CENTRE OF MASS

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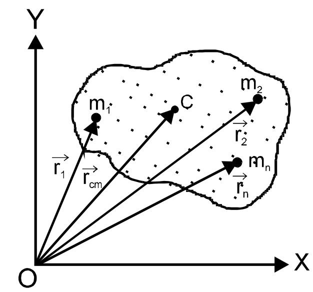
Every physical system has associated with it a certain point whose motion characterises the motion of the whole system. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point for translational motion. This point is called the centre of mass of the system.

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

Consider a system of N point masses m_1, m_2, m_3, ... m_n whose position vectors from origin O are given by $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$,............... $\vec{r}_n$ respectively. Then the position vector of the centre of mass C of the system is given by.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} \quad ; \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$

$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

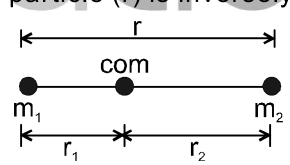
where, $m_i \vec{r}_i$ is called the moment of mass of the particle w.r.t O.

$M = \left(\sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Note: If the origin is taken at the centre of mass then $\sum_{i=1}^{n} m_i \vec{r}_i = 0$. hence, the COM is the point about which the sum of “mass moments” of the system is zero.

POSITION OF COM OF TWO PARTICLES

Centre of mass of two particles of masses m_1 and m_2 separated by a distance r lies in between the two particles. The distance of centre of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)



i.e. $r \propto 1/m$

or $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

or $m_1 r_1 = m_2 r_2$

or $r_1 = \left(\frac{m_2}{m_2 + m_1}\right) r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$

Here, r_1 = distance of COM from m_1
and r_2 = distance of COM from m_2

From the above discussion, we see that

$r_1 = r_2 = 1/2$ if $m_1 = m_2$, i.e., COM lies midway between the two particles of equal masses.

Similarly, $r_1 > r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_2 < m_1$, i.e., COM is nearer to the particle having larger mass.

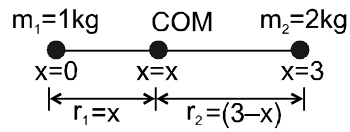
Solved Examples

Example 1. Two particles of mass 1 kg and 2 kg are located at x = 0 and x = 3 m. Find the position of their centre of mass.

Solution :

Since, both the particles lies on x-axis, the COM will also lie on x-axis. Let the COM is located at x = x, then

r_1 = distance of COM from the particle of mass 1 kg = x



and r_2 = distance of COM from the particle of mass 2 kg = (3 – x)

Using $\frac{r_1}{r_2} = \frac{m_2}{m_1}$ or $\frac{x}{3-x} = \frac{2}{1}$ or x = 2 m

Thus, the COM of the two particles is located at x = 2 m. **Ans.**

Example 2. The position vector of three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r}_1 = (\hat{i}+4\hat{j}+\hat{k})m$, $\vec{r}_2 = (\hat{i}+\hat{j}+\hat{k})m$ and $\vec{r}_3 = (2\hat{i}-\hat{j}-2\hat{k})m$ respectively. Find the position vector of their centre of mass.

Solution :

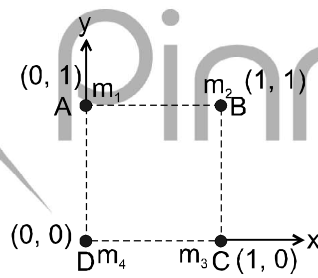
The position vector of COM of the three particles will be given by

$$\vec{\mathbf{r}}_{COM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + m_3\vec{\mathbf{r}}_3}{m_1+m_2+m_3}$$

Substituting the values, we get

$$\vec{r}_{COM} = \frac{(1)(\hat{i}+4\hat{j}+\hat{k})+(2)(\hat{i}+\hat{j}+\hat{k})+(3)(2\hat{i}-\hat{j}-2\hat{k})}{1+2+3} = \frac{1}{2}(3\,\hat{i}+\hat{j}-\hat{k})m$$ **Ans.**

Example 3. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A, B, C and D of a square of side 1 m. Find the position of centre of mass of the particles.



Solution :

Assuming D as the origin, DC as x -axis and DA as y-axis, we have

m_1 = 1 kg, (x_1, y_1) = (0, 1m)

m_2 = 2 kg, (x_2, y_2) = (1m, 1m)

m_3 = 3 kg, (x_3, y_3) = (1m, 0)

and m_4 = 4 kg, (x_4, y_4) = (0, 0)

Co-ordinates of their COM are

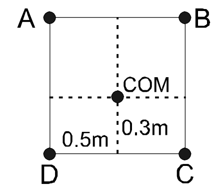
$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3m_3 + m_4x_4}{m_1+m_2+m_3+m_4}$$

$$= \frac{(1)(0)+2(1)+3(1)+4(0)}{1+2+3+4} = \frac{5}{10} = \frac{1}{2}\,m = 0.5 \text{ m}$$

Similarly, $y_{COM} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$

$$= \frac{(1)(1)+2(1)+3(0)+4(0)}{1+2+3+4} = \frac{3}{10} = 0.3 \text{ m}$$

$\therefore$ $(x_{COM}, y_{COM}) = (0.5 \text{ m}, 0.3 \text{ m})$ **Ans.**

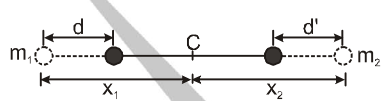


Thus, position of COM of the four particles is as shown in figure.

Example 4. Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position?

Solution :

Consider figure. Suppose the distance of m_1 from the centre of mass C is x_1 and that of m_2 from C is x_2. Suppose the mass m_2 is moved through a distance d′ towards C so as to keep the centre of mass at C.



Then, $m_1x_1 = m_2x_2$ ....(i)

and $m_1(x_1 - d) = m_2(x_2 - d')$. ....(ii)

Subtracting (ii) from (i)

$m_1 d = m_2 d'$

or, $d' = \dfrac{m_1}{m_2} d$,

CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm},\ y_{cm} = \frac{\int y\,dm}{\int dm},\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm.$$

Note: If an object has symmetric mass distribution about x axis then y coordinate of COM is zero and vice-versa

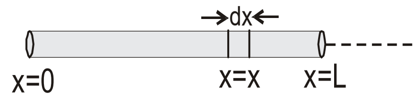
CENTRE OF MASS OF A UNIFORM ROD

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L. Mass per unit length of the rod = $\dfrac{M}{L}$

Hence, dm, (the mass of the element dx situated at x = x is) = $\dfrac{M}{L}dx$

The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be

$$x_{COM} = \frac{\int_0^L x\, dm}{\int dm}$$



$$= \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M}$$

$$= \frac{1}{L}\int_0^L x\, dx = \frac{L}{2}$$

The y-coordinate of COM is

$$y_{COM} = \frac{\int y\, dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e. it lies at the centre of the rod.

Solved Examples

Example 5. A rod of length L is placed along the x-axis between x = 0 and x = L. The linear density (mass/length) λ of the rod varies with the distance x from the origin as $\lambda = Rx$. Here, R is a positive constant. Find the position of centre of mass of this rod.

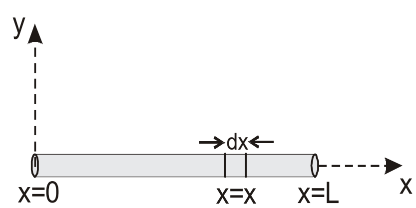
Solution :

Mass of element dx situated at x = x is

$dm = \lambda\, dx = Rx\, dx$

The COM of the element has coordinates (x, 0, 0).

Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\, dm}{\int dm}$$

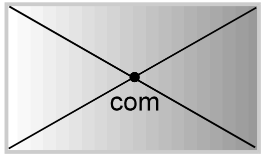
$$= \frac{\int_0^L (x)(Rx)dx}{\int_0^L (Rx)dx} = \frac{R\int_0^L x^2 dx}{R\int_0^L x\, dx} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L} = \frac{2L}{3}$$

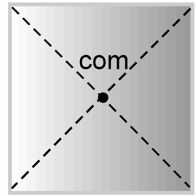
The y-coordinate of COM of the rod is $y_{COM} = \frac{\int y\, dm}{\int dm} = 0$ (as y = 0)

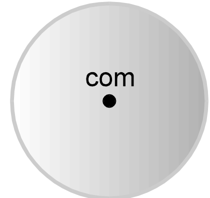
Similarly, $z_{COM} = 0$

Hence, the centre of mass of the rod lies at $\left[\frac{2L}{3}, 0, 0\right]$ **Ans.**

1. Centre of mass of a uniform rectangular, square or circular plate lies at its centre. Axis of symmetry plane of symmetry.







2. For a laminar type (2-dimensional) body with uniform negligible thickness the formulae for finding the position of centre of mass are as follows :

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...} = \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + ...}{\rho A_1 t + \rho A_2 t + ...} \qquad (\because\ m = \rho At)$$

or
$$\vec{r}_{COM} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + ...}{A_1 + A_2 + ...}$$

Here, A stands for the area,

3. If some mass of area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae:

(i) $$\vec{r}_{COM} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2} \quad \text{or} \quad \vec{r}_{COM} = \frac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$$

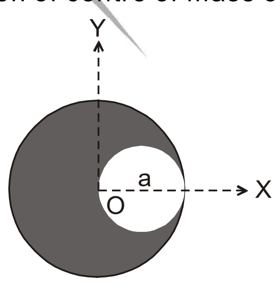
(ii) $$x_{COM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2} \quad \text{or} \quad x_{COM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$y_{COM} = \frac{m_1y_1 - m_2y_2}{m_1 - m_2} \quad \text{or} \quad y_{COM} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

and
$$z_{COM} = \frac{m_1z_1 - m_2z_2}{m_1 - m_2} \quad \text{or} \quad z_{COM} = \frac{A_1z_1 - A_2z_2}{A_1 - A_2}$$

Here, $m_1, A_1, \vec{r}_1, x_1, y_1$ and z_1 are the values for the whole mass while $m_2, A_2, \vec{r}_2, \vec{x}_2, y_2$ and z_2 are the values for the mass which has been removed. Let us see two examples in support of the above theory.

Example 6. Find the position of centre of mass of the uniform lamina shown in figure.



Solution : Here,

A_1 = area of complete circle = πa^2

A_2 = area of small circle = $\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

(x_1, y_1) = coordinates of centre of mass of large circle = (0, 0) and (x_2, y_2) = coordinates of centre of mass of small circle = $\left(\frac{a}{2}, 0\right)$

Using $x_{COM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

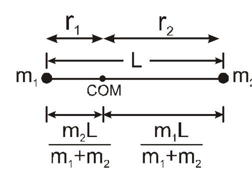
we get $x_{COM} = \dfrac{-\frac{\pi a^2}{4}\left(\frac{a}{2}\right)}{\pi a^2 - \frac{\pi a^2}{4}} = \dfrac{-\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)}\, a = -\dfrac{a}{6}$

and $y_{COM} = 0$ as y_1 and y_2 both are zero.

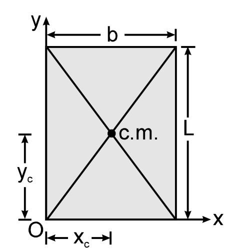
Therefore, coordinates of COM of the lamina shown in figure are $\left(-\frac{a}{6}, 0\right)$ **Ans.**

CENTRE OF MASS OF SOME COMMON SYSTEMS

⇒ A system of two point masses $m_1r_1 = m_2r_2$
The centre of mass lies closer to the heavier mass.

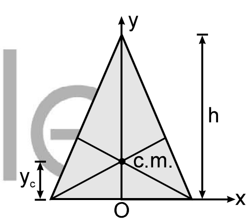


⇒ Rectangular plate (By symmetry)



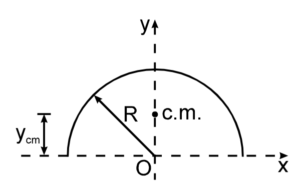
$x_c = \dfrac{b}{2}$ $\quad$ $y_c = \dfrac{L}{2}$

⇒ A triangular plate (By qualitative argument)



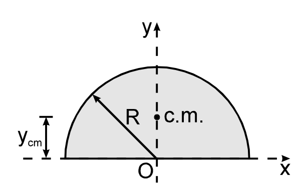
at the centroid : $y_c = \dfrac{h}{3}$

⇒ A semi-circular ring



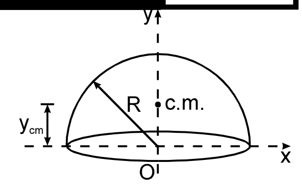
$y_c = \dfrac{2R}{\pi}$ $\quad$ $x_c = 0$

⇒ A semi-circular disc



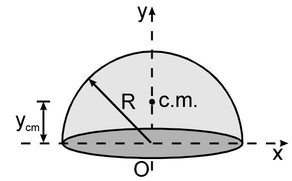
$y_c = \dfrac{4R}{3\pi}$ $\quad$ $x_c = 0$

⇒ A hemispherical shell



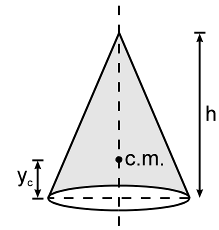
$y_c = \frac{R}{2}$ $\quad x_c = 0$

⇒ A solid hemisphere



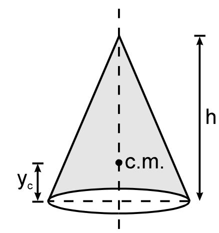
$y_c = \frac{3R}{8}$ $\quad x_c = 0$

⇒ A circular cone (solid)



$y_c = \frac{h}{4}$

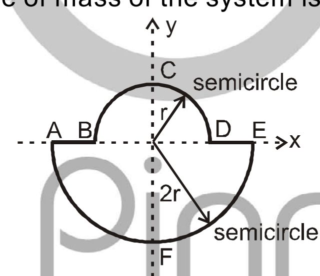
⇒ A circular cone (hollow)



$y_c = \frac{h}{3}$

Solved Examples

Example 7. A uniform thin rod is bent in the form of closed loop ABCDEFA as shown in the figure. The y-coordinate of the centre of mass of the system is



(1) $\frac{2r}{\pi}$ (2) $-\frac{6r}{3\pi+2}$ (3) $-\frac{2r}{\pi}$ (4) Zero

Ans. (2)

Solution. The centre of mass of semicircular ring is at a distance $\frac{2r}{\pi}$ from its centre.

(Let λ = mass/length)

$$\therefore Y_{cm} = \frac{\lambda\pi r \times \frac{2r}{\pi} - \lambda \times 2\pi r \times \frac{4r}{\pi}}{\lambda\pi r + \lambda r + \lambda r + \lambda \times 2\pi r} = -\frac{6r}{3\pi+2}$$

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM :

Velocity of centre of mass of system

$$\vec{v}_{cm} = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} \ldots\ldots\ldots\ldots + m_n \frac{d\vec{r}_n}{dt}}{M} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 \ldots\ldots\ldots + m_n \vec{v}_n}{M}$$

Here numerator of the right hand side term is the total momentum of the system i.e., summation of momentum of the individual component (particle) of the system

Hence velocity of centre of mass of the system is the ratio of momentum of the system to the mass of the system.

$\therefore \quad \vec{P}_{System} = M\vec{v}_{cm}$

Acceleration of centre of mass of system

$$\vec{a}_{cm} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt}\ldots\ldots\ldots\ldots + m_n\frac{d\vec{v}_n}{dt}}{M} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3\ldots\ldots\ldots + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal Force}}{M} = \frac{\text{Net External Force}}{M}$$

($\because$ action and reaction both of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero)

$\therefore \quad \vec{F}_{ext} = M\vec{a}_{cm}$

where $\vec{F}_{ext}$ is the sum of the 'external' forces acting on the system. The internal forces which the particles exert on one another play absolutely no role in the motion of the centre of mass.

If no external force is acting on a system of particles, the acceleration of centre of mass of the system will be zero. If $a_c = 0$, it implies that v_c must be a constant and if v_{cm} is a constant, it implies that the total momentum of the system must remain constant. It leads to the principal of conservation of momentum in absence of external forces.

If $\vec{F}_{ext} = 0$ then $\vec{v}_{cm}$ = constant

"If resultant external force is zero on the system, then the net momentum of the system must remain constant".

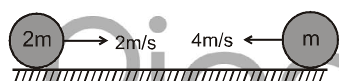
Motion of COM in a moving system of particles:

(1) COM at rest :

If $F_{ext} = 0$ and $V_{cm} = 0$, then COM remains at rest. Individual components of the system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

(i) All the particles of the system are at rest.

