DIFFERENTIAL EVOLUTION

Artificial Intelligence Course

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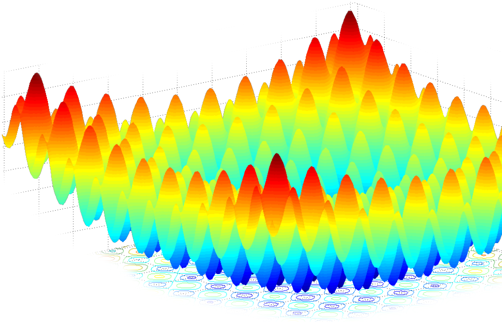

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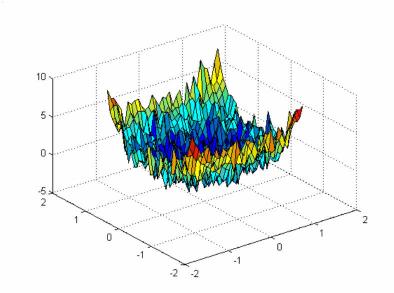
OVERVIEW

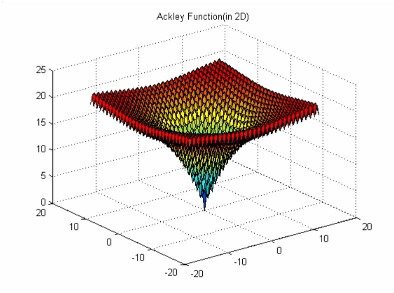
1. Introduction
2. DE Mechanism
3. Conclusion

INTRODUCTION

WHAT'S DIFFERENTIAL EVOLUTION (DE)

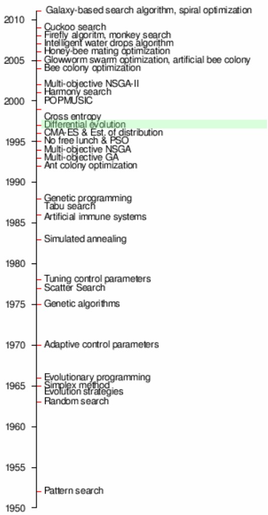
- Metaheuristic and evolutionary algorithm
- Population-based stochastic function minimizer
- Optimization of multi-dimensional real valued functions
- Suitable for noisy, not continuous and changing over time functions





WHO AND WHEN HAS CREATED DE?

- K. Price to solve Chebychev Polynomial fitting Problem posed by R. Storn 1994
- "Simple and Efficient Adaptive Scheme for Global Optimization Over Continuous Spaces" by Storn and Price on 1996



Adapted from Harvard's CCSB Club

IN THE CONTINUE OF THE DE HISTORY..

- Finished 3rd at the first international contest on evolutionary computation (1stICEO)
- Outperformed GA and PSO on a 34-function test suite (Vesterstrom & Thomsen 2004)
- Continually exhibited remarkable performance in competitions on different kinds of optimization problems like dynamic, multi-objective, constrained, and multi-modal problems held under IEEE CEC.

INSPIRATION

- In family of evolutionary algorithms (like genetic algorithm)

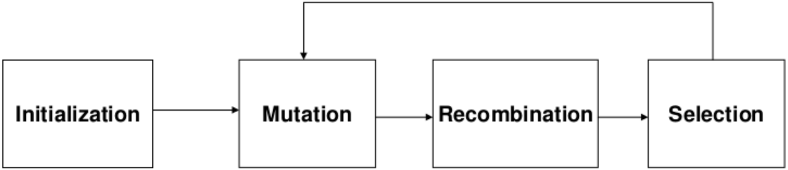
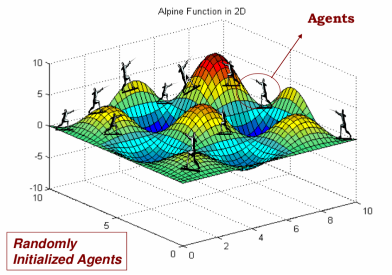


