Q1 - 24 January - Shift 1

If A and B are two non-zero $n \times n$ matrics such that $A^2 + B = A^2 B$, then

(1) $AB = I$

(2) $A^2B = I$

(3) $A^2 = I$ or $B = I$

(4) $A^2B = BA^2$

Space for your notes:

Q2 - 24 January - Shift 2

The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

(3) 150

(4) 125

Space for your notes:

Q3 - 24 January - Shift 2

Let A be a 3×3 matrix such that $\left|\operatorname{adj}\left(\operatorname{adj}(\operatorname{adj} A)\right)\right| = 12^4$. Then $|A^{-1} \operatorname{adj} A|$ is equal to

(1) $2\sqrt{3}$

(2) $\sqrt{6}$

(3) 12

(4) 1

Space for your notes:

Q4 - 25 January - Shift 1

Let $x, y, z > 1$ and

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$$

Then $\left|\operatorname{adj}(\operatorname{adj} A^2)\right|$ is equal to

(1) 6^4
(2) 2^8
(3) 4^8
(4) 2^4

Space for your notes:

Q5 - 25 January - Shift 2

Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^T B A$, then the inverse of the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$
(2) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$
(3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$
(4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Space for your notes:

Q6 - 25 January - Shift 2

Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S1) $A^{13}B^{26}-B^{26}A^{13}$ is symmetric
(S2) $A^{26}C^{13}-C^{13}A^{26}$ is symmetric

Then,

(1) Only S2 is true
(2) Only S1 is true
(3) Both S1 and S2 are false
(4) Both S1 and S2 are true

Space for your notes:

Q7 - 29 January - Shift 1

Let α and β be real numbers. Consider a 3 × 3 matrix A such that $A^2=3A+\alpha I$. If $A^4=21A+\beta I$, then

(1) $\alpha=1$ (2) $\alpha=4$

(3) $\beta=8$ (4) $\beta=-8$

Space for your notes:

Q8 - 29 January - Shift 2

The set of all values of $t\in\mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^t & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is

invertible, is

(1) $\left\{(2k+1)\frac{\pi}{2},k\in\mathbb{Z}\right\}$ (2) $\left\{k\pi+\frac{\pi}{4},k\in\mathbb{Z}\right\}$

(3) $\{k\pi,k\in\mathbb{Z}\}$ (4) $\mathbb{R}$

Space for your notes:

Q9 - 29 January - Shift 2

Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to ______.

Space for your notes:

Q10 - 30 January - Shift 1

Let $A = \begin{pmatrix} & \\ p & q \end{pmatrix}, d = |A| \neq 0$ $|A - d(\operatorname{Adj} A)| = 0$

. Then

(1) $(1+d)^2 = (m+q)^2$

(2) $1+d^2 = (m+q)^2$

(3) $(1+d)^2 = m^2 + q^2$

(4) $1+d^2 = m^2 + q^2$

Space for your notes:

Q11 - 30 January - Shift 2

If P is a 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then

(1) P is a singular matrix

(2) $|\operatorname{Adj} P| > 1$

(3) $|\operatorname{Adj} P| = \frac{1}{2}$

(4) $|\operatorname{Adj} P| = 1$

Space for your notes:

Q12 - 31 January - Shift 1

Let $A=\begin{pmatrix} 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to:

(1) 6144 (2) 4094

(3) 4097 (4) 2050

Space for your notes:

Q13 - 31 January - Shift 2

Let $A=[a_{ij}], a_{ij} \in Z \cap [0,4], 1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2,13)$ is ____________.

Space for your notes:

Q14 - 31 January - Shift 2

Let A be a $n \times n$ matrix such that $|A|=2$. If the determinant of the matrix Adj (2. Adj$(2A^{-1})$). is 2^{84}, then n is equal to _______________.

Space for your notes:

Q15 - 01 February - Shift 2

If $A=\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

(1) $A^{30} - A^{25} = 2I$

(2) $A^{30} + A^{25} + A = I$

(3) $A^{30} + A^{25} - A = I$

(4) $A^{30} = A^{25}$

Space for your notes:

Answer Key

(As per Official NTA Key released on 2 Feb)

Q1 (4)	**Q2** (2)	**Q3** (1)	**Q4** (2)
Q5 (4)	**Q6** (1)	**Q7** (4)	**Q8** (4)
Q9 (5)	**Q10** (1)	**Q11** (4)	**Q12** (3)
Q13 (196)	**Q14** (5)	**Q15** (3)	

