Assignment

Subject: Mathematics

Topic: Continuity

1. Determine the values of a, b, c for which the function $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & \text{for } x<0\\ C & \text{for } x=0\\ \dfrac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} & \text{for } x>0\end{cases}$

is continuous at $x=0$.

2. If $f(x)=\begin{cases}\dfrac{1-\cos x}{x^2} & ,x<0\\ a & ,x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ,x>0\end{cases}$ then correct statement is :

(A)f(x) is discontinuous at x=0 for any value of a
(B)f(x) is continuous at x=0 for a=8
(C)f(x) is continuous at x=0 for a=0
(D)N.O.T

3. Discuss the continuity of the function $f(x)=\begin{cases}\dfrac{a^{|x|+x}-1}{[x]+x}, & x\neq 0\\ \log_e a, & x=0\end{cases}$ at $x=0, a>0$.

4. Discuss the continuity of the function $f(x)\begin{cases}\dfrac{a^{2[x]+\{x\}}-1}{2[x]+\{x\}}, & x\neq 0\\ \log_e a, & x=0\end{cases}$ at $x=0$

where [.] denotes greatest integral part and { . } denotes fractional part of x.

5. If $f(x)=\begin{cases}a+\dfrac{\sin[x]}{x}, & x>0\\ 2, & x=0\\ b+\left[\dfrac{\sin x-x}{x^3}\right], & x<0\end{cases}$

(whre [.] denotes the greatest integer function). If $f(x)$ is continuous at $x=0$, then find b

6. Give a real valued function f such that $f(x)=\begin{cases}\dfrac{\tan^2 x}{(x^2-[x])^2} & \text{for } x>0\\ 1 & \text{for } x=0\\ \sqrt{\{x\}\cot\{x\}} & \text{for } x<0\end{cases}$

where, [x] is the integral part and {x} is the fractional part of x, then

(a) $\lim\limits_{x\to 0} f(x)=1$
(b) $\lim\limits_{x\to 0^-} f(x)=\cot 1$

(c) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$ (d) f is continuous at x = 0

7. If $f(x)=\begin{cases}\dfrac{\sin\{\cos x\}}{x-\pi/2}, & x\neq \dfrac{\pi}{2}\\ 1 & , x=\dfrac{\pi}{2}\end{cases}$, where {.} denotes the fractional part of x hence f(x) is :

(a) continous at $x=\dfrac{\pi}{2}$ (b) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists, but f(x) is not continuous $x=\dfrac{\pi}{2}$

(c) $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ does not exists (d) $\lim\limits_{x\to\frac{\pi}{2}} f(x)=1$

8. If $f(x) = \{x^2\} - (\{x\})^2$, where {x} denotes the fractional part of x, then discuss continuity at x = 2, –2

9. If $f(x)=\begin{cases}\dfrac{\sin[x]}{[x]+1} & , x>0\\ \dfrac{\cos\dfrac{\pi}{2[x]}}{[x]} & , x<0\\ k & , x=0\end{cases}$ is continuous at x = 0,where [ ] denotes the greatest integer function,then

k equal to________

10.. Discuss the continuiuty of $f(x)=\left\{x+\left(x-[x]\right)^2\right\}$ at x = 2& x = 2.5 ,where { } stands for frational part of x and [ ] is greatest integer function.

11. The function f(x) = arc tan $\dfrac{1}{x-5}$ has :

(a) discontinuity of the first kind at x = 5 (b) discontinuity of the second kind at x = 5
(c) removable discontinuity at x = 5 (d) continuous at x = 5

12. Consider $f(x)=\begin{cases} x[x]^2\ \log_{(1+x)} 2 & \text{for } -1<x<0\\ \dfrac{ln\left(e^{x^2}+2\sqrt{\{x\}}\right)}{\tan\sqrt{x}} & \text{for } 0<x<1\end{cases}$

where [*] & {*} are the greatest integer function & fractional part function respectively, then
(A) f(0) = ln2 ⇒ f is continuous at x = 0 (B) f(0) = 2 ⇒ f is continuous at x = 0
(C) f(0) = e^2 ⇒ f is continuous at x = 0 (D) f has an irremovable discontinuity at x = 0

13. If $f(x)=\begin{cases}\dfrac{\sin 3x}{\tan x} & , x<0\\ \dfrac{3}{2} & , x=0\\ \dfrac{\log(1+3x)}{e^{2x}-1} & , x>0\end{cases}$ then f(x) is

(A)continuous at x = 0 (B)discontinuous at x = 0
(C)limit does not exists at x = 0 (D)N.O.T

