1. MATRICES & DETERMINANTS

SYNOPSIS

Matrices:

Definition:

A rectangular array of symbols (which could be real or complex numbers) along rows and columns enclosed by square brackets $[\cdot]$ or round brackets $(\cdot)$ is called a matrix. Thus a system of $m \times n$ symbols arranged in a rectangular formation along m rows and n columns and bounded by the brackets or round brackets is called an m by n matrix (which is written as $m \times n$ matrix)

i.e.,
$$A = \begin{bmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ -- & ---- & ------ & -- \\ -- & ---- & ------ & -- \\ a_{m1} & a_{m2} & .... & a_{mn} \end{bmatrix}$$

Note: Order $m \times n$ should not be read as order mn.
In a compact form the above matrix is represented by where or simply The numbers etc of this rectangular array are called the elements of the matrix. The element belongs to the row and the Column and is called the element of the matrix.

Equal Matrices: Two matrices are said to be equal if they have the same order and each element of one is equal to the corresponding element of the other.

Classification of Matrices:

Row Matrix: A matrix having a single row is called a row matrix. e.g., $\begin{bmatrix} 1 & 3 & 5 & 7 \end{bmatrix}$.

Column Matrix: A matrix having a single column is called a column matrix. e.g., $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$.

Square Matrix: An $m \times n$ matrix A is said to be a square matrix if $m = n$
i.e., number of rows $=$ number of column and the order of square matrix is also said to be m or n.

For example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ -3 & 4 & 5 \end{bmatrix}$ is a square matrix of order 3.

Note: In a square matrix the diagonal form left hand side upper corner to right hand side lower corner is known as leading diagonal or principal diagonal. In the above example square matrix containing the elements 1, 3, 5 is called the leading or principal diagonal.

Trace of a Matrix: The sum of the elements of a square matrix A lying along the principal diagonal is called the trace of A i.e., tr (A). Thus if $A = \left[a_{ij}\right]_{n \times n}$, then $tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + ... + a_{nn}$.

Diagonal Matrix: A square matrix whose non – diagonal elements are zero, is called a diagonal matrix. For a square matrix $A=\left[a_{ij}\right]_{n\times n}$ to be a diagonal matrix, $a_{ij}=0,$ whenever $i\neq j.$

For example: $A=\begin{bmatrix}3&0&0\\0&0&0\\0&0&-1\end{bmatrix}$ is a diagonal matrix of order 3 × 3.

Note: Here A can also be represented as diagonal (3, 2, -1).

Scalar Matrix: A diagonal matrix whose all the elements are equal is called a scalar matrix.

For a square matrix $A=\left[a_{ij}\right]_{n\times n}$ to be a scalar matrix, $a_{ij}=\begin{cases}1, & i\neq j\\ \text{m}, & i=j\end{cases}$, where $m\neq 0.$

For example: $A\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$ is a scalar matrix.

Unit Matrix or Identity Matrix: A diagonal matrix of order n which has unity for all its elements, is called a unit matrix of order n and is denoted by l_n.

Thus a square matrix $A=\left[a_{ij}\right]_{n\times n}$ is a unit matrix if $a_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$

For example: $I_3\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}_{3\times 3}$

Triangular Matrix: A square matrix in which all the elements below the principle diagonal are zero is called an Upper Triangular matrix and a square matrix in which all the elements above the principal diagonal are zero is called a Lower Triangular matrix.

Given a square matrix $A=\left[a_{ij}\right]_{n\times n,}$ for upper triangular matrix, $a_{ij}=0, i>j$ and for lower triangular matrix, $a_{ij}=0, i<j.$

Note:

- Diagonal matrix is both upper and lower triangular
- A triangular matrix $A=\left[a_{ij}\right]_{n\times n}$ is called strictly triangular if $a_{ii}=0\forall 1\leq i\leq n.$

For example: $\begin{bmatrix}a&h&g\\0&b&f\\0&0&c\end{bmatrix}$ and $\begin{bmatrix}1&0&0\\2&3&0\\1&-5&4\end{bmatrix}$ are respectively upper and lower triangular matrices whereas

$\begin{bmatrix}0&1&3\\1&0&2\\-1&0&0\end{bmatrix}$ is a strictly triangular matrix.

Null matrix: If all the elements of a matrix (square or rectangular) are zero, then it is called a null or zero matrix. For $A = \left[a_{ij}\right]$ to be null matrix, $a_{ij} = 0 \forall i, j.$

For example: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a matrix of order 2 × 3

Transpose of a Matrix: The matrix obtained from any given matrix *A*, by interchanging its rows and columns, is called the transpose of A and is denoted by A' or A^T.

If $A = \left[a_{ij}\right]_{m \times n}$ and $A' = \left[b_{ij}\right]_{n \times m}$ then $b_{ij} = a_{ji}, \forall i, j.$

For example: If $A \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}_{3 \times 2}$, then $A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}_{2 \times 3}$.

Properties of Transpose of Matrix:

i. $(A')' = A$

ii. $(A+B)' = A' + B'$, A and B being conformable matrices for addition (see addition of matrices for more)

iii. $(\alpha A)' = \alpha A', \alpha$ being scalar

iv. $(AB)' = B' A'$, A and B being conformable for multiplication (see multiplication of matrices for more)

Conjugate of a Matrix: The matrix obtained from any given matrix *A* containing complex number as its elements, on replacing its elements by the corresponding conjugate complex numbers is called conjugate of *A* and is denoted by $\overline{A}$.

For example: $A = \begin{bmatrix} 1+2i & 2-3i & 3+4i \\ 4-5i & 5+6i & 6-7i \\ 8 & 7+8i & 7 \end{bmatrix}$. Then, $\overline{A} = \begin{bmatrix} 1-2i & 2+3i & 3-4i \\ 4+5i & 5-6i & 6+7i \\ 8 & 7-8i & 7 \end{bmatrix}$

Properties of Conjugate of a Matrix:

i. $\overline{\left(\overline{A}\right)} = A$

ii. $\overline{(A+B)} = \overline{A} + \overline{B}$

iii. $\overline{(\alpha A)} = \overline{\alpha}\,\overline{A}, \alpha$ being any number real or complex

iv. $\overline{(AB)} = \overline{A}\,\overline{B}, A$ and B being conformable for multiplication

Transpose conjugate of a Matrix: The transpose of the conjugate of a matrix A is called transposed conjugate of A and is denoted by A^{θ}. The conjugate of the transpose of A is the same as the transpose of the conjugate of A i.e., $\overline{(A')} = (\overline{A})' = A^{\theta}$.

If $A = [a_{ij}]_{m\times n}$, then $A^{\theta} = [b_{ij}]_{n\times m}$ where $b_{ji} = \overline{a}_{ij}$

i.e., the $(j,i)^{th}$ element of A^{θ} = the conjugate of $(i,j)^{th}$ element of A.

For example: If $A = \begin{bmatrix} 1+2i & 2-3i & 3+4i \\ 4-5i & 5+6i & 6-7i \\ 8 & 7+8i & 7 \end{bmatrix}$ then $A^{\theta} = \begin{bmatrix} 1-2i & 4+5i & 8 \\ 2+3i & 5-6i & 7-8i \\ 3-4i & 6+7i & 7 \end{bmatrix}$.

Properties of Transpose conjugate:

i. $(A^{\theta})^{\theta} = A$

ii. $(A+B)^{\theta} = A^{\theta} + B^{\theta}$

iii. $(kA)^{\theta} = \overline{k}A^{\theta}, k$ being any number

iv. $(AB)^{\theta} = B^{\theta}A^{\theta}$

Algebra of Matrices:

Addition and Subtraction of Matrices:

Any two matrices can be added if they are of the same order and the resulting matrix is of the same order. If two matrices A and B are of the same order, they are said to be conformable for addition.

For example: $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} + \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \\ c_3 & d_3 \end{bmatrix} = \begin{bmatrix} a_1+c_1 & b_1+d_1 \\ a_2+c_2 & b_2+d_2 \\ a_3+c_3 & b_3+d_3 \end{bmatrix}$

Similarly, $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} - \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \\ c_3 & d_3 \end{bmatrix} = \begin{bmatrix} a_1-c_1 & b_1-d_1 \\ a_2-c_2 & b_2-d_2 \\ a_3-c_3 & b_3-d_3 \end{bmatrix}$

Note:

- Only matrices of the same order can be added or subtracted.
- Addition of matrices is commutative as well as associative.
- Cancellation laws hold well in case of addition
- The equation A + X = 0 has a unique solution in the set of all $m \times n$ matrices.

Scalar Multiplication: The matrix obtained by multiplying every element of a matrix A by a scalar λ is called the scalar multiple of A by λ and is denoted by λA i.e., if $A = [a_{ij}]$ then $\lambda A = [\lambda a_{ij}]$.

For example: $A=\begin{bmatrix}2 & -3 & 5\\ 0 & 7 & 8\end{bmatrix}_{2\times 3}$.

Thus, $2A=\begin{bmatrix}4 & -6 & 10\\ 0 & 14 & 16\end{bmatrix}_{2\times 3}$.

Note: All the laws of ordinary algebra hold for the addition or subtraction of matrixes and their multiplication by scalars.

Multiplication of Matrices: Two matrices can be multiplied only when the number of columns in the first, called the prefactor, is equal to the number of rows in the second, called the postfacor. Such matrices are said to be conformable for multiplication.

$$\begin{bmatrix}a_{11} & a_{1} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{m1} & a_{m2} & \dots & a_{mn}\end{bmatrix}_{m\times n} \begin{bmatrix}b_{11} & b_{12} & \dots & b_{1p}\\ b_{21} & b_{22} & \dots & b_{2p}\\ \dots & \dots & \dots & \dots\\ b_{n1} & b_{n2} & \dots & b_{np}\end{bmatrix}_{n\times p} \begin{bmatrix}c_{11} & c_{12} & \dots & c_{1p}\\ c_{12} & c_{22} & \dots & c_{2p}\\ \dots & \dots & \dots & \dots\\ c_{m1} & c_{m2} & \dots & c_{mp}\end{bmatrix}_{m\times p}$$

Where $c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\dots\dots+a_{in}\,b_{nj}=\sum_{k=1}^{n}a_{ik}b_{kj}\,\forall i=1,2,3,\dots\dots,m$ and $j=1,2,3,\dots..,p$.

Properties of Multiplication:

i. Matrix multiplication may or may not be commutative. i.e., AB may or may not be equal to BA
   (a) If $AB = BA$, then matrices A and B are called Commutative Matrices.
   (b) If $AB = BA$, then matrices A and B are called Anti – Commutative Matrixes.

ii. Matrix multiplication is Associative i.e., (AB)C = A(BC).

iii. Matrix multiplication is Distributive over Matrix Addition i.e., (A + B) C = AC + BC

iv. Cancellation Laws not necessarily hold in case of matrix multiplication i.e., if AB = AC need not mean B = C even if A $\neq$ 0.

v. AB = 0 i.e., AB is a Null Matrix, does not necessarily imply that either A or B is a null matrix but either $|A|=0$ or $|B|=0$. In addition, if none of them are null matrices, then $|A|=|B|=0$

Special Matrixes:

Symmetric and Skew Symmetric Matrices: A square matrix $A=\left[a_{ij}\right]$ is said to be Symmetric when $a_{ij}=a_{ji}$ for all *i* and *j*. If $a_{ij}=-a_{ji}$ for all i and j, then the matrix is called a Skew Symmetric Matrix.

For example: $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ is a symmetric matrix and $\begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix}$ is a skew – symmetric matrix

Note: If *A* is a skew – symmetric matrix, then $a_{ii}=-a_{ii} \Rightarrow a_{ii}=0$ i.e., all the elements in the leading diagonal of a skew – symmetric matrix are zero.

Hermitian and Skew – Hermitian Matrices: A square matrix $A=\left[a_{ij}\right]$ is said to be Hermitian matrix if $a_{ij}=\overline{a}_{ji} \forall i, j$ i.e., $A=A^{\theta}$.

For example: $\begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}, \begin{bmatrix} 3 & 3-4i & -2+i \\ 3+4i & 0 & 5 \\ -2-i & 5 & 2 \end{bmatrix}$ are Hermitian matrices.

Note:

- If A is a Hermitian matrix then $a_{ii}=\overline{a}_{ii} \Rightarrow a_{ii}$ is real $\forall$ *i*. Thus every diagonal element of a Hermitian Matrix must be real.
- A Hermitian Matrix over the set of real numbers is actually a real symmetric matrix.
- Every Hermitian matrix is of the form $S+iW$, where S and W are real symmetric and skew symmetric matrices respectively.

  A square matrix, $A=\left[a_{ij}\right]$ is said to be a Skew Hermitian if $a_{ij}=-\overline{a}_{ji}, \forall i, j$ i.e., $A^{\theta}=-A$.

For example: $\begin{bmatrix} 0 & -2+i \\ 2-i & 0 \end{bmatrix}, \begin{bmatrix} 3i & -3+2i & -1-i \\ 3-2i & -2i & -2 \\ 1+i & 2 & 0 \end{bmatrix}$ are Skew Hermitian matrices.

Note:

- If *A* is a skew – Hermitian matrix then $a_{ii}=-\overline{a}_{ii} \Rightarrow a_{ii}+\overline{a}_{ii}=0$ i.e., a_{ii} must be purely imaginary or zero.
- A skew – Hermitian Matrix over the set of real numbers is actually a real skew – symmetric matrix.
- Every Skew Hermitian matrix is of the form *W* + *iS*, where *S* and *W* are real symmetric and skew – symmetric matrices respectively.

Singular and Non – singular Matrices: Any square matrix A is said to be singular if $|A|=0$, otherwise it is said to be non – singular. Here $|A|$ (or det (A) or simply det A) means corresponding determinant of square matrix *A* e.g., if $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then $|A|=\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}=10-12=-2 \Rightarrow A$ is a non – singular matrix

Unitary Matrix: A square matrix is said to be unitary if $A^{\theta} A = I.$ Since $|\overline{A}'| = |A|$ and $|\overline{A}'\, A| = |\overline{A}'||A|,$ we have $|\overline{A}'||A| = 1.$ Thus the magnitude of the determinant of a unitary matrix is one. For a matrix to be unitary it must be non – singular. Hence $\overline{A}' A = i \Rightarrow A\overline{A}' = I.$

Orthogonal Matrix: Any square matrix *A* of order n is said to be orthogonal if $AA' = A'A = I_n$.

Idempotent Matrix: A square matrix *A* is called idempotent provided it satisfies the relation $A^2 = A$.

For example: The matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is idempotent as

$$A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A.$$

Involuntary Matrix: A matrix such that $A^2 = I$ is called an involuntary matrix.

