CIRCULAR MOTION

KINEMATICS OF CIRCULAR MOTION :

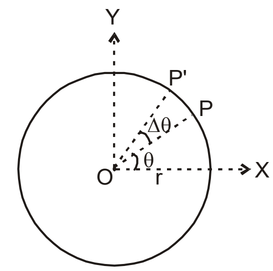
Variables of Motion :

(a) Angular Position :

The angle made by the position vector w.r.t. origin, with the reference line is called angular posi tion. Clearly angular position depends on the choice of the origin as well as the reference line.

(b) Angular Displacement :

Definition: Angle through which the position vector of the moving particle rotates in a given time interval is called its angular displacement. Angular displacement depends on origin, but it does not depends on the reference line. As the particle moves on above circle its angular position θ changes. Suppose the point rotates through an angle $\Delta\theta$ in time Δt, then $\Delta\theta$ is angular displacement.



- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.

(c) Angular Velocity ω

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Angular displacement}}{\text{Total time taken}}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2.

(ii) Instantaneous Angular Velocity

It is the limit of average angular velocity as Δt approaches zero. i.e.

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

Since infinitesimally small angular displacement $\overrightarrow{d\theta}$ is a vector quantity, instantaneous angular velocity $\vec{\omega}$ is also a vector, whose direction is given by right hand thumb rule.

☞ **Important points :**

- Angular velocity has dimension of $[T^{-1}]$ and SI unit rad/s.
- If a body makes 'n' rotations in 't' seconds then average angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t} \Rightarrow \omega_{av} = \frac{2\pi}{T} = 2\pi f$$

where T is the period and 'f' the frequency of uniform circular motion

Solved Examples

Example. If angular displacement of a particle is given by $\theta = a - bt + ct^2$, then find its angular velocity.

Solution : $\omega = \dfrac{d\theta}{dt} = -b + 2ct$

(d) Angular Acceleration α :

(i) Average Angular Acceleration :

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{\Delta t}$$

(ii) Instantaneous Angular Acceleration :

It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

since $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$, $\therefore\ \vec{\alpha} = \dfrac{d\vec{\omega}}{dt} = \dfrac{d^2\vec{\theta}}{dt^2}$, Also $\vec{\alpha} = \omega\dfrac{d\vec{\omega}}{d\theta}$

Important points :

- Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rad/s^2.
- If $\alpha = 0$, circular motion is said to be uniform.

Motion with constant angular velocity

$\theta = \omega t$, $\alpha = 0$

Motion with constant angular acceleration

$\omega_0 \Rightarrow$ Initial angular velocity
$\omega \Rightarrow$ Final angular velocity
$\alpha \Rightarrow$ Constant angular acceleration
$\theta \Rightarrow$ Angular displacement

Circular motion with constant angular acceleration is analogous to one dimensional translational motion with constant acceleration. Hence even here equation of motion have same form.

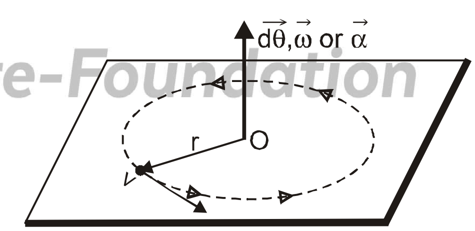
$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\,\theta$$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$$

$$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(\theta_n - \theta_{n-1})$$



RELATION BETWEEN VELOCITY AND ANGULAR VELOCITY :

$\vec{v} = \vec{\omega} \times \vec{r}$

Here, $\vec{v}$ is velocity of the particle, $\vec{\omega}$ is angular velocity about centre of circular motion and '$\vec{r}$' is position of particle w.r.t. center of circular motion.

Since $\vec{\omega} \perp \vec{r}$

v = ω r for circular motion.

Solved Examples

Example . A particle is moving with constant speed in a circular path. Find the ratio of average velocity to its instantaneous velocity when the particle describes an angle $\theta = \frac{\pi}{2}$

Solution : Time taken to describe angle θ, $t = \frac{\theta}{\omega} = \frac{\theta R}{v} = \frac{\pi R}{2v}$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{\sqrt{2}R}{\pi R/2v} = \frac{2\sqrt{2}}{\pi}v$

Instantaneous velocity = v

The ratio of average velocity to its instantaneous velocity = $\frac{2\sqrt{2}}{\pi}$ **Ans.**

Example . A fan is rotating with angular velocity 100 rev/sec. Then it is switched off. It takes 5 minutes to stop. (a) Find the total number of revolution made before it stops. (Assume uniform angular retardation) (b) Find the value of angular retardation (c) Find the average angular velocity during this interval.

Solution : (a) $\theta = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{100+0}{2}\right) \times 5 \times 60 = 15000$ revolution.

(b) $\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 0 = 100 - \alpha\,(5 \times 60) \quad \Rightarrow \alpha = \frac{1}{3}$ rev./sec^2

(c) $\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} = \frac{15000}{50 \times 60} = 50$ rev./sec.

RADIAL AND TANGENTIAL ACCELERATION

There are two types of acceleration in circular motion ; Tangential acceleration and centripetal acceleration.

(a) Tangential acceleration :-

Component of acceleration directed along tangent of circle is called tangential acceleration. It is responsible for changing the speed of the particle. It is defined as,

$a_t = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt}$ = Rate of change of speed.

$a_t = \alpha r$

☞ **IMPORTANT POINT**

(i) In vector form $\vec{a}_t = \vec{\alpha} \times \vec{r}$

(ii) If tangential acceleration is directed in direction of velocity then the speed of the particle increases.

(iii) If tangential acceleration is directed opposite to velocity then the speed of the particle decreases.

(b) Centripetal acceleration :-

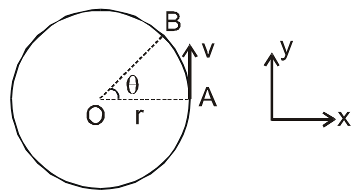
It is responsible for change in direction of velocity. In circular motion, there is always a centripetal acceleration.

Centripetal acceleration is always variable because it changes in direction.

Centripetal acceleration is also called radial acceleration or normal acceleration.

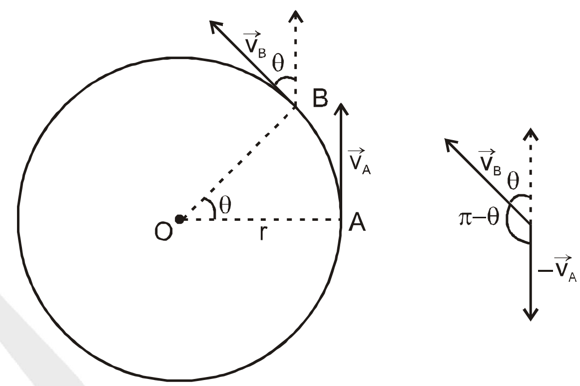
Calculation of centripetal acceleration :

Consider a particle which moves in a circle with constant speed v as shown in figure.



∴ change in velocity between the point A and B is ;

$$\Delta \vec{v} = \vec{v}_B - \vec{v}_A$$



Magnitude of change in velocity.

$$|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 + 2v_A v_B \cos(\pi - \theta)}$$

($v_A = v_B = v$, since speed is same)

$$\therefore |\Delta \vec{v}| = 2v \sin\frac{\theta}{2}$$

Distance travelled by particle between A and B = $r\theta$

Hence time taken, $\Delta t = \dfrac{r\theta}{v}$

Net acceleration , $|\vec{a}_{net}| = \left|\dfrac{\overrightarrow{\Delta v}}{\Delta t}\right| = \dfrac{2v\sin\theta/2}{r\theta/v} = \dfrac{v^2}{r}\dfrac{2\sin\theta/2}{\theta}$

If $\Delta t \to 0$, then θ is small, $\sin(\theta/2) = \theta/2$

$$\lim_{\Delta t\to 0}\left|\frac{\Delta\vec{v}}{\Delta t}\right| = \left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{r}$$

i.e. net acceleration is $\dfrac{v^2}{r}$ but speed is constant so that tangential acceleration, $a_t = \dfrac{dv}{dt} = 0$.

$$\therefore a_{net} = a_r = \frac{v^2}{r}$$

** Through we have derived the formula of centripetal acceleration under condition of constant speed, the same formula is applicable even when speed is variable.