14. Let $f(x)$ be a polynomial of degree one and $f(x)$ be a function defined by $f(x)=\begin{cases} g(x) & ,x\le 0 \\ \dfrac{1+x}{(2+x)^{1/x}}, & x>0 \end{cases}$

If f(x) is continuous at x = 0 and $f(-1)=f'(1)$, then g(x) is equal to :

(a) $-\frac{1}{9}(1+6\log_e 3)x$
(b) $\frac{1}{9}(1+6\log_e 3)x$
(c) $-\frac{1}{9}(1-6\log_e 3)x$
(d) none of these

15. If $(x)=\begin{cases}[x]+[-x], x\neq 2 \\ \lambda, \quad x=2,\end{cases}$ then f is continuous at x = 2, provided λ is equal to :

(a) 1 (b) 0 (c) –1 (d) 2

16. $f(x)=\begin{cases}\dfrac{e^{e/x}-e^{-e/x}}{e^{1/x}+e^{-1/x}} & , x\neq 0 \\ k & , x=1\end{cases}$

(A) f is continuous at x, when k = 0
(B) f is not continous at x = 0 for any real k.
(C) $\lim\limits_{x\to 0} f(x)$ exist infinitely.
(D) None of these

17. If $f(x)=\left(\tan\left(\frac{\pi}{4}+\ell nx\right)\right)^{\log_x e}$ is to be made continuous at x = 1, then f(1) should be equal to

(a) e^2 (b) e (c) 1/e (d) e^{-2}

18. Value of 'a' such that $f(x)=\begin{cases}\dfrac{\left(e^x-1\right)^3.\operatorname{cosec}(ax)}{\ell n\left(1+x^2\right)} & x\neq 0 \\ b, & x=0\end{cases}$ is continuous at x = 0, is equal to

(a) b (b) 1/b (c) –b (d) –1/b

19. Value of f(0) so that $f(x)=\frac{1}{x}\ell n\left(\frac{1+ax}{1-bx}\right)$ can be made continuous at x = 0, is equal to

(a) $a-b$ (b) $a+b$ (c) $\ell n\left(\frac{a}{b}\right)$ (d) $\ell n(ab)$

20. Value of f(0) so that $f(x)=\frac{1}{x^2}(1-\cos(\sin x))$ can be made continuous at x = 0, is equal to

(a) 1/2 (b) 2 (c) 8 (d) 4

21. If $f(x)=\dfrac{(a+x)^2.\sin(a+x)-a^2.\sin a}{x}$ $x\neq 0$,Then the value of f(0) so that f is continuous at x = 0 is

(a) $a^2\cos a+a\sin a$
(b) $a^2\cos a+2a\sin a$
(c) $2a^2\cos a+a\sin a$
(d) N.O.T

22. 'f' is a continuous function on the real line. Given that

$x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

(A) can not be determined (B) is $2(1 - \sqrt{3})$ (C) is zero (D) is $\frac{2(\sqrt{3} - 2)}{\sqrt{3}}$

23. If $f(x) = \left(\frac{\sin x}{\sin \alpha}\right)^{\frac{1}{(x-\alpha)}}$ is continuous at $\alpha \neq m\pi(m \in I)$ then,

(A) $f(\alpha) = e^{\tan \alpha}$ (B) $f(\alpha) = e^{\cot \alpha}$ (C) $f(\alpha) = e^{2\cot \alpha}$ (D) $f(\alpha) = \cot \alpha$

24. The set of points where $f(x) = \sec 2x + \operatorname{cosec} 2x$ is discontinuous :

(A) $\{n\pi : n = 0, \pm 1, \pm 2, ...\}$ (B) $\left\{\frac{n\pi}{2} : n = 0, \pm 1, \pm 2, ...\right\}$

(C) $\left\{\frac{(2n+1)\pi}{4} : n = 0, \pm 1, \pm 2, ...\right\}$ (D) $\left\{\frac{n\pi}{4} : n = 0, \pm 1, \pm 2, ...\right\}$

25. If $f(x) = \sqrt{\sin^{-1} x} + \sqrt{\cos^{-1} x}$ is defined in [0,1] then:

(A) f(x-3) is continuous in the interval [3,4] (B) f(x-3) is continuous in the interval [0,1]
(C) f(x-3) is continuous in the interval [0,1) (D) f(x-3) is continuous in the interval (0,1)

26. The set of points where $f(x) = \frac{\tan x . \ln(x-2)}{x^2 - 4x + 3}$ is discontinuous is :

(A) $(-\infty, 2) \cup \{3\}$ (B) $(-\infty, 2] \cup \left\{3, n\pi + \frac{\pi}{2}, n \in N\right\}$

(C) $(-\infty, 2)$ (D)N.O.T

27. At origin the function $f(x) = |x| + \frac{|x|}{x}$ is:

(A) continuous (B) discontinuous because $\frac{|x|}{x}$ is discontinuous there

(C) discontinuous because | x | is discontinuous there

(D) discontinuous because | x | & $\frac{|x|}{x}$ both are discontinuous there

28. Which of the following function have finite number of points of discontinuity ?

(a) tan x (b) x [x] (c) $\frac{|x|}{x}$ (d) $\sin[n\pi x]$

29. The function $f(x) = \frac{1}{\ell n |x|}$ has non removable discontinuity at x = ______ & removable discontinuity at x = ______ respectively.

