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James F. Peters

# Foundations of Computer Vision

Computational Geometry, Visual Image  
Structures and Object Shape Detection

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James F. Peters

# Foundations of Computer Vision

Computational Geometry, Visual Image Structures and Object Shape Detection



Springer

James F. Peters  
Electrical and Computer Engineering  
University of Manitoba  
Winnipeg, MB  
Canada

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*This book is dedicated to  
Anna Di Concilio, Arturo Tozzi and sweet  $\mathcal{P}$   
for the many shapes and pullbacks they  
have given to me*

# Preface

This book introduces the foundations of computer vision. The principal aim of computer vision (also, called machine vision) is to reconstruct and interpret natural scenes based on the content of images captured by various cameras (see, e.g., R. Szeliski [191]). Computer vision systems include such things as survey satellites, robotic navigation systems, smart scanners, and remote sensing systems. In this study of computer vision, the focus is on *extracting useful information from images* (see, e.g., S. Prince [162]). Computer vision systems typically emulate human visual perception. The hardware of choice in computer vision systems is some form of digital camera, programmed to approximate visual perception. Hence, there are close ties between computer vision, digital image processing, optics, photometry and photonics (see, e.g., E. Stijns and H. Thienpont [188]).

From a computer vision perspective, **photonics** is the science of light in the capture of visual scenes. **Image processing** is the study of digital image formation (e.g., conversion of analogue optical sensor signals to digital signals), manipulation (e.g., image filtering, denoising, cropping), feature extraction (e.g., pixel intensity, gradient orientation, gradient magnitude, edge strength), description (e.g., image edges and texture) and visualization (e.g., pixel intensity histograms). See, e.g., the mathematical frameworks for image processing by B. Jähne [87] and S.G. Hoggar [82], extending to a number of practitioner views of image processing provided, for example, by M. Sonka and V. Hlavac and R. Boyle [186], W. Burger and M.J. Burge [21], R.C. Gonzalez and R.E. Woods [58], R.C. Gonzalez and R.E. Woods and S.L. Eddins [59], V. Hlavac [81], and C. Solomon and T. Breckon [184]. This useful information provides the bedrock for the focal points of computer visionists, namely, image object shapes and patterns that can be detected, analyzed and classified (see, e.g., [142]). In effect, **computer vision** is the study of digital image structures and patterns, which is a layer of image analysis above that of image processing and photonics. Computer vision includes image processing and photonics in its bag of tricks in its pursuit of image geometry and image region patterns.

In addition, it is helpful to cultivate an intelligent systems view of digital images with an eye to discovering hidden patterns such as repetitions of convex enclosures

of image regions and embedded image structures such as clusters of points in image regions of interest. The discovery of such structures is made possible by quantizers. A **quantizer** restricts a set of values (usually continuous) to a discrete value. In its simplest form in computer vision, a quantizer observes a particular target pixel intensity and selects the nearest approximating values in the neighbourhood of the target. The output of a quantizer is called a codebook by A. Gersho and R.M. Gray [55, §5.1, p. 133] (see, also, S. Ramakrishnan, K. Rose and A. Gersho [164]).

In the context of image mesh overlays, the Gersho–Gray quantizer is replaced by geometry-based quantizers. A **geometry-based quantizer** restricts an image region to its shape contour and observes in an image a particular target object shape contour, which is compared with other shape contours that have approximately the same shape as the target. In the foundations of computer vision, geometry-based quantizers observe and compare image regions with approximately the same regions such as mesh maximal nucleus clusters (MNCs) compared with other nucleus clusters. A **maximal nucleus cluster** (MNCs) is a collection of image mesh polygons surrounding a mesh polygon called the nucleus (see, e.g., J.F. Peters and E. İnan on Edelsbrunner nerves in Voronoï tessellations of images [150]). An **image mesh nucleus** is a mesh polygon that is the centre of a collection of adjacent polygons. In effect, every mesh polygon is a nucleus of a cluster of polygons. However, only one or more mesh nuclei are maximal.