(ii) Particles are moving such that their net momentum is zero.
example:



(iii) A bomb at rest suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal & there is no external force on the system for explosion therefore, the COM of the bomb will remain at the original position and the fragment fly such that their net momentum remains zero.

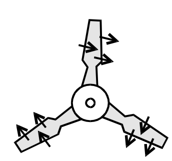
(iv) Two men standing on a frictionless platform, push each other, then also their net momentum remains zero because the push forces are internal for the two men system.

(v) A boat floating in a lake, also has net momentum zero if the people on it changes their position, because the friction force required to move the people is internal of the boat system.

(vi) Objects initially at rest, if moving under mutual forces (electrostatic or gravitation)also have net momentum zero.

(vii) A light spring of spring constant k kept compressed between two blocks of masses m_1 and m_2 on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions, such that the net momentum is zero.

(viii) In a fan, all particles are moving but COM is at rest



(2) COM moving with uniform velocity :

If $F_{ext} = 0$, then V_{cm} remains constant therefore, net momentum of the system also remains conserved. Individual components of the system may have variable velocity and momentum due to mutual forces (internal), but the net momentum of the system remains constant and COM continues to move with the initial velocity.

(i) All the particles of the system are moving with same velocity.
e.g.: A car moving with uniform speed on a straight road, has its COM moving with a constant velocity.

(ii) Internal explosions / breaking does not change the motion of COM and net momentum remains conserved. A bomb moving in a straight line suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal & there is no external force on the system for explosion therefore, the COM of the bomb will continue the original motion and the fragment fly such that their net momentum remains conserved.

(iii) Man jumping from cart or buggy also exert internal forces therefore net momentum of the system and hence, Motion of COM remains conserved.

(iv) Two moving blocks connected by a light spring on a smooth horizontal surface. If the acting forces is only due to spring then COM will remain in its motion and momentum will remain conserved.

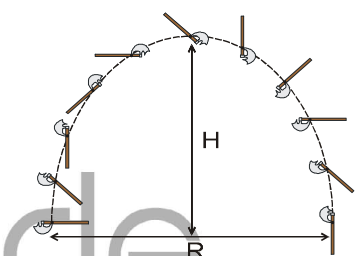
(v) Particles colliding in absence of external impulsive forces also have their momentum conserved.

(3) COM moving with acceleration :

If an external force is present then COM continues its original motion as if the external force is acting on it, irrespective of internal forces.

Example:

Projectile motion : An axe thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation



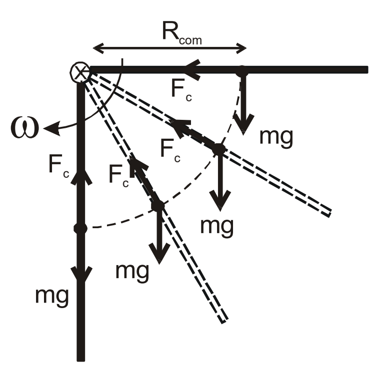
The motion of axe is complicated but the COM is moving in a parabolic motion.

$$H_{com} = \frac{u^2 \sin^2 \theta}{2g}$$

$$R_{com} = \frac{u^2 \sin 2\theta}{g} \qquad T = \frac{2u \sin\theta}{g}$$

Example:

Circular Motion : A rod hinged at an end, rotates, than its COM performs circular motion. The centripetal force (F_c) required in the circular motion is assumed to be acting on the COM.



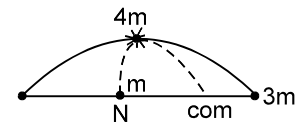
$$F_c = m\omega^2 R_{COM}$$

Solved Examples

Example 8. A projectile is fired at a speed of 100 m/s at an angle of 37º above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the lighter piece coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution : Internal force do not effect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,

$$x_{COM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times10^4 \times\frac{3}{5}\times\frac{4}{5}}{10}\ \text{m}$$



$$= 960\ \text{m}$$

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at x = 480 m. If the heavier block hits the ground at x_2, then

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)} \qquad x_2 = 1120\ \text{m} \qquad \textbf{Ans.}$$

Momentum Conservation :

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass. $\vec{P} = M\ \vec{v}_{cm}$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} \qquad \text{If} \qquad \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \quad ; \quad \vec{P} = \text{constant}$$

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \ldots\ldots\ldots\ldots\ldots + \vec{P}_n = \text{constant.}$$

Solved Examples

Example 9. A shell is fired from a cannon with a speed of 100 m/s at an angle 60º with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

Solution :

As we know in absence of external force the motion of centre of mass of a body remains uneffected. Thus, here the centre of mass of the two fragments will continue to follow the original projectile path. The velocity of the shell at the highest point of trajectory is

$$v_M = u\cos\theta = 100 \times \cos 60º = 50\ \text{m/s}.$$

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2,. which must be along positive x-direction. Now from momentum conservation, we have

$$mv = \frac{-m}{2} v_1 + \frac{m}{2} v_2 \quad \text{or} \quad 2v = v_2 - v_1$$

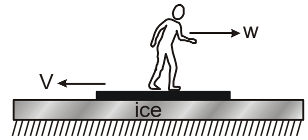
or $\quad v_2 = 2v + v_1 = (2 \times 50) + 50 = 150\ \text{m/s}$

Example 10. A man of mass m is standing on a platform of mass M kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil ?

Solution :

Consider the situation shown in figure. Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is V + w. By the question,

$V + w = v$, or $w = v - V$ ....(i)



Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially both the man and the platform were at rest. Thus,

$0 = MV - mw$ or, $MV = m(v - V)$ [Using (i)]

or, $V = \dfrac{mv}{M+m}$.

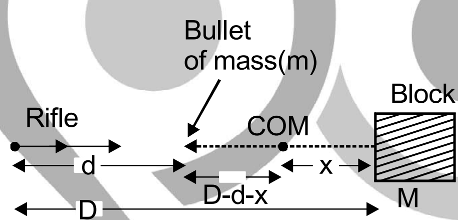
Example 11. In a free space a rifle of mass M shoots a bullet of mass m at a stationary block of mass M distance D away from it. When the bullet has moved through a distance d towards the block the centre of mass of the bullet-block system is at a distance of :

(1) $\dfrac{(D-d)\,m}{M+m}$ from the block (2) $\dfrac{md + MD}{M+m}$ from the rifle

(3) $\dfrac{2\,dm + DM}{M+m}$ from the rifle (4) $(D-d)\dfrac{M}{M+m}$ from the bullet

Ans. (1,2,4)

Solution.



As; $Mx = m(D - d - x)$

$x = \dfrac{m(D-d)}{M+m}$ from the block

and $x' = D - d - x$

$= \dfrac{(D-d)M}{M+m}$ from the bullet.

Example 12. The centre of mass of two masses m & m′ moves by distance $\dfrac{x}{5}$ when mass m is moved by distance x and m′ is kept fixed. The ratio $\dfrac{m'}{m}$ is

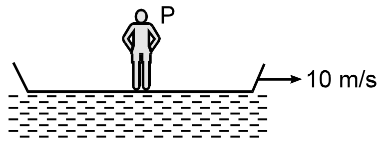
(1) 2 (2) 4 (3) 1/4 (4) None of these

Ans. (2)

Solution $(m + m')\dfrac{x}{5} = mx + m'O$

$\therefore$ $m + m' = 5m$; $m' = 4m$; $\dfrac{m'}{m} = 4$

Example 13. A person P of mass 50 kg stands at the middle of a boat of mass 100 kg moving at a constant velocity 10 m/s with no friction between water and boat and also the engine of the boat is shut off. With what velocity (relative to the boat surface) should the person move so that the boat comes to rest. Neglect friction between water and boat.

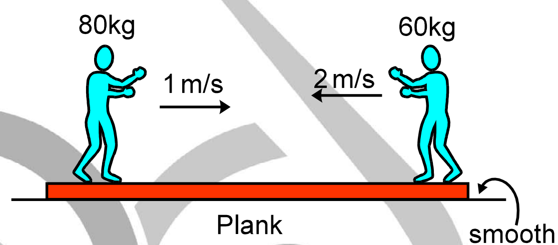


(1) 30 m/s towards right
(2) 20 m/s towards right
(3) 30 m/s towards left
(4) 20 m/s towards left

Ans. (1)

Solution. Momentum of the system remains conserved as no external force is acting on the system in horizontal direction. $\therefore$ $(50 + 100)\,10 = 50 \times V + 100 \times 0 \Rightarrow V = 30$ m/s towards right, as boat is at rest. $V_{P_{boat}} = 30$ m/s

Example 14. Two men of masses 80 kg and 60 kg are standing on a wood plank of mass 100 kg, that has been placed over a smooth surface. If both the men start moving toward each other with speeds 1 m/s and 2 m/s respectively then find the velocity of the plank by which it starts moving.

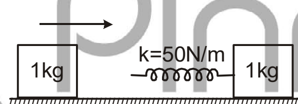


Solution. Applying momentum conservation ;

$(80)\,1 + 60\,(-2) = (80 + 60 + 100)\,v$

$v = \dfrac{-40}{240} = -\dfrac{1}{6}$ m/sec.

Example 15. Each of the blocks shown in figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. Find the maximum compression of the spring. Assume, on a friction less surface



Solution :

Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

$(1\text{ kg})\,(2\text{ m/s}) = (1\text{ kg})V + (1\text{ kg})V$

or, $V = 1$ m/s.

Initial kinetic energy $= \dfrac{1}{2}\,(1\text{ kg})\,(2\text{ m/s})^2 = 2$ J.

Final kinetic energy

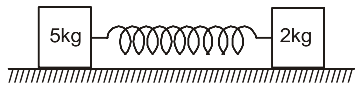
$= \dfrac{1}{2}\,(1\text{ kg})\,(1\text{m/s})^2 + \dfrac{1}{2}\,(1\text{ kg})\,(1\text{ m/s})^2 = 1$ J

The kinetic energy lost is stored as the elastic energy in the spring.

Hence, $\frac{1}{2}$(50 N/m) x^2 = 2J – 1J = 1 J

or, x = 0.2 m.

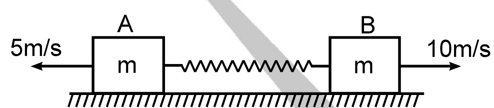
Example 16. Figure shows two blocks of masses 5 kg and 2 kg placed on a frictionless surface and connected with a spring. An external kick gives a velocity 14 m/s to the heavier block towards the lighter one. Find the velocity gained by the centre of mass



Solution : (a) Velocity of centre of mass is

$$v_{cm} = \frac{5 \times 14 + 2 \times 0}{5 + 2} = 10 \text{ m/s}$$

Example 17. The two blocks A and B of same mass connected to a spring and placed on a smooth surface. They are given velocities (as shown in the figure) when the spring is in its natural length :



(1) the maximum velocity of B will be 10 m/s
(2) the maximum velocity of B will be greater than 10 m/s
(3) the spring will have maximum extension when A and B both stop
(4) the spring will have maximum extension when both move towards left.

Ans. (1)

Solution. Suppose B moves with a velocity more than 10 m/s a should move at a velocity greater than 5 m/s and increases the overall energy which is not possible since there is no external force acting on the system. Hence B should move with a maximum velocity 10 m/s.
Also both A and B can never stop so as to keep the momentum constant.
Also both A and B can never move towards left simultaneously for momentum remaining conserved.
Hence only (A) is correct.

IMPULSE

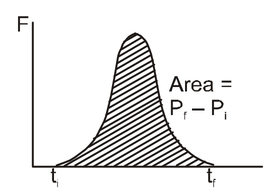
Impulse of a force $\vec{F}$ acting on a body for the time interval t = t_1 to t = t_2 is defined as :-

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt \quad \Rightarrow \quad \vec{I} = \int \vec{F} dt = \int m \frac{d\vec{v}}{dt} dt = \int m d\vec{v}$$

$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta \vec{P}$ = change in momentum due to force $\vec{F}$

Also, $\vec{I}_{Res} = \int_{t_1}^{t_2} \vec{F}_{Res} dt = \Delta \vec{P}$ **(impulse - momentum theorem)**

Note: Impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.



Instantaneous Impulse :

There are many cases when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write.

$$\vec{I} = \int \vec{F} dt = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$$

Important Points :

(1) It is a vector quantity.

(2) Dimensions = [MLT^{-1}]

(3) SI unit = kg m/s

(4) Direction is along change in momentum.

(5) Magnitude is equal to area under the F-t. graph.

(6) $\vec{I} = \int \vec{F}dt = \vec{F}_{av}\int dt = \vec{F}_{av}\Delta t$

(7) It is not a property of a particle, but it is a measure of the degree to which an external force changes the momentum of the particle.

Solved Examples

Example 18. The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period.

Solution : The momentum of each bullet
= (0.050 kg) (1000 m/s) = 50 kg-m/s.
The gun has been imparted this much amount of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

$$= \frac{(50\ \text{kg}-\text{m/s})\times 20}{4\,\text{s}} = 250\ \text{N}.$$

In order to hold the gun, the hero must exert a force of 250 N against the gun.

COLLISION OR IMPACT

Collision is an event in which an impulsive force acts between two or more bodies for a short time, which results in change of their velocities.

Note : (a) In a collision, particles may or may not come in physical contact.

(b) The duration of collision, Δt is negligible as compared to the usual time intervals of observation of motion.

The collision is infact a redistribution of total momentum of the particles. Thus, law of conservation of linear momentum is indispensable in dealing with the phenomenon of collision between particles.

Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of Line of impact can be determined by:

(a) Geometry of colliding objects like spheres, discs, wedge etc.

(b) Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Classification of collisions

(a) On the basis of line of impact

(i) Head-on collision : If the velocities of the colliding particles are along the same line before and after the collision.

(ii) Oblique collision : If the velocities of the colliding particles are along different lines before and after the collision.

(b) On the basis of energy :

(i) Elastic collision : In an elastic collision, the colliding particles regain their shape and size completely after collision. i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus, kinetic energy of system after collision is equal to kinetic energy of system before collision. Thus in addition to the linear momentum, kinetic energy also remains conserved before and after collision.

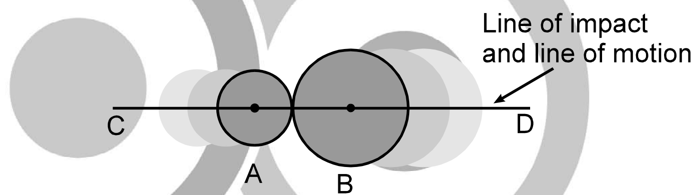
(ii) Inelastic collision : In an inelastic collision, the colliding particles do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles after collision is not equal to that of before collision. However, in the absence of external forces, law of conservation of linear momentum still holds good.

(iii) Perfectly inelastic : If velocity of separation along the line of impact just after collision becomes zero then the collision is perfectly inelastic. Collision is said to be **perfectly inelastic** if both the particles stick together after collision and move with same velocity,

Note : Actually collision between all real objects are neither perfectly elastic nor perfectly inelastic, its inelastic in nature.

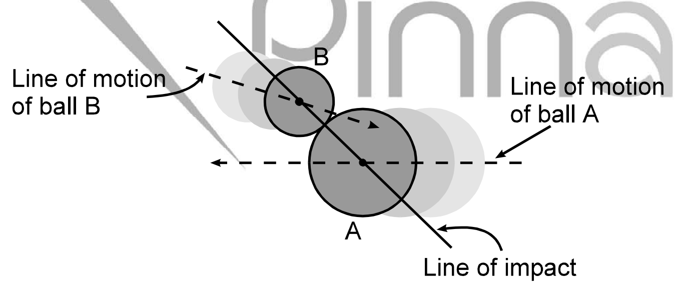
Examples of line of impact and collisions based on line of impact

(i) Two balls A and B are approaching each other such that their centres are moving along line CD.



Head on Collision

(ii) Two balls A and B are approaching each other such that their centre are moving along dotted lines as shown in figure.



Oblique Collision

COEFFICIENT OF RESTITUTION (e)

$$e = \frac{\text{Velocity of seperation along line of impact}}{\text{Velocity of approach along line of impact}}$$

The most general expression for coefficient of restitution is

$$e = \frac{\text{velocity of separation of points of contact along line of impact}}{\text{velocity of approach of point of contact along line of impact}}$$

Note : e is independent of shape and mass of object but depends on the material.
The coefficient of restitution is constant for a pair of materials.

(a) e = 1
- Velocity of separation along the LOI = Velocity of approach along the LOI
- Kinetic energy of particles after collision may be equal to that of before collision.
- Collision is elastic.

(b) e = 0
- Velocity of separation along the LOI = 0
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is perfectly inelastic .

(c) 0 < e < 1
- Velocity of separation along the LOI < Velocity of approach along the LOI
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is Inelastic.

Note : In case of contact collisions e is always less than unity.

