KINEMATICS

1. KINEMATICS

When we observe the nature, we see many objects in motion. From experience we recognize that motion represents the continuous change in position of an object with time.

When we study **mechanics** the first step is to know the details of motion, which is **dynamics** part of the mechanics. The other part is **statics,** which deals with body at rest. We study dynamics in two parts: **Kinematics & Kinetics**. In kinematics we study the motion in terms of space, and time without knowing the cause of motion and in kinetics we include the cause of motion.

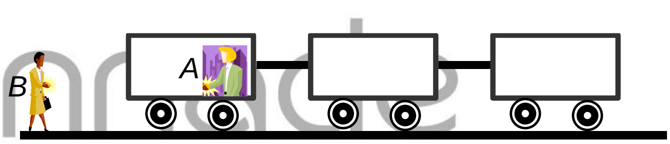
The present lesson is the study about motion irrespective of its cause i.e., **Kinematics**. In physics we study three types of motion: transnational, rotational and vibrational. A car moving on a highway is undergoing transnational motion, the earth's daily spin on its axis is an example of rotational motion and back-and-forth motion of a pendulum is an example of vibrational motion.

1.1 MOTION AND REST

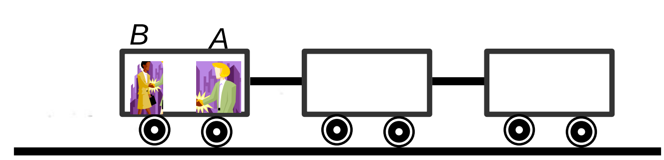
Whether a body or particle is in motion or at rest, it depends upon the frame of reference of the observer. If an object changes its position in space with time relative to an observer, then it is said to be in motion, otherwise at rest. Now we should know "What is frame of reference?"

Frame of Reference

Suppose a person '*B*' who is on the ground observes a person '*A*' who is inside of a moving train. *B* observes that position of *A* is changing, so *A* is in motion



When *B* is also inside the train in the same compartment and train is still in motion, *B* observes that position of *A* is not changing. So *A* is at rest.



We can conclude that whether an object is at rest or motion, it depends from where the object is observed which is called frame of reference.

Here in the first case ground is frame of reference and in the second case, train is frame of reference.

There is no rule or restriction on the choice of a reference frame. We can choose it according to our convenience.

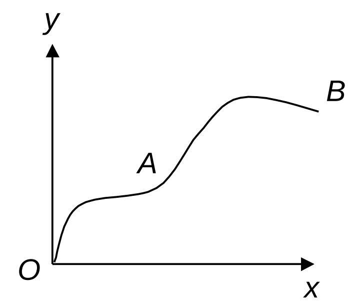
1.2 Distance (PATH LENGTH) AND Displacement

Distance: It is defined as the actual length of path covered by an object.

It is a +ve scalar quantity.

Its SI unit is m (metre).

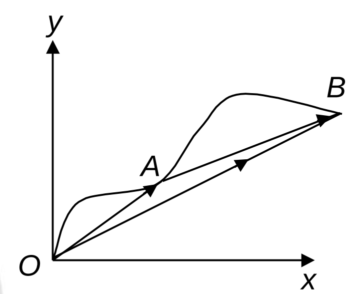
Suppose a particle moves from A to B as shown in figure. The length of curved path from A to B will be distance travelled by that particle.



Displacement: Displacement is the change in position vector i.e., vector joining initial and final position, or we can say it is a minimum possible distance between two positions.

It is a vector quantity.

If a particle moves from A to B in a given reference frame as shown in figure, then



Position vector of $A = \vec{OA} = \vec{r}_1$

Position vector $B = \vec{OB} = \vec{r}_2$

Displacement from position A to $B = \vec{AB}$

$\vec{AB} = \vec{OB} - \vec{OA} = \vec{r}_2 - \vec{r}_1$

1.3 DIFFERENCE BETWEEN DISTANCE AND DISPLACEMENT

Distance	Displacement
1. Distance is length of actual path travelled by the particle	**1.** Displacement is minimum possible distance between two points.
2. It is a scalar quantity	**2.** It is a vector quantity
3. It can have more than two values between any two points	**3.** It has a unique value between any two points
4. It can never decrease with time	**4.** It can decrease with time
5. For a moving particle distance > 0	**5.** For a moving particle displacement >, <, = or 0.

1.4 SPEED

Speed: It is defined as the rate of change of distance, with respect to time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{S}{T} \qquad \dots (1)$$

It is a scalar quantity.

Its S.I. unit is m/s and dimensions is $[LT^{-1}]$

(a) Instantaneous Speed

Instantaneous speed of a particle is speed at a particular instant of time. If ΔS is the distance travelled by a particle in a time interval Δt, then instantaneous speed is the limiting value of $\frac{\Delta S}{\Delta t}$ as Δt approaches zero.

Instantaneous speed (v) = $\underset{\Delta t \to 0}{Lt} \frac{\Delta S}{\Delta t}$

In calculus notation, this limit is called the derivative of s with respect to time, written as ds/dt

Instantaneous speed (v) = $\frac{ds}{dt}$ … (**2**)

(b) Average Speed

The average speed of a particle is defined as ratio of total distance travelled to the total time taken

Average Speed = $\frac{\text{Total Distance travelled}}{\text{Total time taken}}$

If a particle travels distances S_1, S_2, S_3 … etc in time t_1, t_2, t_3 …. etc., respectively.

Average speed = $\frac{S_1 + S_2 + S_3 \ldots S_n}{t_1 + t_2 + t_3 \ldots t_n}$

$$V_{avg} = \frac{\sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} t_i} \quad \ldots (\mathbf{3})$$

☞ *If a particle travels equal interval of distance at speeds v_1, v_2, v_3……… v_n respectively, then*

$$\frac{1}{v_{avg}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} \ldots\ldots + \frac{1}{v_n}\right]$$

Average speed is harmonic mean of individual speeds.

☞ *If a particle travels for equal intervals of time at speed v_1, v_2 …. v_n respectively, then*

$$V_{avg} = \frac{v_1 + v_2 + v_3 \ldots + v_n}{n}$$

In this case average speed is arithmetic mean of individual speed.

1.5 VELOCITY

It is defined as the rate of change in displacement with respect to time.

Velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{\vec{S}}{T}$ … (**4**)

It is a vector quantity.

Its S.I. unit is m/s and dimensions is $[LT^{-1}]$

(a) Instantaneous Velocity

Instantaneous velocity of a particle is velocity at a particular instant of time. It is defined as rate of change of particle's position with time

If the position of a particle changes by $\Delta \vec{r}$ in a small time interval Δt, the limiting value of $\frac{\Delta \vec{r}}{\Delta t}$ as Δt approaches zero, gives the instantaneous velocity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

In calculus notation, this limit is called the derivative of r with respect to time, written as $d\vec{r}/dt$

$$\therefore \qquad v = \frac{d\vec{r}}{dt} \qquad \dots (\mathbf{5})$$

(b) Average Velocity

The average velocity of a particle for a given interval of time is defined as the ratio of its displacement to the time taken.

$$\text{Average velocity} = \vec{v}_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta \vec{S}}{\Delta t} \qquad \dots (\mathbf{6})$$

Average velocity is independent of path taken between any two points. It depends only on the initial and final position of the particle since it depends on displacement.

If a particle starts from some point and returns to the same point, via any path, average velocity in this trip will be zero.

1.6 ACCELERATION

It is defined as rate of change of velocity with respect to time.

$$\text{Acceleration } (\vec{a}) = \frac{\text{change in velocity}}{\text{Time}} \qquad \dots (\mathbf{7})$$

It is a vector quantity

Its S.I. unit is m/s^2 and dimension is [LT^{-2}]

(a) Instantaneous acceleration

Instantaneous acceleration is the limiting value of average acceleration as Δt approaches to zero

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \dots (\mathbf{8})$$

Instantaneous acceleration is equal to the derivative of the velocity with respect to time.

Slope of the tangent on v–t graph gives the instantaneous acceleration.

Instantaneous acceleration also known as acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \ldots (9)$$

When the acceleration is constant, the average acceleration is equal to the instantaneous acceleration

(b) Average acceleration

It is defined as ratio of change in velocity to the time interval in which change takes place.

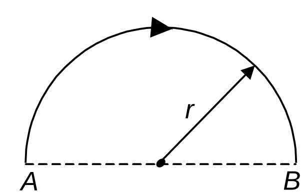
Suppose a particle moving along x-axis has velocity v_1 at time t_1 and velocity v_2 at time t_2 average acceleration a_{av} is given by

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad \ldots (10)$$

Illustration 1

***Question*:** **A person moves on a semi-circular path of radius 20 m as shown in figure.**

If he starts at one end of the path A and reaches other end B. Find the distance covered and displacement during this motion.



***Solution*:** Distance covered by the person

= the length of the path

= πr

= $\pi \times 20$ = **62.8 m**

The displacement = $2\,r$

= 2×20 = **40 m**

Illustration 2

Question: **If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, what will be average speed?**

***Solution*:** When the particle is going with speed v_1 and coming back with speed v_2, distance travelled by the particle in going and coming back is same, so average speed will be harmonic mean

So, $\frac{1}{v_{avg}} = \frac{1}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)$

$$v_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

Illustration 3

Question: **The distance travelled by particle in time t is given by $S = 3t^2 + 2$.**

(a) **Find the instantaneous speed at t = 3 s.**

(b) The average speed of the particle in 0 to 3 s

***Solution*:**

(a) instantaneous speed = $\frac{ds}{dt}$

Here, $S = 3t^2 + 2$

$$\frac{ds}{dt} = 6\,t$$

$= 6 \times 3 =$ **18 m/s**

(b) The distance travelled during time 0 to 3 s is

$S = 3\,(3^2) + 2 - 2 = 27$ m

$\therefore$ Average speed = $\frac{\text{Total distance travelled}}{\text{Total time of motion}} = \boldsymbol{\frac{27}{3}}$ **m/s**

Illustration 4

Question: **The position of an object moving along x-axis is given by $x = 2 + 3t^2$, where t is in seconds (a) what is its instantaneous velocity at t = 2s?(b) what is its average velocity at t = 2s and t = 4s?**

***Solution*:** (a) Instantaneous velocity $(v) = \frac{dx}{dt} = 0 + 6t$

$v_{t=2} = 6 \times 2 =$ **12 m/s**

(b) Average velocity $= \frac{x(4) - x(2)}{4-2} = \frac{\left(2 + 3(4)^2\right) - \left(2 + 3(2)^2\right)}{2}$ = **18 m/s**

2. MOTION

Motion is the change in position with respect to time in a given reference frame. There are different types of motion such as motion in a straight line (Translatory motion), projectile motion, circular motion, simple harmonic motion etc. On the basis of motion with respect to the co-ordinate axes has been catogerized into three types.

(1) One dimensional motion

(2) Two dimensional motion

(3) Three dimensional motion

One dimensional motion is that type of motion in which motion can be described along any one axis of co-ordinate system.

i.e., motion in a straight line

Two dimensional motion when a particle moves in a plane and it requires two axes of co-ordinate system to describe it.

i.e., circular motion, projectile motion.

Three-dimensional motion when a particle moves in space and it requires three axes of coordinate system to describe it.

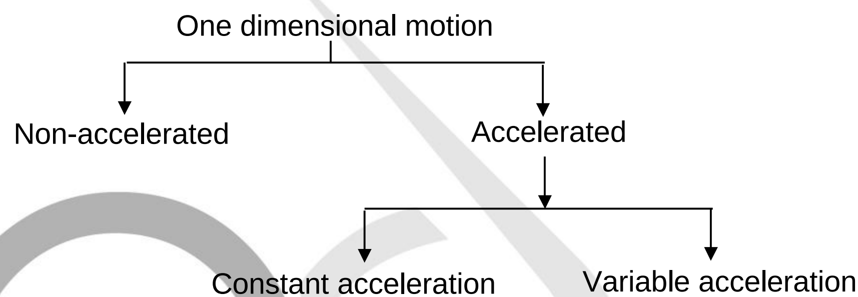
i.e., motion of an aeroplane, a balloon or a kite.

3. ONE DIMENSIONAL MOTION

One-dimensional motion can be classified in two parts

(i) Non-accelerated motion

(ii) Accelerated motion



3.1 NON-ACCELERATED MOTION

Such type of motion in which velocity is uniform, it means acceleration will be zero. In this type of motion

$$\left.\begin{aligned} \text{Distance} &= \text{Velocity} \times \text{time} \\ \text{Velocity} &= \frac{\text{Distance}}{\text{Time}} \end{aligned}\right\} \text{(acceleration should be zero)} \qquad \ldots (\mathbf{11})$$

3.2 ACCELERATED MOTION

If velocity of a particle varies with time then the motion is called accelerated motion. It can be classified in two parts:

(i) Uniformly accelerated motion

(ii) Non-uniformly accelerated motion

3.3 UNIFORMLY ACCELERATED MOTION

Such type of motion in which acceleration is constant or uniform.

In this type of motion the average acceleration equals the instantaneous acceleration.

If initial velocity v_i at time t_i changes to final velocity v_f at time t_f , then

$$\text{acceleration (a)} = \frac{v_f - v_i}{t_f - t_i}$$

Let when $t_i = 0$, $v_i = v_0$ and after time t, $v_f = v$

$$a = \frac{v - v_0}{t}$$

$$v - v_0 = at$$

$$v = v_0 + at \quad \text{(for constant } a\text{)} \qquad \dots (\mathbf{12})$$

This expression enables us to determine the velocity at any time t if the initial velocity, the acceleration, and the elapsed time are given.

From the above equation of motion we can say that velocity varies linearly with respect to time. Therefore the average velocity in any time interval can be expressed as the arithmetic mean of the initial velocity v_0, and final velocity v,

$$v_{avg} = \frac{v_0 + v}{2}$$

Now to find out the displacement as function of time, we can use this formula

$$\Delta s = v_{avg}\, \Delta t$$

Let us choose $t_1 = 0$ at which the initial position is $s_1 = 0$ and in any time t, displacement is s.

$$\Delta s = v_{avg}\, \Delta t$$

$$s - 0 = \frac{1}{2}(v + v_0)t$$

on putting $v = v_0 + at$

$$s - 0 = \frac{1}{2}(v_0 + at + v_0)t$$

$$s = \frac{1}{2}(2v_0 + at)\, t$$

$$s = v_0 t + \frac{1}{2} at^2 \qquad \dots (\mathbf{13})$$

This expression enables us to determine the distances travelled in any time t if the initial velocity, acceleration and the time elapsed are known.

Now to obtain an expression that doesn't contain time, we can substitute t from equation (12) in above equation

$$s = \frac{1}{2}(v + v_0)t$$

$$= \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$= \frac{1}{2}\left(\frac{v^2 - v_0^2}{a}\right)$$

$$v^2 - v_0^2 = 2as$$

$$v^2 = v_0^2 + 2as \qquad \dots (\mathbf{14})$$

Now we want to find out the distance travelled in n^{th} second.

Let distance travels in n second by a particle is S_n and in (n – 1) second is s_{n-1}, where initial velocity at t = 0 is v_0 and acceleration is a

$$s_n = v_0 n + \frac{1}{2} an^2$$

$$s_{n-1} = [v_0(n-1) - \frac{1}{2} a(n-1)^2]$$

$$s_n - s_{n-1} = (v_0 n + \frac{1}{2} an^2) - [v_0(n-1) - \frac{1}{2} a(n-1)^2]$$

$$S_{nth} = v_0 + \frac{a}{2}(2n-1) \qquad \ldots \textbf{(15)}$$

3.4 REACTION TIME AND STOPPING DISTANCE

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapse before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on an individual.

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

***Illustration* 5**

Question: **A car moving along a straight highway with speed of 126 kmh^{-1} is brought to rest within a distance of 200 m. What is the retardation of car (assumed uniform), and how long does it take for the car to stop?**

***Solution*:** Given Initial speed of car (v_0) = 126 km/h = 35 m/s

Distance travelled by the car (s) = 200 m

Final velocity of car (v) = 0

$$v^2 = v_0^2 + 2as$$

$$(0)^2 = (35)^2 + 2 \times a\,(200)$$

$$400\,a = -(35 \times 35)$$

$$a = -\left(\frac{35 \times 35}{400}\right) = \frac{-49}{16} = \mathbf{-3.06\ m/s^2}$$

Here, negative sign shows that the acceleration is opposite to the direction of motion and is called retardation.

So retardation = 3.06 m/s^2

Now retardation calculated above can be used to find the time,

$$v = v_0 + at$$

$$0 = 35 - 3.06\,t$$

$$t = \frac{35}{3.06} = 11.437 \text{ s}$$

So car will take **11.44 s**, to come to rest.

Illustration 6

Question: **A uniformly accelerating train passes successive kilometer stone with speed 18 km/hr and 27 km/hr, respectively. What will its speed be at the next kilometer stone and how long did it take to cover these 2 kilometres?**

Solution: Applying the equation

$v^2 = v_0^2 + 2ax$, we get

$(7.5)^2 = (5)^2 + 2a \times 1000$

$\therefore \quad a = \dfrac{(7.5)^2 - (5)^2}{2000}$ = 0.0156 ms^{-2}

The speed at next kilometer stone is given by

$v_1^2 = v^2 + 2ax$

$= (7.5)^2 + 2 \times 0.0156 \times 1000 = 87.45 \Rightarrow \quad v_1 = 9.35$ m/s

Time to cover the distance of 2 kilometre is obtained by

$t = \dfrac{v_1 - v_0}{a}$

$\Rightarrow \; t = \dfrac{9.35 - 5}{0.0156}$ = **278.8 s**

Illustration 7

Question: **A motorist while driving at a speed of 72 km/hr sees a boy standing on the road at a distance of 52 m. He applies the brake and stops his car at a distance of 2 m from the boy. Find the acceleration caused due to the application of brake and time taken to stop the car**

Solution: $v^2 = v_0^2 + 2ax$,

we get $0^2 = 20^2 + 2a \times 50$ or $a =$ **– 4 ms^{-2}**

Applying equation,

$v = v_0 + at$, we obtain

$0 = 20 + (-4)t$

or ***t* = 5s**

Illustration 8

Question: **The distance covered by a body during the 4th second is twice the distance covered by the body during the 2nd second of this journey. Find the initial velocity of the body. If its acceleration is 3 ms^{-2}.**

Solution: The distance covered in 2nd second is given by

$S_2 = v_0 + \dfrac{a}{2}(2 \times 2 - 1)$

The distance covered in the 4th second is given by

$$S_4 = v_0 + \frac{a}{2}(2 \times 4 - 1)$$

$$v_0 = \frac{a}{2} = \frac{3}{2} = \mathbf{1.5\ ms^{-1}}$$

3.5 MOTION UNDER GRAVITY

(i) Motion under gravity means an object is in motion in space under the force of gravity alone.

(ii) Motion under gravity is a uniformly accelerated motion. So equations of motion for uniformly accelerated motion can be used which are

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = v_0^2 + 2\vec{a}.\vec{s}$$

(iii) Here acceleration will be acceleration due to gravity.

In SI-unit $g = 9.8$ m/s^2

In c.g.s. unit $g = 980$ cm/s^2

(iv) When an object is thrown upward or downward, in both cases same acceleration '*g*' will be experienced by the object, which acts in downward direction.

(v) Here air resistance is neglected. In a real experiment air resistance cannot be neglected. It is an ideal case. Such motion is referred to as free fall.

Case 1:

When an object is thrown in upward direction (taking positive)space with initial velocity v_0.

Acceleration = $-g$ (in downward direction)

So, equation of motion will be

$v = v_0 - gt$

$h = v_0 t - \frac{1}{2} gt^2$

$v^2 = v_0^2 - 2gh$

Case 2:

When an object is thrown in downward direction (taking positive) in space with initial velocity v_0.

Acceleration = $+g$ (in downward direction)

So, equation of motion will be

$v = v_0 + gt$

$s = v_0 t + \frac{1}{2} gt^2$

$v^2 = v_0^2 + 2gh$

Let initial velocity is v_0 which is in vertically upward direction and finally comes down to the ground as shown in figure.

Case 3:

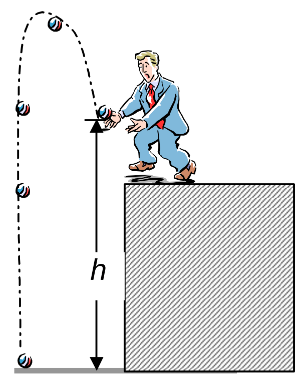
When an object is thrown in space in such a way that at first it goes up and then comes down.

To solve such types of problem, the following sign convention is used:-

Sign Convention

Motional quantities in upward-direction are taken as positive.

Motional quantities, which are in downward-direction, are taken as negative.



Initial velocity = $+v_0$ (in upward directions)

Displacement = $-h$ (in downward direction)

Acceleration = $-g$ (in downward direction)

So using equation of motion, $s = v_0 t + \frac{1}{2}at^2$, we have

$$-h = v_0 t - \frac{1}{2}gt^2$$

Illustration 9

Question: **A player throws a ball upwards with an initial speed of 29.4 m/s^{-1}**

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) To what height does the ball reaches and after how long does the ball return to the player's hands (Take g = 9.8 m/s^2 and neglect air resistance).

Solution:

(a) Direction of acceleration is downward.

(b) At highest point velocity of the ball will be zero and acceleration will be g = **9.8 m/s^2**

(c) The ball will go up till velocity will become zero

$v = 0$

$v_0 = 29.4$ m/s

$a = g = 9.8$ m/s in downward direction (opposite to the motion)

$v^2 = v_0^2 - 2gh$

$o^2 = (29.4)^2 - 2 \times 9.8$ h

$$h = \frac{29.4 \times 29.4}{2 \times 9.8}$$

= **39.6 m**

Let 't' be the time taken by the ball to return to the player's hands. We have,

$v_0 = 29.4$ m/s

$a = 9.8$ m/s^2

$s = 0$

$\therefore\ 0 = 29.4 \times t - \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow t = 29.4 \times 2 =$ **6 s**

Illustration 10

Question: **A rocket is fired vertically and ascends with a constant vertical acceleration of 10 ms^{-2} for 60 s. its fuel is then all used and it continues as a free particle. (a) What is the maximum altitude reached? (b) After how much time from then will the maximum height reached? (c) What is the total time elapsed from take off until the rocket strikes the earth? (*g* = 10 m/s²)**

Solution: (a) The distance traversed by the rocket till its fuel ends is given by

$h_1 = v_0 t + \frac{1}{2} a t^2 = 18000$ m.