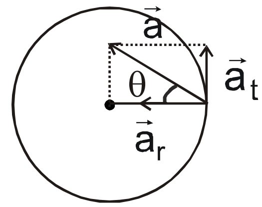
☞ **IMPORTANT POINT**

In vector form $\vec{a}_c = \vec{\omega} \times \vec{v}$

(c) Total acceleration :-

Total acceleration is vector sum of centripetal acceleration and tangential acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_r + \vec{a}_t$$



$$a = \sqrt{a_t^2 + a_r^2}$$

$$\tan\theta = \frac{a_r}{a_t}$$

☞ **IMPORTANT POINT**

$\left|\frac{d\vec{v}}{dt}\right|$ & $\frac{d|\vec{v}|}{dt}$ are not same physical quantity. $\left|\frac{d\vec{v}}{dt}\right|$ is the magnitude of rate of change of velocity, i.e. magnitude of total acceleration and $\frac{d|\vec{v}|}{dt}$ is a rate of change of speed, i.e. tangential acceleration.

Solved Examples

Example. The speed of a particle traveling in a circle of radius 20 cm increases uniformly from 6.0 m/s to 8.0 m/s in 4.0 s, find the angular acceleration.

Solution : Since speed increases uniformly, average tangential acceleration is equal to instantaneous tangential acceleration

$\therefore$ The instantaneous tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{8.0 - 6.0}{4.0}\ \text{m/s}^2 = 0.5\ \text{m/s}^2.$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0.5\,\text{m/s}^2}{20\,\text{cm}} = 2.5\ \text{rad/s}^2.$$

Example. A particle is moving in a circle of radius 10 cm with uniform speed completing the circle in 4s, find the magnitude of its acceleration.

Solution : The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$$v = 2\pi r/t = \frac{2\pi \times 10\,\text{cm}}{4\text{s}} = 5\pi\ \text{cm/s}.$$

The acceleration is $a = \frac{v^2}{r} = \frac{(5\pi\,\text{cm/s})^2}{10\,\text{cm}} = 2.5\pi^2\ \text{cm/s}^2.$

Example. A particle moves in a circle of radius 2.0 cm at a speed given by v = 4t, where v is in cm/s and t is in seconds.

(a) Find the tangential acceleration at t = 1s.

(b) Find total acceleration at t = 1s.

Solution : (a) Tangential acceleration

$$a_t = \frac{dv}{dt} \quad \text{or} \quad a_t = \frac{d}{dt}(4t) = 4\ \text{cm/s}^2$$

$$a_c = \frac{v^2}{R} = \frac{(4)^2}{2} = 8\ \text{cm/s}^2 \quad \Rightarrow \quad a = \sqrt{a_t^2 + a_C^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}\ \text{cm/s}^2$$

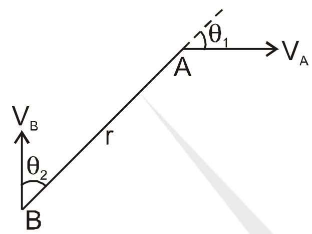
RELATIVE ANGULAR VELOCITY

Angular velocity of a particle ‘A’ with respect to the other moving particle ‘B’ is the rate at which position vector of ‘A’ with respect to ‘B’ rotates at that instant. (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B, ω_{AB} is mathematically define as

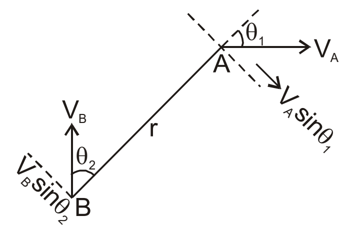
$$\omega_{AB} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicular to line}}{\text{seperation between A and B}} = \frac{(V_{AB})_\perp}{r_{AB}}$$

Solved Examples

Example. Find the angular velocity of A with respect to B in the figure given below:



Solution : Angular velocity of A with respect to B ;



$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

$\Rightarrow \quad (V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$

$\Rightarrow \quad r_{AB} = r$

$$\omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

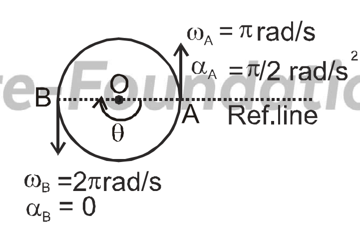
Example. Two particle A and B move on a circle. Initially Particle A and B are diagonally opposite to each other. Particle A move with angular velocity π rad/sec. , angular acceleration $\pi/2$ rad/sec^2 and particle B moves with constant angular velocity 2π rad/sec. Find the time after which both the particle A and B will collide.

Solution : Suppose angle between OA and OB = θ

then, rate of change of θ,

$\dot{\theta} = \omega_B - \omega_A = 2\pi - \pi = \pi$ rad/sec

$\ddot{\theta} = \alpha_B - \alpha_A = -\dfrac{\pi}{2}$ rad/sec^2



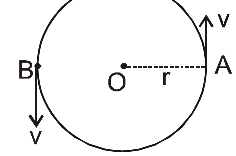
If angular displacement is $\Delta\theta$,

$$\Delta\theta = \dot{\theta}t + \frac{1}{2}\ddot{\theta}t^2$$

for A and B to collide angular displacement $\Delta\theta = \pi$

$\Rightarrow \quad \pi = \pi t + \dfrac{1}{2}\left(\dfrac{-\pi}{2}\right)t^2 \quad \Rightarrow \quad t^2 - 4t + 4 = 0 \quad \Rightarrow \quad t = 2$ sec. **Ans.**

Example. Particles A and B move with constant and equal speeds in a circle as shown, find the angular velocity of the particle A with respect to B, if angular velocity of particle A w.r.t. O is ω.



Solution : Angular velocity of A with respect to O is ; $\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$

Now, $\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$ $\Rightarrow$ $v_{AB} = 2v$,

since v_{AB} is perpendicular to r_{AB},

$\therefore$ $(v_{AB})_\perp = v_{AB} = 2v$; $r_{AB} = 2r \Rightarrow$ $\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{2v}{2r} = \omega$

DYNAMICS OF CIRCULAR MOTION :

If there is no force acting on a body it will move in a straight line (with constant speed). Hence if a body is moving in a circular path or any curved path, there must be some force acting on the body.

If speed of body is constant, the net force acting on the body is along the inside normal to the path of the body and it is called centripetal force.

Centripetal force $(F_c) = ma_c = \frac{mv^2}{r} = m\,\omega^2 r$

However if speed of the body varies then, in addition to above centripetal force which acts along inside normal, there is also a force acting along the tangent of the path of the body which is called tangential force.

Tangential force $(F_t) = Ma_t = M\frac{dv}{dt} = M\,\alpha\, r$; where α is the angular acceleration

☞ **IMPORTANT POINT**

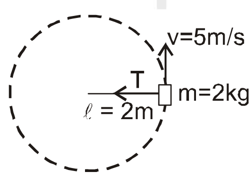
Remember $\frac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal which is responsible for circular motion. This force may be friction, normal, tension, spring force, gravitational force or a combination of them.

So to solve any problem in uniform circular motion we identify all the forces acting along the normal (towards center), calculate their resultant and equate it to $\frac{mv^2}{r}$.

Solved Examples

Example. A block of mass 2kg is tied to a string of length 2m, the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed 5 m/s. Find the tension in the string.