$\therefore \quad 0 \le e \le 1$

Collision in one dimension (Head on)

(a) Before Collision

$u_1 > u_2$

(b) After Collision

$v_2 > v_1$

$e = \dfrac{v_2 - v_1}{u_1 - u_2} \quad \Rightarrow \quad (u_1 - u_2)e = (v_2 - v_1)$

By momentum conservation,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$v_2 = v_1 + e(u_1 - u_2)$

and $\quad v_1 = \dfrac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$

$v_2 = \dfrac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$

Special Case :

(1) e = 0

$\Rightarrow \quad v_1 = v_2$

$\Rightarrow$ for perfectly inelastic collision, both the bodies, move with same vel. after collision.

(2) e = 1

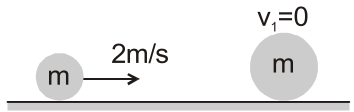
and $\quad m_1 = m_2 = m$,

we get $v_1 = u_2$ and $v_2 = u_1$

i.e., when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.

Before Collision

After Collision



Before Collision

After Collision

(3) $m_1 >> m_2$

$m_1 + m_2 \approx m_1$ and $\frac{m_2}{m_1} \approx 0$

$\Rightarrow$ $v_1 = u_1$ No change

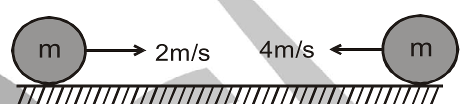
and $v_2 = u_1 + e(u_1 - u_2)$

Now If e = 1

$v_2 = 2u_1 - u_2$

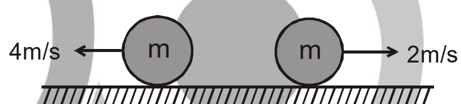
Solved Examples

Example 19. Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.

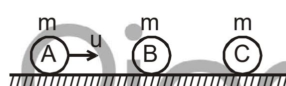


Solution :

The two velocities will be exchanged and the final motion is reverse of initial motion for both.

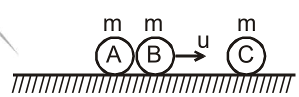


Example 20. Three balls A, B and C of same mass 'm' are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic then, find the final velocities of all the balls.

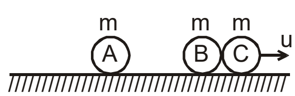


Solution :

A collides elastically with B and comes to rest but B starts moving with velocity u



After a while B collides elastically with C and comes to rest but C starts moving with velocity u

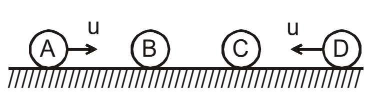


$\therefore$ Final velocities

$V_A = 0$;

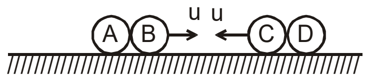
$V_B = 0$ and $V_C = u$ **Ans.**

Example 21. Four identical balls A, B, C and D are placed in a line on a frictionless horizontal surface. A and D are moved with same speed 'u' towards the middle as shown. Assuming elastic collisions, find the final velocities.

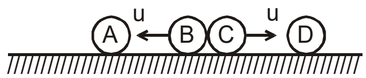


Solution :

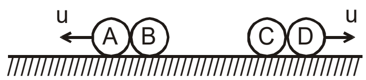
A and D collides elastically with B and C respectively and come to rest but B and C starts moving with velocity u towards each other as shown



B and C collides elastically and exchange their velocities to move in opposite directions



Now, B and C collides elastically with A and D respectively and come to rest but A and D starts moving with velocity u away from each other as shown



$\therefore$ Final velocities $V_A = u\ (\leftarrow)$; $V_B = 0$; $V_C = 0$ and $V_D = u\ (\rightarrow)$ **Ans.**

Example 22. Two particles of mass m and 2m moving in opposite directions on a frictionless surface collide elastically with velocity v and 2v respectively. Find their velocities after collision, also find the fraction of kinetic energy lost by the colliding particles.

Solution :

Let the final velocities of m and 2m be v_1 and v_2 respectively as shown in the figure:

By conservation of momentum:

$m(2v) + 2m(-v) = m(v_1) + 2m\ (v_2)$

or $0 = mv_1 + 2mv_2$

or $v_1 + 2v_2 = 0$ ....(1)

and since the collision is elastic:

$v_2 - v_1 = 2v - (-v)$

or $v_2 - v_1 = 3v$ ....(2)

Solving the above two equations, we get,

$v_2 = v$ and $v_1 = -2v$ Ans.

i.e., the mass 2m returns with velocity v while the mass m returns with velocity 2v in the direction shown in figure:

The collision was elastic therefore, no kinetic energy is lost, KE loss = KE_i - KE_f

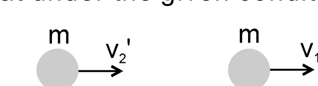
or, $$\left(\frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(-v)^2\right) - \left(\frac{1}{2}m(-2v)^2 + \frac{1}{2}(2m)v^2\right) = 0$$

Example 23. On a frictionless surface, a ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

Solution :

As we have seen in the above discussion, that under the given conditions :

Before Collision



After Collision

By using conservation of linear momentum and equation of e, we get,

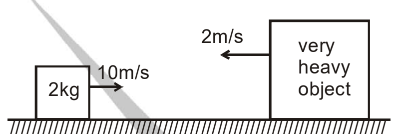
$$v_1' = \left(\frac{1+e}{2}\right)v \quad \text{and} \quad v_2' = \left(\frac{1-e}{2}\right)v$$

Given that $K_f = \frac{3}{4}K_i$ or $\frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$

Substituting the value, we get

$$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4} \quad \text{or} \quad e = \frac{1}{\sqrt{2}}$$ **Ans.**

Example 24. A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surfaces, find the final velocities of the blocks.

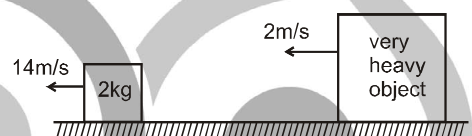


Solution :

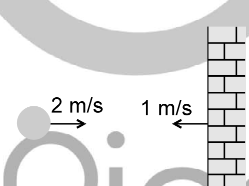
Let v_1 and v_2 be the final velocities of 2kg block and heavy object respectively then,

$v_1 = u_1 + 1\,(u_1 - u_2) = 2u_1 - u_2 = -14$ m/s

$v_2 = -2$m/s

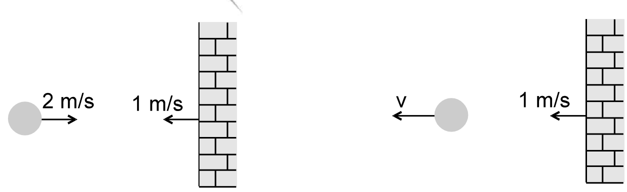


Example 25. A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1m/s as shown in fig. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.



Solution :

The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic (e = 1),



Before Collision After Collision

separation speed = approach speed

or $v - 1 = 2 + 1$

or $v = 4$ m/s **Ans.**

Collision in two dimension (oblique)

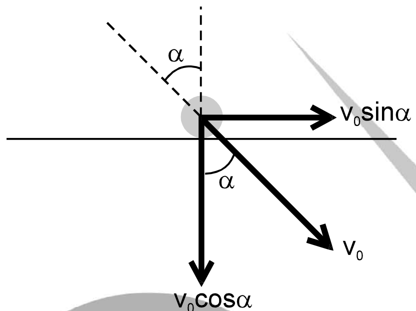
Solved Examples

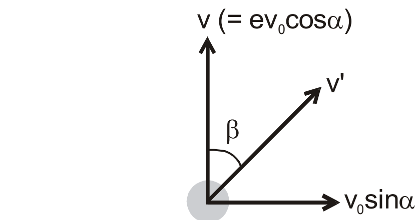
Example 26. A ball of mass m hits a floor with a speed v_0 making an angle of incidence a with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.

Solution :

The component of velocity v_0 along common tangential direction $v_0 \sin \alpha$ will remain unchanged. Let v be the component along common normal direction after collision. Applying, Relative speed of separation = e (Relative speed of approach) along common normal direction, we get

$$v = ev_0 \cos \alpha$$



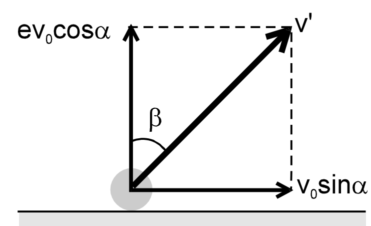


Thus, after collision components of velocity v' are $v_0 \sin \alpha$ and $ev_0 \cos \alpha$

$\therefore$ $v' = \sqrt{(v_0 \sin\alpha)^2 + (ev_0 \cos\alpha)^2}$ **Ans.**

and $\tan \beta = \dfrac{v_0 \sin\alpha}{ev_0 \cos\alpha}$

or $\tan \beta = \dfrac{\tan\alpha}{e}$ **Ans.**



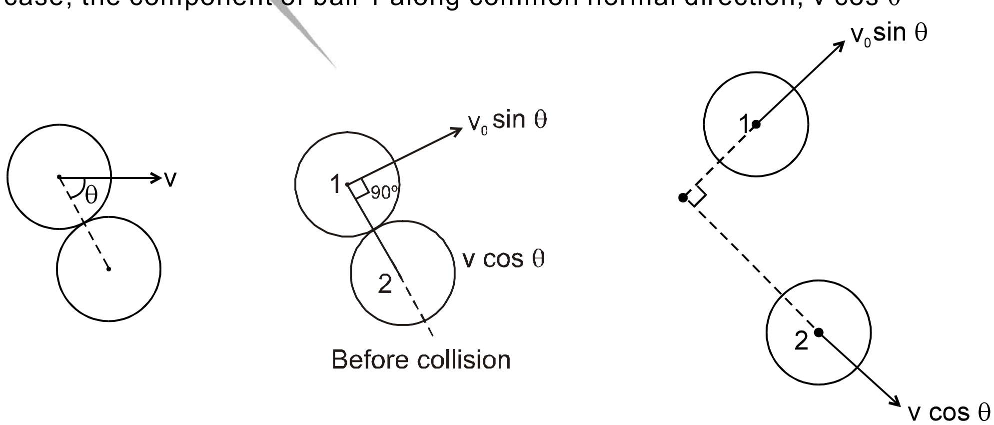
Note : For elastic collision, e = 1

$\therefore$ $v' = v_0$ and $\beta = \alpha$

Example 27. A ball of mass m makes an elastic collision with another identical ball at rest. Show that if the collision is oblique, the bodies go at right angles to each other after collision.

Solution :

In head on elastic collision between two particles, they exchange their velocities. In this case, the component of ball 1 along common normal direction, $v \cos \theta$



Before collision

After collision

becomes zero after collision, while that of 2 becomes $v \cos \theta$. While the components along common tangent direction of both the particles remain unchanged. Thus, the components along common tangent and common normal direction of both the balls in tabular form are given below.

Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	After collision	Before collision	After collision
1	$v \sin\theta$	$v \sin\theta$	$v \cos\theta$	0
2	0	0	0	$v \cos\theta$

From the above table and figure, we see that both the balls move at right angle after collision with velocities $v \sin\theta$ and $v \cos\theta$.

Note : When two identical bodies have an oblique elastic collision, with one body at rest before collision, then the two bodies will go in $\perp$ directions.

VARIABLE MASS SYSTEM :

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{rel}|$.

Thrust Force ($\vec{F}_t$)

$$\vec{F}_t = \vec{v}_{rel}\left(\frac{dm}{dt}\right)$$

Suppose at some moment t = t mass of a body is m and its velocity is $\vec{v}$. After some time at t = t + dt its mass becomes (m – dm) and velocity becomes $\vec{v}+d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass 'dm' is therefore ($\vec{v}$ + $\vec{v}_r$). If no external forces are acting on the system, the linear momentum of the system will remain conserved, or

$$\vec{P}_i = \vec{P}_f$$

or $\quad m\vec{v} = (m - dm)(\vec{v} + d\vec{v}) + dm(\vec{v} + \vec{v}_r)$

or $\quad m\vec{v} = m\vec{v} + md\vec{v} - (dm)\vec{v} - (dm)(d\vec{v}) + (dm)\vec{v} + \vec{v}_r\, dm$

The term (dm) ($d\vec{v}$) is too small and can be neglected.

$\therefore \quad md\vec{v} = -\vec{v}_r\, dm$

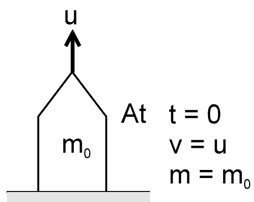
or $\quad m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$

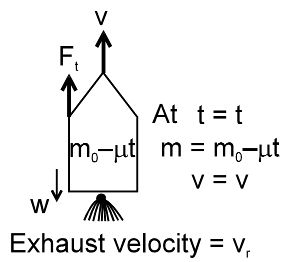
Here, $\quad m\left(-\frac{d\vec{v}}{dt}\right)$ = thrust force $\left(\vec{F}_t\right)$

and $\quad -\frac{dm}{dt}$ = rate at which mass is ejecting $\qquad$ or $\qquad \vec{F}_t = \vec{v}_r\left(\frac{dm}{dt}\right)$

Rocket propulsion :

Let m_0 be the mass of the rocket at time t = 0. m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.





Further, let $\left(\frac{-dm}{dt}\right)$ be the mass of the gas ejected per unit time and v_r the exhaust velocity of the gases with respect to rocket. Usually $\left(\frac{-dm}{dt}\right)$ and v_r are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time t = t,

1. Thrust force on the rocket $\qquad F_t = v_r\left(\frac{-dm}{dt}\right)$ $\qquad$ (upwards)
2. Weight of the rocket $\qquad W = mg$ $\qquad$ (downwards)
3. Net force on the rocket $\qquad F_{net} = F_t - W$ $\qquad$ (upwards)

   or $\qquad F_{net} = v_r\left(\frac{-dm}{dt}\right) - mg$

4. Net acceleration of the rocket $\qquad a = \frac{F}{m}$

   or $\qquad \frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$

   or $\qquad dv = \frac{v_r}{m}(-dm) - g\,dt$

   or $\qquad \int_u^v dv = v_r \int_{m_0}^{m} \frac{-dm}{m} - g\int_0^t dt$

   Thus, $\qquad v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$ $\qquad$ ...(i)

Note : 1. $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.

2. If gravity is ignored and initial velocity of the rocket u = 0, Eq. (i) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$.

Solved Examples

Example 28. A rocket, with an initial mass of 1000 kg, is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning stopsafter one minute. Find the maximum velocity of the rocket. (Take g as at 10 ms^{-2})

Solution :

Using the velocity equation

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

Here $u = 0$, $t = 60s$, $g = 10\ m/s^2$, $v_r = 2000\ m/s$, $m_0 = 1000$ kg

and $m = 1000 - 10 \times 60 = 400$ kg

We get $v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$

or $v = 2000 \ln 2.5 - 600$

The maximum velocity of the rocket is $200(10 \ln 2.5 - 3) = 1232.6\ ms^{-1}$ **Ans.**

LINEAR MOMENTUM CONSERVATION IN PRESENCE OF EXTERNAL FORCE.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$\Rightarrow$ $\vec{F}_{ext}\ dt = d\vec{P}$

$\Rightarrow$ $d\vec{P} = \vec{F}_{ext}\ )_{mpulsive}\ dt$

$\therefore$ If $\vec{F}_{ext}\ )_{mpulsive} = 0$

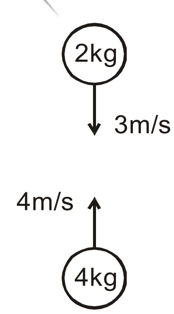
$\Rightarrow$ $d\vec{P} = 0$

or $\vec{P}$ is constant

Note: Momentum is conserved if the external force present is non-impulsive. eg. gravitation or spring force

Solved Examples

Example 29. Two balls are moving towards each other on a vertical line collides with each other as shown. Find their velocities just after collision.



Solution : Let the final velocity of 4 kg ball just after collision be v. Since, external force is gravitational which is non - impulsive, hence, linear momentum will be conserved.

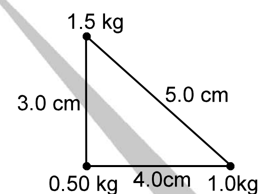
Applying linear momentum conservation:

$$2(-3) + 4(4) = 2(4) + 4(v)$$

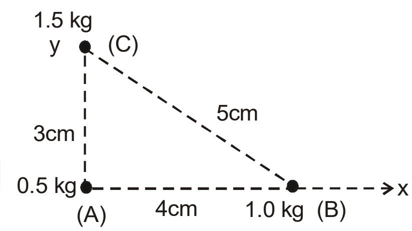
or $$v = \frac{1}{2} \text{ m/s}$$

Solved Miscellaneous Problems

Problem 1. Three particles of masses 0.5 kg, 1.0 kg and 1.5 kg are placed at the three corners of a right angled triangle of sides 3.0 cm, 4.0 cm and 5.0 cm as shown in figure. Locate the centre of mass of the system.