(i) The velocity of the rocket when its fuel ends

$v = v_0 + a t_1 = 600\, ms^{-1}$

(ii) The height the rocket ascends after the fuel ends.

$v_f^2 - v^2 = -2gh \Rightarrow h = 18000$ m $\qquad \therefore$ Maximum height = **36000 m**

(b) The time to reach the maximum height after the burning of the fuel is given by the equation.

$v_f = v - gt^2$

$\therefore\ 0 = 600 - 10 t_2$ or $t_2 = 60$s.

(c) The total time elapsed from the moment the rocket takes off to its touching the ground is given by

$y = \frac{1}{2} g t_3^2$

$\therefore\ t = t_1 + t_2 + t_3 = 204.9$ s

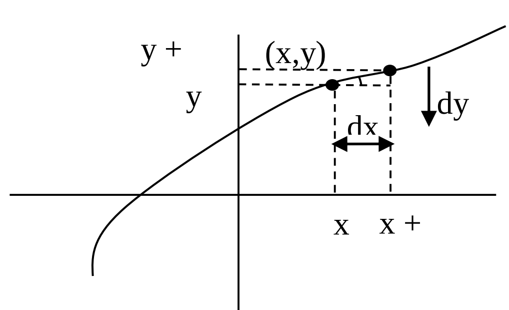
4. DIFFERENTIAL CALCULUS

4.1 MEANING OF DIFFERENTIATION

dy ≡ infinitesimal change in y; dx ≡ infinitesimal change in x

Change in x $\qquad \Delta x = x_f - x_i$

$dx \equiv \Delta x \to 0$



$\tan\theta = \left(\frac{dy}{dx}\right)$

graphical representation of $\frac{dy}{dx} \equiv$ instantaneous slope of curve

Analytical significant of $\frac{dy}{dx} \equiv$ change in y due to per unit change in x

4.2 DIFFERENTIATION OF SOME COMMON EXPRESSIONS

Some frequently used formulae

(i) $\frac{d}{dx}x^n = n\,x^{n-1}$ (ii) $\frac{d}{dx}\sin x = \cos x$

(iii) $\frac{d}{dx}\cos x = -\sin x$ (iv) $\frac{d}{dx}\tan x = \sec^2 x$

(v) $\frac{d}{dx}e^x = e^x$ (vi) $\frac{d}{dx}\ln x = \frac{1}{x}$

(vii) $\frac{d}{dx}(\text{constant}) = 0$

Some basic rules of differentiation

(i) $y = Cf(x) \Rightarrow \frac{dy}{dx} = C\frac{d}{dx}f(x)$

(ii) $y = f(x) \pm g(x) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

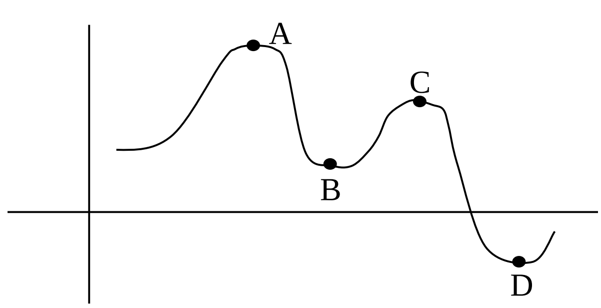
(iii) $y = f(x).g(x) \Rightarrow \frac{dy}{dx} = f(x).\frac{d}{dx}g(x) + g(x).\frac{d}{dx}f(x)$

(iv) Chain rule of differentiation: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$$

4.3 APPLICATION OF DIFFERENTIATION

Maxima and Minima



A, C ≡ Point of local maxima

B, D ≡ Points of local minima

At each maxima or minima slope of curve is zero.

⇒ $\frac{dy}{dx} = 0$ at each local maxima or minima. Then to find maxima or minima value of a function $y = f(x)$

Step (1) Differentiate the function $y = f(x)$ wrt x

Step (2) Solve for $\frac{dy}{dx} = 0$

Step (3) Put all solution of $\frac{dy}{dx} = 0$ in

$$\frac{d^2y}{dx^2}\left[= \frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$$

If $\frac{d^2y}{dx^2} > 0$ for any solution

Then that value belongs to local minima

If $\frac{d^2y}{dx^2} < 0$ for any solution

Then that value belongs to local maxima

5. INTEGRAL CALCULUS

5.1 MEANING OF INTEGRATION

\# **Integration is reverse process of differentiation**

i.e. $\frac{d}{dx}(\sin x + C = \cos x$

⇒ $\int \cos x \, dx = \sin x + C$

5.2 INTEGRATION OF SOME COMMON EXPRESSIONS

\# **Some important formulae**

(i) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$; (where $n \neq -1$)

(ii) $\int \frac{1}{x} dx = \ln x + C$

(iii) $\int \sin x \, dx = -\cos x + C$

(iv) $\int \cos x \, dx = \sin x + C$

(v) $\int e^x \, dx = e^x + C$

(vi) $\int dx = x + C$

\# **<u>Basic rules</u>**

(i) $\int\{f(x) \pm g(x)\}dx = \int f(x)\,dx \pm \int g(x)\,dx$

(ii) **Integration by substitution**

$$\int f(g(x)) \bullet g'(x)\,dx$$

Let $g(x) = u \quad \Rightarrow \quad g'(x) = \frac{du}{dx}$

$\Rightarrow \quad du = g'(x) \bullet dx$

$\Rightarrow \quad \int f(g(x)) \bullet g'(x)\,dx = \int f(u)\,du$

Definite Integration

$$I = \int_a^b f(x)\,dx$$

i.e. Integration of $f(x)$ from a to b

If $\int f(x)\,dx = g(x) + C$ then

$$I = [g(x)]_a^b$$

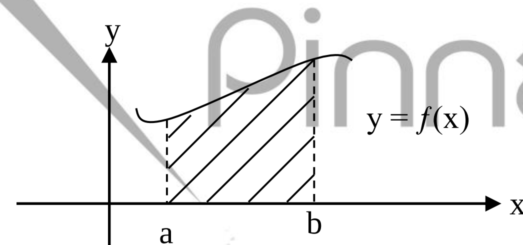
$$= g(b) - g(a)$$

b = Upper limit; a = Lower limit

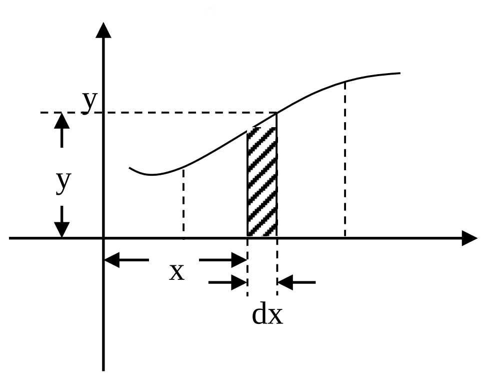
\# **Analytical interperation of definite integration:**

Definite integration is just like summation

5.3 GRAPHICAL INTERPRETATION OF DEFINITE INTEGRATION



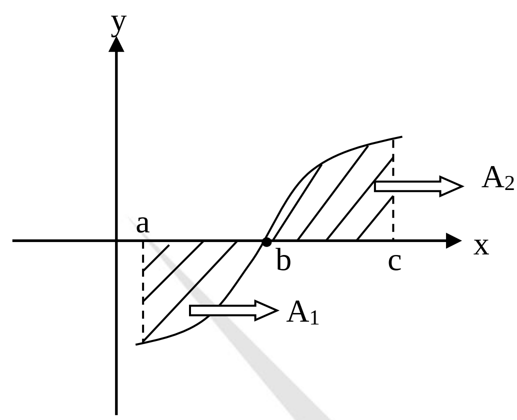
$I = \int_a^b f(x)\,dx$ = Area under $f(x)$ graph from a to b



$y = f(x)$

y.dx = elementary area under curve in dx

$\int_a^b y\,dx$ = summation of all such elementary area from a to b



$$\int_a^c f(x)\,dx = A_2 - A_1$$

where $A_1 = \int_a^b f(x)\,dx$ = Area from a to b

& $A_2 = \int_b^c f(x)\,dx$ = Area from b to c

Example: $\int (\sin x + \cos x)\,dx = \int \sin x\,dx + \int \cos x\,dx$

$= -\cos x + \sin x + C$

Example: $\int (x^2 - 2) + dx = \int x^2\,dx - 2\int dx$

$= \dfrac{x^3}{3} - 2x + C$

Example: $\int (\sin 2x)\,dx$

Let $2x = u \Rightarrow \dfrac{du}{2} = dx$

$\Rightarrow \dfrac{1}{2}\int \sin u\,.du = -\dfrac{1}{2}\cos u + C$

$\Rightarrow \int (\sin 2x)\,dx = \dfrac{1}{2}\cos 2x + C$

Example: $\int \dfrac{1}{x+4}\,dx$

Let $(x + 4) = u \Rightarrow du = dx$

$\Rightarrow \quad \int \frac{1}{x+4} dx = \int \frac{1}{u} du = \ln u + C$

$\Rightarrow \quad \int \frac{1}{(x+4)} dx = \ln(x+4) + C$

Example: $\sin^2 x = \frac{1-\cos 2x}{2}$

$\int \sin^2 x \, dx = \frac{1}{2}\int (1-\cos 2x) dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C \qquad = \frac{2x - \sin 2x}{4} + C$

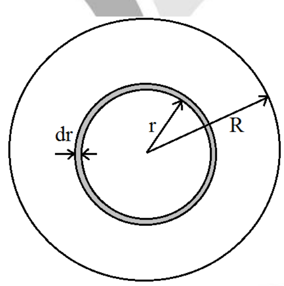
Example: Prove that total surface area of disc is πR^2

Solution: Complete disc can be assumed as a combination of thin concentric rings.

Area of any such elementary ring

$$dA = 2\pi r . dr$$

where r is the radius of elementary ring and dr is thickness of elementary ring.



$(2\pi r . dr = \text{length of periphery} \times \text{thickness})$

Total area of the disc = summation of areas all such elementary rings.

$A = \int dA$

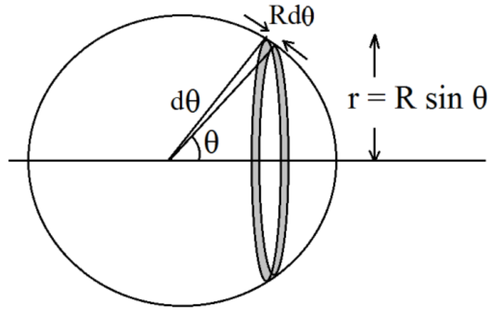
$= \int_0^R 2\pi r . dr \qquad = 2\pi \int_0^R r \, dr$

$= 2\pi \left[\frac{r^2}{2}\right]_0^R \qquad = 2\pi \frac{R^2}{2}$

$\Rightarrow \quad \mathbf{A} = \boldsymbol{\pi R^2}$

Example: Prove that total surface area of sphere is $4\pi R^2$

Solution: Surface area of a sphere can be assumed to be formed by co-axial thin rings as shown in figure.



Consider an elementary ring as shown in figure, radius of this ring subtends angle θ at centre of sphere.

Radius of this ring = $R\sin\theta$

Thickness of this ring subtends angle dθ at centre of sphere.

⇒ Thickness of ring = $Rd\theta$

⇒ Surface area of this elementary ring dA = Length of periphery × thickness

⇒ $dA = 2\pi R\sin\theta\,.\,Rd\theta$

Total surface area of the sphere is summation of all such elementary coaxial rings.

$$A = \int dA = \int_0^\pi 2\pi R\sin\theta\,.\,Rd\theta$$

$$= 2\pi R^2\int_0^\pi \sin\theta\,d\theta$$

$$= 2\pi R^2[-\cos\theta]_0^\pi$$

$$= 2\pi R^2\{(-\cos\pi) - (-\cos 0)\}$$

$$= 4\pi R^2$$

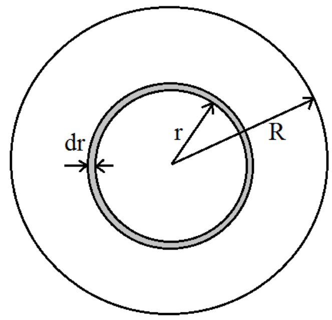
⇒ Total surface area of sphere is $4\pi R^2$.

Example: Prove that total volume of the solid sphere is $\frac{4}{3}\pi R^3$

Solution: Complete solid sphere can be assumed as a combination of thin concentric hollow spheres. Volume of any such elementary hollow sphere is

$$dV = 4\pi r^2.dr$$

where r is the radius of elementary hollow sphere and dr is thickness of elementary hollow sphere.



$(4\pi r^2 . dr = \text{surface area of hollow sphere} \times \text{thickness})$

Total volume of the solid sphere = summation of volumes all such elementary hollow sphere.

$$V = \int dV$$

$$= \int_0^R 4\pi r^2 . dr \qquad = 4\pi \int_0^R r^2\, dr$$

$$= 4\pi \left[\frac{r^3}{3}\right]_0^R \qquad = 4\pi \frac{R^3}{3}$$

⇒ Total volume of a solid sphere is $\boldsymbol{\frac{4}{3}\pi R^3}$**.**

6. CONCEPT OF GRAPHS

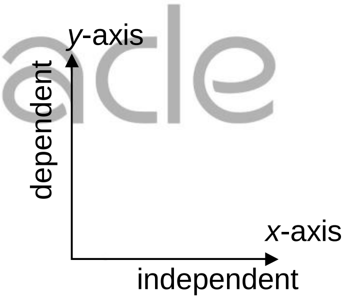
There are two types of variables

(i) Independent variable.

(ii) Dependent variable.

Independent variables are such type of variables which don't depend on other variable like, time and dependent variables are those variables which depend on some other variables i.e., velocity, distance covered, acceleration which depend on time.

When we plot the graph independent variable is plotted on *x*-axis and dependent variable is plotted on *y*-axis.

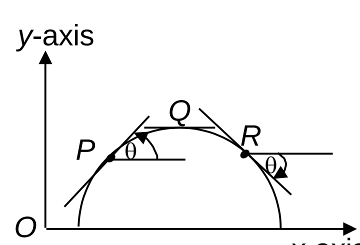


6.1 SLOPE

If a tangent at a particular point on the curve makes an angle θ with positive *x*–axis then tanθ gives the slope of that tangent.

If angle is measured in anticlockwise direction from positive *x*-axis, then slope is taken as positive and if angle is measured in clockwise direction, slope is taken as negative.

Here a curve is shown and at points *P*, *Q* and *R* tangents are drawn.



At *P*, the tangent makes an angle θ in anticlockwise with positive x-axis. So slope at *P* is positive.

At Q, the tangent makes zero angle with the positive x-axis. So slope at Q is zero.

At R, the tangent makes an angle θ in clockwise with positive x-axis. So slope at R is negative.

In this chapter we study mainly three types of graph.

(i) Distance–time graphs

(ii) Velocity–time graph

(iii) Acceleration–time graph

6.2 DISPLACEMENT–TIME GRAPH

(i) Slope of position vs time graph gives velocity

(ii) Slope at a particular point of the graph gives instantaneous velocity

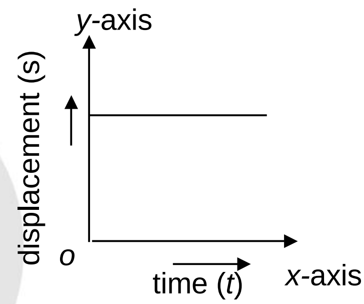
(iii) Slope of a line joining initial position to final position gives average velocity between two points.

(iv) The maximum slope at any point on the graph gives the maximum velocity.

(v) When graph will be straight line parallel to x-axis, it means slope is zero so velocity will be zero.

$$v = \tan\theta = 0 = \tan 0°$$

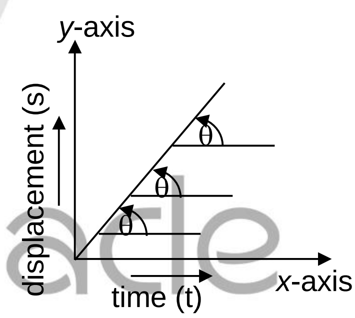
$$\Rightarrow \quad \theta = 0°$$



(vi) Slope of a straight line is constant which means velocity is constant.

(a) As shown in graph slope is positive and constant since it makes an angle θ with positive x-axis in anticlockwise direction.

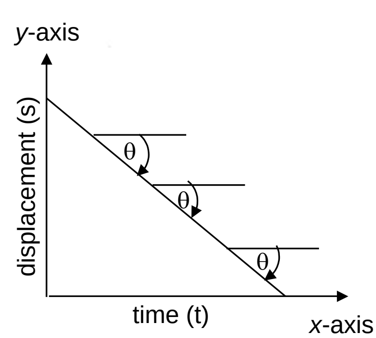
So velocity is positive and constant



(a)

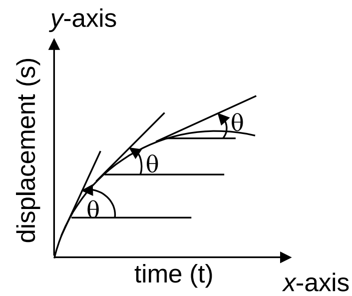
(b) In this graph, slope is negative and constant since θ is in clockwise direction.

So velocity is negative and constant



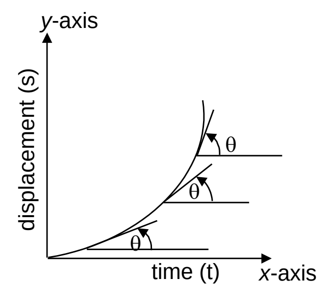
(vii) (a) Slope of a curve as shown in figure, is positive but decreasing.

So velocity is positive and decreases.



(b) In this graph slope is positive and increasing.

So velocity is positive and increases



☞ *Area under curve of a position time graph has no physical significance.*

6.3 VELOCITY–TIME GRAPH

(i) Slope of velocity vs. time graph gives acceleration.

(ii) Area under curve gives displacement and area on negative side gives negative displacement.

As shown in the graph of velocity vs. time

(i) Slope from 0 to A is positive and constant hence, acceleration is positive and constant.

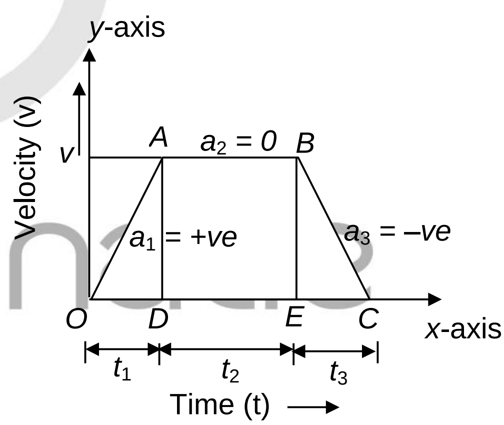
(ii) Slope from A to B is zero, hence acceleration will be zero

(iii) Slope from B to C is negative and constant, hence acceleration is negative and constant.

Distance covered in motion from O to A, A to B and B to C will be area under curve with x-axis.

Here, area of curve = area of trapazium

$$= \frac{1}{2} \times v \times [t_1 + t_2 + t_3 + t_2]$$



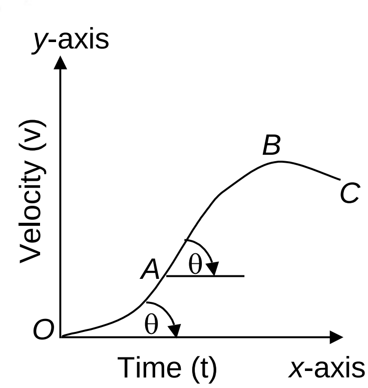
When graph of velocity vs time is in curve form as shown in figure.

(i) Slope from O to A is positive and increasing, hence acceleration is positive and increasing

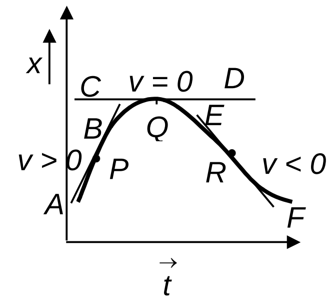
(ii) A to B slope is positive and decreasing, so acceleration is positive and decreasing.

(iii) At B slope is zero, so acceleration is zero

(iv) B to C slope is negative and increasing, so acceleration is also negative and increasing.



☞ The instantaneous velocity can be positive, negative or zero as shown in the position-time graph.



When the slope of position-time graph will be +ve instantaneous velocity will be +ve, such as at point *P*, slope of tangent *AB* is positive as shown in figure. So instantaneous velocity at *P* is positive.

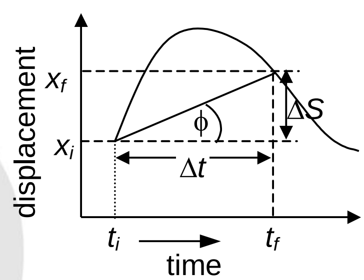
When the slope of this graph will be 0 instantaneous velocity will be 0, such as at *Q* slope of tangent *CD* is 0, so instantaneous velocity at Q is 0.

When the slope of this graph is –ve, instantantaneous velocity is – ve. Such as at point *R,* slope of tangent *EF* is –ve so instantaneous velocity at *R* is –ve.

When the velocity is constant, the average velocity is equal to the instantaneous velocity.

☞ Average velocity of the particle is equal to the slope of the straight line joining the initial and final position on the displacement–time graph as shown in figure.

$$v_{avg} = \frac{\Delta S}{\Delta t} = \tan\phi = \text{slope of chord}$$



6.4 GRAPHICAL METHOD TO PROVE $S = ut + \frac{1}{2}at^2$

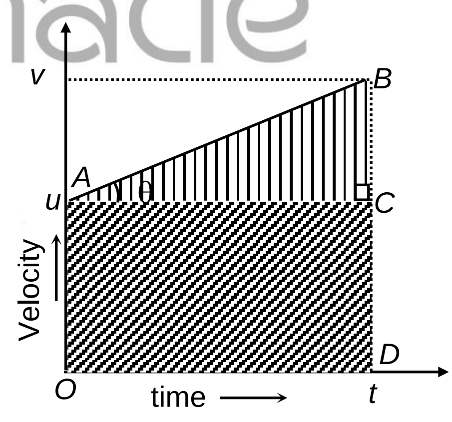
Let a particle starts with velocity *u* and it moves under constant acceleration from *A* to *B* in *t* second. Its velocity versus time graph is as shown in the figure. We know area under velocity-time graph gives the distance *S* covered by the particle in time *t*.

