Challenger I

(1)

$$\sqrt{n} + \sqrt{n+1} < 11$$

Squaring both sides

$$\left(\sqrt{n} + \sqrt{n+1}\right)^2 < (11)^2$$

$$(n) + (n+1) + 2\sqrt{n}\sqrt{n+1} < 121$$

$$2\sqrt{n^2+n} < 120 - 2n$$

$$\sqrt{n^2+n} < 60 - n$$

Squaring both sides

$$\cancel{n^2} + n < 3600 + \cancel{n^2} - 120n$$

$$121n < 3600$$

$$n < \frac{3600}{121}$$

$$n < 29.752 \text{ (approx)}$$

So 29 ~~so 29~~ positive integers are possible

ans. 29

(2) positive integers ~~B~~ < 1000

where sum of the digits of each is divisible by 7 and the number itself is divisible by 3

as we know, a number is divisible by 3, if its sum of the digits is divisible by 3

so now the sum of the digits is divisible by 7 as well as 3 it means sum of the digits is divisible by lcm of 7 & 3 $\rightarrow$ 21

so sum of the digits can be 21, 42, 63, 84 - - - -

Now Case I

when the number is one digit

a one digit positive number can be from 1 to 9

so the sum is not multiple of 21

so no soln.

Case II when the number is two digit $\underline{a}\,\underline{b}$ type

now value of a can be from ~~0~~ 1 to 9

& value of b can be from 0 to 9

the highest possible sum of digits a+b is when both a & b are maximum i.e. a=9, b=9

so sum $a+b = 9+9 = 18$

not a multiple of 21 no soln

Case III when the number is three digit abc type

a can be from 1 to 9, b from 0 to 9, c from 0 to 9

sum $a+b+c$ has to be multiple of 21

first $a+b+c = 21$

let $a=9, b=9$, then $c = 21-18 = 3$

993, 939, 399 three cases as order matters

let $a=9, b=8$, then $c=4$

984 894 948 849 498 489 $\rightarrow$ 6 cases

let $a = 9$, $b = 7$, then $c = 5$

975, 795, 957, 759, 597, 579 $\to$ 6 Cases

let $a = 9$, $b = 6$ then $c = 6$

966, 696, 669 $\to$ 3 Cases

Now let $a = 8$, $b = 8$, then $c = 5$

885, 858, 588 $\to$ 3 Cases

let $a = 8$, $b = 7$, then $c = 6$

876, 867, 786, 687, 768, 678 $\to$ 6 Cases

let $a = 7$, $b = 7$, then $c = 7$

777 $\to$ one Cases

Now next multiple of 21 $\to$ 42

$a+b+c = 42$

if we take highest possible values of a, b, c

$9+9+9 = 27 < 42$ so no solⁿ.

so total Cases $\Rightarrow$ $3+6+6+3+3+6+1 = \underline{\underline{28}}$ ans.

(3) $K(3^3+4^3+5^3) = a^n$ where a & n are positive integers. with $n>1$

we have to find out smallest positive integer K

$K(27+64+125) = a^n$

$K(216) = a^n$

$K(6)^3 = a^n$

if we take smallest Positive integer $K=1$

then $(6)^3 = a^n$ means $a=6$, $n=3$

which satisfies all the Condition

so $K = 1$ ans.

(4) $k^2 < 2020 < (k+1)^2$ where k is a natural number

so if we take ~~k~~ $k = 44$

$(44)^2 < 2020 < (45)^2$

$1936 < 2020 < 2025$ which is true

so $k = 44 = 2 \times 2 \times 11 = 2^2 \times 11$

so largest prime factor of $k = \underline{\underline{11}}$ ans.

(5) unit digit of $4^{217} + 9^{217} + 6^{217} + 7^{217}$?

as we know when we add two numbers $x = (---dcba)$ and

$$\begin{array}{r} ----dcba \\ + \quad ---hgfe \\ \hline \end{array}$$

$y = (--hgfe)$ where d, c, b, a, h, g, f, e are digits, then in the resulting number the unit digit is the result of the sum of unit digits of numbers $(x \& y)$

so we need unit digits of $4^{217}, 9^{217}, 6^{217}, 7^{217}$ then their addition

Unit digit of $4^{217} \longrightarrow 4$

using cyclicity $4^1 \to 4$

$4^2 \to 16$

$4^3 \to 64$

$4^4 \to 256$

so $4\ 6\ 4\ 6\ 4\ 6 ----$

power is → odd

power is → even

$9^{217} \to 9$

$6^{217} \to 6$

$7^{217} \to 7$

so $4+9+6+7 = 2\underline{6}$

so unit digit is $\underline{6}$ ans.