Solution :



taking x and y axes as shown.

coordinates of body A = (0,0)

coordinates of body B = (4,0)

coordinates of body C = (0,3)

$$\text{x - coordinate of c.m.} = \frac{m_A x_A + m_B x_B + M_C r_C}{m_A + m_B + m_C}$$

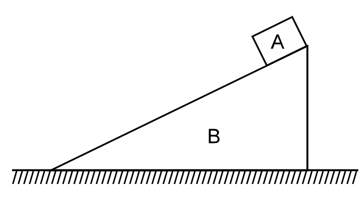
$$= \frac{0.5 \times 0 + 1.0 \times 4 + 1.5 \times 0}{0.5 + 1.0 + 1.5}$$

$$= \frac{4}{3} \frac{\text{cm}}{\text{kg}} = \text{cm} = 1.33\text{cm}$$

$$\text{similarly y - wordinates of c.m.} = \frac{0.5 \times 0 + 1.0 \times 0 + 1.5 \times 3}{0.5 + 1.0 + 1.5} = \frac{4.5}{3} = 1.5 \text{ cm}$$

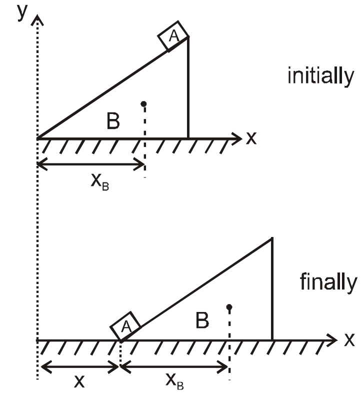
So, certre of mass is 1.33 cm right and 1.5 cm above particle A.

Problem 2. A block A (mass = 4M) is placed on the top of a wedge B of base length l (mass = 20 M) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches at lowest end of wedge. As sume all surfaces are frictionless.



Solution : Initial position of centre of mass

$$= \frac{X_B M_B + X_A M_A}{M_B + M_B} = \frac{X_B .20M + \ell .4M}{24M} = \frac{5X_B + \ell}{6}$$



Final position of centre of mass

$$= \frac{(X_B + x)20M + 4Mx}{24M} = \frac{5(X_B + x) + x}{6}$$

since there is no horizontal force on system

centre of mass initially = centre of mass finally.

$5X_B + \ell = 5X_B + 5x + x$

$\ell = 6x$

$$\boxed{x = \frac{\ell}{6}}$$

Problem 3. An isolated particle of mass m is moving in a horizontal xy plane, along x-axis. At a certain height above ground, it suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later, the smaller fragment is at y = + 15 cm. Find the position of heavier fragment at this instant.

Solution : As particle is moving along x-axis, so, y-coordinate of COM is zero.

$$Y_M M = Y_{\frac{M}{4}} \left(\frac{M}{4}\right) + Y_{\frac{3M}{4}} \left(\frac{3M}{4}\right) \Rightarrow 0 \times M = 15 \left(\frac{M}{4}\right) + Y_{\frac{3M}{4}} \left(\frac{3M}{4}\right)$$

$$\boxed{Y_{\frac{3M}{4}} = -5\text{cm}}$$

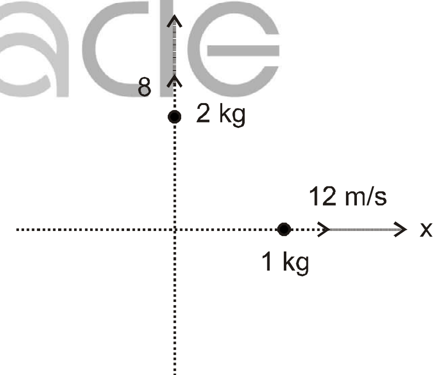
Problem 4. A shell at rest at origin explodes into three fragments of masses 1 kg, 2 kg and m kg. The fragments of masses 1 kg and 2 kg fly off with speeds 12 m/s along x-axis and 8 m/s along y-axis respectively. If m kg flies off with speed 40 m/s then find the total mass of the shell.

Solution : As initial velocity = 0, Initial momentum = (1 + 2 + m) × 0 = 0

Finally, let velocity of M = $\vec{V}$. We know $|\vec{V}|$ = 40 m/s.

Initial momentum = final momentum.

$$0 = 1 \times 12\,\hat{i} + 2 \times 8\,\hat{j} + m\vec{V} \Rightarrow \vec{V} = -\frac{(12\hat{i} + 16\hat{j})}{m}$$



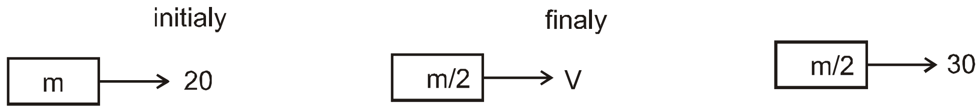
$$|\vec{V}| = \sqrt{\frac{(12)^2 + (16)^2}{m^2}} = \frac{1}{m}\sqrt{(12)^2 + (16)^2} = 40 \text{ \{given\}}$$

$$m = \sqrt{\frac{(12)^2 + (16)^2}{40}} = 0.5 \text{ kg}$$

Total mass = 1 + 2 + 0.5 = 3.5 kg

Problem 5. A block moving horizontally on a smooth surface with a speed of 20 m/s bursts into two equal parts continuing in the same direction. If one of the parts moves at 30 m/s, with what speed does the second part move and what is the fractional change in the kinetic energy of the system.

Solution :



Applying momentum conservation ;

$$m \times 20 = \frac{m}{2} V + \frac{m}{2} \times 30 \quad \Rightarrow \quad 20 = \frac{V}{2} + 15$$

So, $V = 10$ m/s

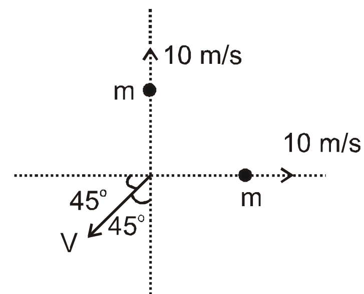
initial kinetic energy $= \frac{1}{2} m \times (20)^2 = 200$ m

final kinetic energy $= \frac{1}{2} . \frac{m}{2} . (10)^2 + \frac{1}{2} \times \frac{m}{2} (30)^2 = 25\text{ m} + 225\text{ m} = 250\text{ m}$

fractional change in kinetic energy $= \frac{(\text{final K.E}) - (\text{initial K.E})}{\text{initial K.E}} = \frac{250m - 200m}{200m} = \frac{1}{4}$

Problem 6. A block at rest explodes into three equal parts. Two parts starts moving along X and Y axes respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.

Solution :



Let total mass = 3 m, initial linear momentum = 3m × 0

Let velocity of third part $= \vec{V}$

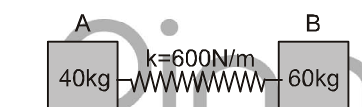
Using conservation of linear momentum :

$m \times 10\ \hat{i} + m \times 10\ \hat{j} + m\ \vec{V} = 0$

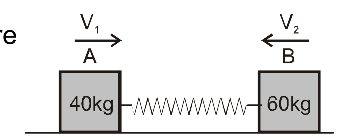
So, $\vec{V} = (-10\ \hat{i} - 10\ \hat{j})$ m/sec.

$|\vec{V}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$, making angle 135° below x-axis

Problem 7. Blocks A and B have masses 40 kg and 60 kg respectively. They are placed on a smooth surface and the spring connected between them is stretched by 1.5m. If they are released from rest, determine the speeds of both blocks at the instant the spring becomes unstretched.



Solution :



Let, both block start moving with velocity V_1 and V_2 as shown in figure Since no horizontal force on system so, applying momentum conservation

$0 = 40\ V_1 - 60\ V_2 \qquad \boxed{2V_1 = 3V_2}$ ....(1)

Now applying energy conservation, Loss in potential energy = gain in kinetic energy

$$\frac{1}{2} kx^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

$$\frac{1}{2} \times 600 \times (1.5)^2 = \frac{1}{2} \times 40 \times V_1^2 + \frac{1}{2} \times 60 \times V_2^2 \qquad ....(2)$$

Solving euation (1) and (2) we get,

$V_1 = 4.5$ m/s, $V_2 = 3$ m/s.

Problem 8. Find the mass of the rocket as a function of time, if it moves with a constant accleration a, in absence of external forces. The gas escaps with a constant velocity u relative to the rocket and its initial mass was m_0.

Solution : Using, $F_{net} = V_{rel}\left(\frac{-dm}{dt}\right)$

$$F_{net} = -u\frac{dm}{dt} \qquad ....(1)$$

$$F_{net} = ma \qquad ......(2)$$

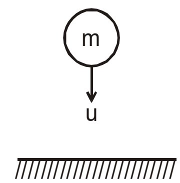
Solving equation (1) and (2)

$$ma = -u\frac{dm}{dt}$$

$$\int_{m_o}^{m}\frac{dm}{m} = \int_{o}^{t}\frac{-adt}{u} \qquad \ell n\frac{m}{m_o} = \frac{-at}{u}$$

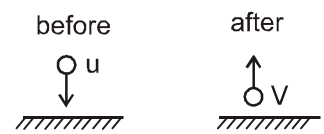
$$\frac{m}{m_o} = e^{-at/u} \qquad \boxed{m = m_0 e^{-\frac{at}{u}}}$$ **Ans.**

Problem 9 . A ball is approaching to ground with speed u. If the coefficient of restitution is e then find out:



(a) the velocity just after collision.
(b) the impulse exerted by the normal due to ground on the ball.

Solution :



$$e = \frac{\text{velocity of separation}}{\text{velocity of approah}} = \frac{v}{u}$$

(a) velocity after collision = V = eu ....(1)

(b) Impulse exerted by the normal due to ground on the ball = change in momentum of ball.

= {final momentum} – {initial momentum}

= {m v} – {– mu}

= mv + mu = m {u + eu} = mu {1 + e} **Ans.**

EXERCISE – 1
BUILDING A FOUNDATION

SECTION-A CALCULATION OF CENTRE OF MASS

A-1. The centre of mass of a body :
(a) Lies always at the geometrical centre (b) Lies always inside the body
(c) Lies always outside the body (d) May lie within or outside the body.

A-2. A body has its centre of mass at the origin. The x-coordinates of the particles
(a) may be all positive
(b) may be all negative
(c) must be all non-negative
(d) may be positive for some particles and negative in other particles

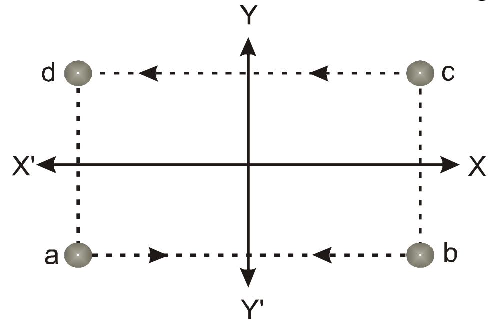
A-3. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is :
(a) = R (b) $\leq$ R (c) > R (d) $\geq$ R.

A-4. A system consists of mass M and m(<M). The centre of mass of the system is ;
(a) At the middle (b) Nearer to M
(c) Nearer to m (d) At the position of large mass.

A-5. Centre of mass is a point
(a) Which is geometric centre of a body
(b) From which distance of particles are same
(c) Where the whole mass of the body is supposed to concentrated
(d) Which is the origin of reference frame

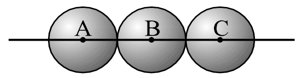
A-6. Choose the correct statement about the centre of mass (CM) of a system of two particles
(a) The CM lies on the line joining the two particles midway between them
(b) The CM lies on the line joining them at a point whose distance form each particle is inversely proportional to the mass of that particle
(c) The CM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle
(d) The CM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

A-7. Four bodies of equal mass start moving with same speed as shown in the figure. In which of the following combination the centre of mass will remain at origin



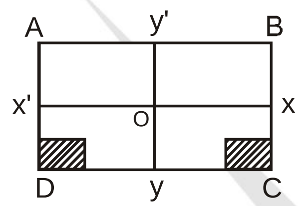
(a) c and b (b) a and b (c) a and c (d) b and c

A-8. Three spheres of same mass are placed touching each other with their centres on a straight line as shown in figure. Their centres are marked A, B and C. The distance of the centre of mass of the system from A is :



(a) $\frac{AB+AC+BC}{3}$ (b) $\frac{AB+AC}{3}$ (c) $\frac{AB+BC}{3}$ (d) $\frac{AC+BC}{3}$

A-9. A uniform square plate ABCD has a mass of 10 kg. If two point masses of 3 kg each are placed at the corners C and D as shown in the adjoining figure, then the centre of mass shifts to the point which is lie on -

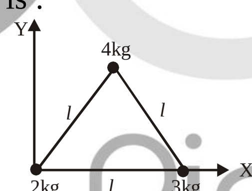


(a) OC (b) OD (c) OY (d) OX

A-10. Two particles whose masses are 10 kg and 30 kg and their position vectors are $\hat{\imath}+\hat{\jmath}+\hat{k}$ and $-\hat{\imath}-\hat{\jmath}-\hat{k}$ respectively would have the centre of mass at -

(a) $-\frac{(\hat{\imath}+\hat{\jmath}+\hat{k})}{2}$ (b) $\frac{(\hat{\imath}+\hat{\jmath}+\hat{k})}{2}$ (c) $-\frac{(\hat{\imath}+\hat{\jmath}+\hat{k})}{4}$ (d) $-\frac{(\hat{\imath}+\hat{\jmath}+\hat{k})}{4}$

A-11. Three masses 2kg, 3kg and 4kg are lying at the corners of an equilateral triangle of side l. The (X) coordinates of centre of mass is :



(a) $\frac{7}{12}l$ (b) $\frac{5}{9}l$ (c) $\frac{7\sqrt{2}}{9}l$ (d) $\frac{\sqrt{5}}{9}l$

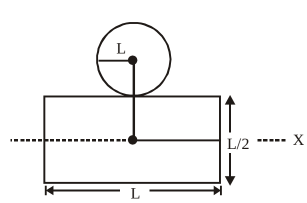
A-12. Three particles of masses 1 kg, 2kg, 1 kg are at the points whose position vectors are i + j, 2i – j, 3i + j. The position of their centre of mass is

(a) $2\hat{\imath}+\hat{\jmath}$ (b) $2\hat{\imath}$ (c) $\frac{1}{3}(6\hat{\imath}+\hat{\jmath})$ (d) $8\hat{\imath}$

A-13. Four particles of masses $m_1 = 2m$, $m_2 = 4m$, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4 so that the centres of mass of all the four particles are exactly at the centre of the square ?

(a) 2 m (b) 8 m (c) 6 m (d) None of these

A-14. Distance of centre of mass from the centre of a rectangular body is (Given mass of disc and rectangular body are equal) :



(a) $X_{cm} = 0, Y_{cm} = \frac{5\pi L}{\pi+1}$ (b) $X_{cm} = 0, Y_{cm} = \frac{5\pi L}{\pi+3}$
(c) $X_{cm} = 0, Y_{cm} = \frac{5\pi L}{\pi+2}$ (d) $X_{cm} = 0, Y_{cm} = \frac{5L}{8}$

A-15. Three point masses of 1 g, 2g and 3g have their centre of mass at (2, 2, 2). A fourth mass of 4g is placed at position vector $\vec{r}$ such that the centre of mass of new system is now at (0, 0, 0) :
(a) $r = (-1, -1, -1)$ (b) $r = (-2, -2, -2)$
(c) $r = (-3, -3, -3)$ (d) $r = (-4, -4, -4)$

SECTION-B MOTION OF CENTRE OF MASS

B-1. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will:
(a) Move vertically upwards and then downwards
(b) Move vertically downwards
(c) Move in irregular path
(d) Move in the parabolic path which the unexploded bomb would have travelled.

B-2. If a ball is thrown upwards from the surface of earth and during upward motion :
(a) The earth remains stationary while the ball moves upwards
(b) The ball remains stationary while the earth moves downwards
(c) The ball and earth both moves towards each other
(d) The ball and earth both move away from each other

B-3. Internal forces can change :
(a) the linear momentum but not the kinetic energy of the system.
(b) the kinetic energy but not the linear momentum of the system.
(c) linear momentum as well as kinetic energy of the system.
(d) neither the linear momentum nor the kinetic energy of the system.