S = Area under velocity – time graph

= area of rectangle OACD + Area of triangle ACB

$$= ut + \frac{1}{2} AC \times CB$$

$$= ut + \frac{1}{2} AC^2 \times \frac{CB}{AC}$$



Also $\frac{CB}{AC}$ = $\tan\theta$ = slope of velocity –time graph = acceleration *a*

$$\therefore \quad S = ut + \frac{1}{2}at^2$$

6.5 ACCELERATION–TIME GRAPH

(1) The slope of acceleration vs. time graphs gives rate of change of acceleration with respect to time, which has no physical significance.

(2) Area under curve of this graph gives change in velocity.

Illustration 11

Question: **The speed-time graph of a particle moving along a fixed direction is shown in figure.**

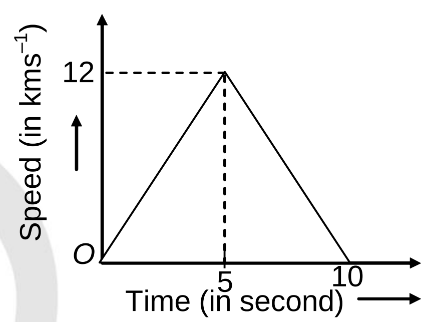
(a) Calculate the distanced traveled by the particle from *t* = 0 to *t* = 10 s.

(b) Calculate the average speed in the time interval from 0 to 10s.

(c) Calculate the distance traveled in the time interval from 2s to 6s.

(d) Calculate the average speed in interval from *t* = 2s to *t* =6s.

Solution:



(a) Distance travelled from $t = 0$ to $t = 10$ s

= area under speed –time graph

$= \frac{1}{2} \times 10s \times 12\,\text{ms}^{-1}$ = **60 m**

(b) Average speed over the interval from

$t = 0$ to $t = 10$ s

$= \frac{60\,\text{m}}{10\,\text{s}} =$ **6 ms^{-1}**

(c) In order to calculate distance from $t = 2$s to $t = 5$s, let us first determine separately the following and then add them up.

(i) distance covered from $t = 2$s to $t = 5$ s

(ii) distance covered from $t = 5$s to $t = 6$ s

(i) Acceleration $= \frac{12\,\text{ms}^{-1}}{5s} = 2.4\ \text{ms}^{-2}$

Velocity at the end of $2s = 2.4 \times 2\,\text{ms}^{-1} = 4.8\ \text{ms}^{-1}$

This will be regarded as initial velocity for motion over the time interval from $t = 2$ second to $t = 5$ second.

If x is the distance travelled in this time interval, then

$$x\left(4.8 \times 3 + \frac{1}{2} \times 2.4 \times 9\right)\text{metre} = (14.4 + 10.8)\text{m} = \mathbf{25.2\ m}$$

(ii) Deceleration = $-2.4\ \text{ms}^{-2}$

For motion from $t = 5$s to $t = 6$s, the initial velocity will be 12 ms^{-1}. If x the distance covered in this interval of time, then

$$x' = 12 \times 1 - \frac{1}{2} \times 2.4 \times 1 \times 1 = 12 - 1.2 = 10.8\,\text{m}$$

Total distance = (25.2 + 10.8) metre = 36 cn

(d) Average speed $\frac{36\,\text{m}}{4s} = \mathbf{9\ ms^{-1}}$

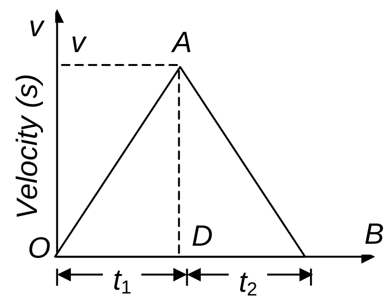
Illustration 12

Question: **A car accelerates from rest at constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, calculate**

(a) the maximum velocity reached

(b) the total distance travelled.

Solution: The situation as shown in figure



Let the car accelerate for time t_1 and decelerate for time t_2, so that total time $t_1 + t_2 = t$ Let v be the maximum velocity reached

(a) For accelerated motion,

$v = 0 + \alpha t_1$

or, $v = \alpha t_1$

For decelerated motion, $0 = v + (-\beta)t_2$

or $v = \beta t_2$

$\therefore \quad \alpha t_1 = \beta t_2$

or $\frac{t_2}{t_1} = \frac{\alpha}{\beta}$

Adding 1 to both sides

$$\frac{t_2 + t_1}{t_1} = \frac{\alpha + \beta}{\beta}$$

or $\frac{t}{t_1} = \frac{\alpha + \beta}{\beta}$

or $t_1 = \frac{\beta t}{\alpha + \beta}$

Hence, $v = \alpha t_1 = \alpha\left(\frac{\beta t}{\alpha + \beta}\right) = \left(\boldsymbol{\frac{\alpha\beta t}{\alpha + \beta}}\right)$

(b) Total distance traveled = Area of ΔOAB

or, $S = \frac{AD \times OB}{2}$

$= \frac{v \times t}{2}$

$= \frac{(\alpha\beta)t}{\alpha + \beta} \cdot \frac{t}{2}$

$= \boldsymbol{\frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2}$

7. MOTION IN A PLANE

In this chapter we deal with the kinematics of a particle moving in a plane, which is two dimensional motion. Some common examples of motion in a plane are the motion of projectiles and satellites and the motion of particles in circular path. As in the case of one dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of displacement, velocity, and acceleration. To define various kinematics quantities like position, displacement, velocity and acceleration for objects moving along a plane, we need to use the language of vectors that already we have learned.

The concept of motion when a particle is moving along a straight line can be used for motion in a plane or three dimensions. When motion of a particle is in a plane, we consider plane of motion as *x*-*y*-plane. We choose the origin at the place from where the motion starts and then we consider motion along any two convenient mutually perpendicular direction as one dimensional motion. Motion in two perpendicular directions are chosen as the *x* and *y*-axes.

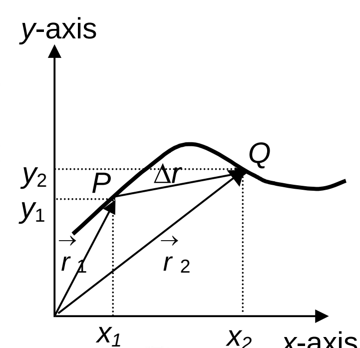
7.1 POSITION VECTOR AND DISPLACEMENT

The displacement of a particle is the difference between its final position, and initial position. Therefore, the **displacement vector** for the particle as shown in figure equals the difference between its final position vector and its initial position vector.

Suppose a particle moves in a plane along the curve as shown in figure. At time t_1, the particle is at point *P*, and at some later time t_2, the particle is at *Q*, where the indices 1 and 2 refer to initial and final values. As the particle moves from *P* to *Q* in the time interval $\Delta t = t_2 - t_1$ the position vector changes from r_1 to r_2.

$\Delta r = r_2 - r_1$ (from triangle law of vector addition). … (**16**)

The direction of Δr is indicated in figure



Let the co-ordinate of the particle at points *P* and *Q* are (x_1, y_1) and (x_2, y_2) respectively.

Then, $\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

$\vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$

$\Delta\vec{r} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j}$

$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$ where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Where, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$; $\Delta x = x_2 - x_1$; $\Delta y = y_2 - y_1$

As we see from figure the magnitude of the displacement vector is less than the distance travelled along the curved path.

7.2 VELOCITY

We define the average velocity of the particle during the time interval Δt as the ratio of the displacement to that time interval:

$$\bar{v} = \frac{\Delta \vec{r}}{\Delta t} \qquad \dots (\mathbf{17})$$

$$\Delta \vec{r} = \Delta x\, \hat{i} + \Delta y \hat{j}$$

$$\frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i}}{\Delta t} + \frac{\Delta y}{\Delta t}\hat{j} \Rightarrow \vec{v} = v_x \hat{i} + v_y \hat{j}$$

Since displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a vector quantity directed along $\Delta \vec{r}$.

☞ *The average velocity between points P and Q is independent of the path between the two points. This is because the average velocity is proportional to the displacement, which in turn depends only on the initial and final position vectors and not on the path taken between those two points.*

☞ *As we did with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip since its displacement is zero.*

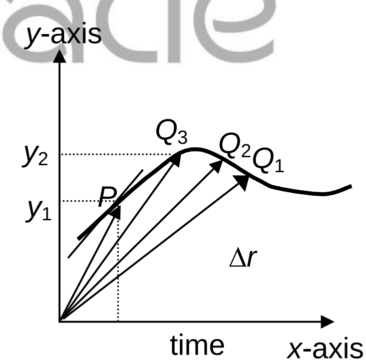
Instantaneous velocity

The instantaneous velocity $\vec{v}$, is defined as the limit of the average velocity, $\Delta \vec{r}$ / Δt, as Δt approaches zero:

i.e. $$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad \dots (\mathbf{18})$$

That is, the instantaneous velocity equals the derivative of the position vector with respect to time.

Consider again the motion of a particle between two points in the *xy* plane, as shown in figure. As the time intervals over which we observe the motion become smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at the point *P.*



$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}\hat{i} + \Delta \vec{y}\hat{j}}{\Delta t} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x \hat{i} + v_y \hat{j}$$

The direction of the instantaneous velocity vector at any point in a particle's path is along a line that is tangent to the path at that point and in the direction of motion; this is illustrated in figure.

☞ *The magnitude of the instantaneous velocity vector is called the speed.*

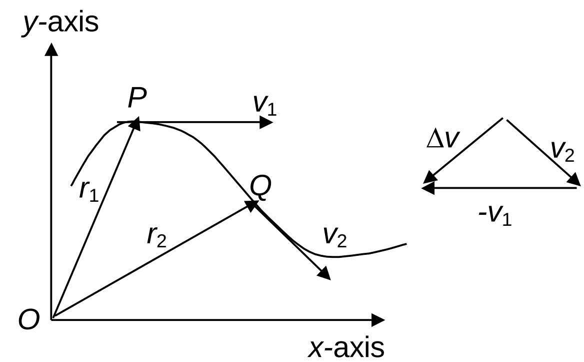
7.3 ACCELERATION

(a) Average acceleration

The average acceleration of a particle as it moves from P to Q is defined as the ratio of the change in the instantaneous velocity vector, Δv, to the elapsed time, Δt

i.e., $$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \quad \dots (\mathbf{19})$$

Since the average acceleration is the ratio of a vector quantity, $\Delta \vec{v}$, and a scalar quantity, Δt, we conclude that $\bar{a}$ is a vector quantity directed along $\Delta \vec{v}$. As is indicated in figure, the direction of $\Delta \vec{v}$ is found by adding the vector $-\vec{v}_1$ (the negative of $\vec{v}_1$) to the vector $\vec{v}_2$, since by definition



$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta\left(\vec{v}_x \hat{i} + \vec{v}_y \hat{j}\right)}{t_2 - t_1} = a_x \hat{i} + a_y \hat{j}$$

(b) Instantaneous acceleration

The instantaneous acceleration $\vec{a}$, is defined as the limiting value of the ratio $\Delta \vec{v} / \Delta t$ as Δt approaches zero:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt} \quad \dots (\mathbf{20})$$

8. TWO DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let us consider two-dimensional motion during which the acceleration remains constant. That is, we assume that the magnitude and direction of the acceleration remain unchanged during the motion.

As we learned, in cause of the motion a particle can be determined by its position vector $\vec{r}$. The position vector for a particle moving in the *xy* plane can be written

$$\vec{r} = x\,\hat{i} + y\hat{j} \quad \dots (\mathbf{21})$$

Where x, y, and $\vec{r}$ change with time as the particle moves. If the position vector is known, the velocity of the particle can be obtained on differentiating equation, which give

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \quad \dots (\mathbf{22})$$

on differentiating equation (25) with respect to time, we get

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j} \qquad \dots (\mathbf{23})$$

Because $\vec{a}$ is assumed constant, its components $\vec{a}_x$ and $\vec{a}_y$ are also constants. Therefore , we can apply the equations of kinematics to the x and y components of the velocity vector and displacement vector as shown below.

$$\vec{v} = \vec{v}_0 + \vec{a}t \begin{cases} v_x = v_{x_0} + a_x t \\ v_y = v_{y_0} + a_y t \end{cases} \qquad \dots (\mathbf{24})$$

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \begin{cases} x = v_{x_0}t + \frac{1}{2}a_x t^2 \\ y = v_{y_0}t + \frac{1}{2}a_y t^2 \end{cases} \qquad \dots (\mathbf{25})$$

$$v^2 = v_0^2 + 2as \begin{cases} v_x^2 = v_{x_0}^2 + 2a_x s_x \\ v_y^2 = v_{y_0}^2 + 2a_y s_y \end{cases} \qquad \dots (\mathbf{26})$$

We can say in other words, two-dimensional motion having constant acceleration is equivalent to two independent motions in the *x* and *y* directions having constant accelerations a_x and a_y.

Examples of motion in plane are projectile motion and circular motion

9. PROJECTILE MOTION

If an object is thrown in air with some initial velocity at some angle with the horizontal, and then allowed to move under the gravity, the object moves in a curved path, such type of motion is called projectile motion and that object is known as projectile. Such a projectile may be football, a cricket ball or any object.

It is an example of two dimensional motion.

☞ *The path of projectile, which is called trajectory, is always a parabola*

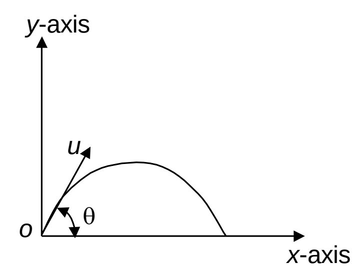
Assumptions of projectile motion

(1) The acceleration due to gravity (‘*g*’) is constant over the motion of projectile.

(2) The effect of air resistances is negligible

(3) The effect due to rotation of earth and curvature of the earth is negligible.

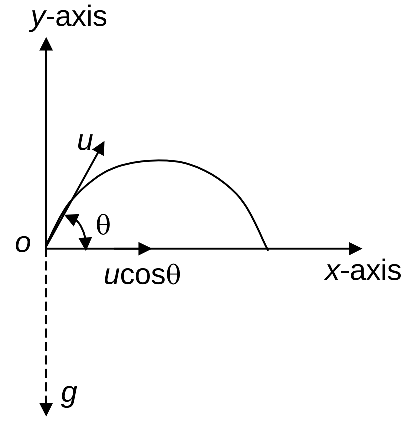
When an object is thrown at some angle to the horizontal, the motion of projectile will be in horizontal direction as well as in vertical direction. Suppose motion of projectile is in *xy*-plane of co-ordinate system as shown in figure, then motion of the object will be along the two mutually perpendicular direction together, say *x* and *y*-axes of the co-ordinate system. Consider motion along horizontal direction as *x*-axis and motion along vertical direction as *y*-axis.



We can study the two-dimensional motion as one-dimensional motion along *x*-axes and *y*-axis since its superposition gives the resultant motion.

Let us assume that at $t = 0$, projectile leaves the origin ($x = 0$, $y = 0$) with velocity u, which makes an angle θ with the horizontal.

Motion along horizontal direction (x-axis): In this direction initial velocity will be horizontal component of initial velocity with which the object is projected i.e.,



$$u_x = u \cos \theta$$

acceleration will be perpendicular component of g (acceleration due to gravity)

$$a_x = 0$$

Now we can say motion in x-direction is with constant velocity since acceleration is zero.

$$v_x = v_{0x} + a_x t$$
$$= u_x$$
$$= u \cos\theta$$

and $$x = v_0 t + \frac{1}{2} a_x t^2$$
$$= u_x t$$
$$= (u\cos\theta)t$$

Motion in vertical direction (y-axis)

In this direction initial velocity will be vertical component of initial velocity with which the object is projected.

$$u_y = u \sin\theta$$

and acceleration will be 'g' which is acting in negative y-axis.

$$a_y = -g \quad \text{(which is constant)}$$

It means motion along y-axis is with uniform acceleration, so equation of kinematics for one-dimensional motion can be used here.

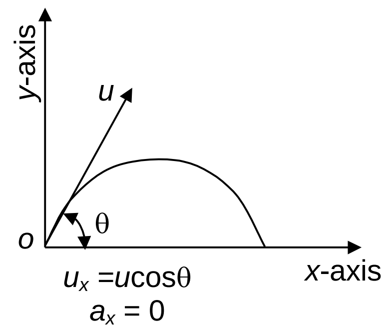
$$v_y = u_y - gt$$
$$= u\sin\theta - gt$$

and $$y = u_y t - \frac{1}{2} g t^2$$
$$= (u\sin\theta)\, t - \frac{1}{2} g t^2$$

Also we have,

$$v_y^2 = u_y^2 - 2gy$$
$$= (u \sin \theta)^2 - 2gy$$

9.1 EQUATION OF TRAJECTORY



Let projected body be in motion for *t* second.

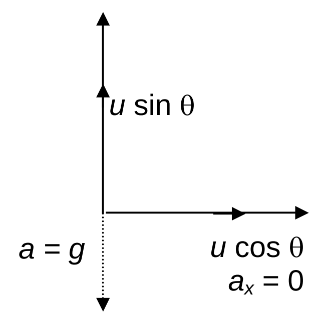
Consider motion in x-direction

$u_x = u\cos\theta$

$a_x = 0$

$x = (u\cos\theta)\,t$

$$t = \frac{x}{u\cos\theta} \qquad \ldots \text{(i)}$$



Now consider motion in *y*-direction

$u_y = u\sin\theta$ and $a_y = -g$

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

on putting value of *t* from equation (i)

$$y = (u\sin\theta)\frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$y = (\tan\theta)x - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta} \qquad \ldots \textbf{(27)}$$

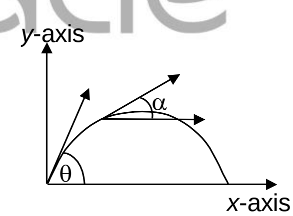
Which is valid for the angles in the range $0 < \theta < \frac{\pi}{2}$

The above equation represents a parabola. So we can say that equation of trajectory is parabola.

9.2 VELOCITY OF PROJECTILE IN SPACE

Now to obtain the speed *v* of the projectile as a function of time, first we calculate speed along *x*-axis and *y*-axis v_x and v_y respectively as a function of time

$$v = \sqrt{v_x^2 + v_y^2}$$



In x-direction velocity will remains constant. So at any time *t*

$v_x = (u\cos\theta)$

In *y*-direction velocity will be $v_y = u\sin\theta - gt$

$$|v| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

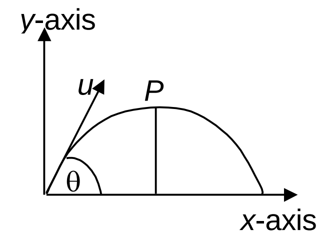
$$v = \sqrt{u^2 - 2ugt\sin\theta + g^2t^2} \qquad \ldots \textbf{(28)}$$

and velocity vector will be tangent to the path at any instant as shown in figure

$$\tan\alpha = \left(\frac{v_y}{v_x}\right).$$

9.3 TIME OF ASCENT

Time of ascent is the time when the projectile will attain the maximum height. At this position the vertical component of velocity vector will become zero.



As shown in figure P is point of maximum height. Now to find the time taken from initial point of projection upto maximum height, we consider motion in y-direction

$$v_y = u_y - gt$$

$$0 = u\sin\theta - gt$$

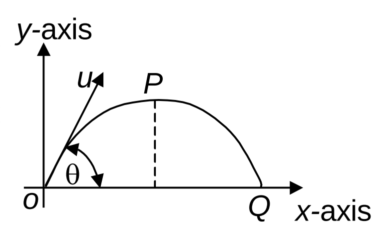
$$t = \frac{u\sin\theta}{g} \qquad \dots (\mathbf{29})$$

9.4 TIME OF FLIGHT

When a particle is projected in air after some time it hits the ground again, time taken in this process is called time of flight or we can say time of flight is time of motion of projectile when it is in air. It is denoted by T.

Let a particle be projected from O and again it hits the ground at point Q. In this motion distance travelled in y-direction is zero.

On applying equation of motion in y-direction.



$$y = ut - \frac{1}{2}gt^2$$

$$0 = u\sin\theta\, T - \frac{1}{2}gT^2$$

$$0 = (u\sin\theta - \frac{1}{2}gT)T$$

$$T = 0 \text{ or } T = \frac{2u\sin\theta}{g}$$

$$T = \frac{2u\sin\theta}{g} \qquad \dots (\mathbf{30})$$

Time of flight can be found out in second way as given below.

The total time of flight is consists of two parts

(i) Time taken by the particle to go from O to highest point P, it is called time of ascent.

(ii) Time taken by the particle to go from the highest point P to Q where it hits the ground again. It is called time of descent.

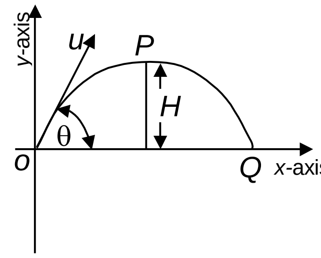
As motion from O to P and P to Q are symmetrical.

$\therefore$ time of ascent = time of descent $= \frac{u\sin\theta}{g}$

Total time of fight = time of ascent + time of descent.

$$T = \frac{2u\sin\theta}{g}$$

9.5 MAXIMUM HEIGHT OF PROJECTILE



It is the maximum vertical height attained by the particle above the point of projection during its flight. it is denoted by *H*.

When the projectile is at maximum height, its vertical component of velocity will be zero.

Initial velocity $v_y = u \sin\theta$

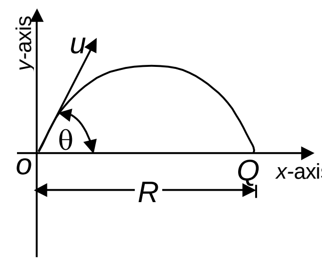
On using equation of motion along *y* axis.

$$v_y^2 = u_y^2 - 2gs$$

$$0 = u^2\sin^2\theta - 2gH$$

$$H = \frac{u^2 \sin^2\theta}{2g} \qquad \ldots (\mathbf{31})$$

9.6 HORIZONTAL RANGE



Range of projectile is horizontal distance travelled by the particle during the time of flight

Motion along *x*-axis is with constant velocity

So distance travelled in this direction will be

$$x = (u\cos\theta)\, t$$

$$R = (u\cos\theta)T$$

Where *T* is time of flight

$$T = \frac{2u\sin\theta}{g}$$

on putting the value of *T*

$$R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$R = \frac{u^2 \sin 2\theta}{g} \qquad \ldots (\mathbf{32})$$

For range to be maximum

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = 1 = \sin 90°$$

$$2\theta = 90°$$

$$\theta = 45°$$

$$R_{\max} = \frac{u^2}{g} \qquad \ldots (\mathbf{33})$$

☞ *A projectile will have maximum range when it is projected at angle of 45° to the horizontal and maximum range will be =* $\frac{u^2}{g}$

☞ *When range is maximum, the maximum height H reached by the projectile* $H = \frac{u^2 \sin^2 45°}{2g}$

$= \frac{u^2}{2g} = \frac{R_{max}}{4}$

☞ *For same range two angle of projectile are possible, such that two angles are complementary to each other.*

$\theta_1 + \theta_2 = 90°$,

where θ_1 and θ_2 are two different angles of projection which gives same horizontal range.

