BLG202E Numerical Methods in Comp. Eng.

Spring 2022 - Homework II

Due: April 12, 2022

By turning in this assignment, II agree by the ITU honor code and declare that all of this is my own work.

Important Notes

- You are required to submit a PDF document and Python source codes to Ninova before the deadline.
- Solve all the questions by hand with necessary explanations of your steps. You may write your answers to a paper by hand, scan the papers and add them to the PDF document. In that case, please make sure that the scans are readable.
- For questions 1, 2 and 5, write necessary `Python` programs and add the screenshots of the execution results to the document. Make sure that the output of the programs are appropriately structured. Submit the `Python` codes as well.
- Please make sure that you write your full name and student identification number to every file you submit.
- If you have any questions, please contact Evren Kanalıcı via kanalici20@itu.edu.tr.

Question 1

The following four methods are expected to compute $21^{1/3}$. Assuming $p_0 = 1$,

1. Rank them in order analytically, based on their apparent speed of convergence.
2. Write a `Python` program to show convergence of each method providing step-by-step output.

$$p_n = \frac{20p_{n-1} + 21/p_{n-1}^2}{21} \tag{a}$$

$$p_n = p_{n-1} - \frac{p_{n-1}^3 - 21}{3p_{n-1}^2} \tag{b}$$

$$p_n = p_{n-1} - \frac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21} \tag{c}$$

$$p_n = (\frac{21}{p_{n-1}})^{1/2} \tag{d}$$

Question 2

Derive root finding iterative estimations for both *Newton* and *Secant* methods to find a solution to equation;

$$exp(x^2 - 2) = 3\ln(x) \tag{1}$$

By using your obtained formulations, write a `Python` program to observe convergence for followings;

1. The *Newton* method starting with $x_0 = 1.5$.
2. The *Newton* method starting with $x_0 = 0.1$.
3. The *secant* method starting with $x_0 = 1.5$, $x_1 = 1.4$.

Each of your program must output step-by-step estimated values for each iteration.

Question 3

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix} \tag{2}$$

1. The matrix A can be decomposed using partial pivoting as $PA = LU$, where L is unit lower triangular, U is upper triangular, and P is a permutation matrix. Find the 4×4 matrices U, L, and P.
2. Given the right-hand-side vector $\mathbf{b} = (26, 9, 1, -3)^T$, find $\mathbf{x}$ that satisfies $A\mathbf{x} = \mathbf{b}$.

Question 4

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{3}$$

1. Obtain the inverse of matrix A using LU decomposition

Question 5

x_i	0	0.15	0.31	0.5	0.6	0.75
y_i	1.0	1.004	1.031	1.117	1.223	1.422

Table 1: Sample points

1. Find the *least squares* polynomials of degrees 1, 2, and 3 for the data in above table 1. Compute the error E for each case.
2. Plot the given data points and the estimated polynomials in your `Python` program.