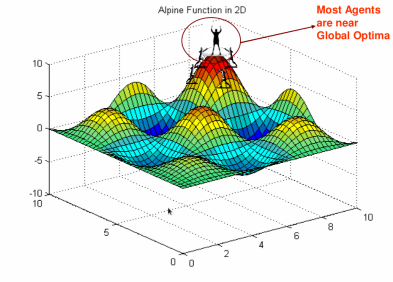
Figure: A basic evolutionary algorithm steps

INSPIRATION

- Search for a missing person in jungle (in my opinion)



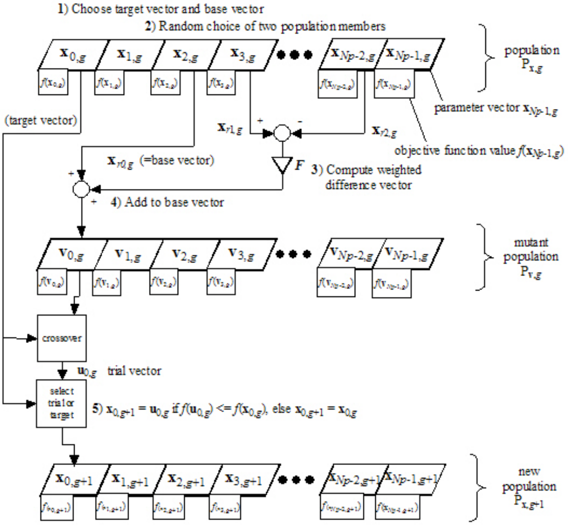
multi-agent optimization in continues space



converged optimization

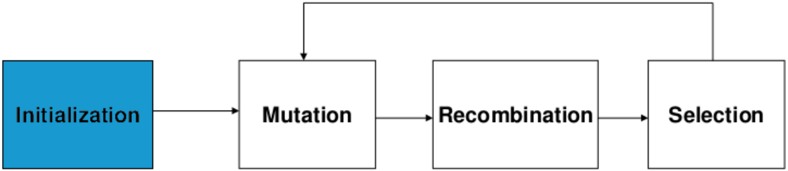
DE MECHANISM

OVERVIEW ON HOW IT WORKS



Adapted from Storn's Website

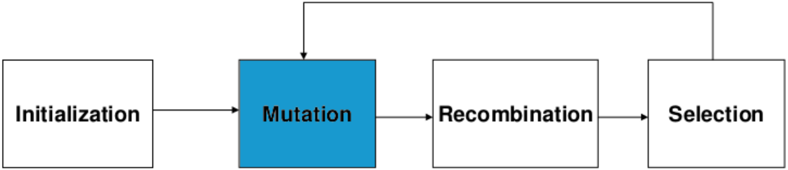
DE INITIALIZATION



Random Population Initialization

$$x_{i,j,0} = x_{j,min} + rand_{i,j}[0,1].(x_{j,max} - x_{j,min}). \quad (1)$$

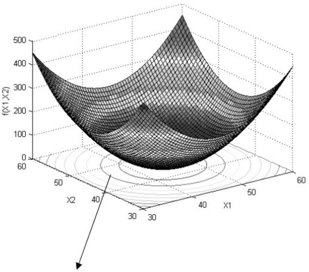
DE MUTATION



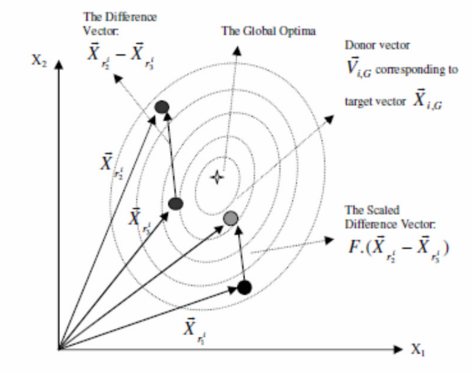
Creating the Donor Vector

$$\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F(\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}). \quad (2)$$

