HERON'S FORMULA

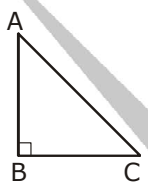
INTRODUCTION

In earlier classes, we have learnt to find perimeter and area of various plane figures such as triangles, quadrilaterals-rectangle, square, parallelogram, trapezium etc. But we know to find the area of a triangle only when its altitude is given or the triangle is right angled, isosceles or equilateral.

In the present chapter, we shall study to find the area of any kind of triangle. Then with the help of area of triangle we will find the area of different plane figures which can be reduced to different triangles or quadrilaterals.

AREA OF SOME SPECIFIC PLANE FIGURES :

1. Right Angled Triangle



Area of a right angled triangle

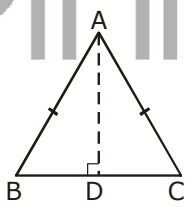
$= \frac{1}{2} \times$ base $\times$ altitude

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2}$ (product of sides forming right angle).

2. Isosceles Triangle :

Let ABC be an isosceles triangle with AB = AC. Let AD $\perp$ BC. Then, by simple geometry, we can prove that AD bisects BC. Then BD = $\frac{1}{2}$BC. Let a be the equal side and b be the base then BD = $\frac{1}{2}$b and AB = a.



$\therefore$ By Pythagoras theorem in ΔABD, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad a^2 = AD^2 + \frac{b^2}{4} \quad \Rightarrow \quad AD = \sqrt{a^2 - \frac{b^2}{4}}$

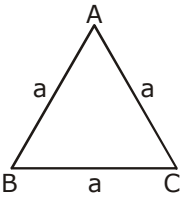
$\therefore$ area of isosceles ΔABC

$= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times b \times \sqrt{a^2 - \frac{b^2}{4}}$

$= \frac{1}{2} \times$ base $\times \sqrt{(\text{equal side})^2 - \frac{(\text{base})^2}{4}}$

3. Equilateral Triangle :

Let a be the side of an equilateral triangle. Then putting b = a in area for isosceles triangle, we get area of equilateral triangle

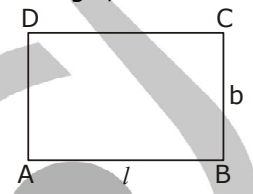


$= \frac{1}{2} \times a \times \sqrt{a^2 - \frac{a^2}{4}}$ $= \frac{1}{2} a \frac{\sqrt{3}}{2} \cdot a$ $= \frac{\sqrt{3}}{4} a^2$

$\therefore$ area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Rectangle :

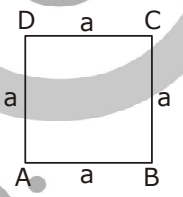
If l and b be the length and breadth of a rectangle, then



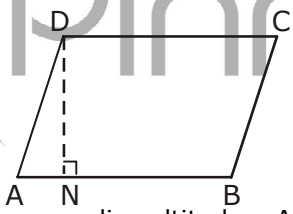
area of rectangle = $l \times b$.

5. Square :

If a be the side of a square, then area of square = $(\text{side})^2 = a^2$.



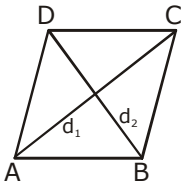
6. Parallelogram:



Area of a parallelogram = base × corresponding altitude = AB × DN.

7. Rhombus :

In d_1 and d_2 are the diagonals of a rhombus then area of a rhombus



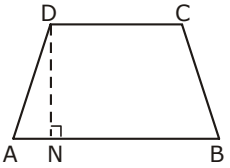
$= \frac{1}{2}(d_1 \times d_2)$

$= \frac{1}{2}(AC \times BD)$

(Note : For rhombus we can use the formula as we have for parallelogram, as rhombus is also a parallelogram, provided the measures of the base and the corresponding altitude are known.)

8. Trapezium :

If in trap. ABCD, AB || CD and DN is the distance between parallel sides then area of trapezium

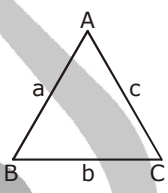


$= \frac{1}{2} \times$ sum of parallel side $\times$ distance between them.

$= \frac{1}{2} \times (AB + DC) \times DN.$

9. Heron's Formula

Heron, a mathematician, gave the famous formula for finding the area of any kind of triangle is terms of its three sides. After his name the formula is known as Heron's formula which is discussed below.



If a, b, c are the sides of a triangle then area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where s is the semiperimeter of triangle that is

$$s = \frac{a+b+c}{2}$$

(Note. In case we know all sides of a triangle as well as altitude corresponding to a particular side, it is better to use the formula.

area of triangle = $\frac{1}{2} \times$ base $\times$ altitude instead of using Heron's formula as it saves a lot of calculations.)

Applicability of Heron's formula will be clear through the following examples.

HERON'S FORMULA

If a,b,c denote the lengths of the sides of a triangle ABC. Then,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$ is the semi–perimeter of ΔABC.

Note: This formula is applicable to all types of triangles whether it is scalene or equilateral or isosceles.

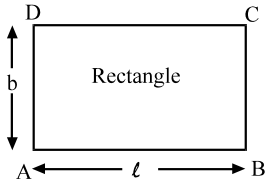
RECTANGLE

If ℓ and b denote respectively the length and breadth of a rectangle, then

(i) Perimeter = 2 (ℓ + b)

(ii) Area = $\ell \times$ b

(iii) $(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$



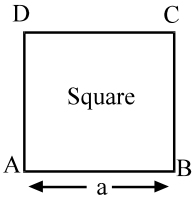
SQUARE

If "a" denote the length of each side of a square then,

(i) Perimeter = 4a

(ii) Area = a^2 = $(side)^2$

(iii) Area = $\frac{1}{2}$ $(diagonal)^2$



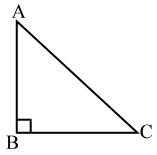
RIGHT-ANGLED TRIANGLE

Let ABC be a right triangle right angled at B. Then,

(i) Perimeter = AB + BC + CA

(ii) Area = $\frac{1}{2}$ (Base × Height)

= $\frac{1}{2}$ × (BC × AB)



ISOSCELES TRIANGLE

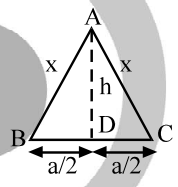
Let ABC be an isosceles triangle such that AB = AC = x and BC = a. Then,

(i) Perimeter = AB + BC + CA

= a + 2x

(ii) Area = $\frac{1}{2}$ (Base × Height)

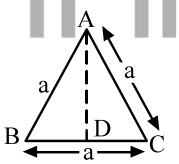
= $\frac{1}{2} \times \left[a \times \sqrt{x^2 - \frac{a^2}{4}} \right]$



(iii) h = $\sqrt{x^2 - (a/2)^2}$

EQUILATERAL TRIANGLE

Let ABC be an equilateral triangle of each side a. Then,



(i) Perimeter = 3a (ii) Altitude = $\frac{\sqrt{3}}{2}$a (iii) Area = $\frac{\sqrt{3}}{4}$$a^2$

SOLVED PROBLEMS

Ex.1 Find the area of a triangle whose sides are 13 cm, 14 cm and15 cm

Sol. Let a, b, c be the sides of the given triangle and s be its semi-perimeter such that

a = 13 cm, b = 14 cm and c = 15 cm

Now, $s = \frac{1}{2}(a + b + c) = \frac{1}{2}(13 + 14 + 15) = 21$cm

$\therefore$ s – a = 21 – 13 = 8cm, s – b = 21 – 14 = 7cm and s – c = 21 – 15 = 6cm

Hence, Area of given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21\times8\times7\times6} = \sqrt{7\times3\times8\times7\times2\times3} = \sqrt{7^2\times4^2\times3^2}$

$= 7 \times 4 \times 3 = 84$ cm^2

Ex.2 Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.

Sol. Let a, b, c be the sides of the given triangle and 2s be its perimeter such that

a = 8 cm, b = 11 cm and 2s = 32 cm i.e. s = 16 cm

Now, a + b + c = 2s

$\Rightarrow$ 8 + 11 + c = 32

$\Rightarrow$ c = 13cm

$\therefore$ s – a = 16 – 8 = 8cm, s – b = 16 – 11 = 5cm and s – c = 16 – 13 = 3cm

Hence, Area of given triangle = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{16\times8\times5\times3} = \sqrt{8\times8\times30} = 8\sqrt{30}$ cm^2

Ex.3 The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of triangle.

Sol. It is given that the sides a,b,c of the triangle are in the ratio 13 : 12 : 5 i.e.,

a : b : c = 13 : 12 : 5 $\Rightarrow$ a = 13x, b = 12x and c = 5x

$\therefore$ Perimeter = 450 $\Rightarrow$ 13x + 12x + 5x = 450 $\Rightarrow$ 30x = 450 $\Rightarrow$ x = 15

So, the sides of the triangle are

a = 13 × 15 = 195 m, b = 12 × 15 = 180 m and c = 5 × 15 = 75 m

It is given that perimeter = 450 $\Rightarrow$ 2s = 450 $\Rightarrow$ s = 225 m

Hence, Area = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{225(225-195)(225-180)(225--75)}$

$\Rightarrow$ Area = $\sqrt{225\times30\times45\times150} = \sqrt{5^2\times3^2\times3\times5\times2\times3^2\times5\times5^2\times2\times3}$

$\Rightarrow$ Area = $\sqrt{5^6\times3^6\times2^2} = 5^3 \times 3^3 \times 2 = 6750$ m^2

Ex.4 Find the area of a triangle having perimeter 32 cm, one side 11 cm and difference of other two sides is 5 cm.

