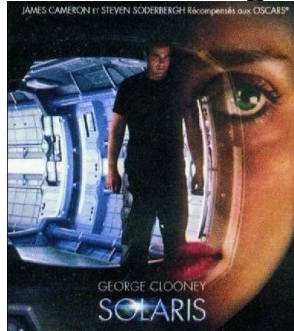


# Artificial Intelligence



## Lesson 6

(From Russell & Norvig)

69

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# Games- Outline

- Optimal decisions
- $\alpha$ - $\beta$  pruning
- Imperfect, real-time decisions

70

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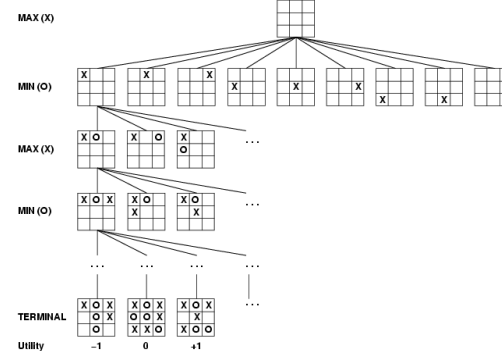
# Games vs. search problems

- "Unpredictable" opponent  $\rightarrow$  specifying a move for every possible opponent reply
- Time limits  $\rightarrow$  unlikely to find goal, must approximate

71

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# Game tree (2-player, deterministic, turns)

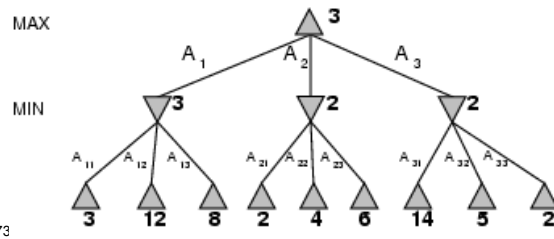


72

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## Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest **minimax value**  
= best achievable payoff against best play
- E.g., 2-ply game:



## Minimax algorithm

**function** MINIMAX-DECISION(*state*) **returns** an action

$v \leftarrow \text{MAX-VALUE}(\text{state})$   
**return** the action in **SUCCESSORS**(*state*) with value *v*

**function** MAX-VALUE(*state*) **returns** a utility value

**if** **TERMINAL-TEST**(*state*) **then** **return** **UTILITY**(*state*)  
 $v \leftarrow -\infty$   
**for** *a, s* in **SUCCESSORS**(*state*) **do**  
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$   
**return** *v*

**function** MIN-VALUE(*state*) **returns** a utility value

**if** **TERMINAL-TEST**(*state*) **then** **return** **UTILITY**(*state*)  
 $v \leftarrow \infty$   
**for** *a, s* in **SUCCESSORS**(*state*) **do**  
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s))$   
**return** *v*

74

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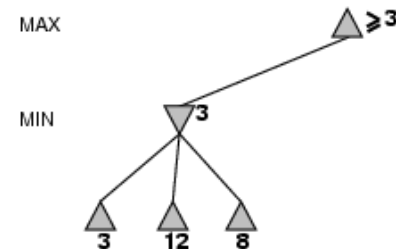
## Properties of minimax

- Complete? (=will not run forever) Yes (if tree is finite)
- Optimal? (=will find the optimal response) Yes (against an optimal opponent)
- Time complexity?  $O(b^m)$
- Space complexity?  $O(bm)$  (depth-first exploration)
- For chess,  $b \approx 35$ ,  $m \approx 100$  for "reasonable" games  
→ exact solution completely infeasible

75

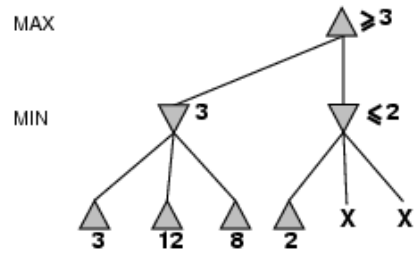
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## $\alpha$ - $\beta$ pruning example



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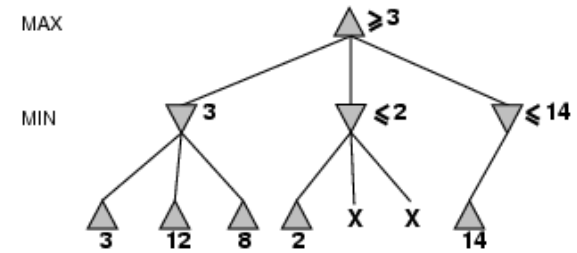
## $\alpha$ - $\beta$ pruning example