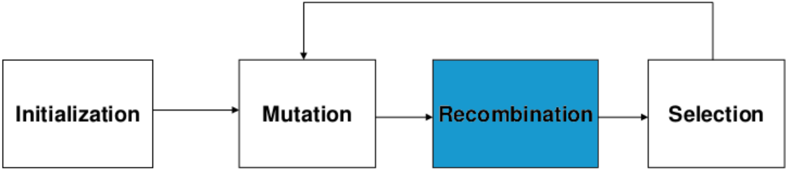
Example of Donor Vector Formation



Constant cost contours of
Sphere function



DE RECOMBINATION



Trial Offspring Vector using Binomial (Uniform) Crossover

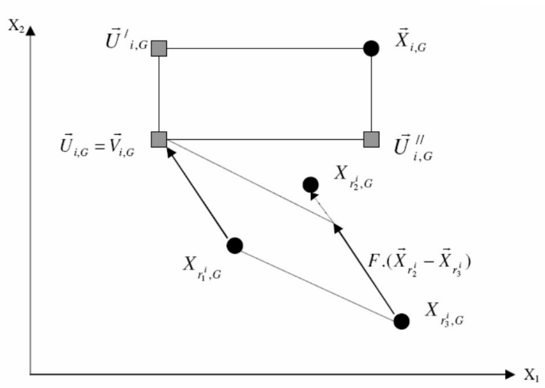
$$u_{j,i,G} = \begin{cases} v_{j,i,G} & : if\,(rand_{i,j}[0,1) \leq CR \text{ or } i = j_{rand}) \\ x_{j,i,G} & : otherwise \end{cases} \quad (3)$$

DE RECOMBINATION

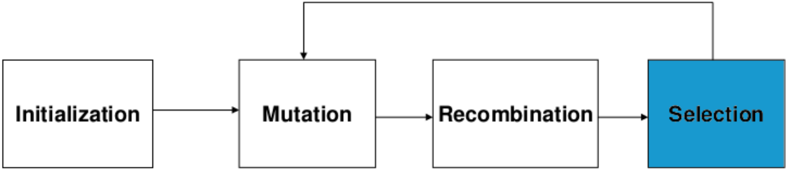
Possible Outcomes:

1. $\overrightarrow{U}_{i,G} = \overrightarrow{V}_{i,G}$ such that both components of $\overrightarrow{U}_{i,G}$ are inherited from $\overrightarrow{V}_{i,G}$.
2. $\overrightarrow{U}_{i,G}^{I}$, in which the first component (j=1) comes from $\overrightarrow{V}_{i,G}$ and the second one (j=2) from $\overrightarrow{X}_{i,G}$
3. $\overrightarrow{U}_{i,G}^{I}I$, in which the first component (j=1) comes from $\overrightarrow{X}_{i,G}$ and the second one (j=2) from $\overrightarrow{V}_{i,G}$

Example of Binomial Crossover in 2D Space



DE SELECTION



Survival of the best

$$
\begin{aligned}
\vec{X}_{i,G+1} &= \vec{U}_{i,G}, \text{ if } f(\vec{U}_{i,G}) \leq f(\vec{X}_{i,G}) \\
\vec{X}_{i,G+1} &= \vec{X}_{i,G}, \text{ if } f(\vec{U}_{i,G}) > f(\vec{X}_{i,G})
\end{aligned}
\tag{4}
$$

PARAMETERS

- The Scale Factor (**F**)
  - A constant from $(0, 2)$
  - Scales donor vector in mutation step
- The Cross Over Rate (**CR**)
  - Leads direction of the search
  - Donor vectors' components enter into the trial offspring vector
  - Recombination step
- The Population Size (**NP**)

