

SIMILAR TRIANGLES

1.1 SIMILAR GEOMETRIC FIGURES

Two geometric figures which are same in shape, such that one is simply a copy of the other on a smaller scale or a larger scale, are called similar geometric figures. Two geometric figures are said to be similar if and only if they have the same shape but not necessarily the same size.

Note: Two congruent geometric figures are always similar but converse may or may not be true.

1.2 SIMILAR POLYGONS

Two polygons of the same number of sides are similar, if

(i) Their corresponding angles are equal and

(ii) Their corresponding sides are in proportion or their corresponding sides are in the same ratio

Note : The same ratio of the corresponding sides is referred to as the representative fraction or the scale factor for the polygons.

1.3 SIMILAR TRIANGLES

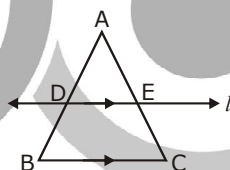
Two triangles are said to be similar, if

(i) Their corresponding angles are equal and

(ii) Their corresponding sides are in proportion (or are in the same ratio)

1.4 BASIC PROPORTIONALITY THEOREM (OR THALES THEOREM)

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.



If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If in $\triangle ABC$, $DE \parallel BC$, intersecting in D and E, then

$$(i) \frac{AD}{DB} = \frac{AE}{EC}$$

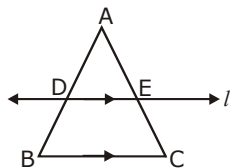
$$(ii) \frac{AD}{AB} = \frac{AE}{AC}$$

$$(iii) \frac{DB}{AB} = \frac{EC}{AC}$$

1.5 CONVERSE OF BASIC PROPORTIONALITY THEOREM

If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

i.e., In $\triangle ABC$, if DE intersects AB in D and AC in E, such



that $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$

1.6 CRITERIA FOR SIMILARITY OF TRIANGLES

Two triangles are said to be similar, if

(i) Their corresponding angles are equal and

(ii) Their corresponding sides are in proportion (or are in the same ratio)

(i) AA or AAA Similarity Criterion :

If two angles of one triangle are equal to two corresponding angles of another triangle, then the triangles are similar.

If two angles of one triangle are respectively equal to the two angles of another triangle, then the third angles of the two triangles are necessarily equal, because the sum of three angles of a triangle is always 180° .

(ii) SAS Similarity criterion :

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

OR

If two sides of a triangle are proportional to two corresponding sides of another triangle and the angles included between them are equal, then the triangles are similar.

(iii) SSS Similarity Criterion :

If in two triangles, sides of one triangle are proportional (or are in the same ratio) to the sides of the other triangle, then the triangles are similar.

Note : If $\triangle ABC \sim \triangle PQR$ by any one similarity criterion, then

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \quad \text{and} \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

i.e. A and P, B and Q, C and R are the corresponding vertices, also AB and PQ, BC and QR, CA and RP are the corresponding sides.

1.7 AREAS OF SIMILAR TRIANGLES

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Note :

- (i) The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.
- (ii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.
- (iii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding angle bisectors.

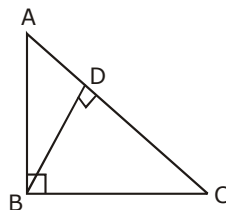
1.8 Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

1.9 Converse of pythagoras Theorem

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Note : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and similar to each other. i.e., If in $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$, then

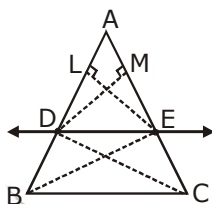


- (i) $\triangle ADB \sim \triangle ABC$ (ii) $\triangle BDC \sim \triangle ABC$ (iii) $\triangle ADB \sim \triangle BDC$

1.10. Basic Proportionality theorem or thales theorem

If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given : $\triangle ABC$ and a line 'l' parallel to BC intersects AB at D and AC at E as shown in figure.



To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Const. : Join BE and CD Draw $EL \perp AB$ and $DM \perp AC$.

Proof : Area of $\triangle ADE = \frac{1}{2} \times AD \times EL \dots (i)$ { \because Area of a $\triangle = \frac{1}{2} \times \text{base} \times \text{corresponding altitude}$ }

Area of $\triangle BDE = \frac{1}{2} \times DB \times EL \dots (ii)$, Now Dividing (i) and (ii), we have

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \dots (iii)$$

Similarly,

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \dots (iv)$$

Since, $\triangle BDE$ and $\triangle CDE$ are triangles on the same base DE and between the same parallels DE and BC

\therefore Area of $\triangle BDE = \text{Area of } \triangle CDE$

From (iii) and (iv), we have $\frac{AD}{DB} = \frac{AE}{EC}$

Corollary : If in a $\triangle ABC$, a line $DE \parallel BC$, intersects AB in D and AC in E, then

$$(i) \frac{AB}{AD} = \frac{AC}{AE}$$

$$(ii) \frac{AB}{DB} = \frac{AC}{EC}$$

Proof :

(i) From the Basic Proportionality Theorem (B.P.T), we have taking reciprocals of both sides adding 1 on both sides]

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) Again, from B.P.T., We have

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

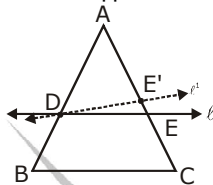
$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

1.11 Converse of Basic proportionality Theorem:

If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

Given : $\triangle ABC$ and a line ℓ' intersects AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$

To Prove : Line ℓ is parallel to BC or $DE \parallel BC$



Proof : If possible, let the line ℓ is not parallel to BC. Through D, draw $\ell' \parallel BC$ intersecting AC in E' . Now, by basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \dots(i) \quad \text{But} \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \dots(ii)$$

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

Adding 1 to both sides, we have

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\Rightarrow \frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C} \Rightarrow \frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow EC = E'C$$

Which is only possible, if E and E' coincide. Hence, line ℓ is parallel to BC or $DE \parallel BC$

1.12 The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given : $\triangle ABC \sim \triangle PQR$

To prove :

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

Const : Draw $AD \perp BC$ and $PM \perp QR$.

Proof: $\triangle ABC \sim \triangle PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

Also, $\angle B = \angle Q$

and $\angle ADB = \angle PMQ$

[\because corresponding angles of similar triangles are equal]
[each = 90°]

$$\Rightarrow \triangle ADB \sim \triangle PMQ \Rightarrow \frac{AD}{PM} = \frac{AB}{PQ}$$

$$\text{From (i) and (ii), we have } \frac{AD}{PM} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

$$\text{Now, } \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM},$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{BC}{QR} \times \frac{AD}{PM} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$$

$$\text{Also, from (i), we have } \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

$$\text{Therefore, from (iv) and (v) we have } \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

1.13 (Pythagoras' theorem) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: A $\triangle ABC$ in which $\angle ABC = 90^\circ$.

To Prove: $AC^2 = AB^2 + BC^2$.

Construction: Draw $BD \perp AC$.

Proof: In $\triangle ADB$ and $\triangle ABC$, we have:

$$\angle A = \angle A \text{ (common)} \quad \angle ADB = \angle ABC$$

$$\therefore \triangle ADB \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$

$$\Rightarrow AD \times AC = AB^2 \quad \dots(i)$$

In $\triangle BDC$ and $\triangle ABC$, we have

$$\angle C = \angle C \text{ (COMMON)}$$

$$\angle BDC = \angle ABC$$

$$\therefore \triangle BDC \sim \triangle ABC \quad \begin{matrix} \text{[each equal to } 90^\circ\text{]} \\ \text{[by AA - similarity]} \end{matrix}$$

$$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC} \Rightarrow DC \times AC = BC^2 \quad \dots(ii)$$

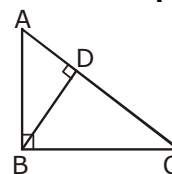
From (i) and (ii), we get

$$AD \times AC + DC \times AC = (AB^2 + BC^2)$$

$$\Rightarrow (AD + DC) \times AC = (AB^2 + BC^2)$$

$$\Rightarrow AC \times AC = (AB^2 + BC^2)$$

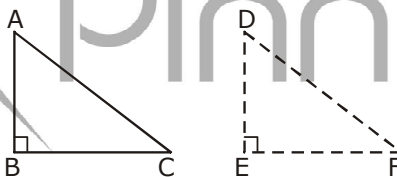
$$\Rightarrow AC^2 = (AB^2 + BC^2)$$


1.14 (Converse of Pythagoras' theorem) In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

Given: A $\triangle ABC$ in which $AC^2 = AB^2 + BC^2$.

To Prove: $\angle B = 90^\circ$.

Construction: Draw a $\triangle DEF$ such that $DE = AB$, $EF = BC$ and $\angle E = 90^\circ$.



Proof: In $\triangle DEF$, we have : $\angle E = 90^\circ$. So, by Pythagora's theorem, we have :

$$DF^2 = DE^2 + EF^2 \Rightarrow DF^2 = AB^2 + BC^2 \quad \dots(i)$$

$$[\because DE = AB \text{ and } EF = BC]$$

$$\text{But, } AC^2 = AB^2 + BC^2 \quad \dots(ii)$$

[given]

From (i) and (ii), we get:

$$AC^2 = DF^2 \Rightarrow AC = DF.$$

Now, in $\triangle ABC$ and $\triangle DEF$, we have

$$AB = DE, BC = EF \text{ and } AC = DF \quad \therefore \triangle ABC \cong \triangle DEF$$

Hence, $\angle B = \angle E = 90^\circ$.

SOLVED PROBLEMS

Ex.1 Prove that the line drawn from the mid-point of one side of a triangle, parallel to another side, bisects the third side.

Sol. $\triangle ABC$ in which D is the mid-point of side AB and the line DE is drawn parallel to BC, meeting AC in E.

To Prove : E is the mid-point of AC i.e, $AE = EC$.

Proof : In $\triangle ABC$, we have $DE \parallel BC$.

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \dots (i)$$

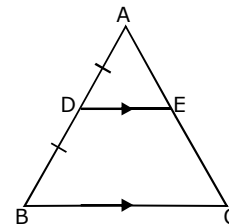
But D is the mid-point of AB.

$$\therefore AD = DB \Rightarrow \frac{AD}{DB} = 1 \quad \dots (ii)$$

From (i) and (ii), we get

$$\frac{AE}{EC} = 1 \Rightarrow AE = EC$$

Hence, E is the mid-point of AC.



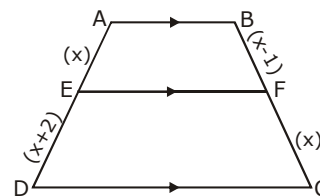
Ex.2 In figure if, $EF \parallel DC \parallel AB$, find x.

Sol. $\therefore EF \parallel DC \parallel AB \therefore \frac{AE}{ED} = \frac{BF}{FC}$

$$\Rightarrow \frac{x}{x+2} = \frac{x-1}{x}$$

$$\Rightarrow x^2 = (x+2)(x-1)$$

$$\Rightarrow x^2 = x^2 + x - 2 \Rightarrow x = 2 \text{ units}$$



Ex.3 If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Sol. A quadrilateral ABCD whose diagonals AC and BD intersect at E such that $\frac{DE}{EB} = \frac{CE}{EA}$.

To prove : ABCD is a trapezium.

Construction : Draw $FE \parallel AB$, meeting AD in F.

Proof : In $\triangle ABD$, we have $FE \parallel AB$.

$$\therefore \frac{DF}{FA} = \frac{DE}{EB} \quad \dots (i)$$

$$\text{But } \frac{DE}{EB} = \frac{CE}{EA} \quad \dots (ii)$$

From (i) and (ii), we get

$$\frac{DF}{FA} = \frac{CE}{EA}$$

which means in $\triangle ACD$, E and F are points on AC and AD respectively such that

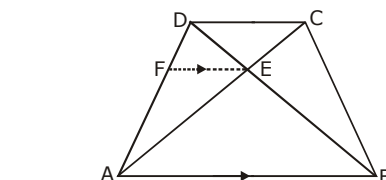
$$\frac{DF}{FA} = \frac{CE}{EA}$$

$$\Rightarrow FE \parallel DC \quad \dots (iii)$$

$$\text{But, } FE \parallel AB \quad \dots (iv)$$

From (iii) and (iv), we get : $AB \parallel DC$.

Hence, **ABCD is a trapezium.**



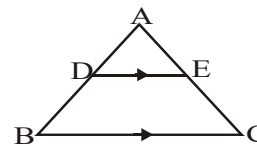
[by converse of B.P.T.]

[by construction]

Ex.4 In the adjoining figure, $DE \parallel BC$.

(i) If $AD = 3.4$ cm, $AB = 8.5$ cm and $AC = 13.5$ cm, find AE .

(ii) If $\frac{AD}{DB} = \frac{3}{5}$ and $AC = 9.6$ cm, find AE .



Sol. (i) Since $DE \parallel BC$, we have $\frac{AD}{AB} = \frac{AE}{AC}$

$$\therefore \frac{3.4}{8.5} = \frac{AE}{13.5} \Rightarrow \frac{3.4 \times 13.5}{8.5} = 5.4$$

Hence, $AE = 5.4$ cm.

(ii) Since $DE \parallel BC$, we have $\frac{AD}{DB} = \frac{AE}{EC}$

$$\therefore \frac{AE}{EC} = \frac{3}{5} \left[\because \frac{AD}{DB} = \frac{3}{5} \text{ (Given)} \right]$$

Let $AE = x$ cm. Then, $EC = (AC - AE) = (9.6 - x)$ cm.

$$\therefore \frac{x}{9.6 - x} = \frac{3}{5} \Rightarrow 5x = 3(9.6 - x)$$

$$\Rightarrow 5x = 28.8 - 3x \Rightarrow 8x = 28.8 \Rightarrow x = 3.6.$$

$\therefore AE = 3.6$ cm.

Ex.5 In the adjoining figure, $AD = 5.6$ cm, $AB = 8.4$ cm, $AE = 3.8$ cm and $AC = 5.7$ cm. Show that $DE \parallel BC$.

Sol. We have, $AD = 5.6$ cm, $DB = (AB - AD) = (8.4 - 5.6)$ cm = 2.8 cm.

