barcelona

February 14, 2024

1 Exploring Spatial Socio-Economic Patterns in Barcelona

1.1 Introduction

This computational essay is a final assignment for the Spatial Data Science for Social Geography (MZ340V17) at Charles University in Prague.

It aims to explore the city of Barcelona's socio-economic patterns, focusing on key variables such as, average children per household, yearly gross taxable income per household (€) and education level in the scope of census districts (seccio censal).

The primary goals of this exploratory analysis are as follows: 1. **Exploration of Socio-Economic patterns:**

```
Utilize the coding infrastructure of the course to visualize the
socio-economic variables. Offering an intro into the city's socio-economic
dynamics.
```

2. **Exploration of spatial autocorrelation:**

   Investigate the degreee of spatial autocorrelation to find patterns of spatial similarity or dissimilarity among neighbouring census districts, preparing the foundation to the subsequent cluster analysis.

3. **Cluster analysis:**

   Utilize k-means, k-means together with spatial lag and regionalization to identify meaningful clusters in Barcelona. The determination of optimal cluster number will be based on silhouette score.

4. **Cluster analysis evaluation metrics**

   Evaluate the quality of the used cluster analysis methods using metrics such as, isoperimetric quotient, Calinski-Harabasz score and adjusted mutual information score, providing insights into effectiveness of each clustering approach.

1.2 Preparations

1.2.1 Libraries and data

[100]:
```python
import geopandas as gpd
import matplotlib.pyplot as plt
import esda
```

```
import numpy as np
import pandas as pd

from sklearn.preprocessing import RobustScaler
from sklearn import metrics
from sklearn import cluster
from libpysal import graph
from splot.esda import lisa_cluster, moran_scatterplot
from IPython.display import display

# Additional library for optimal number of clusters
from kneed import KneeLocator
```

[101]:
```
link = 'https://opendata-ajuntament.barcelona.cat/data/dataset/
↪d8e40c96-9f1f-4fd3-86da-2baa1599616d/resource/
↪edaf6642-a51b-4b2b-a492-fa913d5e8b91/download/2021_atles_renda_bruta_llar.
↪csv'
gross_income = gpd.read_file(link)

children = gpd.read_file('./data/2021_pad_dom_mdbas_edat-0018.csv')

edu = gpd.read_file('./data/2021_pad_mdbas_niv-educa-esta_sexe.csv')

seccio = gpd.read_file('./data/seccio_censal')
```

1.2.2 Data Wrangling

[102]:
```
# Drop unnecessary columns and rename columns for better understanding
children = children.drop(
    columns=['Data_Referencia',
             'Codi_Barri',
             'Nom_Barri',
             'AEB',
             'geometry',
             'Nom_Districte']
             ).rename(columns={'Codi_Districte': 'district_code',
                               'Seccio_Censal': 'section_code',
                               'Valor': 'households',
                               'DOM_00_18': 'children'}
)
# Convert columns to the correct data type
children['households'] = children['households'].astype(int)
children['children'] = children['children'].astype(int)

# Clean the section code by removing the district code infront
children['district_code_length'] = children['district_code'].str.len()
children['section_code'] = children.apply(
```

```python
    lambda row: row['section_code'][row['district_code_length']:], axis=1
)
# Drop the temporary column
children = children.drop(columns=['district_code_length'])

# Convert columns to the correct data type
children['section_code'] = children['section_code'].astype(int)
children['district_code'] = children['district_code'].astype(int)

# Calculate average children per household
children['weighted_children'] = children['children'] * children['households']
grouped = children.groupby(['district_code', 'section_code']).agg(
    total_households=('households', 'sum'),
    total_weighted_children=('weighted_children', 'sum')
)
grouped['average_children'] =(
    grouped['total_weighted_children'] / grouped['total_households']
)
# Reset the index and drop temporary columns
children = grouped.reset_index().drop(
    columns=['total_weighted_children',
             'total_households']
)
```

[103]:
```python
# Drop unnecessary columns and rename columns for better understanding
gross_income = gross_income.drop(
    columns=['Any', 'Codi_Barri', 'Nom_Barri', 'geometry']
    ).rename(columns={'Import_Renda_Bruta_€': 'gross_income',
                      'Codi_Districte': 'district_code',
                      'Nom_Districte': 'district_name',
                      'Seccio_Censal': 'section_code',}
)
# Convert columns to the correct data type
gross_income['section_code'] = gross_income['section_code'].astype(int)
gross_income['district_code'] = gross_income['district_code'].astype(int)
gross_income['gross_income'] = gross_income['gross_income'].astype(float)
```

