TRIGONOMETRIC EQUATIONS

1. TRIGONOMETRIC EQUATION

An equation involving one or more trigonometric ratios of an unknown angle is called a trigonometric equation.

2. SOLUTION OF TRIGONOMETRIC EQUATION

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$, ...

Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature of trigonometric ratios) and can be classified as :

(i) Principal solution (ii) General solution.

2 .1 Principal Solutions

The solutions of a trigonometric equation which lie in the interval $[0, 2\pi)$ are called **Principal solutions.**

e.g Find the Principal solutions of the equation sinx = $\frac{1}{2}$.

Solution :

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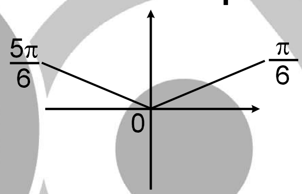


$\because$ $\sin x = \frac{1}{2}$

$\because$ there exists two values i. e. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ which lie in $[0, 2\pi)$ and whose sine is $\frac{1}{2}$

$\therefore$ Principal solutions of the equation $\sin x = \frac{1}{2}$ are $\frac{\pi}{6}, \frac{5\pi}{6}$ **Ans.**

2.2 General Solution

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called **General solution**.

General solution of some standard trigonometric equations are given below.

3. GENERAL SOLUTION OF SOME STANDARD TRIGONOMETRIC EQUATIONS

(i) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], n \in I$.

(ii) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), n \in I$.

(iv) If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$.

(v) If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$.

(vi) If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$.

(iv)–(vi): where $\alpha \in \left[0, \frac{\pi}{2}\right], n \in I$

[**Note:** α is called the principal angle]

Some Important deductions :

(i)	$\sin\theta = 0$	$\Rightarrow$	$\theta = n\pi$,	$n \in I$
(ii)	$\sin\theta = 1$	$\Rightarrow$	$\theta = (4n + 1)\frac{\pi}{2}$,	$n \in I$
(iii)	$\sin\theta = -1$	$\Rightarrow$	$\theta = (4n - 1)\frac{\pi}{2}$,	$n \in I$
(iv)	$\cos\theta = 0$	$\Rightarrow$	$\theta = (2n + 1)\frac{\pi}{2}$,	$n \in I$
(v)	$\cos\theta = 1$	$\Rightarrow$	$\theta = 2n\pi$,	$n \in I$
(vi)	$\cos\theta = -1$	$\Rightarrow$	$\theta = (2n + 1)\pi$,	$n \in I$
(vii)	$\tan\theta = 0$	$\Rightarrow$	$\theta = n\pi$,	$n \in I$

IN CHAPTER EXCERCISE - 1

(Basic formula based questions)

1. Write the general solution of the trigonometric equation:

(A) $\sin\theta = -\frac{\sqrt{3}}{2}$ (B) $\sec 2\theta = -\frac{2}{\sqrt{3}}$

(C) $\tan\theta = 2$ (D) $2\cot^2\theta = \cos ec^2\theta$

(E) $4\tan^2\theta = 3\sec^2\theta$

2. What is the most general value of θ satisfying both the equations:

(A) $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$ (B) $\cot\theta = -\sqrt{3}$ and $\cos ec\theta = -2$

3. If $\cos(A-B) = \frac{1}{2}$ and $\sin(A+B) = \frac{1}{2}$, find the smallest positive values of A and B and also their most general values.

4. If $\tan(A-B) = 1$ and $\sec(A+B) = \frac{2}{\sqrt{3}}$, find the smallest positive values of A and B and also their most general values.

ANSWER KEY

1. (A) $\theta = n\pi + (-1)^n\left(-\frac{\pi}{3}\right)$ (B) $\theta = n\pi \pm \frac{5\pi}{12}$ (C) $\theta = n\pi + \tan^{-1}2$

(D) $\theta = n\pi \pm \frac{\pi}{4}$ (E) $\theta = n\pi \pm \frac{\pi}{3}$

2. (A) $\theta = 2n\pi + \frac{5\pi}{4}$ (B) $\theta = 2n\pi - \frac{\pi}{6}$

3. 105° & 45°; $\left(n+\frac{m}{2}\right)\pi \pm \frac{\pi}{6} + (-1)^m \frac{\pi}{12}$ & $\left(\frac{m}{2}-n\right)\pi \mp \frac{\pi}{6} + (-1)^m \frac{\pi}{12}$

4. $187\frac{1}{2}^\circ$ & $142\frac{1}{2}^\circ$; $\left(n+\frac{m}{2}\right)\pi + \frac{\pi}{8} \pm \frac{\pi}{12}$ & $\left(n-\frac{m}{2}\right)\pi - \frac{\pi}{8} \pm \frac{\pi}{12}$

4. TYPES OF TRIGONOMETRIC EQUATIONS

Type -1 : Trigonometric equations which can be solved by factorization.

Ex. Solve (2sinx – cosx) (1 + cosx) = sin²x.

Sol. $\because$ $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$ $\Rightarrow$ $(2\sin x - \cos x)(1 + \cos x) - \sin^2 x = 0$

$\Rightarrow$ $(2\sin x - \cos x)(1 + \cos x) - (1 - \cos x)(1 + \cos x) = 0$

$\Rightarrow$ $(1 + \cos x)(2\sin x - 1) = 0$

$\Rightarrow$ $1 + \cos x = 0$ or $2\sin x - 1 = 0$

$\Rightarrow$ $\cos x = -1$ or $\sin x = \frac{1}{2}$

$\Rightarrow$ $x = (2n + 1)\pi,\ n \in I$ or $\sin x = \sin\frac{\pi}{6}$ $\Rightarrow$ $x = n\pi + (-1)^n \frac{\pi}{6},\ n \in I$

$\therefore$ Solution of given equation is

$(2n + 1)\pi,\ n \in I$ or $n\pi + (-1)^n \frac{\pi}{6},\ n \in I$ **Ans.**

Type - 2 : Trigonometric equations which can be solved by reducing them in quadratic equations.

Ex. Solve 2 cos²x + 4cosx = 3sin²x

Sol. $\because$ $2\cos^2 x + 4\cos x - 3\sin^2 x = 0$

$\Rightarrow$ $2\cos^2 x + 4\cos x - 3(1 - \cos^2 x) = 0$

$\Rightarrow$ $5\cos^2 x + 4\cos x - 3 = 0$

$\Rightarrow$ $\left\{\cos x - \left(\frac{-2+\sqrt{19}}{5}\right)\right\}\left\{\cos x - \left(\frac{-2-\sqrt{19}}{5}\right)\right\} = 0$ ....(i)

$\because$ $\cos x \in [-1, 1]$ $\forall\ x \in R$ $\therefore$ $\cos x \neq \frac{-2-\sqrt{19}}{5}$

$\therefore$ equation (i) will be true if $\cos x = \frac{-2+\sqrt{19}}{5}$

$\Rightarrow$ $\cos x = \cos\alpha$, where $\cos\alpha = \frac{-2+\sqrt{19}}{5}$

$\Rightarrow$ **$x = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-2+\sqrt{19}}{5}\right)$, $n \in I$ Ans.**

Type - 3 : Trigonometric equations which can be solved by transforming a sum or difference of trigonometric ratios into their product.

Ex. Solve $\cos 3x + \sin 2x - \sin 4x = 0$

Sol. $\cos 3x + \sin 2x - \sin 4x = 0$ or $\cos 3x + 2\cos 3x.\sin(-x) = 0$

$\Rightarrow$ $\cos 3x - 2\cos 3x.\sin x = 0$ or $\cos 3x\,(1 - 2\sin x) = 0$

$\Rightarrow$ $\cos 3x = 0$ or $1 - 2\sin x = 0$

$\Rightarrow$ $3x = (2n+1)\frac{\pi}{2}$, $n \in I$ or $\sin x = \frac{1}{2}$

$\Rightarrow$ $x = (2n+1)\frac{\pi}{6}$, $n \in I$ or $x = n\pi + (-1)^n \frac{\pi}{6}$, $n \in I$

$\therefore$ solution of given equation is

$(2n+1)\frac{\pi}{6}$, $n \in I$ or $n\pi + (-1)^n \frac{\pi}{6}$, $n \in I$ Ans.

Type - 4 : Trigonometric equations which can be solved by transforming a product of trigonometric ratios into their sum or difference.

Ex. Solve $\sin 5x.\cos 3x = \sin 6x.\cos 2x$

Sol. $\because$ $\sin 5x.\cos 3x = \sin 6x.\cos 2x$ $\Rightarrow$ $2\sin 5x.\cos 3x = 2\sin 6x.\cos 2x$

$\Rightarrow$ $\sin 8x + \sin 2x = \sin 8x + \sin 4x$ $\Rightarrow$ $\sin 4x - \sin 2x = 0$

$\Rightarrow$ $2\sin 2x.\cos 2x - \sin 2x = 0$ $\Rightarrow$ $\sin 2x\,(2\cos 2x - 1) = 0$

$\Rightarrow$ $\sin 2x = 0$ or $2\cos 2x - 1 = 0$

$\Rightarrow$ $2x = n\pi$, $n \in I$ or $\cos 2x = \frac{1}{2}$

$\Rightarrow$ $x = \frac{n\pi}{2}$, $n \in I$ or $2x = 2n\pi \pm \frac{\pi}{3}$, $n \in I$ $\Rightarrow x = n\pi \pm \frac{\pi}{6}$, $n \in I$

$\therefore$ Solution of given equation is **$\frac{n\pi}{2}$, $n \in I$ or $n\pi \pm \frac{\pi}{6}$, $n \in I$ Ans.**

Type - 5 : Trigonometric Equations of the form $a\sin x + b\cos x = c$, where $a, b, c \in R$, can be solved by dividing both sides of the equation by $\sqrt{a^2+b^2}$.

Ex.1 Solve $\sin x + \cos x = \sqrt{2}$

Sol. $\because$ $\sin x + \cos x = \sqrt{2}$ ....(i)

Here, $a = 1$, $b = 1$.

$\therefore$ divide both sides of equation (i) by $\sqrt{2}$, we get

$\sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}} = 1$ $\Rightarrow$ $\sin x.\sin\frac{\pi}{4} + \cos x.\cos\frac{\pi}{4} = 1$

$\Rightarrow$ $\cos\left(x - \frac{\pi}{4}\right) = 1$ $\Rightarrow$ $x - \frac{\pi}{4} = 2n\pi$, $n \in I$

$\Rightarrow \quad x = 2n\pi + \frac{\pi}{4}, n \in I \qquad \therefore$ Solution of given equation is

$\mathbf{2n\pi + \frac{\pi}{4}, n \in I}$ **Ans.**

Note : If $x = (2n+1)\pi,\ n \in I$ is not a solution of $a\sin x + b\cos x = c$, then this trigonometric equation can also be solved by changing **sinx** and **cosx** into their **corresponding** $\mathbf{\tan\frac{x}{2}}$ **formulae.**

Ex.2 Solve 3cosx + 4sinx = 5

Sol. $\because \quad 3\cos x + 4\sin x = 5 \qquad$ ....(i)

$\because \quad \cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \quad \& \quad \sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\therefore$ equation (i) becomes

$\Rightarrow \quad 3\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 4\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) = 5 \qquad$ ....(ii)

Let $\tan\frac{x}{2} = t \quad$ equation (ii) becomes, $3\left(\dfrac{1-t^2}{1+t^2}\right) + 4\left(\dfrac{2t}{1+t^2}\right) = 5$

$\Rightarrow \quad 4t^2 - 4t + 1 = 0 \quad \Rightarrow \quad (2t-1)^2 = 0$

$\Rightarrow \quad t = \frac{1}{2} \quad \because \quad t = \tan\frac{x}{2}$

$\Rightarrow \quad \tan\frac{x}{2} = \frac{1}{2} \quad \Rightarrow \quad \tan\frac{x}{2} = \tan\alpha$, where $\tan\alpha = \frac{1}{2}$

$\Rightarrow \quad \frac{x}{2} = n\pi + \alpha \quad \Rightarrow \quad \mathbf{x = 2n\pi + 2\alpha}$ **where** $\mathbf{\alpha = \tan^{-1}\left(\frac{1}{2}\right), n \in I}$ **Ans.**

Type - 6 : Trigonometric equations of the form P(sinx ± cosx, sinx cosx) = 0, where p(y, z) is a polynomial, can be solved by using the substitution sinx ± cosx = t.