(a) x = 0 & x = 1, – 1 (b) x = 2, – 1 & x = 0 (c) x = 1, – 1 & x = 0 (d) None of these

30. $f(x) = [\tan^{-1} x]$, where [.] denotes the greatest integer function, is discontinuous at

(a) $x = \frac{\pi}{4}, -\frac{\pi}{4}$ and 0 (b) $x = \frac{\pi}{3}, -\frac{\pi}{3}$ and 0

(c) $x = \tan 1, -\tan 1$ and 0 (d) None of these

31. Let $f(x) = [x^3 - 3]$, where [.] denotes the greatest integer function. then the number of points in the interval (1, 2) where the function is discontinuous, is

(a) 4 (b) 2 (c) 6 (d) none of these

32. $f(x) = [\sin x]$, where [.] denotes the greatest integer function, is continuous at

(a) $x = \frac{\pi}{2}$ (b) $x = \pi$ (c) $x = \frac{3\pi}{2}$ (d) $x = 2\pi$

33. Let $f(x) = [\sin x + \cos x]$, $0 < x < 2\pi$, (where [.] denotes the greatest integer function) Then the number of point of discontinuity of $f(x)$ is :

(a) 6 (b) 5 (c) 4 (d) 3

34. $f(x) = 1 + x (\sin x) [\cos x]$, $0 < x < \pi/2$ is discontinuous at ....([.] denotes the greatest integer function)

35. Points of discontinuity of $f(x) = [x] \sin \frac{\pi}{[x+1]}$, where [.] denotes the greatest integer function, are

(a) $x \in 1 \sim \{-1, 0\}$ (b) $x \in 1 \sim \{0\}$ (c) $x \in 1 \sim \{-1\}$ (d) None of these

36. Prove that $f(x) = [\tan x] + \sqrt{\tan x - [\tan x]}$. (where [.] denotes greatest integer function) is continuous in $\left[0, \frac{\pi}{2}\right)$.

37. Function $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$ is discontinuous at

(a) every x (b) no x (c) Every intgral point (d) Every non integral point

38. If $f(x) = \left[\frac{x}{2} - 1\right]$, then in the interval $[0, \pi]$:

(A) tan[f(x)] is discontinuous but $\frac{1}{f(x)}$ is continuous

(B) tan[f(x)] is continuous but $\frac{1}{f(x)}$ is discontinuous.

(C) tan[f(x)] & $f^{-1}(x)$ both are continuous

(D) tan[f(x)] & $\frac{1}{f(x)}$ both are discontinuous.

39. Which of the following functions are continuous for all x.

f(x)= | x -1| + | sinx | + cosx , g(x)= $\frac{|x|}{|x|+1}$,

h(x)= $e^{|x|} + |\sin x| + \frac{1}{\sin x + 2}$, k(x)= $|x||x-1| + \frac{x^2}{x^2+1}$

(A) only f & g (B) only g & h (C) only f & h (D) all f , g , h , k

40. Function $f(x) = [x.\sin \pi x]$ is continuous at x = :

(A) 0 (B) 1 (C) –1 (D) (–1, 1)

41. Points of discontinuity of $f(x)=\left[\frac{6x}{\pi}\right].\cos\left[\frac{3x}{\pi}\right]$,where [ ] denotes the greatest integer function

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2},\pi$ (D) N.O.T

42. Which one of the following is not continuous function ?

(A) cos[x] (B) $\sin\pi[x]$ (C) $\sin\frac{\pi}{2}[x]$ (D) All of the above

43. **Matrix match :**

Column-I	Column-II
(A) A function $f(x)=\sin x+\{x\}$ is	(P) Continuous for all x
(B) A function $g(x)=\sin x+\frac{x^2}{1+x^2}$ is	(Q) discontinuous at infinite number of points
(C) A function $h(x)=\|x\|\|x-1\|+\frac{\sin x}{2+\sin x}$ is	(R) discontinuous at finite number of points
(D) A function $k(x)=\sin[x]+\frac{\cos x}{3+\sin x}$ is	(S) discontinuous at all the integres