ALGORITHM

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be the cost/fitness function
Let $\mathbf{x} \in \mathbb{R}^n$ designate a candidate solution (agent) in the population
Initialize all agents $\mathbf{x}$ with random positions in the search-space.
while Criteria is not met **do**
    for each $\mathbf{x}$ in population **do**
        Pick $\mathbf{a}, \mathbf{b}, \mathbf{c}$ at random from population (distinct from eachother and $\mathbf{x}$)
        Pick a random index $R \in \{1, \ldots, n\}$ (n being the dimensionality of the problem)
        Let $\mathbf{y} = [y_1, \ldots, y_n]$ be the agent's potentially new position
        for each i in n **do**
            Pick a uniformly distributed number $r_i \equiv U(0, 1)$
            if $r_i < \mathrm{CR}$ or $i = R$ **then**
                $y_i = a_i + F \times (b_i - c_i)$
            else
                $y_i = x_i$
            end if
        end for
    end for
    if $f(\mathbf{y}) < f(\mathbf{x})$ **then**
        Replace $\mathbf{x}$ with $\mathbf{y}$ in the population
    end if
    Return agent with the best fitness/cost as the best solution found
end while

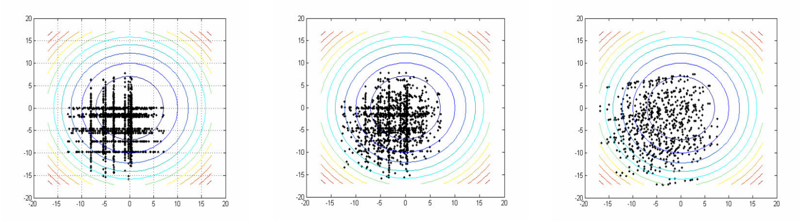
PARAMETERS::SCALE FACTOR

- DE is more sensitive to the choice of F than the CR
- Upper limit of F is empirically taken as 1
- population can converge even in the absence of selection pressure

PARAMETERS::CROSS-OVER RATE

- It controls number of expected parameters to be changed in population
- Low value CR, small portion of parameters changes in each generation
- High value CR, most of the mutant vectors directions inherited

Empirical distribution of trial vectors for three CRs



(a) $Cr = 0$ (b) $Cr = 0.5$ (c) $Cr = 1.0$

Figure: initial population of 10 vectors within 200 generations with selection disabled

PARAMETERS::POPULATION

- influence of NP is yet to be studied to fully understood
- NP should be chosen between 5D and 10D (Storn & Price)
- A method to gradually reduce population size of DE (Brest & Maučec)
- Parallel Populations (Mallipeddi & Suganthan)

CONCLUSION

PERFORMANCE & BENCHMARK

- The stochastic model accurately predicts the behavior of the DE algorithm for a large population size
  - S. Ghosh, S. Das, and A. V. Vasilakos, Convergence Analysis of Differential Evolution over a Class of Continuous Functions with Unique Global Optimum, IEEE Trans. on SMC Part B, 2011.
- Model successfully shows that population vectors converge at global optimum point
- Further research can be undertaken to predict algorithm's behavior for a finite number of vectors

PERFORMANCE & BENCHMARK

Shifted Rastrigin's Function

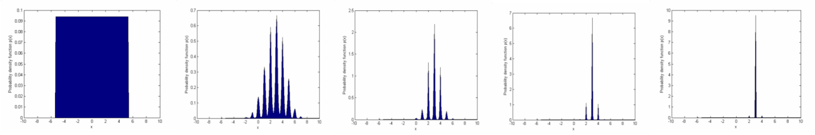


Figure: Prediction for different time instants through stochastic model

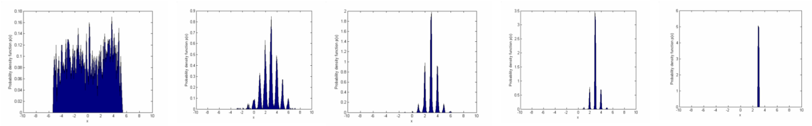


Figure: Estimation by running the DE algorithm

APPLICATIONS

- Parallel Processing
- Multi-objective Optimization
- Constrained Optimization
- Radio Network Design
- Optimal Design of Gas Transmission Network

REFERENCES

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QUESTIONS?