Nilpotent Matrix: A square matrix *A* is called a nilpotent matrix if there exists a positive integer m such that $A^m = O.$ If m is the least positive integer such that $A^m = O,$ then m is called the index of the nilpotent matrix *A*.

Adjoint of a Square Matrix: Let $A = [a_{ij}]$ be a square matrix of order *n* and let C_{ij} be cofactor of a_{ij} in *A*. Then the transpose of the matrix of cofactors of elements of *A* is called the adjoint of *A* and is denoted by *adj A*.

Thus, $adj A = [C_{ij}]^T \Rightarrow (adj\, A)_{ij} = C_{ji}.$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$ then, $adj A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix};$

where C_{ij} denotes the cofactor of a_{ij} in *A*.

For example: $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, C_{11} = s, C_{12} = -r, C_{21} = -q, C_{22} = p$

$$\Rightarrow adj\, A = \begin{bmatrix} s & -r \\ -q & p \end{bmatrix}^T = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}.$$

Theorem: Let A be a square matrix of order $n.$ Then $A(adj\,A)=|A|l_n=(adj\,A)A.$

Proof: Let $A=\left[a_{ij}\right],$ and let C_{ij} be cofactor of a_{ij} in $A.$ Then

$(adj\,A)_{ij}=C_{ji}\ \forall\ 1\le i,i\le n.$

Now, $\left(A(adj\,A)\right)_{ij}=\sum_{r=1}^{n}(A)_{ir}\ \ (adj\ \ A)_{rj}=\sum_{r=1}^{n}a_{ir}\quad C_{jr}=\begin{cases}|A|, & \text{if}\quad i=j\\ 0, & \text{if}\quad i\ne j\end{cases}$

$$\Rightarrow A(adj\,A)=\begin{bmatrix}|A| & 0 & 0 & .... & 0\\ 0 & |A| & 0 & .... & 0\\ 0 & 0 & |A| & .... & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & |A|\end{bmatrix}=|A|l_n.$$

Similarly $\left((adj\ \ A)A\right)_{ij}=\sum_{r=1}^{n}(adj\ \ A)_{ir}\,(A)_{rj}=\sum_{r=1}^{n}C_{ri}a_{rj}=\begin{cases}|A|, & \text{if}\quad i=j\\ 0, & \text{if}\quad i\ne j\end{cases}$

Hence, $A(adj\,A)=|A|l_n=(adjA)A.$

Note: The adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the sign of off – diagonal (left hands side lower corner to right hand side upper corner) elements.

Inverse of a Matrix: A non – singular square matrix of order n is invertible if there exists a square matrix B of the same order such that $AB=l_n=BA.$

In such a case, we say that the inverse of A is B and we write, $A^{-1}=B.$

If A is invertible, then the inverse of A is given by $A^{-1}=\frac{1}{|A|}.adj\,A.$

Properties of Inverse of a Matrix:

i. Every invertible matrix possesses a unique inverse.

ii. (Reversal Law) If A and B are invertible matrices of the same order, then AB is invertible and $(AB)^{-1}=B^{-1}A^{-1}.$ In general, if $A,B,C,....$ are invertible matrices then $(ABC.....)^{-1}=.......C^{-1}B^{-1}A^{-1}.$

iii. If A is an invertible square matrix, then A^T is also invertible and $\left(A^T\right)^{-1}=\left(A^{-1}\right)^T.$

iv. If A is a non – singular square matrix of order n, then $|adj\,A| = |A|^{n-1}$.

v. If A and B are non – singular square matrices of the same order, then $adj\,(AB) = (adj\,B)(adj\,A)$.

vi. If A is an invertible square matrix, then $adj\,(A^T) = (adj\,A)^T$.

If A is a non – singular square matrix, then $adj\,(adjA) = |A|^{n-2}\,A$.

Elementary transformations or elementary operations of a matrix: The following three operations applied on the rows (columns) of a matrix are called elementary row (column) transformations.

i. **Interchange of any two rows (columns):** If ith row (column) of a matrix is interchanged with the jth row (column), it will be denoted by $R_i \leftrightarrow R_j\,(C_i \leftrightarrow C_j)$.

For example: $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, then by applying $R_2 \to R_3$ we get $B = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 4 \\ -1 & 2 & 1 \end{bmatrix}$.

ii. **Multiplying all elements of a row (column)** of a matrix by a non – zero scalar. If the elements of ith row (column) are multiplied by non – zero scalar k, it will be denoted by $R_l \to R_i(k)\left[C_i \to C_i(k)\right]$ or $R_l \to kR_i\,[C_i \to kC_i]$.

If $A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 2 & -3 \end{bmatrix}$, then by applying $R_2 \to 3R_2$, we obtain $B \begin{bmatrix} 3 & 2 & -1 \\ 0 & 3 & 6 \\ -1 & 2 & -3 \end{bmatrix}$.

iii. **Adding to the elements of a row (column),** the corresponding elements of any other row (column) multiplied by any scalar k.

If k times the elements of j^{th} row (column) are added to the corresponding elements of the i^{th} row (column), it will be denoted by $R_i \to R_i + kR_j\,(C_i \to C_i + kC_j)$.

If $A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ -1 & -1 & 0 & 2 \\ 0 & 1 & 3 & 1 \end{bmatrix}$, then the application of elementary operation $R_3 \to R_3 + 2R_1$ lead to

$$B = \begin{bmatrix} 2 & 1 & 3 & 1 \\ -1 & -1 & 0 & 2 \\ 4 & 3 & 9 & 3 \end{bmatrix}$$

Note: In an equation $A = BC$, row operation should only be performed on both A and B but not on C. Column operation should only be performed on A and C but not on B.

System of Simultaneous Linear Equations: Consider the following system of n linear equations in n unknowns:

$$a_{11}X_1 + a_{12}X_2 + ... + a_{1n}X_n = d_1$$

$$a_{21}X_1 + a_{22}X_2 + ... + a_{2n}X_n = d_2$$

$$\begin{matrix} . & . & . & . \\ . & . & . & . \end{matrix}$$

$$a_{n1} + X_1 + a_{n2}X_2 + ... + a_{nn}X_n = d_n$$

This system of equation can be written in the matrix form as

$$\begin{bmatrix} a_{11} & a_{12}..............a_{1n} \\ a_{21} & a_{12}..............a_{1n} \\ . & . \qquad\qquad . \\ & . \qquad\qquad . \\ & . \qquad\qquad . \\ a_{n1} & a_{n2}..............a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} d_1 \\ d_2 \\ . \\ . \\ . \\ d_n \end{bmatrix}_{n\times 1} \text{ or } AX = D$$

The $n \times n$ matrix A is called the coefficient matrix of the system of linear equations.

Homogeneous and Non – Homogeneous System of Linear Equations: A system of equations $AX = D$ is called a homogeneous system if D = 0. Otherwise, it is called a non – homogeneous system of equations.

Solution of a System of Equations: Consider the system of equation $AX = D$. A set of values of the variables $x_1, x_2, \ldots\ldots x_n$ which simultaneously satisfy all the equations is called a solution of the system of equations.

Consistent System: If the system of equations has one or more solutions, then it is said to be a consistent system of equations, otherwise it is an inconsistent system of equations.

Solution of a Non – Homogeneous System of Linear Equations: There are two methods of solving a non – homogeneous system of simultaneous linear equations.

(i) Cramer's Rule and (ii) Matrix Method

i. Cramer's Rule: is discussed in the Chapter Determinants.

ii. Matrix Method: Consider the equations

$$a_1x + b_1y + c_1z = d_1,$$

$$a_2x + b_2y + c_2z = d_2, \qquad ....(i)$$

$$a_3x + b_3y + c_3.z = d_3$$

If $A=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

then the equation (i) is equivalent to the matrix equation

$AX = D.$ $\quad (ii)$

Multiplying both sides of (ii) by the inverse matrix A^{-1}, we get

$A^{-1}(AX) = A^{-1}D \Rightarrow IX = A^{-1}D \quad [\because A^{-1}A = I]$

$$\Rightarrow X = A^{-1}D \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} \times \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad ......(iii)$$

where A_1, B_1 etc. are the cofactors of a_1, b_1 etc. in the determinant

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \ (\Delta \neq 0).$$

i. If A is a non - singular matrix, then the system of equations given by $AX = D$ has a unique solution given by $X = A^{-1}D.$

ii. If A is a singular matrix, and $(adj\ A)D = O,$ then the system of equations given by $AX = D$ is consistent, with infinitely many solutions.

iii. If A is a singular matrix, and $(adj\ A)D \neq O,$ then the system of equation given by $AX = D$ is inconsistent.

Solution of Homogeneous System of Linear Equations: Let $AX = O$ be a homogeneous system of n linear equation with n unknowns. Now if A is non – singular then the system of equations will have a unique solution i.e., trivial solution and if A is singular then the system of equations will have infinitely many solutions.

Formulae and Concepts at a Glance:

Trace of a Matrix:

$$tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + ... + a_{nn}$$

Properties of Transposes:

i. $(A')' = A$

ii. $(A+B)' = A' + B', A$ and B being conformable matrices for addition

iii. $(\alpha A)' = \alpha A', \alpha$ being scalar

iv. $(AB)' = B'A', A$ and B being conformable for multiplication

Properties of Conjugates

i. $\overline{\left(\overline{A}\right)} = A$

ii. $\overline{(A+B)} = \overline{A} + \overline{B}, A$ and B being conformable for addition

iii. $\overline{(\alpha A)} = \overline{\alpha}\overline{A}, \alpha$ being any number

iv. $\overline{(AB)} = \overline{A}\ \ \overline{B}, A$ and B being conformable for multiplication.

Properties of Transpose Conjugate

i. $\left(A^{\theta}\right)^{\theta} = A$

ii. $(A+B)^{\theta} = A^{\theta} + B^{\theta}$

iii. $(kA)^{\theta} = \overline{k}A^{\theta}, k$ being any number

iv. $(AB)^{\theta} = B^{\theta}A^{\theta}$

Algebra of Matrices:

- Only matrices of the same order can be added or subtracted.
- Addition of matrices is commutative as well as associative.
- Cancellation laws hold well in case of addition
- The equation $A + X = 0$ has a unique solution in the set of all $m \times n$ matrices.
- All the laws of ordinary algebra hold for the addition or subtraction of matrices and their multiplication by scalars.

Symmetric and Skew Symmetric Matrices: A square matrix $A = \left[a_{ij}\right]$ is said to be Symmetric when $a_{ij} = a_{ji}$ for all i and j. If $a_{ij} = -a_{ji}$ for all i and j, then the matrix is called a Skew Symmetric matrix.

- If A is a skew – symmetric matrix, then $a_{ii} = -a_{ii} \Rightarrow a_{jj} = 0$ i.e., all the elements in the leading diagonal of a skew – symmetric matrix are zero.

Hermitian and Skew – Hermitian Matrices: A square matrix $A = \left[a_{ij}\right]$ is said to be Hermitina matrix if $a_{ij} = \overline{a}_{ji}\ \ \forall\ i, j$ i.e., $A = A^{\theta}$.

- If A is a Hermitian matrix then $a_{ii} = \overline{a}_{ii}$ is real $\forall\ i$. thus every diagonal element of a Hermitian matrix must be real.
- A Hermitian Matrix over the set of real numbers is actually a real symmetric matrix and a square matrix, $A = \left[a_{ij}\right]$ is said to be a skew – Hermitian if $a_{ij} = -\overline{a}_{ji}, \forall i, j$ i.e., $A^{\theta} = -A$.

- If A is a skew – Hermitian matrix then $a_{ii} = -\overline{a}_{ii} \Rightarrow a_{ii} + \overline{a}_{ii} = 0$ i.e., a_{ii} must be purely imaginary or zero.
- A skew – Hermitian matrix over the set of real numbers is actually a real skew – symmetric matrix.

Orthogonal Matrix: Any square matrix A of order n is said to be orthogonal if $AA' = A'A = I_n$.

Idempotent Matrix: A square matrix A is called idempotent provided it satisfies the relation $A^2 = A$.

Involutary Matrix: A matrix such that $A^2 = I$ is called involuntary matrix.

Nilpotent Matrix: A square matrix A is called a nilpotent matrix if there exists a positive integer m such that $A^m = 0$. If m is the least positive integer such that $A^m = 0$, then m is called the index of the nilpotent matrix A.

Inverse of a Matrix:

The inverse of A is given by $A^{-1} = \frac{1}{|A|}.adj\ A$

i. If A is an invertible square matrix, then $adj\left(A^T\right) = \left(adj\ A\right)^T$

ii. If A is a non – singular square matrix, then $adj\left(adj\ A\right) = |A|^{n-2}\ A$

Matrix Method of Solving non – homogeneous system of linear equations:

i. If A is a non – singular matrix, then the system of equations given by $AX = B$ has a unique solution given by $X = A^{-1}\ B$.

ii. If A is singular matrix, and $\left(adjA\right)D = 0$, then the system of equations given by $AX = D$ is consistent with infinitely many solutions.

iii. If A is singular matrix, and $\left(adj\ A\right)D \neq 0$, then the system of equation given by $AX = D$ is inconsistent.

Solution of Homogeneous System of Linear Equations: Let $AX = 0$ be a homogenous system of n linear equation with n unknowns. Now if A is non – singular then the system of equations will have a unique solution i.e., trivial solution and if A is a singular, then the system of equations will infinitely many solutions.

Determinant:

Definition: Consider the equation $a_1x+b_1y=0, a_2x+b_2y=0$. These give

$\frac{-a_1}{b_1}=\frac{y}{x}=\frac{-a_2}{b_2}\Rightarrow\frac{a_1}{b_1}=\frac{a_2}{b_2}\Rightarrow a_1b_2-a_2b_1=0$.

We express this determinant as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=0$.

The expression $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called a determinant of order two, and equals $a_1b_1-a_2b_1$.

A determinant of order three consisting of 3 rows and 3 column is written as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and is equal to

$a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}-b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}+c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}=a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$.

The number a_i, b_i, c_i (i = 1, 2, 3) are called the elements of the determinant.

The determinants, obtained by deleting the ith row and the column is called the minor of the element at ith row and column. The cofactor of the element is $(-1)^{i+j}$ (minor). Note that: $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=a_1A_1+b_1B_1+c_1C_1$

where A_1, B_1, and C_1 are the cofactor of a_1, b_1 and c_1 respectively.

We can expand the determinant through any row or column. It means we can write

$\Delta=a_2A_2+b_2B_2+c_2C_2 \ or \ \Delta=a_1A_1+a_2A_2+a_3A_3 \ etc.$

Also $\Delta=a_1A_2+b_1B_2+c_1C_2=0$

$\Rightarrow a_iA_j+b_iB_j+c_iC_j \quad =\Delta$ if i=j,

$=0$ if $i\neq j$,

These results are true for determinants of any order.