Ex. Solve sinx + cosx = 1 + sinx.cosx

Sol. $\because \quad \sin x + \cos x = 1 + \sin x.\cos x \qquad$ ....(i)

Let $\sin x + \cos x = t$

$\Rightarrow \quad \sin^2x + \cos^2x + 2\sin x.\cos x = t^2$

$\Rightarrow \quad \sin x.\cos x = \dfrac{t^2-1}{2}$

Now put $\sin x + \cos x = t$ and $\sin x.\cos x = \dfrac{t^2-1}{2}$ in (i), we get $\quad t = 1 + \dfrac{t^2-1}{2}$

$\Rightarrow \quad t^2 - 2t + 1 = 0 \quad \Rightarrow \quad t = 1 \quad \because \quad t = \sin x + \cos x$

$\Rightarrow \quad \sin x + \cos x = 1$ ....(ii)

divide both sides of equation (ii) by $\sqrt{2}$, we get

$\Rightarrow \quad \sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} \quad \Rightarrow \quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

(i) if we take positive sign, we get **$x = 2n\pi + \frac{\pi}{2}$, $n \in I$ Ans.**

(ii) if we take negative sign, we get **$x = 2n\pi$, $n \in I$ Ans.**

Type - 7 : Trigonometric equations which can be solved by the use of boundness of the trigonometric ratios sinx and cosx.

Ex. Solve $\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$

Sol. $\because \quad \sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$ ....(i)

$\Rightarrow \quad \sin x.\cos\frac{x}{4} - 2\sin^2 x + \cos x + \sin\frac{x}{4}.\cos x - 2\cos^2 x = 0$

$\Rightarrow \quad \left(\sin x.\cos\frac{x}{4} + \sin\frac{x}{4}.\cos x\right) - 2(\sin^2 x + \cos^2 x) + \cos x = 0$

$\Rightarrow \quad \sin\frac{5x}{4} + \cos x = 2$ ....(ii)

Now equation (ii) will be true if

$\sin\frac{5x}{4} = 1$ & $\cos x = 1$

$\Rightarrow \quad \frac{5x}{4} = 2n\pi + \frac{\pi}{2}, n \in I$ & $x = 2m\pi, m \in I$

$\Rightarrow \quad x = \frac{(8n+2)\pi}{5}, n \in I$ ....(iii)

$\Rightarrow \quad x = 2m\pi, m \in I$ ....(iv)

Now to find general solution of equation (i)

$\frac{(8n+2)\pi}{5} = 2m\pi \quad \Rightarrow \quad 8n + 2 = 10m \quad \Rightarrow \quad n = \frac{5m-1}{4}$

if	$m = 1$	then	$n = 1$
if	$m = 5$	then	$n = 6$
	....	....	....
	....	....	....
if	$m = 4p - 3, p \in I$	then	$n = 5p - 4, p \in I$

$\therefore$ general solution of given equation can be obtained by substituting either $m = 4p - 3$ in equation (iv) or $n = 5p - 4$ in equation (iii)

$\therefore$ general solution of equation (i) is **$(8p - 6)\pi$, $p \in I$ Ans.**

IN CHAPTER EXCERCISE - 2

Solve the equations

1. $\cos\theta + \cos^2\theta = 1$

2. $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

3. $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

4. $\cos^3 x + \cos^2 x - 4\cos^2\dfrac{x}{2} = 0$

5. $16^{\sin^2 x} + 16^{\cos^2 x} = 10, 0 \le x < 2\pi$

6. $5\sin x + 6\sin 2x + 5\sin 3x + \sin 4x = 0$

7. $\sin 7\theta = \sin 3\theta + \sin\theta$

8. $\sin 5x\cos 3x = \sin 6x\cos 2x$

9. $\sin 3\theta = \sin 2\theta$

10. $\cot\theta = \tan 8\theta$

11. $\cos x\cos 2x\cos 3x = \dfrac{1}{4}$

12. **Solve the equations**

(i) $\sin x + \cos x = \sqrt{2}$

(ii) $\sqrt{3}\cos x - 3\sin x = 4\sin 2x\cos 3x$

(iii) $\cos^{50} x - \sin^{50} x = 1$

(iv) $\cos 4\theta + \sin 5\theta = 2$

(v) $1 + \sin x\sin^2\dfrac{x}{2} = 0$

(vi) $\sin^2 x + \cos^2 y = 2\sec^2 z$ for x,y,z $\in$ R

(vii) $e^{\cos x} = e^{-\cos x} + 4$

(viii) $2\cos^2\dfrac{x^2 + x}{6} = 2^x + 2^{-x}$

13. Find the condition on 'c' for $a\sin x + b\cos x = c$ to have a solution. Is the condition satisfied for $\sin x + \sqrt{3}\cos x = 1$ and hence solve the equation.

14. Find the number of integral values of 'n' so that $\sin x(\sin x + \cos x) = n$ has at least one real solution.

15. Solve for $x \,\&\, y$, if $12\sin x + 5\cos x = 2y^2 - 8y + 21$

ANSWER KEY

1. $\theta = 2n\pi \pm \cos^{-1}\left(\dfrac{\sqrt{5}-1}{2}\right)$

2. $2n\pi \pm \dfrac{5\pi}{6}, n \in I$

3. $x = (2n+1)\pi, n \in I$ or $n\pi + (-1)^n\dfrac{\pi}{6}$

4. $x = (2n+1)\pi, n \in I$

5. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

6. $\dfrac{n\pi}{2}$ or $2n\pi \pm \dfrac{2\pi}{3}$

7. $\dfrac{n\pi}{3}$ or $\dfrac{n\pi}{2} \pm \dfrac{\pi}{12}$

8. $\dfrac{n\pi}{2}$ or $n\pi \pm \dfrac{\pi}{6}$

9. $2n\pi$ or $\dfrac{(2n+1)\pi}{5}$

10. $\left(n+\dfrac{1}{2}\right)\dfrac{\pi}{9}$

11. $x=(2n+1)\dfrac{\pi}{8}$ or $x=m\pi\pm\dfrac{\pi}{3}$

12. **(i)** $2n\pi+\dfrac{\pi}{4}$, **(ii)** $x=\dfrac{n\pi}{3}+\dfrac{\pi}{18}$ or $\dfrac{n\pi}{2}+\dfrac{\pi}{6}$

(iii) $x=n\pi$ **(iv)** $\theta=2n\pi+\dfrac{\pi}{2}$

(v) No Solution **(vi)** $x=(2m+1)\dfrac{\pi}{2}$, $y=n\pi$, $z=k\pi$

(vii) no solution **(viii)** x = 0

13. $|c|\le\sqrt{a^2+b^2}$, YES, $x=2n\pi+\dfrac{\pi}{2}, 2n\pi-\dfrac{\pi}{6}$

14. 2

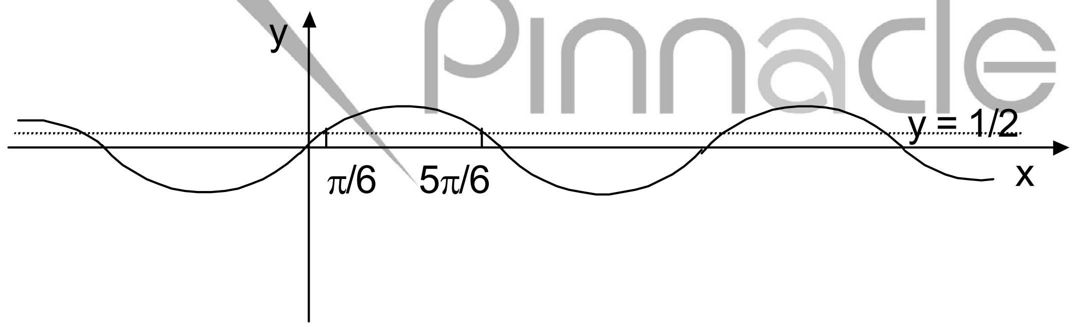
15. $x=2n\pi+\cos^{-1}(\dfrac{5}{13})$ & $y=2$

5. TRIGONOMETRIC INEQUATIONS

Find the solution set of the inequation sin x > 1/2.

Solution :

When sinx = 1/2, the two values of x between 0 and 2π are π/6 and 5π/6.



From, the graph of y = sin x, it is obvious that, between 0 and 2π

sinx > 1/2 for $\dfrac{\pi}{6}<x<\dfrac{5\pi}{6}$.

Hence $\sin x > 1/2$

$\Rightarrow$ $2n\pi+\pi/6 < x < 2n\pi+5\pi/6$;

The required solution set = $\bigcup\limits_{n\in I}\left(2n\pi+\dfrac{\pi}{6}, 2n\pi+\dfrac{5\pi}{6}\right)$

IN CHAPTER EXCERCISE - 3

(Inequalities & graph based questions)

1. Solve the following Inequalities:

(i) $\sin x > \frac{-1}{2}$ **(ii)** $\tan x \le -\sqrt{3}$

(iii) $2\cos^2 x + \sin x \le 2,$ **where** $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ **(iv)** $\sin^4 x + \cos^4 x > \frac{1}{2}$

2. Find the number of solutions of the equation: $\sin x = x^2 + x + 2$

3. Find the number of solutions of equation $\sin x = \frac{x}{10}$

4. Prove that the least positive value of x, satisfying $\tan x = x + 1,$ lies in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

5. Number of solutions of the equation: $\max\{\sin x, \cos x\} = \frac{1}{2}$ in $x \in (-2\pi, 5\pi)$, is equal to

ANSWER KEY

1. **(i).** $2n\pi - \frac{\pi}{6} < x < (2n+1)\pi + \frac{\pi}{6}$ **(ii)** $\frac{(2n-1)\pi}{2} < x \le n\pi - \frac{\pi}{3}$

(iii) $x \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$ **(iv)** $x = R - \left\{\frac{n\pi}{2} + \frac{\pi}{4}\right\}$

2. No Solution **3.** 7 solutions **4.** 1 solution **5.** 7 solutions

6. SOME IMPORTANT POINTS TO REMEMBER

- While solving a trigonometric equation, squaring the equation at any step should be avoided as far as possible. If squaring is necessary, check the solution for extraneous values.
- Never cancel terms containing unknown terms on the two sides, which are in product. It may cause loss of genuine solution.
- The answer should not contain such values of angles, which make any of the terms undefined.
- Domain should not be changed. If it is changed, necessary corrections must be incorporated.
- Check that the denominator is not zero at any stage while solving equations.
- Some times you may find that your answers differ from those in the package in their notations. This may be due to the different methods of solving the same problem. Whenever you come across such situation, you must check their authenticity.
  This will ensure that your answer is correct.

- While solving trigonometric equations you may get same set of solution repeated in your answer. It is necessary for you to exclude these repetitions, e.g. $n\pi+\frac{\pi}{2}$, $(n \in I)$ forms a part of $\frac{k\pi}{5}+\frac{\pi}{10}$, $k \in I$ the second part of the second set of solution (you can check by putting $k = 5m + 2$ $(m \in I)$). Hence the final answer is $\frac{k\pi}{5}+\frac{\pi}{10}, k \in I$.
- Some times the two solution set consist partly of common values. In all such cases the common part must be presented only once.

Solved Problems (Subjective)

1. Solve: $\sin x + \cos x - 2\sqrt{2}\sin x \cos x = 0$

Sol. Here $\sin x + \cos x = 2\sqrt{2}\sin x\cos x = \sqrt{2}\sin 2x$ … (1)

or $\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)=\sqrt{2}\sin 2x$ or $\sin\left(x+\frac{\pi}{4}\right)=\sin 2x$ or, $2x = n\pi + (-1)^n\left(x+\frac{\pi}{4}\right)$

Taking n even, $n = 2m$, $m \in I$, $2x = 2m\pi + x + \frac{\pi}{4}$

$\therefore\ x = 2m\pi + \frac{\pi}{4}$ where $m \in I$

Taking n odd, $n = 2m + 1$, $m \in I$; $2x = (2m+1)\pi - \left(x+\frac{\pi}{4}\right)$

$\therefore\ 3x = (2m+1)\pi - \frac{\pi}{4}$ or $x = \frac{2m+1}{3}\pi - \frac{\pi}{12}$

Thus, $x = \left(2m+\frac{1}{4}\right)\pi$ or $\frac{1}{3}\left(2m+\frac{3}{4}\right)\pi$, where $m \in I$.

