∞/∞ Forms of Limits

1. $\lim\limits_{x\to\infty}\dfrac{x^4+7x^3+46x+a}{x^4+6}$, $a\neq 0$

2. $\lim\limits_{x\to\infty}\dfrac{(2+x)(3x-4)}{(4x-5)(5x-6)}$

3. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}-\sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

4. $\lim\limits_{x\to\infty}\dfrac{(2+x)^{40}(4+x)^5}{(2-x)^{45}}$

5. Find $\lim\limits_{n\to\infty}\dfrac{\left(\sqrt{n^2+1}+n\right)^2}{\left(n^6+1\right)^{1/3}}$

6. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

7. $\lim\limits_{x\to\infty}\dfrac{2x^{1/2}+3.x^{1/3}+4.x^{1/4}+.....+n.x^{1/n}}{(3x-4)^{1/2}+(3x-4)^{1/3}+........+(3x-4)^{1/n}}$

8. $\lim\limits_{x\to\infty}\left[\dfrac{x}{x+\dfrac{\sqrt{x}}{x+\dfrac{\sqrt{x}}{x+\sqrt{x}\quad .....\textit{Infinity}}}}\right]$

9. $\lim\limits_{x\to\infty}\left(\dfrac{x}{x+\dfrac{\sqrt[3]{x}}{x+\dfrac{\sqrt[3]{x}}{x+\dfrac{\sqrt[3]{x}}{.....\infty}}}}\right)=$

(A) 1 (B) 1/2 (C) 2 (D) N.O.T

10. Find $\lim\limits_{n\to\infty}\dfrac{(n+2)!+(n+1)!}{(n+3)!}$

11. If f(x) is continuous in [0, 1] and $f(1/3)=1$, Find $\lim\limits_{n\to\infty}f\left(\dfrac{n}{\sqrt{9n^2+1}}\right)$.

12. Find $\lim\limits_{x\to\infty}\left(\dfrac{1}{x^2}+\dfrac{2}{x^2}+....+\dfrac{x}{x^2}\right)$

13. $\underset{n\to\infty}{\text{Lt}}\dfrac{1}{n^4}(1.4.7+2.5.8+3.6.9+..........\text{to n terms})$

14. $\lim_{n\to\infty} \dfrac{a^n+b^n}{a^n-b^n}$ (a > b)

15. $\lim_{x\to\infty} \left(x-\sqrt{x^2+x}\right)$.

16. $\underset{x\to\infty}{Lim} \left(\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}\right)$

17. $\lim_{x\to\infty} \left(\sqrt{x+\sqrt{x}}-\sqrt{x}\right)$

18. $\lim_{x\to-\infty} \left(3x+\sqrt{9x^2-x}\right)$

19. $\lim_{x\to\infty} \sqrt[4]{(x+a)(x+b)(x+c)(x+d)}-x$, where a,b,c,d are real numbers ,is equal to

(A) $\dfrac{a+b+c+d}{4}$ (B) $\dfrac{abcd}{4}$ (C) $\sqrt{abcd}$ (D) N.O.T

20. Find the constants a and b so that $\lim_{x\to\infty}\left(\dfrac{x^2+1}{x+1}-ax-b\right)=0$

21. If $f(x)=\dfrac{ax^2+b}{x^2+1}$, $\lim_{x\to 0} f(x)=1$ and $\lim_{x\to\infty} f(x)=1$ then prove that $f(-2)=f(2)=1$

22. Find a, b, c and d if $\lim_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right)=4$

23. $\lim_{n\to\infty}\left(\log_{(n-1)} n.\log_n(n+1).\log_{(n+1)}(n+2)........\log_{n^k}\left(n^k+1\right)\right)=$

(A) 1 (B) k (C) 2k (D) DNE

24. Let $P_n=\dfrac{2^3-1}{2^3+1}.\dfrac{3^3-1}{3^3+1}.\dfrac{4^3-1}{4^3+1}.......\dfrac{n^3-1}{n^3+1}$. Prove that $\lim_{n\to\infty} P_n=\dfrac{2}{3}$

25. Let $S_n=\dfrac{1}{1.4}+\dfrac{1}{4.7}+\dfrac{1}{7.10}+........$ to n terms then $\lim_{n\to\infty} S_n=$

(A) 1/3 (B) 3 (C) 1/4 (D) ∞

26. Let the variable x_n be determined by the following low of formation

$x_0=\sqrt{a}$, $x_1=\sqrt{a+\sqrt{a}}$, $x_2=\sqrt{a+\sqrt{a+\sqrt{a}}}$, find $\lim_{n\to\infty} x_n$.

27. If $f_1(x)=\dfrac{x}{2}+10$, $\forall x\in R$ and define $f_n(x)=f(f_{n-1}(x))$, $\forall n\ge 2$. Find $\lim_{n\to\infty} f_n(x)$.

28. If $f(x+y)=f(x).f(y)$ $\forall x, y\in R$; $f(x)>0$ & f(x) is continuous and if $f(1)=1/2$.

Prove that $\lim_{n\to\infty} f(x)+2f(x+1)+3f(x+2)+.....+(n+1)f(x+n))=4f(x)$

Passage 1:

A Square is inscribed in a circle of radius R, acircle is inscribed in the square ,a new square in the circle and so on for n times.

Choose the correct answer:

29. Sum of areas of all the circles is :

(A) $4\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$ (B) $2\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$

(C) $3\pi R^2\left(1-\left(\frac{1}{3}\right)^n\right)$ (D) $\pi R^2\left(1-\left(\frac{1}{2}\right)^n\right)$

30. The limit of sum of areas of all the squares as $n \to \infty$ is :

(A) $2R^2$ (B) $3R^2$ (C) $4R^2$ (D) $8R^2$

31. The limit of sum of areas of all the circles as $n \to \infty$ is :

(A) $2\pi R^2$ (B) $3\pi R^2$ (C) $4\pi R^2$ (D) $8\pi R^2$

<u>Answer key</u>

1. 1 **2.** $\frac{3}{20}$ **3.** 0 **4.** –1 **5.** 4 **6.** d.n.e **7.** $\frac{2}{\sqrt{3}}$

8. 1 **9.a** **10. 0** **11.** 1 **12.** $\frac{1}{2}$ **13.** $\frac{1}{4}$ **14.** 1

15. $-\frac{1}{2}$ **16.** $\frac{1}{2}$ **17.** $\frac{1}{2}$ **18.** $\frac{1}{6}$ **19.a** **20.** 2 **21. 1**

22. (a) a = 1, b = –1, (b) a = 2, c = 5, b, d $\in$R **23.b** **25.a** **26.** $\frac{1+\sqrt{1+4a}}{2}$

29.b **30.c** **31.a**