Properties of Determinants:

i. If rows be changed into columns and columns into the rows, then the values of the determinant remains unaltered.

ii. If any two row (or columns) of a determinant are interchanged, the resulting determinants is the negative of the original determinant.

iii. If two rows (or two columns) in a determinant have corresponding elements that are equal, the value of determinant is equal to zero.

iv. Fi each of the elements of one row (or columns) of a determinant is multiplied by non-zero constant 'k', then the new determinant is k times the original determinants.

v. If each element in a row (or column) of a determinant is written as the sum of two or more terms then the determinant can be written as the sum of two or more determinants.

vi. If to each element of a line (row or column) of a determinant be added the equimultiples of the corresponding element of one or more parallel lines, the determinants remains unaltered

i.e., $\begin{vmatrix} a_1+la_2+ma_3 & a_2 & a_3 \\ b_1+lb_2+mb_3 & b_2 & b_3 \\ c_1+lc_2+mc_3 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

vii. If each element in any row (or any column) of determinant is zero, then the value of determinant is equal to zero.

viii. If a determinant D vanishes for x = a, then (x-a) is a factor of D, In other words, if two rows (or two columns) become identical for x = a. then (x – a) is a factor of D.

In general, if r rows (or r columns) become identical when a is substituted for x, then $(x-a)^{r-1}$ is a factor of D.

Product of Two Determinants:

We can write

$$\begin{vmatrix} a_1b_1c_1 \\ a_2b_2c_2 \\ a_3b_3c_3 \end{vmatrix}\begin{vmatrix} \alpha_1\beta_1\gamma_1 \\ \alpha_2\beta_2\gamma_2 \\ \alpha_3\beta_3\gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1+b_1\beta_1+c_1\gamma_1 & a_1\alpha_2+b_1\beta_2+c_1\gamma_2 & a_1\alpha_3+b_1\beta_1+c_1\gamma_3 \\ a_2\alpha_1+b_2\beta_1+c_2\gamma_1 & a_2\alpha_2+b_2\beta_2+c_2\gamma_2 & a_2\alpha_3+b_2\beta_1+c_2\gamma \\ a_3\alpha_1+b_3\beta_1+c_3\gamma_1 & a_3\alpha_2+b_3\beta_2+c_3\gamma_2 & a_3\alpha_3+b_3\beta_1+c_3\gamma \end{vmatrix}.$$

Here we have multiplied rows by rows. We can also multiply rows by columns or columns by rows, or columns by columns.

Note: If $\Delta = |a_{ij}|$ is a determinant of order n, then the value of the determinant $|A_{ij}|$, where A_{ij} is the cofactor of a_{ij}, is Δ^{n-1}. This is known as power cofactor formula.

Differentiation of a Determinant:

Let $\Delta(x) = \begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$. Then $\Delta'(x) = \begin{vmatrix} a_1'(x) & b_1'(x) \\ a_2(x) & b_2(x) \end{vmatrix} + \begin{vmatrix} a_1(x) & b_1(x) \\ a_2'(x) & b_2'(x) \end{vmatrix}$.

If we write $\Delta(x) = |C_1C_2C_3|$, then $\Delta'(x) = |C_1'C_2C_3| + |C_1C_2'C_3| + |C_1C_2C_3'|$.

Similarly if $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R_1' \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}$.

Summation of Determinants:

Let $\Delta_r = \begin{vmatrix} f(r) & a & l \\ g(r) & b & m \\ h(r) & c & n \end{vmatrix}$, where *a, b, c, l, m* and *n* are constant independent of *r*,

Then $\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} f(r) & a & l \\ \sum_{r=1}^{n} g(r) & b & m \\ \sum_{r=1}^{n} h(r) & c & n \end{vmatrix}$.

Here functions of *r* can be the elements of only one row or column. None of the elements other of than that row or column should be dependent on *r*.

Special Determinants:

1. **Symmetric determinants:** The element situated at equal distance from the diagonal are equal both in magnitude and sign.

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2.$$

2. **Skew symmetric determinants:** All the diagonal elements are zero and the elements situated at equal distance from the diagonal are equal in magnitude but opposite in sign. The value of a skew symmetric determinant of odd order is zero.

$$\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0.$$

3. **Circulant determinant:** The element of the rows (or columns) are in cyclic arrangement.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\left(a^3 + b^3 + c^3 - 3abc\right).$$

4. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$

5. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$

6. $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$

Solution of System of Linear Equations:

Method of solving system of linear equations: Consider a system of simultaneous linear equation in three variable namely x, y, z i.e., $a_1x + b_1y + c_1z = d_1, a_2x + b_2y + c_2z = d_2, a_3x + b_3y + c_3z = d_3.$

Let D $= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$

(Here elements of D are arranged in the same order as they occur as coefficients in the equation). Moreover, let

D $= \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}.$

(D_1 is obtained by replacing 1st column of d by d_1, d_2, & d_3),

$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}.$

(D_2 is obtained by replacing 2nd column of d by d_1, d_2, & d_3),

$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$

(D_3 is obtained by replacing 3rd column of d by d_1, d_2, & d_3).

The following cases can arise:

1. $D \neq 0$: In such case, the system has precisely one solution (unique solution), which is given by 'CRAMER RULE' i.e., x = D_1/D ; y = D_2/D ; z = D_3/D
2. D = 0 : and at least one of the determinants D_1, D_2, or D_3 is non-zero, then the system is inconsistent. i.e., it has no solution.

3. $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the system has either infinite solution (when they are dependent) or they have no solution (when they are independent).

Homogeneous system: If $d_1 = d_2 = d_3 = 0$, then the system is known as a system of homogeneous linear equation. If the system of equation is homogeneous, then $D_1 = D_2 = D_3 = 0$ (value of determinates is zero, if one column has all element = 0). The trivial solution (x = 0, y = 0, z = 0) always exist and for existence of non-trivial solution (infinite solutions) D = 0 and if $D \neq 0$, then it has the unique solution (trivial) x = 0, y = 0 and z = 0.

Note: A system of three linear equations in two known i.e.,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_3x + b_3y + c_3 = 0$

if consistent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

Formulae and Concepts at a Glance:

Properties of Determinants:

- The determinant remains unaltered if its rows are changed into columns and the columns into rows.
- If all the elements of a row (or column) are zero, then the determinant is zero.
- If the elements of a row (column) are propotional (or identical) to the elements of any other row (column), then the determinant is zero.
- The interchange of any two rows (columns) of the determinant changes its sign.
- If all the elements of a row (column) of a determinant are multiplied by a non-zero constant then the determinant gets multiplied by the same constant.
- A determinant remains unaltered under a column (C_i)operation of the form $C_i + \alpha C_j + \beta C_k \left(j,k \neq i\right)$ or a row (R_i) operation of the form $R_i + \alpha R_j + \beta R_k \left(j,k \neq i\right)$.
- If each element in any row (column) is the sum of r terms, then the determinant can be expressed as the sum of r determinants.
- If the determinants $\Delta = f\left(x\right)$ and $f\left(a\right) = 0$, (x – a) is a factor of the determinant. In other word if two rows (or two columns) become proportional (identical) for (x = a) then (x – a) is a factor of determinants. In general, if r rows become identical for x = a then $(x - a)^{r-1}$ is a factor of the determinant.
- If in a determinant (of order three or more) the elements in all the rows (columns) are in A.P. with same or different common difference, the value of the determinant is zero.
- The determinant value of an odd order skew symmetric determinant is always zero.

Product of two Determinants:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_1 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_1 + c_2\gamma \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_1 + c_3\gamma \end{vmatrix}$$

Differentiation of Determinants:

Let $\Delta(x) = \begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$. Then $\Delta'(x)\begin{vmatrix} a_1'(x) & b_1'(x) \\ a_2(x) & b_2(x) \end{vmatrix} + \begin{vmatrix} a_1(x) & b_1(x) \\ a_2'(x) & b_2'(x) \end{vmatrix}$ where a' denotes derivative with respect of x.

Determinants Involving Integrations:

Let $\Delta(x)\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ l & m & n \end{vmatrix}$ where a, b, c, l, m and n are constants.

$$\Rightarrow \int_a^b \Delta(x)\,dx = \begin{vmatrix} \int_a^b f(x)\,dx & \int_a^b g(x)\,dx & \int_a^b h(x)\,dx \\ a & b & c \\ l & m & n \end{vmatrix}$$

Cramer's Rule:

If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0,$ then the solution of the system of non-homogeneous simultaneous linear equations

$a_1x + b_1y + c_1z = d_1$,

$a_2x + b_2y + c_2z = d_2$,

$a_3x + b_3y + c_3z = d_3$ is given by { where $(d_1, d_2, d_3) \neq (0, 0, 0)$}

$$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta} \text{ where}$$

$$\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \qquad \Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \qquad \Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

EXERCISE - I

WORK SHEET - I

MATRICES:

1. **If** $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ **and** $C=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ **then** $C=$

1) $I\cos\theta+B\sin\theta$ 2) $I\sin\theta+B\cos\theta$ 3) $I\cos\theta-B\sin\theta$ 4) $-I\cos\theta+B\sin\theta$

2. **If** $A-2B=\begin{pmatrix}1 & -2\\ 3 & 0\end{pmatrix}$ and $2A-3B=\begin{bmatrix}-3 & 3\\ 1 & -1\end{bmatrix}$ then $B=$

1) $\begin{pmatrix}-5 & 7\\ 5 & 1\end{pmatrix}$ 2) $\begin{pmatrix}-5 & 7\\ -5 & -1\end{pmatrix}$ 3) $\begin{pmatrix}-5 & 7\\ 5 & -1\end{pmatrix}$ 4) $\begin{pmatrix}-5 & -7\\ -5 & -1\end{pmatrix}$

3. **If** $A=\begin{pmatrix}0 & 2\\ 3 & -4\end{pmatrix}$, $kA=\begin{pmatrix}0 & 3a\\ 2b & 24\end{pmatrix}$ **then the values of** k, a, b **are respectively**

1) –6, –12, –18 2) –6, 4, 9 3) –6, –4, –9 4) –6, 12, 18

4. **If** $m\,[-3 \quad 4]+n\,[4 \; -3]=[10 \quad -11]$ **then** $3m+7n=$

1) 3 2) 5 3) 10 4) 1

5. **If** $A=$ **diag(1,–1,2),** $B=$**diag(2,3,–1) then** $3A+4B=$

1) diag (11, 9, 2) 2) diag (11, 9, -2) 3) diag (11, -9, 2) 4) diag (11, -9, -2)

6. **If** $A=\begin{pmatrix}4 & 1 & 0\\ 1 & -2 & 2\end{pmatrix}$, $B=\begin{pmatrix}2 & 0 & -1\\ 3 & 1 & 4\end{pmatrix}$, $C=\begin{pmatrix}1\\ 2\\ -1\end{pmatrix}$ **and** $(3B-2A)\,C+2X=O$ **then** $X=$

1) $\frac{1}{2}\begin{bmatrix}3\\ 13\end{bmatrix}$ 2) $\frac{1}{2}\begin{bmatrix}3\\ -13\end{bmatrix}$ 3) $\frac{1}{2}\begin{bmatrix}-3\\ 13\end{bmatrix}$ 4) $\begin{bmatrix}3\\ -13\end{bmatrix}$

7. **If** $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ $B=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$ $C=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$ **then**

1) $A^2=B^2=C^2=0$ 2) $A^2=B^2=C^2=I$ 3) $A^2=B^2=C^2=-I$ 4) $A^2=B^2=C^2=2I$

8. **If** $\begin{bmatrix}x & y & z\end{bmatrix}\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=$

1) $[ax^2+by^2+cz^2+2hxy+2gxz+2fyz]$ 2) $[ax^2+by^2+cz^2+hxy+gxz+fyz]$

3) $[2ax^2+2by^2+2cz^2+hxy+gxz+fyz]$ 4) $[2ax^2+2by^2+2cz^2+2hxy+2gxz+2fyz]$

9. **The value of** λ **for which the matrix product** $\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}-\lambda & 14\lambda & 7\lambda\\ 0 & 1 & 0\\ \lambda & -4\lambda & -2\lambda\end{bmatrix}$ **is an identity matrix**

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{5}$

10. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then

1) $a = 1$; $b = 1$ 2) $a = \cos 2\theta; b = \sin 2\theta$ 3) $a = \sin 2\theta; b = \cos 2\theta$ 4) $a = \sec^2\theta$; $b = 0$

11. If $\begin{bmatrix} x & 4 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$ then x =

1) $-1+\sqrt{6}$ 2) $8 \pm \sqrt{5}$ 3) $-2 \pm \sqrt{10}$ 4) $3 \pm \sqrt{6}$

12. If A and B are matrices such that $AB = O$ then

1) $A = O, B \neq O$ 2) $A \neq O, B = O$
3) $A = O, B = O$ 4) A, B need not be null matrices

13. If $A(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ then $A(\alpha)\,A(\beta) =$

1) $A(\alpha) - A(\beta)$ 2) $A(\alpha) + A(\beta)$ 3) $A(\alpha - \beta)$ 4) $A(\alpha + \beta)$

14. If $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ then $A^3 - A^2 =$

1) $2A$ 2) $2I$ 3) A 4) I

15. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then $A^3 - 4A^2 - 6A =$

1) 0 2) A 3) -A 4) I

16. If $A = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ and $a^2+b^2+c^2=1$ then $A^2=$

1) 2A 2) A 3) 3A 4) $\frac{1}{2}$A

17. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then

1) $\alpha = a^2 + b^2; \beta = 2ab$ 2) $\alpha = a^2 + b^2; \beta = a^2 - b^2$
3) $\alpha = 2ab; \beta = a^2 + b^2$ 4) $\alpha = a^2 + b^2; \beta = ab$

18. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ then $(A - I)(A - 2I)(A - 3I) =$

1) 1 2) 0 3) A 4) $\frac{1}{2}A$

19. If $AB = A$, $BA = B$ then $A^2 + B^2 =$

1) $A+B$ 2) $A-B$ 3) AB 4) 0

20. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}x & 1\\ y & -1\end{bmatrix}$ **and $(A+B)^2=A^2+B^2$ then (x, y) =**

1) (1, 4) 2) (2, 1) 3) (3, 3) 4) (0, 1)

21. If $A=\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$ **then A^5 =**

1) 243 2) 81A 3) 243A 4) 81

22. If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}, I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ **and $A^2-(a+d)A = KI$ then K =**

1) $bc-ad$ 2) $bc+ad$ 3) $ad-bc$ 4) $ac-bd$

23. If $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, E=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ **then $(aI+bE)^3$=**

1) a^3I+3a^2bE 2) a^3I-3a^2bE 3) a^3E+3a^2bI 4) a^3E-3a^2bI

24. If A = $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ **then A^{2004} =**

1) I 2) O 3) A 4) A^2

25. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ **and $A^2-kA-5I_2=0$ then K =**

1) 3 2) 5 3) –5 4) –3

26. If A and B are two square matrices of order n and A and B commute then for any real number k

1) $A-kI$, $B-kI$ are not commute 2) $A-kI$, $B-kI$ are commute
3) $A-kI=B-kI$ 4) $A-kI$, $k-BI$ are commute

27. If A and B are two matrices such that AB and $A+B$ are both defined then A and B are

1) Square matrices of the same order 2) Square matrices of different order
3) Rectangular matrices of same order 4) Rectangular matrices of different order

28. If A and B are square matrices of size nxn such that $A^2-B^2=(A-B)(A+B)$ then which of the following will be always true ?