2. Solve the equation $\cos^2\left[\frac{\pi}{4}\left(\sin x+\sqrt{2}\cos^2 x\right)\right]-\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)=1$

Sol. Given $\cos^2\left[\frac{\pi}{4}\left(\sin x+\sqrt{2}\cos^2 x\right)\right]-\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)=1$

or, It is possible only when

$$\sin^2\left[\frac{\pi}{4}\left(\sin x+\sqrt{2}\cos^2 x\right)\right]=0 \quad \dots (1)$$

and $$\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)=0 \quad \dots (2)$$

from equation (1)

$$\sin^2\left\{\frac{\pi}{4}(\sin x+\sqrt{2}\cos^2 x)\right\}=0 \qquad \therefore \frac{\pi}{4}(\sin x+\sqrt{2}\cos^2 x)=n\pi,\ n\in I$$

or, $\sin x+\sqrt{2}\cos^2 x=4n$

$\therefore$ The equation has no solution for $n\neq 0$ we consider $n=0$

$\therefore$ $\sin x+\sqrt{2}\cos^2 x=0$ i.e., $\sqrt{2}\sin^2 x-\sin x-\sqrt{2}=0$

or, $(\sin x-\sqrt{2})(\sqrt{2}\sin x+1)=0$

or, $(\sin x-\sqrt{2})(\sqrt{2}\sin x+1)=0$; $\sin x\neq\sqrt{2}$ $\therefore \sin x=-\frac{1}{\sqrt{2}}$

$\Rightarrow$ $x=k\pi+(-1)^k(-\pi/4).\ k\in I$

The values of x that satisfy the equation (2) is $2k\pi-\pi/4$.

Therefore the general solution of given equation is $x=2k\pi-\frac{\pi}{4},\ k\in I$.

3. Find the general solution of the equation $\sin^4 x+\cos^4 x=\sin x\cos x$

Sol. The given equation can be re–written as

$4\sin^4 x+4\cos^4 x=4\sin x\cos x$

or, $(1-\cos 2x)^2+(1+\cos 2x)^2=2\sin 2x$

or, $2(1+\cos^2 2x)=2\sin 2x$

$\Rightarrow$ $1+\cos^2 2x=\sin 2x$

or, $1+1-\sin^2 2x=\sin 2x$

$\Rightarrow$ $\sin^2 2x+\sin 2x=2$

This relation is possible if and only if $\sin 2x=1$ or, $2x=2n\pi+\frac{\pi}{2}$

$$\Rightarrow\quad x=n\pi+\frac{\pi}{4}=\frac{(4n+1)\pi}{4}(n\in I)$$

4. $\tan\theta+\tan(\theta+(\pi/3))+\tan(\theta+(2\pi/3))=3$

Sol. From the given equation,

$$\tan\theta+\frac{\tan\theta+\tan(\pi/3)}{1-\tan\theta\tan(\pi/3)}+\frac{\tan\theta+\tan(2\pi/3)}{1-\tan\theta\tan(2\pi/3)}=3$$

or, $$\tan\theta+\frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}=3$$

or, $$\frac{\tan\theta(1-3\tan^2\theta)+(\tan\theta+\sqrt{3}).(1+\sqrt{3}\tan\theta)+(\tan\theta-\sqrt{3})(1-\sqrt{3}\tan\theta)}{(1-\sqrt{3}\tan\theta)(1+\sqrt{3}\tan\theta)}=3$$

or, $3\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=3$ or, $\tan 3\theta=1=\tan(\pi/4)$

or, $3\theta=n\pi+(\pi/4)$

or, $\theta=(4n+1)(\pi/12)$, where $n\in I$.

5. Solve x and y : $4^{\sin x}+3^{\frac{1}{\cos y}}=11$, $5.16^{\sin x}-2.3^{\frac{1}{\cos y}}=2$

Sol. Let, $4^{\sin x}=\lambda$, $3^{\frac{1}{\cos y}}=\mu$

Then the equation becomes

$\lambda+\mu=11$ … (1)

$5\lambda^2-2\mu=2$ … (2)

On solving we get $\lambda=2,-\frac{12}{5}$

If $\lambda=2,\ 4^{\sin x}=2;\ \therefore\ 2^{2\sin x}=2$;

$\therefore$ $2\sin x=1$; $\therefore$ $\sin x=\frac{1}{2}$

If $\lambda=\frac{-12}{5}$, then $4^{\sin x}=-\frac{12}{5}$ which is impossible as $4^{\sin x}>0$

When $\lambda=2$, we get $\mu=11-2=9$ $\Rightarrow$ $3^{\frac{1}{\cos y}}=9=3^2$

$\therefore$ $\frac{1}{\cos y}=2$; $\cos y=\frac{1}{2}$

Thus we have $\sin x=\frac{1}{2}$, $\cos y=\frac{1}{2}$

$\therefore$ $x=n\pi+(-1)^n\frac{\pi}{6}$ and $y=2m\pi\pm\frac{\pi}{3}$, where $m, n\in I$

6. Find all values of 'α' for which the equation $\sin^4x+\cos^4x+\sin 2x+\alpha=0$ has atleast one solution. Also find the general solution of the equation for that α.

Sol. Here $(\sin^2x+\cos^2x)^2-2\sin^2x.\cos^2x+\sin 2x+\alpha=0$

or, $1-\frac{1}{2}\sin^2 2x+\sin 2x+\alpha=0$ or, $\sin^2 2x-2\sin 2x-2(1+\alpha)=0$

$\therefore$ $\sin 2x=\frac{2\pm\sqrt{4+8(1+\alpha)}}{2}=1\pm\sqrt{2\alpha+3}$ … (1)

But sin 2x is real; so $2\alpha+3\ge 0$, i.e., $\alpha\ge\frac{-3}{2}$

Also, $-1\le\sin 2x\le 1$; $-1\le 1\pm\sqrt{2\alpha+3}\le 1$

As $1+\sqrt{2\alpha+3}\ge 1$; So $1+\sqrt{2\alpha+3}=1\Rightarrow\alpha=\frac{-3}{2}$

Also $-1\le 1-\sqrt{2\alpha+3}\le 1$ $\Rightarrow$ $0\le\sqrt{2\alpha+3}\le 2$

$\Rightarrow$ $\alpha\in\left[\frac{-3}{2},\frac{1}{2}\right]$ $\therefore$ from (1), $\sin 2x=1-\sqrt{2\alpha+3}$

where, $\alpha \in \left[\frac{-3}{2}, \frac{1}{2}\right]$

$\therefore \quad 2x = n\pi + (-1)^n\theta$, $n \in I$ and $\sin\theta = 1 - \sqrt{2\alpha + 3}$

$\therefore \quad x = \frac{n\pi}{2} + (-1)^n \sin^{-1}\left(1 - \sqrt{2\alpha+3}\right)$ where $n \in I$ and $\alpha \in \left[-\frac{3}{2}, \frac{1}{2}\right]$

7. For $x \in (-\pi, \pi)$, solve the equation $(\sqrt{3}\sin x + \cos x)^{\sqrt{(\sqrt{3}\sin 2x - \cos 2x + 2)}} = 4$

Sol. The given equation is

or, $(\sqrt{3}\sin x + \cos x)^{\sqrt{(\sqrt{3}\sin 2x - \cos 2x + 2)}} = 4$

or, $\left[2\sin\left(x + \frac{\pi}{6}\right)\right]^{\sqrt{(3\sin^2 x + \cos^2 x + 2\sqrt{3}\sin x \times \cos x)}} = 4$ or, $\left[2\sin\left(x + \frac{\pi}{6}\right)\right]^{\left|2\sin\left(x + \frac{\pi}{6}\right)\right|} = 4$

Hence, $2\sin\left(x + \frac{\pi}{6}\right) = \pm 2$ or, $\sin\left(x + \frac{\pi}{6}\right) = \pm 1$ or, $x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2}$

or, $x = 2n\pi \pm \frac{\pi}{2} - \frac{\pi}{6}$, As $x \in (-\pi, \pi)$,

$\therefore \ x = \frac{\pi}{3}$ and $x = -\frac{2\pi}{3}$ are the solutions of the given equation

8. Find the solution set of the system of equations, $x + y = \frac{2\pi}{3}$ and $\cos x + \cos y = \frac{3}{2}$, where x and y are real.

Sol. $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$ $\Rightarrow$ $2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\Rightarrow \quad 2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\Rightarrow \quad 2\cos\frac{\pi}{3} \cdot \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ $\qquad \left[x + y = \frac{2\pi}{3}\right]$

$\therefore \quad \cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ which is not possible .

9. Find the values of x, $0 \le x \le 2\pi$, such that

$\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 2x$.

Sol. The given equation can be written as $(\sin x + \sin 3x) + \sin 2x = (\cos x + \cos 3x) + \cos 2x$

$\Rightarrow \quad 2\sin 2x \cos x + \sin 2x = 2\cos 2x \cos x + \cos 2x$

$\Rightarrow \quad \sin 2x\,(2\cos x + 1) - \cos 2x\,(2\cos x + 1) = 0$

$\Rightarrow \quad (\sin 2x - \cos 2x)(2\cos x + 1) = 0$

That is, either $\sin 2x - \cos 2x = 0$ or $2\cos x + 1 = 0$.

In former case $\tan 2x = 1 \Rightarrow 2x = n\pi + \frac{\pi}{4}$ $(n \in I)$

$\Rightarrow \quad x = \frac{(4n+1)\pi}{8} \qquad \Rightarrow \quad x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

If 2 cos x + 1 = 0, then cos x = $-\frac{1}{2}$, that is $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

10. If $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$ and $3\cos 2\theta = 1$, then find the general value of α.

Sol. Given $3\cos 2\theta = 1$ or $\cos 2\theta = \frac{1}{3}$

Now, $\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2}$ ....(i)

Now $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$

or $2\cos^2\alpha - 3\cos\alpha = 32\left(\frac{1}{2}\right)^4 = 2$

or, $2\cos^2\alpha - 3\cos\alpha - 2 = 0$

or $2\cos^2\alpha - 4\cos\alpha + \cos\alpha - 2 = 0$ or, $(\cos\alpha - 2)(2\cos\alpha + 1) = 0$

or $2\cos\alpha + 1 = 0$ $[\because \cos\alpha \neq 2]$ or, $\cos\alpha = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow$ $\alpha = 2n\pi \pm \frac{2\pi}{3}$, where n = 0, ±1, ±2,.........

11. For what value of k the equation sin x + cos(k + x) + cos(k – x) = 2 has real solutions?

Sol. Given equation is sin x + cos(k + x) + cos(k – x) = 2
or sin x + 2 cos k . cos x = 2 or 2 cos k . cos x + sin x = 2
This equation is of the form a cos x + b sin x = c
Here a = 2 cos k, b = 1 and c = 2

Since for real solutions, $|c| \leq \sqrt{a^2+b^2}$

$\therefore$ $|2| \leq \sqrt{1+4\cos^2 k}$ or $2 \leq \sqrt{1+4\cos^2 k}$

$\Rightarrow$ $\cos^2 k \geq \frac{3}{4}$

$\Rightarrow$ $\sin^2 k \leq \frac{1}{4}$

$\Rightarrow$ $\sin^2 k - \frac{1}{4} \leq 0$

or $\left(\sin k + \frac{1}{2}\right)\left(\sin k - \frac{1}{2}\right) \leq 0$

$\Rightarrow$ $-\frac{1}{2} \leq \sin k \leq \frac{1}{2}$

$\Rightarrow$ $n\pi - \frac{\pi}{6} \leq k \leq n\pi + \frac{\pi}{6}$

Solved Problems (Objective)

1. If $x^2 - 4x + 5 - \sin y = 0$, $y \in [0, 2\pi)$, then

(a) $x = 1, y = 0$ (b) $x = 1, y = \pi/2$ (c) $x = 2, y = 0$ (d) $x = 2, y = \pi/2$

Sol. $(x-2)^2 + 1 = \sin y \Rightarrow x = 2, \sin y = 1$

$\Rightarrow \quad x = 2, y = \pi/2$

2. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by

(a) $\left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (b) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (c) $\left(\frac{\pi}{10}, -\frac{3\pi}{10}\right)$ (d) none of these

Sol. We have $|4\sin x - 1| < \sqrt{5}$ $\Rightarrow \quad -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$

$\Rightarrow \quad -\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \left(\frac{\sqrt{5}+1}{4}\right)$ $\Rightarrow \quad -\sin\frac{\pi}{10} < \sin x < \cos\frac{\pi}{5}$

$\Rightarrow \quad \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$ $\Rightarrow \quad \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\frac{3\pi}{10}$

$\Rightarrow \quad x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$

3. The values of x between 0 and 2π, which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in increasing A.P. with common difference

(a) $\pi/4$ (b) $\pi/8$ (c) $3\pi/8$ (d) $5\pi/8$

Sol. We have $\sin x\sqrt{8\cos^2 x} = 1$ $\Rightarrow \quad \sin x\,|\cos x| = \frac{1}{2\sqrt{2}}$

Case –I when $\cos x > 0$

In this case $\sin x \cos x = \frac{1}{2\sqrt{2}}$ $\Rightarrow \quad \sin 2x = +\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$; As x lies between 0 and 2π and $\cos x > 0$, $x = \frac{\pi}{8}, \frac{3\pi}{8}$

Case–II When $\cos x < 0$. In this case $\sin x\,|\cos x| = \frac{1}{2\sqrt{2}}$

$\Rightarrow \quad \sin x \cos x = -\frac{1}{2\sqrt{2}}$ or $\sin 2x = -\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin x \cos x = -\frac{1}{2\sqrt{2}}$ or $\sin 2x = -\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x = \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ $\Rightarrow \quad x = \frac{5\pi}{8}, \frac{7\pi}{8}$ as $\cos x < 0$

Thus the values of x satisfying the given equation which lie between 0 and 2π are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

These are in A.P. with common difference $\pi/4$.