Sol. Let a, b and c be the three sides of ΔABC.

a = 11 cm

a + b + c = 32 cm $\Rightarrow$ 11 + b + c = 32 cm or b + c = 21 cm ... (1)

Also, we are given that b – c = 5 cm ... (2)

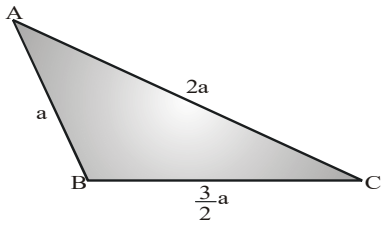
Adding (1) and (2), 2b = 26 cm i.e., b = 13 cm and c = 8 cm

Now, $s = \frac{a+b+c}{2} = \frac{11+13+8}{2} = \frac{32}{2} = 16$ cm

(s – a) = (16 – 11) cm = 5 cm, (s – b) = (16 – 13) cm = 3 cm and (s – c) = (16 – 8) cm = 8 cm

$\therefore$ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{16\times5\times3\times8}$ cm^2 = $\sqrt{64\times30}$ cm^2 = $8\sqrt{30}$ cm^2

Ex.5 In figure, find the area of the ΔABC.



Sol. $s = \frac{BC + CA + AB}{2} = \frac{\frac{3}{2}a + 2a + a}{2} = \frac{9}{4}a$

$\{s - (BC)\} = \frac{9}{4}a - \frac{3}{2}a = \frac{3}{4}a$

$\{s - (CA)\} = \frac{9}{4}a - 2a = \frac{1}{4}a$

$\{s - (AB)\} = \frac{9}{4}a - a = \frac{5}{4}a$

Now, area of $\Delta ABC = \sqrt{\frac{9}{4}a \times \frac{3}{4}a \times \frac{1}{4}a \times \frac{5}{4}a} = \sqrt{\frac{9 \times 3 \times 5}{4 \times 4 \times 4 \times 4}a^4} = \frac{3\sqrt{15}}{4 \times 4}a^2$ sq. units. $= \frac{3\sqrt{15}}{16}a^2$ sq. units.

Ex.6 The sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Sol. Let us take the sides of the triangle as 3x, 5x and 7x because the ratio of the sides is given to be 3 : 5 : 7. Also, we are given that

$3x + 5x + 7x = 300 \Rightarrow 15x = 300 \Rightarrow x = 20$

Hence, the lengths of the three sides are 3 × 20 m, 5 × 20 m, 7 × 20 m. i.e., 60 m, 100 m, 140 m.

Now, $s = \frac{60 + 100 + 140}{2}$ m = 150 m

Area of the triangle

$= \sqrt{50 \times (150 - 60) \times (150 - 100) \times (150 - 140)}$ m^2

$= \sqrt{50 \times 90 \times 50 \times 10}$ m^2 $= \sqrt{15 \times 9 \times 5 \times 10000}$ m^2

$= 15 \times 100 \times \sqrt{3}$ m^2 $= 1500\sqrt{3}$ m^2.

Ex.7 The lengths of the sides of a triangle are 5 cm, 12 cm and 13 cm. Find the length of perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol. Here, a = 5, b = 12 and c = 13.

$\therefore \quad s = \frac{1}{2}(a + b + c) = \frac{1}{2}(5 + 12 + 13) = \frac{30}{2} = 15$ cm

Let A be the area of the given triangle. Then,

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-5)(15-12)(15-13)}$

$\Rightarrow \quad A = \sqrt{15 \times 10 \times 3 \times 2} = 30$ cm^2 ... (i)

Let p be the length of the perpendicular from vertex A on the side BC. Then,

$A = \frac{1}{2} \times (13) \times p$ ... (ii)

From (i) and (ii), we get $= \frac{1}{2} \times 13 \times p = 30 \Rightarrow p = \frac{60}{13}$ cm.

Ex.8 In figure, there is a triangular childern park with sides, AB = 7 m, BC = 8 and AC = 5m, AD ⊥ BC and AD meets BC at D. Trees are planted at A, B, C and D. Find the distance between the trees at A and D.

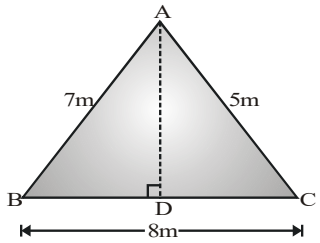
Sol. In figure, a = 8 m, b = 5 m and c = 7 m.

$s = \frac{8+5+7}{2}$ m = $\frac{20}{2}$ = 10 m.

The area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10 \times (10-8) \times (10-5) \times (10-7)}$ m^2

$= \sqrt{10 \times 2 \times 5 \times 3}$ $m^2 = 10\sqrt{3}$ m^2



Now, AD is perpendicular to BC.

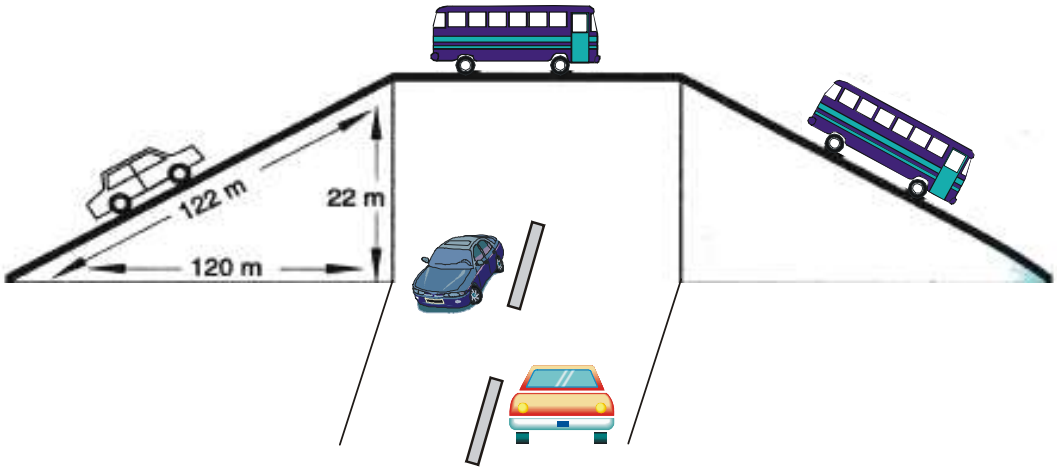
$\Rightarrow \frac{1}{2} \times BC \times AD = 10\sqrt{3}$

$\Rightarrow \frac{1}{2} \times 8 \times AD = 10\sqrt{3}$

$\Rightarrow AD = \frac{10\sqrt{3}}{4} m = \frac{5\sqrt{3}}{2} m$

Hence, the distance between the trees at A and D is $\frac{5\sqrt{3}}{2}$ m.

Ex.9 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22m and 120 m. The advertisements yield an earning of Rs. 5000 per m^2 per year. A company hired both walls for 3 months. How much rent did it pay?



Sol. The lengths of the sides of the walls are 122 m, 22 m and 120 m.

We have, $122^2 = 120^2 + 20^2 = 14884$

So, walls are in the form of a right triangle.

$\therefore$ Area of two walls = 2 x $\left(\frac{1}{2} \times Base \times Height\right)$

$\Rightarrow$ Area of two walls = 2 x $\left(\frac{1}{2} \times 120 \times 22\right)$ = 2640 m^2

We have,

Yearly rent = Rs 5000 per m^2

$\therefore$ Monthly rent = Rs $\left(\frac{5000}{12}\right)$ per m^2

Hence, rent paid by the company for 3 months = Rs $\left(\frac{5000}{12}\times 3\times 2640\right)$ = Rs 3300000.

Ex.10 A traffic signal, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

Sol. (i) Let 2s be the perimeter of the signal board. Then, we have

$2s = a + a + a = 3a \Rightarrow s = \frac{3}{2}a$ [where a is the side of an equilateral triangle]

Let Δ be the area of an equilateral triangle

$\therefore$ A of $\Delta = \sqrt{s(s-a)(s-a)(s-a)} = \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)} = \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}.a^2$.

(ii) If perimeter $\Rightarrow$ 180 = a + a + a

$= 180 = 3a \Rightarrow a = \frac{180}{3} = 60$ cm

and each side of an equilateral Δ(a) = 60 cm

$\therefore$ Area of an equilateral $\Delta = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}$ cm^2.

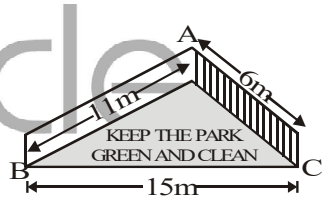
Ex.11 There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" as shown in the figure. If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol. Since the side wall is in the trianglular form with sides a = 15 m, b = 6 m and c = 11 m.

$\therefore$ $2s = a + b + c = (15 + 6 + 11) = 32$

$\Rightarrow$ $s = \frac{32}{2} = 16$ m

$\therefore$ $s - a = 16 - 15 = 1$ m, $s - b = 16 - 6 = 10$ m, $s - c = 16 - 11 = 5$ m



Hence, area to be painted in colour = Area of the side wall

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16\times 1\times 10\times 5} = \sqrt{4\times 4\times 2\times 5\times 5}$

$= 4\times 5\sqrt{2}$ $m^2 = 20\sqrt{2}$ m^2.

Ex.12 An isoscles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. Area of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2-b^2}$ with equal side 'a' and base b.

$\therefore$ $a = 12$ cm $\Rightarrow 2a + b = 30 \Rightarrow 2\times 12 + b = 30, \Rightarrow 24 + b = 30$ $\therefore$ $b = 30 - 24 = 6$ cm

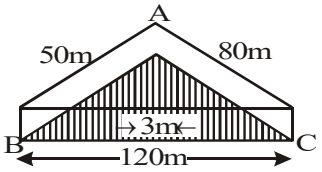
$\therefore$ Area of an isosceles triangle

$= \frac{6}{4}\sqrt{4(12)^2-(6)^2} = \frac{6}{4}\sqrt{4\times 144-36} = \frac{6}{4}\sqrt{576-36} = \frac{6}{4}\sqrt{540} = \frac{6}{4}\sqrt{2^2\times 3^2\times 3\times 5} = 6\sqrt{15}$ cm^2

Ex.13 A triangular park ABC has sides 120 m, 80 m and 50 m shown in the figure. A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant ? Find the cost of fencing it with barbed wire at the rate of Rs. 20 per metre leaving a space 3m wide for a gate on one side.

