BLG202E Numerical Methods in Comp. Eng.

Spring 2022 - Homework I

Due: March 22, 2022

By turning in this assignment, I agree by the ITU honor code and declare that all of this is my own work.

Important Notes

- You are required to submit a PDF document and Python source codes to Ninova before the deadline.
- Solve questions 1, 2 and 3 by hand with necessary explanations of your steps. You may write your answers to a paper by hand, scan the papers and add them to the PDF document. In that case, please make sure that the scans are readable.
- For questions 4 and 5, write necessary `Python` programs and add the screenshots of the execution results to the document. Make sure that the output of the programs are appropriately structured. Submit the `Python` codes for questions 4 and 5 as well.
- Please make sure that you write your full name and student identification number to every file you submit.
- If you have any questions, please contact Rumeysa Aslıhan Ertürk via rumeysa.erturk@itu.edu.tr.

Question 1

Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in parts (ii) and (iii).

(a) $\frac{4}{5} + \frac{1}{3}$

(b) $\frac{4}{5} \cdot \frac{1}{3}$

(c) $\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20}$

(d) $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

Question 2

Use the 64-bit-long real format to find the decimal equivalent of the following floating-point machine numbers.

(a) 0 10000001010 1001001100000000000000000000000000000000000000000000000000

(b) 1 10000001010 1001001100000000000000000000000000000000000000000000000000

(c) 0 01111111111 0101001100000000000000000000000000000000000000000000000000

(d) 0 01111111111 0101001100000000000000000000000000000000000000000000000001

Question 3

Suppose a machine with a floating point system $(\beta, t, L, U) = (10, 8, -50, 50)$ is used to calculate the roots of the quadratic equation

$$ax^2 + bx + c = 0, \tag{1}$$

where a, b, and c are given, real coefficients.
For each of the following, state the numerical difficulties that arise if one uses the standard formula for computing the roots. Explain how to overcome these difficulties (when possible).

(a) $a = 1$; $b = -10^5$; $c = 1$.

(b) $a = 6 \cdot 10^{30}$; $b = 5 \cdot 10^{30}$; $c = -4 \cdot 10^{30}$.

(c) $a = 10^{-30}$; $b = -10^{30}$; $c = 10^{30}$.

Question 4

(a) A fundamental property of real numbers is given by the distributive law

$$(x + y)z = xz + yz. \tag{2}$$

In this question, you are going to check whether floating point numbers obey this law. To do this, you are going to write a `Python` program that runs through a loop 10,000 times and each time draws three random numbers in the interval (0,1) and then checks whether the law holds (whether the two sides of Equation 2 are equal) for these numbers. Count down how many times the law fails, and at the end, print percentage of times that it failed. Print also a set of three numbers for which the law failed.

(b) Repeat (a), but test the associative law $(x + y) + z = x + (y + z)$ instead.

(c) Repeat (a), but test the commutative law $x + y = y + x$ instead.

(d) Repeat (a) and (b), but test the associative and commutative laws for multiplication instead.

Question 5

Implement *Bisection Method* as a recursive function in `Python`. Use your function to find solutions accurate to within 10^{-5} for the following problems.

(a) $3x - e^x = 0, x \in [1, 2]$.

(b) $x + 3\cos x - e^x = 0, x \in [0, 1]$.

(c) $x^2 - 4x + 4 - \ln x = 0, x \in [1, 2]$ and $x \in [2, 4]$, respectively.

(d) $x + 1 - 2\sin(\pi x) = 0, x \in [0, 0.5]$ and $x \in [0.5, 1]$, respectively.