[104]:
```python
# Drop unnecessary columns and rename columns for better understanding
edu = edu.drop(
    columns=['Data_Referencia',
             'Codi_Barri',
             'Nom_Barri',
             'AEB',
             'geometry',
             'Nom_Districte']
             ).rename(columns={'Codi_Districte': 'district_code',
                               'Seccio_Censal': 'section_code',
```

```python
                               'Valor': 'people',
                               'NIV_EDUCA_esta': 'education_level',
                               'SEXE': 'gender'}
)

# Clean the section code by removing the district code infront
edu['district_code_length'] = edu['district_code'].str.len()
edu['section_code'] = edu.apply(
    lambda row: row['section_code'][row['district_code_length']:], axis=1
)
# Drop the temporary column
edu = edu.drop(columns=['district_code_length'])

# Convert columns to the correct data type
edu['section_code'] = edu['section_code'].astype(int)
edu['district_code'] = edu['district_code'].astype(int)
edu['people'] = edu['people'].replace('..', 0).astype(float)

# Group by district code, section code, education level and gender and sum the
# people
grouped = edu.groupby([
    'district_code',
    'section_code',
    'education_level',
    'gender']
    )['people'].sum()

# Unstack the grouped data
unstacked = grouped.unstack(level=[2, 3])
unstacked.reset_index(inplace=True)

# Fill the NaN values with 0
unstacked.fillna(0, inplace=True)

# Get rid of the multiindex by joining the columns with an underscore
unstacked.columns = ['_'.join(col) for col in unstacked.columns.values]

# Cleanup after joining the columns and rename the columns for better
# understanding
column_mapping = {
    col: (
        col.replace('_1', '_f')
        .replace('_2', '_m')
        .replace('district_code_', 'district_code')
        .replace('section_code_', 'section_code')
        .replace('1_', 'no_edu_')
        .replace('2_', 'prim_')
```

```
        .replace('5_', 'high_')
        .replace('6_', 'no_data_')
        )
        for col in unstacked.columns
}
# Rename the columns
unstacked.rename(columns=column_mapping, inplace=True)

# Merge the third and fourth level of education since Spain has tup different
# levels of high school exams
unstacked['sec_f'] = unstacked['3_f'] + unstacked['4_f']
unstacked['sec_m'] = unstacked['3_m'] + unstacked['4_m']
# Drop the third and fourth level of education
unstacked = unstacked.drop(columns=['3_f', '4_f', '3_m', '4_m'])

# Reorder the columns for more logical order
columns_to_move = ['sec_f', 'sec_m']
new_position = 7

original_columns = unstacked.columns.to_list()

for column in columns_to_move:
    original_columns.remove(column)

for column in columns_to_move:
    original_columns.insert(new_position -1, column)
    new_position += 1

edu = unstacked[original_columns]
```

[105]:
```
# Drop unnecessary columns and rename columns for better understanding
seccio = seccio[['DISTRICTE', 'SEC_CENS', 'geometry']]
seccio = seccio.rename(columns={
    'DISTRICTE': 'district_code',
    'SEC_CENS': 'section_code'}
)
# Convert columns to the correct data type
seccio['district_code'] = seccio['district_code'].astype(int)
seccio['section_code'] = seccio['section_code'].astype(int)
```

Merging

[106]:
```
# Merge the dataframes into geopandas dataframe
for df in [children, gross_income, edu]:
    seccio = seccio.merge(df, on=['district_code', 'section_code'])
```

Relative education values

[107]:
```python
# Get the columns that are related to the education
edu_column = [
    col for col in seccio.columns if col.endswith('_f') or col.endswith('_m')
]
# Calculate the total population by gender aged 16+
seccio['pop_f'] = seccio[
    seccio.columns[seccio.columns.str.endswith('_f')]].sum(axis=1)
seccio['pop_m'] = seccio[
    seccio.columns[seccio.columns.str.endswith('_m')]].sum(axis=1)

# Calculate the rate of each education level
for column in edu_column:
    if column.endswith('_f'):
        seccio[column + '_rate'] = seccio[column] / seccio['pop_f'] * 100
    else:
        seccio[column + '_rate'] = seccio[column] / seccio['pop_m'] * 100

seccio.head()
```

[107]:
```
   district_code  section_code  \
0              1             5
1              1             6
2              1             7
3              1             8
4              1             9

                                            geometry  average_children  \
0  POLYGON ((430905.031 4581350.072, 430938.474 4…          1.363542
1  POLYGON ((430874.963 4581396.929, 430870.976 4…          1.318108
2  POLYGON ((430614.207 4581309.336, 430622.668 4…          1.419771
3  POLYGON ((430564.164 4581104.412, 430550.048 4…          1.509542
4  POLYGON ((430275.270 4581082.530, 430331.870 4…          1.433850

  district_name  gross_income  no_edu_f  no_edu_m  prim_f  prim_m  …  \
0  Ciutat Vella       27950.0      29.0      10.0   237.0   318.0  …
1  Ciutat Vella       33086.0      10.0       6.0   128.0   187.0  …
2  Ciutat Vella       32945.0      10.0       0.0   148.0   311.0  …
3  Ciutat Vella       26200.0      21.0      10.0   360.0   686.0  …
4  Ciutat Vella       30306.0      13.0       6.0   224.0   298.0  …

   no_edu_f_rate  no_edu_m_rate  prim_f_rate  prim_m_rate  sec_f_rate  \
0       2.917505       0.873362    23.843058    27.772926   40.140845
1       1.602564       0.722022    20.512821    22.503008   43.108974
2       1.468429       0.000000    21.732746    29.619048   43.465492
3       1.860053       0.562746    31.886625    38.604389   44.906997
4       1.492537       0.579151    25.717566    28.764479   43.857635
```

```
   sec_m_rate  high_f_rate  high_m_rate  no_data_f_rate  no_data_m_rate
0   45.327511    31.388330    23.842795        1.710262        2.183406
1   46.209386    33.814103    29.362214        0.961538        1.203369
2   47.428571    32.305433    22.000000        1.027900        0.952381
3   44.738323    19.574845    14.181204        1.771479        1.913337
4   44.594595    27.669346    25.096525        1.262916        0.965251

[5 rows x 28 columns]
```

