - 7$ 3) $3\pi - 9$ 4) $3\pi - 6$

8. $\sum_{x=-1}^{4} \tan^{-1}(\tan x) =$

1) $7 - 3\pi$ 2) $6 - 3\pi$ 3) $5 - 3\pi$ 4) $9 - 3\pi$

9. $\sin\left[\frac{1}{2}cot^{-1}\left(\frac{-3}{4}\right)\right] =$ ____

1) $\frac{2}{\sqrt{5}}$ 2) $-\frac{2}{\sqrt{5}}$ 3) $\frac{3}{\sqrt{5}}$ 4) $-\frac{3}{\sqrt{5}}$

10. **Which of the following is not a periodic function**

1) $\sin^{-1}(\sin x)$ 2) $\sin(\sin^{-1} x)$ 3) $\cos^{-1}(\cos x)$ 4) $\tan^{-1}(\tan x)$

11. **If** $f(x)=\sin^{-1}\left(\frac{3x-1}{2}\right)+\sec^{-1}\left(\frac{2}{3x+1}\right)$ **then the domain of f is**

1) R 2) R- {-1/3} 3) (-1/3, 1/3] 4) {1/3}

12. **If** $a=\tan^{-1}(1), b=\tan^{-1}(3), c=\tan^{-1}(5)$ **then which of the following is true**

1) a < b < c 2) c < b < a 3) b < a < c 4) c < a < b

13. **The trignometric equation** $\cos^{-1} x=3\cos^{-1}(3a-2)$ **has a solution if**

1) $\frac{5}{6}\le a\le 1$ 2) $\frac{1}{2}\le a\le 5/6$ 3) $\frac{1}{3}\le a\le 1$ 4) $\frac{1}{3}\le a\le 5/6$

14. **If** $\sin^{-1} x+\sin^{-1}\left(\frac{y}{2}\right)+\sin^{-1}\left(\frac{z}{3}\right)=\frac{3\pi}{2}$ **then the value of** $\tan^{-1} x+\tan^{-1} y+\tan^{-1} z=$

1) $\frac{\pi}{2}$ 2) π 3) $\frac{3\pi}{2}$ 4) 2π

15. **The number of quadrant angles in the range of** $f(x)=\sin^{-1} x+\cos^{-1} x+4\cot^{-1} x$ **is**

1) 7 2) 4 3) 3 4) 5

16. **The values of x satisfying** $\sin^{-1} x+\sin^{-1}(1-x)=\cos^{-1} x$ **are**

1) 0 2) $\frac{1}{2}$ 3) 1 4) 2

17. **If** $\frac{1}{2}<|x|<1,$ **then which of the following are real ?**

1) $\sin^{-1} x$ 2) $\tan^{-1} x$ 3) $\sec^{-1} x$ 4) $\cos^{-1} x$

18. **If** $6\sin^{-1}(x^2-6x+8.5)=\pi,$ **then**

1) $x=1$ 2) $x=2$ 3) $x=3$ 4) $x=4$

19. α,β **and** γ **are the angles given by** $\alpha=2\tan^{-1}(\sqrt{2}-1),\ \beta=3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ **and** $\gamma=\cos^{-1}\left(\frac{1}{3}\right),$ **then**

1) $\alpha>\beta$ 2) $\beta>\gamma$ 3) $\gamma>\alpha$ 4) none of these

20. **Indicate the relation which is true**

1) $\tan\left|\tan^{-1} x\right|=|x|$ 2) $\cot\left|\cot^{-1} x\right|=|x|$

3) $\tan^{-1}|\tan x| = |x|$ 4) $\sin|\sin^{-1} x| = |x|$

21. $\cos^{-1}\left(\sqrt{\frac{a-x}{a-b}}\right) = \sin^{-1}\left(\sqrt{\frac{x-b}{a-b}}\right)$ **is possible, if**

1) $a > x > b$ 2) $a < x < b$

3) $a = x = b$ 4) $a > b$ and x, takes any value

22. **If the numerical value of** $\tan\left\{\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right\}$ **is** $\frac{a}{b}$, **then**

1) $a+b=23$ 2) $a-b=11$ 3) $3b=a+1$ 4) $2a=3b$

23. $2\cot^{-1}7+\cos^{-1}\left(\frac{3}{5}\right)$ **is equal to**

1) $\cot^{-1}\left(\frac{44}{117}\right)$ 2) $\cos ec^{-1}\left(\frac{125}{117}\right)$ 3) $\tan^{-1}\left(\frac{4}{117}\right)$ 4) $\cos^{-1}\left(\frac{44}{125}\right)$

