Supply Chain Disruption Risks as a Trade Barrier*

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Abstract

By mapping unobservable supply chain disruption risks to observed inventory and trade patterns through a model, I measure these risks and quantify their impacts on international trade. I develop a tractable multi-country, multi-sector general equilibrium trade model where firms hold inventories to buffer against supply chain disruptions, facing a trade-off between sourcing from expensive but reliable suppliers or carrying inventories. The model rationalizes three empirical regularities on inventories in international trade: (1) inventory intensity increases with import intensity; (2) inventory intensity increases with importing distance; and (3) inventories of sectors that use a wider range of intermediate inputs are more sensitive to importing distance. I calibrate the model parameters that govern the risks of delivery delays and iceberg trade costs using inventory and bilateral trade flow data of 17 economies. Counterfactual analysis shows that eliminating cross-border delivery delay risks would increase global international trade value by 8.2 percent and increase consumption-equivalent welfare by 0.4–2.1 percent across countries. Such risks account for 3.2 percent of total trade barriers, equivalent to imposing a 25.8 percent tariff.

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1 Introduction

Recent dramatic events have highlighted the vulnerability of international supply chains to disruptions. Beyond these headline-grabbing incidents, delivery delays are a constant challenge for importers who rely on the steady flow of intermediate inputs from abroad. At any moment, the failure of specific parts to arrive on schedule can halt production and reduce profits. Businesses manage these risks by sourcing from more reliable—but potentially costlier—suppliers or by maintaining expensive inventories of intermediate inputs.

While the literature on international trade has documented how importers manage supply chain disruption risks, there are two significant challenges that have prevented a thorough quantitative assessment of the size of the trade barriers created by idiosyncratic supply chain disruption risks. First, supply chain disruption risks are inherently difficult to measure directly because they can arise from various sources that are not all observed. Second, even when such risks are directly measured, translating them into economic impacts on trade flows and welfare requires a model.

This paper addresses these challenges by developing and quantifying a multi-country multi-sector general equilibrium model of inventory and trade in the presence of supply chain disruption risks. Given a set of parameters, the model maps the underlying risks to observed inventory and trade patterns that are shaped by those risks. I calibrate the model's parameters governing the supply chain disruption risks to country-sectoral inventory and bilateral trade flow data of 17 major economies. Once the underlying risks are recovered, I then implement quantitative exercise that assess the impact of supply chain disruption risks on the structure of trade and on how these risks affect the gains from trade. The counterfactual analysis shows that removing cross-border supply chain disruption risks would increase international trade value by 8.2 percent and raise consumption-equivalent welfare by 0.4-2.1 percent varying across countries, which implies that such risks account for 3.2 percent of total trade barriers, equivalent to imposing a 25.8 percent tariff.

The core mechanism of the model that links inventories to supply chain disruption risks is simple. In each sector, firms source inputs that are then customized and sold downstream for further processing. These firms face two types of risk. On the one hand, demand for their customized input fluctuates. On the other hand, firms face a source-specific risk that an order for inputs will be delayed until after demand for their customized input has peaked. A firm that is hit by a large demand shock and/or a delivery delay will be forced into a costly stockout situation. This creates the stockout-avoidance motive for holding inventories.

Everything else equal, firms importing inputs from a particular source hold inventories that are increasing in the size of the supply chain disruption risk that they face from that source. Holding inventories is costly, and this discourages the use of risky suppliers. In this respect, supply chain disruption risk raises the effective cost of importing and plays a role like standard iceberg-type trade costs. To sum up, reliance on a source decreases with the source-specific supply chain disruption risks, and greater reliance on high-risk sources implies a higher overall inventory intensity.

These effects are especially pronounced in sectors with complex input requirements. Sectors that rely on a wide array of components are particularly vulnerable because delays in any single input can stall the entire production process. The model assumes that inputs are complementary and that the on-time delivery of an input bundle requires the on-time delivery of every single input. As a result, sectors with more complex input requirements face higher effective supply chain disruption risks when sourcing from the same set of countries.