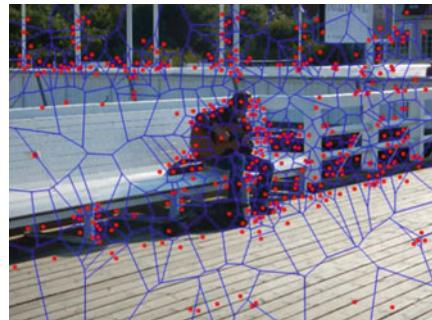
A **maximal image mesh nucleus** is a mesh nucleus with the highest number of adjacent polygons. MNCs are important in computer vision, since what we will call a MNC contour approximates the shape of an underlying image object. A **Voronoi tessellation** of an image is a tiling of the image with polygons. A Voronoï tessellation of an image is also called a Voronoï mesh. A sample tiling of a musician image in Fig. 0.1.1 is shown in Fig. 0.1.2. A sample nucleus of the musician image tiling is shown in Fig. 0.2.1. The red • dots inside each of the tiling polygons are examples of Voronoï region (polygon) generating points. For more about this, see Sect. 1.22.1. This musician mesh nucleus is the centre of a maximal nucleus cluster shown in Fig. 0.2.2. This is the only MNC in the musician image mesh in Fig. 0.1.2. This MNC is also an example of a Voronoï mesh nerve. The study of image MNCs takes us to the threshold of image geometry and image object shape detection. For more about this, see Sect. 1.22.2.

Each **image tiling polygon** is a convex hull of the interior and vertex pixels. A **convex hull** of a set of image points is the smallest convex set of the set of points. A set of image points  $A$  is a **convex set**, provided all of the points on every straight line segment between any two points in the set  $A$  is contained in the set. In other words, knowledge discovery is at the heart of computer vision. Both knowledge and understanding of digital images can be used in the design of computer vision systems. In vision system designs, there is a need to understand the composition and structure of digital images as well as the methods used to analyze captured images.

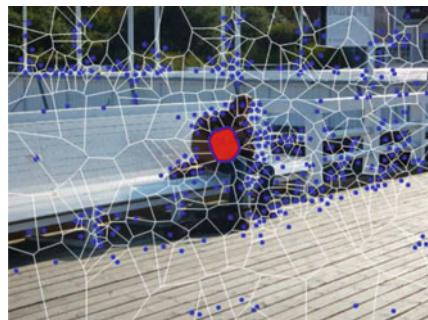
The focus of this volume is on the study of raster images. The sequel to this volume will focus on vector images, which are composed of points (vectors), lines and curves. The basic content of every raster image consists of pixels



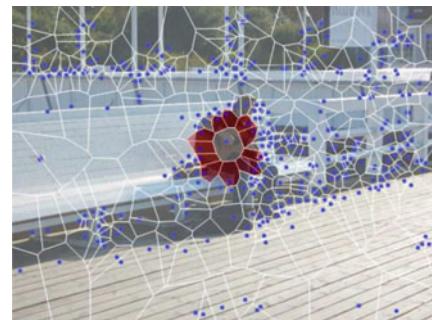
0.1.1: Muscian



0.1.2: Muscian tiling

**Fig. 0.1** Voronoï tessellation of musician image

0.2.1: Musician mesh nucleus



0.2.2: Musician maximal nucleus cluster

**Fig. 0.2** Maximal nucleus cluster on musician image

(e.g., distinguished pixels called sites or mesh generating points), edges (e.g., common, parallel, intersecting, convex, concave, straight, curved, connected, unconnected), angles (e.g., vector angle, angle between vectors, pixel angle), image geometry (e.g., Voronoï regions [141], Delaunay triangulations [140]), colour, shape, and texture. Many problems in computer vision and scene analysis are solved by finding the most probable values of certain hidden or unobserved image variables and structures (see, e.g., P. Kohli and P.H.S. Torr [96]). Such structures and variables include the topological neighbourhood of a pixel, convex hulls of sets of pixels, nearness (and apartness) of image structures and pixel gradient distributions as well as feature vectors that describe elements of captured scenes.