4. The number of points inside or on the circle $x^2+y^2=4$ satisfying $\tan^4 x+\cot^4 x+1=3\sin^2 y$ is

(a) one (b) two (c) four (d) infinite

Sol. $\tan^4 x+\cot^4 x+1=(\tan^2 x-\cot^2 x)^2+3\ge 3$; $3\sin^2 y\le 3 \Rightarrow \tan^2 x=\cot^2 x, \sin^2 y=1$

$\Rightarrow$ $\tan x=\pm 1, \sin y=\pm 1$ $\Rightarrow$ $x=\pm \pi/4, \pm 3\pi/4, \ldots$

But $x^2\le 4$

$\Rightarrow$ $-2\le x\le 2$ $\Rightarrow$ $x=\pm \pi/4$ only. $\sin y=\pm 1$

$\Rightarrow$ $y=\pm \pi/2, \pm 3\pi/2, \ldots..$

But $y^2\le 4$

$\Rightarrow$ $y=\pm \pi/2$ only. So four solutions are possible.

5. The equation $3\sin^2 x+10\cos x-6=0$ is satisfied if $(n\in I)$

(a) $x=n\pi+\cos^{-1}(1/3)$ (b) $x=n\pi-\cos^{-1}(1/3)$

(c) $x=2n\pi\pm\cos^{-1}(1/3)$ (d) $x=\frac{n\pi}{2}-\cos^{-1}(1/3)$

Sol. The given equation is equivalent to $3(1-\cos^2 x)+10\cos x-6=0$

$\Rightarrow$ $3\cos^2 x-10\cos x+3=0$ $\Rightarrow$ $(3\cos x-1)(\cos x-3)=0$

Therefore $\cos x=1/3$ (because $\cos x\neq 3$). Hence $x=2n\pi\pm\cos^{-1}(1/3)$, $n\in I$

6. The equation $2\sin\frac{x}{2}\cos^2 x-2\sin\frac{x}{2}\sin^2 x=\cos^2 x-\sin^2 x$ has a root for which

(a) $\sin 2x=1$ (b) $\sin 2x=-1$ (c) $\cos x=1/2$ (d) $\cos 2x=-1/2$

Sol. The given equation can be written as $2\sin\frac{x}{2}(\cos^2 x-\sin^2 x)=\cos^2 x-\sin^2 x$

or $2\sin\frac{x}{2}\cos 2x=\cos 2x$ $\Rightarrow \left(2\sin\frac{x}{2}-1\right)\cos 2x=0$

Hence $\cos 2x=0$ or $\sin(x/2)=1/2$. That is , $2x=n\pi+\pi/2$ or $x/2=k\pi+(-1)^k\pi/6$ $(n, k\in I)$.

In other words, $x=\frac{n\pi}{2}+\frac{\pi}{4}$ or $x=2k\pi+(-1)^k\frac{\pi}{3}$

If $x=\frac{n\pi}{2}+\frac{\pi}{4}$, then $\sin 2x=\pm 1$, and if $x=2k\pi+(-1)^k\frac{\pi}{3}$,

$\cos x=\cos\frac{\pi}{3}=\frac{1}{2}$ and $\cos 2x=\cos\frac{2\pi}{3}=-\frac{1}{2}$.

7. Number of solutions of the equation $\tan x+\sec x=2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

Sol. The given equation can be written as $\frac{1+\sin x}{\cos x}=2\cos x$

$\Rightarrow$ $1 + \sin x = 2\cos^2 x = 2(1 - \sin^2 x)$

$\Rightarrow$ $2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow$ $(1 + \sin x)(2\sin x - 1) = 0 \Rightarrow \sin x = -1$ or $1/2$

Now, $\sin x = -1$

$\Rightarrow$ $x = \frac{3\pi}{2}$ for which the given equation is not meaningful and $x = \frac{1}{2}$

$\Rightarrow$ $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\therefore$ The required number of solutions are 2.

8. The number of solutions of x for the equation $2^{\cos\alpha} = |\sin x|$, where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $x \in [-2\pi, 2\pi]$ is

(a) zero (b) 2 (c) 4 (d) more than four

Sol. We have $2^{\cos\alpha} = |\sin x|$;

It is true only for $|\sin x| = 1$

$\therefore$ $\sin x = \pm 1$

So, $x = 2n\pi \pm \pi/2$ $\because$ $x \in [-2\pi, 2\pi]$

$\therefore$ $x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{-3\pi}{2}$. Then the no. of solutions = 4.

9. For what and only what values of α lying between 0 and π is the inequality, $\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$ valid ?

(a) $\alpha \in \left(0, \frac{\pi}{4}\right)$ (b) $\alpha \in \left(0, \frac{\pi}{2}\right)$ (c) $\alpha \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) none of these

Sol. We have $\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$

$\Rightarrow$ $\sin\alpha\cos\alpha(\cos^2\alpha - \sin^2\alpha) > 0$

$\Rightarrow$ $\sin\alpha\cos\alpha(1 - \tan^2\alpha) > 0$ $(\because \sin\alpha > 0$ for $0 < \alpha < \pi)$

$\Rightarrow$ $\cos\alpha(1 - \tan^2\alpha) > 0$

$\Rightarrow$ $\cos\alpha > 0$ and $1 - \tan^2\alpha > 0$

$\Rightarrow$ $\cos\alpha < 0$ and $1 - \tan^2\alpha < 0$

$\Rightarrow$ $\alpha \in \left(0, \frac{\pi}{4}\right)$ or $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$

10. $\tan|x| = |\tan x|$ if :

(a) $x \in \left(-\frac{\pi(2k+1)}{2}, -\pi k\right]$ (b) $x \in \left[\pi k, \frac{\pi(2k+1)}{2}\right)$

(c) $x \in \left(-\pi k, -\frac{\pi(2k-1)}{2}\right)$ (d) $x \in \left(\frac{\pi(2k-1)}{2}, \pi k\right)$, $k \in N$

Sol. R.H.S. ≥ 0 for all x, the given condition is true for those values of $|x|$ which lie in the I or III quadrant and the values of x given by (a) and (b) satisfy these conditions.

11. The number of solutions of the equation, $|\cot x| = \cot x + \frac{1}{\sin x}$ $(0 \le x \le 2\pi)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

Sol. If $\cot x \geq 0$

then $\frac{1}{\sin x} = 0$ (impossible)

Now if $\cot x < 0$

then $-\cot x = \cot x + \frac{1}{\sin x}$.

$\Rightarrow \quad \frac{2\cos x + 1}{\sin x} = 0$

$\Rightarrow \quad \cos x = -\frac{1}{2}$

$\therefore \quad x = 2n\pi \pm \frac{2\pi}{3}$, $n \in I$ and $0 \le x \le 2\pi$

$\therefore$ then $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

12. The general solution of, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is

(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$

(c) $(-1)^n\left(\frac{n\pi}{2}\right) + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\left(\frac{3}{2}\right)$

Sol. $\sin x - 3\sin 2x + \sin 3x - \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow \quad 2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x$

$\Rightarrow \quad (2\cos x - 3)\sin 2x = \cos 2x\,(2\cos x - 3)$

$2\cos x - 3 \neq 0$

$\Rightarrow \quad \tan 2x = 1$

$\therefore \quad x = \frac{n\pi}{2} + \frac{\pi}{8}$

EXERCISE 1 (A)

ONLY ONE OPTION IS CORRECT

1. $\sin^3 x + \sin x \cos x + \cos^3 x = 1$, then x =

(A) $n\pi, n\pi \pm \frac{\pi}{3}$ (B) $2n\pi, 2n\pi + \frac{\pi}{2}$ (C) $n\pi, n\pi \pm \frac{\pi}{3}$ (D) $2n\pi, 2n\pi \pm \frac{\pi}{2}$

2. The general value of 'θ' satisfying $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

(A) $(n+1)\frac{\pi}{9}$ (B) $(n+1)\frac{\pi}{3}$ (C) $(3n+1)\frac{\pi}{3}$ (D) $(3n+1)\frac{\pi}{9}$

3. The equation $3(\sin x + \cos x) - 2(\sin^3 x + \cos^3 x) = 8$ has

(A) no solution (B) only one solution (C) two solutions (D) three solutions

4. If $\sin 2x \cos 2x \cos 4x = \lambda$ has a solution. Then λ lies in the interval

(A) $[-1/2, 1/2]$ (B) $[-1/4, 1/4]$ (C) $[-1/3, 1/3]$ (D) None

5. If $\tan\left(\frac{\pi}{4}+\theta\right) + \tan\left(\frac{\pi}{4}-\theta\right) = 4$ then θ =

(A) $n\pi + \frac{\pi}{6}$ (B) $n\pi \pm \frac{\pi}{6}$ (C) $n\pi \pm \frac{\pi}{4}$ (D) None

6. The number of distinct solutions of $\sin 5\theta \cdot \cos 3\theta = \sin 9\theta \cdot \cos 7\theta$ in $[0, \pi/2]$ is

(A) 4 (B) 5 (C) 8 (D) 9

7. The number of solutions of $\sin^2\theta + 3\cos\theta = 3$ in $[-\pi, \pi]$ is

(A) 4 (B) 2 (C) 0 (D) none of these

8. The equation $k\cos x - 3\sin x = k + 1$ is solvable only if k belongs to the interval

(A) $[4, +\infty)$ (B) $[-4, 4]$ (C) $(-\infty, 4]$ (D) none of these

9. If $\sin^2\theta - 2\sin\theta - 1 = 0$ is to be satisfied for exactly 4 distinct values of $\theta \in [0, n\pi]$, $n \in N$, then the least value of n is

(A) 2 (B) 6 (C) 4 (D) 1

10. If $2\tan^2 x - 5\sec x$ is equal to 1 for exactly 7 distinct values of $x \in \left[0, \frac{n\pi}{2}\right], n \in N$, then the greatest value of n is

(A) 6 (B) 12 (C) 13 (D) 15

11. The most general values of θ satisfying $\tan\theta + \tan\left(\frac{3\pi}{4}+\theta\right) = 2$ are

(A) $n\pi \pm \frac{\pi}{3}, n \in z$ (B) $2n\pi + \frac{\pi}{3}, n \in z$ (C) $2n\pi \pm \frac{\pi}{3}, n \in z$ (D) $n\pi + (-1)^n \frac{\pi}{3}, n \in z$

12. The most general solutions of the equation $\sec^2 x = \sqrt{2}(1 - \tan^2 x)$ are given by

(A) $n\pi + \frac{\pi}{8}$ (B) $n\pi \pm \frac{\pi}{4}$ (C) $n\pi \pm \frac{\pi}{8}$ (D) none of these

13. The most general values of x for which $\sin x + \cos x = \min_{a \in R} \{1, a^2 - 4a + 6\}$ are given by

(A) $2n\pi$ (B) $2n\pi + \frac{\pi}{2}$ (C) $n\pi + (-1)^n . \frac{\pi}{4} - \frac{\pi}{4}$ (D) none of these

14. The sum of all the solutions of the equation $\cos x . \cos\left(\frac{\pi}{3} + x\right) . \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}, x \in [0, 6\pi]$ is

(A) 15π (B) 30π (C) $\frac{110\pi}{3}$ (D) none of these

15. If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is

(A) π (B) 2π (C) $\frac{5\pi}{2}$ (D) none of these

16. If $0 \le x \le 3\pi, 0 \le y \le 3\pi$ and $\cos x \sin y = 1$ then the possible number of values of the ordered pair (x, y) is
(A) 6 (B) 12 (C) 8 (D) 15

17. If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$ then the maximum number of values of the pair (r, θ) is
(A) 8 (B) 10 (C) 6 (D) none of these

18. The general solution of $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$ is
(A) $n\pi + \pi/8$ (B) $n\pi/2 + \pi/8$
(C) $(-1)^n (n\pi/2 + \pi/8)$ (D) $2n\pi + \cos^{-1}(3/2)$

19. The number of real solution of $\sin e^x . \cos e^x = 2^{x-2} + 2^{-x-2}$ is
(A) zero (B) one (C) two (D) infinite

20. Total number of solution of the equation $\sin^4 x + \cos^4 x = \sin x . \cos x$ in $[0, 2\pi]$
(A) 2 (B) 4 (C) 6 (D) none of these