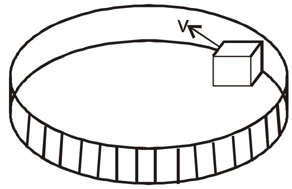
Solution :



here centripetal force is provided by tension.

$T = \frac{mv^2}{r} = \frac{2\times 5^2}{2} = 25$ N

Example. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



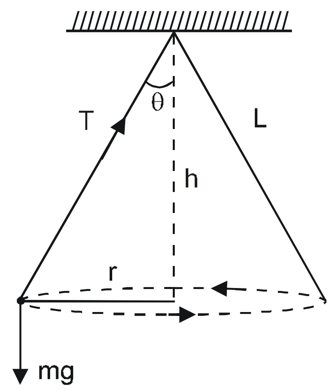
Find :
(i) normal reaction of the floor on the block.
(ii) normal reaction of the vertical wall on the block.

Solution : Here centripetal force is provided by normal reaction of vertical wall.
(i) normal reaction of floor $N_F = mg$

(ii) normal reaction of vertical wall $N_W = \dfrac{mv^2}{r}$.

Example. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.

Solution : The situation is shown in figure. The angle θ made by the string with the vertical is given by



$\sin\theta = r / L$, $\cos\theta = h/L = \dfrac{\sqrt{L^2 - r^2}}{L}$ ...(i)

The forces on the particle are
(a) the tension T along the string and
(b) the weight mg vertically downward.

The particle is moving in a circle with a constant speed v. Thus , the radial acceleration towards the centre has magnitude v^2 / r. Resolving the forces along the radial direction and applying Newton's second law,

$T\sin\theta = m(v^2 / r)$ ....(ii)

As there is no acceleration in vertical direction, we have from Newton's law,

$T\cos\theta = mg$ .....(iii)

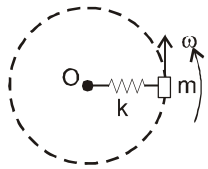
Dividing (ii) by (iii),

$$\tan\theta = \frac{v^2}{rg} \quad \text{or,} \quad v = \sqrt{rg\tan\theta}$$

$$\Rightarrow \quad \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{L\cos\theta}} = \sqrt{\frac{g}{(L^2 - r^2)^{\frac{1}{2}}}} \quad \textbf{Ans.}$$

And from (iii), $T = \dfrac{mg}{\cos\theta} = \dfrac{mgL}{(L^2 - r^2)^{\frac{1}{2}}}$ **Ans.**

Example. A block of mass m is tied to a spring of spring constant k , natural length ℓ, and the other end of spring is fixed at O. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity ω , find tension in the spring.



Solution : Assume extension in the spring is x

Here centripetal force is provided by spring force.

Centripetal force, $kx = m\omega^2(\ell + x)$

$$\Rightarrow \qquad x = \frac{m\omega^2\ell}{k - m\omega^2}$$

therefore,

Tension = $kx = \dfrac{km\omega^2\ell}{k - m\omega^2}$ **Ans.**

Example. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its smooth surface and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

Solution : Let ω be the angular speed of rotation of the bowl. Two force are acting on the ball.

1. Normal reaction N 2. weight mg

The ball is rotating in a circle of radius r (= R sin α) with centre at A at an angular speed ω. Thus,

$N \sin\alpha = mr\omega^2 = mR\omega^2 \sin\alpha$

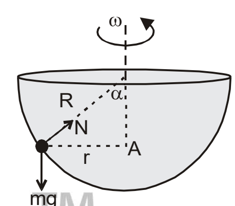
$N = mR\omega^2$ ....(i)

and $N \cos\alpha = mg$ ...(ii)

Dividing Eqs. (i) by (ii),

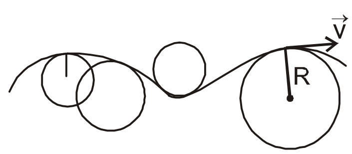
we get $\dfrac{1}{\cos\alpha} = \dfrac{\omega^2 R}{g}$

$\therefore \qquad \boldsymbol{\omega = \sqrt{\dfrac{g}{R\cos\alpha}}}$



RADIUS OF CURVATURE

Any curved path can be assumed to be made of infinite circular arcs. Radius of curvature at a point is the radius of the circular arc at a particular point which fits the curve at that point.



If R is radius of the circular arc at a given point P, where velocity is $\vec{v}$, then centripetal force at that point is,

$$F_c = \frac{mv^2}{R} \qquad \Rightarrow \qquad R = \frac{mv^2}{F_c}$$

Now centripetal force F_c is simply the component of force perpendicular to velocity (let us say $F_\perp$).

$$\therefore \quad R = \frac{mv^2}{F_\perp} \quad \Rightarrow \quad R = \frac{v^2}{a_\perp}$$

Where, $a_\perp$ = Component of acceleration perpendicular to velocity.

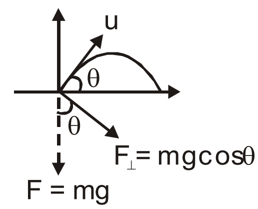
☞ If a particle moves in a trajectory given by $y = f(x)$ then radius of curvature at any point (x, y) of the trajectory is given by $\Rightarrow$

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Solved Examples

Example. A particle of mass m is projected with speed u at an angle θ with the horizontal. Find the radius of curvature of the path traced out by the particle at the point of projection and also at the highest point of trajectory.

Solution : at point of projection



$$R = \frac{mv^2}{F_\perp} = \frac{mu^2}{mg\cos\theta}$$

$$R = \frac{u^2}{g\cos\theta} \quad \textbf{Ans.}$$

at highest point

$$a_\perp = g \ , \ v = u\cos\theta \ : \ R = \frac{v^2}{a_\perp} = \frac{u^2\cos^2\theta}{g} \quad \textbf{Ans.}$$

CIRCULAR TURNING ON ROADS :

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. **By friction only**
2. **By banking of roads only.**
3. **By friction and banking of roads both.**

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

Thus, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN

or $f_L = \mu N = \mu mg \quad (N = mg)$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$

or $\frac{mv^2}{r} \le \mu mg$ or $\mu \ge \frac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

Solved Examples

Example. A bend in a level road has a radius of 100 m. Calculate the maximum speed which a car turning this bend may have without skidding. Given : μ = 0.8.

Solution : $V_{max} = \sqrt{\mu rg} = \sqrt{0.8 \times 100 \times 10} = \sqrt{800}$ = 28 m/s

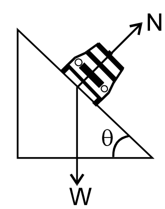
By Banking of Roads Only

Friction is not always reliable at circular turns if high speeds and sharp turns are involved to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction.

$$N \sin\theta = \frac{mv^2}{r}$$

or $$N \cos\theta = mg$$



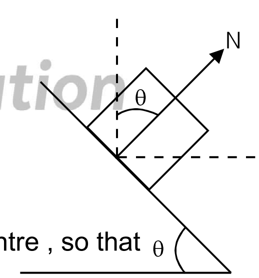
from these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{rg\tan\theta}$$

Solved Examples

Example. What should be the angle of banking of a circular track of radius 600 m which is designed for cars at an average speed of 180 km/hr ?

Solution : Let the angle of banking be θ. The forces on the car are (figure)



(a) weight of the car Mg downward and

(b) normal force N.

For proper banking , static frictional force is not needed.

For vertical direction the acceleration is zero. So,

$N\cos\theta = Mg$ .....(i)

For horizontal direction , the acceleration is v^2 / r towards the centre , so that

$N\sin\theta = Mv^2 / r$ .....(ii)

From (i) and (ii), $\tan\theta = v^2 / rg$

Putting the values , $\tan\theta = \frac{180(km/h)^2}{(600m)(10m/s^2)}$ = 0.4167 $\Rightarrow$ θ = 22.6º.

By Friction and Banking of Road Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.