77

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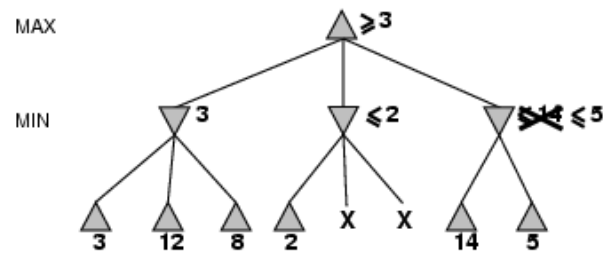
## $\alpha$ - $\beta$ pruning example



78

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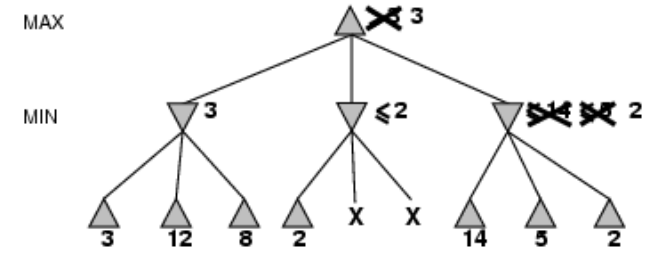
## $\alpha$ - $\beta$ pruning example



79

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## $\alpha$ - $\beta$ pruning example



80

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## Properties of $\alpha$ - $\beta$

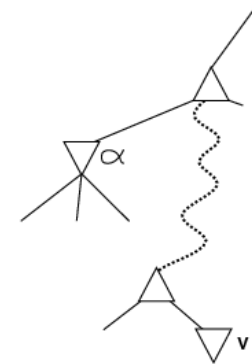
- Pruning **does not** affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering" on binary tree, time complexity =  $O(b^{m/2})$   
 → **doubles** depth of search
- A simple example of the value of reasoning about which computations are relevant (a form of **metareasoning**)

81

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## Why is it called $\alpha$ - $\beta$ ?

- $\alpha$  is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If  $v$  is worse than  $\alpha$ , *max* will avoid it  
 → prune that branch
- Define  $\beta$  similarly for *min*



82

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## The $\alpha$ - $\beta$ algorithm

```

function ALPHA-BETA-SEARCH(state) returns an action
  inputs: state, current state in game
   $v \leftarrow \text{MAX-VALUE}(\text{state}, -\infty, +\infty)$ 
  return the action in SUCCESSORS(state) with value  $v$ 

function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  inputs: state, current state in game
          $\alpha$ , the value of the best alternative for MAX along the path to state
          $\beta$ , the value of the best alternative for MIN along the path to state
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow -\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$ 
    if  $v \geq \beta$  then return  $v$ 
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return  $v$ 
    
```

83

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## The $\alpha$ - $\beta$ algorithm

```

function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  inputs: state, current state in game
          $\alpha$ , the value of the best alternative for MAX along the path to state
          $\beta$ , the value of the best alternative for MIN along the path to state
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow +\infty$ 
  for  $a, s$  in SUCCESSORS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$ 
    if  $v \leq \alpha$  then return  $v$ 
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return  $v$ 
    
```

84

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## Resource limits

Suppose we have 100 secs, explore  $10^4$  nodes/sec  
→  $10^6$  nodes per move

Standard approach:

- **cutoff test:**  
e.g., depth limit (perhaps add **quiescence search**)
- **evaluation function**  
= estimated desirability of position

85

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## Evaluation functions

- For chess, typically **linear** weighted sum of **features**

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

- e.g.,  $w_1 = 9$  with  
 $f_1(s) = (\text{number of white queens}) - (\text{number of black queens}), \text{ etc.}$

86

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## Cutting off search

*MinimaxCutoff* is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

$$b^m = 10^6, b=35 \rightarrow m=4$$

4-ply lookahead is a hopeless chess player!

- 4-ply  $\approx$  human novice
- 8-ply  $\approx$  typical PC, human master
- 12-ply  $\approx$  Deep Blue, Kasparov

87

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## Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.
- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.
- Othello: human champions refuse to compete against computers, who are too good.
- Go: human champions refuse to compete against computers, who are too bad. In go,  $b > 300$ , so most programs use pattern knowledge bases to suggest plausible moves.

88

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## Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about