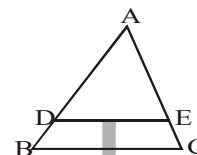
$AE = 3.8$ cm, $EC = (AC - AE) = (5.7 - 3.8)$ cm = 1.9 cm.

$$\therefore \frac{AD}{DB} = \frac{5.6}{2.8} = \frac{2}{1} \text{ and } \frac{AE}{EC} = \frac{3.8}{1.9} = \frac{2}{1}$$

Thus, $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore DE$ divides AB and AC proportionally.

Hence, $DE \parallel BC$



Ex.6 In fig, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

Sol. It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$

So, $ST \parallel QR$ [Theorem]

Therefore, $\angle PST = \angle PQR$ [Corresponding angles] - (1)

Also, it is given that

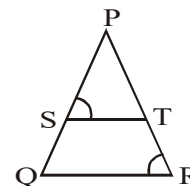
$\angle PST = \angle PRQ$ (2)

So, $\angle PRQ = \angle PQR$ [From 1 and 2]

Therefore $PQ = PR$

[Sides opposite the equal angles]

i.e., PQR is an isosceles triangle.



Ex.7 Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e., in the same ratio).

OR

ABCD is a trapezium with $DC \parallel AB$. E and F are points on AD and BC respectively such that $EF \parallel AB$.

Show that $\frac{AE}{ED} = \frac{BF}{FC}$

Sol. We are given trapezium ABCD.

$CD \parallel BA$

$EF \parallel AB$ and CD both

We join AC.

It meets EF at O.

In $\triangle ACD$, $OE \parallel CD$

$$\Rightarrow \frac{AO}{OC} = \frac{AE}{ED} \quad \dots(i)$$

(Basic Proportionality Theorem)

In $\triangle CAB$, $OF \parallel AB$

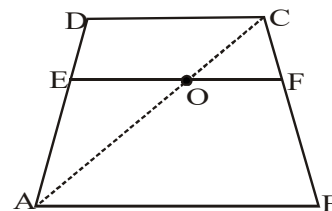
$$\Rightarrow \frac{CO}{OA} = \frac{CF}{FB} \quad [\text{B.P.T.}]$$

$$\Rightarrow \frac{AO}{OC} = \frac{BF}{FC} \quad \dots(ii)$$

From (i) and (ii)

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence, proved.



Ex.8 Any point X inside $\triangle DEF$ is joined to its vertices. From a point P in DX, PQ is drawn parallel to DE meetingXE at Q and QR is drawn parallel to EF meeting XF in R. Prove that $PR \parallel DF$.

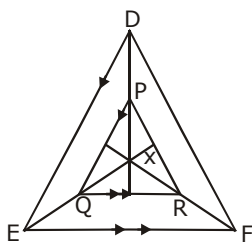
Sol. Given : In figure $PQ \parallel DE$ and $QR \parallel EF$.

To Prove : $PR \parallel DF$.

Proof : In $\triangle XED$; $PQ \parallel DE$.

$$\therefore \frac{XP}{PD} = \frac{XQ}{QE} \quad \dots (i) \quad [\text{by B.P.T.}]$$

Also, in $\triangle XEF$, $QR \parallel EF$ \therefore We have



$$\frac{XQ}{QE} = \frac{XR}{RF} \quad \dots (ii) \quad [\text{by B.P.T.}]$$

From (i) and (ii), we have

$$\frac{XP}{PD} = \frac{XR}{RF}$$

Thus, in $\triangle XFD$, R and P are points dividing sides XF and XD in the same ratio.

Therefore, by converse of B.P.T., we have **$PR \parallel DF$** .

Ex.9 In the given figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. If $AC = 75$ cm, $AB = 1$ m and $BD = 1.25$ m, find AD .

Sol. \therefore In a $\triangle ABC$, $\angle A = 90^\circ$ and $AD \perp BC$, where D is on BC .

$$\therefore \triangle BAC \sim \triangle BDA$$

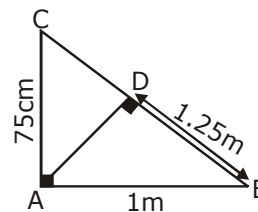
$$\Rightarrow \frac{BA}{BD} = \frac{AC}{DA} = \frac{BC}{BA}$$

$$\Rightarrow \frac{100}{125} = \frac{75}{DA} = \frac{BC}{BA} \quad [\because AB = 1 \text{ m } 100 \text{ cm}]$$

and $BD = 125$ cm

$$\Rightarrow \frac{100}{125} = \frac{75}{DA} \Rightarrow DA = \left(\frac{125 \times 75}{100} \right) \text{ cm}$$

$$AD = 93.75 \text{ cm}$$



Ex.10 In fig, $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\triangle PQS \sim \triangle TQR$.

Sol. $\angle 1 = \angle 2$ (Given)

$$\Rightarrow PR = PQ \quad \dots(i)$$

(Sides opposite to equal angles in $\triangle QRP$)

$$\text{Also } \frac{QT}{PR} = \frac{QR}{QS} \quad (\text{Given}) \quad \dots(ii)$$

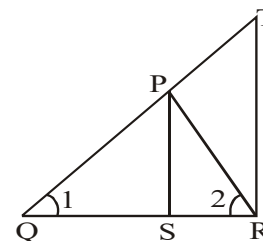
From (i) and (ii), we have

$$\frac{QT}{PR} = \frac{QR}{QS} \Rightarrow \frac{QP}{QT} = \frac{QS}{QR} \quad \dots(iii)$$

Now, in triangles PQR and TQR , we have
 $\angle PQS = \angle TQR$ (each = $\angle 1$)

$$\text{and } \frac{PQ}{TQ} = \frac{QS}{QR} \quad (\text{from (3)})$$

$$\Rightarrow \triangle PQS \sim \triangle TQR \quad (\text{SAS Similarity})$$



Ex.11 If two triangles are equiangular, prove that the ratio of corresponding sides is the same as the ratio of the corresponding angle bisector segments.

Sol. Given : Two \triangle s ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; and AX , DY are the bisectors of A and D respectively.

$$\text{To prove : } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AX}{DY}$$

Proof : Since, equiangular triangles are similar [by AA-similarity]

We have : $\triangle ABC \sim \triangle DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \dots (i)$$

$$\text{Now, } \angle A = \angle D \Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle D$$

$$\Rightarrow \angle BAX = \angle EDY$$

Thus, in \triangle s ABX and DEY , we have : $\angle BAX = \angle EDY$

[proved]

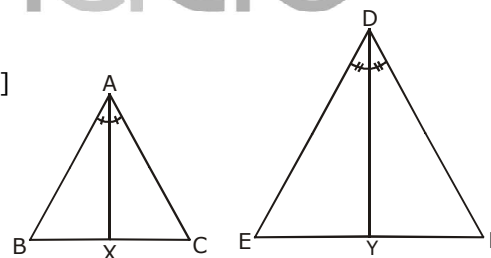
and $\angle B = \angle E$ [given]

$\therefore \triangle ABX \sim \triangle DEY$ [by A.A. similarity]

$$\therefore \frac{AB}{DE} = \frac{AX}{DY} \quad \dots (ii)$$

From (i) and (ii), we get :

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AX}{DY}$$



Ex.12 If two triangles are equiangular, prove that the ratio of the corresponding sides is the same as the ratio of the corresponding medians.

Sol. Given : Two Δ s ABC and DEF in which $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$; AP and DQ are the medians.

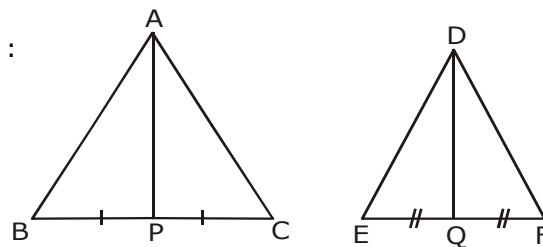
To prove : $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AP}{DQ}$

Proof : Since, equiangular triangles are similar we have :

$\Delta ABC \sim \Delta DEF$ [by A.A. similarity]

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

But, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$ [$\because BC = 2BP$, $EF = 2EQ$]



Now, in Δ s ABP and DEQ, we have

$$\frac{AB}{DE} = \frac{BP}{EQ} \text{ and } \angle B = \angle E \text{ [given]}$$

$\therefore \Delta ABP \sim \Delta DEQ$ [by S.A.S. similarity]

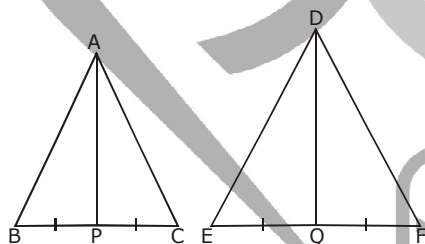
$$\therefore \frac{AB}{DE} = \frac{AP}{DQ}$$

From (i) and (ii), we get :

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AP}{DQ}$$

Ex.13 If two sides and a median bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding median of another triangle then the triangles are similar.

Sol. Given : ΔABC and ΔDEF in which AP and DQ are the medians are such that



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$$

To prove : $\Delta ABC \sim \Delta DEF$

Proof : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$ [given] (i)

$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{AP}{DQ} \text{ [note this step]}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ} \left(\because \frac{1}{2}BC = BP, \frac{1}{2}EF = EQ \right)$$

$\Rightarrow \Delta ABP \sim \Delta DEQ$ [by S.S.S. similarity]

$\Rightarrow \angle B = \angle E$... (ii)

Now, in Δ s ABC and DEF, we have

$$\frac{AB}{DE} = \frac{BC}{EF} \text{ and } \angle B = \angle E \text{ [from (i) and (ii)]}$$

$\therefore \Delta ABC \sim \Delta DEF$ [by S.A.S. similarity]

Ex.14 Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Sol. $\triangle ABC \sim \triangle DEF$

To Prove :

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Proof: Let $BC = a$, $AC = b$, $AB = c$,
 $EF = d$, $DF = e$, $DE = f$

$\therefore \triangle ABC \sim \triangle DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k \text{ (say)} \quad \dots (i)$$

$$\text{or } \frac{c}{f} = \frac{a}{d} = \frac{b}{e} = k \quad \dots (ii)$$

$$\therefore c = fk, a = dk, b = ek$$

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB + BC + AC}{DE + EF + DF}$$

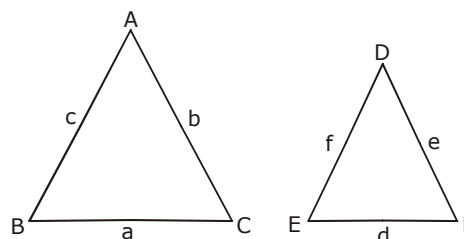
[perimeter of a triangle = sum of its sides]

$$\frac{c + a + b}{f + d + e} = \frac{fk + dk + ek}{f + d + e} \quad [\text{using (ii)}]$$

$$= \frac{k(f + d + e)}{(f + d + e)} = k$$

From (i) and (iii), we get

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



Ex.15 Two isosceles triangles have equal vertical angles and their areas are in the same ratio 16 : 25. Find the ratio of their corresponding heights.

Sol. **Given :** $\triangle ABC$ and $\triangle DEF$ such that $AB = AC$, $DE = DF$, $\angle A = \angle D$.

$$\text{and } \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \frac{16}{25}$$

To Determine : $\frac{AL}{DM} = ?$

Proof : $AB = AC$, $DE = DF$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{DE}{DF} = 1$$

$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

Thus, in $\triangle ABC$ and $\triangle DEF$, we have

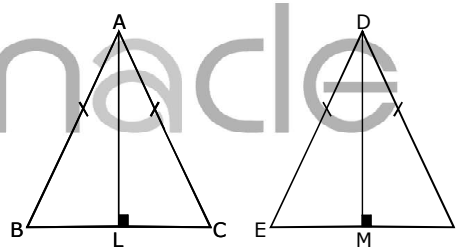
$$\frac{AB}{DE} = \frac{AC}{DF} \text{ and } \angle A = \angle D \text{ [given]}$$

$\therefore \triangle ABC \sim \triangle DEF$ [by S.A.S. similarity]

$$\Rightarrow \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \frac{AL^2}{DM^2}$$

$$\Rightarrow \frac{16}{25} = \frac{AL^2}{DM^2} \quad \left(\because \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DEF)} = \frac{16}{25} \right)$$

$$\Rightarrow \frac{AL}{DM} = \frac{4}{5}$$



Ex.16 Prove that in any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side.

Sol. Given : A $\triangle ABC$ in which AD is a median.

To prove : $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Const. : Draw $AE \perp BC$.

Proof : Since, $\angle AED = 90^\circ$. Therefore, in $\triangle ADE$, we have

$\angle ADE < 90^\circ \Rightarrow \angle ADB > 90^\circ$

Thus, $\triangle ADB$ is an obtuse-angled triangle and $\triangle ADC$ is an acute angled triangle.

Now, $\triangle ABD$ is obtuse-angled at D and

$AE \perp BD$ produced.

we have $AB^2 = AD^2 + BD^2 + 2BD.DE$

... (i)

Again, $\triangle ACD$ is acute-angled at D and

$AE \perp CD$. we have

$AC^2 = AD^2 + DC^2 - 2DC.DE$

$\Rightarrow AC^2 = AD^2 + BD^2 - 2BD.DE$ (ii)

[$\because CD = BD$]

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow AB^2 + AC^2$

$$= 2 \left[AD^2 + \left(\frac{BC}{2} \right)^2 \right] \left[\because BD = \frac{BC}{2} \right]$$

$\Rightarrow AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Ex.17 Prove that three times the square on any side of an equilateral triangle is equal to four times the square on the altitude.

Sol. **Given :** An equilateral $\triangle ABC$ and $AD \perp BC$

To prove : $3AB^2 = 4AD^2$

Proof : We know that in an equilateral triangle perpendicular from a vertex bisect the base.

$\therefore BD = DC = \frac{1}{2}BC$

Since, $\triangle ADB$ is a

right-triangle,

right-angled at D.

$\therefore AB^2 = AD^2 + BD^2$

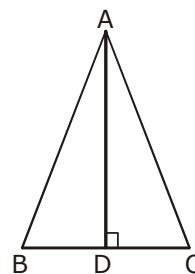
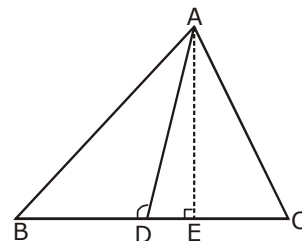
[By Pythagoras Theorem]

$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4}$

$\Rightarrow AB^2 = AD^2 + \frac{AB^2}{4}$ [$\because BC = AB$]

$\Rightarrow \frac{3}{4}AB^2 = AD^2 \Rightarrow 3AB^2 = 4AD^2$



Ex.18 In a $\triangle ABC$, $\angle ABC > 90^\circ$ and $AD \perp (CB \text{ produced})$. Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

Sol. Given: A $\triangle ABC$ in which $\angle ABC > 90^\circ$ and $AD \perp (CB \text{ produced})$.