Standard models of inventory dynamics are notorious for involving a complicated dynamic programming problem. In these models, one needs to keep track of enormous state space for each single agent creating a serious problem due to the curse of dimensionality. As my focus is on the cross-sector, cross-country structure of trade costs induced by idiosyncratic risks, I sidestep this problem in two ways. First, I assume that each sourcing firm is measure zero so that the distribution of shocks hitting sourcing firms is fixed over time. Second, I assume that there is an "off-season" market where firms that find themselves with excess supplies of a particular input can rebalance their inventories by trading in this market. In this way, inventory intensities across firms vary across sectors due to the magnitude of the idiosyncratic risks that they face but is stable within sectors in the model's steady state. It is the variation in inventory intensities across sectors and across foreign suppliers to that sector that are the objects the model is designed to be calibrated to infer the underlying size of the risk and the magnitude of the trade barrier that it creates.

The model rationalizes three empirical regularities that I document on inventories in international trade. First, inventory intensity increases with import intensity. Second, inventory intensity increases with the distance between supplier and buyer. Third, inventories of complex-input sectors are even more sensitive to distance. Additionally, I document that ocean shipping schedule reliability decreases with distance. Consider the link between the model and the facts. First, to enjoy the cheap unit cost of foreign goods, importers must bear delivery delay risks and hold inventories. Second, since delivery delay risks increase with distance, trading with more distant countries entails higher inventory holdings. Third, inventories of complex-input sectors are more sensitive to distance because when inputs are complementary, delay of one input holds up the entire production process.

I assemble a dataset of inventories covering 17 major economies, which I supplement with bilateral trade flows, GDP, and labor endowment. I divide sectors into two types based on their input-complexity, defined as the Herfindahl-Hirschman indexes of input shares. I assume that a bilateral shipping route is captured by three elements, the shipping distance, the exporter port, and the gateways that the route crosses. To reach a destination on time, the shipment must be on time for each three elements. Accordingly, the delivery delay risk of a single shipping route is determined by three parameters each capturing conditions for one element.

The goal of the quantification exercise is to disentangle delivery delay risks from generic iceberg trade costs by relying on their different impacts on inventories, and to quantify them by exploiting the rich variations in inventories and bilateral trade flows across countries and sectors. I jointly calibrate the bilateral delivery delay frictions and iceberg trade costs to match the model moments with bilateral trade flows and inventory data. The quantification results reveal that geographical distance accounts for a significant portion of variations in the average delivery delay risks faced by countries.

I then use the calibrated model to conduct two counterfactual exercises. In the first exercise, to quantify the impact of delivery delay risks on international trade, I remove all cross-border delivery delay risks, and find that global cross-border trade value would increase by 8.2 percent. This suggests that cross-border delivery delay risks account for 3.2 percent of total trade barriers, equivalent to imposing a 25.8 percent tariff. Complex-input sectors respond by both increasing imports and reducing inventory intensity more sharply than simple-input sectors. It also generates gains in consumption-equivalent welfare which vary across countries, ranging from 0.4-2.1 percent, and those who are exposed to more risks in the baseline gain more in the counterfactual.

To further leverage the rich geographical components of the quantitative model, the second exercise investigates the heterogeneous impact of existing maritime infrastructure on the trade and welfare of different countries. Specifically, I increase the delay risks for shipping routes crossing three major gateways—Suez, Panama, and Malacca—by 5 percent, respectively. This analysis not only underscores the critical role of individual maritime infrastructures in shaping global trade and highlights the vulnerability of global supply chains, but also reveals the heterogeneous impacts on countries. For instance, South Korea is geographically distant from the Malacca Strait, yet even minor inefficiencies in the Strait can generate nontrivial welfare change in South Korea. Understanding the contrast between a few key gateways and their extensive effects on numerous countries is therefore of great interest.

Related literature

The paper contributes to four strands of literature. The first literature studies the nature of trade barriers. The international trade economists have paid long-lasting efforts in understanding the components of trade barriers and their magnitudes (Anderson and van Wincoop, 2004). A majority of works in this literature use direct measures (Hummels, 2007; Limão and Venables, 2001; Olken and Barron, 2009; Djankov et al., 2010; Sequeira, 2016), gravity (Anderson and van Wincoop, 2003; Head and Ries, 2001; Johnson and Moxnes, 2023; Yi, 2010), or exogenous shocks (Steinwender, 2018; Feyrer, 2021) to study the size of overall trade costs or specific trade costs. Among them, my work is mostly related to those studying time (Hummels and Schaur, 2013; Djankov et al., 2010; Evans and Harrigan, 2005; Antràs, 2023; Harrigan and Venables, 2006), and uncertainty (Hummels and Schaur, 2010; Leibovici and Adamopoulos, 2024) as trade barriers. My work adds to the literature by studying a new source of trade barriers: the supply chain disruption risk. I make use of inventory to infer this risk, which is hard to measure or observe. To the best of my knowledge, this is the first one to include inventory in both model and quantification in a multi-country multi-sector framework to study trade barriers, and the first one to measure risks as a trade barrier.