44. **Matrix match :**

Column-I	Column-II
(A) A function $f(x)=x[x],1\le x\le 4$ is	(P) Continuous for all x
(B) A function $g(x)=\frac{x}{x^2-1}+\|x\|$ is	(Q) discontinuous at infinite number of points
(C) A function $h(x)=\{\sin x\}+x.\sin x$ is	(R) discontinuous at finite number of points
(D) A function $k(x)=\ln[x]+\frac{\sin x}{\|x\|+1}$ is	(S) discontinuous at all the integres

45. Function $f(x)=a[x+1]+b[x+1]$,(where [ ] denotes the greatest integer function) is continuous at x = 1 ,if (a + b) is equal to_________.

46. If f(x) is continuous for $x\in R$ and range of $f(x)=(2,\sqrt{26})$ & $g(x)=\left[\frac{f(x)}{a}\right]$ is continuous $\forall x\in R$, where [x] denotes the greatest intefer function .then least positive integral value of a is equal to______.

47. If f(x) = sgn (cos 2 x – 2 sin x + 3) , where sgn ( ) is the signum function, then f(x)

(A) is continuous over its domain (B) has a missing point discontinuity

(C) has isolated point discontinuity (D) has irremovable discontinuity.

48. Let $g(x)=\tan^{-1}|x|-\cot^{-1}|x|$, $f(x)=\frac{[x]}{[x+1]}\{x\}$, $h(x)=|g(f(x))|$ where {x} denotes fractional part and [x] denotes the integral part then which of the following holds good ?

(A) h is continuous at x = 0 (B) h is discontinuous at x = 0

(C) $h(0^-)=\frac{\pi}{2}$ (D) $h(0^+)=-\frac{\pi}{2}$

49. The set of points where $f(x)=\dfrac{\tan x.\log x}{1-\cos 4x}$ is discontinuous:

(A) $\left\{\dfrac{n\pi}{2}:n\in Z\right\}$ (B) $\left\{\dfrac{n\pi}{2}:n\in Q\right\}$ (C) $(-\infty,0]\cup\left\{\dfrac{n\pi}{2}:n\in N\right\}$ (D) N.O.T

50. Function $f(x)=[x]^2-[x^2]$,where [ ]denotes the greatest integer function is discontinuous at

(A) all integers (B) all integers except 0&1

(C) all integers except 0 (D) all integers except 1

51. Let $f(x) = x - |x - x^2|, -1 \le x \le 1$.

Discuss the continuity of $f(x)$ in the closed interval $[-1,1]$. Draw the graph of $f(x)$ in the interval.

52. Let $f(x) = \sec^{-1}([1 + \sin^2 x])$; is discontinuous at ....(where [.] denotes greatest integral function.)

53. **Statement-1** : f(x) = sin x + [x] is discontinuous at x = 0.

Statement-2 : If g(x) is continous & h(x) is discontinuous at x = a, then g(x) + h(x) will necessarily be discontinuous at x = a.

54. Let $f(x)=\begin{cases}\dfrac{2x+1}{x-2}, & x\in[0,1]\\ \dfrac{\sqrt{1+ax}-\sqrt{1-ax}}{x}, & x\in[-1,0)\end{cases}$ if f(x) is continuous in [-1, 1], then

(a) $a=-1$ (b) $a=-\dfrac{1}{2}$ (c) $a=-2$ (d) None of these

55. If $f(x)=\begin{cases}|x^2-1|-1 & ,x\le 1\\ |2x-3|-|x-2| & ,x>1\end{cases}$, then f(x) is continous for

(a) R (b) $R-\{0\}$ (c) $R-\{1\}$ (d) None of these

56. Discuss the continuity of f(x) in [0, 2]

$f(x)=\begin{cases}[\cos\pi x], & x\le 1\\ |2x-3|[x-2], & x>1\end{cases}$ where [t] represents the greatest integer function.

57. If $f(x)=\begin{cases}-2\sin x & ,x\le-\dfrac{\pi}{2}\\ a\sin x+b & ,-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\\ \cos x & ,\ x\ge\dfrac{\pi}{2}\end{cases}$ is continuous function ,then a+b is equal to_____.

58. If $f(x)=\begin{cases}1+\sin\dfrac{\pi x}{2} & ,-\infty<x\le 1\\ ax+b & ,1<x<3\\ 6\tan\dfrac{\pi x}{12} & ,3\le x<6\end{cases}$ is continuous in $(-\infty,6)$,then values of a + b is equal to_____.