To Prove: $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

Proof: In $\triangle ABD$, $\angle ADB = 90^\circ$

$$\therefore AB^2 = AD^2 + BD^2 \quad \dots(i)$$

[by Pythagoras' theorem]

In $\triangle ADC$, $\angle ADC = 90^\circ$

$$\therefore AC^2 = AD^2 + CD^2$$

[by Pythagoras's theorem]

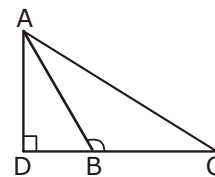
$$= AD^2 + (BC + BD)^2$$

$$[\because CD = (BC + BD)]$$

$$= AD^2 + (BC^2 + BD^2 + 2BC \cdot BD)$$

$$= (AD^2 + BD^2) + BC^2 + 2BC \cdot BD$$

$$= (AB^2 + BC^2 + 2BC \cdot BD) \quad [\text{using (i)}].$$



Ex.19 In $\triangle ABC$, if AD is the median, then prove that $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

Sol. Given: A $\triangle ABC$ in which AD is the median.

To Prove: $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

Construction: Draw $AL \perp BC$.

Proof: In $\triangle ALD$, $\angle ALD = 90^\circ$

$$\therefore \angle ADL < 90^\circ \text{ and therefore,}$$

$$\angle ADB > 90^\circ.$$

Thus, in $\triangle ADB$, $\angle ADB > 90^\circ$ and $AL \perp (BD \text{ produced})$.

$$\therefore AB^2 = AD^2 + BD^2 + 2BD \cdot DL \quad \dots(i)$$

In $\triangle ADC$, $\angle ADC < 90^\circ$ and $AL \perp DC$.

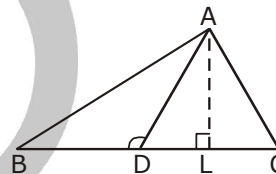
$$\therefore AC^2 = AD^2 + CD^2 - 2CD \cdot DL$$

$$\Rightarrow AC^2 = AD^2 + BD^2 - 2BD \cdot DL \quad \dots(ii)$$

$$[\because CD = BD]$$

Adding (i) and (ii), we get :

$$(AB^2 + AC^2) = 2(AD^2 + BD^2)$$



Ex.20 In the given figure, $\angle B = 90^\circ$. D and E are any points on AB and BC respectively.

Prove that : $AE^2 + CD^2 = AC^2 + DE^2$.

Sol. In $\triangle ABE$, $\angle B = 90^\circ$

$$\therefore AE^2 = AB^2 + BE^2 \quad \dots(i)$$

In $\triangle DBC$, $\angle B = 90^\circ$.

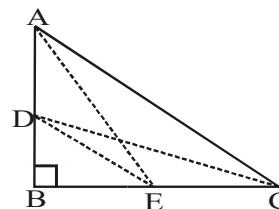
$$\therefore CD^2 = BD^2 + BC^2 \quad \dots(ii)$$

Adding (i) and (ii), we get :

$$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$$

$$= AC^2 + DE^2 \quad [\text{By Pythagoras Theorem}]$$

Hence, $AE^2 + CD^2 = AC^2 + DE^2$.



Ex.21 A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D.

Prove that: $OA^2 + OC^2 = OB^2 + OD^2$.

Sol. Through O, draw $EF \parallel AB$. Then, ABFE is a rectangle.

In right triangles OEA and OFC, we have:

$$OA^2 = OE^2 + AE^2$$

$$OC^2 = OF^2 + CF^2$$

$$\therefore OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2 \dots (i)$$

Again, in right triangles OFB and OED, we have :

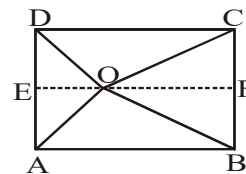
$$OB^2 = OF^2 + BF^2$$

$$OD^2 = OE^2 + DE^2$$

$$\begin{aligned} \therefore OB^2 + OD^2 &= OF^2 + OE^2 + BF^2 + DE^2 \\ &= OE^2 + OF^2 + AE^2 + CF^2 \dots (ii) \quad [\because BF = AE \text{ \& } DE = CF] \end{aligned}$$

From (i) and (ii), we get

$$OA^2 + OC^2 = OB^2 + OD^2.$$



Ex22 In the given figure, $\triangle ABC$ is right-angled at C.

Let $BC = a$, $CA = b$, $AB = c$ and $CD = p$, where $CD \perp AB$.

Prove that: (i) $cp = ab$ (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol. (i) Area of $\triangle ABC = \frac{1}{2} AB \times CD = \frac{1}{2} cp$.

Also, area of $\triangle ABC = \frac{1}{2} BC \times AC = \frac{1}{2} ab$.

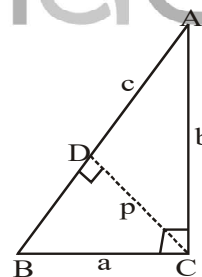
$$\therefore \frac{1}{2} cp = \frac{1}{2} ab. \Rightarrow cp = ab$$

$$(ii) \quad cp = ab \Rightarrow p = \frac{ab}{c}$$

$$\Rightarrow p^2 = \frac{a^2 b^2}{c^2}$$

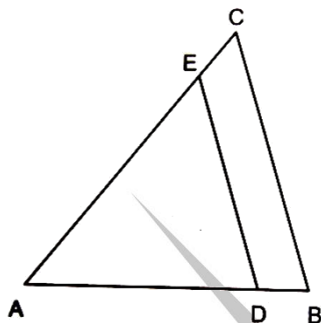
$$\Rightarrow \frac{1}{p^2} = \frac{c^2}{a^2 b^2} = \frac{a^2 + b^2}{a^2 b^2} \quad [\because c^2 = a^2 + b^2]$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

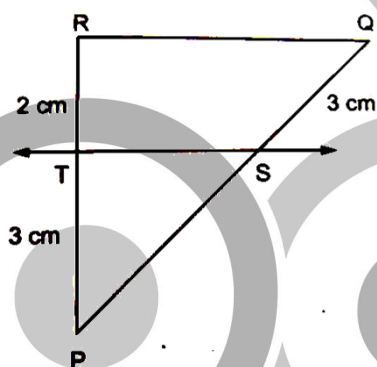


EXERCISE – 1

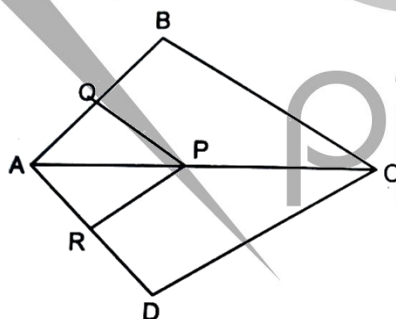
1. In fig. $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x .



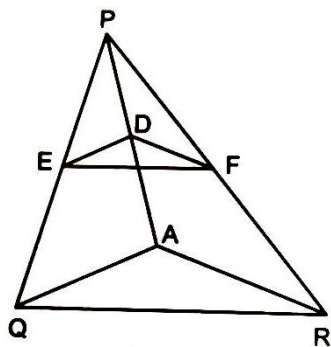
2. In fig., if $ST \parallel QR$. Find PS .



3. In fig., if $PQ \parallel BC$ and $PR \parallel CD$. Prove that (i) $\frac{AR}{AD} = \frac{AQ}{AB}$ (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$



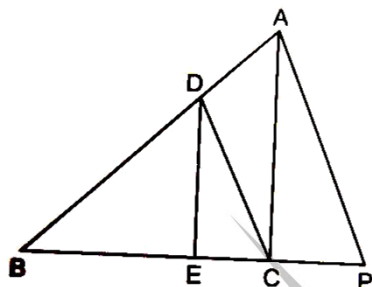
4. In fig., if $DE \parallel AQ$ and $DF \parallel AR$. Prove that $EF \parallel QR$.



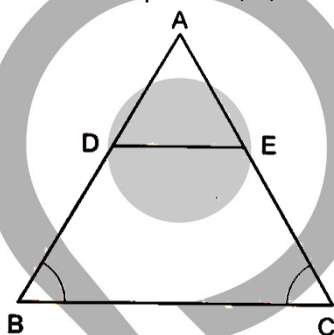
5. ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that

$$(i) \frac{DP}{PL} = \frac{DC}{BL} \quad (ii) \frac{DP}{DP} = \frac{AL}{DC}$$

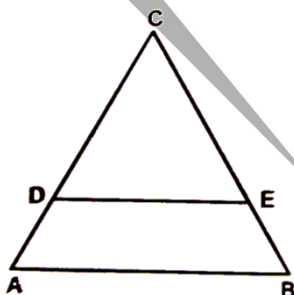
6. In fig, $DE \parallel BC$ and $CD \parallel EF$. Prove that $AD^2 = AB \times AF$.



7. Two triangles ABC and DBC lie on the same side of the base BC. From a point P on BC, $PQ \parallel AB$ and $PR \parallel BD$ are drawn. They meet AC in Q and DC in R respectively. Prove that $QR \parallel AD$.
8. Let ABC be a triangle and D and E be two points on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.
9. In fig., ABC is a triangle in which $AB = AC$. Points D and E are points on the sides AB and AC respectively such that $AD = AE$. Show that the points B, C, E and D are concyclic.



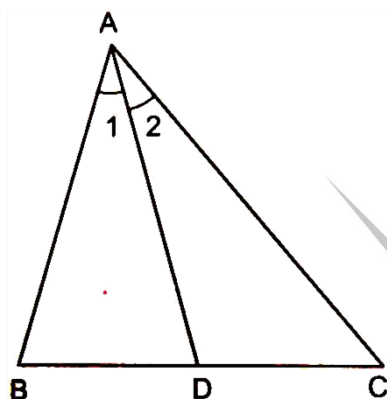
10. In fig., if $\frac{AD}{DC} = \frac{BE}{EC}$ and $\angle CDE = \angle CED$, prove that $\triangle CAB$ is isosceles.



11. In three line segments OA, OB and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear. Show that $LN \parallel AC$.
12. If D and E are points on sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$ and $BD = CE$. Prove that $\triangle ABC$ is isosceles.
13. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.
14. The bisector of interior $\angle A$ of $\triangle ABC$ meets BC in D, and the bisector of exterior $\angle A$ meets BC produced in E. prove that $\frac{BD}{BE} = \frac{CD}{CE}$.
15. AD is a median of $\triangle ABC$. The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC in E and F respectively. Prove that $EF \parallel BC$.

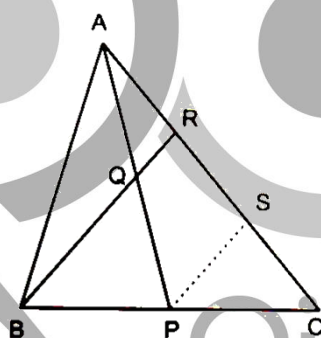
16. In a quadrilateral ABCD, if bisectors of the $\angle ABC$ and $\angle ADC$ meet on the diagonal AC, prove that the bisectors of $\angle BAD$ and $\angle BCD$ will meet on the diagonal BD.

17. In $\triangle ABC$ (fig), if $\angle 1 = \angle 2$, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

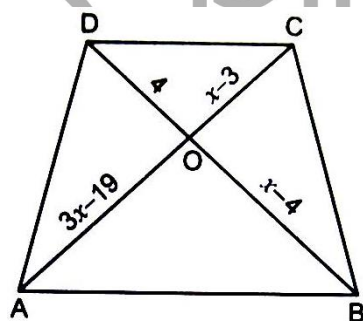


18. D, E and F are the points on sides BC, CA and AB respectively of $\triangle ABC$ such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. if $AB = 5$ cm, $BC = 8$ cm and $CA = 4$ cm, determine AF, CE and BD.

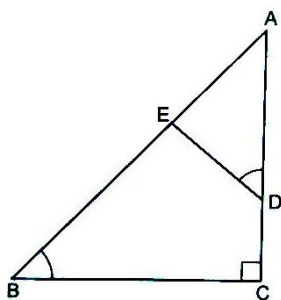
19. In fig., P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that $RA = \frac{1}{3} CA$.



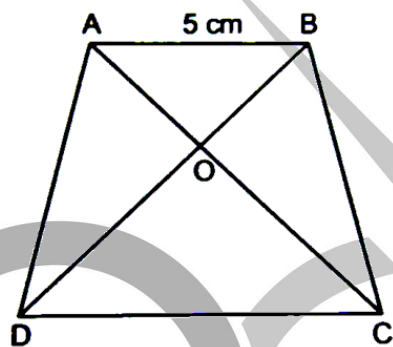
20. In fig., $AB \parallel CD$. If $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$ and $OD = 4$, find x.



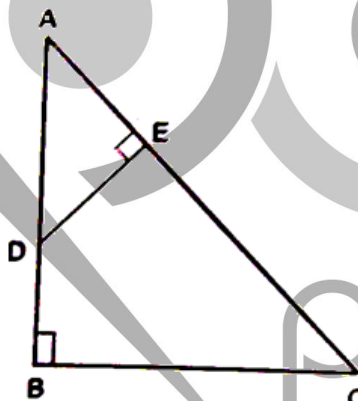
21. In fig., if $\angle ADE = \angle B$ show that $\triangle ADE \sim \triangle ABC$. If $AD = 3.8$ cm, $AE = 3.6$ cm, $BE = 2.1$ cm and $BC = 4.2$ cm, find DE.



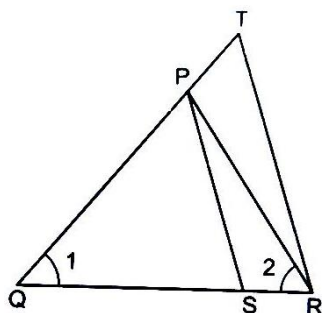
22. In fig., $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 5$ cm. find the value of DC .