If $\theta_1 = 45° + \alpha$,

then $\theta_2 = 45° - \alpha$

Illustration 13

Question: **A ball is thrown from a field with a speed of 12.0 m/s at an angle of 45^0 with the horizontal. At what distance will it hit the field again? Take g =10.0 m/s^2**

Solution: The horizontal range $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(12\,\text{m/s})^2 x \sin(2x45^0)}{10\,\text{m/s}^2}$$

$$= \frac{144\,\text{m}^2/\text{s}^2}{10\,\text{m/s}^2} = 14.4\,\text{m}.$$

Thus, the ball hits the field at 14.4 m from the point of projection.

Illustration 14

Question: **Find the maximum horizontal range when the velocity of projection is 30 m/s. Find the two angles of projection to give a range of 45 m. Take *g* = 10 m/s^2.**

Solution: (i) Maximum range $R_m = \frac{u^2}{g} = \frac{30^2}{10}$ = **90 m**

(ii) Now $\frac{u^2 \sin 2\alpha}{g}$ = 45

or sin 2α = $\frac{45 \times 10}{30 \times 30} = \frac{1}{2}$

or 2α = 30° or 150° [∵ (180 – θ) = sin θ]

or α = **15°** or **75°**

Therefore for a given velocity of projection and for a given range, two angles of projection are possible.

Illustration 15

***Question*:** **What is the least velocity with which a cricket ball can be thrown through a distance of 100 m?**

Solution: Since the range is given, the least velocity of projection is that value when the angle of projection is 45°. For velocity u to be least

$$\frac{u^2 \sin 2\alpha}{g} = 100 \text{ where } \alpha = 45^\circ \quad \text{or} \quad \frac{u^2}{g} = 100$$

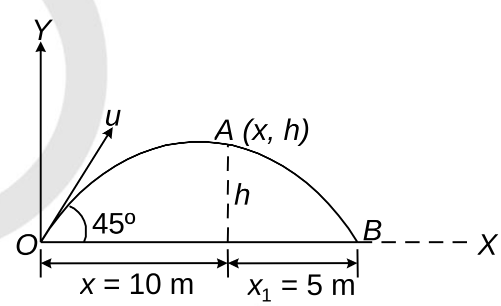
$$u^2 = 100 \times 9.8 = 980$$

$$u = \sqrt{980} = \mathbf{31.3\ m/s}$$

Illustration 16

Question: **From a point on the ground at a distance 10 m from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 5 m on the other side. Find the height of the wall.**

Solution: Let u be the velocity of projection at an angle $\alpha = 45°$ with the horizontal.



$$\text{Horizontal range} = \frac{u^2 \sin 2\alpha}{g} = 15$$

$$\frac{u^2}{g} = 15$$

Equation of the trajectory

$$y = x \tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

or $$y = x \cdot 1 - \frac{1}{2}\frac{gx^2}{u^2 \times \frac{1}{2}} = x - \frac{gx^2}{u^2} = x - \frac{x^2}{15}.$$

When $x = 10$, $y = h$,

$$\therefore \quad h = 10 - \frac{10^2}{15} = \frac{50}{15} = \boldsymbol{3\frac{1}{3}m}$$

9.7 RANGE OF PROJECTILE ON AN INCLINED PLANE THROUGH THE POINT OF PROJECTION

A particle is projected from a point A on an inclined plane, which is inclined at an angle β to the horizon with a velocity u at an elevation α. The direction of projection lies in the vertical plane through AB, the line of the greatest slope of the plane.

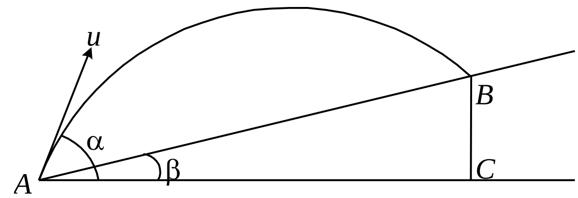


Fig. 3.

Let the particle strike the plane at B so that AB is the range on the inclined plane.

The initial velocity of projection u can be resolved into a component $u \cos(\alpha - \beta)$ along the plane and a component $u \sin(\alpha - \beta)$ perpendicular to the plane. The acceleration due to gravity g which acts vertically down can be resolved into components $g \sin \beta$ up the plane and $g \cos \beta$ perpendicular to the plane. By the principle of physical independence of forces the motion along the plane may be considered independent of the motion perpendicular to the plane. Let T be the time, which the particle takes to go from A to B. Then in this time the distance traversed by the projectile perpendicular to the plane is zero.

$$\therefore \quad 0 = u \sin(\alpha - \beta)\, T - \frac{1}{2} g \cos\beta\, T^2$$

$$\therefore \quad T = \frac{2u \sin(\alpha - \beta)}{g \cos\beta}$$

During this time the horizontal velocity of the projectile ($u \cos \alpha$) remains constant. Hence the horizontal distance described is given by

$$AC = u \cos \alpha T = \frac{2u^2 \sin(\alpha - \beta) \cos\alpha}{g \cos \beta}$$

$$\therefore \quad AB = \frac{AC}{\cos\beta} = \frac{2u^2 \sin(\alpha - \beta) \cos\alpha}{g \cos^2\beta}$$

$$\therefore \quad \text{Range on the inclined plane} = \frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g \cos^2\beta}$$

Maximum range on the inclined plane

$$R = \frac{2u^2 \sin(\alpha - \beta) \cos\alpha}{g \cos^2\beta}$$

$$= \frac{u^2}{g \cos^2\beta} [\sin(2\alpha - \beta) - \sin\beta]$$

For given values of u and β, R is maximum when

$\sin(2\alpha - \beta) = 1$

i.e., $(2\alpha - \beta) = 90^\circ$

$$\alpha = (45^\circ + \beta/2)$$

If R_m represents the maximum range on the inclined plane,

$$R_m = \frac{u^2}{g \cos^2\beta} (1 - \sin\beta)$$

$$R_m = \frac{u^2}{g(1+\sin\beta)}$$

For a given velocity of projection, it can be shown that there are two directions of projection which are equally inclined to the direction of maximum range.

Now $R = \dfrac{u^2}{g\cos^2\beta}$ [sin (2α – β) – sin β]

For given values of u, β and R, sin (2α – β) is constant. There are two values of (2α – β) each less than 180° that can satisfy the above equation.

Let $(2\theta_1 - \beta)$ and $(2\theta_2 - \beta)$ be the two values. Then

$$2\theta_1 - \beta = 180° - (2\theta_2 - \beta)$$

$$\theta_1 - \beta/2 = 90° - (\theta_2 - \beta/2)$$

$$\theta_1 - (45° + \beta/2) = (45° + \beta/2) - \theta_2$$

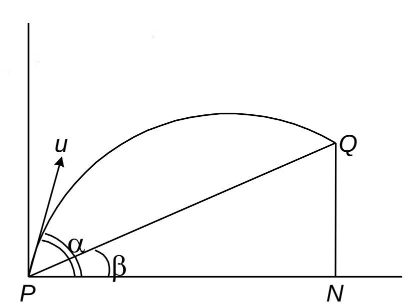
Since (45° + β/2) is the angle of projection giving the maximum range, it follows that the direction giving maximum range bisects the angle between the two angles of projection that can give a particular range.

Illustration 17

Question: **A particle is projected at an angle α =60⁰ with horizontal from the foot of a plane whose inclination to horizontal is β = 15⁰. Find the value of cot β.**

Solution: Let u be the velocity of projection so that u cos (α – β) and u sin (α – β) are the initial velocities respectively parallel and perpendicular to the inclined plane. The acceleration in these two directions are (–g sin β) and (–g cos β).

The initial component of velocity perpendicular to PQ is u sin (α – β) and the acceleration in this direction is (–g cos β). If T is the time the particle takes to go from P to Q then in time T the space described in a direction perpendicular to PQ is zero.



$$0 = u\sin(\alpha-\beta)\cdot T - \frac{1}{2}g\cos\beta\cdot T^2$$

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

If the direction of motion at the instant when the particle hits the plane be perpendicular to the plane, then the velocity at that instant parallel to the plane must be zero.

$\therefore \quad u\cos(\alpha-\beta) - g\sin\beta T = 0$

$$\frac{u\cos(\alpha-\beta)}{g\sin\beta} = T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$\therefore \quad \cot\beta = 2\tan(\alpha-\beta) = \mathbf{2}$

10. RELATIVE VELOCITY

The terms 'rest' and 'motion' are only relative. For example, when we say that a train is moving with velocity 30 m.p.h, we mean is that it is the velocity with which the train moves with respect to an observer on the earth who is regarded as fixed. This is not true strictly since a person on the earth unconsciously partakes the rotatory motion of the earth round its axis and the motion of earth round the sun. In addition, he shares the motion of entire solar system through space with respect to certain fixed stars. Thus there is no absolutely fixed point on the earth about which we can measure motion. Hence a person on the earth can never realise absolute motion or absolute rest.

Let us consider two motor cars *A* and *B* moving in the same direction on a road with equal speed. To a person seated in *A*, if he were unconscious of his motion, the car *B* would appear to be at rest. The line joining the two cars will always remain constant in magnitude and direction. The velocity of *B* relative to *A* or the velocity of *A* relative to *B* is zero.

On the other hand if *A* is moving with 30 m.p.h and *B* with 40 m.p.h in the same direction, a person in A would observe the car *B* to be drawing away from him at the rate of 10 m.p.h. This represents the velocity of *B* relative to *A*. If, however, *B* is moving opposite to the direction of *A* with velocity 40 m.p.h., for a person in *A*, *B* appears to draw away from him at the rate of 70 m.p.h. This, therefore, represents the velocity of *B* relative to *A*.

Definition of Relative velocity

When the distance between two moving points *A* and *B* is altering, either in magnitude or in direction or both, each point is said to possess a velocity relative to the other. The velocity of one of the moving points, say, *A*, relative to the other point *B* is obtained by compounding with the velocity of *A*, the reversed velocity of *B*. The velocity of *A* relative to *B* is the velocity with which *A* will appear to move to *B*, if *B* is reduced to rest.

If velocity *A* is $\vec{v}_A$ and that of *B* is $\vec{v}_B$ with respect to a stationary frame, then from the definition, relative velocity of *A* with respect to *B*, $\vec{v}_{AB}$ is given by

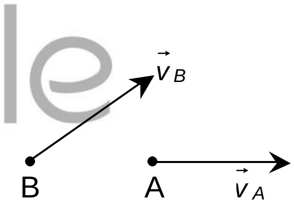


Fig. 8.

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \qquad \ldots(\mathbf{34})$$

If angle between $\vec{v}_A$ and $\vec{v}_B$ is θ then

$$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta} \qquad \ldots(\mathbf{35})$$

Also angle α made by relative velocity with v_A is given by

$$\tan\alpha = \frac{v_B\sin\theta}{v_A - v_B\cos\theta} \qquad \ldots(\mathbf{36})$$

From the above definition of relative velocity it follows that if we impress on both the moving points *A* and *B*, a velocity equal and opposite to that of *B*, then *B* would be reduced to rest and A will have two velocities (i) its own velocity and (ii) the reversed velocity of *B*. These two can be compounded into a single velocity by the parallelogram law, which will give the velocity of *A* relative to *B*.

10.1 RELATIVE VELOCITY IN ONE-DIMENSION

Case I

If $\theta = 0°$,

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B}$$

$$= \sqrt{(v_A - v_B)^2}$$

$$= v_A - v_B$$

If two bodies have velocity in same direction, then their difference gives the relative velocity.

Case II

If $\theta = \pi$ or 180°

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 180°}$$

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_{AB}}$$

$$= v_A + v_B$$

When two bodies have velocity in opposite direction, then their sum gives the relative velocity.

10.2 RELATIVE VELOCITY IN TWO DIMENSIONS

The concept of relative velocity has been discussed in one-dimensional motion. Same concept is used when motion is in a plane. In this case we can consider relative velocity in a plane as superposition of relative velocity in two mutually perpendicular direction considered as motion along x-axis and y-axis.

Suppose two objects A and B are moving with constant velocity $\vec{v}_A$ and $\vec{v}_B$ each with respect to same common frame of reference, say ground, then velocity of A relative to B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

When we will take relative motion along x-axis

$$v_{xAB} = = v_{xA} - v_{xB}$$

When we will take relative motion along y-axis

$$v_{yAB} = v_{yA} - v_{yB}$$

River-Boat Problems

River-boat problem is based on concept of relative velocity and resultant velocity. In this problem we come-across three terms

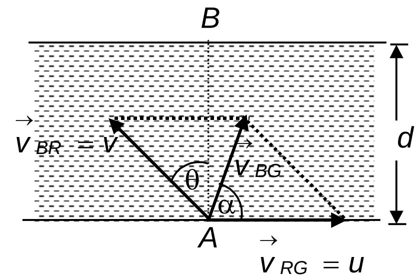
$\vec{v}_{RG}$ = velocity of river/stream with respect to ground. It is denoted by u.

$\vec{v}_{BR}$ = velocity of boatman with respect to river or velocity of boatman in still water. It is denoted by v.

$\vec{v}_{BG}$ = velocity of boatman with respect to ground.

Let the river is flowing along x-axis with velocity $\vec{V}_{RG}$ and width of river is d.

Let boat-man start from one bank of river from the point A with velocity $\vec{v}_{BR}$ along the direction as shown in figure.



$$\vec{v}_{BR} = \vec{v}_{BG} - \vec{v}_{RG}$$

$$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG} \quad \ldots \text{(i)}$$

Taking x-component in equation (i)

$$\vec{v}_{BG} \cos\alpha = u - v\sin\theta \quad \ldots \text{(ii)}$$

Taking y-component in equation (i)

$$\vec{v}_{BG} \sin\alpha = 0 + v\cos\theta \quad \ldots \text{(iii)}$$

On squaring and adding equation (ii) and (iii)

$$v_{BG}^2 = u^2 + v^2 - 2uv\sin\theta$$

$$v_{BG} = \sqrt{u^2 + v^2 - 2uv\sin\theta}$$

on dividing equation (ii) by (i)

$$\tan\alpha = \frac{-v\cos\theta}{u - v\sin\theta}$$

When boatman cross the river, the displacement in y-direction is d

Time taken to cross the river is t

$$t = \frac{d}{v\cos\theta}$$

In this time displacement in x-direction is x

$$x = (u - v\sin\theta)\, t$$

$$x = \frac{(u - v\sin\theta)d}{v\cos\theta}$$

(a) When a boat tends to cross a river in shortest time

$$t = \frac{d}{v\cos\theta}$$

for shortest time , $v\cos\theta$ has to be maximum

$\therefore$ $\cos\theta$ has to be maximum = 1

$$\theta = 0°$$

So the boats should go along AB (vertically opposite direction) to cross the river in shortest time.

When boat-man is crossing the river in shortest time, the horizontal distance traveled along the direction of river = x

$$x = u\,t = \frac{ud}{v}$$

(b) When a boat-man tends to cross the river along a shortest path (wants to reach the point just opposite from where he started)

When boat-man wants to cross the river in shortest path, horizontal distance covered should be zero.

$$x = (u - v\sin\theta)$$

$$x = \frac{(u - v\sin\theta)d}{v\cos\theta} = 0$$

$$u - v\sin\theta = 0\ ,\ \sin\theta = \frac{u}{v}, \qquad \theta = \sin^{-1}\left(\frac{u}{v}\right)$$

Hence, to reach just opposite point the boatman should row at an angle $\theta = \sin^{-1}\frac{u}{v}$ upstream from AB.

$\sin\theta < 1$

$\frac{u}{v} < 1$

$u < v$

so to cross the river directly velocity of river or stream should be less than velocity of boat.

If river velocity is greater than the velocity of boat, then it is impossible to reach the point B.

Illustration 18

Question: **Ram is moving due east with a velocity of 1 m/s and Shyam is moving due west with a velocity of 2 m/s. What is the velocity of Ram with respect to shyam?**

Solution: It is a dimensional motion. So, let us choose the east direction as positive and the west as negative. Now given that

v_S = velocity of Ram = 1 m/s

and v_G = velocity of Shyam = - 2m/s

Thus, v_{SG} = velocity of Ram with respect to Shyam

= $v_S - v_G$ = 1– (-2) = 3 m/s

Hence, velocity of Ram with respect to Shyam is 3 m/s due east.

Illustration 19

Question: **A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?**

Solution: Speed of police van, $v_p = 30\,\text{kmh}^{-1} = \frac{30\times1000}{3600}\text{ms}^{-1} = \frac{25}{3}\text{ms}^{-1}$

Speed of thief's car, $v_t = 192\,\text{kmh}^{-1} = \dfrac{192 \times 1000}{3600}\text{ms}^{-1} = \dfrac{160}{3}\text{ms}^{-1}$

Speed of bullet, v_b = speed of police van + speed with which bullet is actually fired.

$\therefore \quad v_b = \left(\dfrac{25}{3} + 150\right)\text{ms}^{-1} = \dfrac{475}{3}\text{ms}^{-1}$

Relative velocity of bullet w.r.t. thief's car,

$v_{bt} = v_b - v_b = \left(\dfrac{475}{3} - \dfrac{160}{3}\right)\text{ms}^{-1}$ = 105 ms^{-1}

Illustration 20

Question: **A jet air plane traveling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the combustion products w.r.t. an observer on the ground?**

Solution: Speed of combustion products w.r.t. observer on the ground = ?

Velocity of jet air plane w.r.t. observer on ground = 500 km h^{-1}

If $\vec{v}_j$ and $\vec{v}_a$ represent the velocities of jet and observer respectively, then

$v_j - v_a = 500\,\text{kmh}^{-1}$

Similarly, if $\vec{v}_c$ represents the velocity of the combustion products w.r.t. jet plane, then

$v_c - v_f = -1500$ kmh^{-1}

The negative sign indicates that the combustion products move in a direction opposite to that of jet.

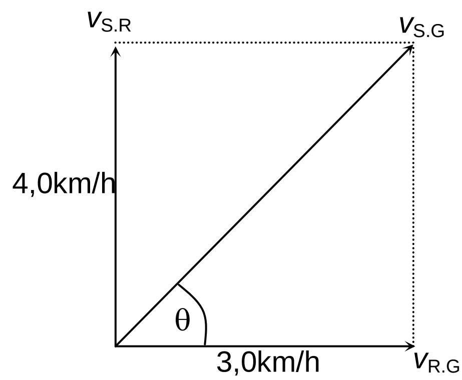
Speed of combustion products w.r.t. observer

$= v_c - v_a = (v_c - v_j) + (v_j - v_a) = (-1500 + 500)\text{kmh}^{-1}$ = – 1000 km h^{-1}

Illustration 21

Question: **A swimmer can swim in still water at a rate 4.0 km/h. If he swims in a river flowing at 3.0 km/h and keeps his direction (with respect to water) perpendicular to the current, find his velocity with respect to the ground.**

Solution: The velocity of the swimmer with respect to water is $\vec{v}_{S,R} = 4.0\,\text{km/h}$ in the direction perpendicular to the river. The velocity of river with respect to the ground is $\vec{v}_{R,G} = 3.0\,\text{km/h}$ along the length of the river. The velocity of the swimmer with respect to the ground is $\vec{v}_{S,G}$ where



$\vec{v}_{S,G} = \vec{v}_{S,R} + \vec{v}_{S,G}$

$v_{S,G} = \sqrt{(4.0\,\text{km/h})^2 + (3.0\,\text{km/h})^2}$ = 5. 0 km/h.

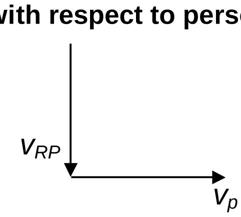
The angle θ made with the direction of flow is

$\tan\theta = \dfrac{4.0\,\text{km/h}}{3.0\,\text{km/h}} = \dfrac{4}{3}$

Illustration 22

Question: **To a standing person, the rains appears to fall vertically downward at 10 km/hr. If the person moves eastward at 5 km/hr, determine the direction of umbrella used to protect the person against rain. Also find the apparent velocity of the rain with respect to person**

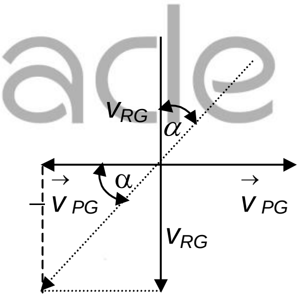
Solution: We have to find out $\vec{V}_{RP}$



$\vec{v}_{RP} = \vec{v}_{RG} - \vec{v}_{PG}$

or, $\vec{v}_{RP} = \vec{v}_{RG} + (-\vec{v}_{PG})$

So to find it, direction of velocity of person should reversed



$\vec{v}_{RP} = \vec{v}_{RG} + (-\vec{v}_{PG})$

$\therefore\ \vec{v}_{RP} = \sqrt{v_{RG}^2 + v_{PG}^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= 5\sqrt{5}$ km/hr

The direction of resultant can be found out by

$\tan\alpha = \dfrac{v_{RG}}{v_{PG}} = \dfrac{10}{5} = 2$

$\alpha = \mathbf{\tan^{-1}2}$

SOLVED OBJECTIVE EXAMPLES

***Example* 1:**

A particle is travelling with velocity of 2 m/s and moves in a straight line with a retardation of 0.1 m/s^2. The time at which the particle is 15m from the starting point is

(a) 10 s **(b) 20 s** **(c) 25 s** **(d) 40 s**

Solution:

$S = ut + \frac{1}{2}at^2; 15 = 2t + \frac{1}{2} \times (-0.1)t^2$

$\Rightarrow$ $20 \times 15 = 40t - t^2$ or $t^2 - 40t + 300 = 0$

$(t - 30)(t - 10) = 0$; $t = 30$ s

or $t = 10$ s

The particle is at *a* distance 15 m from starting point at t = **10 s** and also t = **30 s**.