1.3 Exploration

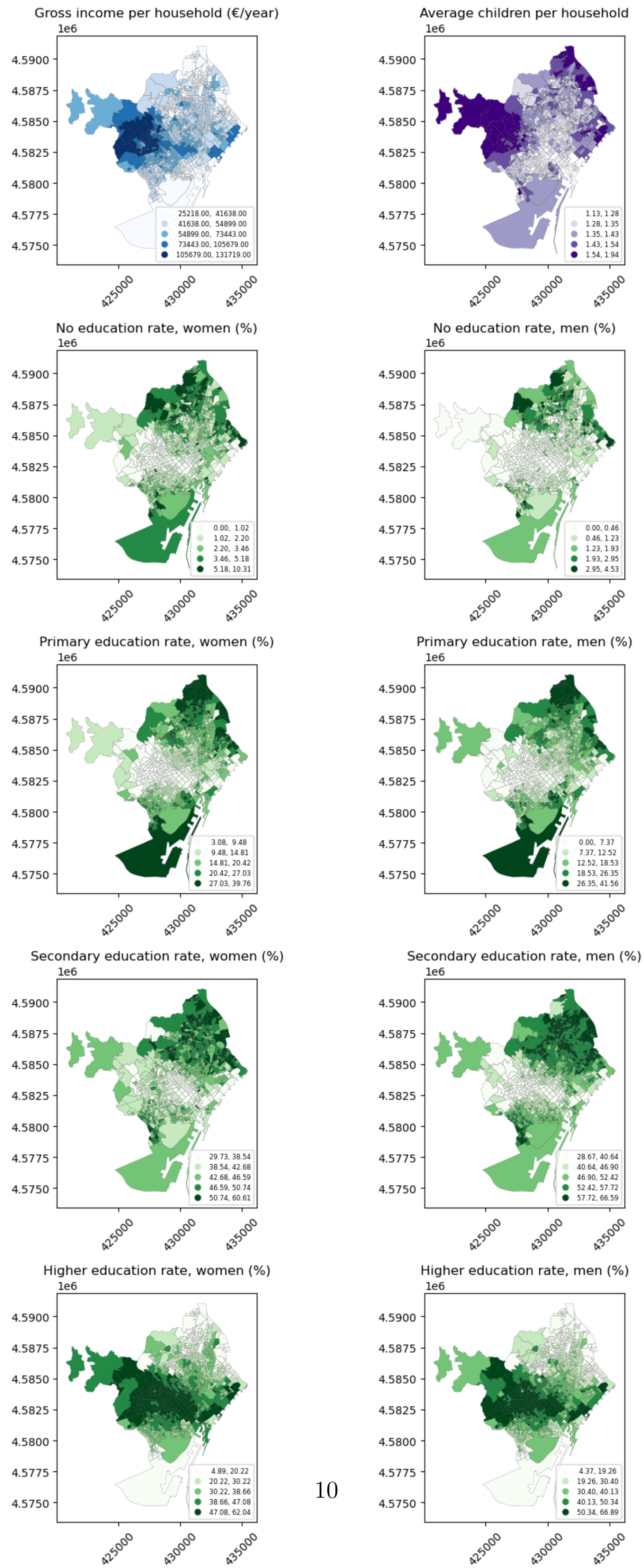
1.3.1 Visualization

[108]:

```
# Plot the data for visual exploration
fig, ax = plt.subplots(5, 2, figsize=(10, 20))

seccio.plot(
    column='gross_income', ax=ax[0, 0],
    legend=True, cmap='Blues',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='average_children', ax=ax[0, 1],
    legend=True, cmap='Purples',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='no_edu_f_rate', ax=ax[1, 0],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='no_edu_m_rate', ax=ax[1, 1],
```

```
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='prim_f_rate', ax=ax[2, 0],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='prim_m_rate', ax=ax[2, 1],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6,
        "loc": 'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='sec_f_rate', ax=ax[3, 0],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={
        "fontsize": 6, "loc":
        'lower right',
        "markerscale": 0.6}
)
seccio.plot(
    column='sec_m_rate', ax=ax[3, 1],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={"fontsize": 6,
                 "loc": 'lower right',
                 "markerscale": 0.6}
)
```

```
seccio.plot(
    column='high_f_rate', ax=ax[4, 0],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={"fontsize": 6,
                 "loc": 'lower right',
                 "markerscale": 0.6}
)
seccio.plot(
    column='high_m_rate', ax=ax[4, 1],
    legend=True, cmap='Greens',
    scheme='naturalbreaks',
    linewidth=.1, edgecolor='black',
    legend_kwds={"fontsize": 6,
                 "loc": 'lower right',
                 "markerscale": 0.6}
)
# Set the titles
ax[0, 0].set_title('Gross income per household (€/year)')
ax[0, 1].set_title('Average children per household')
ax[1, 0].set_title('No education rate, women (%)')
ax[1, 1].set_title('No education rate, men (%)')
ax[2, 0].set_title('Primary education rate, women (%)')
ax[2, 1].set_title('Primary education rate, men (%)')
ax[3, 0].set_title('Secondary education rate, women (%)')
ax[3, 1].set_title('Secondary education rate, men (%)')
ax[4, 0].set_title('Higher education rate, women (%)')
ax[4, 1].set_title('Higher education rate, men (%)')
# Set the x-axis label rotation for better readability
ax[0, 0].tick_params(axis='x', rotation=45)
ax[0, 1].tick_params(axis='x', rotation=45)
ax[1, 0].tick_params(axis='x', rotation=45)
ax[1, 1].tick_params(axis='x', rotation=45)
ax[2, 0].tick_params(axis='x', rotation=45)
ax[2, 1].tick_params(axis='x', rotation=45)
ax[3, 0].tick_params(axis='x', rotation=45)
ax[3, 1].tick_params(axis='x', rotation=45)
ax[4, 0].tick_params(axis='x', rotation=45)
ax[4, 1].tick_params(axis='x', rotation=45)

plt.tight_layout()
```


Gross income per household (€/year)
1e6
4.5900
4.5875
4.5850
4.5825
4.5800
4.5775
4.5750
425000
430000
435000
25218.00, 41638.00
41638.00, 54899.00
54899.00, 73443.00
73443.00, 105679.00