1) $A=B$ 2) $AB=BA$
3) either of A or B is a zero matrix 4) either of A or B is an identity matrix

29. Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}, B=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}$, **$a, b \in N$ then**

1) There cannot exist any B such that $AB=BA$
2) There exist more than one but finite number of $B's$ such that $AB=BA$
3) There exists exactly one B such that AB=BA
4) There exist infinitely many $B's$ such that $AB=BA$

30. If $A=\begin{pmatrix}\cos^2\alpha & \cos\alpha\sin\alpha\\ \cos\alpha\sin\alpha & \sin^2\alpha\end{pmatrix}$ and $B=\begin{pmatrix}\cos^2\beta & \cos\beta\sin\beta\\ \cos\beta\sin\beta & \sin^2\beta\end{pmatrix}$ are two matrices such that the product AB is the null matrix then $\alpha-\beta=$

1) 0 2) multiple of π 3) an odd multiple of $\frac{\pi}{2}$ 4) none

31. If $\alpha-\beta=(2n+1)\frac{\pi}{2};n\in z$ then $\begin{bmatrix}\cos^2\alpha & \cos\alpha\sin\alpha\\ \cos\alpha\sin\alpha & \sin^2\alpha\end{bmatrix}\begin{bmatrix}\cos^2\beta & \cos\beta\sin\beta\\ \cos\beta\sin\beta & \sin^2\beta\end{bmatrix}=$

1) 0 2) I 3) $2I$ 4) -I

32. If $A=\begin{pmatrix}3 & -4\\ 1 & -1\end{pmatrix}$ then $A^n=$

1) $\begin{pmatrix}3n & -4n\\ n & -n\end{pmatrix}$ 2) $\begin{pmatrix}2+n & 5-n\\ n & -n\end{pmatrix}$ 3) $\begin{pmatrix}3^n & (-4)^n\\ 1^n & (-1)^n\end{pmatrix}$ 4) $\begin{pmatrix}1+2n & -4n\\ n & 1-2n\end{pmatrix}$

33. If $n\varepsilon N$ and $A=\begin{pmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix}$ then $A^n=$

1) $\begin{pmatrix}0 & 0 & a^n\\ 0 & b^n & 0\\ c^n & 0 & 0\end{pmatrix}$ 2) $\begin{pmatrix}a^n & 0 & 0\\ 0 & b^n & 0\\ 0 & 0 & c^n\end{pmatrix}$ 3) 0 4) I

34. If $A=\begin{pmatrix}x & 0 & 0\\ 0 & x & 0\\ 0 & 0 & x\end{pmatrix}$ then $A^n=(n\varepsilon N)$

1) $x^n A$ 2) $x^{n-1} A$ 3) $x A$ 4) $-x^n A$

35. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then $A^n=$

1) $\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$ 2) $\begin{bmatrix}\cos^n\theta & \sin^n\theta\\ (-1)^n\sin^n\theta & \cos^n\theta\end{bmatrix}$ 3) $\begin{bmatrix}n\cos\theta & n\sin\theta\\ -n\sin\theta & n\cos\theta\end{bmatrix}$ 4) None of these

36. If $A=\begin{pmatrix}\cosh\theta & \sinh\theta\\ \sinh\theta & \cosh\theta\end{pmatrix}$ then $A^n=$

1) $\begin{pmatrix}\cosh n\theta & \sinh n\theta\\ \sinh n\theta & \cosh n\theta\end{pmatrix}$ 2) $\begin{pmatrix}-\cosh n\theta & \sinh n\theta\\ \sinh n\theta & \cosh n\theta\end{pmatrix}$ 3) $\begin{pmatrix}n\cosh\theta & n\sinh\theta\\ n\sinh\theta & n\cosh\theta\end{pmatrix}$ 4) Does not exist

37. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ and $n\in N$ then $A^n=$

1) 2^nA 2) $2^{n-1}A$ 3) $n A$ 4) $(n+1)A$

38. **If $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (n is positive)then n is**

1) even 2) odd 3) any natural number 4) none of these

39. **If the matrix $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ then $A^{n+1} =$**

1) $2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ 2) $2n\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ 3) $2^n\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ 4) $2^{n+1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

40. **If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction**

1) $A^n = nA - (n-1)I$ 2) $A^n = 2^{n-1}A - (n-1)I$ 3) $A^n = nA + (n-1)I$ 4) $A^n = 2^{n-1}A + (n-1)I$

41. **If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then $A^n =$**

1) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ 2) $\begin{bmatrix} 1 & n & \frac{n(n-1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & -3 & \frac{n}{2} \\ 1 & 1 & n \\ -1 & 0 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 2 & n-1 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

42. **If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $\underset{n\to\infty}{Lt} \frac{1}{n} A^n$**

1) a null matrix 2) an identity matrix 3) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

43. **If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then for $n \geq 4$; $A^n =$**

1) $A^{n-2} + A^3 - A$ 2) $A^{n+1} + I$ 3) $A^n - 2nA + 2I$ 4) $A^{n+3} + A^n + 3I$

44. **If $A^2 = 2A - I$ then for $n \neq 2$, $A^n =$**

1) $nA - (n-1)I$ 2) $nA - I$ 3) $nA - (n-2)I$ 4) $nA - 2I$

45. **If $A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$ then $A^{4n+1} =$ ____, $n \in N$**

1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 3) $\begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$ 4) $\begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$

46. **If $A = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ then the value of $A + A^2 + A^3 + \ldots A^n =$**

1) A 2) nA 3) $(n+1)A$ 4) 0

47. The number of 2 × 2 matrices that can be formed by using 1, 2, 3, 4 when repetitions are allowed is

1) 24 2) 12 3) 6 4) 256

48. The number of 2 × 2 matrices that can be formed by using 1, 2, 3, 4 without repetition is

1) 24 2) 12 3) 6 4) 256

49. If a matrix has 13 elements, then the possible dimensions (orders) of the matrix are

1) 1×13 or 13×1 2) 1×26 or 26×1 3) 2×13 or 13×2 4) 13×13

1) $x^n A$ 2) $x^{n-1} A$ 3) $x A$ 4) $-x^n A$

50. If $A=\begin{pmatrix} x & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{pmatrix}$ such that $A^T = -A$ then x =

1) –1 2) 0 3) 1 4) 4

51. If $A=\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then $A A^T = A^T A$ =

1) O 2) $-I$ 3) I 4) $2I$

52. If $3A=\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ then

1) $AA^T = A^T A = I$ 2) $AA^T = A^T A = -I$ 3) $AA^T = A^T A = 0$ 4) none ot these

53. If $3A + 4B^T = \begin{pmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{pmatrix}$ and $2B - 3A^T = \begin{pmatrix} -1 & 18 \\ 4 & -6 \\ -5 & -7 \end{pmatrix}$ then B =

1) $\begin{pmatrix} 1 & 3 \\ -1 & 0 \\ -2 & -4 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & 3 \\ 1 & 0 \\ 2 & 4 \end{pmatrix}$ 3) $\begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{pmatrix}$ 4) $\begin{pmatrix} -1 & -3 \\ 1 & 0 \\ 2 & 4 \end{pmatrix}$

54. If $A^T B^T = C^T$ then C =

1) AB 2) BA 3) BC 4) ABC

55. If the order of A is 4×3; the order of B is 4×5 and the order of C is 7×3 then the order of $(A^T B)^T C^T$ is

1) 7 × 5 2) 5 × 7 3) 4 × 7 4) 7 × 4

56. If $P + Q = \begin{bmatrix} 1 & 6 \\ 7 & 2 \end{bmatrix}$, P is a symmetric, Q is a skew symmetric then P =

1) $\begin{pmatrix} 1 & \frac{-13}{2} \\ \frac{-13}{2} & 0 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & \frac{13}{2} \\ \frac{13}{2} & 2 \end{pmatrix}$ 3) $\begin{pmatrix} 0 & \frac{13}{2} \\ \frac{13}{2} & 0 \end{pmatrix}$ 4) $\begin{pmatrix} 0 & \frac{-13}{2} \\ \frac{13}{2} & 0 \end{pmatrix}$

57. If $P + Q = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, P is symmetric, Q is a skew symmetric matrix then Q =

1) $\begin{pmatrix} 0 & \frac{-1}{2} & 2 \\ \frac{1}{2} & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$ 2) $\begin{pmatrix} 0 & \frac{1}{2} & 1 \\ \frac{-1}{2} & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ 3) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ 4) $\begin{pmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{pmatrix}$

58. If $A = \begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$ is a symmetric matrix then x =

1) 0 2) 3 3) 6 4) 8

59. If A and B are symmetric matrices then ABA is

1) diagonal matrix 2) symmetric matrix
3) skew symmetric matrix 4) identity matrix

60. If $A = \begin{bmatrix} 0 & a+1 & b-2 \\ 2a-1 & 0 & c-2 \\ 2b+1 & 2+c & 0 \end{bmatrix}$ is skew symmetric then $a + b + c$ =

1) 3 2) –3 3) $\frac{1}{3}$ 4) $-\frac{1}{3}$

61. If $A = [a_{ij}]_{n\times n}$ and a_{ij} = A.M. of (i, j) then A is

1) Triangular matrix 2) diagonal matrix
3) a symmertric matrix 4) skew symmetric matrix

62. If A is a square matrix then $A+A^T$ will be .... matrix.

1) symmetric 2) skew symmetric 3) scalar 4) identity

63. If A is a square matrix then $A - A^T$ is a .... matrix

1) symmetric 2) skew symmetric 3) Hermitian 4) Triangular

64. If A and B are two symmetric matrices then $AB + BA$ is

1) symmetric 2) skew symmetric 3) Diagonal 4) Null matrix

65. If A, B are symmetric matrices of the same order then $AB - BA$ is

1) symmetric matrix 2) skew symmetric matrix
3) Diagonal matrix 4) identity matrix

66. If A is a symmetric matrix and $n \in N$ then A^n is

1) symmetrix matrix 2) skew symmetric matrix
3) Diagonal matrix 4) identity matrix

67. If A is a skew symmetric matrix and n is an even +ve integer then A^n is

1) symmetric matrix 2) skew symmetric matrix
3) identity matix 4) Diagonal matrix

68. **If A is a skew-symmetric matrix and n is odd+ve integer then A^n is**
1) symmetric matrix 2) skew symmetric matrix 3) identity matrix 4) Diagonal matrix

69. **If A is a symmetric matrix or skew symmetric matrix. Then A^2 is**
1) Symmetric matrix 2) skew symmetric matrix
3) an orthogional matrix 4) a diagonal martrix

70. **If A is square matrix then $A A^T$ is..... matrix**
1) symmetric 2) skew symmetric 3) scalar 4) Idempotent

71. **If a matrix A is both symmetric and skew symmetric then A is**
1) I 2) 0 3) A 4) diagonal matrix

72. $A=\left[a_{ij}\right]_{3\times3}$ **is a square matrix so that** $a_{ij} = i^2 - j^2$ **then A is a**
1) symmetric 2) orthogonal 3) involutary 4) skew symmetric

73. **If A is a 3×4 matrix and B is matrix such that A^TB and BA^T are Both defined then order of B is**
1) 3×4 2) 4×3 3) 3×3 4) 4×4

74. **If** $A=\begin{pmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 1 & 0 & 1 \end{pmatrix}$ **then the trace of A is**
1) 1 2) –1 3) 3 4) 2

75. **If $A = [a_{ij}]$ is a scalar matrix then the trace of A is**
1) $\sum_i a_{ij}$ 2) $\sum_i a_{ij}$ 3) $\sum_i \sum_j a_{ij}$ 4) $\sum_i a_{ii}$

76. **If the trace of A is 7 then the trace of $7A$ is**
1) 14 2) 28 3) 73 4) 49

77. **If $tr(A)=2+i$ then $tr((2-i)A)$ =**
1) 5 2) 4 3) 3 4) –4

78. **If the trace of AB is 30 then the trace of BA is**
1) –30 2) 15 3) 30 4) 0

79. **If the traces of A, B are 17 and 8 then the trace of $A + B$ is**
1) 11 2) 25 3) $\frac{17}{8}$ 4) –9

80. **If the traces of A, B are 19 and 8 then the trace of $A–B$ is**
1) 11 2) 25 3) $\frac{17}{8}$ 4) 9

81. **If $tr(A)= 3$, $tr(B)= 5$ then $tr(AB)$ =**
1) 15 2) 8 3) 3/5 4) cannot say

82. **If $A=[a_{ij}]$ is a scalar matrix of order n×nsuch that $a_{ij} = k$ for all i, then trace of A =**
1) nk 2) $n + k$ 3) n/k 4) $n–k$

83. **If** $A=\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ **is symmetric then trace of A is**
1) 5 2) –10 3) 10 4) 15

84. **If A is a skew-symmetric matrix of order 3 then $tr(A)$ =**
1) 1 2) 3 3) 0 4) –1

85. **If $A = [a_{ij}]$ is a skew symmetric matrix of order 'n' then $\sum a_{ii}$ =**

1) 0 2) 1 3) –1 4) n

86. **If $A=\left[a_{ij}\right]_{n\times n}$ and $a_{ij}=i(i+j)$ then trace of A=**

1) $\frac{n(n+1)(2n+1)}{6}$ 2) $\frac{n(n+1)(2n+1)}{3}$ 3) $\frac{n(n+1)}{2}$ 4) $\frac{n^2(n+1)^2}{4}$

87. **If $A=\left[a_{ij}\right]_{n\times n}$ such that $a_{ij}=(i+j)^2$ then trace of A is**

1) $\frac{1}{3}n(n+1)(2n+1)$ 2) $\frac{2}{3}n(n-1)(2n-1)$ 3) $\frac{2}{3}n(n+1)(2n+1)$ 4) $\frac{1}{3}n(n-1)(2n-1)$

88. $A=\begin{bmatrix}1+i & 2-3i & 4\\ 7+2i & -i & 3-2i\end{bmatrix}$ **then the conjugate of A is**