21. If $5\cos 2\theta + 2\cos^2 \theta/2 + 1 = 0, -\pi < \theta < \pi$, then θ
(A) $\pi/3$ (B) $\pi/3, \cos^{-1}(3/5)$
(C) $\cos^{-1}(3/5)$ (D) $\pi/3, \pi - \cos^{-1}(3/5)$

22. The general solution of the equation $\cos x \cos 6x = -1$ is
(A) $x = (2n+1)\pi, n \in I$ (B) $x = 2n\pi, n \in I$
(C) $x = (2n-1)\pi, n \in I$ (D) none of these

23. $6\sin\theta + 7\cos\theta = 9$ if

(A) $\tan\theta = \frac{3}{4}$ (B) $\tan\theta = \frac{7}{8}$ (C) $\tan\theta = \frac{8}{15}$ (D) $\tan\theta = \frac{8}{17}$

24. If $\sin A = \sin B$ and $\cos A = \cos B$ then

(A) $A = B + n\pi, n\in I$ (B) $A = B - n\pi, n\in I$

(C) $A = 2n\pi + B, n\in I$ (D) $A = n\pi - B, n\in I$

25. The equation $4\sin^2 x + 4\sin x + a^2 - 3 = 0$ possesses a solution if 'a' belongs to the interval

(A) $(-1, 3)$ (B) $(-3, 1)$ (C) $[-2, 2]$ (D) $R - (-2, 2)$

26. $\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$ if

(A) $\theta = n\pi + \frac{\pi}{6}$ (B) $\theta = 2n\pi - \frac{\pi}{6}$ (C) $\theta = n\pi + (-1)^n\frac{\pi}{6}$ (D) $\theta = n\pi - \frac{\pi}{6}$

27. Total number of solutions of $\sin^2 x - \sin x - 1 = 0$ in $[-2\pi, 2\pi]$ is equal to

(A) 2 (B) 4 (C) 6 (D) 8

28. $6\tan^2 x - 2\cos^2 x = \cos 2x$ if

(A) $\cos 2x = -1$ (B) $\cos 2x = 1$ (C) $\cos 2x = -1/2$ (D) $\cos 2x = 1/2$

29. The general solution of the trigonometrical equation $\sin x + \cos x = 1$ for $n = 0, \pm 1, \pm 2$ ......is given by

(A) $x = 2n\pi$ (B) $x = 2n\pi + \pi/2$

(C) $x = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$ (D) none of these

30. The complete solution of the equation $7\cos^2 x + \sin x\cos x - 3 = 0$ is given by

(A) $n\pi + \pi/2 (n\in I)$ (B) $n\pi - \pi/4 (n \in I)$

(C) $n\pi + \tan^{-1}(4/3) (n\in I)$ (D) $n\pi + \frac{3\pi}{4}, k\pi + \tan^{-1}(4/3) (k, n\in I)$

31. The equation $a\sin x + b\cos x = c$ where $|c| > \sqrt{a^2+b^2}$ has

(A) one solution (B) two solution

(C) no solution (D) infinite number of sol.

32. The equation $\cos x + \sin x = 2$ has

(A) only one solution (B) two solution

(C) no solution (D) infinite number of solutions

33. General solution of the equation $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta\tan 4\theta\tan 7\theta$

(A) $\theta = \frac{n\pi}{12}, n\in Z$ (B) $\theta = \frac{n\pi}{9}, n\in Z$

(C) $\theta = n\pi + \frac{\pi}{12}, n\in Z$ (D) None of these

34. The total number of solutions of $\tan x + \cot x = 2\operatorname{cosec} x$ in $[-2\pi, 2\pi]$ is
(A) 2 (B) 4 (C) 6 (D) 8

35. If $\sin x + \cos x = \sqrt{\left(y + \frac{1}{y}\right)}$, $x \in [0, \pi]$, then

(A) $x = \frac{\pi}{4}, y = 1$ (B) $x \in R, y = 1$

(C) $x = \frac{\pi}{4}, y = 2$ (D) $x = \frac{3\pi}{4}, y \in R$

36. If $x, y \in [0, 2\pi]$ and $\sin x + \sin y = 2$, then the value of $x + y$ is

(A) π (B) $\frac{\pi}{2}$ (C) 3π (D) None of these

37. The range of 'y' such that the equation in x, $y + \cos x = \sin x$ has a real solution is
(A) $[-2,2]$ (B) $[-\sqrt{2}, \sqrt{2}]$ (C) $[-2,2]$ (D) $[-2,2]$

38. If $3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$, then θ is equal to

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi + \frac{\pi}{8}$ (C) $n\pi + \frac{\pi}{3}$ (D) None of these

EXERCISE 1 (B)

MORE THAN ONE OPTIONS MAY BE CORRECT

1. If $|\cos x|^{\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2}} = 1$, then possible values of x

(A) $n\pi$ or $n\pi + (-1)^n \pi/6,\ n \in I$ (B) $n\pi$ or $2n\pi + \pi/2$ or $n\pi + (-1)^n \pi/6,\ n \in I$

(C) $n\pi + (-1)^n \pi/6,\ n \in I$ (D) $n\pi,\ n \in I$

2. If $\frac{\sin^2 2x + 4\sin^4 x - 4\sin^2 x\cos^2 x}{4 - \sin^2 2x - 4\sin^2 x} = \frac{1}{9}$ and $0 < x < \pi$, then value of x is
(A) $\pi/3$ (B) $\pi/6$ (C) $2\pi/3$ (D) $5\pi/6$

3. If $\sin 5\theta = a\sin^5\theta + b\sin^3\theta + c\sin\theta + d$, then
(A) a + b + c + d = 1 (B) a + b + c = 1 (C) 5a + 4b = 0 (D) b + 4c = 0

4. The number of solution(s) of the equation $\cos\left(\pi\sqrt{x-4}\right)\cos\left(\pi\sqrt{x}\right) = 1$
(A) none (B) one (C) two (D) more than one

5. If $\cos(\theta-\alpha)=a,\ \sin(\theta-\beta)=b$, then

(A) $\sin(\alpha-\beta)=ab-\sqrt{(1-a^2-b^2+a^2b^2)}$ (B) $\cos(\alpha-\beta)=a\sqrt{(1-b^2)}+b\sqrt{(1-a^2)}$

(C) $\sin(\alpha-\beta)=ab+\sqrt{(1-a^2-b^2+a^2b^2)}$ (D) $\cos(\alpha-\beta)=a\sqrt{(1-b^2)}-b\sqrt{(1-a^2)}$

6. A solution (x, y) of the system of equations $x-y=1/3$ and $\cos^2(\pi x)-\sin^2(\pi y)=1/2$ is given by

(A) (7/6, 5/6) (B) (2/3, 1/3) (C) (–5/6, –7/6) (D) (13/6, 11/6)

7. $\sqrt{\cos 2x}+\sqrt{(1+\sin 2x)} = 2\sqrt{(\sin x+\cos x)}$, if

(A) $\sin x+\cos x=0$ (B) $x=2n\pi$ (C) $x=n\pi-\pi/4$ (D) $\sin x-\cos x=0$

8. $|\cos x|=\cos x-2\sin x$ if

(A) $x=n\pi$ (B) $x=2n\pi$ (C) $x=n\pi+\pi/4$ (D) $x=(2n+1)\pi+\pi/4$

9. The number of solutions of the equation $x^3+x^2+4x+2\sin x=0$ in $0\le x\le 2\pi$ is

(A) zero (B) one (C) two (D) four

10. The number of points of intersection of the two curves $y=2\sin x$ and $y=5x^2+2x+3$, is

(A) ∞ (B) 0 (C) 1 (D) 2

11. Number of real roots of the equation $\sec\theta+\operatorname{cosec}\theta=\sqrt{15}$ lying between 0 and 2π is

(A) 8 (B) 4 (C) 2 (D) 0

12. Solve : $4\sin^4 x+\cos^4 x=1$, then x is equal to $(n\in Z)$

(A) $n\pi$ (B) $n\pi\pm\sin^{-1}\sqrt{\dfrac{2}{5}}$ (C) $\dfrac{2n\pi}{3}$ (D) $2n\pi\pm\dfrac{\pi}{3}$

13. A general solution of the equation, $\tan^2\theta+\cos 2\theta=1$ is $(n\in Z)$

(A) $n\pi-\dfrac{\pi}{4}$ (B) $2n\pi+\dfrac{\pi}{4}$ (C) $n\pi+\dfrac{\pi}{4}$ (D) $2n\pi-\dfrac{\pi}{4}$

14. $\sin\theta+\sqrt{3}\cos\theta=6x-x^2-11, 0\le\theta\le 4\pi, x\in R$, holds for

(A) No values of x and θ (B) One value of x and two value of θ

(C) Two values of x and two values of θ (D) Two points of values of (x,θ)

15. For the smallest positive values of x and y the equation, $2(\sin x+\sin y)-2\cos(x-y)=3$, has a solution then which of the following is/are true

(A) $\sin\dfrac{x+y}{2}=1$ (B) $\cos\left(\dfrac{x-y}{2}\right)=\dfrac{1}{2}$

(C) Number of ordered pairs (x, y) is 2 (D) Number of ordered pairs (x, y) is 3

16. for the equation $1-2x-x^2=\tan^2(x+y)+\cot^2(x+y)$.

(A) Exactly one value of x exists (B) exactly two value of x exists

(C) $y=-1+n\pi+\pi/4, n\in Z$ (D) $y=1+n\pi/2+\pi/4, n\in Z$

17. If $x+y=\pi/4$ and $\tan x+\tan y=1$, then

(A) $\sin x=0$ always
(B) when x = $n\pi+\pi/4$ then $y=-n\pi$
(C) when $x=-n\pi$ then $y=n\pi+\dfrac{\pi}{4}$
(D) when $x=n\pi+\pi/4$ then $y=n\pi-\dfrac{\pi}{4}$

18. If $x+y=\dfrac{2\pi}{3}$ and $\dfrac{\sin x}{\sin y}=2$, then

(A) number of values of $x\in[0,4\pi]$ are 4
(B) number of values of $x\in[0,4\pi]$ are 2
(C) number of values of $y\in[0,4\pi]$ are 4
(D) number of values of $y\in[0,4\pi]$ are 8

19. If $\cos\left(x+\dfrac{\pi}{3}\right)+\cos x=a$ has real solutions. Then

(A) number of integral values of 'a' are 3
(B) sum of number of integral values of 'a' is 0
(C) when $a=1$, number of solutions for $x\in[0,2\pi]$ are 3
(D) when $a=1$ number of solutions for $x\in[0,2\pi]$ are 2

COMPREHENSION TYPE

Based upon each passage, three multiple choice questions have to be answered. Each question has four choices A,B,C and D, out of which *only one* is correct.

PASSAGE - 1

Consider the cubic equation $x^3-(1+\cos\theta+\sin\theta)x^2+(\cos\theta\sin\theta+\cos\theta+\sin\theta)x-\sin\theta\cos\theta=0$ whose roots are x_1,x_2 & x_3

20. The value of $x_1^2+x_2^2+x_3^2$ equals

(A) 1
(B) 2
(C) $2\cos\theta$
(D) $\sin\theta(\sin\theta+\cos\theta)$

21. Number of values of θ in $[0,2\pi]$ for which atleast two roots are equal

(A) 3
(B) 4
(C) 5
(D) 6

22. Greatest possible difference between two of the roots if $\theta\in[0,2\pi]$ is

(A) 2
(B) 1
(C) $\sqrt{2}$
(D) $2\sqrt{2}$

PASSAGE - 2

Consider the equation, $\sec\theta+\operatorname{cosec}\theta=a,\theta\in[0,2\pi]-\left\{\dfrac{\pi}{2},\pi,\dfrac{3\pi}{2}\right\}$

23. If the equation has four real roots then

(A) $|a|\geq 2\sqrt{2}$
(B) $|a|<2\sqrt{2}$
(C) $a\geq -2\sqrt{2}$
(D) None of these

24. If the equation has two real roots then

(A) $|a| \geq 2\sqrt{2}$ (B) $a < 2\sqrt{2}$ (C) $|a| < 2\sqrt{2}$ (D) None of these

25. If the equation has no real roots then

(A) $|a| \geq 2\sqrt{2}$ (B) $a < 2\sqrt{2}$ (C) $|a| < 2\sqrt{2}$ (D) None of these