105679.00, 131719.00
Average children per household
1.13, 1.28
1.28, 1.35
1.35, 1.43
1.43, 1.54
1.54, 1.94
No education rate, women (%)
0.00, 1.02
1.02, 2.20
2.20, 3.46
3.46, 5.18
5.18, 10.31
No education rate, men (%)
0.00, 0.46
0.46, 1.23
1.23, 1.93
1.93, 2.95
2.95, 4.53
Primary education rate, women (%)
3.08, 9.48
9.48, 14.81
14.81, 20.42
20.42, 27.03
27.03, 39.76
Primary education rate, men (%)
0.00, 7.37
7.37, 12.52
12.52, 18.53
18.53, 26.35
26.35, 41.56
Secondary education rate, women (%)
29.73, 38.54
38.54, 42.68
42.68, 46.59
46.59, 50.74
50.74, 60.61
Secondary education rate, men (%)
28.67, 40.64
40.64, 46.90
46.90, 52.42
52.42, 57.72
57.72, 66.59
Higher education rate, women (%)
4.89, 20.22
20.22, 30.22
30.22, 38.66
38.66, 47.08
47.08, 62.04
Higher education rate, men (%)
4.37, 19.26
19.26, 30.40
30.40, 40.13
40.13, 50.34
50.34, 66.89

Visualizing the data on maps, we can clearly see a notable spatial variability between the census districts.

Looking at yearly gross income per household, there is notable difference between the census districts. The district Les Corts and Sarria-Sant Gervasi clearly dominate the whole city in terms of gross income.

Based on average children per household, we can clearly see the difference in the city's downtown and surrounding areas.

In the scope of primary education, the mostly industrial districts (namely, Nou Barris, Sant Andreu and Sants Montjuic - Zona Franca port) shine out.

However, to asses the nature of each parts of Barcelona more thoroughly we must have a look at spatial autocorrelation and the cluster analysis.

1.3.2 Global spatial autocorrelation

[109]:
```
# Build the queen contiguity weights matrix and row standardize it
contiguity = graph.Graph.build_contiguity(seccio, rook=False)
contiguity_r = contiguity.transform("r")
```

[110]:
```
# Calculate Moran's I and its p-value for each variable and print the results
mi = esda.Moran(seccio['gross_income'], contiguity_r.to_W())

summary = f"""\
Moran's I and p-value
=====================
Gross income:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""

mi = esda.Moran(seccio['average_children'], contiguity_r.to_W())

summary += f"""\
Children per household:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""
mi = esda.Moran(seccio['high_f_rate'], contiguity_r.to_W())

summary += f"""\
Higher education, women:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""
```

```python
mi = esda.Moran(seccio['high_m_rate'], contiguity_r.to_W())

summary += f"""\
Higher education, men:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""

mi = esda.Moran(seccio['prim_f_rate'], contiguity_r.to_W())

summary += f"""\
Primary education, women:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""

mi = esda.Moran(seccio['prim_m_rate'], contiguity_r.to_W())

summary += f"""\
Primary education, men:
    statistic: {round(mi.I, 3)}
    p-value: {mi.p_sim}
"""

print(summary)
```