1) $\begin{bmatrix}1-i & 2+3i & 4\\ 7-2i & i & 3+2i\end{bmatrix}$ 2) $\begin{bmatrix}1+i & 7+2i & 3-2i\\ -i & 2-3i & 4\end{bmatrix}$ 3) $\begin{bmatrix}2+3i & 1-i & 4\\ 7-2i & 3-2i & 1\end{bmatrix}$ 4) $\begin{bmatrix}1+i & 2-3i & 4\\ -i & 2+3i & -4\end{bmatrix}$

89. **If $A=\begin{bmatrix}1 & 2-3i & 3+4i\\ 2+3i & 0 & 4-5i\\ 3-4i & 4+5i & 2\end{bmatrix}$ then A is**

1) symmetric 2) skew symmetric 3) hermitian 4) skew hermitian

WORK SHEET - II

DETERMINANTS:

1. If $A=\begin{pmatrix}1 & 3\\ 2 & 1\end{pmatrix}$, then the determinant of $A^2 - 2A$ is
   1) 5 2) 25 3) –5 4) –25

2. The minors of 1 and 7 in the matrix $\begin{bmatrix}1 & 4 & 2\\ 2 & -1 & 4\\ -3 & 7 & 6\end{bmatrix}$ are
   1) 34, 0 2) 34, –1 3) –34, 1 4)–34, 0

3. The co factors of 7 and 6 in the matrix $\begin{bmatrix}1 & 2 & 3\\ 4 & -1 & 7\\ 2 & 4 & 6\end{bmatrix}$ are
   1) –22, 0 2) 0, 9 3) 0, –9 4) - 1, –1

4. Elements of $\begin{bmatrix}1 & -1 & 0\\ 0 & 4 & 2\\ 3 & -4 & 6\end{bmatrix}$ with their cofactors and choose the correct answer

Element	Co factor
I) –1	a) –2
II) 1	b) 32
III) 3	c) 4
IV) 6	d) 6
	e) –6

   1) b, d, a, c 2) b, d, c, a 3) d, b, a, c 4) d, a, b, c

5. If $\Delta = \begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$ then $\begin{vmatrix}A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\\ C_1 & C_2 & C_3\end{vmatrix} =$
   1) $\frac{\Delta^2}{2}$ 2) 2Δ 3) Δ^2 4) Δ

6. If $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix} = 5$ then the vaue of $\begin{vmatrix}bc-a^2 & ac-b^2 & ab-c^2\\ ac-b^2 & ab-c^2 & bc-a^2\\ ab-c^2 & bc-a^2 & ac-b^2\end{vmatrix}$ is equal to
   1) 2 2) 25 3) 125 4) 625

7. If $A = \begin{bmatrix}1 & 0 & 1\\ 2 & 1 & 0\\ 3 & 2 & 1\end{bmatrix}$ then det A =
   1) 2 2) 3 3) 4 4) 5

8. If $\begin{pmatrix}1 & 2 & x\\ 4 & -1 & 7\\ 2 & 4 & -6\end{pmatrix}$ is a singular matrix then x =
   1) 0 2) 1 3) –3 4) 3

9. If the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular then θ =

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

10. The matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$ is

1) non singular 2) singular 3) skew sysmatric 4) sysmatric

11. $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} =$

1) –1 2) 1 3) 0 4) 2

12. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

1) 0 2) 1 3) $2\cos 2x - 2\sin 2x$ 4) $\cos 2x$

13. $\begin{vmatrix} x & 1 & y+z \\ y & 1 & z+x \\ z & 1 & x+y \end{vmatrix} =$

1) $1 + x + y + z$ 2) $x + y + z$ 3) 0 4) 1

14. If $x+iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$ then

1) $x = 3, y = 1$ 2) $x = 1, y = 3$ 3) $x = 0, y = 3$ 4) $x = 0, y = 0$

15. $\begin{vmatrix} 1990 & 1991 & 1992 \\ 1991 & 1992 & 1993 \\ 1992 & 1993 & 1994 \end{vmatrix} =$

1) 1992 2) 1993 3) 1994 4) 0

16. $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} =$

1) $15\sqrt{2} - 25\sqrt{3}$ 2) $15\sqrt{5} - 25\sqrt{6}$ 3) $25\sqrt{2} + 15\sqrt{3}$ 4) 0

17. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} =$

1) $(x-p)(x-q)$ 2) $(x-p)(x-q)(x+p+q)$ 3) $(x-p)(x+p+q)$ 4) $(x-q)(x+p+q)$

18. $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} =$

1) $(x+2a)(x-a)$ 2) $(x+2a)^2(x-a)$ 3) $(x+2a)(x-a)^2$ 4) $(x+2a)^2(x-a)^2$

19. **If** $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ **then =**

1) $\Delta_1 = 3\Delta_2^2$ 2) $\frac{d(\Delta_1)}{dx} = 3\Delta_2$ 3) $\frac{d(\Delta_1)}{dx} = 3\Delta_2^2$ 4) $\Delta_1 = 3\Delta_2^{3/2}$

20. $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} =$

1) $a^2b^2c^2$ 2) $4a^2b^2c^2$ 3) $2a^2b^2c^2$ 4) $3a^2b^2c^2$

21. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} =$

1) $(a-1)^3$ 2) $(a-1)^2$ 3) $(a-1)^4$ 4) $(a-1)$

22. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} =$

1) 0 2) 1 3) abc 4) $ab+bc+ca$

23. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2-bc & b^2-ca & c^2-ab \end{vmatrix} =$

1) 0 2) 1 3) abc 4) $ab+bc+ca$

24. **If** $x \neq 0$ **and** $\begin{vmatrix} 1 & x & 2x \\ 1 & 3x & 5x \\ 1 & 3 & 4 \end{vmatrix} = 0$ **then** $x =$

1) 1 2) –1 3) 2 4) –2

25. **If** $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ **then** $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} =$

1) 0 2) abc 3) $-abc$ 4) $2abc$

26. $\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix} =$

1) 0 2) 1 3) 4 loge 4)5 loge

27. $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} =$

1) 0 2) 1 3) 2 4) 3

28. $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} =$

1) pqr 2) $p+q+r$ 3) $2pqr$ 4) 0

29. $\begin{vmatrix} a-b & p-q & x-y \\ b-c & q-r & y-z \\ c-a & r-p & z-x \end{vmatrix} =$

1) 0 2) 1 3) $a\ b\ c$ 4) $x\ y\ z$

30. $\begin{vmatrix} x+y & 0 & 0 \\ 0 & x-y & 0 \\ 0 & 0 & x^2+y^2 \end{vmatrix} =$

1) $x^4 - y^4$ 2) $x^4 + y^4$ 3) $x^9 - y^8$ 4) $x^6 - y^6$

31. $\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} =$

1) $(a-b)\ (b-c)\ (c-a)$ 2) $-(a-b)\ (b-c)\ (c-a)\ (a+b+c)$
3) $abc(a-b)\ (b-c)\ (c-a)$ 4) $2(a-b)\ (b-c)\ (c-a)$

32. $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} =$

1) abc $(a^2+b^2+c^2)$ 2) abc (a+b+c) 3) (a+b+c) $(a^2+b^2+c^2)$ 4)abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

33. If $abc \neq 0$ and if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then $\dfrac{a^3+b^3+c^3}{abc} =$

1) 3 2) –3 3) 2 4) –2

34. If $x \neq 0$ and $\begin{vmatrix} 1 & x & 2x \\ 1 & 3x & 5x \\ 1 & 3 & 4 \end{vmatrix} = 0$ then $x =$

1) –1 2) 1 3) 3 4) –3

35. If $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = 0$ then $x =$

1) a 2) b 3) a or b 4) 0

36. If $\begin{vmatrix} x & 2 & 7 \\ 5 & 0 & 2 \\ 3 & -4 & 6 \end{vmatrix} = -180$ then $x =$

1) 2 2) 1 3) –2 4) –1

37. If $\begin{vmatrix} 1+x & 1-x & 1-x \\ 1-x & 1+x & 1-x \\ 1-x & 1-x & 1+x \end{vmatrix} = 0$ then $x =$

1) 0 o r 3 2) 1 or 2 3) 2 4) -1

38. If $\begin{vmatrix} 1+x & 2 & 3 \\ 1 & 2+x & 3 \\ 1 & 2 & 3+x \end{vmatrix} = 0$ then $x =$

1) 1 2) -1 3) -6 4) 6

39. If $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = K(a+b+c)^2$ then $K =$

1) 2 2) 2 $(a+b+c)$ 3) $2abc$ 4) 2 $(a+b+c)^2$

40. $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} =$

1) xyz 2) $4xyz$ 3) $2xyz$ 4) $3xyz$

41. If $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then $k =$

1) 8 2) 2 3) 3 4) 0

42. $\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix} =$

1) $(a+b+c)^3$ 2) $2(a+b+c)^3$ 3) $(a+b+c)^2$ 4) $2(a+b+c)^2$

43. $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} =$

1) $9b^2(a+b)$ 2) $9a^2(a+b)$ 3) $9(a+b)^3$ 4) $9ab(a+b)$

44. If $a \neq 6$, b, c satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$ then abc =

1) $a+b+c$ 2) 0 3) b^3 4) $ab+bc$

45. If w is a complex cube root of unity then $\begin{vmatrix} 1 & 1+w & 1+w^2 \\ 1+w & 1+w^2 & 1 \\ 1+w^2 & 1 & 1+w \end{vmatrix} =$

1) –2 2) 4 3) 0 4) 2

46. If 1, w, w^2 are the cube roots of unity then $\begin{vmatrix} 1 & w^n & w^{2n} \\ w & w^{2n} & 1 \\ w^{2n} & 1 & w^n \end{vmatrix} =$

1) 0 2) 1 3) w 4) w^2

47. If α, β, γ are the roots of $x^3+px+q = 0$, then $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$

1) 0 2) p 3) q 4) $p^2 - 2q$

48. $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} =$

1) $\log(xyz)$ 2) $\log(xy+yz+zx)$ 3) 0 4) $\log(x+y+z)$

49. $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix} =$

1) 0 2) $\log(xyz)$ 3) $\log(6xyz)$ 4) $6\log(xyz)$

50. $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} =$

1) 0 2) 1 3) abc 4) $(a-b)(b-c)(c-a)$

51. $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} =$

1) $\begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$ 2) $\begin{vmatrix} a & b & c \\ x & y & z \\ y & z & x \end{vmatrix}$ 3) $\begin{vmatrix} ax & by & cz \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$ 4) $\begin{vmatrix} x & y & z \\ a & b & c \\ yz & zx & xy \end{vmatrix}$

52. **If $x + y + z = 0$ and $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then $k =$**

1) $x + y + z$ 2) $xy + yz + zx$ 3) $-xyz$ 4) $x^2 + y^2 + z^2$

53. **If a, b, c are distinct and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ then**

1) $a + b + c = 1$ 2) $ab + bc + ca = 0$ 3) $a + b + c = 0$ 4) $abc = 1$

54. $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, x \neq y \neq z \Rightarrow 1 + xyz =$

1) 0 2) –1 3) 1 4) 2

55. **If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$ then $a\,b\,c\,(ab + bc + ca) =$**

1) $a + b + c$ 2) 0 3) 1 4) –1

56. **If a, b, c are all different and $\begin{vmatrix} 1+a^2 & 1+b^2 & 1+c^2 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = 0$ then $a + b + c =$**

1) abc 2) $a + b + c$ 3) $ab + bc + ca$ 4) 0

57. **If a, b, c are positive and not all equal then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$**

1) ≤ 0 2) < 0 3) ≥ 0 4) > 0

58. $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix} =$

1) $x+a+b+c$ 2) $(x+a^2+b^2+c^2)x^2$ 3) $(a^2+b^2+c^2+x)x$ 4) $(a+b+c+x)x$

59. $\begin{vmatrix} -2a & a+b & a+c \\ a+b & -2b & b+c \\ a+c & b+c & -2c \end{vmatrix} =$

1) $4(a+b)(b+c)(c+a)$ 2) $(a-b)(b-c)(c-a)$ 3) $4(a+b+c)$ 4) $4(ab+bc+ca)$

60. If $a+b+c=0$, and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ then $x =$

1) 0 2) $\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$ 3) $\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$ 4) 0, $\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

61. If $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$ then $\sum_{r=1}^{n} D_r =$

1) 1 2) –1 3) 0 4) n

62. If $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$ then $\sum_{r=1}^{n} D_r =$

1) 0 2) 1 3) -1 4) n

63. If $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$, then $\sum_{r=0}^{m} \Delta_r =$

1) 0 2) m^2-1 3) 2^m 4) $2^m\sin^2(2^m)$

64. If $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ then x =

1) $\frac{2}{3}, \frac{11}{3}$ 2) $\frac{-2}{3}, \frac{11}{3}$ 3) $\frac{2}{3}, \frac{-11}{3}$ 4) $\frac{-2}{3}, \frac{-11}{3}$

65. **If a,b,c are different and** $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ **then** $x =$

1) 1 2) a 3) b 4) 0

66. **If $x = -1$ is a root of the equation** $\begin{vmatrix} 2-x & 3 & 3 \\ 3 & 4-x & 5 \\ 3 & 5 & 4-x \end{vmatrix} = 0$ **then the other roots are**

1) 0, 12 2) 11, 12 3) 0, 11 4) 0, 11/2

67. **If one the roots of** $\begin{vmatrix} 3 & 5 & x \\ 7 & x & 7 \\ x & 5 & 3 \end{vmatrix}$ **=0 is –10, then the other roots are:**

1) 3, 7 2) 4, 7 3) 3, 9 4) 3, 4

68. **If** $\begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 64$ **then** $x =$

1) –1 2) 4 3) 3 4) 1

69. **If a,b,c are in A.P. then** $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} =$

1) 1 2) 0 3) –1 4) 2

70. **If x,y,z are in A.P. then the value of** $\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix} =$

1) 1 2) 0 3) $2a$ 4) a

71. **If l, m, n are the p^{th}, q^{th}, r^{th} terms of G.P. and all positive then** $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} =$

1) log xyz 2) (p-1) (q-1) (r-1) 3) pqr 4) 0

72. **If a, b, c are the p^{th}, q^{th}, r^{th} terms in H.P. then** $\begin{vmatrix} bc & p & 1 \\ ca & q & 1 \\ ab & r & 1 \end{vmatrix} =$

1) abc 2) pqr 3) 0 4) 1

73. If $a_1, a_2, a_3, ...a_n, ...$are in G.P. then the value of $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} =$

1) 0 2) 1 3) –1 4) 2

74. For all values of A, B, C and P, Q, R $\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} =$

1) 0 2) 1 3) –1 4) $\frac{1}{2}$

75. With the usual notation in a ΔABC $\begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} =$

1) $\frac{1}{8R^3}(a-b)(b-c)(c-a)$ 2) $8R^3$

3) $(a-b)(b-c)(c-a)$ 4) $\frac{1}{8R}(a-b)(a-c)(b-c)$

76. If $\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos 2B \\ \sin A & \sin A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix} = 0$ then $B =$