Comprehension # 1

There are two system S_1 and S_2 of definition of limit and contiuity. In system S1 the defintion are as usual. In System S_2 the defintion of limit is as usual but the continuity is defined as follows :

(i) $\left|\lim\limits_{x\to a^-} f(x) - \lim\limits_{x\to a^+} f(x)\right| \le 1$

(ii) f(a) lies between the values of $\lim\limits_{x\to a^-}$ and $\lim\limits_{x\to a^+}$ f(x) if $\lim\limits_{x\to a^-}$ f(x) $\neq$ $\lim\limits_{x\to a^+}$ f(x)

else f(a) = $\lim\limits_{x\to a^-}$ f(x) = $\lim\limits_{x\to a^+}$ f(x). Read the above passage carefully and answer the following

59. If f(x) = $\begin{cases} x+2.7 & , x<0 \\ 2.9 & , x=0 \\ 2x+3 & , x>0 \end{cases}$ amd g(x) = $\begin{cases} 3x+3 & , x<0 \\ 2.8 & , x=0 \\ -x^2+2.7 & , x>0 \end{cases}$ then consider statements

(i) f(x) is discontinuous under the system S_1
(ii) f(x) is continuous under the system S_2
(iii) g(x) is condinuous under the system S_2
which of the following option is correct
(A) only (i) is true
(B) only (i) and (ii) are true
(C) only (ii) and (iii) are true
(D) all (i), (ii), (iii) are true

60. If each of f(x) and g(x) is continuous at x = a in S_2, then in S_2 which of the following is continuoous
(A) f + g
(B) f – g
(C) f.g
(D) None of these

61. Which of the following is correct
(A) A continuous function under the definition in S_1 must also be continuous under the definition in S_2
(B) A continuous function under the definition in S_2 must be continuous under the definition in S_1
(C) A discontinuous function under the definition in S_1 must also be discondinuous under the definition in S_2
(D) A discontinuous function under the definition in S_1 must be continuous under the definition in S_2.

62. Which of the following function contains single point discontinuity :

(A) $f(x) = \begin{cases} 1 & , x \in Q \\ 0 & , x \notin Q \end{cases}$
(B) $g(x) = \begin{cases} x & , x \in Q \\ 1-x & , x \notin Q \end{cases}$
(C) $h(x) = \begin{cases} x & , x \in Q \\ 0 & , x \notin Q \end{cases}$
(D) $k(x) = \begin{cases} x & , x \in Q \\ -x & , x \notin Q \end{cases}$

63. If $f(x) = \begin{cases} x^a . \sin\dfrac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at x = 0 then

(A) a > 0
(B) a < 0
(C) a = 0
(D) $a \ge 0$

64. If $f(x) = \begin{cases} -1, & x<0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$ and $g(x) = x(1-x^2)$, then ; *f*(g(x)) is continuous for ,

(a) R
(b) R – {0}
(c) R – {0, 1}
(d) R – {–1, 0, 1}

65. If $f(x)=\begin{cases}-1, x<0\\ 0,\ x=0\\ 1,\ x>0\end{cases}$ and $g(x)=x(1-x^2)$, then; $g(f(x))$ is continuous form

(a) R (b) $R-\{0\}$ (c) $R-\{0, 1\}$ (d) $R-\{-1, 0, 1\}$

66. If $f(x)=\begin{cases}1-x & ,0\le x\le 1\\ x+2 & ,1<x\le 2\\ 4-x & ,2<x\le 4\end{cases}$,then check the points of discontinuity of *fof*.

67. If $f(x)=\begin{cases}2x^2+12x+16 & ,-4\le x\le -2\\ 2-|x| & ,-2<x\le 1\\ 4x-x^2-2 & ,1<x\le 3\end{cases}$, then check the continuity of | f(x) | & f(| x |).

68. Let $f(x)=\begin{cases}x+2, & -4\le x\le 0\\ 2-x^2, & 0<x\le 4\end{cases}$

then find $f\left(f(x)\right)$, domain of $f\left(f(x)\right)$ and also comment upon the continuity of $f\left(f(x)\right)$.

69. Let $f(x)=\begin{cases}1+x^3 & , x<0\\ x^2-1 & , x\ge 0\end{cases}$; $g(x)=\begin{cases}(x-1)^{1/3} & , x<0\\ (x+1)^{1/2} & , x\ge 0\end{cases}$. Discuss the continuity of g (f(x)).

70.. If $f(x)=\dfrac{x-1}{x+1}, f^{1}(x)=f(x), f^2(x)=fof(x), f^{k+1}(x)=f(f^k(x))$ where k =1,2,3,.........If

$g(x)=f^{1999}(x)$ then check the continuity of g(x).