Other computer vision problems include image matching, feature selection, optimal classifier design, image region measurement, interest point identification, contour grouping, segmentation, registration, matching, recognition, image clustering, pattern clustering in F. Escolano, P. Suau, B. Bonev [45] and in N. Paragios, Y. Chen, O. Faugeras [138], landmark and point shape matching, image warping,

shape gradients [138], false colouring, pixel labelling, edge detection, geometric structure detection, topological neighbourhood detection, object recognition, and image pattern recognition.

In computer vision, the focus is on the detection of the basic geometric structures and object shapes commonly found in digital images. This leads into a study of the basics of image processing and image analysis as well as vector space and computational geometry views of images. The basics of image processing include colour spaces, filtering, edge detection, spatial description and image texture. Digital images are examples of Euclidean spaces (both 2D and 3D). Hence, vector space views of digital images are a natural outcome of their basic character. A **digital image structure** is basically a geometric or a visual topological structure. Examples of image structures are image regions, line segments, generating points (e.g. Lowe keypoints), set of pixels, neighbourhood of a pixel, half spaces, convex sets of pixels and convex hulls of sets of image pixels. For example, such structures can be viewed in terms of image regions nearest selected points or collections of image regions with a specified range of diameters. An **image region** is a set of image points (pixels) in the interior of a digital image. The **diameter** of any image region is the maximum distance between a pair of points in the region). Such structures can also be found in line segments connected between selected points to form triangular regions in 2D and 3D images.

Such structures are also commonly found in 2D and 3D images in the intersection of closed half spaces to form either convex hulls of a set of points or what G.M. Ziegler calls **polytopes** [221]. An **image half space** is the set of all points either above or below a line. In all three cases, we obtain a regional view of digital images. For more about polytopes, see Appendix B.15.

Every image region has a shape. Some region shapes are more interesting than others. The interesting image region shapes are those containing objects of interest. These regional views of images leads to various forms of image segmentations that have practical value when it comes to recognizing objects in images. In addition, detection of image region shapes of interest views lead to the discovery of image patterns that transcend the study of texels in image processing. A **texel** is an image region represented by an array of pixels. For more about shapes, see Appendix B.18 on shape and shape boundaries.

Image analysis focuses on various digital image measurements (e.g., pixel size, pixel adjacency, pixel feature values, pixel neighbourhoods, pixel gradient, closeness of image neighbourhoods). Three standard region-based approaches in image analysis are isodata thresholding (binarizing images), watershed segmentation (computed using a distance map from foreground pixels to background regions), and non-maximum suppression (finding local maxima by suppressing all pixels that are less likely than their surrounding pixels) [212].

In image analysis, object and background pixels are associated with different adjacencies (neighbourhoods) by T. Aberra [3]. There are three basic types of neighbourhoods, namely, Rosenfeld adjacency neighbourhoods [171, 102], Hausdorff neighbourhoods [74, 75] and descriptive neighbourhoods in J.F. Peters [142] and in C.J. Henry [77, 76]. Using different geometries, an adjacency

neighbourhood of a pixel is defined by the pixels adjacent to a given pixel. An image **Rosenfeld adjacency neighbourhood** of a pixel  $p$  is a set of pixels that are adjacent to  $p$ . Adjacency neighbourhoods are commonly used in edge detection in digital images.

A **Hausdorff neighbourhood** of a point  $p$  is defined by finding all pixels whose distance from  $p$  is less than a positive number  $r$  (called the neighbourhood radius). A **descriptive neighbourhood** of a pixel  $p$  (denoted by  $N(img(x, y), r)$ ) is the set of pixels with feature vectors that match or are similar to the feature vector that describes  $img(x, y)$  (the neighbourhood ‘centre’ of a digital image  $img$ ) and which are within a prescribed radius  $r$ .