1) $(2n+1)\frac{\pi}{2}$ 2) $n\pi$ 3) $(2n+1)\pi$ 4) $2n\pi$

77. $\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\beta-\alpha) & \cos(\beta-\gamma) & 1 \end{vmatrix} =$

1) 0 2) 1 3) –1 4) 2

78. If $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x-\pi/2 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then $\frac{df}{dx}$ at $x = \frac{\pi}{2}$ is

1) 2 2) $\frac{\pi}{2}$ 3) 1 4) 8

79. $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x-\frac{\pi}{2} & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix} \Rightarrow f^1(\pi) =$

1) 0 2) 2 3) $\frac{\pi}{2}$ 4) $\pi - 6$

80. If $f(x) = \begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ x-12 & 12 & 2 \end{vmatrix}$ then $f^1\left(\frac{\pi}{2}\right) =$

1) –1 2) 0 3) + 1 4) ± 1

WORK SHEET- III

INVERSE OF MATRIX:

1. **If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, then**

1) $ad - bc = 0$ 2) $ad - bc \neq 0$ 3) $ab - cd \neq 0$ 4) $ab - cd = 0$

2. **If A is a square matrix of order 3 then** $\left|\text{Adj}(\text{Adj}\,A^2)\right| =$

1) $|A|^2$ 2) $|A|^4$ 3) $|A|^8$ 4) $|A|^{16}$

3. **If Adj $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & a & -2 \\ 1 & 1 & 0 \\ -2 & -2 & b \end{bmatrix}$ then [a, b] =**

1) [–4, 1] 2) [–4, –1] 3) [4, 1] 4) [4, –1]

4. **If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$** and $A(\text{Adj}\,A) = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then the value of k is

1) $\sin x \cos x$ 2) 1 3) -1 4) 2

5. **The inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ is**

1) $\frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ 2) $\frac{1}{5}\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$ 3) $\frac{1}{5}\begin{pmatrix} -3 & 1 \\ 1 & 2 \end{pmatrix}$ 4) $\frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

6. **If $A = \begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, $a^2+b^2+c^2+d^2 = 1$ the inverse of A is**

1) $\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$ 2) $\begin{bmatrix} a+ib & c+id \\ c+id & a-ib \end{bmatrix}$ 3) $\begin{bmatrix} a-ib & c-id \\ c-id & a+ib \end{bmatrix}$ 4) $\begin{bmatrix} a+ib & -c-id \\ c-id & a+ib \end{bmatrix}$

7. **If $\begin{pmatrix} x & y^3 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 8 \\ 2 & 0 \end{pmatrix}$ then $\begin{pmatrix} x & y \\ 2 & 0 \end{pmatrix}^{-1} =$**

1) $\begin{pmatrix} 0 & -2 \\ -2 & 1 \end{pmatrix}$ 2) $\begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix}$ 3) $\begin{pmatrix} 0 & -8 \\ -2 & 1 \end{pmatrix}$ 4) $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{4} \end{pmatrix}$

8. $\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1} =$

1) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 2) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 3) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ 4) $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

9. **If the matrix A is such that $A\begin{bmatrix}-1 & 2\\ 3 & 1\end{bmatrix} = \begin{bmatrix}-4 & 1\\ 7 & 7\end{bmatrix}$ then A =**

1) $\begin{bmatrix}1 & 1\\ 2 & -3\end{bmatrix}$ 2) $\begin{bmatrix}-4 & 1\\ 7 & 7\end{bmatrix}$ 3) $\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ 4) $\begin{bmatrix}-1 & 1\\ 2 & 3\end{bmatrix}$

10. **If $A = \begin{bmatrix}3 & 4\\ 7 & 9\end{bmatrix}$ and $AB = I$ then B =**

1) $\begin{bmatrix}9 & 4\\ 7 & -3\end{bmatrix}$ 2) $\begin{bmatrix}-9 & 4\\ 7 & -3\end{bmatrix}$ 3) $\begin{bmatrix}9 & -4\\ 7 & -3\end{bmatrix}$ 4) $\begin{bmatrix}9 & -4\\ 7 & -3\end{bmatrix}$

11. **If $A = \begin{pmatrix}2 & 2\\ -3 & 2\end{pmatrix}$, $B = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ then $(B^{-1}A^{-1})^{-1}$ =**

1) $\begin{pmatrix}2 & -2\\ 2 & 3\end{pmatrix}$ 2) $\begin{pmatrix}3 & -2\\ 2 & 2\end{pmatrix}$ 3) $\frac{1}{10}\begin{pmatrix}2 & 2\\ -2 & 3\end{pmatrix}$ 4) $\frac{1}{10}\begin{pmatrix}3 & 2\\ -2 & 2\end{pmatrix}$

12. **The matrix having the same matrix as its inverse is**

1) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ 2) $\begin{bmatrix}1 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}$ 3) $\begin{bmatrix}0 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$ 4) $\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$

13. **The inverse of $\begin{bmatrix}3 & 5 & 7\\ 2 & -3 & 1\\ 1 & 1 & 2\end{bmatrix}$ is**

1) $\begin{bmatrix}3 & 5 & -7\\ 2 & 3 & 76\\ 2 & 2 & 0\end{bmatrix}$ 2) $\begin{bmatrix}3 & 2 & 1\\ 5 & -3 & 10\\ 7 & 21 & 0\end{bmatrix}$ 3) $\begin{bmatrix}7 & 3 & -26\\ 3 & 1 & -11\\ -5 & -2 & 19\end{bmatrix}$ 4) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

14. **The inverse of $\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$ is**

1) $\begin{bmatrix}1 & -2 & -7\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}$ 2) $\begin{bmatrix}1 & 2 & -7\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}$ 3) $\begin{bmatrix}1 & 2 & -7\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$ 4) $\begin{bmatrix}1 & -2 & 7\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}$

15. **The inverse of $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ is**

1) $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ 2) $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ 3) $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ 4) $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

16. **If** $A = \begin{pmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix} = f(x)$ **then** $A^{-1} =$

1) $f(-x)$ 2) $f(x)$ 3) $-f(x)$ 4) $-f(-x)$

17. **If** $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **and** $G(x) = \begin{bmatrix} \cos x & 0 & \sin x \\ 0 & 1 & 0 \\ -\sin x & 0 & \cos x \end{bmatrix}$ **then** $[F(x)\ G(x)]^{-1}$

1) $G(-x)\ F(-x)$ 2) $\{F(x)\}^{-1}\{G(x)\}^{-1}$ 3) $[G(x)]\ \{F(x)\}$ 4) $F(x).\ G(x)$

18. **If** A **is 4×4 matrix and** $|2A|=64$; $B=\text{Adj } A$ **then** $|\text{Adj } B| =$

1) 2^9 2) 2^{18} 3) 2^{36} 4) 2^6

19. **If** $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ **then** $A^{-1} =$

1) $2A^T$ 2) A^T 3) $3A^T$ 4) $\frac{1}{2}A^T$

20. **Let** $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ **and** $(10)B=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$**. If** B **is the inverse of matrix** A**, then** α **is**

1) –2 2) 5 3) 2 4) –1

21. If $\begin{bmatrix} x & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ has no inverse then x =

1) 0 2) – 1 3) 1 4) 2

22. **If** $\begin{bmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{bmatrix}$ **has no inverse,then the real value of** x **is:**

1) 2 2) 3 3) 0 4) 1

23. **If** $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **then the matrix** A **is**

1) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

24. **If** $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ **then** $A^{-1} + (A - aI)(A - cI) =$

1) $\frac{1}{ac}\begin{bmatrix} a & b \\ 0 & -c \end{bmatrix}$ 2) $\frac{1}{ac}\begin{bmatrix} -a & b \\ 0 & c \end{bmatrix}$ 3) $\frac{1}{ac}\begin{bmatrix} c & -b \\ 0 & a \end{bmatrix}$ 4) $\frac{1}{ac}\begin{bmatrix} c & b \\ 0 & a \end{bmatrix}$

25. **If A is an invertible matirx of order 'n' then the determinant of adj** A **=**

1) $|A|^n$ 2) $|A|^{n+1}$ 3) $|A|^{n-1}$ 4) $|A|^{n+2}$

26. If A is square matrix such that A (Adj A) = $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ then det (Adj A) =

1) 4 2) 16 3) 64 4) 256

27. If $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$ then $|\text{Adj } A|$ =

1) 64 2) 256 3) 8 4) 6

28. If A is a 3×3 matrix and det A =–2 then $|\text{Adj } A|$ =

1) –4 2) 8 3) –8 4) 4

29. If A is a 3×3 matrix and $|\text{Adj } A| = 16$ then $|A|$ =

1) + 4 2) –4 3) ± 4 4) 8

30. If A is a 3×3 matrix and B is its Adjoint matrix. If the determinent of B is 64 then the determinent of A is

1) ± 6 2) ± 8 3) ± 4 4) ± 16

31. The value of a third order determinant is 11 then the value of the square of the determinent formed by the cofactors is

1) 121 2) $(121)^2$ 3) $(121)^3$ 4) $(121)^4$

32. If A is a 4×4 matrix and det A=–2 then $|\text{Adj } A|$=

1) –4 2) 8 3) –8 4) 4

33. If A is a 4×4 matrix and $|\text{Adj } A|$ =–27 then $|A|$ =

1) 2 2) –2 3) – 3 4) 3

34. Let A = $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ then $|\text{AdJ (Adj A)}|$ =

1) 64 2) 256 3) 8 4) 6

35. If $S=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A=\frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$,then SAS^{-1} =

1) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ 2) $\frac{1}{2}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ 3) $2\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ 4) $3\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

36. If A is non-singualr and $A^2–5A + 7I = 0$ then I =

1) $\frac{1}{7}A - \frac{5}{7}A^{-1}$ 2) $\frac{1}{7}A + \frac{5}{7}A^{-1}$ 3) $\frac{1}{5}A + \frac{7}{5}A^{-1}$ 4) $\frac{1}{5}A - A^{-1}$

37. A square non-singular matrix A satisfies $A^2–A+2I = 0$, then A^{-1} =

1) $I - A$ 2) $\frac{1}{2}(I - A)$ 3) $I + A$ 4) $\frac{1}{2}(I + A)$

38. **If A is non Singular and $(A-2I)(A-4I) = 0$ then $\frac{1}{6}A+\frac{4}{3}A^{-1}$ =**

1) I 2) 0 3) 2I 4) 6I

39. **If $A \neq A^2 = I$ then $|I + A|$ =**

1) 1 2) –1 3) 0 4) 2

40. **If A is a nonzero square matrix of order n with det $(I+A)\neq 0$ and $A^3 = 0$, where I, 0 are unit and null matrices of order $n \times n$ respectively then $(I+A)^{-1}$ =**

1) $I - A + A^2$ 2) $I + A + A^2$ 3) $I + A^{-1}$ 4) I + A

41. **If $A \neq I$ is an idempotent matrix then A is**

1) Singular matrix 2) non singular matrix
3) Symmertic 4) Skew symmetric matrix

42. **If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2$ =**

1) 0 2) $A^2 + B^2$ 3) $A^2 + 2AB + B^2$ 4) $A + B$

43. **If A is an orthogonal matrix, then |A| is**

1) 1 2) –1 3) ± 1 4) 0

44. **Let A and B be square matrices of 3rd order and A be an orthogonal matrix and B is a skew symmetric matrix. Then which of the following is not true.**

1) Numerical value of $|A|$ is 1 2) $|B| = 0$ 3) $|AB| = 1$ 4) $|AB| = 0$

45. **Which of the following statements is false :**

1) if $|A| = 0$, then $|\text{adj } A| = 0$
2) adjoint of a diagonal matrix of order 3×3 is a diagonal matrix
3) product of two upper triangular matrices is an upper triangular matrix
4) adj (AB) = adj (A) adj (B)

WORK SHEET - IV

LINEAR EQUATIONS:

1. **The solution of $7x+5y-13z+4=0$, $9x+2y+11z=37$, $3x-y+z=2$ is**
   1) $x=1, y=2, z=3$ 2) $x=1, y=3, z=2$ 3) $y=2, y=3, z=1$ 4) $x=1, y=2, z=-2$

2. **The solution of the system of equations whose Augumented matrix is $\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 4 & 1 & 7 \\ 3 & 2 & 9 & 14 \end{bmatrix}$ is**
   1) $x=1, y=1, z=-1$ 2) $x=-1, y=1, z=1$ 3) $x=1, y=-1, z=1$ 4) $x=1, y=1, z=1$

3. **The equations $x+y+z=0$, $2x-y-3z=0$, $3x-5y+4z=0$ have**
   1) unique solution 2) Infinitely many solutions
   3) no solution 4) none

4. **The equations $x+2y+3z=1$, $2x+y+3z=2$, $5x+5y+9z=4$ have**
   1) no solution 2) one solution
   3) infinitely many solutions 4) none

5. **The equations $2x+y-4z=0$, $x-2y+3z=0$, $x-y+z=0$ have**
   1) Unique solution 2) no solution 3) Infinitely many solutions 4) none

6. **The equations $3x-2y+z=5$, $6x-4y+2z=10$, $9x-6y+3z=15$ have**
   1) No solution 2) one solution
   3) Infinitely many solutions 4) none

7. **The equations $x+4y-2z=3$, $3x+y+5z=7$, $2x+3y+z=5$ have**
   1) unique solution 2) no solution
   3) infinitely many solutions 4) none

8. **The number of solutions of the system of equations $2x+y-z=7$, $x-3y+2z=1$, $x+4y-3z=5$ is**
   1) 3 2) 2 3) 1 4) 0

9. **The number of nontrivial solutions of the system $x-y+z=0$, $x+2y-z=0$, $2x+y+3z=0$ is**
   1) 0 2) 1 3) 2 4) 3

10. **If the system of equations $x+ky+3z=0$, $3x+ky-2z=0$, $2x+3y-4z=0$ has a non trivial solution then K =.......**
    1) -33 2) $-\frac{33}{2}$ 3) $\frac{33}{2}$ 4) 33

11. **If the system of equations $2x+3ky+(3k+4)z=0$, $x+(k+4)y+(4k+2)z=0$, $x+2(k+4)y+(3k+4)z=0$ has a non trivial solution then K =**
    1) -8 or $\frac{1}{2}$ 2) 8 or $-\frac{1}{2}$ 3) -4 or $\frac{1}{2}$ 4) 4 or $-\frac{1}{2}$

12. **If the system of equations $ax+y+z=0$, $x+by+z=0$, $x+y+cz=0$ ($a, b, c \neq 1$) has a non trivial solution then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$**
    1) 1 2) -1 3) 2 4 2

13. **If the system of equations $x - cy - bz = 0$, $y - az - cx = 0$, $z - bx - ay = 0$ has a non trivial solution then $a^2 + b^2 + c^2 + 2abc$ =**

1) 0 2) –1 3) 2 4) 1

14. **If the system of equations $(\sin 3\theta)x - y + z = 0$, $(\cos 2\theta)\, x + 4y + 3z = 0$, $2x + 7y + 7z = 0$ has a non trivial solution then $\sin 3\theta + 2\cos 2\theta$ =**

1) 2 2) –2 3) 0 4) 1

15. **If the system of equations $x + 2y + 3z = \lambda x$, $3x + y + 2z = \lambda y$, $2x + 3y + z = \lambda z$ has a non trivial solution then λ =**

1) 6 2) 12 3) 18 4) 16

16. **If the system of equations $3x - 2y + z = 0$, $\lambda x - 14y + 15z = 0$, $x + 2y - 3z = 0$ has a non - zero solution then λ =**

1) 1 2) 3 3) 5 4) 0

17. **If the system of linear equations $x+2ay+az=0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c**

1) are in G.P. 2) are in H.P. 3) satisfy $a+2b+3c = 0$ 4) are in A.P.