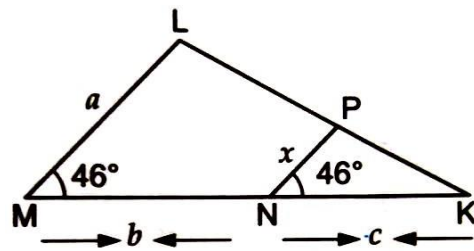
23. In fig., if $AB \perp BC$ and $DE \perp AC$. Prove that $\triangle ABC \sim \triangle AED$.



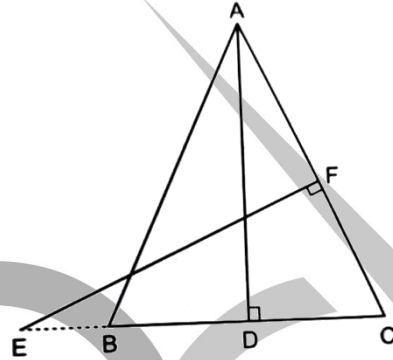
24. In fig., if $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\triangle PQS \sim \triangle TQR$.



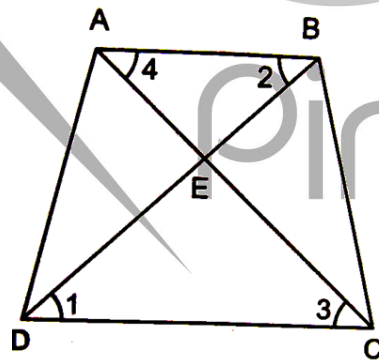
25. D is a point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Prove that $\frac{CA}{CD} = \frac{CB}{CA}$ or, $CA^2 = CB \times CD$.
26. P and Q are points on sides AB and AC respectively of $\triangle ABC$. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3 PQ$.
27. In fig., express x terms of a , b and c .



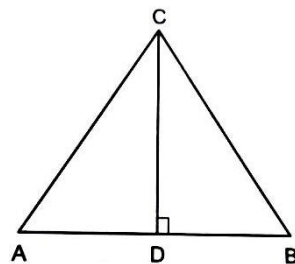
28. In fig., E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$. Prove that (i) $\triangle ABD \sim \triangle ECF$ (ii) $AB \times EF = AD \times EC$.



29. Prove that the line segments joining the mid-points of the sides of a triangle form four triangles, each of which is similar to the original triangle.
30. In $\triangle ABC$, DE is parallel to base BC, with D on AB and E on AC. If $\frac{AD}{DB} = \frac{2}{3}$, find $\frac{BC}{DE}$.
31. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Determine the height of the tower.
32. In fig., ABCD is a trapezium with $AB \parallel DC$. If $\triangle AED$ is similar to $\triangle BEC$. Prove that $AD = BC$.



33. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that $EL = 2 BL$
34. In fig., $\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.



35. ABC is a triangle in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. prove that $BD = BC$.

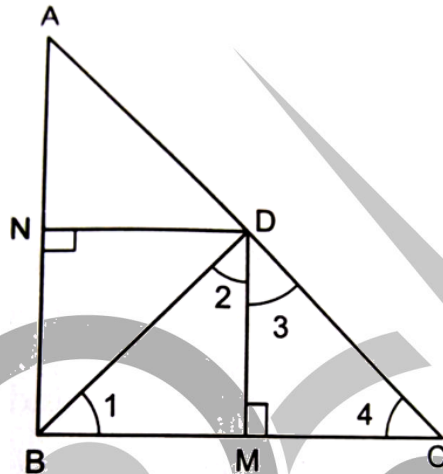
36. In trapezium ABCD, $AB \parallel DC$ and $DC = 2 AB$. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$.

Diagonal DB intersects EF at G. Prove that $7 FE = 10 AB$.

37. In fig., ABC is a right triangle right angled at B and D is the foot of the perpendicular drawn from B on AC. If $DM \perp BC$ and $DN \perp AB$, prove that

(i) $DM^2 = DN \times MC$

(ii) $DN^2 = DM \times AN$

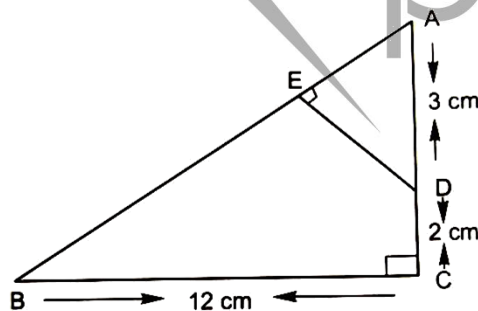


38. In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that $ab = cx$.

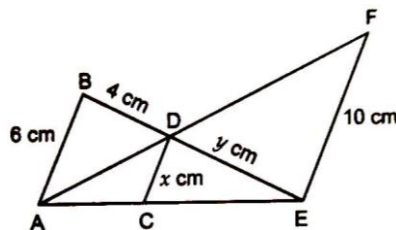
39. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

40. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

41. In fig., $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$ and hence find the lengths of AE and DE.



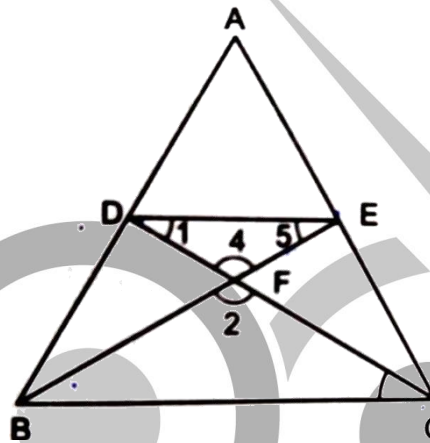
42. In fig., we have $AB \parallel CD \parallel EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 10$ cm, $BD = 4$ cm and $DE = y$ cm, calculate the values of x and y.



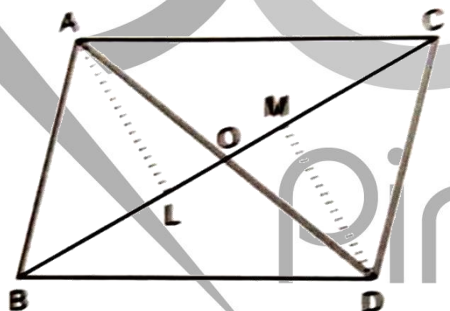
43. If $\triangle ABC$ is similar to $\triangle DEF$ such that $BC = 3$ cm, $EF = 4$ cm and area of $\triangle ABC = 54$ cm. determine the area of $\triangle DEF$.

44. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.
45. Equilateral triangles are drawn on the sides of a right triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the area of triangles on the other two sides.
46. D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$ and divides $\triangle ABC$ into two parts, equal in area, find $\frac{BD}{AB}$.

47. In fig., $DE \parallel BC$ and $AD : DB = 5 : 4$. Find $\frac{\text{Area}(\triangle DEF)}{\text{Area}(\triangle CFB)}$.

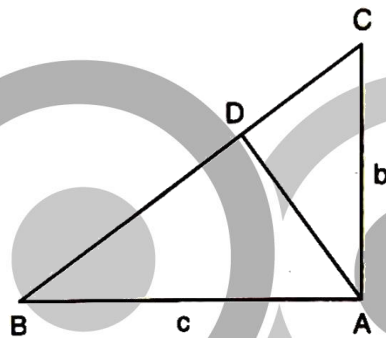


48. In fig., $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD and BC intersect at O, prove that $\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle DBC)} = \frac{AO}{DO}$.



49. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that $\text{Area}(\triangle ADE) : \text{Area}(\triangle ABC) = 3 : 4$.
50. ABC is a right triangle right-angled at B. Let D and E be any points on AB and BC respectively. Prove that $AE^2 + CD^2 = AC^2 + DE^2$.
51. Prove that three times the square of any sides of an equilateral-triangle is equal to four times the square of the altitude.
52. In an equilateral triangle with side a prove that
 (i) Altitude = $\frac{a\sqrt{3}}{2}$ (ii) Area = $\frac{\sqrt{3}}{4} a^2$
53. In an isosceles triangle ABC with $AB = AC$, BD is perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2 CD \cdot AD$.
54. A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$.
55. ABCD is a rhombus. Prove that $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$.

56. In an equilateral triangle ABC the side BC is trisected at D. Prove that $9 AD^2 = 7 AB^2$
57. ABC is an isosceles triangle right-angled at B. similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of $\triangle ABE$ and $\triangle ACD$.
58. ABC is a right-angled triangle right angled at A. A circle is inscribed in it the lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.
59. In a $\triangle ABC$, the angles at B and C are acute. If BE and CF be drawn perpendiculars on AC and AB respectively. Prove that $BC^2 = AB \times BF + AC \times CE$.
60. If A be the area of a right triangle and B one of the sides containing the right angle. Prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.
61. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
62. Using Pythagoras theorem determine the length of AD in terms of b and c shown in fig.,

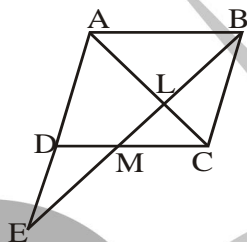


63. The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.
64. In an acute-angled triangle, express a median in terms of its sides.
65. In $\triangle ABC$, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that:
- (i) $AB \times AQ = AC \times AP$ (ii) $BC^2 = (AC \times CP + AB \times BQ)$
66. In a right $\triangle ABC$ right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4 (AD^2 - AC^2)$.

Pinnacle

EXERCISE – 2

1. P and Q are points on the sides CA and CB respectively of a $\triangle ABC$ right-angled at C. Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$.
2. $\triangle ABC$ is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.
3. In $\triangle ABC$, if AD is the median, show that $AB^2 + AC^2 = 2[AD^2 + BD^2]$.
4. In the given figure, M is the mid-point of the side CD of parallelogram ABCD. BM, when joined meets AC in L and AD produced in E. Prove that $EL = 2BL$.



5. $\triangle ABC$ is a right triangle, right-angled at C. If p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) $pc = ab$ (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
6. In an equilateral triangle PQR, the side QR is trisected at S. Prove that $9PS^2 = 7PQ^2$.
7. If the diagonals of a quadrilateral divide each other proportionally, prove that it is trapezium.
8. In an isosceles triangle ABC with $AB = AC$, BD is a perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD \cdot AD$.
9. $\triangle ABC$ and $\triangle DBC$ are two triangles on the same base BC. If AD intersect BC at O. Prove that $\frac{ar.\triangle ABC}{ar.\triangle DBC} = \frac{AO}{DO}$.
10. In $\triangle ABC$, $\angle A$ is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that $AB \times AE = AC \times AD$.
11. Points P and Q are on sides AB and AC of a triangle ABC in such a way that PQ is parallel to side BC. Prove that the median AD drawn from vertex A to side BC bisects the segment PQ.
12. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

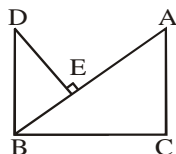
OR

Two $\triangle s$ $\triangle ABC$ and $\triangle DBC$ are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90^\circ$. If CA and BD meet each other at E, show that $AE \cdot EC = BE \cdot ED$.

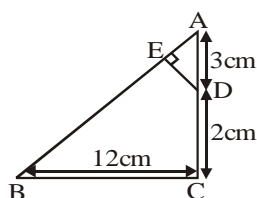
13. D and E are points on the sides CA and CB respectively of $\triangle ABC$ right-angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

OR

In fig. $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.

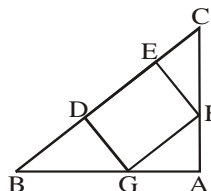


14. E is a point on the side AD produced of a \parallel^{gm} ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.
15. In fig, $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$ and hence find the lengths of AE and DE.

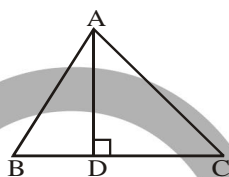


OR

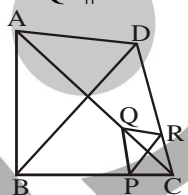
In fig, DEFG is a square and $\angle BAC = 90^\circ$. Show that $DE^2 = BD \times EC$



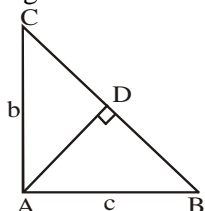
16. In fig, $AD \perp BC$ and $BD = \frac{1}{3} CD$. Prove that $2CA^2 = 2AB^2 + BC^2$.



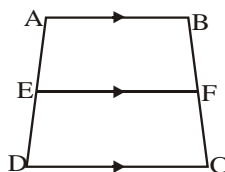
17. In fig, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$.



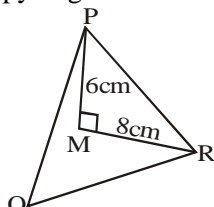
18. In a right triangle ABC, right-angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 1 : 2. Prove that
- $9AQ^2 = 9AC^2 + 4BC^2$
 - $9BP^2 = 9BC^2 + 4AC^2$
 - $9(AQ^2 + BP^2) = 13AB^2$.
19. The ratio of the areas of similar triangles is equal to the ratio of the squares on the corresponding sides, prove. Using the above theorem, prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.
20. Perpendiculars OD, OE and OF are drawn to sides BC, CA and AB respectively from a point O in the interior of a $\triangle ABC$. Prove that :
- $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$.
 - $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.
21. In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides, prove. Using the above theorem, determine the length of AD in terms of b and C.



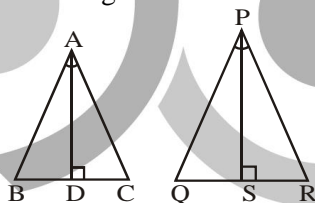
22. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove. Use this result to prove the following: In the given figure, if ABCD is a trapezium in which $AB \parallel DC \parallel EF$, then $\frac{AE}{ED} = \frac{BF}{FC}$.



23. State and prove pythagoras theorem. Use the theorem and calculate area (ΔPMR) from the given figure.

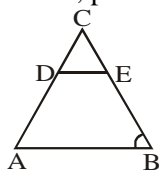


24. In a right-angled triangle, the square of hypotenuse is equal to the sum of the squares of the two sides. Given that $\angle B$ of ΔABC is an acute angle and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.
25. In a right triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using above, solve the following : In quadrilateral ABCD, find the length of CA, if $CD \perp DB$, $AB \perp DB$, $CD = 6$ m, $DB = 12$ m and $AB = 11$ m.
26. Prove that the ratio of the areas of two similar triangles is equal to the squares of their corresponding sides. Using the above, do the following:



In fig. ΔABC and ΔPQR are isosceles triangles in which $\angle A = \angle P$. If $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{9}{16}$, find $\frac{AD}{PS}$.