71. Consider $f(x)=\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\{x\}}$ $x\ne 0$

$g(x)=\cos 2x$ $-\dfrac{\pi}{4}<x<0$

$h(x)=\begin{cases}\dfrac{1}{\sqrt{2}}f(g(x)) & \text{for} x<0\\ 1 & \text{for} x=0\\ f(x) & \text{for} x>0\end{cases}$ then, which of the following holds good.

where {x} denotes fractional part function.

(A) 'h' is continuous at x = 0 (B) 'h' is discontinuous at x = 0

(C) f(g(x)) is an even function (D) f(x) is an even function

72. Let $f(x)=x^3+x$ be function and $g(x)=\begin{cases}f(|x|), & x\ge 0\\ f(-|x|), & x<0\end{cases}$, then prove that g(x) is continuous $\forall\ x\in R$

73. If α, β $(\alpha < \beta)$ are the points of discontinuity of the function $f(f(f(x)))$, where $f(x) = \dfrac{1}{1-x}$, then the set of values of 'a' for which the points (α, β) and (a, a^2) lie on the same side of the line $x + 2y - 3 = 0$, is :

(a) $\left(-\dfrac{3}{2}, 1\right)$ (b) $\left[-\dfrac{3}{2}, 1\right]$ (c) $[1, \infty]$ (d) $\left(-\infty, -\dfrac{3}{2}\right]$

74. Given $f(x) = \begin{cases} \dfrac{1-\sin\left(\frac{\pi x}{2}\right)+\ell n\left(\sin\left(\frac{\pi x}{2}\right)\right)}{\sqrt{\left(2\sin\frac{\pi x}{2}-1\right)+\cos^2\left(\frac{\pi x}{2}\right)}-1} & \text{for } x>1 \\ \frac{1}{3}g(h(x)) & \text{for } x \le 1 \end{cases}$; where $h(x) = \sin^{-1}(\operatorname{sgn}(x))$ &

$g(x) = |x| + \{-x\} + [x]$, where $\{x\}$ is the fractional part of x, $[x]$ is the integral part of x & sgn (x) is the signum of (x). Discuss the continuity of f in $(-\infty, 2)$.

75. If $f(x) = \lim\limits_{n \to 0} \dfrac{(1+\cos \pi x)^n + 1}{(1+\cos \pi x)^n - 1}$, then

(a) $f(1+0) = 1$ (b) $f(1-0) = 2$
(c) f(x) is continuous at x = 1 (d) f(x) is not continuous at x = 1

76. Discuss the continuity of $f(x)$ where $f(x) = \lim\limits_{n \to \infty} \left(\sin \dfrac{\pi x}{2}\right)^{2n}$

77. Examine the continuity of $f(x) = \lim\limits_{n \to \infty} \dfrac{x}{(2\sin x)^{2n} + 1}$ for $x \in R$.

78. If $f(x)$ and g(x) are two functions continuous everywhere and $F(x) = \lim\limits_{n \to \infty} \dfrac{f(x) + x^{2n}.g(x)}{1 + x^{2n}}$ then prove that F(x) is continuous everywhere except at x = 1, –1. Find the condition on $f(x)$, g(x) which makes F(x) continuous everywhere.

79. If $f(x) = \begin{cases} x - [x], & x \notin I \\ 1, & x \in I \end{cases}$, where I is integer and [.] represents the greatest integer function and,

$g(x) = \lim\limits_{n \to \infty} \dfrac{\{f(x)\}^{2n} - 1}{\{f(x)\}^{2n} + 1}$, then

(a) Draw graphs of f (2x), g(x) and g {g(x)} and discuss their continuity.
(b) Find the domain and range of these functions.
(c) Are these function periodic ? If yes, find their periods.

80. Let $g(x) = \lim\limits_{n \to \infty} \dfrac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}, x \ne 1$ and $g(1) = \lim\limits_{n \to 1} \dfrac{\sin^2(\pi.2^x)}{\ln\left((\sec(\pi.2^x)\right)}$ be a continuous function at x = 1, find the value of 4g (1) + 2 f (1) - h(1). Assume that f (x) and h (x) are continuous at x =1.

81. Tick the correct alternatives for the following.

(I). If $f(x)=\dfrac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$ for $x\neq\dfrac{\pi}{4}$, then the value which can be given to f(x) at $x=\dfrac{\pi}{4}$ so that the function becomes continuous every where in $(0,\pi/2)$ is 1/4.

(II) The function f, defined by $f(x)=\dfrac{1}{1+2^{\tan x}}$ is continuous for real x.

(III) $f(x)=\underset{n\to\infty}{\text{Limit}}\ \dfrac{1}{1+n\sin^2\pi x}$ is continuous at x = 1.