Q1 (4)

$$A^2 + B = A^2 B$$

$$\left(A^2 - I\right)\left(B - I\right) = I \ldots\ldots(1)$$

$$A^2 + B = A^2 B$$

$$A^2\left(B - I\right) = B$$

$$A^2 = B\left(B - I\right)^{-1}$$

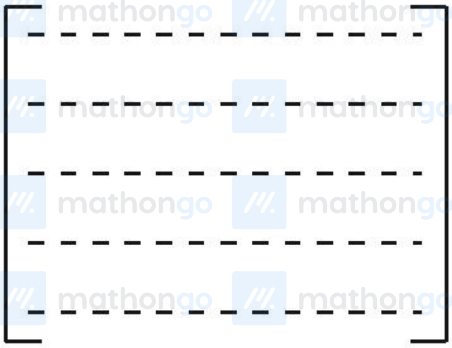
$$A^2 = B\left(A^2 - I\right)$$

$$A^2 = BA^2 - B$$

$$A^2 + B = BA^2$$

$$A^2 B = BA^2$$

Q2 (2)



In each row and each column exactly one is to be placed –

$\therefore$ No. of such materials $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{array}{l} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \to 5 \text{ ways} \\ \to 4 \text{ ways} \\ \to 3 \text{ ways} \\ \to 2 \text{ ways} \\ \to 1 \text{ ways} \end{array}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$3\sqrt{3}$

Q3 (1)

Given $|\text{adj}(\text{adj}(\text{adj}.A))| = 12^4$

$\Rightarrow |A|^{(n-1)^3} = 12^4$

Given n = 3

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A|^2 = 12$

$|A| = 2\sqrt{3}$

We are asked

$|A^{-1}.\text{adj}\,A|$

$= |A^{-1}|.|\text{adj}\,A|$

$= \dfrac{1}{|A|}.|A|^{3-1}$

$= |A| = 2\sqrt{3}$

Q4 (2)

$$|A| = \frac{1}{\log x.\log y.\log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = 2$$

$\Rightarrow |\text{adj}(\text{adj}A^2)| = |A^2|^4 = 2^8$

Q5 (4)

$$AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

.

.

.

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$$M = A^TBA$$

$$M^2 = M.M = A^TBA\ A^TBA = A^TB^2A$$

$$M^3 = M^2.M = A^TB^2AA^TBA = A^TB^3A$$

.

.

.

$$M^{2023} = \ldots\ldots\ldots\ldots\ldots\ A^TB^{2023}A$$

$$AM^{2023}A^T = \underbrace{AA^T}\ B^{2023}\ \underbrace{AA^T} = B^{2023}$$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Q6 (1)

Given, $A^T = A, B^T = -B, C^T = -C$

Let $M = A^{13} B^{26} - B^{26} A^{13}$

Then, $M^T = (A^{13} B^{26} - B^{26} A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13} B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26} C^{13})^T - (C^{13} A^{26})^T$

$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

Q7 (4)

$$A^2 = 3A + \alpha I$$

$$A^3 = 3A^2 + \alpha A$$

$$A^3 = 3(3A + \alpha I) + \alpha A$$

$$A^3 = 9A + \alpha A + 3\alpha I$$

$$A^4 = (9 + \alpha)A^2 + 3\alpha A$$

$$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$$

$$= A(27 + 6\alpha) + \alpha(9 + \alpha)$$

$$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$$

$$\Rightarrow \beta = \alpha(9 + \alpha) = -8$$

Q8 (4)

If its invertible, then determinant value $\neq 0$

So ,

$$\begin{vmatrix} e^{t} & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^{t} & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t) \\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^{t}\cdot e^{-t}\cdot e^{-t}\begin{vmatrix} 1 & \sin t-2\cos t & -2\sin t-\cos t \\ 1 & 2\sin t+\cos t & \sin t-2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying, $R_1 \rightarrow R_1 - R_2$ then $R_2 \rightarrow R_2 - R_3$

We get

$$e^{-t}\begin{vmatrix} 0 & -\sin t-\cos t & -3\sin t+\cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$$e^{-t}\times 1\left(2\sin t\cos t+6\cos^2 t+6\sin^2 t-2\sin t\cos t\right)\neq 0$$

$$\Rightarrow \; e^{-t}\times 6 \neq 0$$

for $\forall\ t \in \mathbb{R}$

Q9 (5)

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}=\begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