Sol. For area of the park, we have

$s = \frac{120 + 80 + 50}{2} = \frac{250}{2}$ = 125 m



Now, s – a = (125 – 120) m = 5m

s – b = (125 – 80) m = 45 m

s – c = (125 – 50) m = 75 m

Therefore, area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}$ m^2 = $\sqrt{25 \times 25 \times 3 \times 15 \times 15 \times 5}$ m^2 = $375\sqrt{15}$ m^2.

Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate) = 247 m.

And the cost of fencing = Rs. 20 × 247 = Rs. 4940.

Ex.14 Find the base of an isosceles triangle whose area is 12 cm^2 and one equal sides is 5 cm.

Sol. Here equal sides : a = 5 cm, b = ?, Area = 12 cm^2

Area of an isosceles triangle = 12 cm^2

$\Rightarrow \frac{b}{4}\sqrt{4a^2 - b^2}$ = 12 $\Rightarrow \frac{b}{4}\sqrt{4 \times 5^2 - b^2}$ = 12 $\Rightarrow b\sqrt{100 - b^2}$ = 12 × 4

On squaring both sides, we get

$b^2 (100 - b^2) = 2304$ or $b^4 - 100b^2 + 2304 = 0$

or $b^4 - 64b^2 - 36b^2 + 2304 = 0$ or $b^2(b^2 - 64) - 36(b^2 - 64) = 0$

or $(b^2 - 36)(b^2 - 64) = 0$

$\Rightarrow$ either $b^2 = 36$ or $b^2 = 64$

$\Rightarrow$ $b = \pm 6$ $\Rightarrow$ $b = \pm 8$

Neglecting the negative sign as length cannot be –ve

$\therefore$ Base(b) = 8 cm or 6 cm

Ex.15 The perimeter of a right triangle is 144 cm and its hypotenuse measures 65 cm. Find the lengths of other sides and calculate its area. Verify the result using Heron's formula.

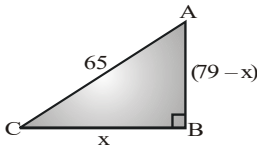
Sol. Perimeter of a right triangle = 144 cm ; Hypotenuse = 65 cm

Sum of the other two sides = 144 – 65 = 79 cm

Let one side be x, then the other side is (79 – x) cm.

In a right angle ΔABC, by pythagorus we have $AC^2 = AB^2 + BC^2$

$(65)^2 = (79 - x)^2 + x^2$



$4225 = 6241 + x^2 - 158x + x^2 \Rightarrow 4225 = 6241 + 2x^2 - 158x$

$2x^2 - 158x + 6241 - 4225 = 0 \Rightarrow 2x^2 - 158x + 2016 = 0$

$x^2 - 79x + 1008 = 0 \Rightarrow x^2 - 16x - 63x + 1008 = 0$

$x(x - 16) - 63(x - 16) = 0 \Rightarrow (x - 16)(x - 63) = 0$

$\Rightarrow$ either $x = 16$ cm or $x = 63$ cm

(i) When $x = 16$ cm $\Rightarrow$ BC = 16 cm and AB = 79 – 16 = 63 cm

(ii) When $x = 63$ cm $\Rightarrow$ BC = 63 cm and AB = 79 – 63 = 16 cm

Hence, the three sides of the triangle are 65 cm, 63 cm and 16 cm

$\therefore$ Area of right triangle $= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 16 \times 63 =$ **504 cm²**

Verification by Heron's formula, we have

$a = 63$ cm, $b = 65$ cm, $c = 16$ cm

$\Rightarrow s = \frac{63 + 15 + 16}{2} = 72$ cm

$\therefore s - a = 72 - 63 = 9$ cm, $s - b = 72 - 65 = 7$ cm, $s - c = 72 - 16 = 56$ cm

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72 \times 9 \times 7 \times 56} = \sqrt{9 \times 8 \times 9 \times 7 \times 7 \times 8} = 9 \times 7 \times 8 =$ **504 cm²**

Ex.16 One side of an equilateral triangle measures 8 cm. Find the area using Heron's formula. What is its altitude ?

Sol. Each side of an equilateral triangle = 8 cm

Here, $a = 8$ cm, $b = 8$ cm, $c = 8$ cm

$\therefore s = \frac{a+b+c}{2} = \frac{8+8+8}{2} = \frac{24}{2} = 12$ cm, $s - a = s - b = s - c = 12 - 8 = 4$ cm

$\therefore$ **(i) Area of the triangle is**

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12(12-8)(12-8)(12-8)}$ cm² $= \sqrt{12 \times 4 \times 4 \times 4}$ cm² $= \sqrt{3 \times 4 \times 4 \times 4 \times 4}$ cm²

$= \sqrt{3} \times 4 \times 4$ cm² $= 16\sqrt{3}$ cm².

and **(ii) Altitude of an equilateral triangle (h)**

$h = \frac{\sqrt{3}}{2} \times$ (side) $= \frac{\sqrt{3}}{2} \times 8$ cm $= 4\sqrt{3}$ cm.

Ex.17 Find the area of an equilateral triangle whose one side x cm.

Sol. $2s = x + x + x, \Rightarrow 2s = 3x \Rightarrow s = \frac{3x}{2}$ $\therefore$ $s - a = s - b = s - c = \frac{3}{2}x - x = \frac{1}{2}x$

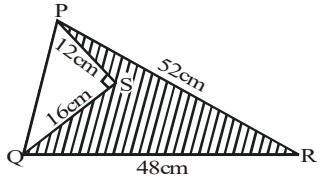
$\therefore$ Area of an equilateral $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{3}{2}x \times \frac{1}{2}x \times \frac{1}{2}x \times \frac{1}{2}x} = \frac{\sqrt{3}}{4} \times x^2 = \frac{\sqrt{3}}{4}.(\text{side})^2$

Ex.18 Calculate the area of the shaded portion of the Δ as shown in figure.

Sol. In right ΔPSQ, by Pythagoras

Theorem $PQ^2 = PS^2 + SQ^2 = (12)^2 + (16)^2 = 144 + 256 = 400$ cm

$\Rightarrow$ PQ $= \sqrt{400}$ cm $= 20$ cm



Now, a = 20 cm, b = 48 cm and c = 52 cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{20\text{cm} + 48\text{cm} + 52\text{cm}}{2} = \frac{120\text{cm}}{2} = 60\text{cm}$

Area of $\Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60 \times 40 \times 12 \times 8} = \sqrt{6 \times 10 \times 10 \times 4 \times 6 \times 2 \times 8}$

$= 6 \times 10 \times 8 = 480 \text{ cm}^2$.

and area of $\Delta PQS = \frac{1}{2} \times PS \times QS = \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$

$\Rightarrow$ area of the shaded portion of the triangle = 480 – 96 = 384 cm^2.

Ex.19 The sides of triangular plate are 8 cm, 15 cm and 17 cm. If its weight is 96 gram, find the weight of the plate per square cm.

Sol. Here a = 15 cm, b = 17 cm, c = 8 cm.

Since $(17)^2 = 289$ and $(15)^2 + (8)^2 = 225 + 64 = 289$

$\therefore$ This is a right triangle with sides 8 cm and 15 cm

$\therefore$ Area of right triangle = $\frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$

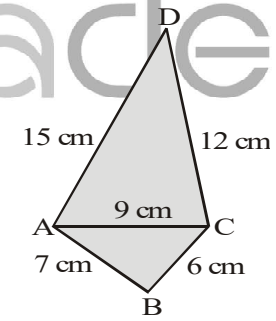
Weight of triangle plate = 96 gram

$\therefore$ Weight per square cm = $\frac{96}{60}$ = 1.6 gm.

Ex.20 Find the area of the quadrilateral ABCD, in which AB = 7 cm, BC = 6 cm, CD= 12 cm, DA = 15 cm and AC = 9 cm.

Sol. The diagonal AC divides the quadrilateral ABCD into two triangles ABC and ACD.

$\therefore$ Area of quad. ABCD = Area of ΔABC + Area of ΔACD,



For ΔABC, we have

Semiperimeter $s = \frac{6+7+9}{2}$ = 11 cm

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ A_1 = Area of $\Delta ABC = \sqrt{11(11-6)(11-7)(11-9)}$

$\Rightarrow$ A_1 = Area of $\Delta ABC = \sqrt{11 \times 5 \times 4 \times 2} = \sqrt{440}$ sq. cm

$\Rightarrow$ A_1 = Area of ΔABC = 20.98 cm^2

For ΔACD, we have $s = \frac{9+12+15}{2}$ = 18 cm

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ A_2 = Area of $\Delta ACD = \sqrt{18(18-9)(18-12)(18-15)}$

$\Rightarrow$ A_2 = Area of $\Delta ACD = \sqrt{18 \times 9 \times 6 \times 3}$

$\Rightarrow$ A_2 = Area of ΔACD

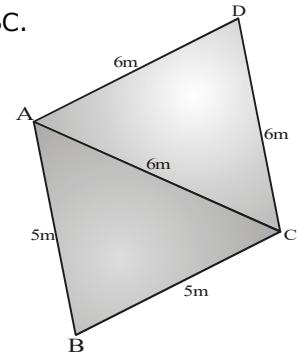
$= \sqrt{2 \times 9 \times 9 \times 2 \times 3 \times 3} = \sqrt{9^2 \times 2^2 \times 3^2}$

$= 9 \times 2 \times 3 = 54 \text{ cm}^2$

Hence, Area of quad. ABCD = $A_1 + A_2$ = (20.98 + 54) cm^2 = 74.98 cm^2

Ex.21 Prove that the area of the quadrilateral ABCD is $3(4+3\sqrt{3})\,m^2$, if AB = 5 m, BC = 5 m, CD = 6 m, AD = 6 m, and diagonal AC = 6 m.