(IV) The function $f(x)=\begin{cases}2x+1 & \text{if } -3<x<-2\\ x-1 & \text{if } -2\le x<0\\ x+2 & \text{if } 0\le x<1\end{cases}$ is continuous everywhere in (–3 , 1).

(A) T T T T (B) T F T F (C) T T F F (D) F F F F

82. The function defined as $f(x)=\underset{n\to\infty}{\text{Limit}}\ \dfrac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$

(A) is discontinuous at x = 1 because $f(1^+)\neq f(1^-)$
(B) is discontinuous at x = 1 because f(1) is not defined
(C) is discontinuous at x = 1 because $f(1^+)=f(1^-)\neq f(1)$
(D) is continuous at x = 1

83. Discuss the continuity of the function $f(x)=\underset{n\to\infty}{\text{Lt}}\ \dfrac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$ at $x=1$

84.c The set of points of discontinuity of the function $f(x)=\lim\limits_{n\to\infty}\dfrac{(2\sin x)^{2n}}{3^n-(2\cos x)^{2n}}$ is given by

(a) R (b) $\left\{n\pi\mp\dfrac{\pi}{3},\ n\in I\right\}$ (c) $\left\{n\pi\pm\dfrac{\pi}{6},\ n\in I\right\}$ (d) none of these

85. Let $y=f(x)$ be defined parametrically as $y=t^2+t|t|$, $x=2t-|t|$, $t\in R$, find f(x) and discuss continuiuty. at x = 0.

86. Discuss the continuity for ; $f(x)=\dfrac{1-u^2}{2+u^2}$ where $u=\tan x$

87. If $y=\dfrac{1}{t^2-t-6}$ and $t=\dfrac{1}{x-2}$, then the values of x which make the function y discontinuous, are

(a) $2,\dfrac{2}{3},\dfrac{7}{3}$ (b) $2,\dfrac{3}{2},\dfrac{7}{3}$ (c) $2,\dfrac{3}{2},\dfrac{3}{7}$ (d) none of these

88. If $f(x)=\min(\tan x,\cot x)$, then

(a) $f(x)$ is discontinuous at $x=0,\ \dfrac{\pi}{4},\dfrac{5\pi}{4}$ (b) $f(x)$ is continuous at $x=0,\ \dfrac{\pi}{2},\dfrac{3\pi}{2}$

(c) $\int_0^{\pi/2}f(x)\,dx=2\ln\sqrt{2}$ (d) $f(x)$ is periodic with period π

89. Let $f(x) = 1 + 4x - x^2, \forall\ x \in R$

$$g(x) = \begin{cases} \max.\ \{f(t);\ x \le t \le (x+1);\ 0 \le x < 3\} \\ \min.\ \{(x+3);\ 3 \le x \le 5\} \end{cases}$$

Verify continuity of g(x) for all $x \in [0,5]$

90. Let $f(x) = \{\min(x, y)\}^{\{\max(y,x)\}}$ where $y = \sqrt{1-x^2}$ discuss the continuiuty of f(x) in[0,1].

91. If $f(x) = \begin{cases} \left(\frac{1}{2^n}(1+\cos x)\left(1+\cos\frac{x}{2}\right)\left(1+\cos\frac{x}{4}\right).......\left(1+\cos\frac{x}{2^{n+1}}\right)\right)^{1/2}, & x \ne 0 \\ & x = 0 \end{cases}$

Discuss the continuity of f(x) at x = 0,

92. Given $f(x) = \sum_{r=1}^{n} \tan\left(\frac{x}{2^r}\right)\sec\left(\frac{x}{2^{r-1}}\right); r, n \in N$

$$g(x) \underset{n\to\infty}{\text{Limit}} \frac{\ln\left(f(x)+\tan\frac{x}{2^n}\right) - \left(f(x)+\tan\frac{x}{2^n}\right)^n\left[\sin\left(\tan\frac{x}{2}\right)\right]}{1+\left(f(x)+\tan\frac{x}{2^n}\right)^n} \quad = k \text{ for } x = \frac{\pi}{4}$$

and the domain of $g(x)$ is $(0, \pi/2)$ .where [ ] denotes the greatest integer function.

Find the value of k, if possible, so that g(x) is continuous at $x = \pi/4$. Also state the points of discontinuity of $g(x)$ in $(0, \pi/4)$, if any.

93. Let f be a function satisfying $f(x+y) = f(x).f(y) - \sqrt{4 - f(y)}$ and $f(h) \to 4$ as $h \to 0$ .discuss the continuity of f(x) if f(x) is continuous at x = 0.

94. Let $f(x+y) = f(x) + f(y) \forall x, y \in R$ .If f(x) is continuous at x = 0 then show that f(x) is also continuous other than zero.