Now $ac-b^2=2$ and $2a+b=1$

and $2b+c=2$

solving all these above equations we get

$$\frac{1-b}{2}\times\left(\frac{2-2b}{1}\right)-b^2=2$$

$$\Rightarrow (1-b)^2-b^2=2$$

$$\Rightarrow 1-2b=2$$

$$\Rightarrow b=-\frac{1}{2} \text{ and } a=\frac{3}{4} \text{ and } c=3$$

Hence $\alpha=3a+\frac{3b}{2}=\frac{9}{4}-\frac{3}{4}=\frac{3}{2}$

and $\beta=3b+\frac{3c}{2}=-\frac{3}{2}+\frac{9}{2}=3$

also $s=a+c=\frac{15}{4}$

$$\therefore \frac{\beta s}{\alpha^2}=\frac{3\times 15}{4\times\frac{9}{4}}=5$$

Q10 (1)

$$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \qquad |A - d(\text{adj}\,A)| = 0$$

$$\Rightarrow \quad |A - d(\text{adj}\,A)| = \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right|$$

$$= \begin{vmatrix} m - qd & n(1+d) \\ p(1+d) & q - md \end{vmatrix} = 0$$

$\Rightarrow \quad (m - qd)(q - md) - np(1 + d)^2 = 0$

$\Rightarrow \quad mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 = 0$

$(mq - np) + d^2(mq - np) - d(m^2 + q^2 + 2np) = 0$

$\Rightarrow \quad d + d^3 - d((m + q)^2 - 2d) = 0$

$\Rightarrow \quad 1 + d^2 = (m + q)^2 - 2d$

$\Rightarrow \quad (1 + d)^2 = (m + q)^2$

$\therefore$ Option (1) is correct.

Q11 (4)

$$P^{T} = aP + (a-1)I$$

$$\Rightarrow P = aP^T + (a-1)I$$

$$\Rightarrow P^T - P = a(P - P^T)$$

$$\Rightarrow P = P^T, \text{ as } a \neq -1$$

$$\text{Now, } P = aP + (a-1)I$$

$$\Rightarrow P = -I \Rightarrow |P| = 1$$

$$\Rightarrow |\text{Adj } P| = 1$$

Q12 (3)

$$A^2=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=A$$

$\Rightarrow A^3=A^4=\ldots\ldots=A$

$(A+I)^{11}={}^{11}C_0A^{11}+{}^{11}C_1A^{10}+\ldots..{}^{11}C_{10}A+{}^{11}C_{11}I$

$=\left({}^{11}C_0+{}^{11}C_1+\ldots..{}^{11}C_{10}\right)A+I$

$=\left(2^{11}-1\right)A+I=2047A+I$

$\therefore$ Sum of diagonal elements $=2047(1+4-3)+3$

$=4094+3=4097$

Q13 (196)

As given $a+b+c+d=3$ or 5 or 7 or 11

if sum = 3

$(1+x+x^2+.....+x^4)^4 \rightarrow x^3$

$(1-x^5)^4(1-x)^{-4} \rightarrow x^3$

$\therefore {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1-4x^5)(1-x)^{-4} \rightarrow x^5$

$\Rightarrow {}^{4+5-1}C_5 - 4x\,{}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1-4x^5)(1-x)^{-4} \rightarrow x^7$

$\Rightarrow {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 11

$(1-4x^5+6x^{10})(1-x)^{-4} \rightarrow x^{11}$

$\Rightarrow {}^{4+11-1}C_{11} - 4.{}^{4+6-4}C_6 + 6.{}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4.{}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

Q14 (5)

$$\left|\operatorname{Adj}\left(2\operatorname{Adj}(2A^{-1})\right)\right|$$

$$=\left|2\operatorname{Adj}\left(\operatorname{Adj}(2A^{-1})\right)\right|^{n-1}$$

$$=2^{n(n-1)}\left|\operatorname{Adj}(2A^{-1})\right|^{n-1}$$

$$=2^{n(n-1)}\left|(2A^{-1})\right|^{(n-1)(n-1)}$$

$$=2^{n(n-1)}2^{n(n-1)(n-1)}\left|A^{-1}\right|^{(n-1)(n-1)}$$

$$=2^{n(n-1)+n(n-1)(n-1)}\frac{1}{|A|^{(n-1)^2}}$$

$$=\frac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}}$$

$$=2^{n(n-1)+n(n+1)^2-(n-1)^2}$$

$$=2^{(n-1)(n^2-n+1)}$$

Now, $2^{(n-1)(n^2-n+1)}$

$2^{(n-1)(n^2-n+1)}=2^{84}$

So, $n=5$

Q15 (3)

$$A=\frac{1}{2}\begin{bmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{bmatrix}$$

$$A=\begin{bmatrix}\cos 60^\circ & \sin 60^\circ\\ -\sin 60^\circ & \cos 60^\circ\end{bmatrix}$$

$$\text{If } A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix} \text{ Here } \alpha=\frac{\pi}{3}$$

$$A^2=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$$

$$A^{30}=\begin{bmatrix}\cos 30\alpha & \sin 30\alpha\\ -\sin 30\alpha & \cos 30\alpha\end{bmatrix}$$

$$A^{30}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$$A^{25}=\begin{bmatrix}\cos 25\alpha & \sin 25\alpha\\ -\sin 25\alpha & \cos 25\alpha\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$$

$$A^{25}=A$$

$$A^{25}-A=0$$