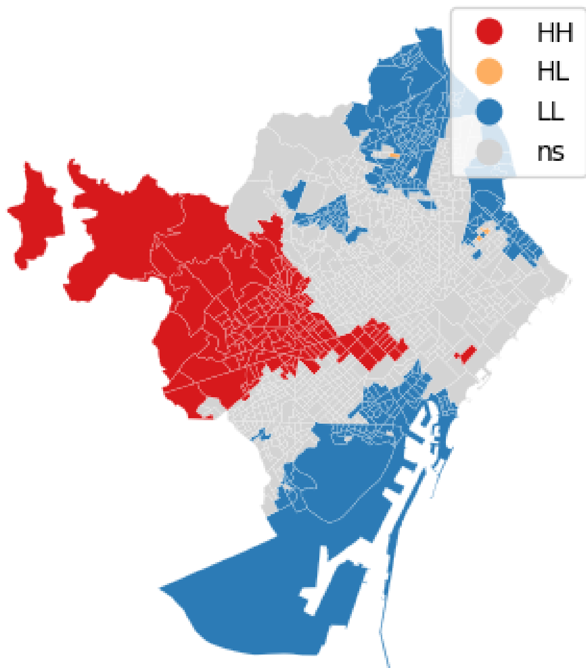
```
Moran's I and p-value
=====================
Gross income:
    statistic: 0.833
    p-value: 0.001
Children per household:
    statistic: 0.635
    p-value: 0.001
Higher education, women:
    statistic: 0.866
    p-value: 0.001
Higher education, men:
    statistic: 0.886
    p-value: 0.001
Primary education, women:
    statistic: 0.804
    p-value: 0.001
Primary education, men:
    statistic: 0.8
    p-value: 0.001
```

The global spatial autocorrelation using Moran's I clearly shows us, that the chosen variables do seem to be positively correlated over Barcelona. Except the average children per household variable, the variables show a high positive rate of spatial autocorrelation. But even the average children per household shows a notable correlation.

Given we recieved p-value of 0.001, the lowest possible value we could recieve, we can deduce with high probability, that the values are not arranged randomly.

1.3.3 Local spatial autocorrelation

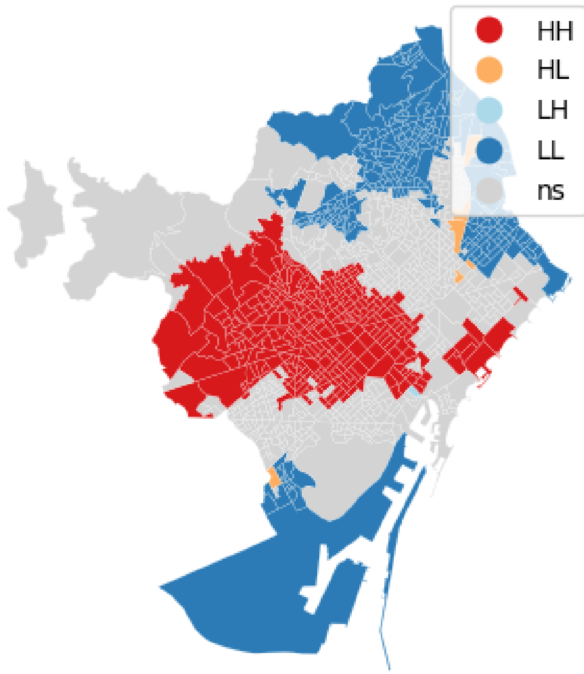
[111]:
```python
# Plot the LISA cluster map for the gross income
lisa = esda.Moran_Local(seccio['gross_income'], contiguity_r.to_W())
_ = lisa_cluster(lisa, seccio)
```



The LISA (Local Indicators of Spatial Association) cluster map extracts statistically significant areas with local association - those that are higly unlikely to have come from pure randomness. The HH (high-high) cluster groups together the areas with high values surrounded by high values, the LL (low-low) cluster, on the other hand groups together areas with low values, surrounded by low values.

Reading the LISA cluster map of yearly gross taxable income clearly shows cluster of high gross income in the previously mentioned districts. Similarly the LL cluster copies the industrial districts or districts close to industrial areas.

[112]:
```
# Plot the LISA cluster map for the higher education of women
lisa = esda.Moran_Local(seccio['high_f_rate'], contiguity_r.to_W())
_ = lisa_cluster(lisa, seccio)
```



Similar result can be seen while looking at higher education rate. The HH cluster is located in the downtown area, transitioning into the high gross income districts. The LL cluster in this example still outlines the highly industrial areas.