Sol. Diagonal AC divides the quadrilateral ABCD into two triangles ΔACD and ΔABC.



For ΔACD, side are 6m, 6 m and 6 m.

Semiperimeter, s = $\frac{6m+6m+6m}{2}$ = 9 m

∴ Area of ΔACD = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{9\times(9-6)(9-6)\times(9-6)}$ m^2

= $\sqrt{9\times3\times3\times3}$ = $9\sqrt{3}$ m^2

For ΔABC, side are 5 m, 5 m and 6 m.

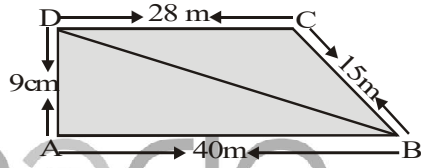
Semiperimeter, s = $\frac{5m+5m+6m}{2}$ = 8 m

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{8(8-5)(8-5)(8-6)}$ = $\sqrt{8\times3\times3\times2}$ m^2 = $\sqrt{16\times9}$ m^2 = 12 m^2

Thus, the area of the quadrilateral ABCD = $(12+9\sqrt{3})$ m^2 = $3(4+3\sqrt{3})$ m^2.

Ex.22 In fig. ABCD is a field in the form of a quadrilateral whose sides are indicated in the figure. If ∠DAB = 90°, find the area of the field.

Sol. Clearly, ΔDAB is a right-angled triangle. Therefore,



$DB^2 = DA^2 + AB^2$ [Using Pythagoras Theorem]

⇒ $DB^2 = 9^2 + 40^2$

⇒ DB = $\sqrt{81+1600}$ m = $\sqrt{1681}$ = 41m

For ΔDAB, we have s = $\frac{9+40+41}{2} = \frac{90}{2}$ = 45 m

Therefore, Area of ΔDAB = $\sqrt{s(s-a)(s-b)(s-c)}$

⇒ A_1 = Area of ΔDAB = $\sqrt{45\times(45-9)\times(45-40)\times(45-41)}$ m^2

⇒ A_1 = Area of ΔDAB = $\sqrt{45\times36\times5\times4}$ m^2

⇒ A_1 = Area of ΔDAB = $\sqrt{5\times9\times36\times5\times4}$ m^2 = $\sqrt{5^2\times3^2\times6^2\times2^2}$ m^2

⇒ A_1 = Area of ΔDAB = $(5\times3\times6\times2)$ m^2 = 180 m^2

For ΔDCB, we have s = $\frac{28+15+41}{2} = \frac{84}{2}$ = 42 m.

⇒ A_2 = Area of ΔDCB = $\sqrt{42\times(42-28)\times(42-15)\times(42-41)}$ m^2

⇒ A_2 = Area of ΔDCB = $\sqrt{42\times14\times27\times1}$ m^2

⇒ A_2 = Area of ΔDCB = $\sqrt{7\times2\times3\times7\times2\times3\times3\times3}$ m^2

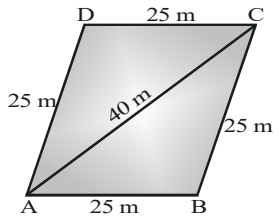
⇒ A_2 = Area of ΔDCB = $\sqrt{7^2\times2^2\times3^4}$ m^2

⇒ A_2 = Area of ΔDCB = $(7\times2\times3^2)$ m^2 = 126 m^2

Hence, Area of the field = $A_1 + A_2$ = (180 + 126) m^2 = 306 m^2

Ex.23 A rhombus has perimeter 100 m and one of its diagonal is 40 m. Find the area of the rhombus.

Sol. ABCD is the rhombus having perimeter = 100 m and AC = 40 m.



Now, we have AB = BC = CD = AD = $\frac{100}{4}$ = 25 m

We know that, ar (ΔABC) = ar (ΔADC)

Sides of ΔABC are 25 m, 25 m and 40 m.

Semi perimeter of ΔABC (s) = $\frac{25\text{m} + 25\text{m} + 40\text{m}}{2}$ = 45 m.

The area of ΔABC = $\sqrt{45 \times (45-25) \times (45-25) \times (45-40)}$ m^2 = $\sqrt{45 \times 20 \times 20 \times 5}$ m^2

= $\sqrt{9 \times 25 \times 20 \times 20}$ = 3 × 5 × 20 m^2 = 300 m^2

Also, we have area of ΔADC = 300 m^2.

Hence, the area of the rhombus ABCD = ar (ΔABC) + ar (ΔADC) = 300 m^2 + 300 m^2 = 600 m^2.

Ex.24 Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.

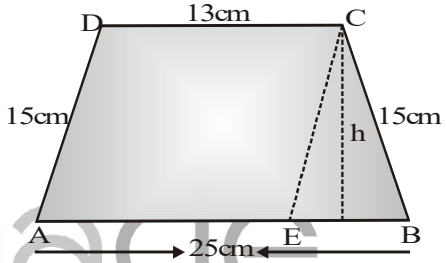
Sol. Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm.

Draw CE||AD.

Now, ADCE is a parallelogram in which AD||CE and AE||CD.

AE = DC = 13 cm and BE = AB - AE = 25 - 13 = 12 cm

In ΔBCE, we have



s = $\frac{15+15+12}{2}$ = 21 cm

$\therefore$ Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21(21-15)(21-15)(21-12)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21 \times 6 \times 6 \times 9}$ = $18\sqrt{21}$ sq. cm ...(i)

Let h be the height of ΔBCE, then

Area of ΔBCE = $\frac{1}{2}$ (Base × Height) = $\frac{1}{2}$ × 12 × h = 6h ...(ii)

From (i) and (ii), we have,

6h = $18\sqrt{21}$ $\Rightarrow$ h = $3\sqrt{21}$ cm

Clearly, the height of trapezium ABCD is same as that of ΔBCE.

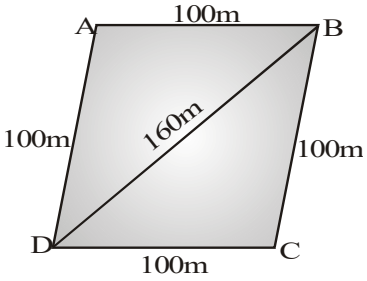
$\therefore$ Area of trapezium = $\frac{1}{2}$ (AB + CD) × h

$\Rightarrow$ Area of trapezium = $\frac{1}{2}$ (25 + 13) × $3\sqrt{21}$ cm^2 = $57\sqrt{21}$ cm^2

Ex.25 Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equals parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get?

Sol. Let ABCD be the field which is divided by the diagonal BD = 160 m into two equal parts.

Since ABCD is a rhombus of perimeter 400 m. Therefore,



AB = BC = CD = DA = $\frac{400}{4}$ m = 100 m

Let s be the semi-perimeter of ΔBCD.

Then, s = $\frac{BC + CD + BD}{2} = \frac{100 + 100 + 160}{2}$ m = 180 m

∴ Area of ΔBCD

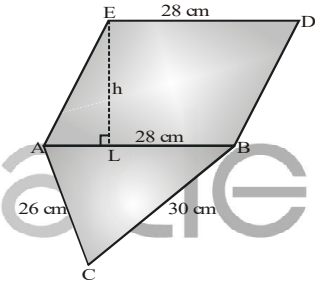
= $\sqrt{180 \times (180 - 100) \times (180 - 100) \times (180 - 160)}$ m^2

= $\sqrt{180 \times 80 \times 80 \times 20}$ m^2 = 4800 m^2

Hence, each of the two children will get an area of 4800 m^2.

Ex.26 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Sol. Semiperimeter of ΔABC



s = $\frac{26 + 28 + 30}{2}$ = 42 cm

s – a = 42 – 26 = 16 cm

s – b = 42 – 28 = 14 cm

s – c = 42 – 30 = 12 cm

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{42 \times 16 \times 14 \times 12}$ cm^2 = $\sqrt{14 \times 3 \times 4 \times 4 \times 14 \times 4 \times 3}$ cm^2

= 14 × 4 × 3 × 2 cm^2 = 336 cm^2

∴ Area of parallelogram = Area of triangle [Given]

h × AB = 336

h × 28 = 336 cm^2

h = $\frac{336}{28} = 12\text{cm}$

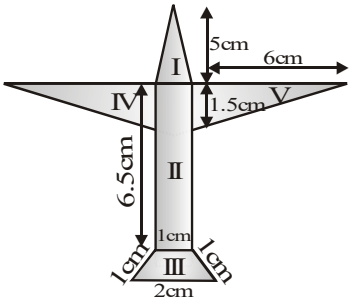
Ex.27 Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.

Sol. **Area of I in figure**

It is triangle part and its sides are 5 cm, 5 cm, 1 cm. Here, semiperimeter of the triangle

$= \frac{5\,cm + 5\,cm + 1\,cm}{2} = \frac{11}{2}\,cm$

Area of part I $= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}\ cm^2 = \frac{3}{4}\sqrt{11}\ cm^2$



$= \frac{3}{4} \times 3.31\ cm^2 = 2.482$ (approx)

Area of II in figure = area of rectangle = L × B = 6.5 × 1 = 65 cm^2

Area of III in figure

Area of $\Delta BEC = \frac{\sqrt{3}}{4}(1)^2\ cm^2 = \frac{\sqrt{3}}{4}\ cm^2$

Let h be the height of the ΔBEC

$\frac{1}{2} \times BE \times h = \frac{\sqrt{3}}{4} \Rightarrow \frac{1}{2} \times 1 \times h = \frac{\sqrt{3}}{4} \Rightarrow h = \frac{\sqrt{3}}{2}\,cm$

Area of Region III $= \frac{1}{2}(1 + 2) \times \frac{\sqrt{3}}{2}\,cm^2 = \frac{3}{4}\sqrt{3}\ cm^2$

$= \frac{3}{4} \times 1.732\ cm^2 = 1.3\ cm^2$ (approx)

Area of IV in figure $= \frac{1}{2} \times 6 \times \frac{3}{2}\,cm^2 = \frac{9}{2}\,cm^2$

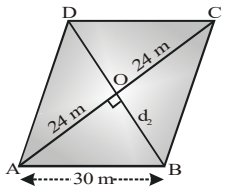
Area of V in figure $= \frac{1}{2} \times 6 \times \frac{3}{2}\,cm^2 = \frac{9}{2}\,cm^2$

Total area of the paper used = 2.482 cm^2 + 6.5 cm^2 + 1.3 cm^2 + $\frac{9}{2}\,cm^2$ + $\frac{9}{2}\,cm^2$

= (10.282 + 9) cm^2 (approx.) = 19.282 cm^2 (approx.) = 19.3 cm^2 (approx.)