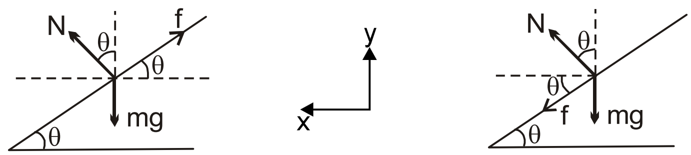


Figure (i) Figure (ii)

The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component of weight mg $\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if

$$v > \sqrt{rg\tan\theta}$$

(iii) Friction f will be outwards if

$$v < \sqrt{rg\tan\theta} \text{ and}$$

(iv) Friction f will be zero if

$$v = \sqrt{rg\tan\theta}$$

(v) For maximum safe speed (figure (ii)

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \quad \text{........(i)}$$

$$N\cos\theta - f\sin\theta = mg \quad \text{.......(ii)}$$

As maximum value of friction

$$f = \mu N$$

$$\therefore \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \quad \therefore \quad v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

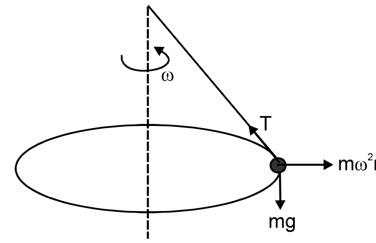
Similarly ; $v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$

CENTRIFUGAL FORCE :

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body which is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force. $= \frac{mv^2}{r} = m\omega^2 r$

Direction of centrifugal force, it is always directed radially outward.

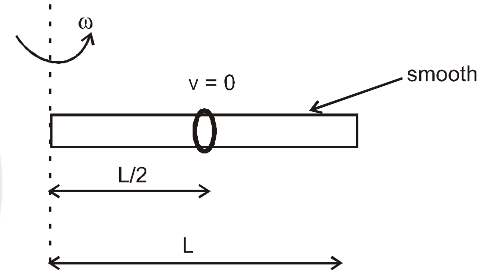


Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame. FBD of ball w.r.t. non inertial frame rotating with the ball.

Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyses the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

Solved Examples

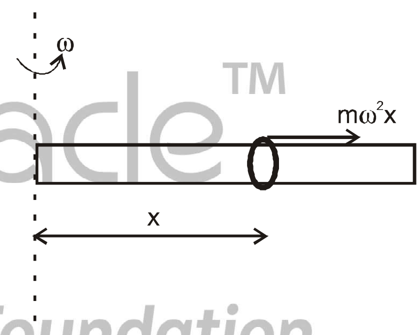
Example. A ring which can slide along the rod are kept at mid point of a smooth rod of length L. The rod is rotated with constant angular velocity ω about vertical axis passing through its one end. Ring is released from mid point. Find the velocity of the ring when it just leave the rod.



Solution : Centrifugal force

$m\omega^2 x = ma$

$\omega^2 x = \frac{vdv}{dx}$



$\int_{L/2}^{L} \omega^2 x\, dx = \int_0^v v\, dv$ (integrate both side.)

$\omega^2 \left(\frac{x^2}{2}\right)_{L/2}^{L} = \left(\frac{v^2}{2}\right)_0^v$

$\omega^2 \left(\frac{L^2}{2} - \frac{L^2}{8}\right) = \frac{v^2}{2} \quad \Rightarrow \quad v = \frac{\sqrt{3}}{2}\omega L.$

Velocity at time of leaving the rod

$v' = \sqrt{(\omega L)^2 + \left(\frac{\sqrt{3}}{2}\omega L\right)^2} = \frac{\sqrt{7}}{2}\omega L$ **Ans.**

EXERCISE – 1
BUILDING A FOUNDATION

SECTION-A KINEMATICS OF CIRCULAR MOTION

A-1. A wheel is at rest. Its angular velocity increases uniformly and becomes 80 radian per second after 5 second. The total angular displacement is:
(a) 800 rad (b) 400 rad (c) 200 rad (d) 100 rad

A-2. When a particle moves in a circle with a uniform speed
(a) Its velocity and acceleration are both constant
(b) Its velocity is constant but the acceleration changes
(c) Its acceleration is constant but the velocity changes
(d) Its velocity and acceleration both change

A-3. A wheel is of diameter 1m. If it makes 30 revolutions/sec., then the linear speed of a point on its circumference will be.
(a) 30π m/s (b) π m/s (c) 60 π m/s (d) π/2 m/s

A-4. The angular speed of a fly wheel making 120 revolutions/minute is.
(a) 2 π rad/s (b) $4\pi^2$ rad/s (c) π rad/s (d) 4π rad/s

A-5. The angular velocity of the second's needle in watch is-
(a) $\frac{\pi}{30}$ (b) 2 π (c) π (d) $\frac{60}{\pi}$

A-6. A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this:
(a) its velocity is changed in magnitude (b) its velocity is not changed
(c) its speed only is changed (d) its velocity is changed

A-7. Angular displacement of any particle is given q = w_0t + αt^2 where w_0 & α are constant if w_0 = 1 rad/sec, α = 1.5 rad/sec^2 then in t = 2 sec. angular velocity will be (in rad/sec)
(a) 1 (b) 5 (c) 3 (d) 4

A-8. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:
(a) 160 π m/s^2 (b) 40 m/s^2 (c) 40 π m/s^2 (d) 640 π m/s^2

A-9. An aeroplane revolves in a circle above the surface of the earth at a fixed height with speed 100 km/hr. The change in velocity after completing half revolution will be
(a) 200 km/hr (b) 150 km/hr (c) 300 km/hr (d) 400 km/hr

A-10. Two cars of masses m_1 and m_2 are moving along the circular paths of radius r_1 and r_2 respectively. The speed are such that they complete one round at the same time. The ratio of angular speeds of the two cars is
(a) $m_1 : m_2$ (b) $r_1 : r_2$ (c) 1 : 1 (d) $m_1r_1 : m_2r_2$

A-11. A wheel is of diameter 1m. If it makes 30 revolution/sec. then the linear speed of a point on its circumference will be
(a) 30π m/s (b) π m/s (c) 60π m/s (d) $\pi/2$ m/s

A-12. The angular velocity of earth's rotation about its axis is
(a) $\frac{12}{\pi}$ rad/hr (b) $\frac{\pi}{12}$ rad/hr (c) 48π rad/hr (d) 0.5 degree/min

A-13. If a particle moves on a circle, describing equal angle in equal time intervals, the velocity vector
(a) remains constant (b) changes in magnitude
(c) changes in direction (d) changes both in magnitude and direction

A-14. The ratio of angular speed of hours hand and seconds hand of a clock is
(a) 1 : 1 (b) 1 : 60 (c) 1 : 720 (d) 3600 : 1

A-15. Two particles having mass M and m are moving in a circular path having radius R and r. If their periods are same, then the ratio of angular velocity will be
(a) r/R (b) R/r (c) 1 (d) $\sqrt{R/r}$

A-16. The magnitude of the centripetal force acting on the particle of mass m executing uniform motion in a circle of radius R and with speed v is
(a) mvR (b) $\frac{mv^2}{R}$ (c) $\frac{v^2}{mR}$ (d) $\frac{v}{mR}$

SECTION-B RADIAL AND TENGENTIAL ACCELERATION

B-1. Let a_r and a_t represent radial and tangential acceleration. The motion of a particle may be circular if :
(a) $a_r = 0, a_t = 0$ (b) $a_r = 0, a_t \neq 0$ (c) $a_r \neq 0, a_t = 0$ (d) none of these

B-2. The formula for centripetal acceleration in a circular motion is.
(a) $\vec{\alpha} \times \vec{r}$ (b) $\vec{\omega} \times \vec{v}$ (c) $\vec{\alpha} \times \vec{v}$ (d) $\vec{\omega} \times \vec{r}$

B-3. In the uniform circular motion
(a) Acceleration and velocity both remains constant
(b) Acceleration and speed both remains constant
(c) Acceleration and velocity both keep on changing
(d) Acceleration and speed both change