PASSAGE - 3

Consider the system of equations

$\sin x \cos 2y = (a^2 - 1)^2 + 1$, $\cos x \sin 2y = a + 1$

26. Number of values of 'a' for which the system has a solution is

(A) 1 (B) 2 (C) 3 (D) Infinite

27. Number of values of $x \in [0, 2\pi]$ when the system has solution for permissible values of 'a'

(A) 1 (B) 2 (C) 3 (D) 4

28. Number of values of $y \in [0, 2\pi]$ when the system has solution for permissible values of 'a'

(A) 2 (B) 3 (C) 4 (D) 5

PASSAGE - 4

Consider the system of equations

$x\cos^3 y + 3x\cos y \sin^2 y = 14$, $x\sin^3 y + 3x\cos^2 y \sin y = 13$

29. The value(s) of x are

(A) $\pm 5\sqrt{5}$ (B) $\pm\sqrt{5}$ (C) $\pm\frac{1}{\sqrt{5}}$ (D) None of these

30. The number of values of $y \in [0, 6\pi]$ are

(A) 5 (B) 3 (C) 4 (D) 6

31. The values of $\sin^2 y + 2\cos^2 y$ is

(A) 4/5 (B) 9/5 (C) 2 (D) None of these

REASONING TYPE

(A) If both the statements are TRUE and STATEMENT 2 is the correct explanation of STAEMENT1

(B) If both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT1

(C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE

(D) If STATEMENT 1 is FALSE and STATEMENT 2 is TRUE

32. **Statement 1:** The value of x for which $(\sin x + \cos x)^{1+\sin 2x} = 2$, when $0 \leq x \leq \pi$ is $\pi/4$

Statement 2: The maximum value of $\sin x + \cos x$ occurs when $x = \frac{\pi}{4}$

33. **Statement 1**: Equation, $x\sin x = 1$, has four roots for $x \in (-\pi, \pi)$

Statement 2: The graph of $y = \sin x$ and $y = \frac{1}{x}$ cuts exactly two times for $x \in (0, \pi)$

34. **Statement 1**: $\sin x = a$, where $-1 < a < 0$, then for $x \in [0, n\pi]$ has $2(n-1)$ solutions $\forall n \in N$

Statement 2: $\sin x$ takes value a exactly two time when we take one complete rotation covering all the quadrant starting from $x = 0$

35. **Statement 1**: Equation $\sqrt{1-\sin 2x} = \sin x$ has 1 solution for $x \in [0, \frac{\pi}{4}]$

Statement 2: $\cos x > \sin x$ when $x \in [0, \frac{\pi}{4}]$

36. **Statement 1:** General solution of $\frac{\tan 4x - \tan 2x}{1+\tan 4x \tan 2x} = 1$ is $x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in I$

Statement 2: General solution of $\tan\alpha = 1$ is $\alpha = n\pi + \frac{\pi}{4}$

MATRIX MATCH

37. **Column I (Equation)** **Column II (solution)**

(A) $\cos^2 2x + \cos^2 x = 1$ (P) $x = \left\{n\pi + \frac{\pi}{4}\right\} \cup \left\{n\pi + \frac{\pi}{6}\right\} n \in Z$

(B) $\cos x + \sqrt{3}\sin x = \sqrt{3}$ (Q) $x = \frac{2n\pi}{3}, n \in Z$

(C) $1 + \sqrt{3}\tan^2 x = \left(1+\sqrt{3}\right)\tan x$ (R) $x = (2n-1)\frac{\pi}{6}, n \in Z$

(D) $\tan 3x - \tan 2x - \tan x = 0$ (S) $x = \left\{2n\pi + \frac{\pi}{2}\right\} \cup \left\{2n\pi - \frac{\pi}{6}\right\}, n \in Z$

38. **Column - I** **Column - II**

(A) The number of real roots of the equation $\cos^7 x + \sin^2 x = 1$ in $(-\pi, \pi)$ is (P) 1

(B) The value of $\sqrt{3}\operatorname{cosec} 20° - \sec 20°$ is (Q) 4

(C) $4\cos 36° - 4\cos 72° + 4\sin 18° \cdot \cos 36°$ equals (R) 3

(D) The number of values of x where $x \in [-2\pi, 2\pi]$. Which satisfy $\operatorname{cosec} x = 1 + \cot x$ (S) 2

39. **Column - I** | **Column - II**

Column - I		Column - II	
(A)	If $\sin\theta + \sin\phi = \frac{1}{2}$ and $\cos\theta + \cos\phi = 2$ then $\cot\left(\frac{\theta+\phi}{2}\right)$	(P)	No solution
		(Q)	$\frac{1}{3}$
(B)	$\sin^2\alpha + \sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right) = \sec\alpha$	(R)	1
(C)	If $\tan\theta = 3\tan\phi$, then maximum value of $\tan^2(\theta-\phi)$ is	(S)	4

EXERCISE 2 (A)

ONLY ONE OPTION IS CORRECT

1. If $\tan(\pi\cos x) = \cot(\pi\sin x)$, then $\cos\left(\frac{\pi}{4}-x\right)$ is equal to

(A) $\pm\frac{1}{2}$ (B) ± 1 (C) $\pm\frac{1}{2\sqrt{2}}$ (D) $\pm\frac{1}{\sqrt{2}}$

2. The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have at least one solution if

(A) $|a| \le 4$ (B) $|a| \le 2$ (C) $|a| \le \sqrt{3}$ (D) None of these

3. If the equation $x^2 + 4 + 3\sin(ax+b) - 2x = 0$ has at least one real solution where $a, b \in [0, 2\pi]$. Then one possible value of (a + b) can be equal to

(A) $\frac{7\pi}{2}$ (B) $\frac{5\pi}{2}$ (C) $\frac{9\pi}{2}$ (D) none of these

4. If $3\sin x + 4\cos ax = 7$ has atleast one solution, then 'a' has to be necessarily a

(A) odd integer (B) even integer
(C) rational integer (D) irrational integer

5. Total number of solutions of $|\cot x| = \cot x + \frac{1}{\sin x}$, $x \in [0, 3\pi]$ is equal to

(A) 1 (B) 2 (C) 3 (D) zero

6. Total number of values of x in $(-2\pi, 2\pi)$ and satisfying $\log_{|\cos x|}|\sin x| + \log_{|\sin x|}|\cos x| = 2$ is equal to

(A) 2 (B) 4 (C) 6 (D) 8

7. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$ if

(A) $\theta\in\left(0,\dfrac{\pi}{2}\right)$ (B) $\theta\in\left(\dfrac{\pi}{2},\pi\right)$ (C) $\theta\in\left(\pi,\dfrac{3\pi}{2}\right)$ (D) $\theta\in\left(\dfrac{3\pi}{2},2\pi\right)$

8. The number of solutions of $|\cos x| = \sin x$, $0 \le x \le 4\pi$, is
(A) 8 (B) 4 (C) 2 (D) none of these

9. The most general solutions of $2^{\sin x} + 2^{\cos x} = 2^{1-1/\sqrt{2}}$ are

(A) $n\pi - \dfrac{\pi}{4}$ (B) $(2n+1)\pi+\dfrac{\pi}{4}$ (C) $n\pi + (-1)^n\dfrac{\pi}{4}$ (D) $2n\pi \pm \dfrac{\pi}{4}$

10. The least positive non integral solution of $\sin\pi(x^2+x) - \sin\pi x^2 = 0$ is
(A) rational
(B) irrational of the form $\sqrt{p}$
(C) irrational of the form $\dfrac{\sqrt{p}-1}{4}$, where p is an odd integer
(D) irrational of the form $\dfrac{\sqrt{p}+1}{4}$, where p is an even integer

11. If the equation $2\cos x + \cos 2\lambda x = 3$ has only one solution then λ is
(A) 1 (B) a rational number (C) an irrational number (D) none of these

12. If $\cos^4 x + a\cos^2 x + 1 = 0$ has at least one solution, then
(A) $a \in [2, \infty)$ (B) $a \in [-2, 2]$ (C) $a \in (-\infty, -2]$ (D) $a \in R-(-2, 2)$

13. The number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are
(A) one (B) three (C) two (D) four

14. The number of solutions of the equation $\sin\left(\dfrac{\pi x}{2\sqrt{3}}\right) = x^2 - 2\sqrt{3}x + 4$

(A) is zero (B) is only one (C) is only two (D) is greater than 2

15. The number of pairs (x, y) satisfying the equations $\sin x + \sin y = \sin(x+y)$ and $|x| + |y| = 1$ is
(A) 2 (B) 4 (C) 6 (D) infinite

16. The smallest positive integral value of p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ in x has a solution in $[0, 2\pi]$ is
(A) 2 (B) 1 (C) 3 (D) none of these

17. If $0 < x, y < 2\pi$, the number of solutions of the system of equations $\sin x \sin y = 3/4$ and $\cos x \cos y = 1/4$ is
(A) 0 (B) 1 (C) 2 (D) infinite

18. If $|\sin x + \cos x| = |\sin x| + |\cos x|$, then x belongs to the quadrant
(A) I or III (B) II or IV (C) I or II (D) III or IV

19. If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x =$
(A) $(2n+1)\frac{\pi}{4}$ (B) $\frac{4}{(2n+1)\pi}$ (C) $(2n+1)\frac{\pi}{2}$ (D) $\frac{4\pi}{(2n+1)}$

20 The number of solutions of the equation $\sin^5 x - \cos^5 x = \frac{1}{\cos x} - \frac{1}{\sin x} (\sin x \neq \cos x)$ is
(A) 0 (B) 1 (C) infinite (D) none of these

21. The number of solutions of the equation $12\cos^3 x - 7\cos^2 x + 4\cos x = 9$ is
(A) 0 (B) 2 (C) Infinite (D) None of these

22. Which of the following is not the general solution of the equation $2^{\cos 2x} + 1 = 3.2^{-\sin^2 x}$
(A) $n\pi$ (B) $(n+\frac{1}{2})\pi$ (C) $(n-\frac{1}{2})\pi$ (D) None of these

23. The number of solution of the equation, $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$ in $0 \le x \le 3\pi$ is
(A) 3 (B) 4 (C) 5 (D) 6

24. For $n \in Z$, the general solution of the equation $(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$
(A) $\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$ (B) $\theta = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$
(C) $\theta = 2n\pi \pm \frac{\pi}{4}$ (D) $\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$

25. $\tan\left(\frac{p\pi}{4}\right) = \cot\left(\frac{q\pi}{4}\right)$ if $n \in Z$
(A) $p+q=0$ (B) $p+q=2n+1$ (C) $p+q=2n$ (D) $p+q=2(2n+1)$

26. Number of roots of the equation, $(1-\tan\theta)(1+\sin 2\theta) = 1+\tan\theta$ for $\theta \in [0, 2\pi]$
(A) 3 (B) 4 (C) 5 (D) None of these

27. One of the general solutions of the equation, $4\sin\theta\sin 2\theta\sin 4\theta = \sin 3\theta$, is
(A) $(3n\pm 1)\frac{\pi}{12}$ (B) $(4n\pm 1)\frac{\pi}{9}$ (C) $(3n\pm 1)\frac{\pi}{9}$ (D) $(3n\pm 1)\frac{\pi}{3}$

28. The general solution of the equation, $\sin 3\alpha = 4\sin\alpha\sin(x+\alpha)\sin(x-\alpha)$ is
(A) $n\pi \pm \pi/4$ (B) $n\pi \pm \pi/3$ (C) $n\pi \pm \pi/9$ (D) $n\pi \pm \pi/12$

29. One of the general solution of the equation $\sqrt{3}\cos\theta - 3\sin\theta = 4\sin 2\theta\cos 3\theta$, is
(A) $m\pi + \pi/18$ (B) $m\pi/2 + \pi/6$
(C) $m\pi/3 + \pi/18$ (D) None of these

30. The general solution of the equation $8\cos x\cos 2x\cos 4x=\sin 6x/\sin x$, is
(A) $x=n\pi/7+\pi/21$ (B) $x=n\pi/7+\pi/14$
(C) $x=n\pi/7+\pi/14$ (D) $x=n\pi+\pi/14$

31. The total number of solutions of $\cos x=\sqrt{1-\sin 2x}$ in $[0,2\pi]$ is equal to
(A) 2 (B) 3 (C) 5 (D) None of these

32. The number of solutions of $\sum_{r=1}^{5}\cos rx=5$ in the interval $[0,2\pi)$ is
(A) 0 (B) 1 (C) 5 (D) 10

33. If $\tan 3\theta+\tan\theta=2\tan 2\theta$, then θ is equal to ($n\in Z$)
(A) $n\pi$ (B) $\frac{n\pi}{4}$ (C) $2n\pi$ (D) None of these

34. The set of all x in $(-\pi,\pi)$ satisfying $|4\sin x-1|<\sqrt{5}$ is given by
(A) $\left(\frac{-\pi}{10},\frac{3\pi}{10}\right)$ (B) $\left(\frac{\pi}{10},\frac{3\pi}{10}\right)$ (C) $\left(\frac{\pi}{10},\frac{-3\pi}{10}\right)$ (D) None of these

35. The number of solutions of the equation $\sin^3 x\cos x+\sin^2 x\cos^2 x+\sin x\cos^3 x=1$, in the interval $[0,2\pi]$, is
(A) 4 (B) 2 (C) 1 (D) 0

EXERCISE 2 (B)

SUBJECTIVE TYPE

Find the genaral solution of the equations :- (Q.1 to Q.8)

1. $\sin\theta+\sin 5\theta=\sin 3\theta$, where $0\le\theta\le\pi$.