24. **If the equation** $\sin^{-1}\left(x^2+x+1\right)+\cos^{-1}\left(\lambda x+1\right)=\frac{\pi}{2}$ **has exactly two solutions, then** λ **cannot have the integral value**

1) -1 2) 0 3) 1 4) 2

25. **Let** $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)=\alpha,\ \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)=\beta.$ **Then**

1) $\alpha > \beta$ 2) $4\alpha-3\beta=0$ 3) $\alpha+\beta=\frac{7\pi}{12}$ 4) $\alpha+\beta=\frac{\pi}{12}$

26. **If** $0<x<1$ **then** $\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}$ **is equal to**

1) $\frac{1}{2}\cos^{-1}x$ 2) $\cos^{-1}\sqrt{\frac{1-x}{2}}$ 3) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ 4) none of these

27. **One of the values of x satisfying** $\tan\left(\sec^{-1}x\right)=\sin\cos^{-1}\frac{1}{\sqrt{5}}$ **is**

1) $\frac{\sqrt{5}}{3}$ 2) $\frac{3}{\sqrt{5}}$ 3) $-\frac{\sqrt{5}}{3}$ 4) $-\frac{3}{\sqrt{5}}$

28. **If** $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3},\ \cos^{-1}x-\cos^{-1}y=\frac{\pi}{3}$ **then the number of values of** (x,y) **is**

1) two 2) four 3) zero 4) one

29. **A solution of** $\sin^{-1}(1)-\sin^{-1}\left(\sqrt{3}/x^2\right)-\pi/6=0$ **is**

1) $x=-\sqrt{2}$ 2) $x=1$ 3) $x=\sqrt{2}$ 4) $x=1/\sqrt{2}$

30. **If** $\theta=\cos^{-1}(4/5)+\tan^{-1}(2/3)$ **then**

1) $\sin\theta=\dfrac{17}{5\sqrt{13}}$ 2) $\cos\theta=\dfrac{6}{5\sqrt{13}}$ 3) $\tan\theta=17/6$ 4) $\cot\theta=17/6$

31. **If** $A=\sin^{-1}(\sin 10), B=\cos^{-1}(\cos 10)$ **then**

1) $A=3\pi-10$ 2) $A=3\pi+10$ 3) $A>B$ 4) $A<B$

32. $\cos^{-1}x$ **is equal to**

1) $2\sin^{-1}\sqrt{\dfrac{1-x}{2}}$ 2) $2\cos^{-1}\sqrt{\dfrac{1-x}{2}}$ 3) $2\cos^{-1}\sqrt{\dfrac{1+x}{2}}$ 4) $2\sin^{-1}\sqrt{\dfrac{1+x}{2}}$

33. $2\tan^{-1}(-3)$ **is equal to**

1) $-\cos^{-1}(-4/5)$ 2) $-\pi+\cos^{-1}(4/5)$

3) $-\pi/2+\tan^{-1}(-4/3)$ 4) $\cot^{-1}(4/3)$

34. **The principal value of** $\sin^{-1}\left\{\cos\dfrac{33\pi}{10}\right\}$ **must**

1) lie between 0 and $\dfrac{\pi}{2}$ 2) lie between $-\dfrac{\pi}{2}$ and 0

3) be equal to $-\dfrac{\pi}{5}$ 4) be equal to $-\dfrac{2\pi}{5}$

35. **The equation** $\sin^{-1}x+\sin^{-1}2x=\dfrac{\pi}{3}$

1) has two solutions 2) has only one solution

3) has a rational number as solution 4) has an irrational number as solution

WORK SHEET (HW) - II

(Linked Comprehension type questions)

Passage - I :

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{x_r - x_{r-1}}{1+x_{r-1}x_r}\right)=\sum_{r=1}^{n}\left(\tan^{-1}x_r-\tan^{-1}x_{r-1}\right)=\tan^{-1}x_n-\tan^{-1}x_0,\forall n\in N$$

1. **The sum to infinite terms of the series** $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+...+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)+...$ **is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) none of these

2. **The value of** $\cos ec^{-1}\sqrt{5}+\cos ec^{-1}\sqrt{65}+\cos ec^{-1}\sqrt{(325)}+...\infty\sqrt{2}$ **is**