(6)

the product $a^b \cdot b^a$ is divisible by 2000 (where a, b are positive integers)

so $a^b \cdot b^a$ is a multiple of 2000

let $k(2000) = a^b \cdot b^a$

$K(2^4 \times 5^3) = a^b \cdot b^a$

as ab is least

$\Rightarrow$ $k(4^2 \times 5^3) = a^b \cdot b^a$

if we take $a = 4$, $b = 5$

then K has to be $4^3 \times 5^1$

so $(4^3 \times 5^1)(4^2 \times 5^3) = a^b \cdot b^a$

$4^5 \times 5^4 = a^b \cdot b^a$

$a = 4$, $b = 5$ so $ab = \underline{20}$ ans.

(7)

last digit is 6, & if this is moved to the front of the number, the number becomes 4 times larger

Case I if two digit number, so the number is $\underline{a6}$ type where a, & 6 are digits

now
$$\begin{array}{r} a6 \\ \times 4 \\ \hline 6a \end{array}$$

$4 \times 6 = 2\underline{4}$
so $\underline{a}$ has to be 4

so
$$\begin{array}{r} 46 \\ \times 4 \\ \hline 184 \end{array} \neq \underline{64}$$

in this we will get each succeeding digits from right hand side

so now
$$\begin{array}{r} c46 \\ \times 4 \\ \hline --84 \end{array}$$
means $\underline{c = 8}$

so now
$$\begin{array}{r} d846 \\ \times 4 \\ \hline 384 \end{array}$$
means $d = 3$

so now
$$\begin{array}{r} e3846 \\ \times 4 \\ \hline 5384 \end{array}$$
means $e = 5$

so now
$$\begin{array}{r} f53846 \\ \times 4 \\ \hline 15384 \end{array}$$
means $f = 1$

so now
$$\begin{array}{r} g153846 \\ \times 4 \\ \hline 615384 \end{array}$$
~~so g~~

as we can see the condition is now satisfied so need of $\underline{g}$

so the required number is $\underline{\underline{153846}}$ ans.

(8)

$$\frac{5}{x} + \frac{6}{y} = 1$$

& x & y are natural numbers

and $\underline{x > y}$

$$\Rightarrow \frac{5}{x} = 1 - \frac{6}{y}$$

$$\Rightarrow \frac{5}{x} = \frac{y-6}{y}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{y-6}$$

$$\Rightarrow x = \frac{5y}{y-6}$$

$$x = \frac{5y}{y-6}$$

now we have to check y's value (as y is a natural number) so that x is also a natural number & $x > y$

now if $y = 1, 2, 3, 4, 5$

$x = -ve$

if $y = 6 \Rightarrow x = \frac{5 \times 6}{6-6}$ = not ~~df~~ define

if $y = 7$ $x = \frac{5 \times 7}{1} = 35$ $(35 > 7)$ so possible

if $y = 8$ $x = \frac{5 \times 8}{2} = 20$ $(20 > 8)$ so possible

if $y = 9$ $x = \frac{5 \times 9}{3} = 15$ $(15 > 9)$ so possible

if $y = 10$ $x = \frac{5 \times 10}{4}$ = not a natural number so not possible

if $y = 11$ $x = \frac{5 \times 11}{5} = 11$ (now $11 = 11$) so not possible as $x > y$

now if we increases y further x's value will be less than y

so 3 possible answers

(9)

if $\frac{1}{\sqrt{2011 + \sqrt{2011^2 - 1}}} = \sqrt{m} - \sqrt{n}$ where m & n are positive integers

$$\Rightarrow \frac{1}{\sqrt{2011 + \sqrt{2010 \times 2012}}}$$

$a^2 - b^2 = (a-b)(a+b)$

$$= \frac{1}{\sqrt{2011+\sqrt{2010 \times 2012}}}$$

$$= \frac{1}{\sqrt{2011+2\sqrt{1005 \times 1006}}}$$

$$= \frac{1}{\sqrt{1005+1006+2\sqrt{1005}\sqrt{1006}}}$$

$$= \frac{1}{\sqrt{(\sqrt{1006}+\sqrt{1005})^2}}$$

$$= \frac{1}{\sqrt{1006}+\sqrt{1005}} \times \frac{\sqrt{1006}-\sqrt{1005}}{\sqrt{1006}-\sqrt{1005}}$$

rationalize this

$$= \frac{\sqrt{1006}-\sqrt{1005}}{1006-1005} = \sqrt{1006}-\sqrt{1005}$$

$$= \sqrt{m}-\sqrt{n}$$

so $m = 1006$

$n = 1005$

so $m+n = 2011$ ans.