27. In a right-angled triangle, prove that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Using the above result, find the length of the second diagonal of a rhombus whose side is 5 cm and one of the diagonals is 6 cm.
28. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides prove that the angle opposite the first side is a right angle. Use the above theorem and prove that following: In triangle ABC, $AD \perp BC$ and $BD = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.
29. In a right triangle, prove that the square on hypotenuse is equal to sum of the squares on the other two sides. Using the above result, prove the following: PQR is a right triangle, right angled at Q. If S bisects QR, show that $PR^2 = 4PS^2 - 3PQ^2$.
30. If a line is drawn parallel to one side of a triangle prove that the other two sides are divided in the same ratio. Using the above result, prove from fig. that $AD = BE$ if $\angle A = \angle B$ and $DE \parallel AB$.

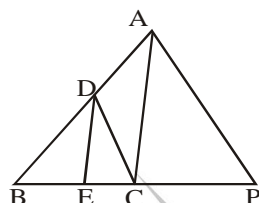


31. Prove that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Apply the above theorem on the following: ABC is a triangle and PQ is a straight line meeting AB in P

and AC in Q. If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm, $QC = 4.5$ cm, prove that area of $\triangle APQ$ is one-sixteenth of the area of $\triangle ABC$.

32. If a line is drawn parallel to one side of a triangle, prove that the other two sides are divided in the same ratio.

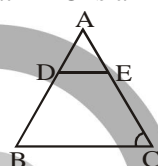
Use the above to prove the following : In the given figure $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.



33. In a triangle if the square on one side is equal to the sum of squares on the other two sides, prove that the angle opposite to the first side is a right angle. Use the above theorem to prove the following :

In a quadrilateral ABCD, $\angle B = 90^\circ$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.

34. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following : In figure, $DE \parallel BC$ and $BD = CE$. Prove that ABC is an isosceles triangle.



35. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Use the above for the following : If the areas of two similar triangles are equal, prove that they are congruent.

36. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Using the above result, prove the following :

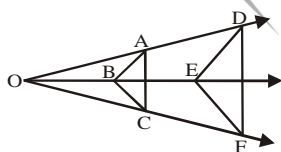
In a $\triangle ABC$, XY is parallel to BC and it divides $\triangle ABC$ into two parts of equal area. Prove that $\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

37. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides. Using the above, do the following:

The diagonals of a trapezium ABCD, with $AB \parallel DC$, intersect each other at the point O. If $AB = 2$ CD, find the ratio of the area of $\triangle AOB$ to the area of $\triangle COD$.

38. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following :

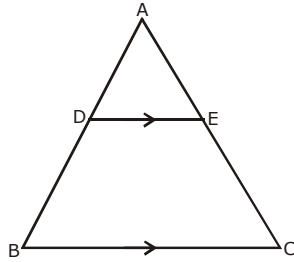
In the fig, $AB \parallel DE$ and $BC \parallel EF$. Prove that $AC \parallel DF$.



39. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Using the above, do the following: In a trapezium ABCD, AC and BD are intersecting at O, $AB \parallel DC$ and $AB = 2$ CD. If area of $\triangle AOB = 84$ cm², find the area of $\triangle COD$.

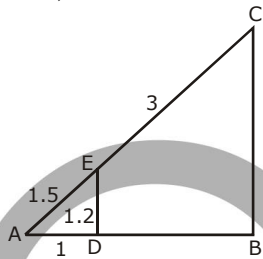
EXERCISE – 3

1. In $\triangle ABC$, $DE \parallel BC$, $AD = 2.4\text{cm}$, $AE = 3.2\text{ cm}$, and $EC = 4.8\text{ cm}$. The length of AB is:



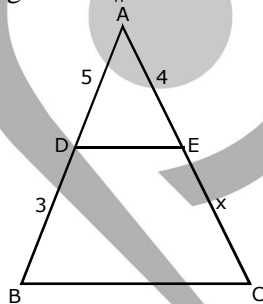
- (a) 3.6 cm (b) 6cm (c) 6.4cm (d) 1.6cm

2. If $\triangle ADE \sim \triangle ABC$, then $BC = ?$

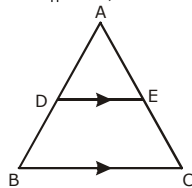


- (a) 4.5 (b) 3 (c) 3.6 (d) 2.4

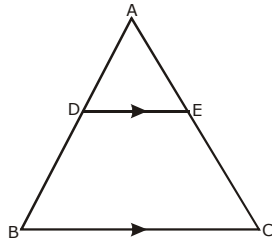
3. In the given figure $ED \parallel BC$. The value of x is:



- (a) 2.8 (b) 2.5 (c) 2.4 (d) 4
4. The line segments joining the midpoints of the sides of a triangle form four triangles, each of which is
- (a) congruent to the original triangle
 (b) similar to the original triangle
 (c) an isosceles triangle
 (d) an equilateral triangle
5. In the $\triangle ABC$ $DE \parallel BC$, $AD = 1.7\text{ cm}$, $AB = 6.8\text{ cm}$ and $AC = 9\text{ cm}$. The length of AE is

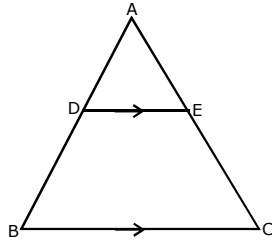


- (a) 2.5 cm (b) 4.5 cm (c) 2.2 cm (d) 7.3 cm
6. In the $\triangle ABC$, $DE \parallel BC$, $AD = (7x - 4)\text{ cm}$, $AE = (5x - 2)\text{ cm}$, $DB = (3x + 4)\text{ cm}$, and $EC = 3x\text{ cm}$. The value of x is:?



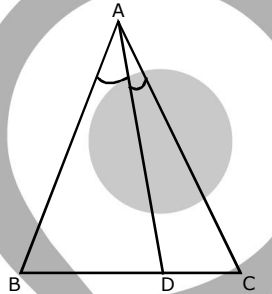
- (a) 3 (b) 5 (c) 4 (d) 2.5

7. In the $\triangle ABC$, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm, then the length of AE is :



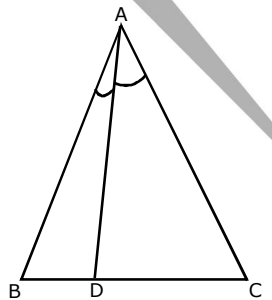
- (a) 4.2 cm (b) 3.1 cm (c) 2.1 cm (d) 2.8 cm

8. In $\triangle ABC$, AD is the internal bisector of $\angle A$. If $BD = 5$ cm, $BC = 7.5$ cm, then $AB : AC$ is equal to :



- (a) 2 : 1 (b) 1 : 2 (c) 4 : 5 (d) 3 : 5

9. In the $\triangle ABC$, AD is the internal bisector of $\angle A$. If $BD = 4$ cm, $DC = 5$ cm and $AB = 6$ cm, then the length of $AC = ?$

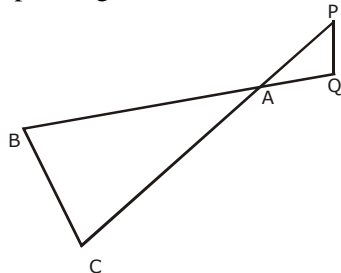


- (a) 3 cm (b) 8 cm (c) 4.5 cm (d) 7.5 cm

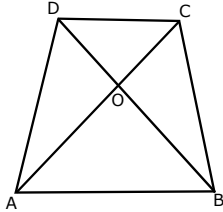
10. In a right triangle PQR , $PR^2 + PQ^2 = QR^2$. Which angle is equal to 90° ?

- (a) $\angle P$ (b) $\angle Q$ (c) $\angle R$ (d) none of these

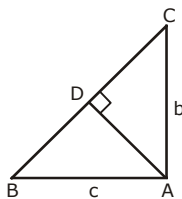
11. The perimeter of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of the second triangle is:



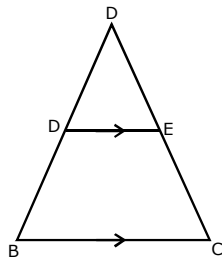
- (a) 4.5 cm (b) 5cm (c) 3.5cm (d) 5.4 cm
12. In the given figure, $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm and $AP = 2.8$ cm, then the length of AQ is:
 (a) 3.25 cm (b) 4cm (c) 4.25cm (d) 3 cm
13. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, the longest side of the smaller triangle is
 (a) 21 cm (b) 22cm (c) 24 cm (d) 20 cm
14. In the given figure, $AB \parallel DC$ and diagonals AC and BD intersect at O . If $AO = (3x - 1)$ cm, $BO = (2x + 1)$ cm, $OC = (5x - 3)$ cm and $OD = (6x - 5)$ cm, then the value of x is :



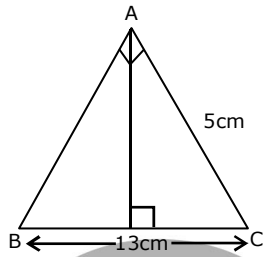
- (a) 2 (b) 3 (c) 2.5 (d) 4
15. $\triangle ABC \sim \triangle DEF$ and the perimeters of $\triangle ABC$ and $\triangle DEF$ are 30 cm and 18 cm respectively. If $BC = 9$ cm, then EF is equal to:
 (a) 6.3 (b) 5.4 (c) 7.2 (d) 4.5
16. $\triangle ABC \sim \triangle DEF$ such that $AB = 9.1$ cm and $DE = 6.5$ cm. If the perimeter of $\triangle DEF$ is 25 cm, then the perimeter of $\triangle ABC$ is :
 (a) 35 cm (b) 28 cm (c) 42 cm (d) 40 cm
17. If D is a point on the side AB of $\triangle ABC$ such that $AD : DB = 3 : 2$ and E is a point on BC such that $DE \parallel AC$. The ratio of areas of $\triangle ABC$ and $\triangle BDE$ is
 (a) 4 : 25 (b) 25 : 4 (c) 5 : 4 (d) 4 : 5
18. In an equilateral triangle ABC , if $AD \perp BC$, then:
 (a) $2AB^2 = 3AD^2$ (b) $4AB^2 = 3AD^2$ (c) $3AB^2 = 4AD^2$ (d) $3AB^2 = 2AD^2$
19. The line segments joining the mid points of the adjacent sides of a quadrilateral form a:
 (a) Parallelogram (b) square (c) rhombus (d) rectangle
20. If the diagonals of a quadrilateral divide each other proportionally, then it is a:
 (a) Parallelogram (b) trapezium (c) rectangle (d) square
21. A right triangle has hypotenuse of length p cm and one side of length q cm. If $p - q = 1$, then the length of the third side of the triangle is :
 (a) $\sqrt{2q+1}$ (b) $\sqrt{2p+1}$ (c) $2p$ (d) $1 + q$
22. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, then $\triangle ABC$ is right angled at:
 (a) $\angle A$ (b) $\angle B$ (c) $\angle C$ (d) none of these
23. In the figure, $DABC$ is a right triangle, right angled at A and $AD \perp BC$. If $AB = c$ and $AC = b$, then AD is equal to:



- (a) $\frac{bc}{\sqrt{b^2 + c^2}}$ (b) $\frac{bc}{b^2 + c^2}$ (c) $\frac{b^2c}{\sqrt{b^2 + c^2}}$ (d) none of these
24. A triangle has sides 5cm, 12 cm and 13 cm. The length of the perpendicular from the opposite vertex to the side whose length is 13 cm is:
 (a) 4.9 (b) 3.6 cm (c) 5.5 cm (d) 4.6 cm
25. In the given figure, $DE \parallel BC$. If $DE = 3$ cm, $BC = 6$ cm and $\text{ar}(\triangle ADE) = 15 \text{ cm}^2$. Area of $\triangle ABC$ is:

(a) 30 cm^2 (b) 60 cm^2 (c) 40 cm^2 (d) 50 cm^2

26. $\triangle ABC$ is right angled at A and $AD \perp BC$. If $BC = 13 \text{ cm}$ and $AC = 5 \text{ cm}$, the ratio of the areas of $\triangle ABC$ and $\triangle ADC$ is :



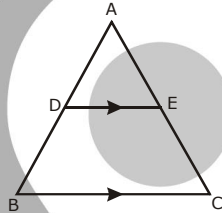
(a) 25 : 169

(b) 169 : 25

(c) 5 : 13

(d) 13 : 5

27. In the given figure, $DE \parallel BC$ and $DE : BC = 3 : 5$. The ratio of the areas of $\triangle ADE$ and the trapezium BCED is :



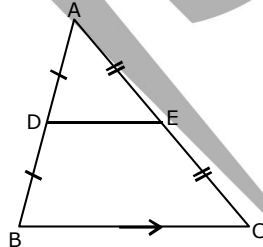
(a) 9 : 16

(b) 16 : 9

(c) 3 : 5

(d) 5 : 3

28. In $\triangle ABC$, D and E are the mid points of AB and AC respectively. The ratio of the areas of $\triangle ADE$ and $\triangle ABC$ is:



(a) 1 : 2

(b) 2 : 1

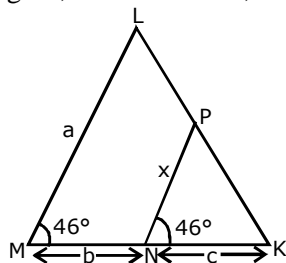
(c) 1 : 4

(d) 4 : 1

29. In a trapezium ABCD, O is the point of intersection of AC and BD, $AB \parallel CD$ and $AB = 2 \times CD$. If the area of $\triangle AOB = 84 \text{ cm}^2$, then the area of $\triangle COD$ is :

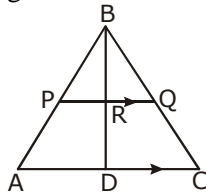
(a) 25 cm^2 (b) 21 cm^2 (c) 24 cm^2 (d) 32 cm^2

30. In the given figure, x in terms of a, b and c is :

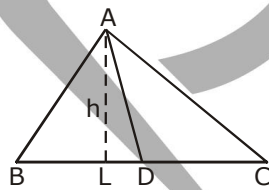
(a) $x = \frac{ac}{b+c}$ (b) $x = \frac{ab}{b+c}$ (c) $x = \frac{ac}{a+b}$ (d) $x = \frac{bc}{a+c}$

31. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, the distance between their tops is

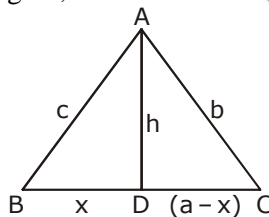
- (a) 12 m (b) 13 m (c) 15 m (d) 11 m
32. Two isosceles triangles have their corresponding angles equal and their areas are in the ratio 25 : 36. The ratio of their corresponding height is
 (a) 25 : 36 (b) 36 : 25 (c) 5 : 6 (d) 6 : 5
33. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is:
 (a) 100 m (b) 120 m (c) 25 m (d) 200 m
34. The length of altitude AD of an isosceles $\triangle ABC$, in which $AB = AC = 2a$ units and $BC = a$ units, is :
 (a) $\frac{a\sqrt{15}}{4}$ (b) $\frac{a\sqrt{15}}{2}$ (c) $\frac{\sqrt{15}a}{4}$ (d) none of these
35. In the given figure, if BD is the bisector of $\angle B$, $PQ \parallel AC$, then :



- (a) $PR \times QR = BQ \times BP$
 (b) $PR \times BQ = QR \times BP$
 (c) both (a) and (b)
 (d) None of these
36. $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = 36 \text{ cm}^2$ and $\text{ar}(\triangle DEF) = 49 \text{ cm}^2$. Then, the ratio of their corresponding sides is
 (a) 36 : 49 (b) 6 : 7 (c) 7 : 6 (d) $\sqrt{6} : \sqrt{7}$
37. In $\triangle ABC$, if AD is the bisector of $\angle A$, then $\frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle ADC)}$ is equal to



- (a) $\frac{BD}{DC}$ (b) $\frac{AB}{AC}$ (c) both A and B (d) none of these
38. In the given figure, $\angle B < 90^\circ$ and segment $AD \perp BC$, then :



- (a) $b^2 = h^2 + a^2 + x^2 - 2ax$
 (b) $b^2 = h^2 - a^2 - x^2 + 2ax$
 (c) $b^2 = h^2 + a^2$
 (d) none of these
39. If ABC is an isosceles triangle and D is a point on BC such that $AD \perp BC$, then :
 (a) $AB^2 - AD^2 = BD \cdot DC$
 (b) $AB^2 - AD^2 = BD^2 - DC^2$
 (c) $AB^2 + AD^2 = BD \cdot DC$
 (d) $AB^2 + AD^2 = BD^2 \cdot DC^2$
40. $\triangle ABC$ is a right angle right-angled at A and $AD \perp BC$. Then is equal to :

- (a) $\left(\frac{AB}{DC}\right)^2$ (b) $\frac{AB}{AC}$ (c) $\left(\frac{AB}{AD}\right)^2$ (d) $\frac{AB}{AD}$

41. If each side of a rhombus is 10 cm and one of its diagonals is 16 cm, then the length of the other diagonal is :
 (a) 16 cm (b) 14 cm (c) 12 cm (d) 10 cm
42. D, E and F are the mid-points of side AB, BC and CA respectively of $\triangle ABC$. The ratio of the areas of $\triangle DEF$ and $\triangle ABC$ is :
 (a) 1 : 2 (b) 4 : 1 (c) 3 : 4 (d) 1 : 4
43. An aeroplane leaves an airport and flies due north at a speed of 1000 km/h. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/h. After $1\frac{1}{2}$ hours, the distance between the two planes is :
 (a) $300\sqrt{61}$ km (b) $61\sqrt{300}$ km (c) $\sqrt{36100}$ km (d) $30\sqrt{61}$ km
44. ABC is a right triangle, right angled at C. If p is the length of the perpendicular from C to AB, $AB = c$, $BC = a$ and $AC = b$, then :
 (a) $\frac{1}{a^2} = \frac{1}{b^2} - \frac{1}{p^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$ (c) $\frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{a^2}$ (d) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
45. A vertical pole 5m long casts a shadow 2m long on the ground. At the same time, a tower casts a shadow 25m long on the ground. The height of the tower is
 (a) 12.5 m (b) 50 m (c) 100 m (d) 75 m
46. A vertical stick 1.2m long casts a shadow 40cm long on the ground. At the same time a pole 6m high casts a shadow x on the ground, the value of x is
 (a) 3m (b) 12m (c) 2m (d) 18m
47. Two poles 6m and 11m high stand vertically on the ground. If the distance between their feet is 12m, then the distance between their tops is
 (a) 11m (b) 14m (c) 12m (d) 13m
48. $\triangle ABC$ and $\triangle DEF$, are two similar triangles such that $\angle A = 36^\circ$ and $\angle E = 74^\circ$, then $\angle C$ is
 (a) 70° (b) 50° (c) 60° (d) 80°
49. Corresponding sides of two similar triangles are in the ratio of 5 : 7. Areas of these triangles are in the ratio of
 (a) $\sqrt{5} : \sqrt{7}$ (b) 7 : 5 (c) 25 : 49 (d) 49 : 25
50. The area of two similar triangles are 25 sq cm and 121 sq cm. The ratio of their corresponding sides is
 (a) 5 : 11 (b) 11 : 5 (c) $\sqrt{5} : \sqrt{11}$ (d) $\sqrt{11} : \sqrt{5}$
51. In $\triangle ABC$, $AB = 2$ cm, $BC = 3$ cm and $AC = 2.5$ cm. If $\triangle DEF \sim \triangle ABC$ and $EF = 6$ cm, then perimeter of $\triangle DEF$ is
 (a) 7.5 cm (b) 15cm (c) 22.5cm (d) 30cm
52. In $\triangle ABC$, and $\triangle DEF$, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{3}{4}$, then area ($\triangle ABC$) : Area ($\triangle DEF$) is equal to
 (a) 3 : 4 (b) 16 : 9 (c) 9 : 16 (d) 27 : 64
53. DE is drawn parallel to base BC of $\triangle ABC$ meeting AB at D and AC at E. If $\frac{AB}{BD} = 4$ and $CE = 2$ cm, then AE is equal to
 (a) 2cm (b) 4cm (c) 6cm (d) 8cm
54. In $\triangle PQR$, G and H are points on PQ and PR respectively such that $GH \parallel QR$ and $PG : GQ = 3 : 1$. If $PH = 3.3$ cm then PR is equal to
 (a) 1.1 cm (b) 4cm (c) 5.5 cm (d) 4.4 cm
55. $\triangle ABC$ and $\triangle BDE$ are two equilateral triangles such that D is mid-point of BC. The ratio of areas of triangles ABC and BDE is
 (a) 4 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4
56. In a $\triangle ABC$, D and E are points on sides AB and AC respectively such that BCED is a trapezium. If $\frac{DE}{BC} = \frac{3}{5}$, then $\frac{\text{Area}(\triangle ADE)}{\text{Area}(\text{Trap. BCED})}$ is equal to

(a) $\frac{3}{4}$

(b) $\frac{9}{16}$

(c) $\frac{3}{5}$

(d) $\frac{9}{25}$

57. Two isosceles triangles have equal angles and their areas are as 16 : 25. The ratio of their corresponding heights is

(a) 3 : 2

(b) 5 : 4

(c) 5 : 7

(d) 4 : 5

58. ABCD is a trapezium in which $BC \parallel AD$. If $AB = 4\text{cm}$ and the diagonals AC and BD intersect at O such that

$$\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}, \text{ then BC is equal to}$$

(a) 7 cm

(b) 8 cm

(c) 6 cm

(d) 9 cm

59. In a $\triangle ABC$, $\angle A = 90^\circ$, $AB = 5\text{cm}$ and $AC = 12\text{cm}$. If $AD \perp BC$, then AD is equal to

(a) $\frac{13}{2}\text{cm}$

(b) $\frac{13}{60}\text{cm}$

(c) $\frac{60}{13}\text{cm}$

(d) $\frac{2\sqrt{15}}{13}\text{cm}$

60. A man goes 24 m due west and then 10 m due north. How far is he from the starting point?

(a) 34 m

(b) 17 m

(c) 26 m

(d) 28 m

61. A man goes 12 m due south and then 35 m due west. How far is he from the starting point?

(a) 47 m

(b) 23.5 m

(c) 23 m

(d) 37 m

62. Two poles of height 13 m and 7 m respectively stand vertically on a plane ground at a distance of 8 m from each other. The distance between their tops is

(a) 9 m

(b) 10 m

(c) 11 m

(d) 12 m

63. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high?

(a) 2.4 m

(b) 1.35 m

(c) 1.5 m

(d) 13.5 m

64. A vertical pole 6 m long casts a shadow of length 3.6 m on the ground. What is the height of a tower which casts a shadow of length 18m at the same time?

(a) 10.8 m

(b) 28.8 m

(c) 32.4 m

(d) 30 m

65. A ladder 25 m long just reaches the top of building 24 m high from the ground. What is the distance of the foot of the ladder from the building?

(a) 7 m

(b) 14 m

(c) 21 m

(d) 24.5 m

66. A ladder 15 m long reaches a window which 9 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. The width of the street is

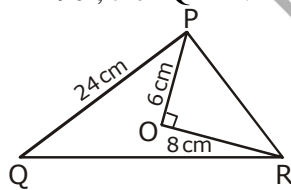
(a) 27 m

(b) 21 m

(c) 24 m

(d) 18 m

67. In the given figure, O is a point inside a $\triangle PQR$ such that $\angle PQR = 90^\circ$, $OP = 6\text{ cm}$ and $OR = 8\text{ cm}$. If $PQ = 24\text{ cm}$ and $\angle QPR = 90^\circ$, then $QR = ?$



(a) 28 cm

(b) 25 cm

(c) 26 cm

(d) 32 cm

68. The hypotenuse of a right triangle is 25 cm. The other two sides are such that one is 5 cm longer than the other. The length of these sides are

(a) 10 cm, 15 cm

(b) 15 cm, 20 cm

(c) 12 cm, 17 cm

(d) 13 cm, 18 cm

69. The height of an equilateral triangle having each side 12 cm, is

(a) $6\sqrt{2}\text{ cm}$

(b) $6\sqrt{3}\text{ cm}$

(c) $3\sqrt{6}\text{ cm}$

(d) $6\sqrt{6}\text{ cm}$

70. $\triangle ABC$ is an isosceles triangle with $AB = AC = 13\text{ cm}$ and the length of altitude from A on BC is 5 cm. Then, $BC = ?$

(a) 12 cm

(b) 16 cm

(c) 18 cm

(d) 24 cm

71. The measures of three angles of a triangle are in the ratio 1 : 2 : 3. Then, the triangle is

(a) right-angled

(b) equilateral

(c) isosceles

(d) obtuse-angled

72. For a $\triangle ABC$, which is true?

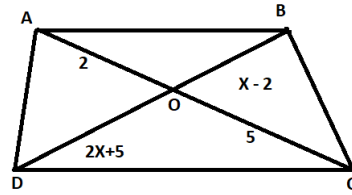
- (a) $AB - AC = BC$ (b) $(AB - AC) > BC$ (c) $(AB - AC) < BC$ (d) None of these
73. In a triangle, the perpendicular from the vertex to the base bisect the base. The triangle is
(a) right-angled (b) isosceles (c) scalene (d) obtuse-angled
74. In a rhombus of side 10 cm, one of the diagonals is 12 cm long. The length of the second diagonal is
(a) 20 cm (b) 18 cm (c) 16 cm (d) 22 cm
75. The length of the diagonals of a rhombus are 24 cm and 10 cm. The length of each side of the rhombus is
(a) 12 cm (b) 13 cm (c) 14 cm (d) 17 cm
76. If the diagonals of a quadrilateral divide each other proportionally, then it is a
(a) parallelogram (b) trapezium (c) rectangle (d) square
77. The line segment joining the midpoints of the adjacent sides of a quadrilateral form
(a) a parallelogram (b) a rectangle (c) a square (d) rhombus
78. If the bisector of an angle of a triangle bisects the opposite side, then the triangle is
(a) scalene (b) equilateral (c) isosceles (d) right-angled
79. In $\triangle ABC$, it is given that $AB = 9$ cm, $BC = 6$ cm and $CA = 7.5$ cm. Also, $\triangle DEF$ is given such that $EF = 8$ cm and $\triangle DEF \sim \triangle ABC$. Then, perimeter of $\triangle DEF$ is
(a) 22.5 cm (b) 25 cm (c) 27 cm (d) 30 cm
80. It is given that $\triangle ABC \sim \triangle DEF$. If $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm, then which of the following is true?
(a) $DE = 12$ cm, $\angle F = 50^\circ$
(b) $DE = 12$ cm, $\angle F = 100^\circ$
(c) $EF = 12$ cm, $\angle D = 100^\circ$
(d) $EF = 12$ cm, $\angle D = 30^\circ$
81. In $\triangle ABC$ and $\triangle DEF$, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$, then
(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$
82. In $\triangle DEF$ and $\triangle PQR$, it is given that $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?
(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QE}$
83. In $\triangle ABC$ and $\triangle DEF$, it is given that $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$, then the two triangles are
(a) congruent but not similar
(b) similar but not congruent
(c) neither congruent nor similar
(d) similar as well as congruent
84. If in $\triangle ABC$ and $\triangle PQR$, we have: $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then
(a) $\triangle PQR \sim \triangle CAB$ (b) $\triangle PQR \sim \triangle ABC$ (c) $\triangle CBA \sim \triangle PQR$ (d) $\triangle BCA \sim \triangle PQR$
85. It is given that $\triangle ABC \sim \triangle DEF$ and the corresponding sides of these triangles are in the ratio 3 : 5. Then $\text{ar}(\triangle ABC) : \text{ar}(\triangle DEF) =$
(a) 3 : 5 (b) 5 : 3 (c) 9 : 25 (d) 25 : 9
86. It is given that $\triangle ABC \sim \triangle PQR$ and $\frac{BC}{QR} = \frac{2}{3}$, then $\frac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle ABC)} = ?$
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$
87. In $\triangle ABC$ and $\triangle DEF$, we have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then $(\triangle ABC) : \text{ar}(\triangle DEF) = ?$
(a) 5 : 7 (b) 25 : 49 (c) 49 : 25 (d) 125 : 343
88. In right $\triangle ABC$, $BC = 7$ cm, $AC - AB = 1$ cm and $\angle B = 90^\circ$. The value of $\cos A + \cos B + \cos C$ is
(a) $\frac{1}{7}$ (b) $\frac{32}{24}$ (c) $\frac{31}{25}$ (d) $\frac{25}{31}$
89. Given that $\triangle ABC \sim \triangle PQR$ and $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{16}{9}$. If $PQ = 18$ cm and $BC = 12$ cm. Then, AB and QR are, respectively