$\therefore$ **(a)**

***Example* 2:**

A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as

(a) S^{-3} **(b) $S^{2/3}$** **(c) S^2** **(d) $S^{5/2}$**

Solution:

$S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\frac{dS}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right) \times a - \frac{(at + b)(at + b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at + b)^2}{\sqrt{at^2 + 2bt + c} \times (at^2 + 2bt + c)}$$

$$= \frac{(ac - b^2)}{S \cdot S^2}$$

$\therefore$ $\frac{d^2S}{dt^2} \propto \frac{1}{S^3}$ $\Rightarrow$ **acceleration $\propto S^{-3}$**

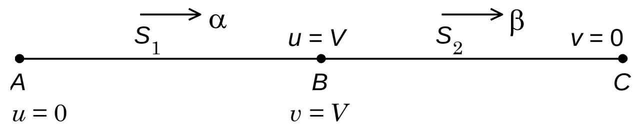
$\therefore$ **(a)**

***Example* 3:**

A car accelerates from rest at a constant rate α for sometime, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is *t*, the maximum velocity acquired by car is

(a) $\frac{\alpha\beta t}{\alpha+\beta}$ (b) $\frac{(\alpha+\beta)}{\alpha\beta}t$ (c) $\frac{\alpha^2+\beta^2}{\alpha\beta}t$ (d) $\frac{\alpha^2-\beta^2}{\alpha\beta}t$

Solution:



For motion from A to B,

$V=\alpha t_1$ or $t_1=\frac{V}{\alpha}$

For motion from B to C, $0=V-\beta t_2$ or $t_2=\frac{V}{\beta}$

$\therefore \quad t=t_1+t_2=\frac{V}{\alpha}+\frac{V}{\beta}=\frac{V(\alpha+\beta)}{\alpha\beta}$

or, $\quad \boldsymbol{V=\frac{\alpha\beta}{(\alpha+\beta)}t}$

$\therefore$ **(a)**

***Example* 4:**

Two particles *P* and *Q* start simultaneously from *A* with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different accelerations. When *P* overtakes *Q* at *B*, velocity of *P* is 30 m/s. The velocity of *Q* at *B* is

(a) 30 m/s (b) 25 m/s (c) 20 m/s (d) 15 m/s

Solution:

As displacement (in uniformly accelerated motion) = average velocity × time

The average velocity is the same, when overtaking takes place.

15 + 30 = 20 + v

or, v = **25 m/s**

$\therefore$ **(b)**

***Example* 5:**

A stone *A* is dropped from rest from a height *h* above the ground. A second stone *B* is simultaneously thrown vertically up with velocity *v*. The value of *v* which would enable the stone *B* to meet the stone *A* midway between their initial positions is

(a) 2 *gh* (b) $2\sqrt{gh}$ (c) $\sqrt{gh}$ (d) $\sqrt{2gh}$

Solution:

Time of travel of each stone = t

Distance travelled by each stone = $\frac{h}{2}$

For stone A, $\frac{h}{2}=\frac{1}{2}gt^2 \quad i.e., t=\sqrt{\frac{h}{g}}$

For stone B, $\frac{h}{2} = ut - \frac{1}{2}gt^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$

$\Rightarrow \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2}$

or, $u\sqrt{\frac{h}{g}} = h$

$\therefore \quad u = h\sqrt{\frac{g}{h}} = \sqrt{\boldsymbol{gh}}$

$\therefore$ **(c)**

Example **6:**

A body is dropped from rest from a height *h*. It covers a distance 9*h*/25 in the last second of fall. The height *h* is

(a) 102.5 m **(b) 112.5 m** **(c) 122.5 m** **(d) 132.5 m**

Solution:

t is time to reach ground.

$h=\frac{1}{2}at^2; \left(1-\frac{9}{25}\right)h=\frac{1}{2}a(t-1)^2$

$\left(1-\frac{9}{25}\right)=\frac{(t-1)^2}{t^2}; \frac{16}{25}=\frac{(t-1)^2}{t^2}$ or $\frac{4}{5}=\frac{t-1}{t}$ $\therefore t = 5$ sec

$h = \frac{1}{2} \times 9.8 \times 5^2 = \mathbf{122.5\ m}$

$\therefore$ **(c)**

Example **7:**

A particle *P* is projected vertically upward from a point *A*. Six seconds later, another particle *Q* is projected vertically upward from *A*. Both *P* and *Q* reach A simultaneously. The ratio of maximum heights reached by *P* and *Q* = 64 : 25. Find the velocity of the projection of *Q* in m/s

(a) 7 *g* **(b) 6 *g*** **(c) 5 *g*** **(d) 4 *g***

Solution:

$\frac{1}{2}g(t+3)^2 : \frac{1}{2}gt^2 = 64 : 25$

or $(t + 3)^2 : t^2 = 64 : 25$; or $(t + 3) : t = 8 : 5$

$5t + 15 = 8t$ or $3t = 15$; $t = 5$ seconds

$v = g \times t = g \times 5 = \mathbf{5g\ m/s}$

$\therefore$ **(c)**

Example **8:**

A stone is dropped from rest from the top of a cliff. A second stone is thrown vertically down with a velocity of 30 m/s two seconds later. At what distance from the top of a cliff do they meet?

(a) 60 m **(b) 120 m** **(c) 80 m** **(d) 44 m**

Solution:

The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u(t-2) + \frac{1}{2}g(t-2)^2$

i.e., $\frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2gt + 2g$

$0 = (u-2g)t - 2(u-g)$; $t = \frac{2(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4$ s

Distance S at which they meet $= \frac{1}{2} \times gt^2 = \frac{1}{2} \times 10 \times 16$

= 80 m from top of cliff

∴ **(c)**

***Example* 9:**

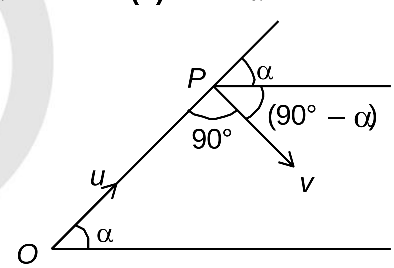
A particle is projected from a point *O* with velocity *u* in a direction making an angle α upward with the horizontal. At *P*, it is moving at right angles to its initial direction of projection. Its velocity at *P* is

(a) $u \tan \alpha$ **(b) $u \cot \alpha$** **(c) $u \operatorname{cosec} \alpha$** **(d) $u \sec \alpha$**

Solution:

$v \cos(90 - \alpha) = v \sin \alpha = u \cos \alpha$; $v = \boldsymbol{u \cot \alpha}$

∴ **(b)**



***Example* 10:**

In the previous example the time of flight from *O* to *P* is

(a) $\frac{u \operatorname{cosec} \alpha}{g}$ **(b) $\frac{u \sin \alpha}{g}$** **(c) $\frac{u \tan \alpha}{g}$** **(d) $\frac{u \sec \alpha}{g}$**

Solution:

$v = g't = (g \cos\alpha)t = u \cot \alpha$

$t = \frac{u\cot\alpha}{g\cos\alpha} = \frac{u}{g} \cdot \frac{1}{\sin\alpha} = \boldsymbol{\frac{u \operatorname{cosec} \alpha}{g}}$

∴ **(a)**

***Example* 11:**

A particle is projected from *O* and is moving freely under gravity and strikes the horizontal plane through *O* at a distance *R* from it. Then which of the following is incorrect?

(a) There will be two angles of projection if $Rg < u^2$

(b) There will be more than two angles of projection if $Rg = u^2$

(c) The two possible angles of projection are complementary

(d) The products of the times of flight for two angles of projection is $2R/g$

Solution:

The range $R = \dfrac{2u^2 \sin\alpha\cos\alpha}{g}$

$\sin 2\alpha = \dfrac{Rg}{u^2} \Rightarrow Rg = u^2$, then $\sin 2\alpha = 1$, so $\alpha = 45^0$.

$\sin 2\alpha_1 = \sin 2\alpha_2 \Rightarrow \dfrac{Rg}{u^2}$ $\sin 2\alpha_1 - \sin 2\alpha_2 = 0$

or, $2\cos(\alpha_1 + \alpha_2)\sin(\alpha_1 - \alpha_2) = 0$

If $Rg < u^2$, $\alpha_1 + \alpha_2 = \pi/2$; if $Rg = u^2$, $\alpha_1 = \alpha_2 = \pi/2$

$t_1 t_2 = \dfrac{2u\sin\alpha_1}{g} \times \dfrac{2u\sin\alpha_2}{g} = \dfrac{4u^2\sin\alpha_1\cos\alpha_1}{g^2} = \dfrac{2R}{g}$ since $\alpha_1 + \alpha_2 = \pi/2$

$\therefore$ **(b)**

***Example* 12:**

The velocity of a particle *P* moving freely under gravity is 4.9 m/s, the direction being 30° with the downward normal

(a) its acceleration normal to the direction of motion at *P* = 9.8 m/s^2

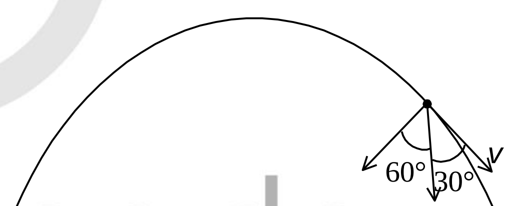
(b) the radius of curvature of *P* of the parabolic trajectory of particle is 4. 9 m

(c) the particle has no acceleration normal to the direction of motion

(d) the radius of curvature at *P* of the path depends upon the initial velocity of projection

Solution:

The component of acceleration perpendicular to velocity is $g\cos 60°$ and this provide the necessary centripetal acceleration.



$g\cos 60° = \dfrac{g}{2} = 4.9$ m/s^2

The radius of curvature is $\dfrac{v^2}{r} = 4.9$;

$r = \dfrac{v^2}{4.9} = \mathbf{4.9\ m}$

$\therefore$ **(b)**

***Example* 13:**

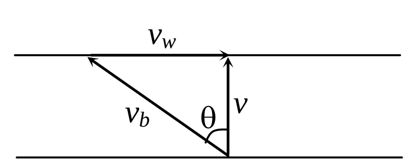
A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in kilometres per hour is

(a) 1 **(b) 3** **(c) 4** **(d) $\sqrt{41}$**

Solution:

For shortest possible path

$$v = \frac{1\text{km}}{15\,\text{min}} = 4\,\text{km/h}$$



$$v_w^2 + v^2 = v_b^2$$

$$v_w = \sqrt{v_b^2 - v^2} = \sqrt{5^2 - 4^2} = 3\text{kmh}^{-1}$$

$\therefore$ **(b)**

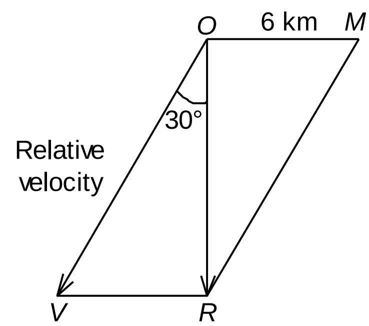
***Example* 14:**

A man running at 6 km/hr on a horizontal road in vertically falling rain observes that the rain hits him at 30° from the vertical. The actual velocity of rain has magnitude

(a) 6 km/hr **(b) $6\sqrt{3}$ km/hr** **(c) $2\sqrt{3}$ km/hr** **(d) 2 km/hr**

Solution:

Velocity of rain = Velocity of man + Relative velocity of rain *OR* gives the actual velocity.



$$\tan 30° = \frac{VR}{OR} = \frac{1}{\sqrt{3}} = \frac{6}{OR}$$

or, $OR = \mathbf{6\sqrt{3}}$ **km/hr**

$\therefore$ **(b)**

***Example* 15:**

A boat which has a speed of 5 km/hr in still water crosses a river of width 3 km along the shortest possible path in *t* min. The river flows at the rate of 3 km/hr. The time taken *t* is

(a) 20 min **(b) 25 min** **(c) 45 min** **(d) 55 min**

Solution:

$$t = \frac{AB}{\sqrt{5^2 - 3^2}} = \frac{3}{4} = \textbf{45 minutes}$$

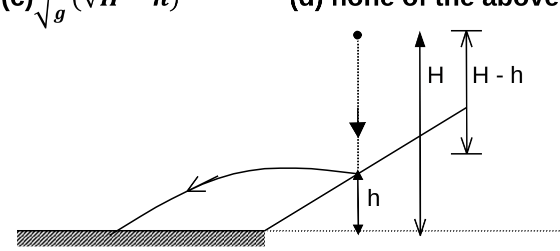
$\therefore$ **(c)**

***Example* 16:**

A body falling freely from a given height H hits an inclined plane in its path at a height 'h'. As a result of this impact the direction of the velocity of the body becomes horizontal. Find the total time the body will take to reach the ground.

(a) $\sqrt{\frac{2}{g}}(\sqrt{h} + \sqrt{H-h})$ **(b)** $\sqrt{2gh}$ **(c)** $\sqrt{\frac{2}{g}}(\sqrt{H-h})$ **(d) none of the above**

Solution: (A) Time taken by the body to strike the inclined plane



t1 = $\sqrt{\frac{2(H-h)}{g}}$

Now as after impact the velocity of the body is horizontal, so time taken to reach the ground -

t2 = $\sqrt{\frac{2h}{g}}$

So total time of motion -

t = t1 + t2 = $\sqrt{\frac{2}{g}}[\sqrt{h}+\sqrt{H-h}]$

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

A particle moves along a straight path *ABC* with a uniform acceleration of 0.5 m/s^2. While it crosses *A* its velocity is found to be 5 m/s. It reaches *C* with a velocity 40 m/s, 30 seconds after it has crossed B in its path. Find the distance *AB*.

Solution:

The velocity while it crosses the point *A* is 5 m/s

Considering the displacement *AC*,

initial velocity u = 5 m/s

final velocity v = 40 m/s

acceleration a = 0.5 m/s^2

$\therefore$ time of motion $t = \frac{v-u}{a} = \frac{40-5}{0.5} = 70$ s

For the displacement *AB*,

initial velocity u = 5 m/s

acceleration a = 0.5 m/s^2

time of motion t = 70 – 30 = 40 s

$\therefore \quad AB = S = ut + \frac{1}{2}at^2 = (5 \times 40) + \left(\frac{1}{2} \times 0.5 \times 40^2\right) = 200 + 400 =$ **600 m**

***Example* 2:**

A particle moving with uniform acceleration in a straight line covers a distance of 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement (in cm) of the particle from the beginning of the 6th second to the end of 15th second?

Solution:

The distance traveled during the nth second of motion of a body is given by

$$S_n = u + \frac{1}{2}a\ (2n-1)$$

For the motion during the 8th second,

$$3 = u + \frac{1}{2}a\ (16-1) = u + \frac{15a}{2} \qquad \text{... (i)}$$

For the motion during the 16th second,

$$5 = u + \frac{1}{2}a\,(32-1) = u + \frac{31a}{2} \qquad \text{... (ii)}$$

Subtracting equations (i) from (ii)

$8a = 2$

or acceleration $a = \frac{1}{4}$ ms^2

From equation (1), $u = 3 - \left(\frac{15}{2}\times\frac{1}{4}\right)=\frac{9}{8}$ ms^2

Now, the velocity at the end of 5 s (velocity at the beginning of 6th second)
$v_1 = u + 5a$

The velocity at the end of 15th s, $v_2 = u + 15a$

Average velocity during this interval of 10 seconds = $\frac{v_1 + v_2}{2}$

$$=\frac{(u+5a)+(u+15a)}{2} = u + 10a$$

Distance travelled during this interval

S = average velocity x time = $(u + 10a) \times t$

$$= \left(\frac{9}{8}+\frac{10}{4}\right)\times 10 = \frac{290}{8} = 36.25 \text{ m } \mathbf{= 3625\ cm}$$

***Example* 3:**

An automobile can accelerate or decelerate at a maximum value of $\frac{5}{3}$ m/s^2 and can attain a maximum speed of 90 km/hr. If it starts from rest, what is the shortest time in which it can travel one kilometre, if it is to come to rest at the end of the kilometre run?

Solution:

In order that the time of motion be shortest, the automobile should attain the maximum velocity with the maximum acceleration after the start, maintain the maximum velocity for as along as possible and then decelerate with the maximum retardation possible, consistent with the condition that, the automobile should come to rest immediately after covering a distance of 1 km.

Let t_1 be the time of acceleration, t_2 be the time of uniform velocity and t_3 be the time of retardation.

Now, maximum velocity possible = 90 km/hr = $90 \times \frac{5}{18}$ m/s = 25 m/s

$$t_1 = \frac{v-u}{a} = \frac{25-0}{\frac{5}{3}} = 15 \text{ s}$$

Similarly, the time of retardation is also given by

$$t_3 = \frac{0-25}{-\frac{5}{3}} = 15 \text{ s}$$

During the period of acceleration, the distance covered

= average velocity x time

$$= \frac{25+0}{2} \times 15 = 187.5 \text{ m}$$

During the period of retardation, the distance covered is the same and hence

= 187.5 m

$\therefore$ the total distance covered under constant velocity = 1000 – 375 = 625 m

Time of motion under constant velocity, $t_2 = \frac{625}{25} = 25s$

$\therefore$ the shortest time of motion = $t_1 + t_2 + t_3$ = 15 + 25 + 15 = **55 seconds**

***Example* 4:**

A stone is dropped into a well and the sound of the splash is heard $3\frac{1}{8}$ seconds later. If the velocity of sound in air is 352.8 m/s, find the depth of the well (in cm). g = 9.8 m/s^2.

Solution:

Let x metres be the depth of the well and t the time taken by the stone to reach the surface of water.

In this case $u = 0$, $a = 9.8$ m/s^2

Now in the relation, $S = ut + \frac{1}{2}at^2$,

we have

$$x = 0 + \frac{1}{2}(9.8)\,t^2$$

or $x = 4.9t^2$ ... (i)

Time taken by the sound to travel the distance x up (Motion of sound wave is not affected by gravity) is

$\left(3\frac{1}{8} - t\right)$ seconds

Distance traveled = Velocity of sound × Time taken

$$x = 352.8\left(\frac{25}{8} - t\right) \quad \text{... (ii)}$$

From equations (i) and (ii),

$$4.9t^2 = 352.8\left(\frac{25}{8} - t\right)$$

or, $4.9t^2 + 352.8t - 1102.5 = 0$

or, $t^2 + 72t - 225 = 0$

Solving, t = 3 or – 75 s.

Since the negative value of t has no meaning, t = 3 s.

This gives $x = 4.9t^2$ = 4.9 x 9 = 44.1 m

Hence the depth of the well = 44.1 m = **4410 cm**

***Example* 5:**

A circus artist maintains four balls in motion making each in turn rise to a height of 5 m from his hand. With what velocity (in m/s) does he project them and the height (in cm) of the other three balls at the instant when the fourth one is just leaving his hand? (take g = 10 m/s^2.)

Solution:

Obviously, to maintain proper distances, the artists must throw the balls after equal intervals of time. Let the interval of time be t, so that when the fourth ball is just leaving his hand, the first ball would have travelled for time $3t$, the second for time $2t$ and the third for time t. The second obviously would just have reached the maximum height of 5 m.

If v be the initial velocity of throw of each ball, then for the second ball we have,

$$v_2 = 0 = v - g\,(2t) \qquad \text{... (i)}$$

and $$s_2 = 5 = v\,(2t\,) - \frac{1}{2}g(2t)^2 \qquad \text{... (ii)}$$

These gives, $v = 2gt$

and $v \cdot 2t = 2t^{\,2} + 5$

or $v = 20t$

and $v \cdot 2t = 20t^{\,2} + 5$

Solving for t, we get $20t^2 = 5$ or $t = \frac{1}{2}$ second.

Therefore, $v = 20 \times \frac{1}{2} = 10$ m/s. Thus each ball is thrown up with initial velocity of **10 m/s**.

For the first ball, which would have come down for time (3t – 2t) = t, we have

$$S = 0 + \frac{1}{2} g \cdot t^{\,2}$$

$$= \frac{1}{2} \cdot 10 \cdot \left(\frac{1}{2}\right)^2 = \frac{5}{4} = \mathbf{125\ cm}$$

Therefore, it will be at a height of (5 – 1.25) = 3.75 m = **375 cm** from the hand and going downwards.

For the third ball, which will have risen up for time t,

$$S_3 = vt - \frac{1}{2}gt^2 = 10\left(\frac{1}{2}\right) - \frac{1}{2}\cdot 10 \cdot \left(\frac{1}{2}\right)^2 = 5 - 1.25 = \mathbf{375\ cm}$$

***Example* 6:**

A stone is projected from the point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height $2h$ above the ground. If at the instant of projection the bird were to fly away horizontally with uniform speed v = $2\left(\sqrt{2}-1\right)$ m/s. Find the horizontal velocity of the stone if the stone still hits the bird while descending.

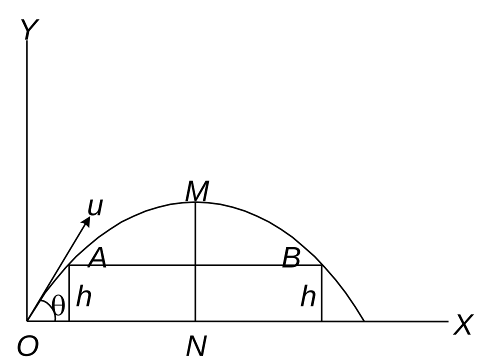
Solution:

The situation is shown in Figure. Let θ be the angle of projection and u the velocity of projection.

Maximum height $MN = 2h$

$$MN = 2h = \frac{u^2 \sin^2\theta}{2g}$$

$$\therefore\ u\sin\theta = 2\sqrt{gh} \qquad \text{... (i)}$$



Let t be the time taken by stone to attain the vertical height h above the ground.

$$\therefore \qquad h = (u\sin\theta)t - \frac{1}{2}gt^{\,2}$$

$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} - \frac{2h}{g}}$$

Substituting the value of u sin θ from (i),

$$t = \frac{2\sqrt{gh}}{g} \pm \sqrt{\frac{4gh}{g^2} - \frac{2h}{g}} \quad = \sqrt{\frac{4h}{g}} \pm \sqrt{\frac{2h}{g}}$$

$$t_1 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}} \qquad t_2 = \sqrt{\frac{4h}{g}} + \sqrt{\frac{2h}{g}}$$

where t_1 and t_2 are time to reach A and B respectively shown in the figure. If v is the horizontal velocity of bird, then

$AB = vt_2$.

