

Sandwich Theorem

1. $\lim_{n \rightarrow \infty} \frac{[1^2 x] + [2^2 x] + [3^2 x] + \dots + [n^2 x]}{n^3}$, where $[.]$ denotes greatest integer function.
2. Evaluate : $\lim_{x \rightarrow \infty} \frac{x^2 + x + 1}{[x^2]} = \dots\dots\dots$ (where $[x]$ represents greatest integer function)
3. $\lim_{x \rightarrow \infty} \frac{\log x - [x]}{[x]}$
4. $\lim_{n \rightarrow \infty} \frac{[x] + [2x] + \dots + [nx]}{n^2}$
5. $\lim_{x \rightarrow 0} \left(\lim_{n \rightarrow \infty} \frac{[1^2 |\sin x|^x] + [2^2 |\sin x|^x] + \dots + [n^2 |\sin x|^x]}{n^3} \right)$,
6. If $f(x)$ is differentiable $\forall x > 0$ and $f(x) > 0$ which satisfies

$$f(x) = \lim_{n \rightarrow \infty} \frac{[1^2 (f(x))^x] + [2^2 (f(x))^x] + \dots + [n^2 (f(x))^x]}{n^3}$$
, where $[.]$ denotes greatest integer function,
 find $f'(x)$.
7. Evaluate $\lim_{n \rightarrow \infty} \frac{a^n}{n!}$, $a \in R^+$.
8. Solve $\lim_{n \rightarrow \infty} x_n$, when $x_n^2 = a + x_{n-1}$ and $x_0 = \sqrt{a}$.
9. If $a_n = 1$ and $a_{n+1} = \frac{4+3a_n}{3+2a_n}$, $n \geq 1$ then show $a_{n+2} > a_{n+1}$ and if a_n has a limit l as $n \rightarrow \infty$, then evaluate $\lim_{n \rightarrow \infty} a_n$.

ANSWER KEY

- 2.1 3. has value -1 4. $\frac{x}{2}$ 7.0 9. $\sqrt{2}$