- (a) 9 cm, 24 cm
 - (b) 24 cm, 9 cm
 - (c) 32 cm, 6.75 cm
 - (d) 13.5 cm, 16 cm
90. $\triangle ABC$ is an equilateral triangle of side $2\sqrt{3}$ cm, O is any point in the interior of $\triangle ABC$. If x, y, z are the distances of O from the sides of the triangle, then $x + y + z$ is equal to
- (a) $2 + \sqrt{3}$ cm (b) 3 cm (c) 4 cm (d) 5 cm

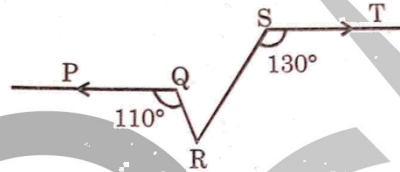


EXERCISE – 4

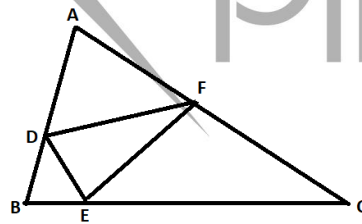
1. In a given figure in trapezium ABCD if $AB \parallel CD$ then value of x is : (NTSE Stage – 1 = 2013)



- (a) $\frac{29}{8}$ (b) $\frac{8}{29}$ (c) 20 (d) $\frac{1}{20}$
2. In a given figure $PQ \parallel ST$, $\angle PQR = 110^\circ$, $\angle RST = 130^\circ$ then value of $\angle QRS$ is (NTSE Stage – 1 = 2013)

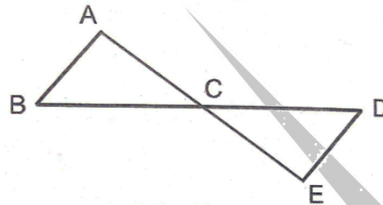


- (a) 20° (b) 50° (c) 60° (d) 70°
3. $\triangle ABC \sim \triangle PQR$ and $\frac{\text{area } \triangle ABC}{\text{area } \triangle PQR} = \frac{16}{9}$. If $PQ = 18$ cm and $BC = 12$ cm, then AB and QR are respectively: (NTSE Stage – 1 = 2013)
- (a) 9 cm, 24 cm (b) 24 cm, 9 cm
(c) 32 m, 675 cm (d) 135 cm, 16 cm
4. E and F are respectively, the mid points of the sides AB and AC of $\triangle ABC$ and the area of the quadrilateral BEFC is k times the area of $\triangle ABC$. The value of k is: (NTSE Stage – 1 = 2013)
- (a) $\frac{1}{2}$ (b) 3 (c) $\frac{3}{4}$ (d) 4
5. In the figure $AD = DB$, $BE = \frac{1}{2} EC$ and $CF = \frac{1}{3} AF$. If the area of $\triangle ABC = 120$ cm^2 , the area (in cm^2) of $\triangle DEF$ is : (NTSE – 1 = 2013)

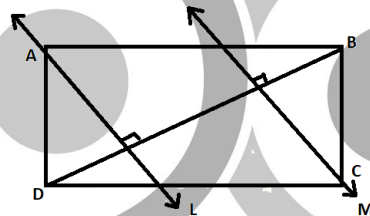


- (a) 21 (b) 35 (c) 40 (d) 45
6. In $\triangle ABC$, $\angle A = 100^\circ$, $\angle B = 50^\circ$, AHBC, BM is a median and MH is joined. Then $\angle MHC =$ (NTSE Stage – 1 = 2013)
- (a) 15° (b) 30° (c) 45° (d) 60°
7. In $\triangle ABC$ and $\triangle DEF$, $AC = BC = DF = EF$, length $AB = 2FH$, where $FHDE$. Which of the following statemnets is (are) true?
- I. $\angle ACB$ and $\angle DFE$ are complementary
II. $\angle ACB$ and $\angle DFE$ are supplementary
III. Area of $\triangle ABC = \text{Area of } \triangle DEF$
IV. Area of $\triangle ABC = 1.5x$ (Area of $\triangle DEF$) (NTSE Stage – 2)

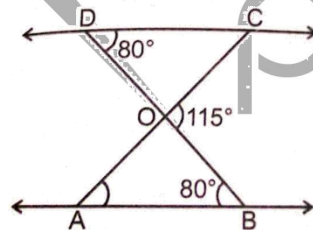
- (a) II only (b) III only (c) I and III only (d) II and III only
8. The ratio of the areas of two similar triangles is equal to: (NTSE Stage – 1 = 2013)
- (a) The ratio of corresponding medians
(b) The ratio of corresponding sides
(c) The ratio of the squares of corresponding sides
(d) None of these
9. In the figure, $\triangle ABC$ is similar to $\triangle EDC$. If we have $AB = 4\text{ cm}$, $ED = 3\text{ cm}$, $CE = 4.2\text{ cm}$ and $CD = 4.8\text{ cm}$, then the values of CA and CB respectively are: (NTSE Stage – 1 = 2013)



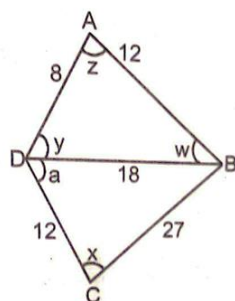
- (a) 6 cm, 6.6 cm (b) 4.8 cm, 6.6 cm (c) 5.4 cm, 6.4 cm (d) 5.6 cm, 6.4 cm
10. In the figure, DB is diagonal of rectangle $ABCD$ and line l through A and line m each of length 1 cm and are perpendicular each of length 1 cm and are perpendicular to DB . Area (in cm^2) of rectangle $ABCD$ is : (NTSE Stage – 1 = 2014)



- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $3\sqrt{2}$ (d) $3\sqrt{3}$
11. In the given figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 115^\circ$ and $\angle CDO = 80^\circ$. Then $\angle OAB$ is equal to – (NTSE Stage – 1 = 2014)

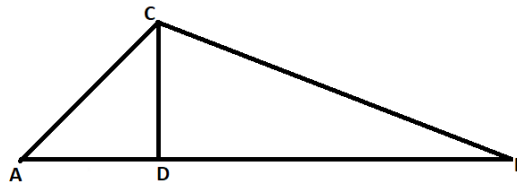


- (a) 80° (b) 35° (c) 45° (d) 65°
12. In the quadrilateral $ABCD$: (NTSE Stage – 1 = 2015)

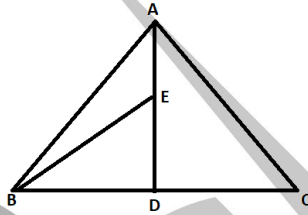


- (a) $x = y, a = z$ (b) $x = z, a = y$ (c) $x = z, a = y$ (d) $x = y, a = w$

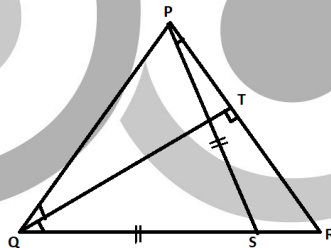
13. In the following figure $\angle ACB = 90^\circ$ and $CD \perp AB$. If $AD = 4\text{ cm}$ and $BD = 9\text{ cm}$ then the ratio $BC : AC$ is : (NTSE Stage – 1 = 2017)



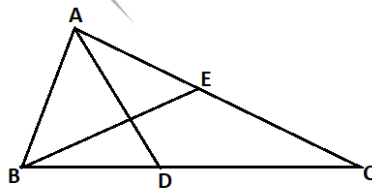
- (a) 3 : 2 (b) 2 : 3 (c) 16 : 81 (d) 81 : 16
14. In the following figure of triangle ABC, E is the midpoint of median AD. The ratio of areas of the triangles ABC and BED is : (NTSE Stage – 1 = 2017)



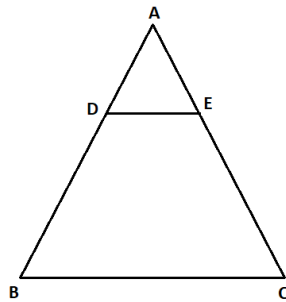
- (a) 1 : 4 (b) 3 : 4 (c) 4 : 1 (d) 4 : 3
15. In the following figure $QT \perp PR$ and $QS = PS$. If $\angle TQR = 40^\circ$ and $\angle RPS = 20^\circ$ the value of x is (NTSE Stage – 1 = 2018)



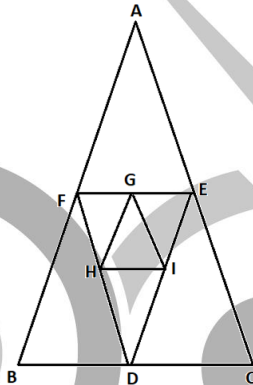
- (a) 80° (b) 25° (c) 15° (d) 35°
16. If ratio of heights of two similar triangles is 4 : 9, then ratio between their areas is (NTSE Stage – 1 = 2018)
- (a) 2 : 3 (b) 3 : 2 (c) 81 : 16 (d) 16 : 81
17. In given $\triangle ABC$, AD and BE are medians of triangle which intersect each other at point G. If area of $\triangle BDG$ is 1 cm^2 , then what is the area of DCEG? (NTSE Stage – 1 = 2018)



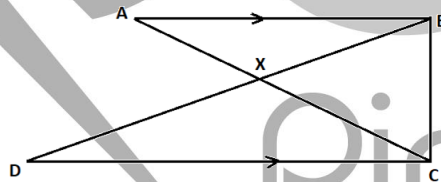
- (a) 2 cm^2 (b) 3 cm^2 (c) 4 cm^2 (d) 1 cm^2
18. If $\triangle ABC$, $m\angle B = 90^\circ$, $AB = BC$. Then $AB : AC =$ _____. (NTSE Stage – 1 = 2018)
- (a) 1 : 3 (b) 1 : 2 (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$
19. If the sides of a triangle are parallel respectively to the sides of another triangle, then the triangles are necessarily –
- (a) Similar (b) Congruent (c) Equal in area (d) None of these
20. Sides AB and AC of $\triangle ABC$ are trisected at D and E such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. $\triangle ADE$ and trapezium DECB have their areas in the ratio of



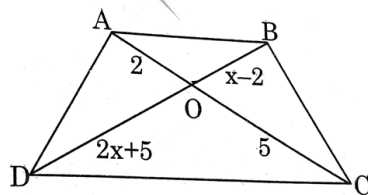
- (a) 1 : 4 (b) 1 : 8 (c) 1 : 9 (d) 1 : 2
21. D, E, F are midpoints of BC, CA and AB respectively. G, H, I are midpoints of FE, FD, DE respectively. Area of triangles DHI and AFE are in the ratio



- (a) 1 : 3 (b) 1 : 4 (c) 1 : 9 (d) 1 : 16
22. In the given diagram, AB is parallel to DC. AC and BD intersect at X. if $AX = 4$ cm, $XC = 6$ cm and $BD = 14$ cm. Find BX.



- (a) 4.5 cm (b) 5.6 cm (c) 8.5 cm (d) 12 cm
23. In a given figure in trapezium ABCD if $AB \parallel CD$ then value of x is –



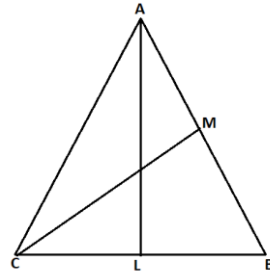
- (a) $\frac{29}{8}$ (b) $\frac{8}{29}$ (c) 20 (d) $\frac{1}{20}$
24. If $\triangle ABC \sim \triangle EDF$ and $\triangle ABC$ is not similar to $\triangle DEF$, then which of the following is not true?
- (a) BC. EF = AC. FD (b) AB. EF = AC. DE
- (c) BC. DE = AB. EF (d) BC. DE = AB. FD
25. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3 DE$. Then, the two triangles are –
- (a) Congruent but not similar
- (b) Similar but not congruent
- (c) Neither congruent nor similar

(d) Congruent as well as similar

26. In a right triangle with sides a and b , and hypotenuse c , the altitude drawn on the hypotenuse is x . Then which one of the following is correct?