1) π 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

3. **The sum to infinite terms of the series** $\cot^{-1}\left(2^2+\frac{1}{2}\right)+\cot^{-1}\left(2^3+\frac{1}{2^2}\right)+\cot^{-1}\left(2^4+\frac{1}{2^3}\right)+...$ **is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\cot^{-1}2$ 4) $-\cot^{-1}2$

Passage - II :

$\alpha=\tan^{-1}(1/2)+\tan^{-1}(1/3),\beta=\cos^{-1}(2/3)+\cos^{-1}(\sqrt{5}/3)$ **and**

$\gamma=\sin^{-1}(\sin(2\pi/3))+\frac{1}{2}\cos^{-1}(\cos(2\pi/3))$

4. $\cos(\alpha+\beta+\gamma)$ **is equal to**

1) $\cos(5\pi/12)$ 2) $\cos(7\pi/12)$ 3) $-\cos(5\pi/12)$ 4) $-\cos(7\pi/12)$

5. $\tan\alpha-\tan(\beta/2)+\sqrt{3}\tan(\gamma/4)$ **is equal to**

1) 4 2) 3 3) 2 4) 1

6. $\sin\cot^{-1}\tan\cos^{-1}(\sin\gamma)$ **is equal to**

1) $\sin\gamma$ 2) $\sin(\gamma/2)$ 3) $(1/2)\sin\gamma$ 4) $\cos\gamma$

Passage -III :

The function $\sin^{-1}x$ **is defined if** $|x|\le 1$**, and it denotes an angle in** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **whose sine is x. Thus from the equality** $y=\sin x$ **follows the equality** $x=\sin^{-1}y$ **if** $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ **and** $|y|\le 1$**. If x does not**

belong to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **We weite** $y=\sin x$ **as** $y=\sin x'$ **where** $x'\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **since for every** $x\in(-\infty,\infty)$, **there exists** $x'\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **such that** $\sin x=\sin x'$.**Hence we can write** $x'=\sin^{-1}y$.**Similarly** $y=\cos x$ **is invertible if** $|y|\le 1$ **and** $x\in[0,\pi]\tan^{-1}x$ **is defined for all x and has a range** $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

7. **The value of** $\sin^{-1}\left(\cos\frac{23\pi}{6}\right)$ **must be**

1) $\frac{\pi}{2}-\frac{23\pi}{6}$ 2) $-\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $-\frac{\pi}{3}$

8. **If** $x<0$ **then** $\cos^{-1}\frac{1-x^2}{1+x^2}$ **must be equal to**

1) $2\tan^{-1}x$ 2) $\pi+2\tan^{-1}x$ 3) $-2\tan^{-1}x$ 4) $\pi-2\tan^{-1}x$

9. **If** $0<x<\frac{1}{2}$ **then derivative of** $\cos^{-1}\left(4x^3-3x\right)$

1) $\frac{1}{\sqrt{1-x^2}}$ 2) $\frac{-3}{\sqrt{1-x^2}}$ 3) $2\pi-\frac{3}{\sqrt{1-x^2}}$ 4) $\frac{3}{\sqrt{1-x^2}}$

Passage - IV :

While defining inverse trigonometric functions, a new system is followed where domains and ranges have been redefined as follows.

Function	**Domain**	**Range**
$\sin^{-1}x$	[-1, 1]	$\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$
$\tan^{-1}x$	R	$\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$
$\cos^{-1}x$	$[-1,1]$	$[\pi,2\pi]$
$\cot^{-1}x$	R	$[\pi,2\pi]$

10. $\sin^{-1}(-x)=$

1) $-\sin^{-1}x$ 2) $\pi+\sin^{-1}x$ 3) $2\pi-\sin^{-1}x$ 4) $3\pi-\cos^{-1}\sqrt{1-x^2},x>0$

11. **If** $f(x)=3\sin^{-1}x-2\cos^{-1}x,$ **then f(x) is**

1) even function
2) odd function
3) neither even nor odd
4) even as well as odd function

12. **The value of** $\sin^{-1}x+\cos^{-1}x$ **is equal to**

1) $\frac{\pi}{2}$ 2) $\frac{3\pi}{2}$ 3) $\frac{5\pi}{2}$ 4) $\frac{7\pi}{2}$

WORK SHEET (HW) - III

(Matrix Matching Type Questions)

1.

Column -I	Column - II
1) $\sin^{-1}x+\sin^{-1}2x=\pi/3$	p) 4/3
2) $\tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36}$	q) $\frac{1}{2}\sqrt{\frac{3}{7}}$
3) $\tan^{-1}\cos x=\sin\left(\tan^{-1}2\right)$	r) $\sqrt{5}/3$
4) $\sin\tan^{-1}x=-3/\sqrt{73}$	s) - 3/8

2.