1.4 Clustering

1.4.1 Preparation

[113]:
```
# Prepare the variables for the clustering
subranks = [
    'average_children', 'gross_income',
    'high_f_rate', 'sec_f_rate', 'prim_f_rate', 'no_edu_f_rate',
    'high_m_rate', 'sec_m_rate', 'prim_m_rate', 'no_edu_m_rate'
    ]
# Scale the data using RobustScaler to handle outliers
scaler = RobustScaler()
data_scaled = scaler.fit_transform(seccio[subranks])

# Calculate the silhouette score for different number of clusters
scores = []
```

```
for k in range(2, 11):
    kmeans = cluster.KMeans(n_clusters=k, random_state=42)
    kmeans.fit(data_scaled)
    scores.append(metrics.silhouette_score(data_scaled, kmeans.labels_))

# Plot the silhouette score for different number of clusters
plt.plot(range(2, 11), scores)
plt.title('Silhouette Score vs Number of Clusters')
plt.xlabel('Number of Clusters')
plt.ylabel('Silhouette Score')
plt.show()
```

[114]:
```
K = [i for i in range(2, 11)]

# Find the optimal number of clusters using the elbow method
# 'K' is the number of clusters and 'scores' is the silhouette score
knee_locator = KneeLocator(K, scores, curve='convex', direction='decreasing')

optimal_clusters = knee_locator.knee
```

```python
# Print the optimal number of clusters
print(f'The optimal number of clusters is {optimal_clusters}')
```

```
The optimal number of clusters is 7
```

KMeans

[115]:
```python
# Perform k-means clustering with the optimal number of clusters
kmeans = cluster.KMeans(n_clusters=optimal_clusters, random_state=42)
kmeans.fit(data_scaled)
# Add the cluster labels to the dataframe
seccio['kmeans'] = kmeans.labels_
```

Spatially-lagged cluster

[116]:
```python
# Create new columns for the spatially lagged values of the variables and define
# the variables for the spatially lagged clustering
for column in subranks:
    seccio[column + "_lag"] = contiguity_r.lag(seccio[column])

subranks_lag = [column + "_lag" for column in subranks]
subranks_spatial = subranks + subranks_lag
# Scale the data using RobustScaler to handle outliers
data_scaled_spatial = scaler.fit_transform(seccio[subranks_spatial])

kmeans_lag = cluster.KMeans(n_clusters=optimal_clusters, random_state=42)
kmeans_lag.fit(data_scaled_spatial)
# Add the cluster labels to the dataframe
seccio['kmeans_lag'] = kmeans_lag.labels_
```

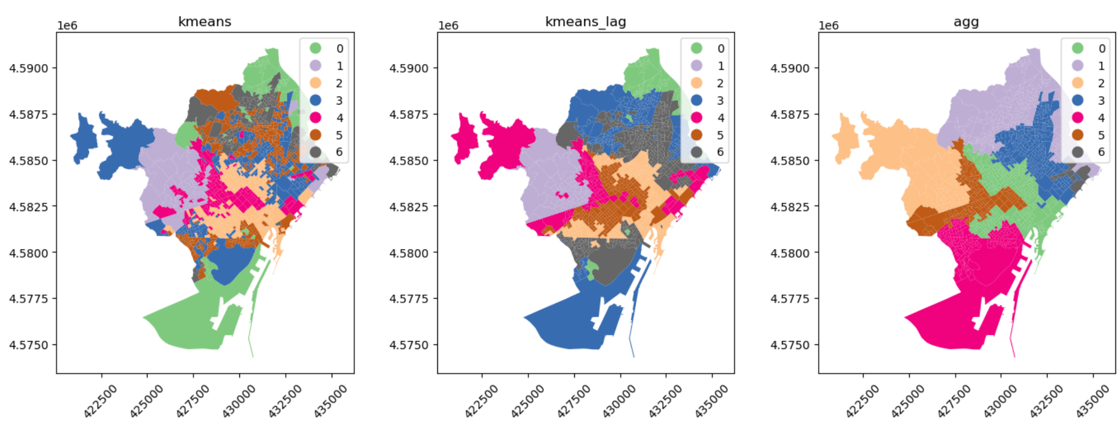
Regionalisation

[117]:
```python
# Perform Agglomerative Clustering with the optimal number of clusters
agg = cluster.AgglomerativeClustering(
    n_clusters=optimal_clusters, connectivity=contiguity.sparse
)

agg.fit(data_scaled)
# Add the cluster labels to the dataframe
seccio['agg'] = agg.labels_
```

1.4.2 Visualization

[118]:
```python
# Plot the different clustering methods on the map
fig, ax = plt.subplots(1, 3, figsize=(14, 5))

seccio.plot(
    column='kmeans',
    ax=ax[0],
    legend=True,
```

```
    categorical=True,
    cmap='Accent'
)
seccio.plot(
    column='kmeans_lag',
    ax=ax[1],
    legend=True,
    categorical=True,
    cmap='Accent'
)
seccio.plot(
    column='agg',
    ax=ax[2],
    legend=True,
    categorical=True,
    cmap='Accent'
)
# Set the titles
ax[0].set_title('kmeans')
ax[1].set_title('kmeans_lag')
ax[2].set_title('agg')
# Set the x-axis label rotation for better readability
ax[0].tick_params(axis='x', rotation=45)
ax[1].tick_params(axis='x', rotation=45)
ax[2].tick_params(axis='x', rotation=45)
plt.tight_layout()
plt.show()
```



Each method algorithm groups observations into pre-specified number of clusters. This analysis uses silhouette score to find the optimal number of clusters to use. The score is an average standardized distance from each observation to its "next best fit" cluster. When employing the elbow method, we seek the point where adding more clusters would no longer result in a significant decrease in the average standardized distance from each observation. This 'elbow' point serves as an indication of the optimal number of clusters, striking a balance between cluster cohesion and the overall reduction in distance.