2. 4 sin x . sin 2x . sin 4x = sin 3x.

3. $\sin^6 2x+\cos^6 2x=7/16$..

4. $\sin x+\sin 2x+\sin 3x=\frac{1}{2}\cot\frac{x}{2}$.

5. $5\sin^2 x-7\sin x\cos x+16\cos^2 x=4$.

6. $\sin^2\theta-\cos\theta=\frac{1}{4}$, where $0\le\theta\le 2\pi$..

7. $\sin^4 x-\cos^7 x=1$.

8. $\sin\frac{2x+1}{x}+\sin\frac{2x+1}{3x}-3\cos^2\frac{2x+1}{3x}=0$.

9. In a right angled ΔABC, the hypotenuse is four times the perpendicular distance of the opposite vertex from it. Determine the other two angles.

10. Solve : $\cos 3x \cos^3 x + \sin 3x \sin^3 x = 0$.

11. Solve : $\sin 3\theta - \sin\theta = 4\cos^2\theta - 2$.

12. Solve the : **(i)** $\sec 4\theta - \sec 2\theta = 2$ **(ii)** $\sin^2 n\theta - \sin^2(n-1)\theta = \sin^2\theta$.

13. Solve : $2^{\cos 2x} = 3.2^{\cos^2 x} - 4$.

14. **(i)** $\cos\frac{\sqrt{2}+1}{2}\theta \cos\frac{\sqrt{2}-1}{2}\theta = 1$ **(ii)** $\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)=2$

15. **(i)** $\cos(\sin\theta) = \sin(\cos\theta)$. **(ii)** $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$.

16. **(i)** $\left(2+\sqrt{3}\right)\cos\theta = 1 - \sin\theta$.

(ii) $\cos^2\left(\frac{\pi}{4}\left(\sin x + \sqrt{2}\cos^2 x\right)\right) - \left(\tan^2\left(x + \frac{\pi}{4}\tan^2 x\right)\right) = 1$

17. Solve for x, y, & z, if $\tan^2 x + \cot^2 x = 2\sin^2 y$ and $\sin^2 y + \cos^2 z = 1$.

18. Solve the following system of equations $\cos(2\theta + 3\phi) = \frac{1}{2}$ and $\cos(3\theta + 2\phi) = \frac{\sqrt{3}}{2}$.

19. Solve : $\frac{\tan x}{\tan 2x} + \frac{\tan 2x}{\tan x} + 2 = 0$.

20. Find the most general values of θ which satisfies the equations $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$.

21. Solve : $\cos^5 x + \sin^3 x = 1$ in the interval $[0, 2\pi]$.

22. Solve : $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$, $0 \le x \le 2\pi$.

23. Solve for θ and φ the equations are $\sec\theta = \sqrt{2}\sec\phi$, and $\tan\theta = \sqrt{3}\tan\phi$.

24. Solve : $3\tan 2x - 4\tan 3x = \tan^2 3x.\tan 2x$.

25. Show that $1 + \sin^2 ax = \cos x$, where a is irrational, has only one solution. Find that solution.

26. If α & β the roots of the equation $a\cos\theta + b\sin\theta = C$, prove that

(i) $\cos\alpha + \cos\beta = \frac{2ac}{a^2+b^2}$ **(ii)** $\sin\alpha.\sin\beta = \frac{c^2-a^2}{a^2+b^2}$

27. Solve : $\frac{3}{2} - \sin 2x = \sqrt{9 + 10\sin 2x}$

28. Solve : $3\sin\left(\frac{\pi}{2}-x\right) - 4\sin(\pi+x)\sin\left(\frac{5\pi}{2}+x\right) + 8\cos^2\frac{x}{2} = 4$.

29. Solve : $\sqrt{\sin x}+\sqrt{\cos x}>1$.

30. Determine all values of 'a' for which the equation $\cos^4 x - (a+2)\cos^2 x - (a+3) = 0$ possesses solutions and find these solutions.

31. Find all values of 'a' for which the equation $\cos\left(x+\frac{\pi}{3}\right)+\cos x = a,$ has real solutions.

When a =1, find all the solutions in $(0, 2\pi)$.

32. Find the general values of x and y satisfying the equations $5 \sin x \cos y = 1$, $4 \tan x = \tan y$.

33. Solve the equation $2(\sin x + \sin y) - 2\cos(x - y) = 3$ for smallest positive values of x and y.

34. Find the coordinates of the points of intersection of the curves $y = \cos x$, $y = \sin 3x$, $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$

35. Find the range of y such that the equation in x, $y + \cos x = \sin x$ has a real solution. For $y = 1$, find x such that $0 < x < 2\pi$.

36. Find the general solution of the equation $1+2\,\text{cosec}\,x+\dfrac{\sec^2\left(\frac{x}{2}\right)}{2}=0$.

37. For what values of a does the equation $\dfrac{a\cos x}{2\cos 2x-1}=\dfrac{a+\sin x}{(1-4\sin^2 x)\tan x}$ possess a solution ?

Also find the solution.

38. Find the common roots of the equations $2\sin^2 x + \sin^2 2x = 2$ and $\sin 2x + \cos 2x = \tan x$.

39. Find all pairs (x, y) which will satisfy $x^2 + 2x \sin(xy) + 1 = 0$.

40. Solve for x, $(-\pi \le x \le \pi)$ the equation ; $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$.

41. Find the sum of all the roots of the equation, $\sin\sqrt{x}=-1$, which are less than $100\pi^2$.
Also find the sum of the square roots of these roots.

Now, can we conclude that all the roots $\cos\sqrt{x}=0$ are also the roots of $\sin\sqrt{x}=-1$?
Justify your answer.

EXERCISE 3 (A)

ONLY ONE OPTION IS CORRECT

1. The number of all possible 5-tuples (a_1, a_2, a_3, a_4,a_5) such that $a_1 + a_2 \sin x + a_3 \cos x + a_4 \sin 2x + a_5 \cos 2x = 0$ holds for all x is
(A) zero (B) 1 (C) 2 (D) infinite

2. Let $\theta \in [0, 4\pi]$ satisfying the equation $(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$. If the sum of all the values of θ is of the form $k\pi$ then the values of k is
(A) 6 (B) 5 (C) 4 (D) 2

3. If the inequality $\sin^2 x + a\cos x + a^2 > 1 + \cos x$ holds for $x \in R$ then the largest negative integral value of a is
(A) – 4 (B) –3 (C) – 2 (D) –1

4. The values of $\cos y \cos\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - y\right)\cos x + \sin y \cos\left(\frac{\pi}{2} - x\right) + \sin x \sin\left(\frac{\pi}{2} - y\right)$ is zero if
(A) $x = 0$ (B) $y = 0$ (C) $x = y$ (D) $n\pi + y - \frac{\pi}{4}$

5. One root of the equation $\cos x - x + 1/2 = 0$ lies in the interval
(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) $\left(\frac{\pi}{2}, \pi\right)$ (D) $\left(\pi, \frac{3\pi}{2}\right)$

6. If $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$, then the number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ are
(A) 1 (B) 2 (C) 3 (D) 4

7 The equation, $a\sin x + \cos 2x = 2a - 7$, possesses a solution, if
(A) a < 2 (B) $2 \le a \le 6$ (C) a > 6 (D) None of these

8 $\sin x + \cos x = y^2 - y + a$ has no values of x for any y if a belongs to
(A) $\left(0, \sqrt{3}\right)$ (B) $(-\sqrt{3}, 0)$ (C) $(-\infty. -\sqrt{3})$ (D) $(\sqrt{3}, \infty)$

9 The solution of the equation $4\sin^2 x + \tan^2 x + \cos ec^2 x + \cot^2 x - 6 = 0$ is
(A) $n\pi \pm \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{4}$ (C) $n\pi + \frac{\pi}{3}$ (D) $n\pi - \frac{\pi}{6}$

10 If both the distinct roots of the equation $|\sin^2 x| + |\sin x| + b = 0$ in $[0.\pi]$ are real, then the values of b are
(A) [–2,0] (B) (–2,0) (C) [–2,0) (D) (–2,0]

11 The total number of ordered pairs (x,y) satisfying $|x| + |y| = 4, \sin\left(\frac{\pi x^2}{3}\right) = 1$; is equal to
(A) 2 (B) 3 (C) 4 (D) 6

EXERCISE 3 (B)

SUBJECTIVE TYPE

1. For which values of the parameter a are the systems of equation $\sin(x + y) = 0$, $x^2 + y^2 = a$ and $x + y = 0$, $x^2 + y^2 = a$ are equivalent.

2. If $0<\alpha,\ \beta < \pi/2$ and $\cos\alpha\cos\beta\cos(\alpha+\beta)=-1/8$, find α and β.

3. Indicate in the coordinate plane all the points whose coordinates (x, y) are such that the expression $\left(2\cos t+\frac{1}{2}\cos x\cos y\right)\cos x\cos y+1+\cos x-\cos y+\cos 2t$ is positive for any value of t and depict the region formed by these points.

4. **Solve for x & y:** $\cos x+\cos y-\cos(x+y)=3/2$.

5. **Solve** $\left(\sin^2 x+\frac{1}{\sin^2 x}\right)^2+\left(\cos^2 x+\frac{1}{\cos^2 x}\right)^2=12+\frac{1}{2}\sin y$.

6. Find the general solution of the equations :

   (i) $\sin^2 x+1/4\ \sin^2 3x=\sin x.\sin^2 3x$. **(ii)** $\sin x+2\sin 2x=3+\sin 3x$.

   (iii) $\frac{\sqrt{3}}{2}\sin x-\cos x=\cos^2 x$.

7. Find all the number a such that for every root of the equation
$2\sin^7 x-(1-a)\sin^3 x+(2a^3-2a-1)\sin x=0$ is a root of the equation
$2\sin^6 x+\cos 2x=1+a-2a^3+a\cos^2 x$.

8 For which values of a, does the equation $4\sin(x+\pi/3)\cos(x-\pi/6)=a^2+\sqrt{3}\sin 2x-\cos 2x$ have solution? Find solution for a = 0, if exists.

9 Find the smallest positive root of the equation $\sqrt{\sin(1-x)}=\sqrt{\cos x}$

10 Prove that the equation $2\sin x=|x|+a$ has no solution for $a\in\left(\frac{3\sqrt{3}-\pi}{3},\infty\right)$

11 Solve $\sin x+\sin\frac{\pi}{8}\left(\sqrt{(1-\cos 2x)^2+\sin^2 2x}\right)=0, x\in\frac{5\pi}{2},\frac{7\pi}{2}$

12. Find all the numbers ‘a’ for which any root of the equation, $a\cos 2x+|a|\cos 4x+\cos 6x=1$, is a root of the equation $\sin x\cos 2x=\sin 2x\cos 3x-1/2\ \sin 5x$ and, conversely, every root of the latter equation is a root of the former.

13. Find all numbers ‘a’ for each of which every root of $4\cos^2 x-\cos 3x=a\cos x-|a-4|(1+\cos 2x)$ is a root of $2\cos x\cos 2x=1+\cos 2x+\cos 3x$ and conversely, every root of the latter equation is a root of the former.

WINDOW TO I.I.T. - JEE

1. Given $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

(A) $1 \le A \le 2$ (B) $\frac{3}{4} \le A \le 1$ (C) $\frac{13}{16} \le A \le 1$ (D) $\frac{3}{4} \le A \le \frac{13}{16}$

2. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$.

3. If the expression $\dfrac{\left[\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) + i\tan(x)\right]}{\left[1 + 2i\sin\left(\frac{x}{2}\right)\right]}$ is real, then the set of all possible values of x is....

4. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \ge 0$, is

5. The general solution of (exactly one choice is correct) :
sin x – 3 sin 2x + sin 3x = cos x - 3 cos 2x + cos 3x is :

(A) $n\pi + \frac{\pi}{8}$ (B) $\frac{n\pi}{2} + \frac{\pi}{8}$ (C) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (D) $2n\pi + \cos^{-1}\frac{3}{2}$

6. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ in the variable x , has a real roots. Then p can take any value in the interval

(A) $(0, 2\pi)$ (B) $(-\pi, 0)$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $(0, \pi)$

7. The numbers of solutions of the equation tan x+ sec x = 2 cos x lying in the interval $[0, 2\pi]$ is
(A) 0 (B) 1 (C) 2 (D) 3

8. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is

9. The number of values of x in the interval $(0, 5\pi)$, satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is
(A) 0 (B) 5 (C) 6 (D) 10

10. The number of integral values of k, for which the equation 7 cos x + 5 sin x = 2 k + 1 has a solution
(A) 4 (B) 8 (C) 10 (D) 12

11. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6}, \pi\right)$

12. Find all the solution of $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$.

13. Find the values of $x(-\pi, +\pi)$ which satisfy the equation $8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+........)} = 4^3$.

14. Solve for x and y, $\begin{cases} x\cos^3 y + 3x\cos y\sin^2 y = 14 \\ x\sin^3 y + 3x\cos^2 y\sin y = 13 \end{cases}$.

15. Solve the equation : $4\sin^4 x + \cos^4 x = 1$.

16. Find all the values of α for which the equation $\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$ is valid. Also find the general solution of the equation.

17. The number of values of θ in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ± 1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is

18. A triangle ABC is such that $\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}$. If A, B and C are in arithmatical progression, then find the values of A, B, C.

19. In a triangle ABC, the angle B is greater than angle A and if the values of angle A and B satisfy the equation $3\sin x - 4\sin^3 x - K = 0$, $0 < K < 1$. Then show that the value of C is $\frac{2\pi}{3}$.

20. Show that $2\sin x + \tan x > 3x$, where $0 < x < \frac{\pi}{2}$.

21. If $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + ...)\log_e 2\}$ satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$.

22. Choose the correct answer from C_2 to match with C_1, $\frac{\sin 3\alpha}{\cos 2\alpha}$ is

	C_1	C_2
(a) positive	**(i)** $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$	**(ii)** $\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
(b) negative	**(iii)** $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$	**(iv)** $\left(0, \frac{\pi}{2}\right)$

23. Solve the equation $\sec\theta - \operatorname{cosec}\theta = \frac{4}{3}$.

24. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

(A) 0 (B) 5 (C) 6 (D) 10

25. Find the general values of x and y satisfying the equations
$5 \sin x \cos y = 1$, $4 \tan x = \tan y$

26. Find real values of x for which, $27^{\cos 2x} . 81^{\sin 2x}$ is minimum . Also find this minimum value.

27. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$, where $\alpha, \beta \in [-\pi, \pi]$, numbers of pairs of α, β which satisfy both the equations is
(A) 0 (B) 1 (C) 2 (D) 4

28. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is
(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

29. The number of solutions of the pair of equations
$2 \sin^2\theta - \cos 2\theta = 0$
$2 \cos^2\theta - 3 \sin \theta = 0$
in the interval $[0, 2\pi]$ is
(A) zero (B) one (C) two (D) four

ANSWER KEY

EXERCISE 1 (A)

1. B	2. D	3. A	4. B	5. B	6. D	7. D
8. C	9. C	10. D	11. A	12. C	13. C	14. B
15. B	16. A	17. C	18. B	19. A	20. A	21. D
22. A	23. C	24. C	25. C	26. C	27. B	28. D
29. C	30. D	31. C	32. C	33. D	34. B	35. A
36. A	37. B	38. A				

EXERCISE 1 (B)

MORE THAN ONE ARE CORRECT

1. C, D	2. B, D	3. A, B, C, D	4. B
5. A, B	6. A, C, D	7. A, B, C	8. B, D
9. B	10. B	11. B	12. A, B
13. A, C	14. B,D	15. A, B, C	16. A, D
17. B, C	18. A, C	19. A, B, D	

COMPREHENSION TYPE

20. B	21. C	22. A	23. A	24. C	25. D	26. A
27. B	28. D	29. A	30. D	31. B		

REASONING TYPE

32. A 33. A 34. D 35. A 36. D

MATRIX MATCH TYPE

37. A - S, B - S, C - P, D - Q

38. A - R, B - Q, C - R, D - S

39. A - P, B - P, C - Q

EXERCISE 2 (A)

1. C	2. C	3. A	4. C	5. B	6. D	7. B
8. B	9. B	10. C	11. C	12. C	13. B	14. B
15.C	16. A	17. C	18. A	19. B	20. A	21. C
22. D	23. B	24. A	25. D	26. C	27. C	28. B
29. C	30. C	31. B	32. B	33. A	34. A	35. D

EXERCISE 2 (B)

1. $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ & π

2. $x = n\pi$ or $\frac{m\pi}{3} \pm \frac{\pi}{9}, n \in I$

3. $x = \frac{n\pi}{4} \pm \frac{\pi}{12}, n \in I$

4. $x = \frac{2n\pi}{7} + \frac{\pi}{7}, n \in I$

5. $n\pi + \tan^{-1} 3$ *or* $m\pi + \tan^{-1} 4, m, n \in I$

6. $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

7. $x = 2n\pi + \pi, n \in I$, $x = n\pi + \frac{\pi}{2}, n \in I$

8. $x = \frac{2}{6n\pi + 3\pi - 4}$ or $\frac{1}{3n\pi + 3(-1)^n \sin^{-1}\frac{3}{4} - 2}$ where $n \in I$

9. $15^\circ, 75^\circ$

10. $x = (2n+1)\frac{\pi}{4}, n \in I$

11. $\theta = (2n+1)\frac{\pi}{4}$ or $\theta = 2n\pi + \frac{\pi}{2}$

12.(i) $\theta = (2n+1)\frac{\pi}{2}$ or $\theta = (2m+1)\frac{\pi}{10}, m, n \in I$ **(ii)** $\theta = m\pi, m \in I$ or $\begin{cases} \theta = \frac{k\pi}{2(n-1)}, & \text{k is even integer} \\ \theta = \frac{k\pi}{2n}, & \text{k is odd integer} \end{cases}$

13. $x = n\pi, n \in I$ **14.** **(i)** **0** **(ii)** $n\pi$, $n \in I$

15. **(i)** ϕ **(ii)** $\theta = 2n\pi$ *or* $2n\pi + \pi, n \in I$

16. **(i)** $\theta = 2p\pi - \frac{\pi}{3}, p \in I$ **(ii)** $x = 2n\pi - \frac{\pi}{4}, n \in I$

17. $x = (2k+1)\frac{\pi}{4}$; $y = (2\ell+1)\frac{\pi}{2}$; $z = (2m+1)\frac{\pi}{2}$; ℓ, m, $k \in I$

18. $\theta = (6m-4n)\frac{\pi}{5} \pm \frac{\pi}{10} \mp \frac{2\pi}{15}, \phi = (6n-4m)\frac{\pi}{5} \pm \frac{\pi}{5} \mp \frac{\pi}{15}$

19. $x = n\pi \pm \frac{\pi}{3}$ **, where n is not a multiple of 3**

20. $\theta = 2n\pi + \frac{7\pi}{6}$, $n \in I$

21. **x = 0,** $\frac{\pi}{2}, 2\pi$

22. $\frac{\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{13\pi}{7}$

23. $\theta = n\pi \pm \frac{\pi}{3}, n \in z$ $\phi = n\pi \pm \frac{\pi}{4}, n \in z$

24. $n\pi; n\pi \pm \frac{1}{2}\cos^{-1}\left(\frac{1}{4}\right)$, $n \in I$

25. 0

27. $\frac{n\pi}{2} - (-1)^n\left(\frac{\pi}{12}\right)$ where $n \in I$

28. $(2n+1)\frac{\pi}{2}$, $n \in I$

29. $x \in \left(2k\pi, 2k\pi + \frac{\pi}{2}\right), k \in I$

30. $-3 \le a \le -2, x = n\pi \pm \cos^{-1}\sqrt{(a+3)}, n \in I$

31. $-\sqrt{3} \le a \le \sqrt{3}, \cos^{-1}(1/\sqrt{3}) - \frac{\pi}{6}, 11\pi/6 - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

32. $x = (n-m)\pi + \frac{\pi}{4} - \frac{\alpha}{2}, (n-m)\pi - \frac{\pi}{4} + \frac{\alpha}{2},\ y = (n+m)\pi + \frac{\pi}{4} + \frac{\alpha}{2}, (n+m)\pi + \frac{3\pi}{4} - \frac{\alpha}{2}, n, m \in I$

where $\alpha = \sin^{-1}\left(-\frac{3}{5}\right)$

33. $(x, y) = (5\pi/6, \pi/6), (\pi/6, 5\pi/6)$

34. $\left(\frac{\pi}{8}, \frac{\sqrt{\sqrt{2}+1}}{2\sqrt{2}}\right), \left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(-\frac{3\pi}{8}, \frac{\sqrt{2-1}}{2\sqrt{2}}\right)$

35. $-\sqrt{2} \le y \le \sqrt{2}, x = \frac{\pi}{2}, \pi$

36. $2n\pi - \frac{\pi}{2}$

37. $n\pi + (-1)^{-1}\sin^{-1}\left(\frac{a}{a-1}\right), -\infty < a < \frac{1}{2}$

38. $\left[(2n+1)\frac{\pi}{4}, \forall\ n \in I\right]$

39. $(x, y) = \left(\pm 1, 2n\pi - \frac{\pi}{2}\right)$

40. $-\pi, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$

41. $\frac{765\pi^2}{4}, \frac{55\pi}{2}$. All the roots of $\cos\sqrt{x} = 0$ are not the same as thoese of $\sin\sqrt{x} = -1$

EXERCISE 3 (A)

1. B	2. B	3. B	4. D	5. A	6. B	7. B
8. D	9. A	10. B	11. C			

EXERCISE 3 (B)

1. $a < \frac{\pi^2}{2}$

2. $a = \beta = \frac{\pi}{3}$

3. $\cos x > \cos y$

4. $x = 2n\pi \pm \frac{\pi}{3}; y = 2(n-k)\pi \pm \frac{\pi}{3}$, n, $k \in I$

5. $x = \frac{k\pi}{2} + \frac{\pi}{4}; y = 2n\pi + \frac{\pi}{2}; k, n \in I$

6. (i) $x = k\pi, x = k\pi + (-1)^k \frac{\pi}{6}, k \in I$

(ii) **no solution** **(iii)** $2n\pi + \pi, 2n\pi + \frac{\pi}{3}$

7. $a = -1, 0$

8. $-2 \le a \le 2$ **and for a = 0,** $x = n\pi + \pi/2$

9. $x = \frac{7\pi}{4} + 1/2$

11. $x = 3\pi$

12. $a < -1, a = 0$

13. $a = 4, a > 5$

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1. B

2. False

3. $x = 2n\pi$ or $2n\pi + \alpha$, where $\tan^3\frac{\alpha}{2} = 2 + \tan\left(\frac{\alpha}{2}\right)$

4. $x \in \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

5. B

6. D

7. C

8. $\theta = n\pi,\ m\pi \pm \frac{\pi}{3},\ m, n \in I$

9. C

10. B

11. B

12. $x = m\pi + (-1)^m \sin^{-1}\left(\frac{-1 \pm \sqrt{5}}{4}\right)$, $x = n\pi,\ m\pi + (-1)^m \sin^{-1}\left(\frac{-1 \pm \sqrt{5}}{4}\right)$, m and n are integer

13. $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

14. $x = \pm 5\sqrt{5}$ and $y = n\pi + \tan^{-1}\frac{1}{2}$

15. $x = n\pi$ or $x = 2n\pi \pm \frac{1}{2}\cos^{-1}\frac{1}{5}$

16. $\left[\frac{-3}{2} \le \alpha \le \frac{1}{2},\ x = \frac{n\pi}{2} + (-1)^n \frac{\beta}{2},\ \beta = \sin^{-1}\left(1 - \sqrt{3 + 2\alpha}\right)\right]$.

17. 3

18. $A = 45^0, B = 60^0, C = 75^0$

19. $C = \pi - (A + B) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

21. $\frac{\sqrt{3} - 1}{2}$

22. $a \to iii, b \to i$

23. $\theta = n\pi + \frac{\pi}{4} + (-1)^n \sin^{-1}\frac{1}{2\sqrt{2}}, n \in Z$

24 C

25. $y = (n - m)\frac{\pi}{2} + (-1)^n \frac{\pi}{4} - (-1)^m \frac{\alpha}{2}$; $x = (m + n)\frac{\pi}{2} + (-1)^n \frac{\pi}{4} + (-1)^m \frac{\alpha}{2}$ where $\alpha = \sin^{-1}\left(-\frac{3}{5}\right)$, m, n∈I

26. min. value $= 3^{-5}$ for $x = (4n - 1)\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$, n∈I ;

max. value $= 3^5$ for $x = (4n + 1)\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$, n∈I

27. D

28. A

29. C