B-4. If the external forces acting on a system have zero resultant, the centre of mass
(a) must not move (b) must accelerate (c) may move (d) may accelerate

B-5. Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)
(a) depends on the direction of the motion of the balls
(b) depends on the masses of the two balls
(c) depends on the speeds of the two balls
(d) is equal to g

B-6. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speed of 2m/s and 6m/s respectively on a smooth horizontal surface. The speed of centre of mass of the system is :
(a) $\frac{10}{3}$m/s (b) $\frac{10}{7}$ m/s (c) $\frac{11}{2}$ m/s (d) $\frac{12}{3}$m/s

B-7. Two objects of masses 200 gm and 500 gm posses velocities $10\hat{\imath}$ m/s and $3\hat{\imath} + 5\hat{\jmath}$ m/s respectively. The velocity of their centre of mass in m/s is :
(a) $5\hat{\imath} - 25\hat{\jmath}$ (b) $\frac{5}{7}\hat{\imath} - 25\hat{\jmath}$ (c) $5\hat{\imath} + \frac{25}{7}\hat{\jmath}$ (d) $25\hat{\imath} - \frac{5}{7}\hat{\jmath}$

B-8. 2 bodies of different masses of 2kg and 4kg are moving with velocities 20m/s and 10m/s towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass ?

(a) 5 m/s (b) 6 m/s (c) 8 m/s (d) zero

B-9. A 2 kg body and a 3 kg body are moving along the x-axis. At a particular instant the 2 kg body has a velocity of 3 ms–1 and the 3 kg body has the velocity of 2 ms–1. The velocity of the centre of mass at that instant is
(a) 5 ms–1 (b) 1 ms–1 (c) zero (d) None of these

B-10. Two particle of masses m1 and m2 initially at rest start moving towards each other under their mutual force of attraction. The speed of the centre of mass at any time t, when they are at a distance r apart, is
(a) Zero (b) $\left(G\frac{m_1m_2}{r^2}\cdot\frac{1}{m_1}\right)t$ (c) $\left(G\frac{m_1m_2}{r^2}\cdot\frac{1}{m_2}\right)t$ (d) $\left(G\frac{m_1m_2}{r^2}\cdot\frac{1}{m_1+m_2}\right)t$

B-11. Two bodies A and B have masses M and m respectively, where M > m and they are at a distance d apart. Equal force is applied to them so that they approach each other. The position where they hit each other is
(a) Nearer to B (b) Nearer to A
(c) At equal distance from A and B (d) Cannot be decided

SECTION-C CONSERVATION OF LINEAR MOMENTUM

C-1. A bullet of mass m is being fired from a stationary gun of mass M. If the velocity of the bullet is v, the velocity of the gun is-
(a) $\frac{Mv}{m+M}$ (b) $\frac{Mv}{M}$ (c) $\frac{(M+m)v}{M}$ (d) $\frac{M+m}{Mv}$

C-2. A bomb explodes in air in two equal fragments. If one of the fragments is moving vertically upwards with velocity v_0, then the other fragment is moving-
(a) Vertically up with velocity v_0 (b) Vertically downwards with velocity v_0
(c) In any arbitrary direction (d) None of these

C-3. Two particles with equal kinetic energies are having masses in the ratio of 1 : 2. Then linear momenta will be in the ratio-
(a) 1 (b) 4 (c) 0.707 (d) 2

C-4. If a shell fired from a canon is exploded in air then-
(a) Momentum decreases (b) Momentum increases
(c) Kinetic energy increases (d) K.E. decreases

C-5. Under the effect of mutual internal attractions-
(a) The linear momentum of a system increases
(b) The linear momentum of a system decrease
(c) The linear momentum of the system is conserved
(d) The angular momentum increases

C-6. The kinetic energies of a lighter body and a heavier body are same. Then the value of momentum is-
(a) Higher for lighter body
(b) Higher for heavier body
(c) Same for both
(d) Additional information is needed for replying this question

C-7. A bomb is projected with 200m/s at an angle 60° with horizontal. At the highest point, it explodes into three particles of equal masses. One goes vertically upward with velocity 100m/sec, second particle goes vertically downward with the same velocity as the first. Then what is the velocity of the third one-

(a) 120 m/sec with 60° angle (b) 200 m/sec with 30° angle
(c) 50 m/sec, in horizontal direction (d) 300 m/sec, in horizontal direction

C-8. A space craft of mass M is travelling in space with velocity v. It then breaks up into two parts such that the smaller part m comes to the rest, then the velocity of the remaining part is-

(a) $\frac{Mv}{M-m}$ (b) $\frac{Mv}{M+m}$ (c) $\frac{mv}{M-m}$ (d) $\frac{Mv}{m}$

C-9. A bomb at rest has mass 60 kg. It explodes and a fragment of 40 kg has kinetic energy 96 joule. Then kinetic energy of other fragment is-

(a) 180 J (b) 190 J (c) 182 J (d) 192 J

C-10. When a U^{238} nucleus originally at rest, decays emitting an alpha particle having a speed 'u', the recoil speed of the residual nucleus is-

(a) $\frac{4u}{234}$ (b) $-\frac{4u}{238}$ (c) $\frac{4u}{238}$ (d) $-\frac{4u}{234}$

C-11. A boy of mass m is standing on a block of mass M kept on a rough surface. When the boy walks from left to right on the block, the centre of mass (boy + block) of system :

(a) remains stationary
(b) shifts towards left
(c) shifts towards right
(d) shifts towards right if M > m and towards left if M < m.

C-12. A boy of mass m standing on a plank of mass M over a rough surface as shown. If boy starts running towards right over the plank in such a way that plank starts sliding , find the direction in which the centre of mass of boy-plank system will be displaced, when boy will reach the other end of the plank

(a) not displaced (b) towards left
(c) towards right (d) cannot be predicted

C-13. Two bodies having masses m_1 and m_2 and velocities $\vec{u}_1$ and $\vec{u}_2$ collide and form a composite system of

$$m_1\vec{u}_1 + m_2\vec{u}_2 = 0(m_1 \neq m_2).$$

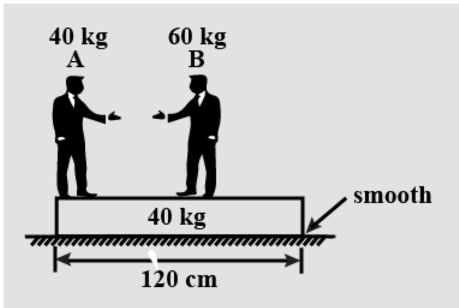
The velocity of the composite system is :

(a) 0 (b) $\vec{u_1} + \vec{u_2}$ (c) $\vec{u_1} - \vec{u_2}$ (d) $\frac{\vec{u_1}+\vec{u_2}}{2}$.

C-14. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to the other end of the plank. If the mass of the plank is 3M, the distance that the man moves relative to the ground is :

(a) L/4 (b) 3L/4 (c) 2L/3 (d) L/3

C-15. Two men 'A' and 'B' are standing on a plank. 'B' is at the middle of the plank and 'A' is at the left end of the plank. The lower surface of the plank is smooth. The system is initially at rest and masses are as shown in the figure. 'A' and 'B' start moving such that the position of 'B' remains fixed with respect to ground. A and B meets at some position. Then the point where A meets B is located at



(a) The middle of the plank
(b) 30 cm from the left end of the plank
(c) The right end of the plank
(d) None of these

C-16. A man of mass m is standing on a plank of equal mass m resting on a smooth horizontal surface. The man starts moving on the plank with speed u relative to the plank. The speed of the man relative to the ground is
(a) u (b) u/2 (c) 2u (d) 0

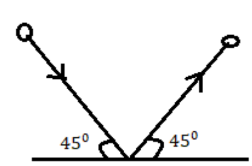
C-17. Two skaters A and B of mass 50 kg and 70 kg respectively stand facing each other 6 metres apart. They then pull on a light rope stretched between them. How far has each moved when they meet?
(a) Both have moved 2 metres
(b) A moves 2.5 metre and B moves 3.5 metres.
(c) A moves 3.5 metre and B moves 2.5 metres.
(d) A moves 2 metre and B moves 4 metres.

SECTION-D IMPULSE

D-1. A force of 50 dynes is acted on a body of mass 5gm which is at rest for an interval of 3 sec, then impulse is-
(a) 0.16×10^{-3} N-S (b) 0.98×10^{-3} N-S
(c) 1.5×10^{-3}N-S (d) 2.5×10^{-3}-S

D-2. A body of mass M moving with a speed V collides on a surface at an angle 45^0 degree without changing its speed the change in momentum
of the body will be-

(a) $MV(\hat{j} - \hat{\imath})$ (b) $MV(\hat{j} + \hat{\imath})$
(c) $2MV\hat{j}$ (d) $\sqrt{2}MV\hat{j}$



D-3. The area of F-t curve is A, where 'F' is the force on one mass due to the other. If one of the colliding bodies of mass M is at rest initially, its speed just after the collision is :

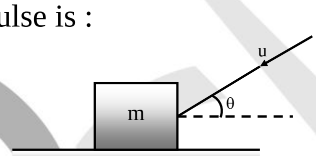
(a) A/M (b) M/A (c) AM (d) $\sqrt{\frac{2A}{M}}$

D-4. A body of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 60° with the vertical. The change in momentum [in magnitude] of the body when it returns on ground is-
(a) 24.5 N-s (b) 49.0 N-s (c) 98.0 N-s (d) $49\sqrt{3}$ N-s

D-5. A body of mass 'M' collides against a wall with a velocity u and retraces its path with the same speed. The change in momentum is (take initial direction of velocity as positive) :
(a) zero (b) 2Mu (c) Mu (d) –2Mu

D-6. If two balls, each of mass 0.06 kg, moving in opposite directions with speed of 4m/s, collide and rebound with the same speed, then the impulse imparted to each ball due to other (in kg-m/s) is :
(a) 0.48 (b) 0.53 (c) 0.81 (d) 0.92

D-7. A bullet of mass m fired with velocity u forming an angle with the horizontal and get embedded in a wooden block of same mass m. The block is placed on smooth surface. The magnitude of horizontal component of impulse is :



(a) $\frac{mu\cos\theta}{2}$ (b) $\frac{mu\sin\theta}{2}$ (c) $mu\cos\theta$ (d) $mu\sin\theta$

D-8. Choose the correct definition of impulse :
(a) impulse is defined as rate of change of momentum of particle
(b) impulse is defined as change in momentum of a particle
(c) impulse is defined as the integral of force with respect to time
(d) both B and C are correct.

SECTION-E COLLISION

E-1. In head on elastic collision of two bodies of equal masses, it is not possible :
(a) the velocities are interchanged
(b) the speeds are interchanged
(c) the momenta are interchanged
(d) the faster body speeds up and the slower body slows down

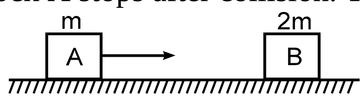
E-2. A massive ball moving with speed v collides head-on with a tiny ball at rest having a mass very less than the mass of the first ball. If the collision is elastic, then immediately after the impact, the second ball will move with a speed approximately equal to:
(a) v (b) 2v (c) v/2 (d) ∞

E-3. A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is
(a) m (b) 2m (c) 4m (d) infinity

E-4. In a collision between two solid spheres, velocity of separation along the line of impact (assume no external forces act on the system of two spheres during impact) :

(a) cannot be greater than velocity of approach
(b) cannot be less than velocity of approach
(c) cannot be equal to velocity of approach
(d) none of these

E-5. In the figure shown the block A collides head on with another block B at rest. Mass of B is twice the mass of A. The block A stops after collision. The co-efficient of restitution is :



(a) 0.5 (b) 1 (c) 0.25 (d) it is not possible

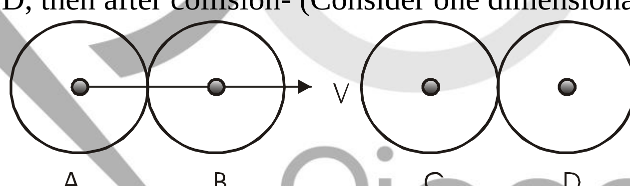
E-6. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after collision will be :

(a) $\frac{1-e}{1+e}$ (b) $\frac{1+e}{1-e}$ (c) $\frac{e+1}{e-1}$ (d) $\frac{e-1}{e+1}$

E-7. A ball rebounds after colliding with the floor, then in case of inelastic collision-
(a) The momentum of the ball before and after collision is same
(b) The mechanical energy of the ball is conserved
(c) The total momentum of the earth-ball system is conserved
(d) The total kinetic energy of earth and ball is conserved

E-8. A ball is allowed to fall from a height of 8cm, if the ball is perfectly elastic, how much it rise after rebound-
(a) 8 cm (b) 1 cm (c) 0.5 cm (d) 0

E-9. Two identical smooth spheres A and B are moving with same velocity and collides with similar spheres C and D, then after collision- (Consider one dimensional collision)



(a) D will move with greater speed
(b) C and D will move with same velocity v
(c) C will stop and D will move with velocity v
(d) All spheres A, B, C & D will move with velocity v/2

E-10. A ball is allowed of fall from a height 1.0 m. If the value of the coefficient of restitution is 0.6, then after the impact ball will go up to-
(a) 0.16 m (b) 0.36 m (c) 0.40 m (d) 0.60 m

E-11. A sphere of mass M moving with velocity u collides head on elastically with a sphere of mass m at rest. After collision their respective velocities are V and v. The value of v is-
(a) $2u\frac{M}{m}$ (b) $2u\frac{m}{M}$ (c) $\frac{2u}{1+\frac{m}{M}}$ (d) $\frac{2u}{1+\frac{M}{m}}$

E-12. A scooter of 40 kg mass moving with velocity 4 m/s collides with another scooter of 60 kg mass and moving with velocity 2 m/s. After collision the two scooters stick to each other the loss in kinetic energy-
(a) 392 J (b) 440 J (c) 48 J (d) 110 J

E-13. Two spheres approaching each other collides elastically. Before collision the speed of A is 5m/s and that of B is 10m/s. Their masses are 1kg and 0.5kg. After collision velocities of A and B are respectively-

(a) 5 m/s –10 m/s　　(b) 10 m/s, –5 m/s
(c) –10 m/s, –5 m/s　　(d) –5 m/s, 10 m/s

E-14. One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is 1/2, the ratio of their speeds after collision shall be-

(a) 1 : 2　　(b) 2 : 1　　(c) 1 : 3　　(d) 3 : 1

E-15. Which of the following statements is true for collisions-

(a) Momentum is conserved in elastic collisions but not in inelastic collisions
(b) Total kinetic energy is conserved in elastic collisions but momentum is not conserved
(c) Total kinetic energy is not conserved in inelastic collisions but momentum is conserved
(d) Total kinetic energy and momentum both are conserved in all types of collisions

E-16. For a two particle collision, the following quantities are conserved in general-

(a) Kinetic energy　　(b) Momentum
(c) Both kinetic energy and momentum　　(d) Neither kinetic energy nor momentum

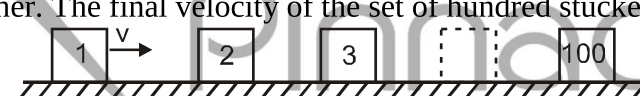
E-17. In a perfectly inelastic direct collision maximum transfer of energy takes place if-

(a) m1 >> m2　　(b) m1 << m2　　(c) m1 = m2　　(d) m2 = 0

E-18. Which of the following statement is true for collisions-

(a) Momentum is conserved in elastic collisions but not in inelastic collisions.
(b) Total K.E. is conserved in elastic collisions but momentum is not conserved.
(c) Total K.E. is not conserved in inelastic collisions but momentum is conserved.
(d) Total K.E. and momentum both are conserved in all types of collisions.