Ex.28 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 cm, how much area of grass field will each cow be getting ?

Sol. Since the diagonals of a Rhombus bisect each other at right angles



$\therefore$ OB $= \sqrt{(30)^2 - (24)^2} = \sqrt{324} = 18$ cm

$\Rightarrow$ diagonal of d_2 = 2 × OB = 2 × 18 = 36 cm

$\therefore$ Area of a Rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 48 \times 36 = 864\ m^2$

[d_1 = 48 cm (given)]

Total area of grass field for 18 cows = 864 m^2

Area of grass grazed by each cow $= \frac{864}{18} = 48\ m^2$.

Ex.29 An umbrealla is made by stitching 10 triangular pieces of cloth of two different colour (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella ?

Sol. Sides of triangular piece of coloured cloth are 20 cm, 50 cm, 50 cm

Semiperimeter, s = $\frac{20+50+50}{2} = \frac{120}{2}$ = 60 cm

Then, area of one triangular piece

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{60(60-20)(60-50)(60-50)} = \sqrt{60\times40\times10\times10} = \sqrt{10\times6\times10\times4\times10\times10}$

= $10 \times 10 \times 2\sqrt{6}$ cm^2 = $200\sqrt{6}$ cm^2

There are 5 triangular pieces of one colour and 5 of the other colours

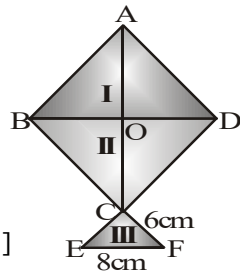
Then, total area of cloth of each colour = $5 \times 200\sqrt{6}$ cm^2 = $1000\sqrt{6}$ cm^2

Ex.30 A kite in the shape of a square is made of three different shades marked as I, II and III as shown in figure. How much paper of each shade has been used in it, if each diagonal of this kite is 32 cm and lower potion has the sides 6 cm, 6 cm and 8 cm ?

Sol. Each diagonal of the square = 32 cm

Area of I = $\frac{1}{2} \times 32 \times 16$ = 256 cm^2

Area of II = $\frac{1}{2} \times 32 \times 16$ = 256 cm^2

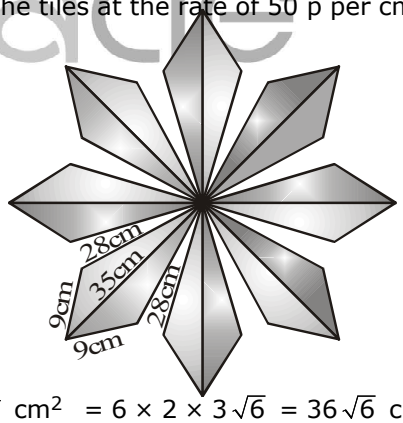


Area of III = $\sqrt{10(10-6)(10-6)(10-8)}$ [$\because s = \frac{6+6+8}{2} = \frac{20}{2}$ = 10 cm]

= $\sqrt{10\times4\times4\times2}$ = $8\sqrt{5}$ cm^2 = 17.92 cm^2

Ex.31 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm shown in the figure. Find the cost of polishing the tiles at the rate of 50 p per cm^2.

Sol. Sides of triangular tiles are 9 cm, 28 cm and 35 cm



Its semiperimeter, s = $\frac{35+28+9}{2} = \frac{72}{2} = 36$ cm

s – a = 36 – 35 = 1 cm

s – b = 36 – 28 = 8 cm

s – c = 36 – 9 = 27 cm

Area of one triangular tile

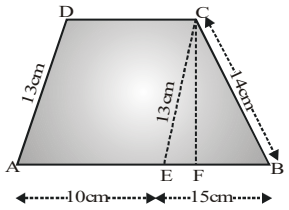
= $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{36\times1\times8\times27}$ cm^2 = $\sqrt{36\times4\times2\times3\times9}$ cm^2 = $6 \times 2 \times 3\sqrt{6}$ = $36\sqrt{6}$ cm^2

Total area of 16 tiles = $16 \times 36\sqrt{6}$ cm^2

= $576\sqrt{6}$ cm^2 = 576 × 2.45 cm^2 (approx.) = 1411.20 cm^2 (approx.)

Total cost of polishing at the rate of 50p per cm^2 = Rs. 1411.20 × $\frac{50}{100}$ = Rs. 705.60

Ex.32 A field is in the shape of a tapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.



Sol. Through C draw CE || DA

Draw CF ⊥ AB

In ΔBCE, we have

$s=\frac{15+14+13}{2}=21$ m

s – a = 21 – 15 = 6 cm, s – b = 21 – 14 = 7 cm, s – c = 21 – 13 = 8 cm

Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{21\times6\times7\times8}$ cm^2 = 7 × 4 × 3 = 84 cm^2

Now, area of ΔBCE = 84 m^2

$\Rightarrow \quad \frac{1}{2}$ × Base × Altitude = 84 m^2 $\Rightarrow \quad \frac{1}{2}$ × 15 × h = 84 $\Rightarrow \quad h=\frac{84\times2}{15}$ m

$\Rightarrow$ Distance between parallel sides of trapezium = $\frac{168}{15}$ m

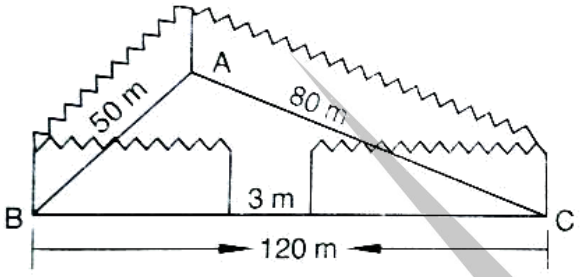
Area of parallelogram, AECD = base × height = 10 × $\frac{168}{15}$ = 56 × 2 = 112 m^2

$\therefore$ Area of trapezium ABCD = Area of parallelogram

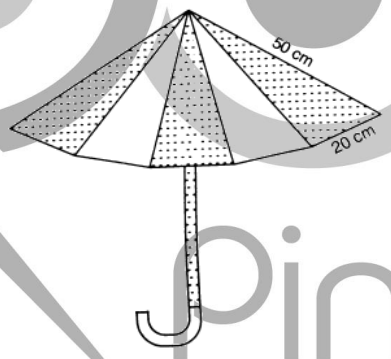
AECD + Area of ΔBCE = 112 + 84 = 196 m^2.

EXERCISE – 1

1. The lengths of the sides of a triangle are 5 cm, 12 cm and 13 cm. find the length of perpendicular from the opposite vertex to the side whose length is 13 cm.
2. A triangular park ABC has sides 120m, 80 m and 50 m. (shown in fig.). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of Rs. 20 per metre leaving a space 3m wide for a gate on one side.



3. In a Δ ABC, AB = 15 cm, BC = 13 cm and AC = 14 cm. find the area of Δ ABC and hence its altitude on AC.
4. The perimeter of a triangular field is 540 m and its sides are in the ratio 25 : 17 : 12 . Find the area of the triangle.
5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be grazing?
6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colors (shown in fig,) each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each color is required for the umbrella?

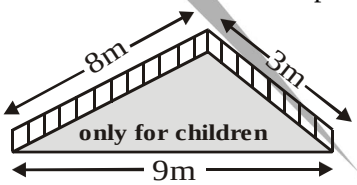


7. A park, in shape of a quadrilateral ABCD, has $\angle C = 90^0$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 ml. How much area does it occupy?
8. A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs. 5 per m^2. Find the cost of painting.
9. Find the area of a quadrilateral ABCD in which AD= 24 cm, $\angle BAD = 90^0$ and BCD forms an equilateral triangle whose each side is equal to 26 cm. (take $\sqrt{3} = 1.73$)
10. The adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. find the area of the parallelogram.
11. Find the area of a triangle, two of whose sides are 18 cm and 10 cm and perimeter is 42 cm.
12. Find the area of a triangle two of whose sides are 8 cm and 11 cm and perimeter is 32 cm. Also find the altitude corresponding to the base 11 cm.
13. Sides of triangle ABC are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.
14. An isosceles triangle has perimeter 30 cm and each of the equal is 12 cm. Find the area of the triangle .