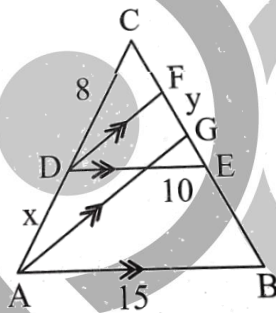
- (a) $Ab = x^2$ (b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$ (c) $a^2 + b^2 = 2x^2$ (d) $\frac{1}{x^2} = \frac{1}{a^2} + \frac{1}{b^2}$

27. In the figure below, AL is perpendicular to BC and CM is perpendicular to AB . If $CL = AL = 2BL$, find MC/BM .



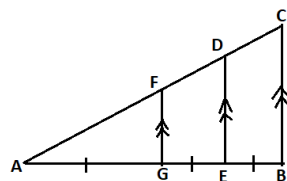
- (a) 2 (b) 3 (c) 4 (d) Cannot be determined

28. In the figure $DF \parallel AG$, $DE \parallel AB$, $AB = 15$, $CD = 8$, $AD = x$, $DE = 10$, $FG = y$ and $CG = 6$. The ratio $x : y$ equal to:



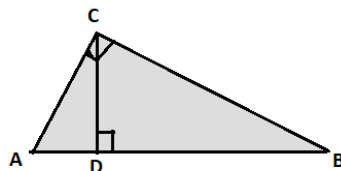
- (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 2

29. In the given figure, ABC is a right angled triangle. Also $GF \parallel ED \parallel BC$ and $AG = GE = EB$. If $DE = 12$ cm, then the measure of BC is :



- (a) 12 cm (b) 18 cm (c) 24 cm (d) 30 cm

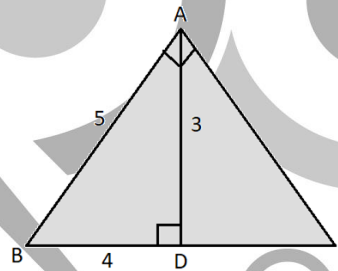
30. In a right angled $\triangle ABC$, $\angle C = 90^\circ$ and CD is perpendicular on hypotenuse AB . If $BC = 15$ cm and $AC = 20$ cm then CD is equal to:



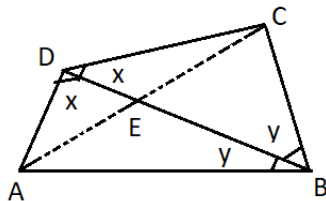
- (a) 18 cm (b) 12 cm (c) 17.5 cm (d) Can't be determined

31. What is the ratio of side and height of an equilateral triangle?

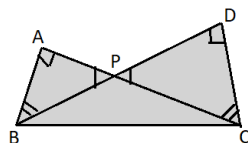
- (a) 2 : 1 (b) 1 : 1 (c) $2 : \sqrt{3}$ (d) $\sqrt{3} : 2$
32. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is:
 (a) Centroid (b) Incentre (c) Circumcentre (d) Orthocentre
33. Incentre of a triangle lies in the interior of:
 (a) An isosceles triangle only (b) A right angled triangle only
 (c) Any equilateral triangle only (d) Any triangle
34. In a triangle PQR, PQ = 20 cm and PR = 6cm, the side QR is:
 (a) Equal to 14 cm (b) Less than 14 cm
 (c) Greater than 14 cm (d) None of these
35. If ABC is a right angled triangle at B and M, N are the mid – points of AB and BC, then $4(AN^2 + CM^2)$ is equal to –
 (a) $4AC^2$ (b) $6AC^2$ (c) $5AC^2$ (d) $\frac{5}{4} AC^2$
36. If $\triangle ABC$ and $\triangle DEF$ are so related that $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$, then which of the following is true?
 (a) $\angle A = \angle F$ and $\angle B = \angle D$ (b) $\angle C = \angle F$ and $\angle A = \angle D$
 (c) $\angle B = \angle F$ and $\angle C = \angle D$ (d) $\angle A = \angle E$ and $\angle B = \angle D$
37. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to:
 (a) $\left(\frac{AB}{AC}\right)^2$ (b) $\left(\frac{AB}{AD}\right)^2$ (c) $\frac{AD}{AC}$ (d) $\frac{AB}{AD}$
38. In the adjoining figure the $\angle BAC$ and $\angle ADB$ are right angles. BA = 5cm, AD = 3 cm and BD = 4 cm, what is the length of DC?



- (a) 2.5 (b) 3 (c) 2.25 (d) 2
39. The diagonal BD of quadrilateral ABCD bisects $\angle B$ and $\angle D$, then:

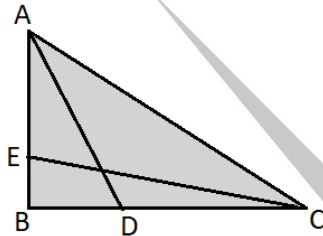


- (a) $\frac{AB}{CD} = \frac{AD}{BC}$ (b) $\frac{AB}{BC} = \frac{AD}{CD}$ (c) $AB = AD \times BC$ (d) None of these
40. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC. If AC and DB intersect at P, then

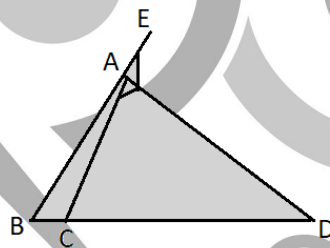


- (a) $\frac{AP}{PC} = \frac{BP}{DP}$ (b) $AP \times DP = PC \times BP$
 (c) $AP \times PC = BP \times DP$ (d) $AP \times BP = PC \times PD$

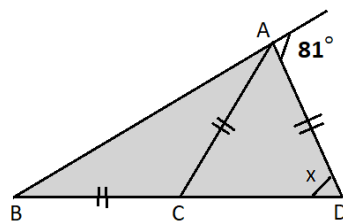
41. A vertical stick 15 m long casts its shadow 10 cm long on the ground. At the same time a flag pole casts a shadow 60 cm long. Find the height of the flag pole.
 (a) 40 cm (b) 45 cm (c) 90 cm (d) None
42. Vertical angles of two isosceles triangles are equal. Then corresponding altitudes are in the ratio 4 : 9. Find the ratio of their areas:
 (a) 16 : 49 (b) 16 : 81 (c) 16 : 65 (d) None
43. In figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



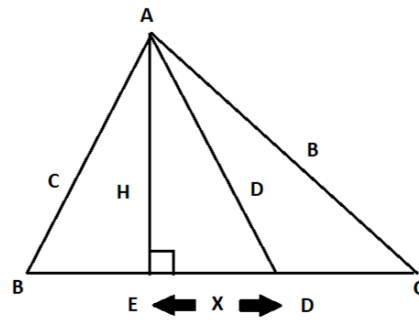
- (a) $2\sqrt{5}$ cm (b) 2.5 cm (c) 5 cm (d) $4\sqrt{2}$ cm
44. Area of $\triangle ABC = 30 \text{ cm}^2$. D and E are the mid-points of BC and AB respectively. Find ar ($\triangle BDE$).
 (a) 10 cm (b) 7.5 cm (c) 15 cm (d) None
45. In the figure, AD is the external bisector of $\angle EAC$, intersects BC produced to D. If $AB = 12$ cm, $AC = 8$ cm and $BC = 4$ cm, find CD.



- (a) 10 cm (b) 6 cm (c) 8 cm (d) 9 cm
46. In $\triangle ABC$, $AB^2 + AC^2 = 2500 \text{ cm}^2$ and median $AD = 25$ cm, find BC.
 (a) 25 cm (b) 40 cm (c) 50 cm (d) 48 cm
47. In the given fig, $BC = AC = AD$, $\angle EAD = 81^\circ$. Find the value of x.



- (a) 45° (b) 54° (c) 63° (d) 36°
48. If in triangle ABC, D is the mid-point of side BC, $\angle ADB = 45^\circ$ and $\angle ACD = 30^\circ$, then $\angle BAD$ and $\angle ABC$ are respectively equal to – (NTSE Stage – 2 = 2016)
 (a) $15^\circ, 105^\circ$ (b) $30^\circ, 105^\circ$ (c) $30^\circ, 100^\circ$ (d) $60^\circ, 100^\circ$
49. In the following figure, $AE \perp BC$, D is the mid point of BC, then x is equal to:



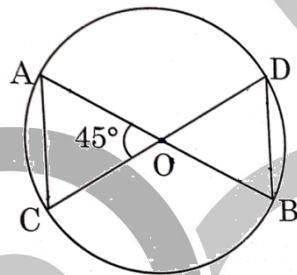
(a) $\frac{1}{a} \left[b^2 - d^2 - \frac{a^2}{4} \right]$

(b) $\frac{h+d}{3}$

(c) $\frac{c+d-h}{2}$

(d) $\frac{a^2+b^2+d^2-c^2}{4}$

50. If in Fig, O is the point of intersection of two chords AB and CD such that $OB = OD$, the triangles OAC and ODB are



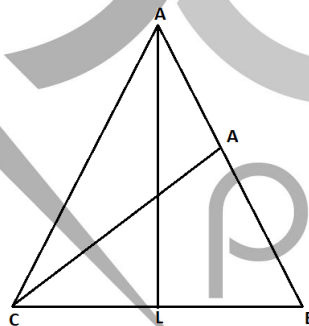
(a) Equilateral but not similar

(b) Isosceles but not similar

(c) Equilateral and similar

(d) Isosceles and similar

51. In the figure below, AL is perpendicular to BC and CM is perpendicular to AB. If $CL = AL = 2BL$, find MC/AM .



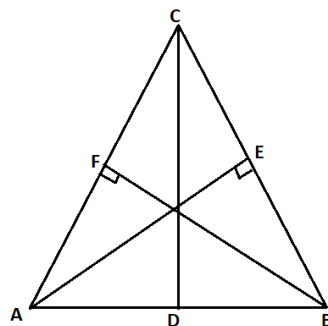
(a) 2

(b) 3

(c) 4

(d) Cannot be determined

52. In the given triangle ABC, CD, BF and AE are the altitudes. If the ratio of $CD : AE : BF = 2 : 3 : 4$, then the ratio of $AB : BC : CA$ is –



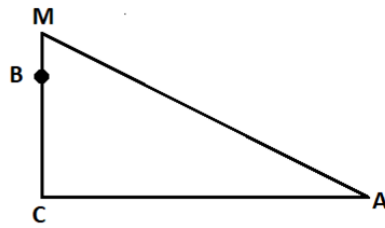
(a) 4 : 3 : 2

(b) 2 : 3 : 4

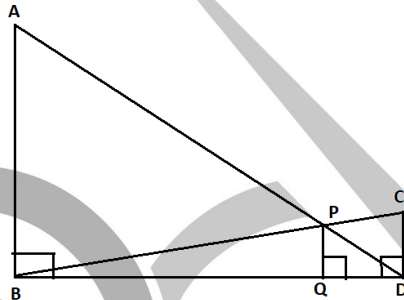
(c) 4 : 9 : 16

(d) 6 : 4 : 3

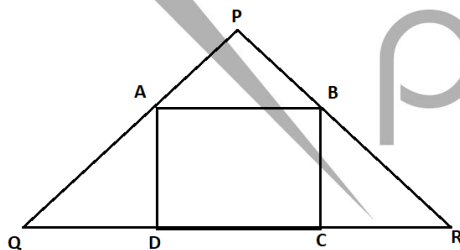
53. In the right triangle shown the sum of the distances BM and MA is equal to the distances BC and CA. If $MB = x$, $CB = h$ and $CA = d$, then x equals.



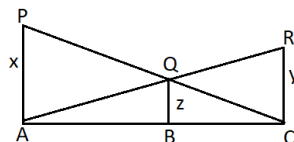
- (a) $\frac{hd}{ah+d}$ (b) $d - h$ (c) $h + d - \sqrt{2d}$ (d) $\sqrt{h^2 + d^2} - h$
54. In the diagram given below, $\angle ADB = \angle CDB = \angle PQD = 90^\circ$. If $AB : CD = 3 : 1$, the ratio of $CD : PQ$ is :



- (a) 1 : 0.69 (b) 1 : 0.75 (c) 1 : 0.72 (d) None of the above
55. A line drawn from vertex A of an equilateral $\triangle ABC$ meets BC at D and the circumcircle at P. Then :
- (a) $PA = PB + PD$ (b) $\frac{1}{PD} = \frac{1}{PB} + \frac{1}{PC}$ (c) $\frac{1}{PA} = \frac{1}{PB} + \frac{1}{PC}$ (d) None of these
56. The area of a right angled triangle is 40 sq. cm and its perimeter is 40 cm, the length of its hypotenuse is :
- (a) 16 cm (b) 18 cm (c) 17 cm (d) Data insufficient
57. A square ABCD is constructed inside a triangle PQR having sides 10 cm, 17 cm and 21 cm as shown in figure. Find the perimeter of the square ABCD.



- (a) 28 cm (b) 23.2 cm (c) 25.4 cm (d) 28.8 cm
58. If AD, BE, CF are the altitudes of $\triangle ABC$ whose orthocentre is H, then C is the orthocentre of:
- (a) $\triangle ABH$ (b) $\triangle BDH$ (c) $\triangle ABD$ (d) $\triangle BEA$
59. O is orthocentre of a triangle PQR, which is formed by joining the mid points of the sides of a $\triangle ABC$, O is:
- (a) Orthocentre (b) Incentre (c) Circumcentre (d) Centroid
60. In the adjoining figure PA, QB and RC are each perpendicular to AC. Which one of the following is true?



- (a) $x + y = z$ (b) $xy = 2z$ (c) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (d) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z} = 0$

ANSWER KEY

EXERCISE – 1

1. $x = 4$

2. $PS = \frac{9}{2} \text{ cm}$

18. $AF = \frac{5}{3} \text{ cm}, CE = \frac{32}{13} \text{ cm}, BD = \frac{40}{9} \text{ cm}$

20. 11 or, 8

21. $DE = 2.8$

22. $DC = 10 \text{ cm}$

40. 1.6 m

41. $DE = \frac{36}{13} \text{ cm}$ and $AE = \frac{15}{13} \text{ cm}$

42. $x = 3.75 \text{ cm}; y = 6.67 \text{ cm}$

64. $\frac{2AB^2 + 2AC^2 - BC^2}{4}$

EXERCISE – 2

2. $2\sqrt{5} \text{ cm}$

15. $AE = \frac{15}{13}, DE = \frac{36}{13}$

21. $\frac{bc}{\sqrt{b^2 + c^2}}$

23. 24 cm^2

25. 13 cm

26. 3 : 4

27. 8 cm

37. 4 : 1

39. 21 cm^2

EXERCISE – 3

Ques.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Ans.	b	c	c	b	c	c	c	a	d	a
Ques.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Ans.	d	a	b	a	b	a	b	c	a	b
Ques.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
Ans.	a	c	a	d	b	b	a	c	b	a
Ques.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
Ans.	b	c	a	a	b	b	c	a	a	a
Ques.	41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
Ans.	c	d	a	d	b	c	d	a	c	a
Ques.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
Ans.	b	c	c	d	a	a	d	b	c	c
Ques.	61.	62.	63.	64.	65.	66.	67.	68.	69.	70.
Ans.	d	b	c	d	a	b	c	b	b	d
Ques.	71.	72.	73.	74.	75.	76.	77.	78.	79.	80.
Ans.	a	c	b	c	b	b	a	c	d	b
Ques.	81.	82.	83.	84.	85.	86.	87.	88.	89.	90.
Ans.	c	a	b	a	c	d	b	c	b	b

EXERCISE – 4

Ques.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Ans.	c	c	b	c	b	b	d	c	d	c
Ques.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Ans.	b	a	a	c	c	d	a	c	a	b
Ques.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
Ans.	b	b	c	d	b	d	a	c	b	b
Ques.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
Ans.	c	b	d	c	c	a	a	c	b	c
Ques.	41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
Ans.	c	b	a	b	c	c	b	b	a	d
Ques.	51.	52.	53.	54.	55.	56.	57.	58.	59.	60.
Ans.	b	d	a	b	b	b	b	a	c	c