18. **If a, b, c are all different and the equations $ax + a^2y + (a^3 + 1)\, z = 0$, $bx + b^2y + (b^3+1)\, z = 0$, $cx + c^2y + (c^3+1)z = 0$ has a non zero solution then abc =**

1) –1 2) 1 3) $a+b+c$ 4) 0

19. **If the system of equations $x+y+z=6$, $x+2y+\lambda z=0$, $x+2y+3z = 10$ has no solution then λ =**

1) 2 2) 3 3) 4 4) 5

20. **The system of equations $\alpha x+y+z=\alpha-1$, $x+\alpha y+z=\alpha-1$, $x+y+\alpha z=\alpha-1$ has no solution, if α is** 1) 1 2) not –2 3) either –2 or 1 4) –2

21. **If the system of equations $x + y + z = 6$, $x+2y+kz=0$, $x+2y+3z=10$ has no solution then k=**

1) 2 2) 3 3) 4 4) 5

22. **If $x^2 + y^2 + z^2 \neq 0$, $x = cy+bz$, $y = az+cx$ and $z=bx+ay$ then $a^2+b^2+c^2+2abc$=**

1) 0 2) 1 3) 2 4) –1

23. **By eliminating a, b, c from the Homogeneous Equations $x = \dfrac{a}{b-c}$, $y = \dfrac{b}{c-a}$, $z = \dfrac{c}{a-b}$ where a, b, c not all zero then $xy + yz + zx$ =**

1) 1 2) –1 3) 2 4) 0

24. **The equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x+2y+\lambda z = \mu$ have unique solution if**

1) $\lambda=3, \mu=10$ 2) $\lambda=3, \mu \neq 10$ 3) $\lambda \neq 3$ 4) none

25. **The system of linear equations $x+y+z=2$, $2x+y-z=3$, $3x+2y+kz=4$ has a unique solution if**

1) $k = 0$ 2) $-2 < k < 2$ 3) $-1 < k < 1$ 4) $k \neq 0$

26. **The values of λ for which the system of equations $x + y - 3 = 0$, $(1+\lambda)\, x+(2+\lambda)y-8=0$, $x-(1+\lambda)\, y+(2+\lambda) = 0$ is consistent are**

1) –5/3, 1 2) 2/3, –3 3) –1/3, –3 4) 0, 1

27. **If the system of equations : $(k + 1)^3 x + (k + 2)^3 y = (k + 3)^3$, $(k + 1)\, x + (k + 2)\, y = k + 3$, $x + y = 1$ is consistent, then the value of k is :**

1) 2 2) –2 3) –1 4) 1

28. **The system of equations $-2x + y + z = a$, $x-2y+z=b$, $x+y-2z = c$ is consistent if**

1) $a + b + c = 0$ 2) $a + b + c = 1$ 3) $a + b + c \neq 0$ 4) $a + b + c \neq 1$

29. **The system of equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$ is inconsistent if**

1) $a + b + c = 0$ 2) $a + b + c = 1$ 3) $a + b + c \neq 0$ 4) $a + b + c \geq 0$

30. **The system of equations $2x+6y+11=0$, $6y-18z+1= 0$, $6x+20y-6z+3 = 0$ are**

1) consistent 2) inconsistent
3) can not be determined 4) none

31. **The system of equations $x+y+z =6$, $x+2y+3z=10$, $x+2y+\lambda z = k$ is inconsistent if λ = ..., $k \neq$ ....**

1) 3, 7 2) 3, 10 3) 7, 10 4) 10, 3

32. **The Rank of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is**

1) 1 2) 2 3) 0 4) 3

33. **The Rank of $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ is**

1)1 2) 2 3) 0 4) 3

34. **The Rank of $\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$ is**

1) 1 2) 2 3) 0 4) 3

35. **The Rank of $\begin{bmatrix} -1 & 2 \\ -2 & 4 \\ 3 & 6 \end{bmatrix}$ is**

1)1 2) 2 3) 0 4) 3

36. **The Rank of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is**

1) 1 2) 2 3) 0 4) 3

37. **The Rank of $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ is**

1) 0 2) 1 3) 2 4) 3

38. **The Rank of $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is**

1) 3 2) 2 3) 1 4) 0

39. **The Rank of** $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ **is**

1) 3 2) 2 3) 1 4) 0

40. **The Rank of** $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ **is**

1) 3 if $a = 6$ 2) 1 if $a = -6$ 3) 3 if $a = 2$ 4) 2 if $a = -6$

41. **The Rank of** $\begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 2 \\ 1 & 1 & -1 & 3 \end{bmatrix}$ **is**

1) 4 2) 3 3) 2 4) 1

42. **The Rank of** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 1 & -1 & 4 \\ 2 & 2 & 8 \end{bmatrix}$ **is**

1) 1 2) 2 3) 0 4) 3

43. **The Rank of** $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$ **is**

1)1 2) 2 3) 0 4) 3

44. **If** A **be a matrix of Rank** r**. Then Rank of** A^T **is**

1) r 2) $r-1$ 3) $r+1$ 4) 0

45. **If** I_n **is the identity matrix of order n then the Rank of** I_n **is**

1) 1 2) $n+1$ 3) n 4) $n-1$

EXERCISE - I / ANSWERS

MATRICES

WORK SHEET- I

1) 1	2) 2	3) 3	4) 4	5) 1	6) 2	7) 3	8) 1	9) 4	10) 4
11) 3	12) 4	13) 4	14) 1	15) 3	16) 2	17) 1	18) 2	19) 1	20) 1
21) 2	22) 1	23) 1	24) 1	25) 2	26) 2	27) 1	28) 2	29) 4	30) 3
31) 1	32) 4	33) 2	34) 2	35) 1	36) 1	37) 2	38) 1	39) 3	40) 1
41) 2	42) 1	43) 1	44) 4	45) 3	46) 2	47) 4	48) 1	49) 1	50) 2
51) 3	52) 1	53) 3	54) 2	55) 2	56) 2	57) 1	58) 3	59) 2	60) 3
61) 3	62) 1	63) 2	64) 1	65) 2	66) 1	67) 1	68) 2	69) 1	70) 1
71) 2	72) 4	73) 1	74) 1	75) 4	76) 4	77) 1	78) 3	79) 2	80) 1
81) 4	82) 1	83) 3	84) 3	85) 1	86) 2	87) 3	88) 1	89) 3	

DETERMINANTS

WORK SHEET - II

1) 2	2) 4	3) 3	4) 3	5) 3	6) 2	7) 1	8) 3	9) 4	10) 2
11) 3	12) 1	13) 3	14) 4	15) 4	16) 1	17) 2	18) 3	19) 2	20) 2
21) 1	22) 1	23) 1	24) 2	25) 2	26) 1	27) 1	28) 4	29) 1	30) 1
31) 2	32) 3	33) 1	34) 1	35) 3	36) 2	37) 1	38) 3	39) 2	40) 2
41) 2	42) 1	43) 1	44) 3	45) 2	46) 1	47) 1	48) 3	49) 1	50) 1
51) 1	52) 3	53) 4	54) 1	55) 1	56) 1	57) 2	58) 2	59) 1	60) 4
61) 3	62) 1	63) 1	64) 1	65) 4	66) 3	67) 1	68) 2	69) 2	70) 2
71) 4	72) 3	73) 1	74) 1	75) 1	76) 1	77) 1	78) 1	79) 2	80) 1

INVERSE OF MATRIX

WORK SHEET - III

1) 2	2) 3	3) 3	4) 2	5) 4	6) 1	7) 4	8) 1	9) 3	10) 2
11) 1	12) 1	13) 3	14) 4	15) 2	16) 1	17) 1	18) 2	19) 2	20) 2
21) 3	22) 4	23) 1	24) 3	25) 3	26) 2	27) 1	28) 4	29) 3	30) 2
31) 2	32) 3	33) 3	34) 2	35) 1	36) 3	37) 2	38) 1	39) 3	40) 1
41) 1	42) 2	43) 3	44) 3	45) 4					

LINEAR EQUATIONS

WORK SHEET - IV

1) 2	2) 4	3) 1	4) 2	5) 3	6) 3	7) 2	8) 4	9) 1	10) 3
11) 1	12) 1	13) 4	14) 1	15) 1	16) 3	17) 2	18) 1	19) 2	20) 4
21) 2	22) 2	23) 2	24) 3	25) 4	26) 1	27) 2	28) 1	29) 3	30) 2
31) 2	32) 1	33) 2	34) 2	35) 2	36) 4	37) 4	38) 1	39) 2	40) 2
41) 2	42) 4	43) 2	44) 1	45) 3					

EXERCISE - II

WORK SHEET (HW) - I

1. If A and B are symmetric matrices and $AB=BA$, then $A^{-1}B^{-1}$ is a

   1) symmetirc matrix 2) skew-symmetirc matrix 3) unit matrix 4) null matrix

2. If A is a skew-symmetric matrix , then $I-A$ is

   1) skew symmetric 2) symmetric 3) singular 4) non-singular

3. Let A is $n\times n$ matrix such that $A^n=\alpha A$, Where α is a real number different from 1 and -1. Thus the matrix $A+I_n$ is

   1) Singular 2) non-singular(invertible)
   3) scalar matrix 4) None of these

4. If abc = p and $A=\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$ and $AA^1=1$, then a,b,c are the roots of the equation

   1) $x^3+p=0$ 2) $x^3+x^2=0$ 3) $x^3-2x^2+p=0$ 4) $x^3 \mp x^2+p=0$

5. If A & B are two matrices such that $AB = BA$ then $\forall n\in N$

   1) $A^nB=BA^n$ 2) $(AB)^n=A^nB^n$
   3) $(A+B)^n={}^nC_0A^n+{}^nC_1A^{n-1}B+.....{}^nC_nB^n$ 4) $A^{2n}-B^{2n}=(A^n-B^n)(A^n+B^n)$

6. If α,β,γ are the roots of an equation $x^3-12x^2+47x-60=0$ and $p(\alpha,\beta,\gamma)$ lies in the plane 8x + 4y + 3z = 20 and $A=\begin{bmatrix} \alpha & 3 & 2 \\ 1 & \beta & 4 \\ 2 & 2 & \gamma \end{bmatrix}$ then A . adj A =

   1) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ 2) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$ 3) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ 4) $\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$

7. Let $\phi_1(x)=x+a_1, \phi_2(x)=x^2+b_1x+b_2, x_1=2, x_2=3$ and $x_3=5$ and $\Delta=\begin{vmatrix} 1 & 1 & 1 \\ \phi_1(x_1) & \phi_1(x_2) & \phi_1(x_3) \\ \phi_2(x_1) & \phi_2(x_2) & \phi_3(x_3) \end{vmatrix}$ then $\Delta=$

   1) 2 2) 3 3) 4 4) 6

8. If $e^{ix}=\cos x+i\sin x$ and $x+iy=\begin{vmatrix} 1 & e^{\pi i/4} & e^{\pi i/3} \\ e^{-\pi i/4} & 1 & e^{2\pi i/3} \\ e^{-\pi i/3} & e^{-2\pi i/3} & e^{-2\pi i} \end{vmatrix}$, then

   1) $x=-1, y=\sqrt{2}$ 2) $x=1, y=-\sqrt{2}$ 3) $x=-\sqrt{2}, y=\sqrt{2}$ 4) y = 0

9. If $a, b \in R$ and $a^2+b^2-ab-a-b+1\leq 0$ and $\alpha+\beta+\gamma=0$,then $\Delta=\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & a & \cos\alpha \\ \cos\alpha & \cos\beta & b \end{vmatrix}$ equals

1) ab   2) 1   3) 0   4) -1

10. If A,B are two $n\times n$ matrices with real entries and AB= O_n , then det $(I_n+A^{2p}+B^{2q})$ for any positive intergers p and q is

1) ≤ 0 2) ≥ 0 3) = 0   4) None

11. The coeffiecient of x in the expansion of $\begin{vmatrix} (1+x)^{22} & (1+x)^{44} & (1+x)^{66} \\ (1+x)^{33} & (1+x)^{66} & (1+x)^{99} \\ (1+x)^{44} & (1+x)^{88} & (1+x)^{44} \end{vmatrix}$

1) 22   2) -22   3) 0   4) none

12. If $\Delta=\begin{vmatrix} 1/z & 1/z & -(x+y)/z^2 \\ -(y+z)/x^2 & 1/x & 1/x \\ -y(y+z)/x^2z & (x+2y+z)/xz & -(x+y)/xz^2 \end{vmatrix}$ then

1) Δ is independent of x 2) Δ is independent of y

3) Δ depends only on z 4) Δ =0

13. Let $\Delta(x)=\begin{vmatrix} e^x & \sin x & 1 \\ \cos x & \log_e(1+x^2) & 1 \\ x & x^2 & 1 \end{vmatrix}=Ax^2+Bx+C$, then

1) $A=0$ 2) B= –1   3) C = 0   4) $\Delta'(x)=-1$

14.. Let $\Delta(x)=\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}=Ax^4+Bx^3+Cx^2+Dx+E$. Then the value of 5A+4B+3C+2D+E is

1) 0   2) 16   3) -11   4) -17

15. If f (x) = $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$ and g(x) = $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}$, then

a) $f^1(x)+g^1(x) = f^1(x)-g^1(x)$ 2) $f^1(x)+g^1(x) = \left(f(x)+g(x)^1\right)$

3) $f^1(x)-g^1(x) = \left(f(x)-g(x)^1\right)$ 4) $2x\ f^1(x)=(x+1)\ g^1(x)$

16. If $f_n(x) = (n^x + n^{-x})^2$ and $g_n(x) = (n^x - n^{-x})^2$, then the value of $\begin{vmatrix} f_2(x) & g_2(x) & 1 \\ f_3(y) & g_3(y) & 1 \\ f_5(z) & g_5(z) & 1 \end{vmatrix}$ is

1) equal to zero   2) independent of x,y,z   3) 2^x3^y5 4) $\frac{1}{2^x3^y5^z}$

17. If $\Delta_i = \begin{vmatrix} 2^{i-1} & \dfrac{1}{i(i+1)} & \sin i\theta \\ x & y & z \\ 2^n - 1 & \dfrac{n}{n(n+1)} & \dfrac{\sin\left(\dfrac{n+1}{2}\right)\theta \sin n\theta}{\sin\dfrac{\theta}{2}} \end{vmatrix}$, then $\sum_{i=1}^{n} \Delta_i$ is

1) 0 2) independent of n 3) independent of θ 4) independent of x,y and z

18. Let $\alpha = \pi/5$ and $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, then $B = A + A^2 + A^3 + A^4$

1) singular 2) non-singular 3) skew -symmetric 4) |B|=1

19. Let $a, b, c \in R$ such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0$, then equation $4ax^2 + 4bx + c = 0$ has

1) atleast one root in [0, 1] 2) atleast one root in $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

3) atleast one root in [–1, 2] 4) atleast two roots in [0, 2]

20. If adj(B)=A and P and Q are two unimodular matrices i.e.,$|P|=|Q|=1$, then $(Q^{-1}BP^{-1})^{-1}$ is=

1) PAQ 2) PBQ 3) QAP 4) QBP

21. Let $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$. Then the number of column vectors $X = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ such that $AX = \lambda X$ for some scalar is

1) 1 2) 2 3) 3 4) infinite

22. Let $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ If θ is the angle between two nonzero column vectors X such that Ax= λX for some scalar λ , then $\tan\theta$ is .