Column -I	Column - II
1) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\,(0<x<\pi/2)$	p) x
2) $\tan^{-1}\frac{\cos x\pm\sin x}{\cos x\pm\sin x}$	q) $\frac{\pi}{4}-x$
3) $2\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$	r) $x/2$
4) $\sin^{-1}\frac{\cos 2x}{\sqrt{2}(\cos x\pm\sin x)}$	s) $\pi/4+x$

3. **Match the following numerical quantities with theri principal values :**

Column -I	Column - II
1) $\sin^{-1}(\sin 21)$	p) $7\pi-21$
B) $\cos^{-1}(\cos 20)$	q) $-\frac{\pi}{6}$
3) $\sin^{-1}\left(\sin\frac{23\pi}{6}\right)$	r) $\frac{\pi}{14}$
4) $\cos^{-1}\left(\sin\frac{46\pi}{7}\right)$	s) $20-6\pi$

WORK SHEET (HW)- IV

(Integer type questions)

1. $0 < A < \frac{\pi}{4}$ **and** $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}\left(\cot A\right) + \tan^{-1}\left(\cot^3 A\right) = 4\tan^{-1} x$ **Then x=_____**

2. $Tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7) + \pi$ **then x=_____**

3. **When** $0 < x < \frac{\pi}{2}$ **then the value of** $\tan\left\{\sin^{-1}\left(\cos\left(\sin^{-1} x\right)\right)\right\}.\tan\left\{\cos^{-1}\left(\sin\left(\cos^{-1} x\right)\right)\right\} =$ _____

4. **If** $\cos^{-1}\left(\frac{n}{2\pi}\right) > \frac{2\pi}{3}$ **then maximum integer value of n is** λ **then** $|\lambda|$ **=** _____

5. **If** $\log_2 x \geq 0$ **then** $\log_{1/\pi}\left\{\sin^{-1}\frac{2x}{1+x^2} + 2\tan^{-1} x\right\} = \lambda$ **then** $|\lambda|$ _____

6. **If** $\theta \neq 0^0, \theta \in I$ **quadrant and** $\theta = \tan^{-1}\left(2\tan^2\theta\right) - \tan^{-1}\left(\frac{1}{3}\tan\theta\right)$ **then** $\tan\theta =$ ____

7. $\cos^{-1} x_1 + \cos^{-1} x_2 + \cos^{-1} x_3 + \cos^{-1} x_4 = 4\pi$ **then** $\sum x_1 x_2 =$

8. $2\tan^{-1}(2x-1) = \cos^{-1} x$ **then x=_____**

9. **In** $\Delta ABC, \angle C = 90^0$ *and* $\sin^{-1} x = \sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c}$ **then +ve integer value of x =** _____

10. $\tan^{-1}\left(\frac{1}{1+0+0^2}\right) + \tan^{-1}\left(\frac{1}{1+1+1^2}\right) + \tan^{-1}\left(\frac{1}{1+2+2^2}\right) + \tan^{-1}\left(\frac{1}{1+3+3^2}\right) + \ldots\infty = \frac{\pi}{x}$ **then x**

EXERCISE - II / ANSWERS

WORK SHEET (HW) - I

1) 1	2) 2	3) 1	4) 2	5) 4	6) 1	7) 1	8) 4	9) 1	10) 2
11) 3	12. 3	13) 1	14) 2	15) 4	16) 1,2	17) 1,2,4	18) 2,4	19) 2,3	20) 1,2,4
21) 1,2	22) 1,2,3	23) 1,2,4	24) 1,3,4	25) 2,3	26) 1,2,3	27) 2,4	28) 4	29) 1,3	30) 1,2,3
31) 1,4	32) 1,3	33) 1,2,3	34) 2,3	35) 2,4					

WORK SHEET (HW) - II

1) 1	2) 4	3) 3	4) 2,3	5) 4	6) 1,2	7) 3	8) 3	9) 4	10) 3
11) 2	12) 3								

WORK SHEET (HW) - III

1) $A \to q; B \to p; C \to r; D \to s$ **2)** $A \to r; B \to q,s; C \to p; D \to q,s$

3) $A \to p; B \to s; C \to q; D \to r$

WORK SHEET (HW) - IV

1) 1	2) 2	3) 1	4) 4	5) 1	6) 1	7) 6	8) 0	9) 1	10) 2