E-19. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity v towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be :



(a) $\frac{v}{99}$　　(b) $\frac{v}{100}$　　(c) zero　　(d) v

E-20. The co-efficient of restitution depends upon-

(a) The masses of the colliding bodies
(b) The direction of motion of the coliding bodies
(c) The inclination between the colliding bodies
(d) The materials of the colliding bodies

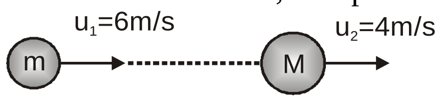
E-21. In an elastic collision of two particles the following is conserved :

(a) Momentum of each particle　　(b) Speed of each particle
(c) Kinetic energy of each particle　　(d) Total kinetic energy of both the particles

E-22. Two putty balls of equal mass moving with equal velocity in mutually perpendicular directions, stick together after collision. If the balls were initially moving with a velocity of ms^{-1} each, the velocity of their combined mass after collision is :

(a) $45\sqrt{2}$ ms^{-1}　　(b) 45 ms^{-1}　　(c) 90 ms^{-1}　　(d) $22.5\sqrt{2}$ms^{-1}

E-23. A particle of mass m moving with horizontal speed 6 m/sec. as shown in figure. If m << M then for one dimensional elastic collision, the speed of lighter particle after collision will be :



(a) 2 m/sec in original direction
(b) 2 m/sec opposite to the original direction
(c) 4 m/sec opposite to the original direction
(d) 4 m/sec in original direction

SECTION-F VARIABLE MASS

F-1. If the force on a rocket which is ejecting gases with a relative velocity of 300 m/s, is 210 N. Then the rate of combustion of the fuel will be :
(a) 10.7 kg/sec (b) 0.07 kg/sec (c) 1.4 kg/sec (d) 0.7 kg/sec

F-2. A belt is moving horizontally with a speed of 2m/s and sand is falling on it at the rate of 150 gm/sec. The additional force require to keep constant the speed of belt, is-
(a) 0.015 N (b) 0.30 N (c) 3N (d) 300 N

F-3. A rocket with a life-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. The initial thrust of the blast is-
(a) 14.0×10^5 N (b) 1.76×10^5 N (c) 3.5×10^5 N (d) 7.0×10^5 N

F-4. Fuel is consumed at the rate of 100 kg/sec. in a rocket. The exhaust gases are ejected as a speed of 4.5×10^4m/s. What is the thrust experience by the rocket-
(a) 3×10^6 N (b) 4.5×10^6 N (c) 6×10^6 N (d) 9×10^6 N

F-5. A 6000 kg rocket is set for vertical firing. If the exhaust speed is 1000 m/sec. How much gas must be ejected each second to supply the thrust needed to give the rocket an initial upward acceleration of 20 m/sec^2- (consider g = 9.8 m sec^{-2} acceleration due to gravity)
(a) 92.4 kg/sec (b) 178.8 kg/sec (c) 143.2 kg/sec (d) 47.2 kg/sec

F-6. The rocket works on the principle of conservation of-
(a) Energy
(b) Angular momentum
(c) Momentum
(d) Mass

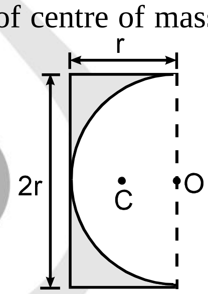
EXERCISE – II
READY FOR CHALLENGES

1. Two homogenous spheres A and B of masses m and 2m having radii 2a and a respectively are placed in touch. The distance of centre of mass from first sphere is :
(a) a (b) 2a (c) 3a (d) none of these

2. A non–uniform thin rod of length L is placed along x-axis as such its one of ends at the origin. The linear mass density of rod is l = l0x. The distance of centre of mass of rod from the origin is :
(a) L/2 (b) 2L/3 (c) L/4 (d) L/5

3. The centre of mass of the shaded portion of the disc is : (The mass is uniformly distributed in the shaded portion) :
(a) $\frac{R}{20}$ to the left of A (b) $\frac{R}{12}$ to the left of A
(c) $\frac{R}{20}$ to the right of A (d) $\frac{R}{12}$ to the right of A

4. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remainin g plate, from point 'O' is :

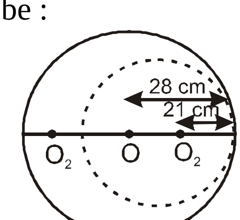


(a) $\frac{2r}{3-\pi}$ (b) $\frac{3r}{2(4-\pi)}$
(c) $\frac{2r}{4+\pi}$ (d) $\frac{2r}{3(4-\pi)}$

5. Four particle of masses m, 2m, 3m and 4m are arranged at the corners of a parallelogram with each side equal to a and one of the angle between two adjacent sides is 60°. The parallelogram lies in the x-y plane with mass m at the origin and 4m on the x-axis. The centre of mass of the arrangement will be located at
(a) $\left(\frac{\sqrt{3}}{2}a, 0.95a\right)$ (b) $\left(0.95a, \frac{\sqrt{3}}{4}a\right)$ (c) $\left(\frac{3a}{4}, \frac{a}{2}\right)$ (d) $\left(\frac{a}{2}, \frac{3a}{4}\right)$

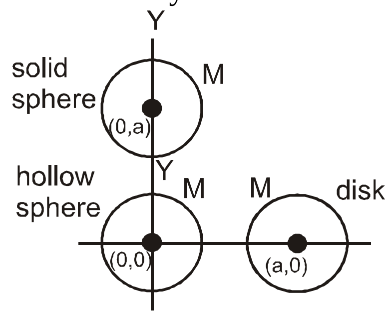
6. Masses 8, 2, 4, 2 kg are placed at the corners A,B,C,D respectively of a square ABCD of diagonal 80 cm. The distance of centre of mass from A will be
(a) 20 cm (b) 30 cm (c) 40 cm (d) 60 cm

7. A circular plate of uniform thickness has a diameter 56 cm. A circular portion of diameter 42 cm is removed from one edge as shown in the figure. The centre of mass of the remaining portion from the centre of plate will be :



(a) 5 cm (b) 7 cm (c) 9 cm (d) 11 cm

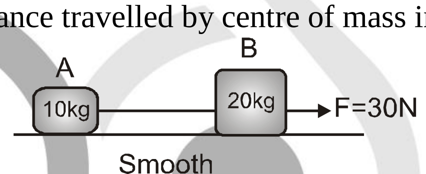
8. The coordinate of the centre of mass of a system as shown in figure : -



(a) $\left(\frac{a}{3}, 0\right)$ (b) $\left(\frac{a}{2}, \frac{a}{2}\right)$ (c) $\left(\frac{a}{3}, \frac{a}{3}\right)$ (d) $\left(0, \frac{a}{3}\right)$

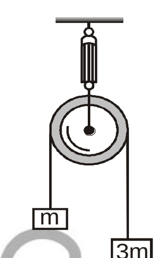
9. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The centre of mass :
(a) of the box remains constant
(b) of the box plus the ball system remains constant
(c) of the ball remains constant
(d) of the ball relative to the box remains constant

10. Two blocks A and B are connected by a massless string (shown in figure) A force of 30 N is applied on block B. The distance travelled by centre of mass in 2s starting from rest is:



(a) 1m (b) 2m (c) 3m (d) none of these

11. If the system is released, then the acceleration of the centre of mass of the system :



(a) $\frac{g}{4}$
(b) $\frac{g}{2}$
(c) g
(d) 2g

12. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the minimum energy of explosion is

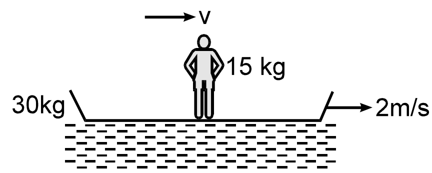
(a) $\frac{P^2}{2(m_1+m_2)}$ (b) $\frac{P^2}{2\sqrt{m_1 m_2}}$ (c) $\frac{P^2(m_1+m_2)}{2m_1 m_2}$ (d) $\frac{P^2}{2(m_1-m_2)}$

13. A bomb dropped from an aeroplane explodes in air. Its total :
(a) momentum decreases
(b) momentum increases
(c) kinetic energy increases
(d) kinetic energy decreases

14. A man of mass 'm' climbs on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move?

(a) downwards, $\frac{mv_{rel}}{m+M}$
(b) upwards, $\frac{Mv_{rel}}{m+M}$
(c) downwards, $\frac{mv_{rel}}{M}$
(d) downwards, $\frac{(M+m)v_{rel}}{M}$

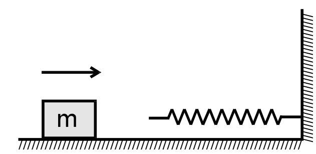
15. In the figure shown the initial velocity of boat (30 kg) + person (15 kg) is 2 m/s. Find velocity of person w.r.t. boat so that velocity of boat will be 1 m/s in right (Neglect friction between boat and water)



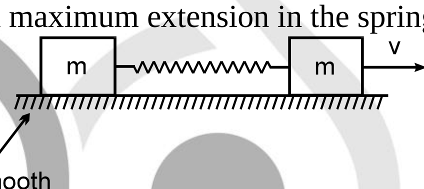
(a) 3 m/s towards right
(b) 3 m/s towards left
(c) 4 m/s towards right
(d) 4 m/s towards left

16. In the figure shown the change in magnitude of momentum of the block when it comes to its initial position if the maximum compression of the spring is x_0 will be :

(a) $2\sqrt{km}\,x_0$
(b) $\sqrt{km}\,x_0$
(c) zero
(d) none of these



17. Two masses are connected by a spring as shown in the figure. One of the masses was given velocity v = 2 k, as shown in figure where 'k' is the spring constant. Then maximum extension in the spring will be

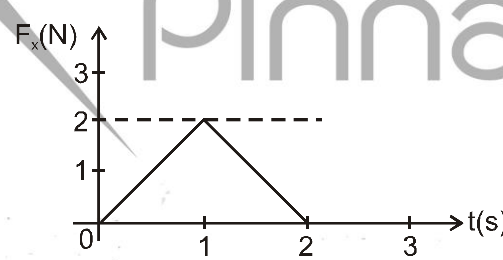


(a) 2 m
(b) m
(c) $\sqrt{2mk}$
(d) $\sqrt{3mk}$

18. A ball of mass 50 gm is dropped from a height h = 10 m. It rebounds losing 75 percent of its kinetic energy. If it remains in contact with the ground for Δt = 0.01 sec., the impulse of the impact force is :

(a) 1.3 N–s
(b) 1.05 N-s
(c) 1300 N–s
(d) 105 N–s

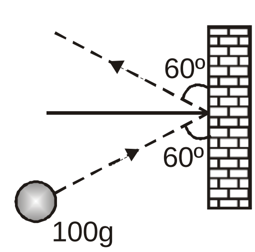
19. The given figure shows a plot of the time dependent force Fx acting on a particle in motion along the x-axis. What is the total impulse delivered by this force to the particle from time t = 0 to t = 2second?



(a) 0
(b) 1 kg-m/s
(c) 2 kg-m/s
(d) 3 kg-m/s

20. A mass of 100g strikes the wall with speed 5m/s at an angle as shown in figure and it rebounds with the same speed. If the contact time is 2×10^{-3} sec., what is the force applied on the mass by the wall :

(a) $250\sqrt{3}$ to right
(b) 250 N to right
(c) 250N $\sqrt{3}$ to left
(d) 250 N to left



21. A particle of mass ‘m’ and velocity ‘$\vec{v}$’ collides oblique elastically with a

stationary particle of mass ‘m’. The angle between the velocity vectors of the two particles after the collision is :
(a) 45° (b) 30° (c) 90° (d) None of these

22. When two bodies collide elastically, the force of interaction between them is :
(a) conservative (b) non–conservative
(c) either conservative or non–conservative (d) zero

23. During the head on collision of two masses 1 kg and 2 kg the maximum energy of deformation is $\frac{100}{3}$J. If before collision the masses are moving in the opposite direction, then their velocity of approach before the collision is :
(a) 10 m/sec. (b) 5 m/sec. (c) 20 m/sec. (d) $10\sqrt{2}$ m/sec.

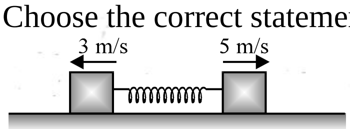
24. A block A of mass m moving with a velocity ' v ' along a frictionless horizontal track and a blocks of mass m/2 moving with 2 v collides with plank elastically. Final speed of the block A is :

(a) $\frac{5v}{3}$ (b) v (c) $\frac{2v}{3}$ (d) none of these

25. Consider the following two statements :
A. Linear momentum of a system of particles is zero
B. Kinetic energy of a system of particles is zero,
Then,
(a) A does not imply B and B does not imply A (b) A implies B but B does not imply A
(c) A does not imply B but B implies A (d) A implies B and B implies A

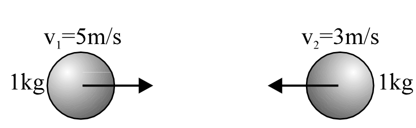
26. Two blocks of masses 10kg and 4kg are connected by a spring of negligible mass and are placed on a frictionless horizontal surface. An impulse gives a speed of 14 ms–1 to the heavier block in the direction of the lighter block. Then, the velocity of the centre of mass is
(a) 30 ms^{-1} (b) 20 ms^{-1} (c) 10 ms^{-1} (d) 5 ms^{-1}

27. Two identical blocks each of mass 1kg are joined together with a compressed spring. When the system is released the two blocks appear to be moving with unequal speeds in the opposite directions as shown in figure. Choose the correct statement (s) :



(a) It is not possible
(b) Whatever may be the speed of the blocks the centre of mass will remain stationary
(c) The centre of mass of the system is moving with a velocity of 2 m/s.
(d) The centre of mass of the system is moving with a velocity of 1 m/s.

28. A t = 0, the positions and velocities of particles are as shown in figure. They are kept on a smooth surface and being mutually attached by gravitational force. Find the position of centre of mass at t = 2s



(a) X = 5 m (b) X = 7 m (c) X = 3 m (d) X = 2 m

29. A bomb of mass 3 kg moving with velocity $\vec{v} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ m/s explodes into 3 parts of masses 1 kg each. The three parts move along x, y and z-axis respectively. The gain in kinetic energy is :

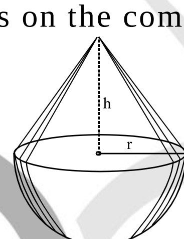
(a) 150 J (b) 250 J (c) 350 J (d) zero

30. A bag of mass M hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined system (bag + bullet) :

(a) Momentum is $\frac{mMv}{(M+m)}$ (b) KE is $\frac{1}{2}Mu^2$

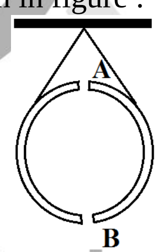
(c) Momentum is $\frac{mv(M+m)}{M}$ (d) KE is $\frac{m^2v^2}{2(M+m)}$

31. A uniform solid right cone of base radius r is joined to a uniform solid hemisphere of radius r and of the same density, so as to have a common face. The centre of gravity of the composite solid lies on the common face. The height of the cone is :



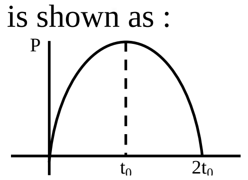
(a) 3r/2 (b) $r\sqrt{6}$ (c) $\frac{r}{\sqrt{3}}$ (d) None of these

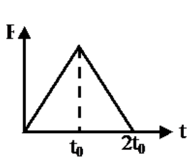
32. A uniform metallic spherical shell is suspended by threads. It has two holes A & B at top & bottom respectively, as shown in figure :

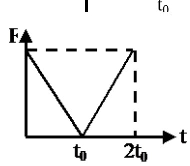


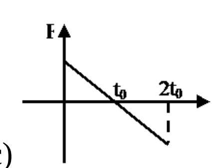
(a) If B is closed & sand is poured from A, centre of mass first rises & then falls.
(b) If shell is completely filled with sand & B is opened, then centre of mass falls, initially
(c) If shell is slightly filled with sand & B is opened, then centre of mass falls.
(d) All of these.

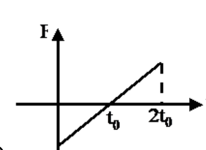
33. The magnitude of the momentum of a particle varying with time is shown in figure. The variation of force acting on the particle is shown as :



(a) 

(b) 

(c) 

(d) 

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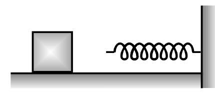
34. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L . The maximum momentum of the block after collision is:

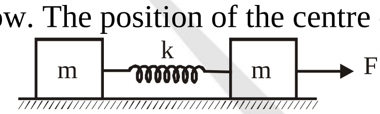


(a) $\frac{KL^2}{2M}$ (b) $\sqrt{MK}L$ (c) $\frac{ML^2}{K}$ (d) zero

35. A stick is standing vertically on a frictionless floor. The trajectory of the centre of mass during its fall, after it starts slipping is :

(a) hyperbolic (b) parabolic
(c) straight line at an angle 45° to the vertical (d) vertical straight line

36. Two blocks of equal mass 'm' are connected by an unstretched spring and the system is kept on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown below. The position of the centre of mass at time t is :



(a) $\frac{Ft}{2m}$ (b) $\frac{Ft^2}{4m}$ (c) $\frac{Ft^2}{3m}$ (d) $\frac{Fm}{3t^2}$

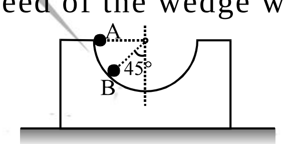
37. In a gravity free space, a man of mass M standing at a height h above the floor, throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be :

(a) $h\left(1+\frac{m}{M}\right)$ (b) $h\left(2-\frac{m}{M}\right)$
(c) 2h (d) a function of m, M, h and u

38. Two particles of masses m_1 and m_2 in projectile motion have collision at time t_0.Their velocities become $\vec{v}_1$ and $\vec{v}_2$ at time $2t_0$ while still moving in air . The value of change in momentum of the system during the time interval from t_o to $2t_0$ is :

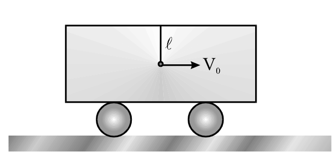
(a) Zero (b) $(m_1+m_2)gt_0$ (c) $2(m_1+m_2)gt_0$ (d) $4(m_1+m_2)gt_0$

39. A ball of mass m is released from A inside a smooth wedge of mass m as shown in the figure. What is the speed of the wedge when the ball reaches point B?