1) $\dfrac{19}{\sqrt{3}}$ 2) $\dfrac{\sqrt{3}}{19}$ 3) $\dfrac{19}{\sqrt{364}}$ 4) $\dfrac{\sqrt{3}}{\sqrt{364}}$

23. If $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$, Then $A^8-2^8(A-I)=$

1) I - A 2) 2I-A 3) I + A 4) A-2I

24. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then $8A^{-4}=$

1) $145A^{-1}+27I$
2) $145A^{-1}-27I$
3) $27I-145A^{-1}$
4) $29A^{-1}+9I$

25. If x,y,z are in A.P with common difference d and the rank of matrix $\begin{pmatrix}4 & 5 & x\\ 5 & 6 & y\\ 6 & k & z\end{pmatrix}$ is 2 ,then values of d and k are

1) x,5 2) x/2,6 3) arbitrary no,7 4) x/4 ,7

26. The rank of a matrix $\begin{bmatrix}-1 & 2 & 5\\ 2 & -4 & a-4\\ 1 & -2 & a+1\end{bmatrix}$ is

1) 1, if a = 6 2) 2, if a = 1 3) 3, if a = 2 4) 1, if a = –6

27. The system of equations $(a\alpha+b)x+ay+bz=0;\ (b\alpha+c)x+by+cz=0;\ (a\alpha+b)y+(b\alpha+c)z=0$ has a non -trivial solution,if

1) a ,b,c are in A.P
2) a,b,c are in G.P
3) a ,b,c are in H.P
4) α is a root of $ax^2+2bx+c=0$

28. The system of equations –2x + y + z = a; x–2y + z = b; x + y –2z = c has

1) no solution if $a+b+c\neq 0$
2) unique solution if a+b+c=0
3) infinite number of solutions if a+b+c=0
4) none of these

29. $x+y=1;(c+2)x+(c+4)y=6;(c+2)^2x+(c+4)^2y=36$ are consistent then c=

1) 1 2) 2 3) 3 4) 4

30. If a,b and c are all different from zero such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$,then the matrix $A=\begin{pmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{pmatrix}$ is

1) symmetric
2) non-singular
3) can be written as sum of a symmetric and a skew symetric matrix
4) none of these

WORK SHEET (HW) - II

Linked - Comprehension Type

Passage - I :

Given $f(x)=\begin{vmatrix} 5+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 5+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 5+4\sin 2x \end{vmatrix}$, **then**

1. The domain of the function f(x)

   1) $(-\infty, 0)$ 2) $(-\infty, \infty)$ 3) $(0, \infty)$ 4) (0, 1)

2. The range of the function f(x)

   1) [50, 250] 2) [250, 250] 3) [50, 50] 4) None of these

3. The period of the function f(x)

   1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{8}$

Passage : II

Let $\Delta(x)=\begin{vmatrix} 2x^3-3x^2 & 5x+7 & 2 \\ 4x^3-7x & 3x+2 & 1 \\ 7x^3-8x^2 & x-1 & 3 \end{vmatrix}=a_0+a_1x+...+a_4x^4$ To evaluate a_i we differentiate $\Delta(x)$ i times w.r.t x and put x=0 or divide $\Delta(x)$ by x^4 put 1/x=t,differentiate (4-i) time w.r.t t and put t=0

4. a_0 equals

   1) 0 2) 1 3) 2 4) 3

5. a_1 equals

   1) 0 2) 61 3) 161 4) 191

6. a_4 equals

   1) 41 2) -43 3) -41 4) 43

Passage - III :

Consider the equation x + y + z = 6; x + 2y + 3z = 10; x + 2y + az = b

7. The system has no solution when

   1) a = 2, b $\neq$ 3 2) b = 2, a = 3 3) a = 3, b $\neq$ 10 4) b = 3, a $\neq$ 10

8. The system has a unique solution when

   1) a $\neq$ 2 whatever b may be 2) a $\neq$ 3 whatever 'b' may be
   3) b $\neq$ 2 whatever a may be 4) b $\neq$ 3 whatever a may be

9. The system possesses infinite number of solution when

1) a = 2, b = 10 2) a = 10, b = 2 3) a = 3, b = 10 4) a = 10, b = 3

Passage : IV

For $\alpha,\beta,\gamma,\theta \in R$ **,Let** $A_\theta(\alpha,\beta,\gamma)=\begin{vmatrix}\cos(\alpha+\theta) & \sin(\alpha+\theta) & 1\\ \cos(\beta+\theta) & \sin(\beta+\theta) & 1\\ \cos(\gamma+\theta) & \sin(\gamma+\theta) & 1\end{vmatrix}$

10 If $a=A_{\pi/2}(\alpha,\beta,\gamma), b=A_{\pi/3}(\alpha,\beta,\gamma)$ which of the following is true ?

1) a=b 2) a<b 3) a>b 4) 2a=b

11 $dA_\theta/d\theta$ when $\theta=\pi/6$ equals

1) -1 2) 0 3) 1 4) none of these

12 If $\alpha=\beta+2\pi/3,$ then A_θ is maximum when γ equals

1) $\alpha+\pi/3$ 2) $\alpha-\pi/3$ 3) $\alpha+2\pi/3$ 4) none of these

Passage : V

Consider the matrix A= $\begin{bmatrix}5 & -2 & 0\\ -2 & 6 & 2\\ 0 & 2 & 7\end{bmatrix}$ **and the homogeneous system** $AX=\lambda X, X\neq 0$

13 The number of distinct values of λ is

1) 1 2) 2 3) 3 4) 4

14 The sum of all the values of λ is

1) 12 2) 15 3) 18 4) 4

15 The product of all the values of λ is

1) 140 2) 144 3) 152 4) 162

Passage: VI :

Consider the matrix A= $\begin{bmatrix}8 & -8 & -2\\ 4 & -3 & -2\\ 3 & -4 & 1\end{bmatrix}$ **then**

16 $A^3-6A^2+10A-5I=$

1) 0 2) I+A 3) A-I d)I-A

17 $6A^{-1}=$

1) $A^2-4A+8I$ 2) $A^2-5A+9I$ 3) $A^2-6A+10I$ 4) $A^2-6A+11I$

18 $A^4=$

1) $25A^2-66A+42I$ 2) $25A^2-60A+36I$ 3) $25A^2+60A-42I$ 4) $25A^2+60A-36I$

WORK SHEET (HW) - III

Matrix matching

1 Let $p(x)=\begin{vmatrix} 3 & 3x & 3x^2+2 \\ 3x & 3x^2+2 & 3x^3+6x \\ 3x^2+2 & 3x^3+6x & 3x^4+12x^2+2 \end{vmatrix}$ $q(x)=2\begin{vmatrix} 1 & 1 & 1 \\ 2+x & 3+x & 4+x \\ (2+x)^2 & (3+x)^2 & (4+x)^2 \end{vmatrix}$

$r(x)=\begin{vmatrix} \cos(x+\pi/4) & \sin(x+\pi/4) & 1 \\ \cos(x+\pi/2) & \sin(x+\pi/2) & \sqrt{2} \\ \cos(x+3\pi/4) & \sin(x+3\pi/4) & 1 \end{vmatrix}$ and $s(x)=\begin{vmatrix} 1 & 2x & 2x+1 \\ 4x & 2x(2x-1) & 2x(2x+1) \\ 6x(2x-1) & 4x(2x-1)(x-1) & 2x(4x^2-1) \end{vmatrix}$

Match the expressions on the left with their properties /values on the right.

1) p(x)	p) is independent of x
2) q(x)	q) equals 4
3) r(x)	r) equals 0
4) s(x)	s) equals x^2-x+1

2 Suppose $f(x)$ is a function satisfying the following conditions

1) $f(o)=2$, $f(1)=1$ 2) $f(x)$ has a maximum at x=5/2 3) for all $x\in R$, and

$f'(x)=\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$ where a,b are constants, then

1) a	p) 1/2
2) b	q) -5/4
3) f(2)	r) 1/4
4) $f'(0)$	s) 1/8

3. Let A = $\begin{bmatrix} 0 & -\tan\alpha \\ \tan\alpha & 0 \end{bmatrix}$, $B(\alpha)=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

1) $(I+A)(I-A)^{-1}$	p) $A-B(\alpha)$
B) $(I-A)(I+A)^{-1}$	q) $B(2\alpha)$
3) $B(\alpha)^2$	r) $B(-2\alpha)$
4) $B(\alpha)^{-2}$	s) $AB(-\alpha)$

4. Let x and y be real numebrs .Match the determinants on the left with their values on the right

1) $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$ p) 4xy

2) $\begin{vmatrix} 1 & 2x & 2y \\ 1 & 2x+2y & 2y \\ 1 & 2x & 2x+2y \end{vmatrix}$ q) x+y+z

3) $\begin{vmatrix} 2 & x & y \\ 1 & 1+x & y \\ 1 & x & 1+y \end{vmatrix}$ r) $-2\left(x^3+y^3\right)$

4) $\begin{vmatrix} 1+x & y & y \\ 1 & x+y & 1 \\ x & x & 1+y \end{vmatrix}$ s) $-4(x+y)$

5. The number of real roots of the equation

1) $(x-1)(x-3)(x-5)+\frac{2}{3}(x-2)(x-4)(x-6)=0$ p) 0

2) $\begin{vmatrix} (x+1)(x+2) & x+2 & 1 \\ (x+2)(x+3) & x+3 & 1 \\ (x+3)(x+4) & x+4 & 1 \end{vmatrix}=0$ q) 1

3) $\begin{vmatrix} x & 2 & 6 \\ x+2 & 4 & 2-x \\ x-4 & 4+x & 2 \end{vmatrix}=0$ r) 2

4) $\begin{vmatrix} 1-x & 4-x & 2 \\ 1-x & 2 & 4-x \\ 1 & 4-x & 2-x \end{vmatrix}=0$ s) 3

6. Let $\Delta_1=\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$, $\Delta_2=\begin{vmatrix} 1 & y & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$,

$\Delta_3=\begin{vmatrix} x & x+y & x+2y \\ -x & x & 0 \\ 0 & -x & x \end{vmatrix}$, $\Delta_4=\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & 1 & 1 \end{vmatrix}$ then

1) Δ_1 p) $3x^2(x+y)$

2) Δ_2 q) xy

3) Δ_3 r) $-2(x^3+y^3)$

4) Δ_4 s) $(x-4)(y-1)(1-x)$

WORK SHEET (HW) - IV

Integer type answer

1 If a matrix A given by $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$ r =1,2,3,...10 and $|A_1|+|A_2|+|A_3|+......|A_{10}| = P$ then $E_3(P!)$ = 16k, then 'k' is

2. If the number of idempotent diagonal matrices of order 4. by taking the elements 0,1 is 4p then'p' is

3. The set of natural numbers N is partitioned into arrays of rows and columns in the form of matrices as $M_1 = (1),\ M_2 = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix},\ M_3 = \begin{pmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{pmatrix} ....M_n = (\ )$ and so on. Then the unit digit in the sum of the elements of the diagonal in M_6 is

4. If $f_r^{||}(x) = a,\ a \in R$ where r = 1,2,3 then the degree of g(x) = $\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f'_1(x) & f_2'(x) & f_3'(x) \\ f_1^{||}(x) & f_2^{n}(x) & f_3^{n}(x) \end{vmatrix}$ is

5. Given f(x)=$\log_{10}$x and g(x)= $e^{\pi i x}$ and ϕ (x) = $\begin{vmatrix} f(x).g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2).g(x^2) & [f(x)^2]^{g(x^2)} & 0 \\ f(x^3).g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$ then the value of ϕ (10) is

EXERCISE - II / ANSWERS

WORK SHEET (HW)- I

1) 1 2) 4 3) 2 4) 4 5) 1234 6) 3 7) 4 8) 4 9) 3 10) 2

11) 3 12) 124 13) 1234 14) 3 15) 4 16) 1 17) 1234 18) 23 19) 2 20) 1

21) 2 22) 2 23) 2 24) 3 25) 3 26) 24 27) 24 28) 13 29) 24 30) 123

WORK SHEET (HW) - II

1) 2 2) 1 3) 1 4) 1 5) 3 6) 2 7) 3 8) 2 9) 3 10) 1

11)2 12)4 13)3 14)3 15)4 16)4 17)4 18)2

WORK SHEET (HW) - III

1) $1 \to p,q; 2 \to p,q; 3 \to p,r; 4 \to p,r$

2) $1 \to r; 2 \to q; 3 \to p; 4 \to q$

3) $1 \to r; 2 \to q; 3 \to q; 4 \to s$

4) $1 \to r, 2 \to p, 3 \to q, 4 \to p$

5) $1 \to s, 2 \to q, 3 \to q, 4 \to s$

6) $1 \to r, 2 \to q, 3 \to p, 4 \to s$

WORK SHEET (HW) - IV

1) 3 2) 4 3) 1 4) 0 5) 0