AB is also equal to $u\cos\theta\,(t_2 - t_1)$, where $u\cos\theta$ is constant horizontal velocity of stone.

$$t_2 - t_1 = 2\sqrt{\frac{2h}{g}}$$

$$\therefore \qquad u\cos\theta \cdot 2\sqrt{\frac{2h}{g}} = vt_2$$

$$\frac{v}{u\cos\theta} = \frac{2\sqrt{\frac{2h}{g}}}{t_2} = \frac{2\sqrt{\frac{2h}{g}}}{\sqrt{\frac{2h}{g}}\cdot(\sqrt{2}+1)}$$

$$\frac{v}{u\cos\theta} = \frac{2}{\sqrt{2}+1}$$

$$\therefore \qquad u\cos\theta = \textbf{1 m/s}$$

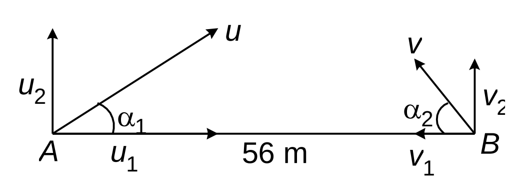
***Example* 7:**

Two particles are projected at the same instant from two points *A* and *B* on the same horizontal level where *AB* = 56 m, the motion taking place in a vertical plane through *AB*. The particle from *A* has an initial velocity of 39 m/s at an angle $\sin^{-1}\left(\frac{5}{13}\right)$ with *AB* and the particle from *B* has an initial velocity of 25 m/s at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ with BA. Show that the particles would collide in mid-air and find when and where the impact occurs.

Solution:

AB = 56 m.

At A, a particle is projected with velocity u = 39 m/s. u_1 and u_2 are its horizontal and vertical components respectively. The angle u makes with AB is α_1.



Given that $\sin\alpha_1 = \frac{5}{13} \therefore \cos\alpha_1 = \frac{12}{13}$.

Similarly, for the particle projected from B, with velocity v = 25 m/s, v_1 and v_2 are the horizontal and vertical components respectively.

$\sin \alpha_2 = \frac{3}{5} \ \therefore \ \cos \alpha_2 = \frac{4}{5}$.

Now $u_2 = u \sin \alpha_1 = 39 \times \frac{5}{13} = 15$ m/s.

$v_2 = v \sin \alpha_2 = 25 \times \frac{3}{5} = 15$ m/s.

The vertical components of the velocities are the same at the start. Subsequently at any other instant t their vertical displacement are equal and have a value

$h = 15t - 5t^2$

which means that the line joining their positions at the instant t continues to be horizontal and the particles come closer to each other.

Their relative velocity in the horizontal direction

$$= 39 \cos \alpha_1 + 25 \cos \alpha_2$$

$$= 39 \times \frac{12}{13} + 25 \times \frac{4}{5} = 36 + 20 = \mathbf{56\ m/s}$$

Time of collision $= \frac{AB}{56} = \frac{56}{56}$ = **1 s**, after they were projected.

Height at which the collision occurs $= ut - \frac{1}{2}at^2 = 15(1) - \frac{1}{2}(10)(1)^2$

$$= \mathbf{10\ m}$$

The horizontal distance of the position of collision from A

$$= 39 \times \frac{12}{13} \times 1 \text{ s} = \mathbf{36\ m}$$

***Example* 8:**

A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is α = 30° and the angle of barrel to the horizontal is β = 60°. The initial velocity of shell is 21 m/s. Find the distance from the gun to the point at which the shell falls.

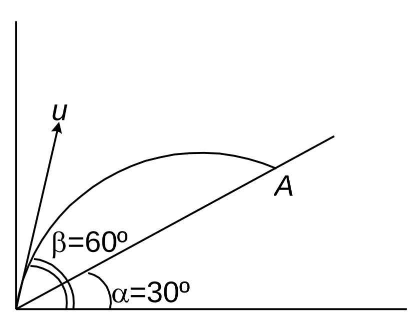
Solution:

We can write the equation of motion as

$$x = ut \cos \beta$$

$$y = ut \sin \beta - \frac{gt^2}{2}$$

$$OA = \ell$$



At the moment the shell falls to the ground

$$x = \ell \cos \alpha = \ell \cos 30°$$

$$y = \ell \sin \alpha = \ell \sin 30°$$

$$\ell \cos \alpha = ut \cos \beta \quad \text{... (i)}$$

$\ell \sin\alpha = ut \sin\beta - \dfrac{gt^2}{2}$ ... (ii)

$\therefore \quad t = \dfrac{\ell\cos\alpha}{u\cos\beta}$

$$\ell\sin\alpha = \frac{\ell\cos\alpha\sin\beta}{\cos\beta} - \frac{g\ell^2\cos^2\alpha}{2u^2\cos^2\beta} \Rightarrow \ell = \frac{2u^2\sin(\beta-\alpha)\cos\beta}{g\cos^2\alpha}$$

Substituting u = 21 m/s, $\alpha = 30^\circ$, $\beta = 60^\circ$ and g = 9.8 m/s^2, we get ℓ **= 30 m**

***Example* 9:**

A man can swim at a velocity V_1 relative to water in a river flowing with speed V_2. Show that it will take him $\dfrac{V_1}{\sqrt{V_1^2 - V_2^2}}$ times as long to swim a certain distance upstream and back as to swim the same distance and back perpendicular to the direction of the stream ($V_1 > V_2$). (given $\sqrt{24}V_1 = \sqrt{25}V_2$)

Solution:

Suppose the man swims a distance x up and the same distance down the stream.

Velocity of man upstream relative to the ground = $V_1 - V_2$.

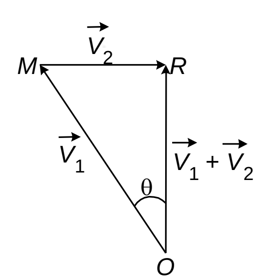
Time taken for this, $t_1 = \dfrac{x}{V_1 - V_2}$

Velocity of man downstream relative to the ground = $V_1 + V_2$

Time taken for this, $t_2 = \dfrac{x}{V_1 + V_2}$

Total time taken $t_1 + t_2 = \dfrac{x}{(V_1 - V_2)} + \dfrac{x}{(V_1 + V_2)} = \dfrac{2V_1 x}{(V_1^2 - V_2^2)}$

Next the man intends crossing the river perpendicular to the direction of the stream. If he wants to cross the river straight across he must swim in a direction OM such that the vector sum of velocity of man + velocity of river will give him a velocity relative to the ground in a direction perpendicular to the direction of the stream. In the Figure the velocity relative to the ground is $\overrightarrow{OR}$ and the magnitude of $\overrightarrow{OR} = \sqrt{V_1^2 - V_2^2}$



Now the man swims a distance x up and x down perpendicular to the river flow. Time taken for this, $t = \dfrac{2x}{\sqrt{V_1^2 - V_2^2}}$

Then the ratio, $\dfrac{t_1 + t_2}{t} = \dfrac{2V_1 x}{(V_1^2 - V_2^2)} \div \dfrac{2x}{\sqrt{V_1^2 - V_2^2}}$

$$= \frac{2V_1 x}{(V_1^2 - V_2^2)} \times \frac{\sqrt{V_1^2 - V_2^2}}{2x} = \frac{V_1}{\sqrt{V_1^2 - V_2^2}} = \mathbf{5}$$

***Example* 10:**

A man walking eastward at 6 km/hr finds that the wind seems to blow directly from north. On doubling his velocity, the wind appears to come *N* 30° *E*. Find the speed of the wind.

Solution:

Actual velocity of the man = 6 km/hr eastward.

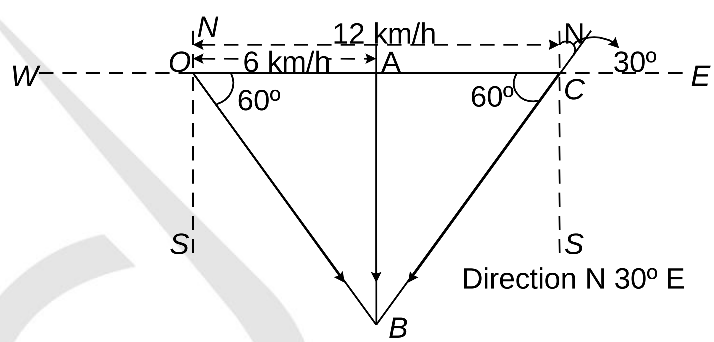
The direction of the relative velocity of the wind in this case is North to South.

If $\vec{OA}$ represents the velocity of man and $\vec{AB}$ represents the relative velocity of the wind, then

velocity of man + relative velocity of wind = velocity of wind $= \vec{OB}$ (say)

It is also given that when the velocity of the man is doubled (i.e., 12 km/hr) the wind seems to blow from a direction *N* 30°E. Representing this by vector $\vec{OC}$ = New velocity of man = 12 km/hr.

The direction of the relative velocity of the wind in this case is *CB*. The two directions of the relative velocity meet at *B*. Hence $\vec{OB}$ should give the real velocity of the wind. From the geometry of the Figure, it is clear that *OBC* is an equilateral triangle. Hence the magnitude of the real velocity of the wind = **12 km/hr**.



MIND MAP

KINEMATICS

1. Relation between kinematic variables for motion in one dimension

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{vdv}{dx}$$

2. Equations of motion in one dimension

- Motion with uniform velocity $S = vt$
- Motion with uniform acceleration,

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

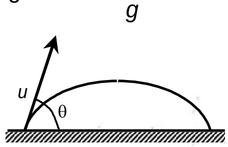
$$S_n = u + (2n-1)\frac{a}{2}$$

3. Graphical representation of motion

- Slope of tangent to position time graph gives velocity.
- Slope of tangent to v–t curve gives acceleration.
- Area enclosed between v–t curve and time axis between an interval of time gives displacement.
- Slope of tangent to a–t curve gives rate of change of acceleration
- Area enclosed between a–t curve and time axis between an interval of time gives change in velocity.

4. Projectile on horizontal plane

- Time of flight, $T = \dfrac{2u\sin\theta}{g}$
- Range $R = \dfrac{u^2 \sin 2\theta}{g}$



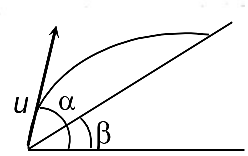
- Maximum Height, $H = \dfrac{u^2 \sin^2\theta}{2g}$
- Equation of trajectory ,

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\theta$$

- For maximum range, $\theta = 45°$
- For a given speed and given range, there are two possible angles of projection; θ and $(90° - \theta)$.

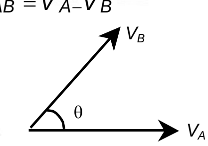
5. Projectile on inclined plane

- Time of flight, $T = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$
- Range, $R = \dfrac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$



6. Relative velocity

- $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$



- $|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$
- If relative velocity makes an angle α with V_A then,

$$\tan\alpha = \frac{V_B\sin\theta}{V_A - V_B\cos\theta}$$

EXERCISE – 1
BUILDING A FOUNDATION

SECTION-A MOTION, PARAMETERS OF MOTION, ACCELERATED MOTION

A-1. Consider the motion of the tip of the minute hand of a clock. In one hour
(a) the displacement is zero (b) the distance covered is zero
(c) the average speed is zero (d)none

A-2. A particle moves along a circular path of diameter 40m. The distance and displacement of the particle in completing one rotation is :
(a) 40πm, 40m (b) 40π m, 0 (c) 40π m, 20m (d) 20π m,0

A-3. A carrom board (4 ft × 4 ft square) has the queen at the centre. The queen hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. The displacement of the queen from the centre to the front edge is :
(a) $\frac{2}{3}\sqrt{10}$ ft (b) $\frac{4}{3}\sqrt{10}$ ft (c) $2\sqrt{2}$ ft (d) 4ft

A-4. A particle moving on a straight line moves half a distance with a velocity of 10m/s and next half distance with a constant velocity 5m/s. What is the average velocity of the particle during the entire journey?
(a) 7.5 m/s (b) 3.33 m/s (c) 2.5 m/s (d) 6.67 m/s

A-5. A particle moving on a straight line moves with a velocity of 10m/s for some time interval and with a velocity of 5 m/s for the same time interval. What is the average velocity of particle during the whole journey?
(a) 7.5 m/s (b) 3.33 m/s (c) 2.5 m/s (d) 1.67 m/s

A-6. A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this:
(a) its velocity is changed in magnitude (b) its velocity is not changed
(c) its speed only is changed (d) its velocity is changed

A-7. A train was moving at a rate of 36 km/h. When the brakes were applied, it came to rest at a distance 200m. The retardation produced in the train is :
(a) 0.20 m/s^2 (b) 0.3 m/s^2 (c) 0.25 m/s^2 (d) 0.50 m/s^2

A-8. A body starts from rest and is uniformly accelerated for 30s. The distance travelled in the first 10s is x_1, next 10s is x_2 and the last 10s is x_3. Then $x_1 : x_2 : x_3$ is the same as :
(a) 1 : 2 : 4 (b) 1 : 2 : 5 (c) 1 : 3 : 5 (d) 1 : 3 : 9

A-9. A body is moving in a straight line with uniform acceleration. It covers distance of 10 m and 12 m in third and fourth seconds respectively. Then the initial velocity in m/sec is :
(a) 2 (b) 3 (c) 4 (d) 5

A-10. A car, moving with a speed of u km/h can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 2 u km/h, the minimum stopping distance is:
[assuming retardation remain same]
(a) 12 m (b) 18 m (c) 24 m (d) 6 m

A-11. A particle moves with constant acceleration for 6 seconds after starting from rest. The distances travelled during the consecutive 2 seconds interval are in the ratio
(a) 1 : 1 : 1 (b) 1 : 2 : 3 (c) 1 : 3 : 5 (d) 1 : 5 : 9

SECTION-B MOTION UNDER GRAVITY

B-1. Body 1 of mass 2 kg is dropped from a height h and body 2 of mass 6 kg is dropped from a height 4 h . Ratio of time taken by the bodies 1 and 2 is :
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 3

B-2. A stone is released from an elevator going up with acceleration 5 m/s^2. The acceleration of the stone after the release is :
(a) 5 ms^{-2} (b) 4.8 ms^{-2} upward
(c) 4.8 down ward (d) 9.8 ms^{-2} down ward

B-3. A particle is projected vertically upward from A with a speed of 50 m/s and another is dropped simultaneously from B, which is 200 m vertically above A. They cross each other after :
(a) 4 s (b) 5 s (c) 6 s (d) 8 s.

B-4. One body is dropped while a second body is thrown downwards with an initial velocity of 1 m/s simultaneously. The separation between these two bodies is 18 m after a time:
(a) 18 s (b) 9 s (c) 4.5 s (d) 36 s

B-5. A trolley runs down a slope from rest with constant acceleration. In the first second of its motion it travels 1.6 m. Its acceleration (in m/s2) is
(a) 3.2 (b) 1.6 (c) 0.8 (d) 2.4

SECTION-C DIFFERENTIATION AND ITS APPLICATION

C-1. $y = x^2 + 6x + 10$; then minimum value of y is
(a) 1 (b) 2 (c) 3 (d) None of these

C-2. $y = 3\sin x + 4\cos x$; then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{7}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $-\frac{7}{\sqrt{2}}$

C-3. $y = x^5 + x^3 + x$; then $\frac{dy}{dx}$ at x = 1 is
(a) 8 (b) 9 (c) 12 (d) 15

C-4. $y = x^4 - 4x^{-2} + 5$; then $\frac{dy}{dx}$ at x = 1 is
(a) 12 (b) 8 (c) – 4 (d) – 8

C-5. $y = x^3 + 5$; then slope of the curve at x = 2 is
(a) 12 (b) 8 (c) – 4 (d) – 8

C-6. $y = x^{-3/2}$; find slope of the curve at x = 4 is
(a) $\frac{3}{64}$ (b) $-\frac{3}{64}$ (c) $\frac{5}{64}$ (d) $-\frac{5}{64}$

C-7. $y = x^{-3} - x^{-2}$; rate of change of y wrt x at x = 1 is
(a) 1 (b) – 1 (c) – 5 (d) 5

C-8. $y = x^2 e^x$; then $\frac{dy}{dx}$ at $x = 1$
(a) e (b) 2e (c) 3e (d) 4e

C-9. $y = x^2 \ln x$; then $\frac{dy}{dx}$ at $x = e$
(a) e (b) 2e (c) 3e (d) 4e

C-10. $y = \sin x \cos x$; then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$
(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) zero

C-11. $y = x\sin x$; then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$
(a) $\frac{1}{\sqrt{2}}\left(1 + \frac{\pi}{4}\right)$ (b) $\frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$ (c) $\frac{1}{\sqrt{2}}\left(\frac{\pi}{4} - 1\right)$ (d) $-\frac{1}{\sqrt{2}}\left(1 + \frac{\pi}{4}\right)$

C-12. $y = (x^2 + a^2)^{10}$; then $\frac{dy}{dx}$ is

(a) $10(x^2 + a^2)^9$ (b) $10\,x(x^2 + a^2)^9$
(c) $20\,x\,(x^2 + a^2)^9$ (d) $20\,x\,(x^2 + a^2)^{10}$

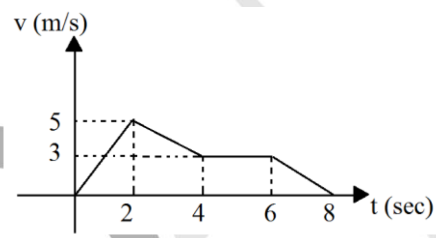
C-13. $y = (x+1)^3 + 5$; then $\frac{dy}{dx}$ is
(a) $3(x+1)^2$ (b) $2(x+1)^3$ (c) $3(x+1)$ (d) Zero

C-14. $y = \sin 2x$; then $\frac{dy}{dx}$ is
(a) $\cos 2x$ (b) $\frac{\cos 2x}{2}$ (c) $2\cos 2x$ (d) None of these

C-15. $y = \sin^2 x$; then $\frac{dy}{dx}$ is
(a) $\sin 2x$ (b) $\cos 2x$ (c) $\frac{\sin 2x}{2}$ (d) $2\sin 2x$

Passage (for question no. 16-19) :

Acceleration is rate of change in velocity. If acceleration is constant, then velocity of body increases linearly. Velocity time graph of a particle is shown in figure.



C-16. Acceleration of body at t = 1 sec
(a) $\frac{5}{2}$ m/s^2 (b) 1 m/s^2 (c) Zero (d) $\frac{3}{2}$ m/s^2

C-17. Acceleration of body at t = 3 sec
(a) $-\frac{5}{2}$ m/s^2 (b) – 1 m/s^2 (c) Zero (d) $-\frac{3}{2}$ m/s^2

C-18. Acceleration of body at t = 5 sec
(a) $\frac{5}{2}$ m/s^2 (b) 1 m/s^2 (c) Zero (d) $-\frac{3}{2}$ m/s^2

C-19. Acceleration of body at t = 7 sec
(a) $-\frac{5}{2}$ m/s^2 (b) – 1 m/s^2 (c) Zero (d) $-\frac{3}{2}$ m/s^2

C-20. The displacement x of particle along a straight line at time t is given by $x = a_0 + a_1 t + a_2 t^2$. The acceleration of the particle is:
(a) a_0 (b) a_1 (c) $2a_2$ (d) a_2

C-21. The displacement of a particle is proportional to the first power of time $s \propto t$, then acceleration of the particle is :
(a) infinite (b) zero (c) small finite value (d) large finite value

C-22. A particle moves along x–axis in such a way that its coordinate x varies with time t according to the expression, $x = 2 - 5\,t + 1.25\,t^2$. The initial velocity of the particle is:
(a) – 5 m/s (b) – 3 m/s (c) 3 m/s (d) 6 m/s

SECTION-D INTEGRATION AND ITS APPLICATION

D-1. (a) 2x (b) x^2 (c) $x^2 - 2x + 1$

D-2. (a) $\frac{1}{x^2}$ (b) $\frac{5}{x^2}$ (c) $2 - \frac{5}{x^2}$

D-3. (a) $\frac{3}{2}\sqrt{x}$ (b) $\frac{3}{2\sqrt{x}}$ (c) $\sqrt{x} + \frac{1}{\sqrt{x}}$

D-4. (a) $\frac{4}{3}\sqrt[3]{x}$ (b) $\frac{1}{3\sqrt[3]{x}}$ (c) $\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}$

D-5. $(1-x^2-3x^5)$

D-6. 3 sin x

D-7. $\frac{4}{3}x^3+\frac{7}{x^2}+x$

D-8. X^8+9

D-9. x^{-7}

D-10. $\frac{1}{3x}$

D-11. $\int_{-2}^{1} 5\,dx$

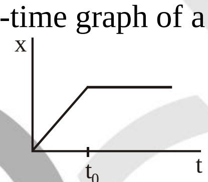
D-12. $\int_{-4}^{-1}\frac{\pi}{2}\,d\theta$

D-13. $\int_{-2}^{4}\left(\frac{x}{2}+3\right)\,dx$

D-14. $\int_{0}^{2\pi}\sin\theta\,\,d\theta$

SECTION-E CONCEPTS OF GRAPHS

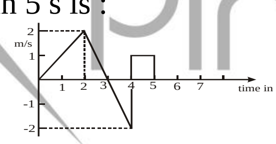
E-1. Figure shows the displacement-time graph of a particle moving on the X-axis.



(a) the particle is continuously going in positive x direction
(b) the particle is at rest
(c) the velocity increases up to a time t_0, and then becomes constant
(d) the particle moves at a constant velocity up to a time t_0, and then stops.

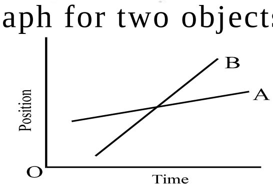
E-2. On a displacement-time graph two straight lines make 30º and 60º with the time-axis. The ratio of the velocities represented by them is :
(a) $1:\sqrt{3}$ (b) 1 : 3 (c) $\sqrt{3}:1$ (d) 3 : 1.

E-3. The velocity-time graph of a body moving along a straight line is as follows:
The displacement of the body in 5 s is :



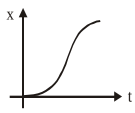
(a) 5 m (b) 2 m (c) 4 m (d) 3 m

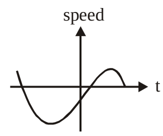
E-4. The position time graph for two objects A and B is shown in figure :

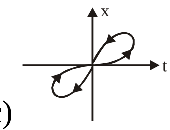


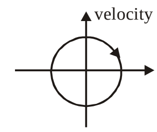
(a) both have same velocities
(b) velocity of A is greater than that of B
(c) velocity of B is greater than that of A
(d) initially velocity of A is greater and after some time velocity of B is greater

E-5. Which of these possibly represent one-dimensional motion of the particle ?