B-4. The quantity may be constant in circular motion is
(a) Linear speed (b) Centripetal force (c) Acceleration (d) Momentum

B-5. A body is moving with a constant speed v in a circle of radius r. Its angular acceleration is
(a) zero (b) $\frac{v}{r}$ (c) $\left(\frac{v}{r}\right)^2$ (d) $\frac{v^2}{r}$

B-6. A particle is performing non uniform circular motion then :
(a) velocity and acceleration are in the same direction
(b) angular velocity and angular acceleration must be along same line
(c) radial acceleration and total acceleration are in the same direction

(d) tangential acceleration & total force on the particle are in same direction

SECTION-C CIRCULAR MOTION IN HORIZONTAL PLANE

C-1. A string breaks if its tension exceeds 10 newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be.
(a) 20 rad/s (b) 40 rad/s (c) 100 rad/s (d) 200 rad/s

C-2. A particle moving along a circular path due to a centripetal force having constant magnitude is an example of motion with :
(a) constant speed and velocity (b) variable speed and velocity
(c) variable speed and constant velocity (d)Constant speed and variable velocity

C-3. A stone of mass 0.5 kg tied with a string of length 1 metre is moving in a circular path with a speed of 4 m/sec. The tension acting on the string in newton is -
(a) 2 (b) 8 (c) 0.2 (d) 0.8

C-4. A stone is moved round a horizontal circle with a 20 cm long string tied to it. If centripetal acceleration is 9.8 m/sec2, then its angular velocity will be
(a) 7 rad/s (b) 22/7 rad/s (c) 49 rad/s (d) 14 rad/s

C-5. A particle of mass m is executing a uniform motion along a circular path of radius r. If the magnitude of its linear momentum is p, the radial force acting on the particle will be.
(a) pmr (b) rm/p (c) mp^2/r (d) p^2/mr

C-6. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to –k/r2. The total kinetic energy of the particle is-
(a) –k/r (b) k/r (c) k/2r (d) –k/2r

C-7. A particle is moving in a horizontal circle with constant speed. It has constant
(a) Velocity (b) Acceleration (c) Kinetic energy (d) Displacement

C-8. A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be.
(a) Double (b) Half (c) 4 times (d) 1/4 times

C-9. When a body moves with a constant speed along a circle
(a) No work is done (b) No acceleration is produced in the body
(c) No force acts on the body (d) Its velocity remains constant

C-10. A particle under the action of force F moves in a circular path of radius r with a constant speed. Its speed will be
(a) $\sqrt{\frac{rF}{m}}$ (b) $\sqrt{mrF}$ (c) $\sqrt{\frac{F}{r}}$ (d) $\frac{F}{mr}$

C-11. A body of mass 100 gm is tied to one end of 2m long string. The other end of the string is at the centre of the horizontal circle. The maximum revolution in one minute is 200. The maximum tensile strength in the string is approximately

(a) 8.942 dyne (b) 8.942 N (c) 89.42 dyne (d) 89.42 N

C-12. A stone is moved round a horizontal circle with a 20 cm long string tied to it. If centripetal acceleration is 9.8 m/s^2, then its angular velocity will be
(a) 7 rad/s (b) 22/7 rad/s (c) 49 rad/s (d) 14 rad/s

C-13. A mass of 2 Kg is whirled in a horizontal circle by means of a string with initial speed 5 revolution per minute. Keeping the radius constant the tension in the string is doubled. The new speed is nearly
(a) 14 rpm (b) 10 rpm (c) 2.25 rpm (d) 7 rpm

C-14. A string breaks if its tension exceeds 10 Newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be
(a) 20 rad/s (b) 40 rad/s (c) 100 rad/s (d) 200 rad/s

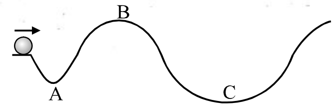
C-15. A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now
(a) T_0 (b) $T_0/2$ (c) $4T_0$ (d) $8T_0$

C-16. A car of mass m is taking circular turn of radius r on a rough horizontal road with a speed v. In order that the car does not skid
(a) $\frac{mv^2}{r} \geq \mu mg$ (b) $\frac{mv^2}{r} \leq \mu mg$ (c) $\frac{mv^2}{r} = \mu mg$ (d) $\frac{v}{r} = \mu mg$

C-17. Two coins are placed on a rough horizontal disc of radius R. Distances of coins are r_1 and r_2 from the centre of disc ($r_1 < r_2$). Disc starts rotating about its centre with a constant angular acceleration. After some time
(a) the coin which is closer from centre of disc will slip first
(b) the coin which is farther from centre of disc will slip first
(c) both the coins will start slipping simultaneously
(d) massive coin will start slipping first, irrespective of the distance

C-18. A motorcycle is going on an over bridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the over bridge, the normal force on it
(a) increases (b) decreases (c) remains the same (d) fluctuates

C-19. A body moves along an uneven horizontal road surface with constant speed at all points. The normal reaction of the road on the body is :



(a) maximum at A (b) maximum at B
(c) maximum at C (d) the same at A, B & C

C-20. The roadway of a bridge over a canal in the form of circular arc of radius 18 m. What is the greatest constant speed with which a motor bike can cross the bridge without leaving ground?
(a) $\sqrt{9.8}\ m/s$ (b) $\sqrt{9.8 \times 18}\ m/s$ (c) $9.8 \times 18\ m/s$ (d) $\frac{18}{9.8}\ m/s$

SECTION-D BANKING OF ROAD

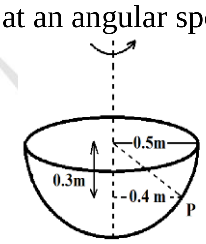
D-1. Maximum safe speed on a rough banked road of coefficient of friction 0.2 and angle of banking is 45^0, made for a turn of radius of curvature 60 m is

(a) 25 m/s (b) 30 m/s (c) 35 m/s (d) 40 m/s

D-2. Minimum safe speed on a rough banked road of coefficient of friction 0.2 and angle of banking is 45^0, made for a turn of radius of curvature 60 m is

(a) 15 m/s (b) 20 m/s (c) 25 m/s (d) 50 m/s

D-3. A particle will be equilibrium inside a hemispherical bowl of radius 0.5 m at a height 0.2 m from the bottom when the bowl is rotated at an angular speed, ($g = 10\ m/s^2$)



(a) $\frac{10}{\sqrt{3}}$ rad/sec (b) $10\sqrt{3}$ rad/sec (c) 10 rad/sec (d) $\sqrt{20}$ rad/sec

SECTION-E RADIUS OF CURVATURE

E-1. A particle of mass m is moving with constant velocity v on smooth horizontal surface. A constant force F starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is:

(a) $\frac{mv^2}{F}$ (b) $\frac{mv^2}{F\cos\theta}$ (c) $\frac{mv^2}{F\sin\theta}$ (d) none of these

E-2. If the radii of circular paths of two particles of same masses are in the ratio of 1:2, then in order to have same centripetal force, their speeds should be in the ratio of :

(a) 1 : 4 (b) 4 : 1 (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

E-3. A particle is projected at 45° with horizontal. Its radius of curvature when it reaches to the top most point is :

(a) equal to max height of projectile
(b) equal to range of projectile
(c) equal to twice of maximum height of projectile
(d) equal to twice of the range of projectile

E-4. A projectile is fired with speed 40 m/s at an angle 60^0 with horizontal. The radius of curvature of path of projectile at highest point is ($g = 10\ m/s^2$)

(a) 40 m (b) 20 m (c) 10 m (d) None of these

EXERCISE – II
READY FOR CHALLENGES

1. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane . Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the pivoted ends.
(a) $T_1 > T_2$
(b) $T_2 > T_1$
(c) $T_1 = T_2$
(d) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise

2. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is :(Assume that the string remains straight)

