DPP: 5
Subject: Mathematics
Topic: Limits

1. $\lim_{x\to 0} \frac{1-\cos^3 x}{x \sin x \cos x}$

2. $\lim_{x\to 0} \frac{2\sin x - \sin 2x}{x^3}$

3. $\lim_{x\to 0} \frac{\sin(2+x) - \sin(2-x)}{x}$

4. $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$

5. $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

6. $\lim_{h\to 0} \left[\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}\, h\, (\sqrt{3}\cos h - \sin h)} \right]$

7. $\lim_{x\to 0} \frac{\tan[e^2]\, x^2 - \tan[-e^2]\, x^2}{\sin^2 x}$

8. $\lim_{h\to 0} \frac{\cos^2(x+h) - \cos^2 x}{h}$

9. $\lim_{x\to 0} \frac{8}{x^8}\left[1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$

10. $\lim_{x\to 0}\left(\frac{\sin(a+3h) - 3\sin(a+2h) + 3\sin(a+h) - \sin a}{h^3}\right) =$

(A) 1 (B) 0 (C) -2 (D) -cosa

11. $\lim_{x\to 0}\left(\frac{1-\cos(1-\cos x)}{x^4}\right) =$

(A) 1/8 (B) 1/16 (C) 1/24 (D) N.O.T

12. $\lim_{x\to 0} \frac{\sqrt[3]{1+\sin x} - \sqrt[3]{1-\sin x}}{x}$

13. $\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

14. $m, n \in I^+$, then $\lim_{x\to 0} \frac{\sin x^n}{\sin x^m} =$

(A) ∞, *if* $n < m$ (B) 1, *if* $n = m$ (C) $\frac{n}{m}$ (D) 0, *if* $n > m$

15. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

16. $\lim_{x\to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2} =$

(A) $\frac{\sin 2y}{2y}$ (B) $\frac{\cos 2y}{2y}$ (C) $\frac{\tan 2y}{2y}$ (D) N.O.T

17. $\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi - 4x}$ where $x < \frac{\pi}{4}$

18. $\lim_{x\to \pi/2} \frac{\sin(\cos x)\cos x}{\sin x - \text{cosec } x}$

19. $\lim\limits_{0\to\pi/4}\dfrac{\left(\sqrt{2}-\cos\theta-\sin\theta\right)}{(4\theta-\pi)^2}$

20. $\lim\limits_{x\to-1}\dfrac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$

21. If α,β are the roots of the quadratic equation $ax^2+bx+c=0$ then $\lim\limits_{x\to\alpha}\left(\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}\right)=$

(A) 0 (B) $\dfrac{(\alpha-\beta)^2}{2}$ (C) $\dfrac{a^2(\alpha-\beta)^2}{2}$ (D) $-\dfrac{a^2(\alpha-\beta)^2}{2}$

22. If α,β are the roots of $ax^2+bx+c=0$, then $\lim\limits_{x\to\frac{1}{\alpha}}\sqrt{\dfrac{1-\cos^2\left(cx^2+bx+a\right)}{4(1-\alpha x)^2}}=$

(A) $\left|\dfrac{c}{2\alpha}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)\right|$ (B) $\left|\dfrac{c}{2\alpha}\left(\dfrac{1}{\alpha}-\dfrac{1}{\beta}\right)\right|$ (C) $\left|\dfrac{c}{2}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)\right|$ (D) Not possible

23. $\lim\limits_{x\to\pi/4}\dfrac{\operatorname{cosec}x-\sec x}{\cot x-\tan x}$

24. $\lim\limits_{x\to-\frac{\pi}{4}}\dfrac{1+\tan x}{\cos 2x}$

25. $\lim\limits_{x\to\pi/2}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$

26. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}$

27. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\cos x+\sin x)^3-2\sqrt{2}}{1-2\sin x\cos x}$

28. $\lim\limits_{x\to0}\dfrac{(\cos x)^{1/2}-(\cos x)^{1/3}}{\sin^2 x}$

29. $\lim\limits_{x\to\infty}x\left[\tan^{-1}\dfrac{x+1}{x+2}-\cot^{-1}\dfrac{x+2}{x}\right]$

30. $\operatorname{Lt}\limits_{x\to\infty}x\left(\tan^{-1}\dfrac{x+1}{x+2}-\dfrac{\pi}{4}\right)$

31. $\lim\limits_{x\to0}\dfrac{\tan^{-1}x-\sin^{-1}x}{\sin^3 x}$

32. $\lim\limits_{x\to0}\dfrac{1}{x\sin^{-1}x}-\dfrac{1-x^2}{x^2}$

33. $\lim\limits_{x\to0}x^{1/10}\sin(1/x)$

34. $\lim\limits_{x\to\infty}\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$

35. $\lim\limits_{x\to\frac{\pi}{2}}\tan^{-1}\left(\dfrac{\sin(a\tan^3 x+b\tan^2 x+c\tan x)}{a\tan^3 x+b\tan^2 x+c\tan x}\right)$

36. $\lim\limits_{x\to\frac{\pi}{2}}\sqrt{\dfrac{\tan x-\sin\left\{\tan^{-1}(\tan x)\right\}}{\tan x+\cos^2(\tan x)}}$