95. If $f(x)$ be a continuous function in $[0, 2\pi]$ and $f(0) = f(2\pi)$ then prove that there exists point $c \in (0, \pi)$ such that $f(c) = f(c + \pi)$.

96. If f(x) is continuous in [0 , 1] & f(x) = 1 for all rational numbers in [0 , 1] then $f\left(\frac{1}{\sqrt{2}}\right)$ = ______.

(A) 0 **(B)** 1 **(C)** 2 **(D)**-1

97. Tick the correct alternatives for the following.

(I) The function defined by $f(x) = \frac{x}{|x| + 2x^2}$ for $x \ne 0$ & f(0) = 1 is continuous at x = 0.

(II) The function $f(x) = 2^{-2^{1/(1-x)}}$ if $x \ne 1$ & f(1) = 1 is not continuous at x = 1.

(III) The function $f(x) = 2x\sqrt{(x^3-1)} + 5\sqrt{x}\ \sqrt{(1-x^4)} + 7x^2\sqrt{(x-1)} + 3x + 2$ is continuous at x = 1.

(IV) There exists a continuous function f: [0, 1] $\longrightarrow$ [0, 10], but there exists no continuous function g : [0, 1] $\longrightarrow$ (0, 10).

(A) T T T T (B) T F T F (C) F T F T (D) F F F F

98. Let ‘f’ be a continuous function on R. If f $(1/4^n) = \left(\sin e^n\right)e^{-n^2} + \dfrac{n^2}{n^2+1}$ then f(0) is :

(A) not unique
(B) 1
(C) data sufficient to find f(0)
(D) data insufficient to find f(0)

99. If g: [a, b] onto [a, b] is continuous show that there is some c $\in$ [a, b] such that g (c) = c.

100. Which of the following functions are continuous in $(0, \pi)$:

(A) tanx

(B) $\displaystyle\int_0^x t.\sin\frac{1}{t}\,dt$

(C) $\begin{cases} 1 & ,0 < x \le \dfrac{3\pi}{4} \\ 2\sin\dfrac{2}{9}x & ,\dfrac{3\pi}{4} < x < \pi \end{cases}$

(D) $\begin{cases} x.\sin x & ,0 < x \le \dfrac{\pi}{2} \\ \dfrac{\pi}{2}\sin(\pi + x) & ,\dfrac{\pi}{2} < x < \pi \end{cases}$

101. Discuss the continuity of $f(x) = \displaystyle\int_{-1}^{1} \frac{\sin x}{1 - 2t\cos x + t^2}\,dx$

102. Draw the graph of $f(x) = (-1)^{[x]}$,where [] denotes the greatest integer function.

103. Let (x) denote the positive or negative excess of x over the nearest integer, and when x exceeds an integer by $\frac{1}{2}\cdot$ let (x) = 0. What do you say about the continuity of (x) ? Draw the graph.

Answer Key

2.a	**3.** Disc	**4.** Disc	**5.** a +1	**6.c**	**7.c**	
8. continuous at x = 2 but not at x = –2			**9.** 0	**11.a**	**12.d**	**13.bc**
15.c	**16.b**	**17.a**	**18.b**	**19.b**	**20.a**	**21.b**
22.b	**23.b**	**24.d**	**25.a**	**26.b**	**27.b**	**28.c**
29.c	**30.c**	**31.c**	**32.c**	**33.b**	**34.** no point	**35.c**
37.b	**38.d**	**39.d**	**40.ad**	**41.abc**	**42.ac**	**45.** 0
46. 6	**47.c**	**48.**a	**49.c**	**50.d**	**52.** $\left\{(2n-1)\frac{\pi}{2}, n \in I\right\}$	
53.a	**54.b**	**55.c**	**56.** 0, 1/2, 2	**57.** 0	**58.** 2	**59.d**
60.d	**61.b**	**62.bcd**	**63.a**	**64.c**	**65.c**	**71.a**
73.a	**75.c**	**76.** f(x) discontinuous at $x = (2n+1),\ n \in I$				**80.** 5
81.d	**82.d**	**83.** Discontinous		**84.c**		

85. $f(x) = \begin{cases} 2x^2 & , \quad x \ge 0 \\ 0 & \quad x < 0 \end{cases}$ f(x) is continuous at x = 0 **87.b** **88.cd**

92. $k = 0; g(x) = \begin{bmatrix} \ln(\tan x) & \text{if } 0 < x < \dfrac{\pi}{4} \\ 0 & \text{if } \dfrac{\pi}{4} \le x < \dfrac{\pi}{2} \end{bmatrix}$ .Hence g(x) is continuous everywhere. **96.b**

97.c **98.b** **100.bc**