This paper also contributes to a literature that studies inventories in international trade. Two-country single-sector models have been dominating the field and generated fruitful results in understanding long-term trends and short-term dynamics of inventories (Alessandria et al., 2010, 2013, 2022; Carreras-Valle, 2021). What is less understood is the cross-country (Guasch and Kogan, 2001) and cross-sector variations in inventory and especially how they interact with different trade barriers. I contribute to this literature by documenting novel facts about the relationship between inventory, geography, and sectoral heterogeneity in input structure, and by developing a multi-country, multi-sector model to explain these facts and to quantify the contributors to inventory variations across countries and sectors.

The paper also engages with the broad literature on sectoral heterogeneities in global supply chains (Yi, 2010; Caliendo and Parro, 2015; Antràs and Gortari, 2020; Johnson and Moxnes, 2023). Among these works, my research is closely related to the subset that examines the role of trade barriers in the geography of global value chains (Yi, 2010; Antràs and Gortari, 2020; Johnson and Moxnes, 2023). Most of this research focuses on the position or upstreamness of a sector in a vertical supply chain (often described as a "snake" input-output structure), centering around the intuition that iceberg trade costs accumulate along the value chain. My work differs by employing a flat (or "spider") input-output structure and by focusing on the coordination difficulties between inputs. Through this lens, I move beyond iceberg trade costs and shed light on the role of supply chain disruption risks in trade patterns. A growing subset of this literature studies the optimal policies in increasing the resilience in supply chains (Grossman et al., 2023, 2024). My work differs from theirs in evaluating the magnitude of idiosyncratic supply chain disruption risks per se.

Finally, my work is related to a classic literature in macroeconomics on "time-to-build". Among recent works in this branch, my work speaks to Liu and Tsyvinski (2024) by modeling intertemporal adjustment costs in input-output linkages within a dynamic framework. They remain within a closed economy and focus on impulse responses following TFP shocks, while I use a multi-country setup and emphasize the differences in stationary equilibrium across countries.

The rest of the paper proceeds as follows. Section 2 describes the empirical regularities. Section 3 describes the model. Section 4 describes the implementation and the results of the quantitative exercise. Section 5 conducts the counterfactual exercises. Section 6 concludes.

2 Empirical Regularities

To guide a model, I begin by examining the relationship between inventory and trade in data. The main data and empirical findings are discussed below, and further details can be found in Appendix A.

2.1 Data

Inventory. The data of inventories, intermediate inputs, and output come from the OECD input-output table 2020 version. I get the input-output table covering 64 countries from 1995 to 2018.

Each I-O table covers the input flows of two-digit sectors in the ISIC version 4. I focus on mining and manufacturing sectors.

Since the OECD input-output table provides only changes in inventories, I need to first recover the stock levels. I digitize and assemble the values of inventory stocks and output data for two-digit ISIC during 1987 and 1991 for over 25 countries from the Industrial Statistics Yearbooks released by the United Nations. I then calculate the country-sector inventory-to-output ratios in 1990, and use them to calculate the initial inventory stock using the OECD input-output table. With the initial stocks and changes in inventories, I recover forward the inventory stocks from 1995 to 2018. See A.1 for the data construction.

Bilateral trade flows. I get the bilateral trade data from UN Comtrade. I use the data for HS six-digit industries, and aggregate them into two-digit ISIC level, so that I can compute the import shares for an importing country over all the sourcing countries.

Shipping data. The shipping delay data comes from the Sea-Intelligence Global Liner Performance August 2018 Report. It contains the monthly records of schedule reliability of 69 container carriers (e.g., ACL, Evergreen, COSCO) operating on 34 trade lanes each linking two subcontinents (e.g., East Asia - North America West Coast, Indian Subcontinent - Northern Europe) in July 2017 to July 2018.