(a) 3 : 5 : 7 (b) 3 : 4 : 5 (c) 7 : 11 : 6 (d) 3 : 5 : 6

3. A heavy & big sphere is hang with a string of length l, this sphere moves in a horizontal circular path making an angle q with vertical then its time period is -
(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{l\, sin\theta}{g}}$ (c) $T = 2\pi\sqrt{\frac{l\, cos\theta}{g}}$ (d) $T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

4. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as k = as^2 where a is a constant. The force acting on the particle is
(a) $2a\frac{s^2}{R}$ (b) $2as\left(1+\frac{s^2}{R^2}\right)^{\frac{1}{2}}$ (c) 2as (d) $2a\frac{R^2}{s}$

5. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{\imath}$ m/s and $\vec{a} = 2\hat{\imath} + 4\hat{\jmath}$ m/s^2 respectively at an instant of time. The radius of the circle is
(a) 1m (b) 2m (c) 3m (d) 4m

6. If mass speed & radius of rotation of a body moving on a circular path are increased by 50% then to keep the body moving in circular path, increase in force required will be –
(a) 225% (b) 125% (c) 150% (d) 100%

7. A train A runs from east to west and another train B runs from west to east at the same speed along the equator. A presses the track with a force of F_1 and B presses the track with a force F_2.
(a) $F_1 > F_2$ (b) $F_1 < F_2$ (c) $F_1 = F_2$ (d) none of these

8. For a particle in uniform circular motion, the acceleration at a point P (R, θ) on the circle of radius R is
(Here θ is measured from the x-axis)
(a) $-\frac{v^2}{R}\cos\theta\,\hat{\imath} + \frac{v^2}{R}\sin\theta\,\hat{\jmath}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{\imath} + \frac{v^2}{R}\cos\theta\,\hat{\jmath}$
(c) $-\frac{v^2}{R}\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$ (d) $\frac{v^2}{R}\hat{\imath} + \frac{v^2}{R}\hat{\jmath}$

9. If the angle θ between velocity vector and the acceleration vector is 90 <θ< 180, the body is moving on a :
(a) Straight path with retardation (b) Straight path with acceleration
(c) Curvilinear path with acceleration (d) Curvilinear path with retardation.

10. Two cars of masses m1 and m2 are moving in circles of radii r1 and r2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :
(a) $m_1\, r_1 : m_2\, r_2$ (b) $m_1 : m_2$ (c) $r_1 : r_2$ (d) 1 : 1

11. An aeroplane revolves in a circle above the surface of the earth at a fixed height with speed 100 km/hr. The change in velocity after completing half revolution will be
(a) 200 km/hr (b) 150 km/hr (c) 300 km/hr (d) 400 km/hr

12. A wheel is of diameter 1m. If it makes 30 revolution/sec. then the linear speed of a point on its circumference will be
(a) 30π m/s (b) π m/s (c) 60π m/s (d) $\pi/2$ m/s

13. The angular velocity of earth's rotation about its axis is
(a) $\frac{12}{\pi}$ rad/hr (b) $\frac{\pi}{12}$ rad/hr (c) 48π rad/hr (d) 0.5 degree/min

14. If a particle moves on a circle, describing equal angle in equal time intervals, the velocity vector
(a) remains constant (b) changes in magnitude
(c) changes in direction (d) changes both in magnitude and direction

15. The ratio of angular speed of hours hand and seconds hand of a clock is
(a) 1 : 1 (b) 1 : 60 (c) 1 : 720 (d) 3600 : 1

16. A particle moves in a circular orbit under the action of central attractive force inversely proportional to the distance r. The speed of the particle is proportional to
(a) r^2 (b) r^0 (c) r (d) r^{-1}

17. Radius of the curved road on national highway is R. Width of the road is b. The outer edge of the road is raised by h wrt inner edge so that a car with velocity v can pass safe over it. The value of h is
(a) $\frac{v^2 b}{Rg}$ (b) $\frac{v}{Rbg}$ (c) $\frac{v^2 R}{g}$ (d) $\frac{v^2 b}{R}$

18. A block of mass M is whirled with a constant angular speed ω by spring of stiffness K and natural length L. Block is attached at one end of spring and other end is fixed. Extension in spring in steady state is
(a) $\frac{ML\omega^2}{K}$ (b) $\frac{ML\omega^2}{K+M\omega^2}$ (c) $\frac{ML\omega^2}{K-M\omega^2}$ (d) $\frac{ML\omega^2}{2K-M\omega^2}$

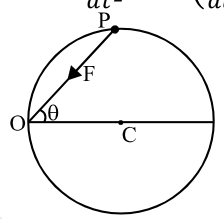
19. A light hollow thin cylinder of length L is filled with a liquid of density ρ. Cylinder is closed at one end and it is whirled about open end in horizontal plane with constant angular speed ω. Pressure at the closed end of cylinder is
(a) $\rho L^2\omega^2$ (b) $\frac{\rho L^2\omega^2}{2}$
(c) $\frac{\rho L^2\omega^2}{3}$ (d) Area of cylinder is required

20. A ring of mass M, radius R is executing circular motion about its centre with a constant angular velocity ω. Tension in wire of ring is
(a) $MR\omega^2$ (b) $\frac{MR\omega^2}{\pi}$ (c) $\frac{MR\omega^2}{2\pi}$ (d) $2\pi MR\omega^2$

21. A projectile is fired with speed 40 m/s at an angle 30^0 with horizontal. The tangential acceleration of projectile at the time of projection is ($g = 10\ m/s^2$)

(a) $10\ m/s^2$ (b) $8\ m/s^2$ (c) $6\ m/s^2$ (d) None of these

22. A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference. The ratio of $\frac{d^2\theta}{dt^2}$ and $\left(\frac{d\theta}{dt}\right)^2$.



(a) $\frac{\tan\theta}{2}$ (b) $4\tan\theta$ (c) $\tan\theta$ (d) $2\tan\theta$

23. A body starts revolving from rest in a circular path of radius 2 m with tangential acceleration a = 2t m/s^2. Its total acceleration at t = 2 sec is :

(a) $4\ m/s^2$ (b) $8\ m/s^2$ (c) $2\sqrt{10}\ m/s^2$ (d) $2\sqrt{20}\ m/s^2$

24. A car is travelling with linear velocity v on a circular road of radius r. If the speed is increasing at the rate a m/s^2, then the resultant acceleration will be

(a) $\sqrt{\left(\frac{v}{r}\right)^2 - a^2}$ (b) $\sqrt{\frac{v^4}{a^2} + a^2}$ (c) $\sqrt{\frac{v^4}{a^2} - a^2}$ (d) $\sqrt{\frac{v^2}{a^2} + a^2}$

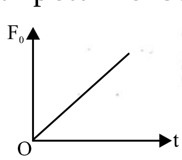
25. A point moves along a circle with speed, v = at, where a = $0.5\ m/s^2$. Then what is its total acceleration when it has travelled a distance 1/10 of circle from initial point?