Unlike an adjacency neighbourhood, a descriptive neighbourhood can have holes in it, i.e., pixels with feature vectors that do not match the neighbourhood centre and are not part of the neighbourhood. Other types of descriptive neighbourhoods are introduced in [142, Sect. 1.16, pp. 29–34].

The chapters in this book grew out of my notes for an undergraduate class in Computer Vision taught over the past several years. Many topics in this book grew out my discussions and exchanges with a number of researchers and others, especially, S. Ramanna (those many shapes, especially in crystals), Anna Di Concilio (those proximities, region-free geometry, and seascape shapes like those in Fig. 0.3), Clara Guadagni (those flower nerve structures), Arturo Tozzi (those Borsuk-Ulam Theorem insights and Gibson shapes, Avenarius shapes), Romy Tozzi (remember 8,  $\infty$ ), Zdzisław Pawlak (those shapes in paintings of the Polish countryside), Lech Polkowski (those mereological, topological and rough set structures), Piotr Artiemjew (those dragonfly wings), Giangiacomo Gerla (those tips (points)–vertices–of UNISA courtyard triangles and spatial regions), Gerald Beer (those moments in Som Naimpally’s life), Giuseppe Di Maio (those insights about proximities), Somashekhar (Som) A. Naimpally (those topological structures), Chris Henry (those colour spaces, colour shape sets), Macek Borkowski (those 3D views of space), Homa Fashandi, Dan Lockery, Irakli Dochviri, Ebubekir İnan (those nearness relations and near groups), Mehmet Ali Öztürk (those beautiful algebraic structures), Mustafa Uçkun, Nick Friesen (those shapes of dwellings), Özlem Umdu, Doungarat Chitcharoen, Çenker Sandoz (those Delaunay triangulations), Surabi Tiwari (those many categories), Kyle Fedoruk (application of computer vision: Subaru EyeSight®), Amir H. Meghdadi, Shabnam Shahfar, Andrew Skowron (those proximities at Banacha), Alexander Yurkin, Marcin Wolksi (those sheaves), Piotr Wasilewski, Leon Schilmoeler, Jerzy W. Grzymala-Busse (those insights about rough sets and L<sup>A</sup>T<sub>E</sub>X hints), Zbigniew Suraj (those many Petri nets), Jarosław Stepaniuk, Witold Pedrycz, Robert Thomas (those shapes of tilings), Marković G. oko (polyforms), Miroslaw Pawlak, Pradeepa Yahampath, Gabriel Thomas, Anthony (Tony) Szturm, Sankar K. Pal, Dean McNeill, Giuseppe (Joe) Lo Vetri, Witold Kinsner, Ken Ferens, David Schmidt (set theory), William Hankley (time-based specification), Jack Lange (those chalkboard topological doodlings), Irving Sussman (gold nuggets in theorems and proofs) and Brian Peters (those fleeting glimpses of geometric shapes on the walls).



**Fig. 0.3** Seascape shapes along the coastline of Vietri, Italy

A number of our department technologists have been very helpful, especially, Mount-First Ng, Ken Biegun, Guy Jonatschick and Sinisa Janjic.

And many of my students have given important suggestions concerning topics covered in this book, especially, Drew Barclay, Braden Cross, Binglin Li, Randima Hettiarachchi, Enoch A-iyeh, Chidoteremndu (Chido) Chinonyelum Uchime, D. Villar, K. Marcynuk, Muhammad Zubair Ahmad, and Armina Ebrahimi.

Chapter problems have been classified. Those problems that begin with are the kind you can run with, and probably will not take much time to solve. Problems that begin with are the kind you can probably solve in about the time it takes to drink a cup of tea or coffee. The remaining problems will need varying lengths of time to solve.

Winnipeg, Canada

James F. Peters

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