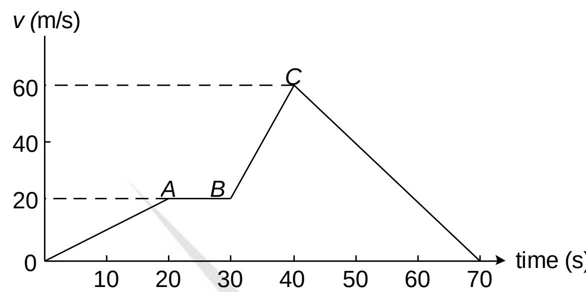
(a) 

(b) 

(c) 

(d) 

E-6. The velocity-time graph of a body is given below.



The maximum acceleration in m/s^2 is

(a) 4 (b) 3 (c) 2 (d) 1

SECTION-F PROJECTILE MOTION

F-1. A projectile is projected at an angle of 30° from the horizontal with a velocity of 98 m/s. Calculate
(a) time to reach the maximum height, (b) maximum height,
(c) time of flight, (d) horizontal range.

F-2. A cricketer throws a ball with a velocity of 60 m/s at an angle of 60°. He then runs to catch the same ball. What should be his minimum velocity so that he may catch the ball before it strikes the ground?

F-3. A man can throw a ball to a maximum horizontal distance of 100 m Then what would be maximum vertical distance upto which he can throw the ball assuming the ball to have same initial velocity.

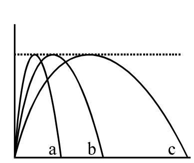
F-4. What should be the angle of projection so that horizontal range of a projectile is equal to its maximum vertical height.

F-5. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw it will be :
(a) h/2 (b) h (c) 2 h (d) 4 h.

F-6. If the maximum horizontal range for a projectile is R, the greatest height attained by it is :
(a) 4R (b) R/2 (c) 2R (d) R/4.

F-7. Galileo writes that for angles of projection of a projectile at angle (45 + θ) and (45 – θ) the horizontal ranges described by the projectile are in the ratio of (if $\theta \le 45°$) :
(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 2 : 3

F-8. Trajectories are shown in figure for three kicked footballs. Ignoring air resistance, order the trjectories according to figure. Order of time of flight of the tragectory is given by :



(a) $t_a = t_b = t_c$ (b) $t_a > t_b > t_c$ (c) $t_a < t_b < t_c$ (d) none of these

F-9. A stone projected with a velocity u at an angle θ with the horizontal reaches maximum height H_1. When it is projected with velocity u at an angle (π/2 – θ) with the horizontal reaches maximum height H_2. The relation between horizontal range R of the projectile, H_1 and H_2 is :

(a) $R = 4\sqrt{H_1H_2}$ (b) $R = 4(H_1 - H_2)$ (c) $R = 4(H_1 + H_2)$ (d) $R = \frac{H_1^2}{H_2^2}$

F-10. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time ?

(a) The one with the farthest range.
(b) The one which reaches maximum height.
(c) The one with the greatest initial velocity.
(d) The one leaving the bat at 45º with respect to the ground.

F-11. A projectile is fired with a speed u at angle θ with the horizontal. Its speed when its direction of motion makes an angle 'α' with the horizontal is :

(a) $u\ \sec\theta\cos\alpha$ (b) $u\ \sec\theta\sin\alpha$ (c) $u\ \cos\theta\sec\alpha$ (d) $u\ \sin\theta\sec\alpha$

SECTION-G RELATIVE MOTION IN ONE DIMENSION

G-1. A person standing on the floor of an elevator drops a coin . The coin reaches the floor of the elevator in a time t_1 if the elevator is stationary and in time t_2 , if it is moving uniformly. Then:

(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$

(d) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

G-2. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum when the angle between their velocities is :

(a) zero (b) π/4 (c) π/2 (d) π

G-3. A 100 m long train running with uniform velocity overtakes a man running in the same direction on the platform at a speed of 5 m/s in 10 seconds. Find the velocity of the train.

G-4. When two bodies move uniformly towards each other, the distance between them decreases by 6 metres/second. If both the bodies move in the same direction with the same speeds, the distance between them increases by 4 metres/second. What are the speeds of the two bodies.

SECTION-H RELATIVE MOTION IN TWO DIMENSION

H-1. During a rainstorm, raindrops are observed to be striking the ground at an angle of θ with the vertical. A wind is blowing horizontally at the speed of 5.0 m/s. The speed of raindrop is (in m/s):

(a) 5 sinθ (b) 5/sinθ (c) 5 cosθ (d) 5/cosθ

H-2. A swimmer wishes to reach directly opposite bank of a river, flowing with velocity 8 m/s. The swimmer can swim 10 m/s in still water. The width of the river is 480 m. Time taken by him to do so:

(a) 60 sec (b) 48 sec (c) 80 sec (d) none of these.

H-3. A helicopter is flying south with a speed of 50 km/h. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be $50\sqrt{2}$ km/h towards:

(a) north east (b) south east (c) north west (d) south west

H-4. A car is going eastwards with a velocity of 8 m/s. To the passengers in the car, a train appears to be moving northwards with a velocity 15 m/s. What is the actual velocity of the train ?

(a) 7 m/s (b) 17 m/s (c) 23 m/s (d) None of these

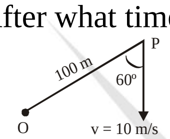
H-5. A man who can swim at the rate of 2 km/h crosses a river to an exactly opposite point on the other bank by swimming in a direction of 120º to the flow of the water in the river. The velocity of the water current in km/h is :

(a) 1 (b) 2 (c) 1/2 (d) 3/2

H-6. A river is flowing from west to east at a speed of 5 meters per minute. A man on the south bank of the river, capable of swimming at 10 meters per minute in still water, wants to swim across the river in shortest time. He should swim in a direction :

(a) due north (b) 30º east of north (c) 30º north of west (d) 60º east of north

H-7. P is a point moving with constant speed 10 m/s such that its velocity vector always maintains an angle 60º with line OP as shown in figure (0 is a fixed point in space). The initial distance between O and P is 100 m. After what time shall P reach O ?



(a) 10 sec (b) 15 sec (c) 20 sec (d) $20\sqrt{3}$ sec

H-8. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is:

(a) 10 m/s (b) $10\sqrt{3}$ m/s (c) 20 m/s (d) $20\sqrt{2}$ m/s

H-9. A river flows at 3 metres per second and is 300 meters wide. A man swims across the river with a velocity of 2 meters per second directed always perpendicular to the flows of current. How long does it take the man to cross the river?

(a) 150 second (b) 2 minute (c) 90 second (d) None

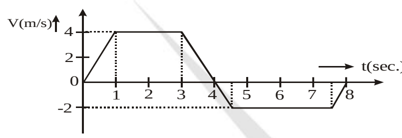
H-10. A monkey is climbing a vertical tree with a velocity of 12 m/s while a dog runs towards the tree chasing the monkey with a velocity of 16 m/s. Find the magnitude of velocity of the dog relative to the monkey.

H-11. A man is walking on a level road at a speed of 3.0 km/h. Raindrops fall vertically with a speed of 4.0 km/h. Find the velocity of the raindrops with respect to the man.

EXERCISE – II
READY FOR CHALLENGES

1. A sail boat sails 2 km due East, 5 km 37° south of East and finally an unknown displacement. If the final displacement of the boat from the starting point is 6 km due East, determine the third displacement.
(a) 2 km towards east (b) 2 km towards south
(c) 4 km towards east (d) 3 km towards north

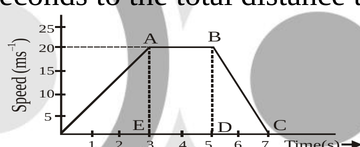
2. The velocity - time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is :



(a) 5 m (b) 16 m (c) 8 m (d) 6 m.

3. A bird flies for 6 seconds with a velocity of $|t - 3|$ ms^{-1} in a straight line, where t is time in seconds. It covers a distance of :
(a) 9 m (b)6m (c) 18 m (d)12m

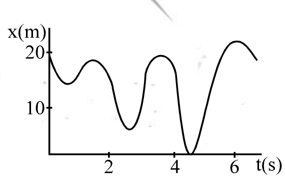
4. Figure gives the speed-time graph of motion of a car. What the ratio of the distance travelled by the car during the last two seconds to the total distance travelled in seven seconds?



(a) 1/9 (b) 2/9 (c) 1/3 (d) 4/9

5. The position of a particle moving along x-axis is given by $x = 3t^2 - t^3$ where x is in m and t is in s . Then which of the following is correct?
(a) Displacement of the particle after 4 sec is 16 m.
(b) Distance traveled by the particle upto 4s is 20 m.
(c) Displacement of the particle after 4 s is –16 m.
(d) Distance covered by the particle upto 4 s is 22 m.

6. In the figure is shown the position of a particle moving on the X-axis as a function of time.



(a) the particle has come to rest 6 times.
(b) the maximum speed is at t = 6 s.
(c) the velocity remains positive for t = 0 to t = 6 s.
(d) the average velocity for the total period shown is negative.

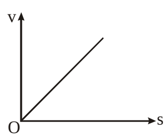
7. A truck travels with speed v_1 for $(1/3)^{rd}$ of its total journey time and with speed v_2 for rest, then average speed of the truck is :
(a) $\frac{v_1 + v_2}{2}$ (b) $\frac{v_1 + 2v_2}{3}$ (c) $\frac{3v_1 v_2}{2v_1 + v_2}$ (d) $\frac{2v_1 + v_2}{3}$

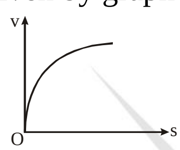
8. Choose the wrong statement .
(a) Zero velocity of a particle does not necessarily mean that its acceleration is zero.

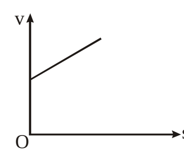
(b) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
(c) If speed of a particle is constant, its acceleration must be zero.
(d) none of these.

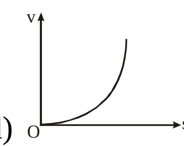
9. If the angle θ between velocity vector and the acceleration vector is $90 < \theta < 180$, the body is moving on a :
(a) Straight path with retardation (b) Straight path with acceleration
(c) Curvilinear path with acceleration (d) Curvilinear path with retardation.

10. A body starts from rest moves along a straight line with constant acceleration. The variation of speed v with distance s is given by graph :

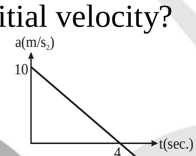
(a) 

(b) 

(c) 

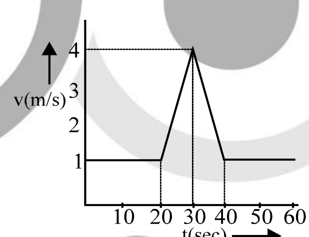
(d) 

11. The acceleration-time graph of a particle moving along a straight line is as shown in figure. At what time the particle acquires its initial velocity?



(a) 12 sec. (b) 5 sec. (c) 8 sec. (d) 16 sec.

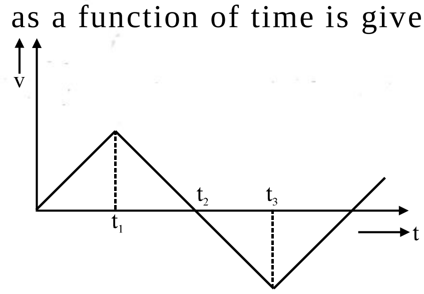
12. Velocity-time graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non zero acceleration and retardation is

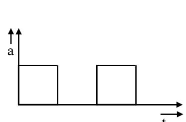


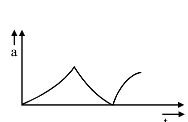
(a) 60 m (b) 50 m (c) 30 m (d) 40 m

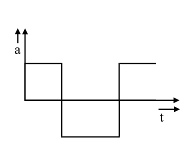
13. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$. Its acceleration will be :
(a) – 16 m/s² (b) – 8 m/s² (c) 4 m/s² (d) 32 m/s²

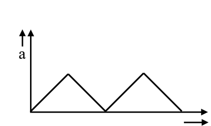
14. Given the graph of velocity of an point object as a function of time. The plot of acceleration of the particle as a function of time is given as



(a) 

(b) 

(c) 

(d) 

15. The acceleration of a particle travelling along a straight line varies linearly from zero to $4 m/s^2$ in 5sec. and then linearly decreases to zero in next 5sec. The maximum speed of the particle is :

(a) 20 m/s (b) 30 m/s (c) 40 m/s (d) 60 m/s

16. If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then the :

(a) velocity of the particle is inversely proportional to t
(b) velocity of the particle is proportional to t
(c) velocity of the particle is proportional to $\sqrt{t}$
(d) the particle moves with a variable acceleration

17. A, B, C and D are points in a vertical line such that AB = BC = CD. If a body falls from rest at A, then the times of descent through AB, BC and CD are in the ratio :

(a) $1 : \sqrt{2} : \sqrt{3}$ (b) $\sqrt{2} : \sqrt{3} : 1$ (c) $\sqrt{3} : 1 : \sqrt{2}$ (d) $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

18. A body released from the top of a tower falls through a height of 5m during the first second of its fall and 35m during the last second of its fall. The height of the tower is :

(a) 80 m (b) 60 m (c) 40 m (d) 20 m

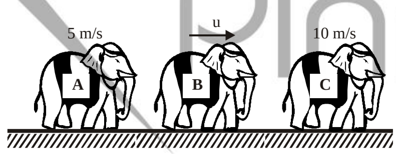
19. Two particles start moving along the same straight line at the same moment from the same point. The first moves with constant velocity u and the second with initial velocity zero and constant acceleration f. The greatest distance between the particles before they meet once again is :

(a) $\frac{u}{f}$ (b) $\frac{u^2}{2f}$ (c) $\frac{f}{2u^2}$ (d) $\frac{u^2}{f}$

20. A ball '*A*' is thrown up vertically with speed u. At the same instant another ball '*B*' is released from rest from a height h. At time t, the velocity of *A* relative to *B* is :

(a) u (b) u – 2gt (c) $\sqrt{u^2 - 2gh}$ (d) u-gt

21. Three elephants A, B and C are moving along a straight line with constant speed in same direction as shown in the figure. Spedd of A is 5 m/s and speed of C is 10 m/s. Initially separation between A & B is 'd' and between B and C is also d. When B catches C separation between A and C becomes 3d. Then the speed of B will be :



(a) 7.5 m/s (b) 15 m/s (c) 20 m/s (d) 5 m/s

22. The co-ordinates of a moving particle at any time t are given by $x = ct^2$ and $y = bt^2$. The speed of the particle is given by:

(a) 2t(c+b) (b) $2t\sqrt{c^2 - b^2}$ (c) $t\sqrt{c^2 + b^2}$ (d) $2t\sqrt{c^2 + b^2}$

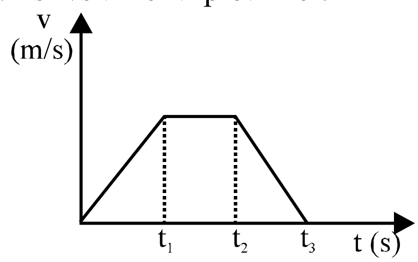
23. The co−ordinates of a moving particle at a time t , are given by , x = 5 sin 10 t , y = 5 cos 10 t . The speed of the particle is :

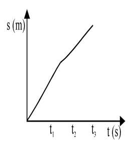
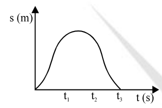
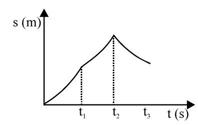
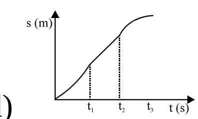
(a) 25 (b) 50 (c) 10 (d) None

24. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again 5 steps forward and 3 steps backward and so on. Each step is 1 m long and requires 1 second. The time of fall of drunkard in a pit 13m away from the start is :

(a) 52 sec (b) 48 sec (c) 37 sec (d) 26 sec

25. The v–t plot of a particle in motion over a time period t_3 seconds is shown in figure . What would be the distance travelled 's' vs time 't' plot like ?

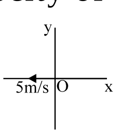


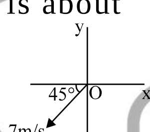
(a)  (b)  (c)  (d) 

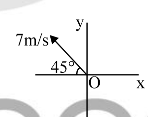
26. The x and y-coordinates of a particle in motion, as functions of time t, are given by : $x = 7t^2 - 4t + 6$, $y = 3t^3 - 3t^2 - 12t - 5$ (x and y are in m and t is in sec.)
The x and y-components of the average velocity, in the interval from t = 0 sec to t = 5 s are:
(a) $v_x = 32.2\ ms^{-1}$, $v_y = 47\ ms^{-1}$ (b) $v_x = -32.2\ ms^{-1}$, $v_y = -47\ ms^{-1}$
(c) $v_x = 31\ ms^{-1}$, $v_y = 48\ ms^{-1}$ (d) $v_x = -31\ ms^{-1}$, $v_y = -48\ ms^{-1}$

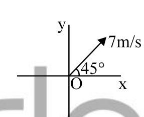
27. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The retardation is:
(a) $2av^3$ (b) $2bv^2$ (c) $2abv^2$ (d) $2b^2v^3$.

28. A particle is moving along the positive y-axis at a speed of 5 m/s. Then it turns by 90° in the anticlockwise sense without changing the speed. The change in the velocity of the particle is about

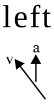
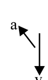
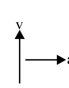
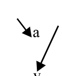
(a) 

(b) 

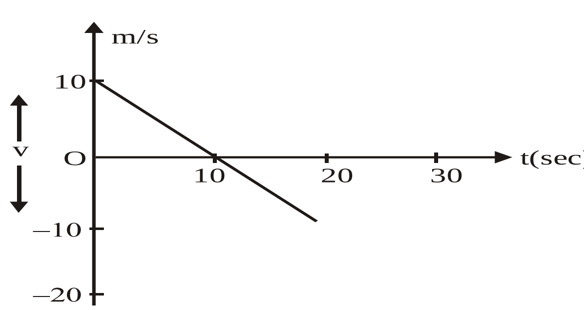
(c) 

(d) 

29. Shown here are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object slowing down and turning to the left ?

(a)  (b)  (c)  (d) 

30. The velocity-time plot for a particle moving on a straight line is shown in figure. Then select incorrect statements:



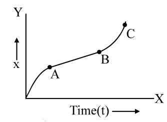
(a) the particle has a constant acceleration
(b) the particle has never turned around
(c) the particle has zero displacement
(d) the average speed in the interval 0 to 10s is the same as the average speed in interval 10 s to 20 s.

31. The initial velocity of a particle at t = 0 is u . The particle is subjected to an acceleration which increases linearly with time i.e., a = c t , where c is a constant . The velocity v at any time is given by :

(a) v = u + at (b) $v = u + ct^2$ (c) $v = u + \frac{1}{2}ct^2$ (d) $v = ut + \frac{1}{2}ct^2$

32. A particle starts from rest at the origin and moves along X−axis with acceleration a = 12 – 2 t . The time after which the particle arrives at the origin is :
(a) 6 sec (b) 18 sec (c) 12 sec (d) 4 sec

33. The graph between the displacement x and time t for a particle moving in a straight line is shown in figure . During the interval OA , AB , BC and CD, the acceleration of the particle is :

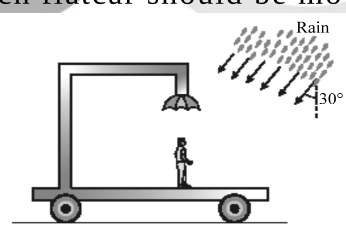


	OA	AB	BC
(a)	+	0	+
(b)	–	0	+
(c)	+	0	–
(d)	–	0	–

34. The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6)\hat{\imath} + t^2\hat{\jmath}$. The time after which the velocity vector and acceleration vector becomes perpendicular to each other is equal to:
(a) 1 sec (b) 2 sec (c) 1.5 sec (d) not possible

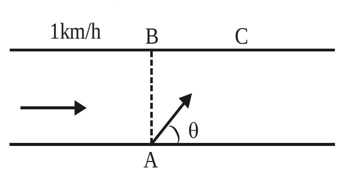
35. A parachutist drops freely from an aeroplane for 10 s before the parachute opens out. Then he descends with a net retardation of 2.5 m/s^2. If he falls out of the plane at a height of 2495 m and g = 10 m/s^2, hit velocity on reaching the ground will be :
(a) 5 m/s (b) 10 m/s (c) 15 m/s (d) 20 m/s

36. A man standing on flat car under the shed of a small umbrella as shown in figure. Rainfall starts at 30 m/s at an angle 30° with the vertical as shown. Find the speed and direction in which flatcar should be moved in order to save man from rain fall .



(a) 15 m/s toward right (b) 15 m/s toward left
(c) $15\sqrt{3}$ m/s toward right (d) $15\sqrt{3}$ m/s toward left

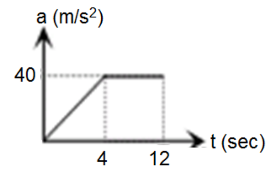
37. A river is flowing with a speed of 1km/h. A swimmer wants to go to point C starting from A. He swims with a speed of 5km/h at an angle θ w.r.t. the river. If AB = BC = 400 m. Then the value of θ is :



(a) 37º (b) 30º (c) 53º (d) 45º

38. Six person are initially at the six corner of a hexagon of side a. Each person now moves with a uniform speed v in such manner that person 1st is always directed toward 2nd, 2nd toward 3rd, 3rd toward 4th and so-on. The time after which they meet is
(a) $\frac{2a}{V}$ (b) $\frac{a}{V}$ (c) $\frac{3a}{V}$ (d) $\frac{a}{2V}$

39. An experiment on the take off performance of an aeroplane shows that the acceleration varies as shown in the figure, and that it taken 12 s to take off from a rest position. The distance along the runway covered by the aeroplane is approximately:



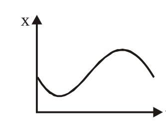
(a) 210 m (b) 2127 m (c) 2027 m (d) 1900 m

40. A body is thrown vertically upwards in air. When air resistance is taken into consideration let the time of ascent be t_1 and time of descent be t_2, then :

(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 > = < t_2$

41. The graph of position x versus time t represents the motion of a particle. If b and c are both positive constants, which of the following expressions best describes the acceleration a of the particle ?