(a) $\left(\frac{gR}{3\sqrt{2}}\right)^{1/2}$ (b) $\sqrt{2gR}$ (c) $\left(\frac{5gR}{2\sqrt{3}}\right)^{1/2}$ (d) $\sqrt{\frac{3}{2}gR}$

40. A bob of mass m is suspended by a string from a train of mass M, free to move on a horizontal surface. The bob is given a horizontal velocity V_0. The maximum height attained by the bob is :



(a) $\frac{V_0{}^2}{2g}\left(\frac{M+m}{M}\right)$ (b) $\frac{V_0{}^2 m}{2gM}$ (c) $\frac{V_0{}^2 M}{2g(M+m)}$ (d) $\frac{V_0{}^2}{2g}$

41. Three particles A, B,C of equal mass move with equal speed v along the medians of an equilateral triangle. They collide at the centroid of the triangle and A comes to rest after collision while B retraces its path with speed v. The velocity of C will be :

(a) Zero (b) $v/2$ in the direction of A
(c) v in the same direction as that of B (d) v in the direction opposite to that of B

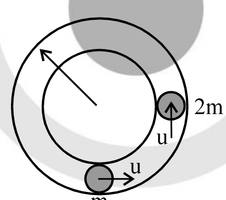
42. A man of mass 'm'stands on a plane plank of mass $\frac{m}{2}$, lying on a smooth horizontal floor . Initially both are at rest. Then the man starts walking on the plank towards the east and stops after a distance 'l' on the plank. Then:

(a) The plank will slide to the west by a distance l
(b) The plank will continue to move towards the west over smooth floor
(c) The plank will slide to the west by $\frac{2l}{3}$ and then stop
(d) The centre of mass of the man will remain unchanged on the floor

43. A ball impinges directly on a similar ball at rest. The first ball is brought to rest by the impact. If half the kinetic energy is lost by impact, what is the value of the coefficient of restitution ?

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) None of these

44. Two small bodies of masses 'm' and '2m' are placed in a fixed smooth horizontal circular hollow tube of mean radius 'r' as shown. The mass 'm' is moving with speed 'u' and the mass '2m' is stationary. After their first collision, the time elapsed for next collision is: [e = 1/2]



(a) $\frac{2\pi r}{u}$ (b) $\frac{4\pi r}{u}$ (c) $\frac{3\pi r}{u}$ (d) $\frac{12\pi r}{u}$

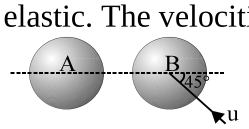
45. A steel ball strikes a steel plate placed on a horizontal surface at an angle θ with the vertical. If the co-efficient of restitution is e, the angle at which the rebound will take place is :

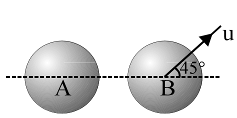
(a) θ (b) $\tan^{-1}\left[\frac{\tan\theta}{e}\right]$ (c) $e\tan\theta$ (d) $\tan^{-1}\left[\frac{e}{\tan\theta}\right]$

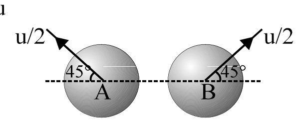
46. A smooth sphere is moving on a horizontal surface with velocity vector $3\hat{\imath}+\hat{\jmath}$ immediately before it hits a vertical wall. The wall is parallel to the vector $\hat{\jmath}$ and the coefficient of restitution between the wall and sphere is 1/3. The velocity vector of the sphere after it hits the wall is :

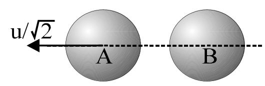
(a) $\hat{\imath}+\hat{\jmath}$ (b) $\frac{3\hat{\imath}-\hat{\jmath}}{3}$ (c) $-\hat{\imath}+\hat{\jmath}$ (d) $\hat{\imath}-\hat{\jmath}$

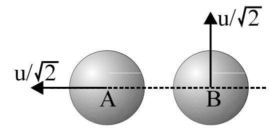
47. The diagram shows the velocities just before collision of two smooth spheres of equal radius and mass. The impact is perfectly elastic. The velocities just after impact are :



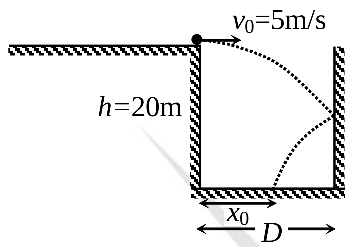
(a) 

(b) 

(c) 

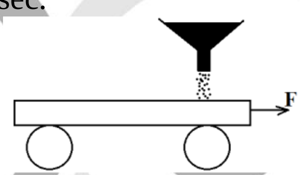
(d) 

48. A ball rolls off a horizontal table with velocity $v_0 = 5$ m/s . The ball bounces elastically from a vertical wall at a horizontal distance D (= 8) m from the table, as shown in figure. The ball then strikes the floor a distance x_0 from the table (g = 10 m/s^2). The value of x_0 is :



(a) 6 m (b) 4 m (c) 5 m (d) 7 m

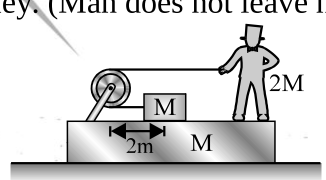
49. A flat cart of mass m_0 starts moving to the right due to a constant horizontal force F at t = 0 . Sand spills on the flat cart from a stationary hopper as shown in figure .The rate of loading is constant and is equal to μ Kg/sec.



(a) Initial acceleration of the cart is $\frac{F}{m_0}$ (b) Acceleration at time is $\frac{F}{m_0+\mu t}$

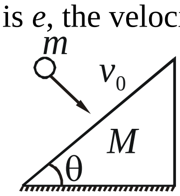
(c) Initial acceleration is less than $\frac{F}{m_0}$ (d) Acceleration at time is $\frac{2F}{m_0}$

50. A block of mass M is tied to one end of a massless rope. The other end of the rope is in the hands of a man of mass 2M as shown in figure. The block and the man are resting on a rough wedge of mass M as shown in the figure. The whole system is resting on a smooth horizontal surface. The man pulls the rope. Pulley is massless and frictionless. What is the displacement of the wedge when the block meets the pulley. (Man does not leave his position during the pull) ?



(a) 0.5 m (b) 1 m (c) zero (d) $\frac{2}{3}$ m

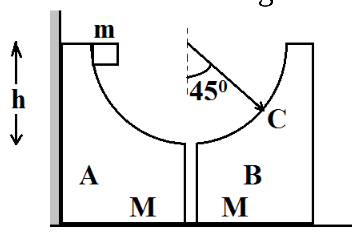
51. A ball of mass m collides perpendicularly on a smooth stationary wedge of mass M. If the coefficient of restitution of collision is e, the velocity of the wedge after collision is :



(a) $\frac{(1+e)mv_0}{M}$ (b) $\frac{emv_0}{M+m}$

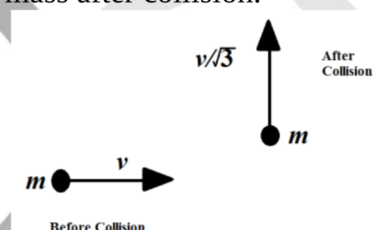
(c) $\dfrac{M v_0 e}{M+m}$ (d) $\dfrac{(1+e) m v_0 \sin\theta}{M + m\sin^2\theta}$

52. Two identical blocks having mass M each are smoothly conjugated and placed on a smooth horizontal floor as shown in fig. On left of block A, there is a wall. A small block of mass m is released from the position shown in the fig. Velocity of block B is maximum,



(a) When m is at highest position on B
(b) When m is at lowest position & moving towards left.
(c) When m is at point C
(d) is equal to $\frac{m\sqrt{2gh}}{m+M}$

53. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of 2nd mass after collision.

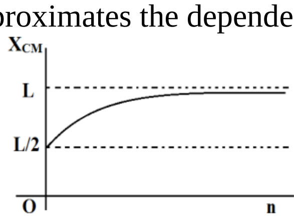


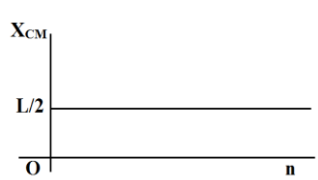
(a) v (b) $\sqrt{3}v$ (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{\sqrt{3}}$

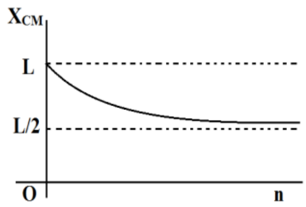
54. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is
(a) 96 J (b) 144 J (c) 288 J (d) 192 J

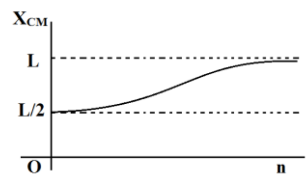
55. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?
(a) d (b) $\frac{m_2}{m_1}d$ (c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$

56. A thin rod of length L is lying along the x-axis with its ends x = 0 and x = L. Its linear density (mass/length) varies with x as $\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position X_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of X_{CM} on n?

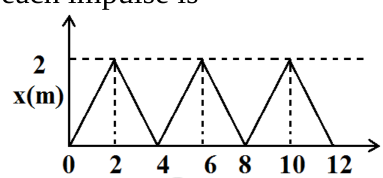
(a) 

(b) 

(c) 

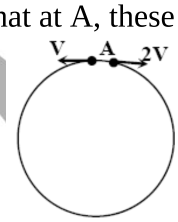
(d) 

57. The figure shows the position-time (x – t) graph of one dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



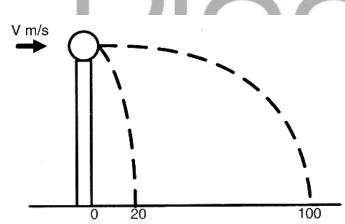
(a) 0.4 Ns (b) 0.8 Ns (c) 1.6 Ns (d) 0.2 Ns

58. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(a) 4 (b) 3 (c) 2 (d) 1

59. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



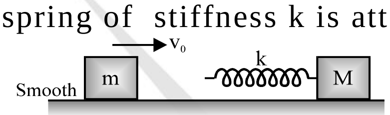
(a) 250 m/s (b) 250 $\sqrt{2}$ m/s (c) 400 m/s (d) 500 m/s

EXERCISE – III
CROSSING THE HURDLES

MORE THAN ONE CORRECT

1. A particle of mass m moving with a velocity $3\hat{\imath}+2\hat{\jmath}$ m/s collides with stationary body of mass M and finally moves with velocity $-2\hat{\imath}+\hat{\jmath}$ m/s. Then
(a) Impulse received by $m=(5\hat{\imath}-5\hat{\jmath})$
(b) Impulse received by $m=(-5\hat{\imath}-\hat{\jmath})$
(c) Impulse received by $M=(5\hat{\imath}+\hat{\jmath})$
(d) No impulse is received by M

2. A block of mass m moving with a velocity v_0 collides with a stationary block of mass M at the back of which a spring of stiffness k is attached, as shown in figure.

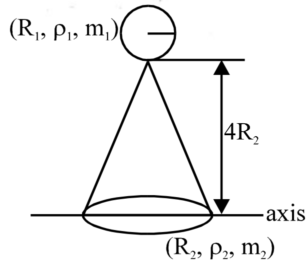


Choose the correct alternative(s).
(a) The velocity of the centre of mass is $v_0/2$
(b) The initial kinetic energy of the system in the centre of mass frame is $\frac{1}{4}\left(\frac{mM}{M+m}\right)v_0^2$
(c) The maximum compression in the spring is $v_0\sqrt{\frac{mM}{(m+M)}\frac{1}{k}}$
(d) When the spring is in the state of maximum compression, the kinetic energy in the centre of mass frame is zero

3. In an elastic collision :
(a) the kinetic energy remains constant at each instant.
(b) the linear momentum remains constant
(c) the final kinetic energy is equal to the initial kinetic energy
(d) the final linear momentum is equal to the initial linear momentum

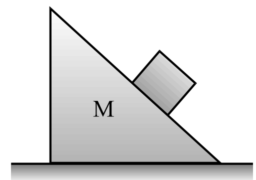
4. If the external forces acting on a system have zero resultant, the centre of mass :
(a) must not move
(b) must not accelerate
(c) may move
(d) may accelerate

5. A solid cone and a solid sphere is arranged as shown in the figure. The centre of mass is :



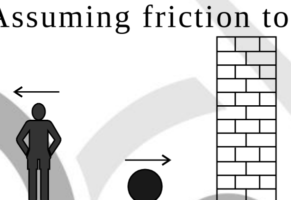
(a) at 3R if $m_1 = m_2 = m$ and $R_1 = R_2 = R$
(b) at 2R from the centre of mass of solid cone if $\rho_1=\rho_2$ and $R_1 = R_2 = R$
(c) If $\rho_1=2\rho_2$ and $R_1 = R_2 = R$ then distance from the centre of base of solid cone is 11R/3
(d) None of these

6. A block is kept at the top of a smooth wedge, which in turn is kept on a smooth horizontal surface. Then:



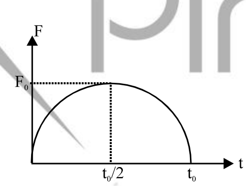
(a) The centre of mass will not shift in the horizontal direction.
(b) Centre of mass moves vertically
(c) Centre of mass shifts in both directions horizontally as well as vertically
(d) None of these

7. A man of mass M is carrying a ball of the mass M/2. The man is initially in the state of rest at a distance D from fixed vertical wall. He throws the ball towards the wall with a velocity V with respect to earth at $t = 0$. As a result of throwing, the man also starts moving backwards. The ball rebounds elastically from the wall. The man finally collects the ball. Assuming friction to be absent,



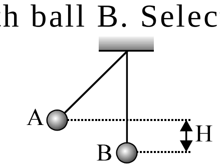
(a) The velocity of the man + ball system after the man has collected the ball is $\frac{2V}{3}$
(b) Impulse by ball on man is $\frac{MV}{3}$
(c) Impulse by ball on man is $\frac{MV}{6}$
(d) He catches the ball again at $t = \frac{4D}{V}$

8. A variable force acts on a particle of mass m (initially at rest) from $t = 0$ to $t = t_0$. The plot of F versus t is a semi-circle as shown in figure. Which of the following is correct?



(a) Impulse imparted to the particle is infinite
(b) Impulse imparted to the particle is $\frac{\pi F_0^2}{2}$
(c) the velocity acquired by the particle is $\frac{\pi F_0^2}{2m}$
(d) The momentum gain is $\frac{\pi F_0^2}{2}$

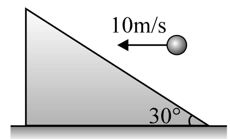
9. Two small balls A and B of mass M and 3M hang from the ceiling by strings of equal length. The ball is drawn aside so that it is raised to a height H. If the ball A is released and collides with ball B. Select the correct answer(s).



(a) If collision is perfectly elastic, ball B will rise to a height H/4

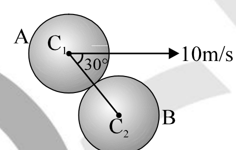
(b) If the collision is perfectly elastic ball A will rise upto a height H/4
(c) If the collision is perfectly inelastic, the combined mass will rise to a height H/16
(d) If the collision is perfectly inelastic, the combined mass will rise to a height H/4.

10. A ball of mass 1 kg strikes a wedge of mass 4 kg horizontally with a velocity of 10 m/s. Just after collision velocity of wedge becomes 4 m/s. Friction is absent everywhere and collision is elastic. Select the correct alternative(s).