37. $\lim\limits_{x\to\infty}\left(\dfrac{x^2\sin\frac{1}{x}-x}{1-|x|}\right)$

38. $f(x)=\begin{cases}x\sin\left(1/x\right) & x\neq0\\ 0 & x=0\end{cases}$. Then prove that $\lim\limits_{x\to0}f(x)=0$

39. $\lim\limits_{x\to0}\dfrac{e^{x^2}-\cos x}{x^2}$

40. $\lim\limits_{x\to0}\dfrac{64^x-32^x-16^x+4^x+2^x-1}{\left(\sqrt{15+\cos x}-4\right)\sin x}$

41. $\lim_{x\to 0}\dfrac{-1+\sqrt{(\tan x-\sin x)+\sqrt{(\tan x-\sin x)+\sqrt{(\tan x-\sin x)}.....to\infty}}}{-1+\sqrt{x^3+\sqrt{x^3+\sqrt{x^3+.....to\infty}}}}=$

(A) 1 (B) 1/2 (C) 2 (D) N.O.T

42. $\lim_{n\to\infty} n^2\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)}.....\infty}}=$

(A) 1/2 (B) 1/3 (C) 1/4 (D) DNE

43 Find $\lim_{n\to\infty}\dfrac{\sqrt{1-x_0^{\,2}}}{x_1x_2x_3x_4.....x_n}$, where $x_{r+1}=\sqrt{\dfrac{1+x_r}{2}}$

44. Let $P_n=\cos\frac{x}{2}\cdot\cos\frac{x}{2^2}\cdot\cos\frac{x}{2^3}\cdots\cos\frac{x}{2^n}$. Show that $P_n=\dfrac{1}{2^n}\sin x\cdot\operatorname{cosec}\dfrac{x}{2^n}$.

Hence, show that $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{2^r}\tan\left(\frac{x}{2^r}\right)=\frac{1}{x}-\cot x.$

45. Let $a_1, a_2, ..., a_n$ be sequency of real numbere with $a_{n+1}=a_n+\sqrt{1+a_n^2}$ and $a_0=0$.

Prove that $\lim_{n\to\infty}\left(\dfrac{a_n}{2^{n-1}}\right)=$

46. Let $a=\min\{x^2+2x+3, x\in R\}\ \&\ b=\lim_{\theta\to 0}\dfrac{(1-\cos\theta)}{\theta^2}$. Then the value of $\sum_{r=0}^{n}a^r.b^{n-r}=$

(A) $\dfrac{4^{n+1}-1}{3.2^n}$ (B) $\dfrac{2^{n+1}-1}{3.2^n}$ (C) $\dfrac{2^{n+1}-1}{2^n}$ (D) N.O.T

47. $f(x+y)=f(x)+f(y)\forall x,y\in R\ \&\ f(1)=1$ Then the value of $\lim_{x\to 0}\dfrac{2^{f(\tan x)}-2^{f(\sin x)}}{x^2.f(\sin x)}=$

(A) log2 (B) (log2)/2 (C) 2log2 (D) N.O.T

48. If $2f(\sin x)+\sqrt{2}f(\cos x)=\tan x$ then find $\lim_{x\to 1/2}\sqrt{1-xf(x)}$

49. $L=\lim_{x\to 0}\dfrac{\frac{\sqrt{\cos 2x+(1+3x)^{1/3}}}{2}-\frac{\sqrt[3]{4\cos^3 x-\ln(1+x)^4}}{4}}{x}$ If L = a/b where 'a' and 'b' are relatively primes find (a + b).

50. $f(x)$ is the function such that $\lim_{x\to 0}\dfrac{f(x)}{x}=1$. If $\lim_{x\to 0}\dfrac{x(1+a\cos x)}{(f(x))^3}=1$, then find the value of a and b.

51. Let $(\tan\alpha)x+(\sin\alpha)y=\alpha$ and $(\alpha\operatorname{cosec}\alpha)x+(\cos\alpha)y=1$ be two variable striaght lines, a being the paramater. Let P be the point of intersection of the lines. In the limiting position when $\alpha\to 0$. Then find the point of intersection of straight lines.

Answer Key

1. $\frac{3}{2}$ **2.** 1 **3.** 2 cos 2 **4.** $\frac{1}{2}$ **5.** π **6.** $\frac{4}{3}$ **7.** 15 **8.** – sin x **9.** 1/32 **10.**D **11.**A

12. $\frac{2}{3}$ **13.** $a^2 \cos a + 2a \sin a$ **14.**ABD **15.** $\frac{2}{\pi}$ **16.**A **17.** $\frac{1}{4}$ **18.** –1 **19.** $\frac{1}{16\sqrt{2}}$ **20.** $\frac{1}{\sqrt{2\pi}}$

21.C **22.**B **23.** $\frac{1}{\sqrt{2}}$ **24.** 1 **25.** $\frac{1}{12}$ **26.** $\frac{1}{2}$ **27.** No Answer **28.** 1 / 6 **29.** $\frac{1}{2}$ **30.** $-\frac{1}{2}$