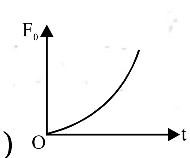
(a) $0.5\ m/s^2$ (b) $0.6\ m/s^2$ (c) $0.7\ m/s^2$ (d) $0.8\ m/s^2$

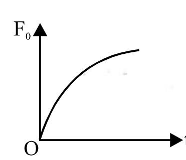
26. During a sharp turn, a fast moving car should not

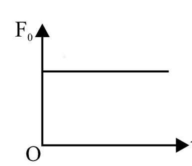
(a) Apply breaks (b) accelerate (c) both of above (d) None of these

27. The speed of a particle moving along a horizontal circular path is increasing at a constant rate a_0. Identify the correct graph, which shows the variation of magnitude of centripetal force F_0 with time t,

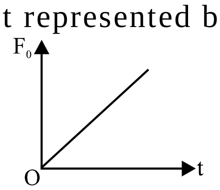
(a) 

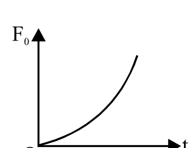
(b) 

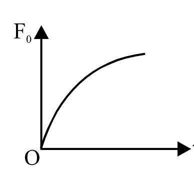
(c) 

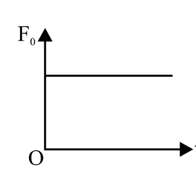
(d) 

28. In the above problem, the variation of magnitude of tangential force F_t with time t is best represented by

(a) 

(b) 

(c) 

(d) 

29. A particle is thrown from origin at an angle 30° with the horizontal (x-axis) at speed 20 m/s. Find the angular speed of particle about the origin after t = 1 second from projection

(a) 2 rad/sec (b) $2\sqrt{3}$ rad/sec (c) 4 rad/sec (d) None of these

30. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v. The centrifugal force on it is

(a) $\frac{mv^2}{r}$ towards the centre

(b) $\frac{mv^2}{r}$ away from the centre

(c) $\frac{mv^2}{r}$ along the tangent through the particle

(d) zero

31. A particle of mass m rotates in a circle of radius a with a uniform angular speed ω. It is viewed from a frame rotating about the z- axis with a uniform angular speed ω_0. The centrifugal force on the particle is

(a) $m\omega^2 a$ (b) $m\omega_0{}^2 a$ (c) $m\left(\frac{\omega+\omega_0}{2}\right)$ (d) $m\,\omega\omega_0 a$

32. A particle is kept fixed on a turntable rotating uniformly. As seen from the ground, the particle goes in a circle, its speed is 20cm/s and acceleration is 20cm/s^2. The particle is now shifted to a new position to make the radius half of the original value. The new values of the speed and acceleration will be

(a) 10 cm/s, 10cm/s^2 (b) 10 cm/s, 80cm/s^2
(c) 40 cm/s, 10cm/s^2 (d) 40 cm/s, 40cm/s^2

33. A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the center. The speed of the stone is v. If the string breaks, the stone will move

(a) towards the centre (b) away from the center
(c) along the tangent (d) will stop.

34. A coin placed on a rotating on a turntable just slips if it is placed at a distance of 4 cm from the center. If the angular velocity of the turntable is doubled, it will just slip at a distance of

(a) 1 cm (b) 2 cm (c) 4 cm (d) 8cm

EXERCISE – III
CROSSING THE HURDLES

MORE THAN ONE CORRECT

1. An object follows a curved path. The following quantities may remain constant during the motion
(a) speed (b) velocity (c) acceleration (d) magnitude of acceleration

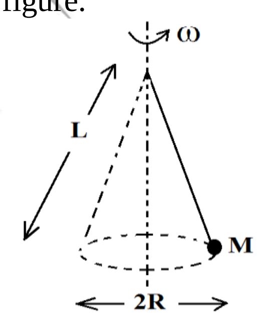
2. A car of mass M is moving on a horizontal circular path of radius r. At an instant its speed is v and is increasing at a rate a.
(a) the acceleration of the car is towards the centre of the path
(b) the magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$
(c) the friction coefficient between the ground and the car is not less than $\frac{a}{g}$
(d) the friction coefficient between the ground and the car is μ = $\tan^{-1}\frac{v^2}{rg}$

3. A circular road of radius r is banked for a speed v=40 km/hr. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.
(a) the car cannot make a turn without skidding.
(b) if the car turns at a speed less than 40 km/hr, it will slip down.
(c) if the car turns at the correct speed of 40 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$
(d) if the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$

4. A person applies a constant force F on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.
(a) This is not possible.
(b) There are other forces on the particle.
(c) The resultant of the other forces is $\frac{mv^2}{r}$ towards the center.
(d) The resultant of the other forces varies in magnitude as well as in direction.

COMPREHENSION TYPE QUESTION

Passage 1
A bob of mass M is whirled by a string of length L so that bob moves in horizontal circular path of radius R, as shown in figure.



5. Angular speed of the circular motion of the bob is
(a) $\sqrt{\frac{g}{L}}$ (b) $\sqrt{\frac{g}{R}}$ (c) $\sqrt{\frac{g}{\sqrt{L^2-R^2}}}$ (d) $\sqrt{\frac{g}{\sqrt{L^2+R^2}}}$

6. Tension in the string is
(a) $MR\omega^2$ (b) $ML\omega^2$ (c) $\frac{ML^2\omega^2}{R}$ (d) $\frac{MR^2\omega^2}{L}$

7. If the breaking strength of the string is equal to 2Mg and angular speed of revolution is increased gradually, angular speed just before breaking the string is

(a) $\sqrt{\frac{g}{L}}$ (b) $\sqrt{\frac{2g}{L}}$ (c) $2\sqrt{\frac{g}{L}}$ (d) $\sqrt{\frac{g}{2L}}$

8. If the breaking strength of the string is equal to 2Mg and angular speed of revolution is increased gradually, semi vertical angle of cone formed by string and bob, just before breaking the string is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

Passage 2

A coin is placed on a rough horizontal disc at a distance 1 metre from centre of disc. Disc starts rotating from rest with a constant angular acceleration 1 rad/s^2. If coefficient of friction between disc and coin is 0.6, then answer the following questions

9. Initial acceleration of coin is

(a) 1 m/s^2 (b) 6 m/s^2 (c) zero (d) None of these

10. The time after which frictional force is directed at angle $\frac{\pi}{4}$ with velocity vector, is

(a) 1 sec (b) $\sqrt{2}$ sec (c) 2 sec (d) 6 sec

11. Time after which coin starts sliding on disc is

(a) 1 sec (b) $\sqrt{5}$ sec (c) $\sqrt{35}$ sec (d) None of these

12. Distance moved by coin just before sliding is

(a) $\frac{5}{2}$ metres (b) $\frac{35}{2}$ metres (c) 5 metres (d) $\sqrt{35}$ metres

Passage 3

A man of mass 50 kg stands in a rough vertical cylinder of radius R metre, closed at the bottom. Coefficient of friction of all surfaces of cylinder is μ. Cylinder starts rotating about its axis with angular acceleration α rad/sec^2. If initially man was very close to the vertical wall without touching it then answer the following questions.

13. The time after which man has to take support of the vertical wall of cylinder is

$R = \sqrt{3}; \alpha = \sqrt{3}; \mu = 0.6$

(a) 1sec (b) $\sqrt{3}$ sec (c) $3^{1/4}$ sec (d) $\sqrt{27}$ sec

14. After some time the angular acceleration of cylinder is sufficient so that man will not fall if bottom of cylinder is removed, then angular speed becomes constant. Minimum time so that bottom can be removed safely $R = \frac{3}{2}; \alpha = 2; \mu = 0.6$

(a) 1 sec (b) 0.6 sec (c) 1.5 sec (d) 1.67 sec

15. In the above question, man will fall down if

(a) cylinder starts accelerating again

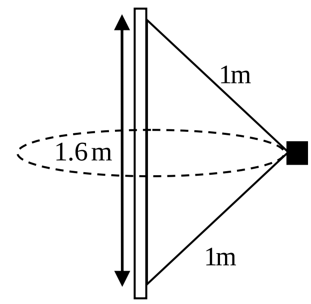
(b) cylinder starts decelerating

(c) man carries some extra weight at same speed

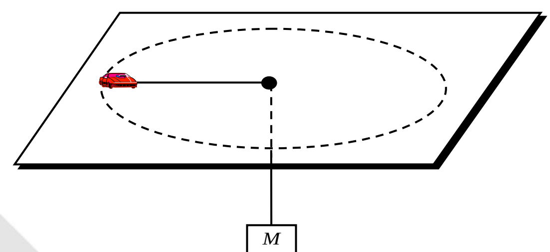
(d) both of (a) and (b)

EXERCISE – IV

1. A block of mass 0.4 kg is attached to a vertical rotating spindle by two strings of equal length, as shown in the figure. The time period of rotation is 1.2s. Determine the tensions in the strings.