The K-means method groups together census districts solely on the statistics, meaning the spatial distribution has no effect on the defined clusters. However because of this limitation, this method is very efficient. We can clearly see it on the visualization, the clusters are not spatially connected.

By adding the information about spatial lag into the K-means algorithm, we can introduce the needed context of spatial distribution. We can see that clusters defined by K-means with spatial lag, are much more spatially connected because of this additional information.

In some cases we might require that all the defined clusters to be spatially connected. For instance, when the objective is to identify new neighborhoods or districts, ensuring that clusters are not spatially fragmented is crucial, unlike the outcome observed in the K-means algorithm results. For this reason we can turn to regionalization. As evidenced by the resulting clusters above, the reginalization output showcases clusters interconnected in a visually cohesive and presentable way.

1.4.3 Comparison

Statistics

[126]:
```
k_means = seccio.groupby('kmeans')[subranks].median()
kmeans_stat = k_means.T

k_means_lag = seccio.groupby('kmeans_lag')[subranks].median()
kmeans_lag_stat = k_means_lag.T

agg_clustering = seccio.groupby('agg')[subranks].median()
agg_stat = agg_clustering.T

display(kmeans_stat)
display(kmeans_lag_stat)
display(agg_stat)
```

```
kmeans                       0              1             2             3  \
average_children      1.497529       1.592155      1.275821      1.348606
gross_income      31356.000000  131719.000000  53619.000000  52671.000000
high_f_rate          12.914286      51.107473     45.020601     37.049180
sec_f_rate           51.165981      40.494234     40.269170     44.677661
prim_f_rate          30.395137       5.709845     11.793111     14.840989
no_edu_f_rate         3.762136       0.000000      1.604048      2.072539
high_m_rate           9.466264      56.817074     43.708791     34.126984
sec_m_rate           56.081081      36.666052     45.497939     52.325581
prim_m_rate          30.715005       3.112222      8.657890     11.353712
no_edu_m_rate         1.624549       0.000000      0.000000      0.000000

kmeans                       4             5             6
average_children      1.385787      1.326637      1.349734
gross_income      75594.000000  44993.000000  37161.000000
high_f_rate          51.612903     29.258260     18.097015
sec_f_rate           39.484979     47.661897     50.533808
```

prim_f_rate	7.012195	19.054196	24.688279
no_edu_f_rate	0.733496	2.667279	4.941176
high_m_rate	53.682720	25.211880	14.216478
sec_m_rate	39.778449	55.750472	58.504673
prim_m_rate	4.539386	16.316583	22.520420
no_edu_m_rate	0.000000	1.063645	2.435530

kmeans_lag	0	1	2	3 \
average_children	1.511468	1.569359	1.299413	1.350000
gross_income	31015.000000	130474.000000	49908.000000	38271.000000
high_f_rate	10.893855	51.057157	39.097802	18.558559
sec_f_rate	51.366743	40.494234	43.341613	50.574713
prim_f_rate	31.019830	5.822862	14.391169	24.400000
no_edu_f_rate	3.942181	0.000000	1.947364	4.404568
high_m_rate	8.219178	56.779123	37.281994	14.450867
sec_m_rate	55.900621	37.150902	49.795833	59.187621
prim_m_rate	31.721195	3.112222	11.026666	22.093023
no_edu_m_rate	1.624549	0.000000	0.000000	2.210884

kmeans_lag	4	5	6
average_children	1.401924	1.299054	1.344538
gross_income	77330.000000	59231.000000	46802.000000
high_f_rate	51.295030	48.180678	30.619266
sec_f_rate	40.206471	39.341421	46.977547
prim_f_rate	6.700337	10.446009	18.104907
no_edu_f_rate	0.708942	1.343101	2.526003
high_m_rate	54.016903	46.843854	26.782609
sec_m_rate	40.257440	44.114002	55.371901
prim_m_rate	4.368130	7.324365	14.747475
no_edu_m_rate	0.000000	0.000000	0.984529

agg	0	1	2	3 \
average_children	1.283096	1.375361	1.577405	1.346995
gross_income	53406.000000	37910.500000	130898.000000	47999.000000
high_f_rate	43.868922	18.275673	51.011840	32.192846
sec_f_rate	41.012216	50.576932	40.494234	47.643979
prim_f_rate	12.181303	24.799784	5.912937	16.736402
no_edu_f_rate	1.602959	4.243114	0.000000	2.435312
high_m_rate	42.509363	14.285714	56.685291	28.380386
sec_m_rate	46.557971	58.393285	37.429756	55.584082
prim_m_rate	8.864697	22.259093	3.468872	13.241107
no_edu_m_rate	0.000000	2.077815	0.000000	0.937500