10 $a+1 = b+2 = c+3 = d+4 = e+5 = a+b+c+d+e+3$

a, b, c, d, e. are real numbers.

let $k = a+1 = b+2 = c+3 = d+4 = e+5 = a+b+c+d+e+3$

so $a+1 = k$

$a = k-1$ —(i)

$b+2 = k$

$b = k-2$ —(ii)

$c+3 = k$

$c = k-3$ —(iii)

$$d + 4 = K$$
$$d = K - 4 \quad -(iv)$$
$$e + 5 = K$$
$$e = K - 5 \quad -(V)$$

now $\quad a + b + c + d + e + 3 = K \quad -(VI)$

put a, b, b, d, e, values from eqn (i), (ii), (iii), (iv), (V) in terms of K in eq (VI)

$$K - 1 + K - 2 + K - \cancel{3} + K - 4 + \cancel{K} - 5 + \cancel{3} = K$$
$$4K = 12$$
$$K = 3$$

so $\quad a = K - 1 = 3 - 1 = 2 \qquad d = K - 4 = 3 - 4 = -1$
$\quad b = K - 2 = 3 - 2 = 1 \qquad e = K - 5 = 3 - 5 = -2$
$\quad c = K - 3 = 3 - 3 = 0$

so $\quad a^2 + b^2 + c^2 + d^2 + e^2 = 2^2 + 1^2 + 0^2 + (-1)^2 + (-2)^2$
$$= \underline{10} \quad \underline{\text{ans.}}$$

(11)

15549#325 is divisible by 3

it will happen when the sum of the digits is divisible by 3

so $\quad 1 + 5 + 5 + 4 + 9 + \# + 3 + 2 + 5$ is divisible by 3

$\Rightarrow \quad 34 + \#$ is divisible by 3

& # is a digit which can take value from 0 to 9

so if # 2 then $34 + 2 = 36$ divisible by 3

if # 5, then $34 + 5 = 39$ divisible by 3

if # 8, then $34 + 8 = 42$ divisible by 3

so sum $\quad 2 + 5 + 8 = \underline{\underline{15}} \quad \underline{\underline{\text{ans.}}}$

(12) total people = 20

for the first person there are 19 othe people, ~~whome~~ with whom, he can sake his hand,

so total hand sakes = 19,

now suppose he left (as he ~~also~~ has done his job)

now there are total people = 19

for the second person there are 18 other people

so total hand sake = 18

in this way total hand sake

$= 19+18+17+16+ \cdots +1$

$= \frac{n(n+1)}{2} = \frac{19 \times \cancel{20}^{10}}{\cancel{2}} = \underline{\underline{190}}$

(13) total digits = 107

↳ Consist of 58 natural number

means.

number (x) = 1 2 3 4 5 6 7 8 9 10 11 12 - - - - - - 56 57 58

we have to find out remainder when this is divided by 8

as we know for a number to check divisibility by 8

we need last three digits so last three digits = 758

so now ~~so~~ our question is ~~now~~ remainder when 758 is divided by 8 which is equal to $\underline{\underline{6}}$ ans.

(14)

$$x^2+y^2=2015$$

$$y^2=2015-x^2$$

as sum = odd, so one of them has to be odd and one has to be even

and as this equation is symmetric

so we need to check $x \rightarrow 1$ to 31

as $(31)^2 = 961$

$$(961)+961 < 2015$$

& we get there are no integers for which $x^2+y^2=2015$

so $\underline{0}$ ans.

(15)

natural number $< 10^7$

which have exactly 77 divisors

$$77 = 7 \times 11$$

as we know for a number $p_1^x \times p_2^y \times p_3^z$ total number of divisors $= (x+1)(y+1)(z+1)$ (where p_1, p_2, p_3 are prime numbers)

so our number has to be

$$p_1^x p_2^y$$

$$(x+1)(y+1) = 7 \times 11$$

$$x = 6, \; y = 10$$

$$(p_1)^6 (p_2)^{10} < (10)^7$$

$$(p_1)^6 (p_2)^{10} < 2^7 \times 5^7$$

if $p_1 = 2, \; p_2 = 3$ $\qquad (2)^6 (3)^{10} < (10)^7$

and $p_1 = 3, \; p_2 = 2$ $\qquad (3)^6 (2)^{10} < (10)^7$

so $\underline{\underline{2}}$ $\underline{\underline{\text{ans.}}}$