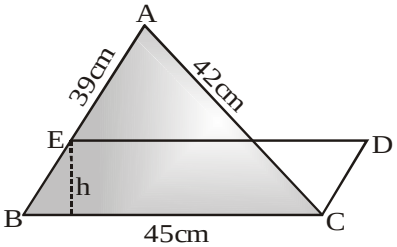
15. A triangular field has dimensions 24 m, 7 m and 25 m. It has a road running around it so that the triangular region including road has dimensions 26 m, 9 m and 27 m. Find the cost of fitting tiles on the road at the rate of Rs. 80 per m^2. (use $\sqrt{34100} = 58.40$)
16. The perimeter of a right angled triangle is 60 cm. Its hypotenuse is 26 cm, find the other two sides and the area of triangle. Verify the result using Heron's formula.
17. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.
18. A park, in the shape of a quadrilateral ABCD has $\angle C = 90^o$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?
19. If the perimeter of a triangle is 300 cm and its sides are in the ratio 5 : 12 : 13, find area of the triangle.
20. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 6 cm, DA = 5 cm and diagonal AC = 5 cm.
21. If the area of an equilateral triangle is $81\sqrt{3}$ cm^2, find its height.
22. If the area of an equilateral triangle $36\sqrt{3}$ cm^2, find its perimeter.
23. Using Heron's formula find the area of an isosceles triangle whose one of the equal sides is 16 cm and third side is 10 cm.
24. The perimeter of a right triangle is 144 cm and its hypotenuse measures 65 cm. Find the lengths of other sides and calculate its area. Verify the result using Hero's formula.
25. The base of an isosceles triangle is 14 cm and one of its equal sides is 12 cm. Find its area using Hero's formula.
26. An isosceles right triangle has an area 200 cm^2. What is the length of its hypotenuse?
27. The perimeter of a right triangles is 12 cm and its hypotenuse is of length 5 cm. Find the other two sides and calculate its area.
28. Find the base of an isosceles triangle whose area is 12 cm^2 and one of the equal sides is 5 cm.
29. The lengths of the sides of triangle ABC are in the ratio 4 : 3 : 5, and its perimeter is 144 cm. Find the height corresponding to the longest side.

EXERCISE – 2

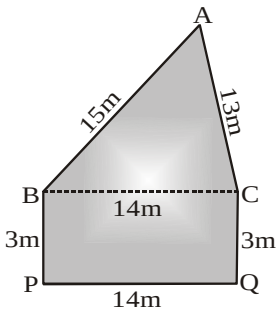
1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?
2. Find the percentage increase in the area of a triangle if its each side is doubled.
3. A triangle has sides 35 cm, 54 cm and 61 cm long. Find its area. Also, find the smallest of its altitudes.
4. The perimeter of an isosceles triangle is 42 cm and its base is (3/2) times each of the equal sides. Find the length of each side of the triangle, area of the triangle and the height of the triangle.
5. A field is in the shape of a trapezium whose parallel sides are 25 m and 10m. The non – parallel sides are 14 m and 13 m. find the area of the field.
6. Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.
7. Kamla has a triangular field with sides 240 m, 200m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the pervious field she wanted to grow potatoes and onions as shown in fig. She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 1000 m^2)
8. Radha made a picture of an aeroplane with coloured paper as shown in fig., Find the total area of the paper used.
9. Two parallel side of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm. find the area of the trapezium.
10. Find the perimeter and area of the quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, $\angle$ ACB = 90^0 and AC = 15 cm.
11. A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown in fig. The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.
12. There is a slide in a children park. The front side of the slide has ben painted and a message "ONLY FOR CHILDREN" is written on it as shown in figure. If the sides of the triangular front wall of the slide are 9 m, 8 m and 3 m, then find the area which is painted in colour.



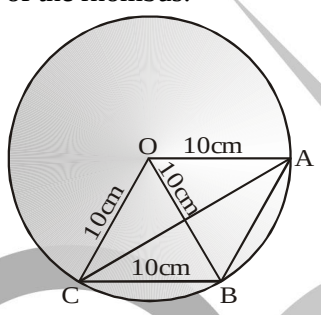
13. The sides of a triangle are 39 cm, 42 cm and 45 cm. A parallelogram stands on the greatest side of the triangle and has the same area as that of the triangle. Find the height of the parallelogram.



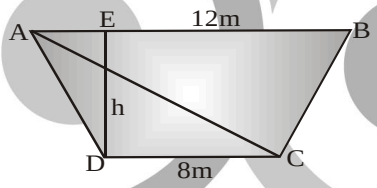
14. A municipal corporation wall on road side has dimensions as shown in fig. The wall is to be used for advertisements and it yields an earning of Rs. 400 per m^2 in a year. Find the total amount of revenue earned in a year.



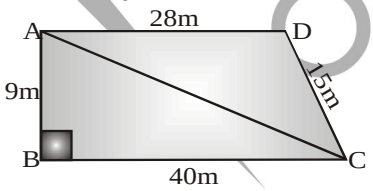
15. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm, find the area of the rhombus.



16. The cross-section of a canal is in the shape of a trapezium. If the canal is 12 m wide at the top and 8 m wide at the bottom and the area of its cross-section is 84 m^2, determine its depth.



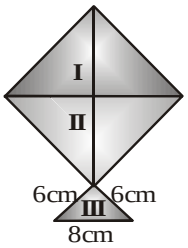
17. Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA ; while the other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m. CD = 15 m, DA = 28 m and □B = 90°, which group cleaned more area and by how much? Find the total area cleaned by the students.



18. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN". If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

19. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours, each piece measuring 20cm, 50cm and 50cm. How much cloth of each colour is required for the umbrella?

20. A kite in the shape of a square with a diagonal 32cm and an isosceles triangle of base 8cm and sides 6cm each is to be made of three different shades as shown in fig. How much paper of each shade has been used in it?

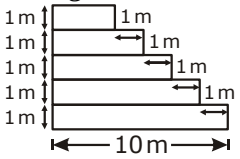


21. Using Heron's Formula, find the area of an isosceles triangle, the measure of one of its equal sides being a units and the third side 2b units
22. From a point in the interior of an equilateral triangle perpendiculars drawn to the three sides are 8 cm, 10 cm and 11 cm respectively. Find the area of the triangle to the nearest cm. (use $\sqrt{3} = 1.73$)
23. ABCD is a quadrilateral such that AB = 5 cm, BC = 4 cm, CD = 7cm, AD = 6 cm and diagonal BD = 5 cm. Prove that the area of the quadrilateral ABCD is $4\left(3+\sqrt{6}\right)cm^2$.
24. Find the area of the quadrilateral ABCD in which AB = 7 cm, BC = 6 cm, CD = 12 cm, DA = 15 cm and AC = 9 cm. $\left(Take\ \sqrt{110} = 10.5\ approx.\right)$
25. A rhombus has perimeter 64 m and one of the diagonals is 22 m. Prove that the area of the rhombus is $66\sqrt{15}\,m^2$
26. A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°$, AB = 9m, BC = 12m, CD = 5m and AD = 8m. How much area does it occupy?
27. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30m and its longer diagonal is 48m, how much area of grass field will each cow be getting?

EXERCISE – 3

1. The height of an equilateral triangle is 6 cm. Its area is
   (a) $12\sqrt{3}$ cm^2 (b) $6\sqrt{3}$ cm^2 (c) $12\sqrt{2}$ cm^2 (d) 18 cm^2
2. The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is
   (a) 480 m^2 (b) 320 m^2 (c) 384 m^2 (d) 360 m^2
3. The sides of a triangle are in the ratio 5 : 12 : 13 and its perimeter is 150 cm. The area of the triangle is
   (a) 375 cm^2 (b) 750 cm^2 (c) 250 cm^2 (d) 500 cm^2
4. The lengths of the three sides of a triangle are 30 cm, 24 cm and 18 cm respectively. The length of the altitude of the triangle corresponding to the smallest side is
   (a) 24 cm (b) 18 cm (c) 30 cm (d) 12 cm
5. The base of an isosceles triangle is 16 cm and its area is 48 cm^2. The perimeter of the triangle is
   (a) 41 cm (b) 36 cm (c) 48 cm (d) 324 cm
6. Each of the equal sides of an isosceles triangle is 13 cm and its base is 24 cm. The area of the triangle is
   (a) 156 cm^2 (b) 78 cm^2 (c) 60 cm^2 (d) 120 cm^2
7. The base of a right triangle is 48 cm and its hypotenuse is 50 cm long. The area of the triangle is
   (a) 168 cm^2 (b) 252 cm^2 (c) 336 cm^2 (d) 504 cm^2
8. The difference between the semi-perimeter and the sides of a ΔABC are 8 cm, 7 cm and 5 cm respectively. The area of the triangle is
   (a) $20\sqrt{7}$ cm^2 (b) $10\sqrt{14}$ cm^2 (c) $20\sqrt{14}$ cm^2 (d) 140 cm^2
9. The perimeter of a triangular field is 144 m and ratio of the sides is 3 : 4 : 5. Then the area of the field is
   (a) 864 sq m (b) 764 sq m (c) 854 sq m (d) 754 sq m
10. One side of an equilateral triangle is 8 cm. Its area is
    (a) $16\sqrt{3}$ cm^2 (b) $12\sqrt{3}$ cm^2 (c) $8\sqrt{3}$ cm^2 (d) $4\sqrt{3}$ cm^2
11. The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Then its area is
    (a) 48 sq cm (b) 36 sq cm (c) 24 sq cm (d) 12 sq cm
12. Two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm. Then the area of parallelogram is
    (a) $5\sqrt{3}$ cm^2 (b) $10\sqrt{3}$ cm^2 (c) $15\sqrt{3}$ cm^2 (d) $20\sqrt{3}$ cm^2
13. Two adjacent sides of a parallelogram are 51 cm and 37 cm. One of its diagonals is 20 cm, then the its area is
    (a) 412 cm^2 (b) 512 cm^2 (c) 612 cm^2 (d) 712 cm^2
14. The sides of a triangle are in the ratio of 13 : 14 : 15 and its perimeter is 84 cm. Then the area of the triangle is
    (a) 136 cm^2 (b) 236 cm^2 (c) 336 cm^2 (d) 436 cm^2
15. The area of a parallelogram whose diagonals is 6.8 cm and the perpendicular distance of this diagonal from an opposite vertex is 7.5 cm is
    (a) 25.5 cm^2 (b) 11.9 cm^2 (c) 12.5 cm^2 (d) 51 cm^2
16. The adjacent sides of a parallelogram are 4 cm and 9 cm. The ratio of its altitudes is
    (a) 16 : 81 (b) 9 : 4 (c) 2 : 3 (d) 3 : 2
17. The perimeter of a rhombus is 52 cm and one of its diagonals is 24 cm. The length of the other diagonals is
    (a) 24 cm (b) 10 cm (c) $2\frac{1}{6}$ (d) 12 cm
18. In quadrilateral ABCD given that AB = 7 cm, BC = 12 cm, CD = 12 cm, DA = 9 cm and diagonals AC = 15 cm. It's area is
    (a) $(10\sqrt{34} + 54)$ sq cm
    (b) $(10\sqrt{34} - 54)$ sq cm
    (c) data insufficient
    (d) none of these

19. Adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm. Then the area of parallelogram is
(a) $8\sqrt{3}$ cm^2 (b) $9\sqrt{3}$ cm^2 (c) $10\sqrt{3}$ cm^2 (d) $12\sqrt{3}$ cm^2
20. The perimeter of a rhombus is 146 cm. One of its diagonals is 55 cm. Then the length of the other diagonal and the area of the rhombus is
(a) 48 cm, 1320 sq cm
(b) 45 cm, 660 sq cm
(c) 27.5 cm, 660 sq cm
(d) none of these
21. In a quadrilateral the sides are 9, 40, 28, 15 units and the angle between first sides is a right angle. The area of quadrilateral is
(a) 106 sq units (b) 206 sq units (c) 306 sq units (d) 406 sq units
22. In a quarilateral ABCD, AB = 7 cm, BC = 6 cm, CD = 12 cm, DA = 15 cm, AC = 9 cm. Its area is
(a) $(\sqrt{440}+54)$ sq cm (b) $(\sqrt{440}+44)$ sq cm
(c) $(\sqrt{110}+44)$ sq cm (d) $(\sqrt{340}+64)$ sq cm
23. The area of a rhombus is 28 cm^2 and one of its diagonals is 4 cm. Its perimeter is
(a) $4\sqrt{53}$ cm (b) $36\sqrt{53}$ cm (c) 2cm (d) none
24. The adjacent sides of a parallelogram are 8 cm and 9 cm. The diagonal joining the ends of these sides is 13 cm. Its area is
(a) 72 cm^2 (b) $12\sqrt{53}$ cm^2 (c) $2\sqrt{53}$ cm^2 (d) 150 cm^2
25. The sides of a triangle are 11 cm, 15 cm and 16 cm. The altitude to largest side is
(a) $30\sqrt{7}$ (b) $\frac{15\sqrt{7}}{2}$ cm (c) $\frac{15\sqrt{7}}{4}$ cm (d) 30 cm
26. The perimeter of a triangular field is 144 m and the ratio of the sides is 3 : 4 : 5. The area of the field is
(a) 864 m^2 (b) 468 m^2 (c) 824 m^2 (d) none
27. If the altitude of an equilateral triangle is cm, its area is
(a) $2\sqrt{3}$ cm^2 (b) $2\sqrt{2}$ cm^2 (c) $3\sqrt{3}$ cm^2 (d) $6\sqrt{2}$ cm^2
28. The area of triangle whose sides are 18 cm, 10 cm and 14 cm is
(a) 241 cm^2 (b) $21\sqrt{11}$ cm^2 (c) $21\sqrt{15}$ cm^2 (d) none of these
29. The cost of turfing a triangular field at the rate of Rs. 5 per sq. m is Rs. 1350. If the sides of the field are in the ratio 5 : 12 : 13, then the sides of the field are
(a) 5, 12, 13 (b) 10, 24, 26 (c) 15, 36, 39 (d) none of these
30. From a point in the interior of an equilaterial triangle, perpendiculars drawn to the three sides are of length 8 cm, 10 cm and 11 cm respectively. Then the area of triangle is
(a) $\frac{841\sqrt{3}}{3}$ cm^2
(b) $841\sqrt{3}$ cm^2
(c) $\frac{841}{3}$ cm^2
(d) cannot be found with given data
31. Area of the given figure is



(a) 50 m^2 (b) 40 m^2 (c) 45 m^2 (d) 48 m^2
32. The area of a rectangular field is 150 sq. units. If its perimeter is 50 units, its dimensions are
(a) 2,75 (b) 3,50 (c) 5,30 (d) 10,15
33. If each side of a rectangle is increased by 50%, its area will increase by

(a) 50% (b) 125% (c) 100% (d) none of these

34. The perimeter of a right angled triangle is 24 cm and its hypotenuse 10 cm. The area of the triangle is

(a) 240 cm^2 (b) 24 cm^2 (c) 120 cm^2 (d) 48 cm^2

35. The length of shortest altitude of triangle with sides 52 cm, 56 cm and 60 cm is

(a) 44.8 cm (b) 51.7 cm (c) 48 cm (d) none of these

36. A rhombus ABCD is such that AB = 10 cm and $\angle B = 120^o$. Then area of rhombus ABCD =

(a) 40 cm^2 (b) 400 cm^2 (c) 200 cm^2 (d) $50\sqrt{3}$ cm^2

37. The area of a right angled triangle is 600 sq. cm. If base of the triangle exceeds the altitude by 10 cm, the dimensions of the triangle are

(a) 120, 100, 130 (b) 30, 40, 60

(c) 30, 40, 50 (d) none of these

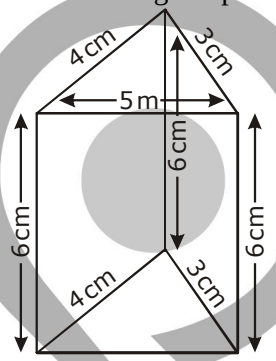
38. The parallel sides AB and DC of a trapezium ABCD are 15 cm and 25 cm respectively. If each of non-parallel sides is 10 cm, the area of trapezium is

(a) 100 cm^2 (b) $100\sqrt{3}$ (c) 300 cm^2 (d) none of these

39. A triangle has its sides as 13 cm, 14 cm and 15 cm. A parallelogram having double the area of this triangle stands on side 14 cm of this triangle. Then height of parallelogram is

(a) 12 cm (b) 10 cm (c) 8 cm (d) none of these

40. Total surface are of rectangular prism as shown in the figure is



(a) 20 cm^2 (b) 12 cm^2 (c) 96 cm^2 (d) none of these

41. A diagonal of a square field is 40 m, then area of the field is

(a) 800 m^2 (b) 1600 m^2 (c) 400 m^2 (d) none of these

42. The sides of a triangle are 35 cm, 54 cm and 61 cm respectively. The length of its longest altitude is

(a) $16\sqrt{5}$ cm (b) $10\sqrt{5}$ cm (c) $24\sqrt{5}$ cm (d) 28 cm

43. An isosceles right angled triangle has area 8 cm^2. The length of its hypotenuse is

(a) $\sqrt{32}$ cm (b) $\sqrt{16}$ cm (c) $\sqrt{48}$ cm (d) $\sqrt{24}$ cm

44. In a ΔABC it is given that base = 12 cm and height = 5 cm. Its area is

(a) 60 cm^2 (b) 30 cm^2 (c) $15\sqrt{3}$ cm^2 (d) 45 cm^2

45. The length of three sides of a triangle is 20 cm, 16 cm and 12 cm. The area of the triangle is

(a) 96 cm^2 (b) 120 cm^2 (c) 144 cm^2 (d) 160 cm^2

46. The base of an isosceles triangle is 8 cm long and each of its equal sides measures 6 cm. The area of the triangle is

(a) $16\sqrt{5}$ cm^2 (b) $8\sqrt{5}$ cm^2 (c) $16\sqrt{3}$ cm^2 (d) $8\sqrt{3}$ cm^2

47. The base of an isosceles triangle is 6 cm and each of its equal is 5 cm. The height of the triangle of the triangle is

(a) 8 cm (b) $\sqrt{30}$ cm (c) 4 cm (d) $\sqrt{11}$ cm

48. Each of the two equal sides of an isosceles right triangle is 10 cm long. Its area is

(a) $5\sqrt{10}$ cm^2 (b) 50 cm^2 (c) $10\sqrt{3}$ cm^2 (d) 75 cm^2

49. Each side of an equilateral triangle is 10 cm long. The height of the triangle is

(a) $10\sqrt{3}$ cm (b) $5\sqrt{3}$ cm (c) $10\sqrt{2}$ cm (d) 5 cm

EXERCISE – 4

1. The side of a regular hexagon is a. Its area is

   (a) $\frac{3\sqrt{3}}{2}$ a^2 sq. units (b) $\frac{\sqrt{3}}{2}$ a^2 sq. units

   (c) $2\sqrt{3}$ a^2 sq. units (d) $6a^2$ sq. units

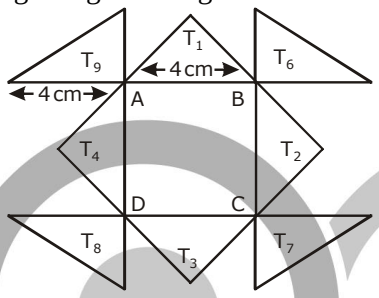
2. If every side of a triangle is doubled, the area of the new triangle is k times the area of the old one. k =

   (a) 2 (b) 4 (c) 3 (d) $\sqrt{2}$

3. The perimeter of a rhombus is 160 cm and one diagonal is 10 cm long. The length of other diagonal is

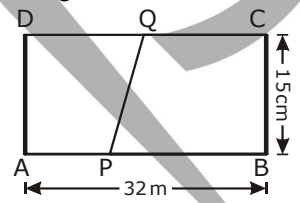
   (a) $50\sqrt{35}$ cm (b) $150\sqrt{7}$ cm (c) $15\sqrt{41}$ cm (d) $150\sqrt{10}$ cm

4. In the given figure ABCD is a square of side 4 cm. T_1, T_2, T_3, T_4 are equilateral triangles and T_5, T_6, T_7, T_8 are isosceles right angled triangles with base as 4 cm. Total area of figure is