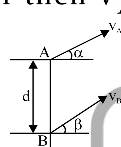
(a) a = b – ct (b) a = + b
(c) a = –c (d) a = b + ct



42. Displacement of a particle varies as $S = t^3 - 3t^2 + 4t$ where t is in seconds and S in meters. The displacement of the particle when the velocity of particle is minimum :

(a) 3m (b) 2m (c) 4m (d) 6m

43. Two particles A and B are thrown simultaneously from two different floors of tower having distance d between them. Velocity of particle A is v_A at an angle α from horizontal while velocity of particle B is v_B at an angle β from horizontal. If two particles collide in mid air then v_A/v_B =



(a) $\frac{\cos\beta}{\cos\alpha}$ (b) $\frac{\sin\alpha}{\sin\beta}$ (c) $\frac{\tan\alpha}{\tan\beta}$ (d) 1 : 1

44. $y = \sin^2 3x$; then $\frac{dy}{dx}$ is

(a) $\sin 6x$ (b) $6\sin 6x$ (c) $3\sin 6x$ (d) $\frac{\sin 6x}{6}$

45. $y = \frac{a^2}{a^2+x^2}$; then $\frac{dy}{dx}$ is

(a) $\frac{2a^2x}{a^2+x^2}$ (b) $\frac{2a^2x}{(a^2+x^2)^2}$ (c) $-\frac{2a^2x}{(a^2+x^2)^2}$ (d) $\frac{a^2x}{2(a^2+x^2)^2}$

46. $y = \ln(x^2+2x+2)$; then $\frac{dy}{dx}$ is

(a) $\frac{1}{(x^2+2x+2)}$ (b) $\frac{2x+2}{(x^2+2x+2)}$ (c) (x^2+2x+2) (d) $\frac{(x^2+2x+2)}{2x+2}$

47. $y = (\tan 2x + \sin 3x)^2$; then $\frac{dy}{dx}$ is

(a) $2(\tan 2x + \sin 3x)$ (b) $2(\tan 2x + \sin 3x)^2(\sec^2 2x + \cos 3x)$
(c) $2(\tan 2x + \sin 3x)\,(2\sec^2 2x + 3\cos 3x)$ (d) $2(2\sec^2 2x + 3\cos 3x)$

48. If sum of two numbers is constant (= K) then product of these two numbers will take maximum value is equal to

(a) K^2 (b) $\frac{K^2}{2}$ (c) $\frac{K^2}{4}$ (d) None of these

49. $y = 3\sin x + 4\cos x$; then maximum value of y is

(a) 3 (b) 4 (c) 5 (d) None of these

50. $P = \frac{Kx}{(x^2+a^2)^{3/2}}$; where K and a are positive constants. The value of x for which P becomes maximum is

(a) $\pm\frac{a}{3}$ (b) $\pm\frac{a}{2}$ (c) $\pm\frac{a}{\sqrt{2}}$ (d) $\pm a$

51. $\int_0^1 (x^2+x+1)\,dx$

(a) 1 (b) 1/6 (c) 5/6 (d) 11/6

52. $\int_0^{\frac{\pi}{4}} \sin\left(2x+\frac{\pi}{4}\right)dx$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) Zero

53. $\int_0^2 \frac{1}{4-x}dx$

(a) $\ln 2$ (b) $\ln 4$ (c) $-\ln 2$ (d) $-\ln 4$

54. $\int_{-\infty}^0 e^{3x}\,dx$

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) e^3

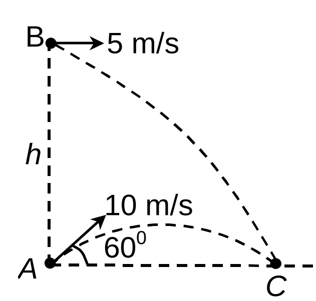
55. $\int_{-1}^1 (3x^2-2x+1)\,dx$

(a) 1 (b) 2 (c) 3 (d) 4

56. Velocity and acceleration of a particle at some instant of time are $\vec{v} = (3\hat{i}+4\hat{j})$m/s and $\vec{a} = -(6\hat{i}+8\hat{j})$m/s^2 respectively. At the same instant particle is at origin. Maximum x-co-ordinate of particle will be

(a) 1.5 m (b) 0.75 m (c) 2.25 m (d) 4.0 m

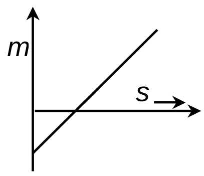
57. A particle A is projected from the ground with an initial velocity of 10 m/s at an angle of 60^0 with horizontal. From what height h should an another particle B be projected horizontally with velocity 5 m/s so that both the particles collides at point C if both are projected simultaneously (g = 10 m/s^2)

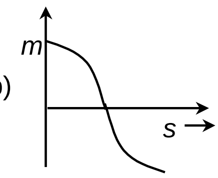


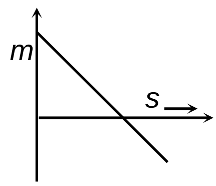
(a) 10 m (b) 30 m

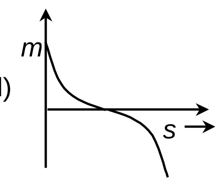
(c) 15 m (d) 25 m

58. The graph which represents the variation of slope m of the trajectory of a projectile with horizontal displacement s is

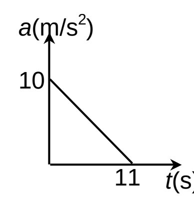
(a) 

(b) 

(c) 

(d) 

59. A particle starting from rest undergoes a rectilinear motion with acceleration a. The variation of a with time t is shown in the figure. The maximum velocity attained by the particle during the motion is



(a) 55 m/s (b) 550 m/s

(c) 110 m/s (d) 650 m/s

60. A particle has a velocity u towards east at t=0. Its acceleration is towards west and is constant. Let x_a and x_b be the magnitude of displacements in the first 10 seconds and the next 10 seconds.

(a) $x_a < x_b$ (b) $x_a = x_b$ (c) $x_a > x_b$

(d) the information is insufficient to decide the relation of x_a with x_b.

61. In a projectile motion the velocity

(a) Is always perpendicular to the acceleration

(b) Is never perpendicular to the acceleration

(c) Is perpendicular to the acceleration for one instant only

(d) Is perpendicular to the acceleration for two instants.

62. Two bullets are fired simultaneously, horizontally and with different speed from the same place. Which bullet will hit the ground first.

(a) the faster one

(b) the slower one

(c) both will reach simultaneously

(d) depends on the masses

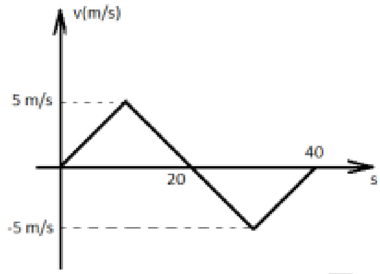
63. A river is flowing from west to east at a speed of 5 meters per minutes. A man on the south bank of the river, capable of swimming at 10 meters per minutes in still water, wants to swim acoss the river in the shortest time. He should swim in a direction

(a) due south (b) 30^0 east of north (c) 30^0 north of west (d) 60^0 east of north.

EXERCISE – III
CROSSING THE HURDLES

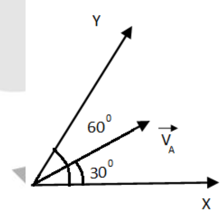
MORE THAN ONE CORRECT

1. The velocity time graph of a particle moving along a straight line is shown in the figure. For this situation mark the correct statement(s).



(a) The average velocity of the particle for 0 to 40 sec is zero.
(b) The distance travelled by the particle and magnitude of displacement for any time interval 0 to 20 sec are same.
(c) The distance travelled by the particle in 0 to 30 sec is 75m.
(d) The distance travelled by the particle in 0 to 30 sec is 25m.

2. Oblique co-ordinate axes are considered as shown in figure. A particle is having velocity $\vec{v}_A$at an angle 300 to the X – axis. For this situation mark the correct statement(s).



(a) Its component of velocity along Y – axis is $v_A\sin30^0$
(b) Its component of velocity along Y – axis is v_A $\cos30^0$
(c) Its component of velocity along Y – axis is v_A $\tan30^0$
(d) Its projection of velocity on Y – axis is $v_A\sin30^0$

3. Two particles A and B are initially 20m apart, A is behind B. Particle A starts moving with a uniform velocity of 5 m/s towards B. Particle B starting from rest has an acceleration 1 m/s2 in the direction of velocity A. Mark the correct statement(s).
(a) The minimum distance between the two particles is 10 m.
(b) The minimum distance between the particles is 7.5 m.
(c) The particle will be closest at t = 5s
(d) The particle will be closest at t = 10s

4. A particle starts moving from rest along a straight line. The velocity of the particle as function of time is given by v = t2 – t where t is in second and v is in m/s. For this situation mark the correct statement.
(a) The particle first speed up, then slows down and then again speeds up forever.
(b) The particle slows down for $\frac{1}{2} < t < 1$s.
(c) The particle speeds up for $0 < t < \frac{1}{2}$ s.
(d) The acceleration of particle is +ve for $\frac{1}{2} < t < 1$s.

5. At time t = 0, a car moving along a straight line has a velocity of 16 m/s. It slows down with an acceleration of – 0.5t m/s2, where t is in second. Mark the correct statement(s).
(a) The direction of velocity changes at t = 8s.
(b) The distance travelled in 4 s is approximately 59 m.
(c) The distance travelled in 10 s is approximately 94 m.
(d) The velocity at t = 10 s is 9 m/s.

6. An object moves in X-Y plane with an acceleration that has +ve Y component. At time t = 0, the object has a velocity given by $\vec{v} = 3\hat{\imath} + 4\hat{\jmath}$. Mark the correct statements. [take g = 10 m/s^2]
(a) Magnitude of velocity of the particle is continuously increasing.
(b) X component of velocity is continuously increasing.
(c) Y component of velocity is continuously increasing.
(d) Y component of velocity is remains constant.

7. A projectile is projected with an initial speed of 20 m/s at an angle 370 with horizontal it is just able to clear two hurdles of height 2 m each. For this situation mark the correct statements. [take g = 10 m/s2]
(a) The time elapsed to cross the hurdles is 2.04 s.
(b) The horizontal separation between the hurdles is 32.64 m.
(c) The distance of 1st hurdle from point of projection is 2.88 m.
(d) The time taken by projectile to reach 2nd hurdle is 2.22 s.

8. A ball is thrown upward from the ground with velocity u. It is at a height 100 m at two times t1 and t2 respectively. If g = 10 m/s2, then
(a) $t_1 t_1$= 20 (b) $t_1 + t_2$= 20 (c) $t_1 t_1 = \frac{u}{5}$ (d)) $t_1 + t_2 = \frac{u}{5}$

9. The maximum horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration a = g/4 is imparted to the projectile due to wind, then
(a) its horizontal range is R
(b) its horizontal range is R + H
(c) its maximum height is $\frac{H}{2}$
(d) its maximum height is H

10. A particle moves along the X-axis as $x = u(t - 2s) + a(t - 2s)^2$.
(a) the initial velocity of the particle is u.
(b) the acceleration of the particle is a.
(c) the acceleration of the particle is 2a.
(d) at t=2 s particle is at the origin.

11. An object may have
(a) varying speed without having varying velocity
(b) varying velocity without having varying speed
(c) nonzero acceleration without having varying velocity
(d) nonzero acceleration without having varying speed

12. Mark the correct statements :
(a) The magnitude of the velocity of a particle is equal to its speed
(b) The magnitude of average velocity in an interval is equal to its average speed in that interval
(c) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero
(d) It is possible to have a situation in which the speed of a particle is never zero but the average in an interval is zero

COMPREHENSION TYPE QUESTION

PASSAGE I

A particle moving with uniform acceleration along a straight line passes three successive points A , B and C where the distances AB : BC is 3 : 5 and the time taken from A to B is 40 sec. If the velocities at A and C are 5 m/s and 15 m/s respectively.

13. The velocity of the particle at B is:
(a) 10 m/s (b) 12 m/s (c) 20 m/s (d) 22.5 m/s

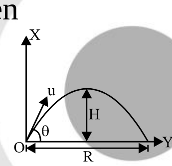
14. Acceleration of the particle:
(a) 1/6 m/s^2 (b) 1/8 m/s^2 (c) 1/10 m/s^2 (d) 1/4 m/s^2

15. Time taken to cover B to C:
(a) 10 sec (b) 20 sec (c) 30 sec (d) 40 sec

16. Total distance from A to C:
(a) 600 m (b) 800 m (c) 1000 m (d) 1200 m

PASSAGE II

A particle is projected from O and is moving freely under gravity and strikes the horizontal plane through O at a distance R from it . Then

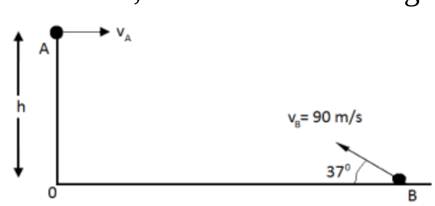


17. The maximum height H and the range R are related to each other as,
(a) R = 4Hcotθ (b) R = 2Hcotθ (c) R = 4Htanθ (d) R = 2Htanθ

18. The angle of elevation □ of the highest point of the projectile and the angle of projection □ are related to each other as
(a) tanφ = (tanθ)/2 (b) tanφ = (tanθ)/4 (c) tanφ = 2tanθ (d) tanφ = tan(θ/2)

PASSAGE III

Two particles are projected simultaneously from two points A and B at t = 0. The stone from A is projected horizontally with speed vA and stone from B is projected with a speed 90 m/s at an angle of 370 with the horizontal. During their course of motion, the particle collides in mid of air at t = 6 s and at the time of collision the velocities of the two particles are mutually perpendicular. [take g = 10 m/s2] Based on the above information, answer the following questions.



19. The value of l is
(a) 462 m (b) 324 m (c) 5 m (d) 825 m

20. The value of h is

(a) 462 m (b) 324 m (c) 5 m (d) 825 m

21. The value of v_A is

(a) 10 m/s (b) 5 m/s (c) 15 m/s (d) 20 m/s

MATRIX MATCH TYPE

22. A particle is moving along a straight line. Its velocity varies with time as v = 2 –t where v is in m/s and t in seconds. In time from t = 0 to t = 4s, match the following.

	Column (I)		Column (II)
A	Displacement (m)	P	0
B	Distance (m)	Q	1
C	Average velocity (m/s)	R	2
D	Average speed (m/s)	S	4

23. Trajectory of a particle in projectile motion is given as:$y = x - \frac{x^2}{80}$. Here x and y are in meters. For this projectile motion, with $g = 10\frac{m}{s^2}$.

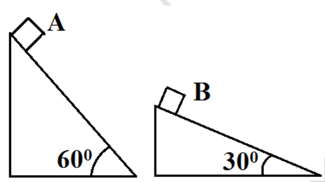
	Column (I)		Column (II)
A	Angle of projection (in degree)	P	20 m
B	Angle of velocity with horizontal after 4s	Q	80 m
C	Maximum height	R	45^0
D	Horizontal range	S	30^0

24. A balloon starts rising up with constant net acceleration of . After 2s, a particle drops from the balloon. After further 2s, match the following.

	Column (I)		Column (II)
A	Height of the particle from ground	P	zero
B	Speed of particle	Q	10 SI units
C	Displacement of particle(after leaving balloon)	R	40 SI units
D	Acceleration of particle at last moment	S	20 SI units

EXERCISE – IV

1. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{dv}{dt} = -\frac{5}{2}\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be
2. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be
3. Two fixed frictionless inclined plane making an angle 30^0 and 60^0 with the vertical are shown in the figure. Two block A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



4. A parachutist after bailing out falls 50 m without friction. When parachute open, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

5. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on a horizontal road with an acceleration of $1m/s^2$ and the projection velocity in the horizontal direction is 9.8m/s. How far behind the boy will the ball fall on the car.

6. A river 400 m wide is flowing at a rate of 2m/s. A boat is sailing at a velocity of 10m/s. with respect to water , in a direction perpendicular to the river.

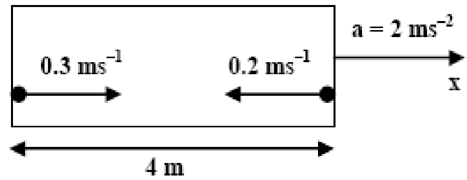
   (a) Find the time taken by the boat to reach the opposite bank.

   (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.

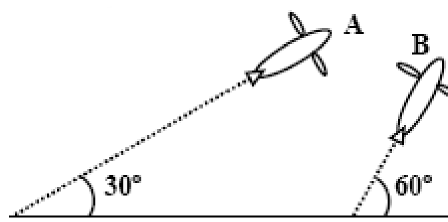
7. An NCC parade is gone at a uniform speed of 6km/h through a place under a berry tree on which a bird is sitting at a height of 12.1 m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform.

8. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60º to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

9. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along +x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in - x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



10. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30^0 and 60^0 with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



11. A gun kept on a straight horizontal road is used to hit a car travelling along the same road away from the gun with a uniform speed of 72 km/hr. The car is at a distance of 500 m from the gun which the gun is fired at an angle of 45º with the horizontal. Find (i) the distance of the car from the gun where the shell hits it and (ii) the speed of projection of the shell from the gun.

12. Two trains having a speed of 30 km/hr are headed at each other on the same straight track. A bird that can fly at 60 km/hr files off one train when they are 60 km apart and heads directly for the other train. On reaching the other train it files directly back to the first and sot forth. Find
(a) How many trips can the bird make from one train to the other before the trains collide?

(b) What is the total distance travelled by the bird?

13. Two particles moves in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point in space and moved with velocities v_1= 3.0 m/s and v_2= 4.0 m/s horizontally in opposite directions. Find the separation between the particles at the moment when their velocity vectors become mutually perpendicular.

14. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particle will take to meet each other.

ANSWERS

EXERCISE – I
BUILDING A FOUNDATION

SECTION-A

A-1. (a)	A-2. (b)	A-3. (a)	A-4. (d)
A-5. (a)	A-6. (d)	A-7. (c)	A-8. (c)
A-9. (d)	A-10. (c)	A-11. (c)	

SECTION-B

B-1. (b)	B-2. (d)	B-3. (a)	B-4. (a)
B-5. (a)			

SECTION-C

C-1. (a)	C-2. (c)	C-3. (b)	C-4. (a)
C-5. (a)	C-6. (a)	C-7. (b)	C-8. (c)
C-9. (c)	C-10. (d)	C-11. (a)	C-12. (c)
C-13. (a)	C-14. (c)	C-15. (a)	C-16. (a)
C-17. (b)	C-18. (c)	C-19. (d)	C-20. (c)
C-21. (b)	C-22. (a)		

SECTION-D

D-1. (a) $x^2 + C$ (b) $\frac{x^3}{3} + C$ (c) $\frac{x^3}{3} - x^2 + x + C$

D-2. (a) $-\frac{1}{x} + c$ (b) $-\frac{5}{x} + c$ (c) $2x + \frac{5}{x} + c$

D-3. (a) $x^{\frac{3}{2}} + c$ (b) $3\sqrt{x} + c$ (c) $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$

D-4. (a) $x^{\frac{4}{3}} + c$ (b) $\frac{x^{\frac{2}{3}}}{2} + c$ (c) $\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} + c$

D-5. $x - \frac{x^3}{3} - \frac{x^6}{2} + c$

D-6. $-3\cos x + c$ **D-7.** $\frac{x^4}{9} - \frac{7}{x} + \frac{x^2}{2} + c$

D-8. $\frac{x^9}{9} + 9x + c$ **D-9.** $\frac{x^{-6}}{-6} + c$

D-10. $\frac{1}{3}\ln x + c$ **D-11.** 15 **D-12.** $\frac{3\pi}{2}$

D-13. 21 **D-14.** 0

SECTION-E

E-1. (d)	E-2. (b)	E-3. (d)	E-4. (c)
E-5. (a)			

SECTION-F

F-1. (a) 5s (b) 122.5 m (c) 10 s (d) $490\sqrt{3}$ m

F-2. 30 m/s **F-3.** 50 m

F-4. $\tan^{-1}(4)$

F-5. (c)	F-6. (d)	F-7. (c)	F-8. (a)
F-9. (a)	F-10. (b)	F-11. (c)	

SECTION-G

G-1. (a) **G-2.** (d)

G-3. 15 m/s **G-4.** U = 5 m/s and v = 1 m/s

SECTION-H

H-1. (b) **H-2. (c)** **H-3. (d)** **H-4. (b)**
H-5. (a) **H-6. (a)** **H-7. (c)** **H-8. (b)**
H-9. (a)
H-10. 20 m/s **H-11. 5 km/h**

EXERCISE – II
READY FOR CHALLENGES

1. (d)	2. (a)	3. (a)	4. (b)	5. (c)
6. (a)	7. (b)	8. (c)	9. (d)	10. (b)
11. (c)	12. (b)	13. (b)	14. (c)	15. (a)
16. (b)	17. (d)	18. (A)	19. (b)	20. (a)
21. (b)	22. (d)	23. (b)	24. (c)	25. (d)
26. (c)	27. (a)	28. (b)	29. (b)	30. (b)
31. (C)	32. (b)	33. (b)	34. (a)	35. (a)
36. (b)	37. (C)	38. (a)	39. (a)	40. (b)
41. (a)	42. (b)	43. (a)	44. (c)	45. (c)
46. (b)	47. (c)	48. (c)	49. (c)	50. (c)
51. (d)	52. (c)	53. (a)	. 54. (b)	55. (d)
56. (b)	57. (c)	58. (c)	59. (a)	60. (d)
61. (c)	62. (c)	63. (a)		

EXERCISE – III
CROSSING THE HURDLES

MORE THAN ONE CORRECT

1. (a,b,c)	2. (c)	3. (b,c)	4. (a,b,c,d)	5. (a,b,c)
6. (a,c)	7. (a,b,c,d)	8. (a,d)	9. (b,d)	10. (c,d)

11. (b,d)	12. (a)	13. (a)	14. (b)	15. (d)
16. (b)	17. (a)	18. (a)	19. (a)	20. (b)
21. (b)				

MATCH THE COLUMN

22. (A-P, B-S, C-P, D-Q)
23. (A-R, B-R, C-P, D-Q)
24. (A-R, B-P, C-S, D-Q)

EXERCISE – IV

1. 2sec
2. 20m
3. 4.9ms^{-2}
4. 293m
5. 2m
6. (a) 40sec (b) 80m
7. 2.62m
8. 5ms^{-2}
9. 2sec
10. 5sec
11. (i) 746.82m (ii)85.55
12. (a) infinity (b) 60km
13. 2.5m
14. $\frac{2a}{v}$