agg	4	5	6
average_children	1.320616	1.353075	1.641819
gross_income	44300.500000	72284.500000	73103.000000
high_f_rate	32.777553	50.031763	50.961956
sec_f_rate	44.695850	40.077967	37.682039
prim_f_rate	18.919437	7.718815	8.075002

```
no_edu_f_rate          2.400857      0.847888      0.781780
high_m_rate           29.149767     53.532314     46.479111
sec_m_rate            51.144370     40.396678     44.180710
prim_m_rate           16.286653      4.873945      6.019503
no_edu_m_rate          0.923792      0.000000      0.000000
```

Examining the cluster statistics generated by each clustering method reveals noteworthy similarities. Notably, cluster `0` of K-means method, cluster `3` of K-means with spatial lag and cluster `4` of regionalization, all group together the south industrial part of the city sharing somewhat similar observations.

Geographical and cluster coherence

[132]:
```
results = []
for cluster_type in ("kmeans", "kmeans_lag", "agg"):
    # compute the region polygons using a dissolve
    regions = seccio[[cluster_type, "geometry"]].dissolve(by=cluster_type)
    # compute the actual isoperimetric quotient for these regions
    ipqs = (
        regions.area * 4 * np.pi / (regions.boundary.length ** 2)
    )
    # cast to a dataframe
    result = ipqs.to_frame(cluster_type)
    results.append(result)
# stack the series together along columns
compactness = pd.concat(results, axis=1)

ch_scores = []
for cluster_type in ("kmeans", "kmeans_lag", "agg"):
    # compute the CH score
    ch_score = metrics.calinski_harabasz_score(
        # using scaled variables
        data_scaled,
        # using these labels
        seccio[cluster_type],
    )
    # and append the cluster type with the CH score
    ch_scores.append((cluster_type, ch_score))

# re-arrange the scores into a dataframe for display
chscore = pd.DataFrame(
    ch_scores, columns=["cluster type", "CH score"]
).set_index("cluster type")

ami_scores = []
# for each cluster solution
for i_cluster_type in ("kmeans", "kmeans_lag", "agg"):
    # for every other clustering
    for j_cluster_type in ("kmeans", "kmeans_lag", "agg"):
```

```python
        # compute the adjusted mutual info between the two
        ami_score = metrics.adjusted_mutual_info_score(
            seccio[i_cluster_type], seccio[j_cluster_type]
        )
        # and save the pair of cluster types with the score
        ami_scores.append((i_cluster_type, j_cluster_type, ami_score))
# arrange the results into a dataframe
results = pd.DataFrame(
    ami_scores, columns=["source", "target", "similarity"]
)
# and spread the dataframe out into a square
ami_score = results.pivot(index="source", columns="target", values="similarity")

display(compactness)
display(chscore)
display(ami_score)
```

```
     kmeans  kmeans_lag       agg
0  0.033081    0.108564  0.055013
1  0.075555    0.232582  0.106769
2  0.021103    0.027805  0.167446
3  0.013306    0.033720  0.101856
4  0.023645    0.034611  0.108407
5  0.009960    0.049002  0.109718
6  0.019825    0.037184  0.113265
```

```
                CH score
cluster type
kmeans        559.844660
kmeans_lag    458.645083
agg           328.245361
```

```
target           agg    kmeans  kmeans_lag
source
agg         1.000000  0.484849    0.542482
kmeans      0.484849  1.000000    0.593148
kmeans_lag  0.542482  0.593148    1.000000
```

Looking at isoperimetric quotient or "compactness" we can see that the overall shape measures for the clusters are notably superior under the regionalization, underscoring the advantage of this particular method in terms of cluster cohesion. Intriguingly the cluster `0` and `1` show significantly better results when employing the K-means method with spatial lag.

Considering the Calinski-Harabasz score, which evaluates the "within-cluster-variance", the K-means method is clearly the best. This is based on the nature of the algorithm putting together areas sharing similar values.

From the solution similarity, we can observe that K-means and K-means method with spatial lag exhibit the highest degre of self-similarity. Interestingly the regionalization and K-means with spatial lag demonstrate a relatively similar level of self-similarity, resembling the K-means methods.

This underscores the effectiveness of incorporating spatial distribution into the K-means algorithm in this particular case.

1.5 Summary

In this comprehensive exploration of Barcelona's socio-economic patterns, the computational essay employs the infrastructure of the course. Focusing on variables like average children per household, yearly gross taxable income per household, and education level, the study sets out the new rehions based on these socio-economic observations.

The essay concludes that the choice of clustering method depends on the specific goals. K-means, efficient and straightforward, excels in minimizing within-cluster variance. K-means with spatial lag enhances spatial coherence, particularly in clusters 0 and 1. Regionalization stands out for its ability to create spatially contiguous and visually cohesive clusters.