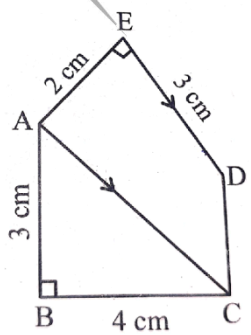
   (a) 16($\sqrt{3}$ +1) cm^2 (b) 16 $\sqrt{3}$ ($\sqrt{3}$ +1) cm^2 (c) 15($\sqrt{3}$ – 1) cm^2 (d) 48($\sqrt{3}$ + 1) cm^2

5. ABCD is a rectangle. A point P on AB and another point Q on DC is taken such that PB : QC = 3 : 1. If area of shaded region of $\frac{3}{8}$ times the area of rectangle then lengths PB and QC respectively are



   (a) 6 cm, 2 cm (b) 12 cm, 4 cm (c) 24 cm, 8 cm (d) none of these

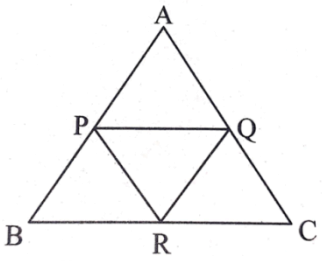
6. In the given figure, ABCDE is a pentagon in which <B = <E = 90°, AB = ED = 3 cm, BC = 4cm and AE = 2 cm. The area of pentagon is:



   (a) 21 cm² (b) 18 cm² (c) 14 cm² (d) 15 cm²

7. The sides of a triangle are in the ratio 6 : 8 : 9, then :
   (a) Angles of the triangle are in the ratio 6: 8: 9.
   (b) It is an acute angled triangle.
   (c) It is a right angled triangle.
   (d) It is an obtuse angled triangle.

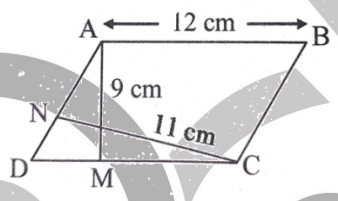
8. ABC is an equilateral triangle of side $4\sqrt{3}$ cm. P, Q and R are midpoints of AB, CA and BC respectively. The area of triangle PQR (in cm²) is:



(a) $\frac{\sqrt{3}}{4}$ (b) $3\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

9. Two parallel sides of a trapezium are 60 cm and 77 cm and other sides are 25 cm and 26 cm, then its area is:
(a) 1604 cm² (b) 1644 cm² (c) 1504 cm² (d) None of these

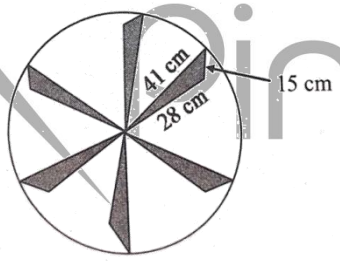
10. In parallelogram ABCD, AB = 12 cm. The altitudes corresponding to the sides DC and AD are respectively 9 cm and 11 cm. Then the length of AD is:



(a) $\frac{108}{11}$ cm (b) $\frac{108}{10}$ cm (c) $\frac{99}{10}$ cm (d) $\frac{108}{17}$ cm

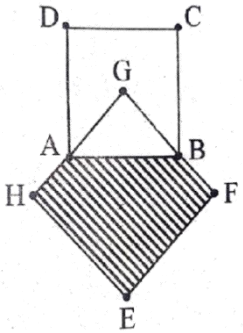
11. The area of a parallelogram ABCD in which AB = 12 cm, BC = 9 cm and diagonal AC = 15 cm is A cm². Then the value of $\frac{A-100}{4}$ is:
(a) 1 (b) 2 (c) 3 (d) 4

12. Two identical circles with same inside design as shown in the given figure are to be made at entrance. The identical triangular leaves are to be painted red the remaining are to be painted green. Find the total area to be painted red.



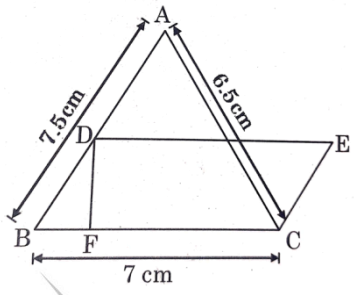
(a) 1266 cm² (b) 1546 cm² (c) 1512 cm² (d) 1682 cm²

13. Squares ABCD and EFGH are congruent, AB = 10 cm, and G is the centre of square ABCD. The area of the shaded region in the plane is:



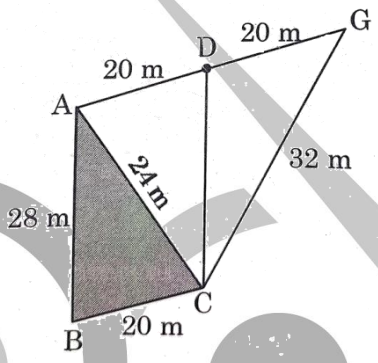
(a) 75 cm² (b) 100 cm² (c) 125 cm² (d) 175 cm²

14. In the given figure, ΔABC has sides AB = 7.5 cm, AC = 6.5 cm and BC =7 cm. On the base BC a parallelogram DBCE of area same as that of ΔABC is constructed. Find the height DF of the parallelogram.



(a) 3 cm (b) 5 cm (c) 6 cm (d) 7 cm

15. Find the ratio of the shaded area to the area of the quadrilateral ABCD.



(a) $2+\sqrt{6}:\sqrt{6}$ (b) $3:2+\sqrt{6}$ (c) $\sqrt{6}:2+\sqrt{6}$ (d) $\sqrt{6}:4+\sqrt{6}$

16. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m. the advertisements yield an earning of ₹ 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?

(a) ₹ 3300000 (b) ₹ 1650000 (c) ₹ 1600000 (d) ₹ 19800000

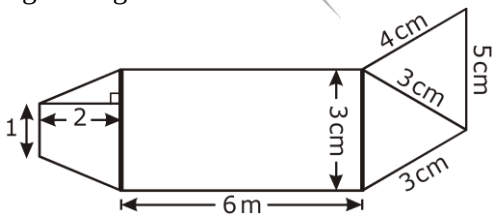
17. The sides of a triangle are 11 m, 60 m and 61 m. The altitude of the smallest side is –

(a) 11 m (b) 66 m (c) 60 m (d) 50 m

18. If the lengths of the sides of a triangle are in proportion 3 : 4 : 5, then the area of triangle is _____sq. units, where perimeter of the triangle is 144.

(a) 64 (b) 264 (c) 564 (d) 864

19. The area of the given figure is



(a) $24+\frac{9}{4}\sqrt{3}$ (b) $34+\frac{9}{4}\sqrt{3}$ (c) $28+\frac{9}{4}\sqrt{3}$ (d) none of these

20. Sides of triangle are in the ratio 13 : 14 : 15 and its perimeter is 84 cm. Find its area –

(a) 226 cm² (b) 412 cm² (c) 162 cm² (d) 336 cm²

ANSWER KEY

EXERCISE – 1

1. p= $\frac{60}{13}$ cm

2. Rs: 4940

3. 84 cm^2, 12 cm

4. 9000 m^2

5. 48m^2

7. 65.52 m^2

8. Rs. 625.00

9. 412.37 cm^2

10. 672 cm^2

11. $21\sqrt{11}$ cm^2

12. $8\sqrt{30}$ cm^2; $\frac{16\sqrt{30}}{11}$ cm

13. 9000 cm^2

14. $9\sqrt{15}$ cm^2

15. Rs. 2624

16. Sides = 24cm, 10 cm Area = 120 cm^2

17. 15.16 cm^2

18. 65.496 m^2

19. 3000 cm^2

20. 18 cm^2

21. $9\sqrt{3}$ *cm*

22. 36 cm

23. $5\sqrt{231}$ cm^2

24. 504 cm^2

25. $7\sqrt{95}$ cm^2

26. $20\sqrt{2}$ cm

27. 3 cm, 4 cm; 6 cm^2

28. 8 cm or 6 cm

29. 28.8 cm

EXERCISE – 2

1. $900\sqrt{3}$ cm^2
2. 300%
3. 939.14 cm^2, 3079 cm
4. 12 cm, 12 cm, 18 cm; 71.42 cm^2, 7.94 cm
5. 196m^2
6. $=57\sqrt{21}$ cm^2
7. 1.92 hectares.
8. 19.29 cm^2
9. 1644 cm^2
10. 46 cm, 114 cm^2
11. 840 cm^2 of paper of each type
12. $2\sqrt{35}$ m^2
13. 16.8 cm
14. Rs. 50400
15. $50\sqrt{3}$ cm^2
16. 8.4 m
17. I group cleaned more area by 54 m^2 ; 306 m^2
18. $20\sqrt{2}$ m^2
19. $1000\sqrt{6}\,cm^2$, $1000\sqrt{6}\,cm^2$
20. Area of shade I = Area of shade II, = 256 cm^2 and area of shade III = 17.92 cm^2
21. $b\sqrt{a^2-b^2}$ sq. units
22. 485 cm^2
24. 75 cm^2 (approx)
26. 65.5 m^2 (approx)
27. 48 m^2

EXERCISE – 3

Ques.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Ans.	a	c	b	a	b	c	c	c	a	a
Ques.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Ans.	a	b	c	c	d	b	b	a	c	a
Ques.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
Ans.	c	a	a	b	c	a	a	b	c	a
Ques.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
Ans.	b	d	b	b	a	d	c	b	a	d
Ques.	41.	42.	43.	44.	45.	46.	47.	48.	49.	
Ans.	a	a	a	b	a	b	c	b	b	

EXERCISE – 4

Ques.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Ans.	a	b	b	b	d	c	b	b	b	a
Ques.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Ans.	b	c	a	a	c	b	c	d	c	d