(a) speed of ball after collision is 6 m/s
(b) impulse between ball and wedge during collision is 16 N-s
(c) impulse between ball and wedge during collision is 32 N-s
(d) None of these

11. A ball A collides elastically with an another identical ball B with velocity 10 m/s at an angle of 30° from the line joining their centres C_1 and C_2. Select the correct alternative(s).



(a) velocity of ball A after collision is 5 m/s
(b) velocity of ball B after collision is $5\sqrt{3}$ m/s
(c) both the balls move at right angles after collision
(d) kinetic energy will not be conserved here, because collision is not head on

12. A ball of mass m collides elastically with an identical ball at rest with some impact parameter.
(a) 100% energy transfer can never take place
(b) 100% energy transfer may take place
(c) angle of divergence between the two balls must be 90°
(d) angle of divergence between the two balls depend on impact parameter

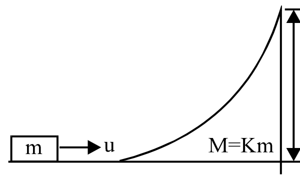
COMPREHENSION TYPE QUESTION

PASSAGE I

Collision is the transfer of momentum due to only the internal forces between the particles taking part in collision. When exchange of a momentum takes place between two physical bodies due to their mutual interactive force, it is defined as collision between two bodies.

Two bodies move in different directions interact each other at the point of intersection of their line of motion and the reaction due to their physical contact is the interaction force which is the cause of transfer of momentum from one body to another. Collision may be either elastic or inelastic. In case of elastic collision momentum and K. E. are both conserved but in case of inelastic collision only momentum is conserved and K. E. is not conserved.

13. A block of mass m is moved towards a movable wedge of mass M = km and height h with velocity u (All the surface are smooth). If the block just reaches the top of the wedge, the value of u is :



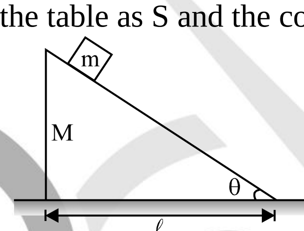
(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{2ghK}{1+K}}$ (c) $\sqrt{\frac{2gh(1+K)}{K}}$ (d) $\sqrt{2gh\left(1-\frac{1}{K}\right)}$

14. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. A projected along the groove and at the end of time t impinges on B. If e is the coefficient of restitution, the second impact will occur after a time

(a) 2t (b) 2et (c) 2t/e (d) $2t/e^2$

PASSAGE II

A block of mass m slides down a smooth incline of mass M and length l, solely as a result of the force of gravity. The incline is placed on a smooth horizontal table as shown in figure. Let us denote the coordinate system relative to the table as S and the coordinate system relative to the incline as S¢.



15. The acceleration of m in the S¢ frame is :

(a) $\frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$ (b) $\frac{(M+m)g\sin\theta}{m-M\sin^2\theta}$ (c) $\frac{(M-m)g\sin\theta}{M+m\sin^2\theta}$ (d) $\frac{(M+m)g\sin\theta}{M+m\sin\theta}$

16. The acceleration of the incline in the S frame is :

(a) $\left(\frac{mg\sin\theta\cos\theta}{M-m\sin^2\theta}\right)$ (b) $\left(\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$ (c) $\left(\frac{Mg\sin\theta\cos\theta}{M-m\sin^2\theta}\right)$ (d) $\left(\frac{Mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$

17. The force exerted by the small m on the wedge of mass M is :

(a) mg cos θ (b) $\frac{Mmg\cos\theta}{M+m\sin^2\theta}$ (c) $\frac{mg}{\cos\theta}$ (d) None

PASSAGE III

Two friends A and B (each weighing 40 kg) are sitting on a frictionless platform some distance d apart. A rolls a ball of mass 4 kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back and forth between A and B. The ball has a fixed speed of 5 m/s on the platform.

18. The speed of A after he rolls the ball for the first time is :

(a) 0.5m/s (b) 1 m/s (c) 1.5 m/s (d) 0.25 m/s

19. The speed of A after he catches the ball for the first time is :

(a) 10/11 m/s (b) 0.5 m/s (c) 50/11 m/s (d) none

20. The speeds of A and B after the ball has made 4 round trips and is held by A are :

(a) 50/11 m/s, 4.5 m/s (b) 40/11 m/s, 4.0 m/s
(c) 30/11 m/s, 3.5 m/s (d) 50/11 m/s, 30/11 m/s

PASSAGE IV

A piece of wood of mass 0.03 kg is dropped from the top of a building 100 m high. At the same time a bullet of mass 0.02 kg is fired vertically upwards with a velocity of 100 m/s from the ground. The bullet gets embeded in the wooden piece after striking it. [Take:g =10m/s^2]

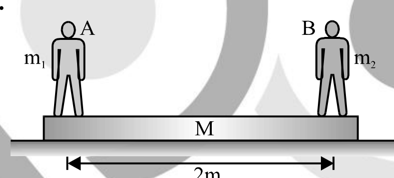
21. The time after which bullet strikes the wooden block is :
(a) 1 sec (b) 0.5 sec (c) 1.5 sec (d) 2 sec

22. If the time of collisions is 100 μ sec and $\vec{u}_1, \vec{u}_2 \& \vec{v}_1, \vec{v}_2$ are velocities of block , bullet before and after the collision respectivelly, then the value of $|(m_1\vec{u}_1 + m_2\vec{u}_2) - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is :
(a) 5×10^{-4} Ns (b) 5×10^{-5} Ns (c) 5×10^{-3} Ns (d) 5×10^{-6} Ns

23. The velocity of the (block + bullet) system after the collision is :
(a) 14 m/sec (b) 21 m/sec (c) 42 m/sec (d) 30 m/sec

MATRIX MATCH TYPE

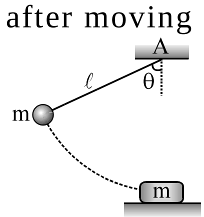
24. In the figure shown, when the persons A and B exchange their positions, then match the Column I and Column II.



m_1=50kg, m_2 = 70 kg, M = 80 kg

	Column I		Column II
A	the distance moved by the centre of mass of the system is	P	20 cm
B	the distance moved by the plank is	Q	1.8 m
C	the distance moved by A with respect to ground is	R	zero
D	the distance moved by B with respect to ground is	S	2.2 m

25. In the arrangement shown in figure, ball and block have the same mass m = 1kg each, θ = 60° and length l = 2.50 m. Coefficient of friction between block and floor is 0.5. When the ball is released from the position shown in the figure, it collides with the block and the block stops after moving a distance 2.50 m.



	Column I		Column II
A	velocity of ball just before collision (m/sec)	P	5
B	velocity of block just after collision (m/sec)	Q	3
C	velocity of block after travelling 1.6 m (m/sec)	R	1
D	Coefficient of restitution (dimensionless)	S	0

EXERCISE – IV

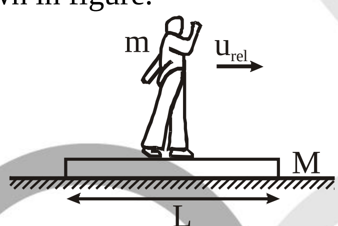
1. A projectile of mass 3 m explodes at highest point of its path. It breaks into three equal parts. One part retraces its path, the second one comes to rest. The range of the projectile was 100 m if no explosion would have taken place. The distance of the third part from the point of projection when it finally lands on the ground is :

(a) 100 m (b) 150 m (c) 250 m (d) 300

2. Two persons standing on a floating boat run in succession along its length with a speed 4.2 m/s relative to the boat and dive off from the end. The mass of each man is 80 kg and that of boat is 400 kg. If the boat was initially at rest, find the final velocity of the boat. Neglect friction :

(a) 0.6 m/s (b) 0.7 m/s (c) 0.1 m/s (d) 1.3 m/s

3. A man of mass m moves on a plank of mass M with a constant velocity $u_{rel.}$ with respect to the plank, as shown in figure.



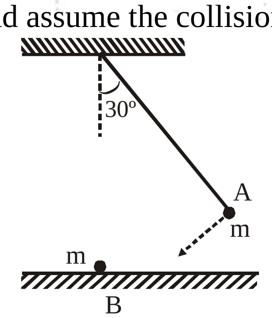
(i) If the plank rests on a smooth horizontal surface, then determine its velocity with respect to ground.

(ii) If the man travels a distance L with respect to the plank, then find the distance travelled by the plank with respect to ground.

4. Two identical blocks A and B of mass M each are kept on each other on a smooth horizontal plane. There exists friction between A and B. If a bullet of mass m hits the lower block with a horizontal velocity v and gets embedded into it. Find the work done by friction between A and B.

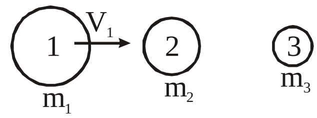
5. A 75.2 kg man is riding on a 38.6 kg cart travelling at a speed of 2.33 m/s. He jumps off in such a way as to land on the ground with zero horizontal speed. Find the resulting change in the speed of the cart.

6. The bob A of a pendulum released from 30^0 to the vertical hits another bob B of the same mass at rest on a table as shown in fig. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.

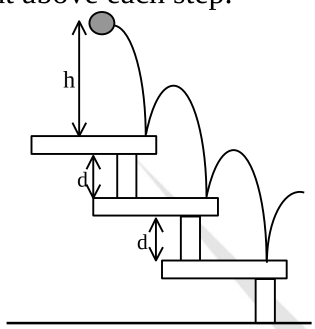


7. The centres of the spheres 1, 2 and 3 lie on a single straight line. Sphere 1 is moving with an (initial) velocity v_1 directed along this line and hits sphere 2. Sphere 2, acquiring after collision a velocity v_2, hits sphere 3. Both collisions are absolutely elastic . What must be the mass of sphere

2 for the sphere 3 to acquire maximum velocity (the masses m_1 and m_3 of spheres 1 and 3 are known)?



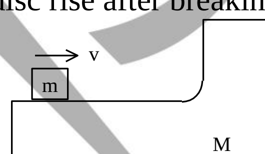
8. A ball is dropped from a height h above the landing and bounces down the flight of stairs. Denoting by e the coefficient of restitution, determine the value of h for which the ball will bounce the same height above each step.



9. Sphere of mass m_1 = 3 kg impinges with a velocity of 7 m/s directly on another sphere of mass m_2 = 5 kg. The velocities after impact are in the ratio 2 : 3. Find the coefficient of restitution and loss of energy.

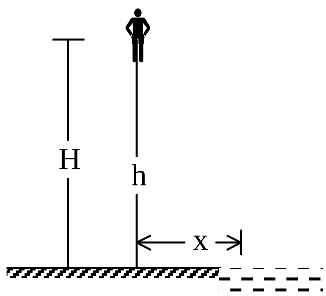
10. A projectile is fired with a speed u at an angle θ above a horizontal field. The coefficient of restitution of collision between the projectile and the field is e. How far from the starting point, does the projectile makes its second collision with the field?

11. A body of mass M with a small disc m placed on it rests on a smooth horizontal surface. The disc is set in motion in the horizontal direction with a velocity v. To what height relative to the initial level will the disc rise after breaking off from the body M. Friction can be assumed to be absent.

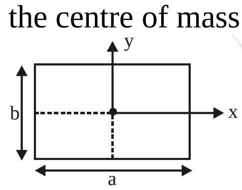


12. A particle A of mass m moving on a smooth horizontal surface collides with a stationary particle B of mass 2m directly, situated at a distance d from a wall. The coefficient of restitution between A and B and between B and the wall is e = 1/4. Calculate the distance from the wall where they collide again. Assume that the entire motion takes place along a straight line perpendicular to the wall.

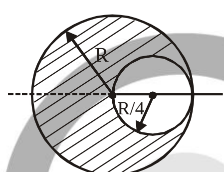
13. A girl of mass M holding a bag of mass m slips from the roof of a tall building of height H and starts falling vertically as shown in figure. When at a height h from the ground, she notices that the ground below her is pretty hard, but there is a pond at a horizontal distance x from the line of fall. In order to save herself she throws the bag horizontally (with respect to herself) in the direction opposite to the pond. Calculate the minimum horizontal velocity imparted to the bag so that the girl lands in the water. If the girl just succeeds to avoid the hard ground, where will the bag land ?



14. Consider a rectangular plate of dimensions a × b. If this plate is considered to be made up of four rectangles of dimensions a/2 × b/2 and we now remove one out of four rectangles, find the position where the centre of mass of the remaining system will be ?



15. Find the centre of mass of the shaded portion of disc.



ANSWERS

EXERCISE – I
BUILDING A FOUNDATION

SECTION-A

A-1. (d)	A-2. (d)	A-3. (b)	A-4. (b)
A-5. (c)	A-6. (b)	A-7. (c)	A-8. (b)
A-9. (c)	A-10. (a)	A-11. (b)	A-12. (b)
A-13. (d)	A-14. (d)	A-15. (c)	

SECTION-B

B-1. (d)	B-2. (b)	B-3. (b)	B-4. (c)
B-5. (d)	B-5. (a)	B-5. (c)	B-5. (d)
B-5. (d)	B-5. (a)	B-5. (b)	

SECTION-C

C-1. (b)	C-2. (b)	C-3. (c)	C-4. (c)
C-5. (c)	C-6. (b)	C-7. (d)	C-8. (a)
C-9. (d)	C-10. (a)	C-11. (c)	C-12. (c)
C-13. (a)	C-14. (b)	C-15. (c)	C-16. (b)
C-17. (c)			

SECTION-D

D-1. (C)	D-2. (d)	D-3. (a)	D-4. (d)
D-5. (d)	D-6. (a)	D-7. (a)	D-8. (d)

SECTION-E

E-1. (d)	E-2. (b)	E-3. (a)	E-4. (a)
E-5. (a)	E-6. (a)	E-7. (c)	E-8. (a)
E-9. (b)	E-10. (b)	E-11. (c)	E-12. (c)
E-13. (d)	E-14. (c)	E-15. (c)	E-16. (b)
E-17. (c)	E-18. (c)	E-19. (b)	E-20. (d)
E-21. (d)	E-22. (b)	E-23. (a)	

SECTION-F

E-1. (d)	E-2. (b)	E-3. (d)	E-4. (b)
E-5. (b)	E-6. (c)		

EXERCISE – II
READY FOR CHALLENGES

1. (b)	2. (b)	3. (a)	4. (d)	5. (b)
6. (b)	7. (c)	8. (c)	9. (b)	10. (b)
11. (a)	12. (c)	13. (c)	14. (a)	15. (a)
16. (a)	17. (c)	18. (B)	19. (c)	20. (c)
21. (c)	22. (a)	23. (a)	24. (b)	25. (c)
26. (c)	27. (d)	28. (b)	29. (a)	30. (d)

31. (D)	32. (b)	33. (c)	34. (b)	35. (d)
36. (b)	37. (A)	38. (b)	39. (a)	40. (c)
41. (d)	42. (c)	43. (d)	44. (b)	45. (b)
46. (c)	47. (d)	48. (a)	49. (a)	50. (a)
51. (d)	52. (b)	53. (c)	. 54. (c)	55. (c)
56. (a)	57. (b)	58. (c)	59. (d)	

EXERCISE – III
CROSSING THE HURDLES

MORE THAN ONE CORRECT

1. (b,c)	2. (c,d)	3. (b,c,d)	4. (b,c)	5. (a,b,c)
6. (a,b)	7. (a,c,d)	8. (b,c,d)	9. (a,b,c)	10. (d)
11. (a,b,c)	12. (a,c)	13. (c)	14. (c)	15. (a)
16. (b)	17. (b)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (d)		

MATCH THE COLUMN

24. (A-R, B-P, C-S, D-Q)

25. (A-P, B-P, C-Q, D-R)

EXERCISE – IV

1. 150
2. 1.3 m/s
3. (i) $\frac{mu_{rel}}{m+M}$, (ii) $\frac{ML}{m+M}$
4. $\frac{1}{2}\frac{M^2 m V^2}{(m+2M)^2}$
5. Increases by $4.54\,\frac{\text{m}}{\text{s}}$
6. Bob will not rise because whole momentum is transferred to the body at rest.
7. $m_2 = \sqrt{m_1 m_3}$

8. $\frac{d}{1-e^2}$

9. e = 1/7, 45 J

10. $\frac{(1+e)u^2 \sin 2\theta}{g}$

11. $h = \frac{Mv^2}{2g(M+m)}$

12. x = $\frac{3d}{13}$

13. $\frac{Mx\sqrt{g}}{m[\sqrt{2H}-\sqrt{2(H-h)}]}$, $\frac{Mx}{m}$ left to the line of fall

14. $x_{CM} = -a/12$, $y_{CM} = -b/12$

15. –R/20