2. A toy car of mass *m* can travel at a fixed speed. It moves in a circle on a horizontal table. The centripetal force is provided by a string attached to a block of mass *M* that hanges as shown in the figure. The coefficient of static friction is μ. Find the ratio of the maximum radius to the minimum radius possible.



3. A cyclist rides along the circumference of a circle in a horizontal plane of radius $R = 100m$, the friction coefficient being dependent only on distances r from the centre O of the plane as $\mu = \mu\left(1 - \frac{r}{R}\right)$, where $\mu_O = 0.4$. Find the radius of the circle with the centre at the point O along which the cyclist can ride with the maximum velocity. What is this maximum velocity?

4. Two block of mass $M_1 = 10kg$ and $M_2 = 5kg$ connected to each other by a massless inextensible string of length 0.3 m are placed alone a diameter of table. The coefficient of friction between the table and M_1 is 0.5 will be there is no friction between M_2 and table:

The table is rotating is with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of table on either side of the centre O. The masses are observed to be a rest with respect to an observer on the turn table. **(a)** Calculate the frictional force on M_1 **(b)** What should be the minimum angular speed of the turn table so that the masses will slip from this position.

5. A very small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of v rev/sec. The wall of the funnel makes an angle θ with the horizontal. If the coefficient of static friction between the cube and the funnel is μ_s and the centre of cube is at a distance r from the axis of rotation, what are the largest and smallest values of v for the which the block will not move with respect to funnel?

6. A block of mass m moves on a horizontal circle against the wall of cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_o. As a function of the speed v find (a) The normal force by the wall on the block (b) The frictional force by the wall and (c) The tangential acceleration of the block. (d) Integrate the tangential acceleration $\left(\frac{dv}{dt} = v\frac{dv}{ds}\right)$ to obtain the speed.

7. A circular road of radius 50m has the angle of banking equal to 30^0. At what speed should a vehicle go on this road so that the friction is not used?

8. A stone is fastened to one end of a string and is whirled in a vertical circle of radius R. Find the minimum speed the stone can have at the highest point of the circle..

9. A car goes on a horizontal circular road of radius R, the speed increases at a constant rate $\frac{dv}{dt} = a$. The friction coefficient between the road and the tyre is μ. Find the speed at which the car will skid.

10. In a children's park a heavy rod is pivoted at the centre and is made to rotate about the pivot so that the rod always remains horizontal. Two kids hold the rod near the ends and thus rotate with the rod. Let the mass of each kid be 15 kg, the distance between the points of the rod where the two kids hold it be 3m and suppose that the rod rotates at the rate of 20 revolutions per minute. Find the force of friction exerted by the rod on one of the kids.

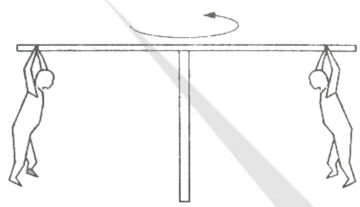


Figure 7-E2

11. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R. A smooth groove AB of length L (<<R) is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.

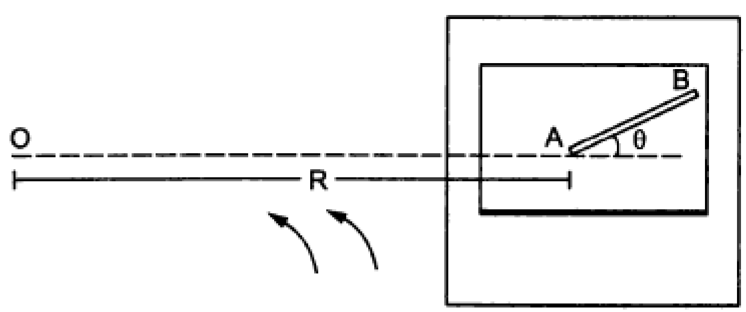


Figure 7-E3

12. A table with a smooth horizontal surface is placed in a cabin which moves in a circle of large radius. A smooth pulley of small radius is fastened to the table. Two masses m and 2m placed on the table are connected through a string going over the pulley. Initially the masses are held by a person with the springs along the outward radius and then the system is released from rest (with respect to cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.

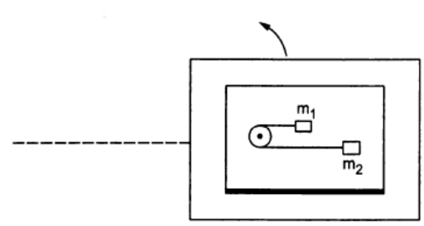


Figure 7-E5

ANSWERS

EXERCISE – I

BUILDING A FOUNDATION

SECTION-A

A-1. (c)	A-2. (d)	A-3. (a)	A-4. (d)
A-5. (a)	A-6. (d)	A-7. (d)	A-8. (b)
A-9. (a)	A-10. (c)	A-11. (a)	A-12. (b)
A-13. (c)	A-14. (c)	A-15. (c)	A-16. (b)

SECTION-B

B-1. (c)	B-2. (d)	B-3. (c)	B-4. (a)
B-5. (a)	B-6. (b)		

SECTION-C

C-1. (a)	C-2. (d)	C-3. (b)	C-4. (a)
C-5. (d)	C-6. (c)	C-7. (c)	C-8. (c)
C-9. (a)	C-10. (a)	C-11. (d)	C-12. (a)
C-13. (d)	C-14. (a)	C-15. (d)	C-16. (b)
C-17. (a)	C-18. (b)	C-19. (a)	C-20. (b)

SECTION-D

D-1. (b)	D-2. (b)	D-3. (a)

SECTION-E

E-1. (a)	E-2. (c)	E-3. (c)	E-4. (a)

EXERCISE – II
READY FOR CHALLENGES

1. (a)	2. (d)	3. (c)	4. (b)	5. (a)
6. (b)	7. (a)	8. (c)	9. (d)	10. (c)
11. (a)	12. (a)	13. (b)	14. (c)	15. (c)
16. (b)	17. (a)	18. (C)	19. (b)	20. (c)
21. (b)	22. (d)	23. (d)	24. (b)	25. (d)
26. (c)	27. (b)	28. (d)	29. (d)	30. (d)
31. (b)	32. (a)	33. (c)	34. (a)	

EXERCISE – III
CROSSING THE HURDLES

MORE THAN ONE CORRECT

1. (a,d)	2. (b,c)	3. (b,d)	4. (b,d)	5. (c)
6. (b)	7. (b)	8. (c)	9. (a)	10. (a)
11. (c)	12. (b)	13. (c)	14. (d)	15. (d)

EXERCISE – IV

1. $T_1 = \frac{55}{6}$, $T_2 = \frac{25}{6}$
2. $\dfrac{M + \mu m}{M - \mu m}$
3. $r = \frac{R}{2}$, $v_{max} = \sqrt{\frac{1}{4}\mu_0 g R}$
4. (A) F=50N, (B) $\omega = 11.78\, rad/s$
5. $v_{max} = \frac{1}{2\pi}\sqrt{\frac{g}{r}\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}\right)}$, $v_{min} = \frac{1}{2\pi}\sqrt{\frac{g}{r}\left(\frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}\right)}$
6. (a) $\dfrac{mv^2}{R}$ (b) $\mu\dfrac{mv^2}{R}$ (c) $-\dfrac{\mu v^2}{R}$ (d) $v_0 e^{-2\pi\mu}$
7. $v = 17 m/s$
8. $v = \sqrt{rg}$
9. $v = \left(r\sqrt{\mu^2 g^2 - a^2}\right)^{\frac{1}{2}}$
10. $f = 10\pi^2$
11. $t = \sqrt{\frac{2L}{\omega^2 R \cos\theta}}$
12. $a = \frac{\omega^2 R}{3}$, $\frac{4}{3} m\omega^2 R$