Sea distance and gateways. I get the data via the route planning function of VesselFinder.[1] Each route search requires the names of the departure and the arrival ports, and returns the distance of the route measured in nautical miles, the duration, and crossing gateways. For instance, from Rotterdam to Singapore, the length of the route is 8399 nm, the duration of the trip is 29 days 4 hours given the speed of the ship is 12 knots (22.22 kilometers per hour), and the route crosses the Dover Strait, the Gibraltar Strait, the Suez Canal, the Bab-el-Mandab Strait, and the Malacca Strait.

2.2 Empirical Facts

Fact 1: Inventory intensity increases in import intensity.

I start by examining the cross-sectional relationship between inventory and import. I run the following regression

$$\ln\left(IS_{jdt}\right) = \beta_0 + \beta_1 \ln\left(ImpShare_{jdt}\right) + \eta_{jt} + \mu_{dt} + \varepsilon_{jdt} \tag{1}$$

where IS_{jdt} denotes the inventory-sales ratios of sector j in country d in year t, $ImpShare_{jdt}$ denotes the ratio between the imported intermediate inputs and the sales revenue. η_{jt} denotes the sector-year fixed effects, μ_{dt} denotes the country-year fixed effects, and ε_{jdt} is the error term. The specification uses the destination-country-year fixed effects to control for unobserved time-varying country-specific components, such as a nationwide improvement in storage technology, that can affect the overall inventory intensity in a country, and it uses the sector-year fixed effects to control

[1] See its website at https://route.vesselfinder.com/.

for unobserved time-varying sector-specific components that can affect the inventory intensity in a sector, e.g., the shift to "just-in-time" manner in the global car producing sector.

The results are shown in column (2) in Table 1. The slope is 0.62, implying that there is strong positive correlation between inventory intensity and import intensity.

Note that the same positive correlation between inventory and import has been documented in the literature using data of a single country (Alessandria et al., 2010; Carreras-Valle, 2021). My first exercise here verifies this fact using a dataset covering more countries.

Fact 2: Inventory intensity increases in importing distance.

I then examine if inventory behaviors are systematically different due to the geography of the source countries. I run the following regression,

$$\ln\left(IS_{jdt}\right) = \beta_0 + \beta_1 \ln\left(ImpDistance_{jdt}\right) + \beta_2 \ln\!\left(ImpShare_{jdt}\right) + \eta_{jt} + \mu_{dt} + \varepsilon_{jdt}, \tag{2}$$

where $ImpDistance_{jdt}$ denotes the import-value weighted importing distance. Notice that this specification differs from equation (1) by adding the trade-weighted importing distance. I am interested in the coefficient β_1 and the variation driving it arises from the cross-sectional differences between destination countries and sectors in their geographic distances to their source countries.

The results are shown in column (4) in Table 1. The result indicates that the same extent of reliance on imports, those sectors importing from faraway countries are more inventory-intensive than those importing from close countries.

Fact 3: Inventory intensity of sectors using more complex inputs is more sensitive to importing distance.

Guided by the conjecture that sectors that use a wider range of inputs are intrinsically more sensitive to reliable flows of supply, I explore inventory patterns in international trade along one particular dimension of sectoral heterogeneity: input complexity.

Following Levchenko (2007), I define input complexity of a sector using the Herfindahl-Hirschman Index aggregating the share of each sector's expenditure over its upstream inputs.

$$HHI_j = \sum_k s_{kj}^2 \tag{3}$$

where s_{kj} denotes the share of sector j's expenditure on goods produced by sector k. I then decide a complex-input sector to be those whose HHI is above median

$$\mathbf{1}\left(complex\right)_j = \begin{cases} 1, & HHI_j \leq median\left(HHI_k\right) \\ 0, & HHI_j > median\left(HHI_k\right) \end{cases}.$$

The results are shown in column (6) in Table 1. Intuitively, a higher HHI of input use indicates

Table 1: Inventory Intensity, Trade, and Geography

	(1)	(2)	(3)	(4)	(5)	(6)
ln(import share)	0.592***	0.624***	0.577***	0.610***	0.575***	0.607***
	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)	(0.020)
ln(distance)			0.268***	0.261***	0.205***	0.193***
			(0.038)	(0.038)	(0.039)	(0.039)
complexity=1 × ln(distance)					0.134***	0.147***
					(0.039)	(0.037)
Constant	-0.732***	-0.686***	-3.018***	-2.913***	-3.075***	-2.980***
	(0.034)	(0.034)	(0.319)	(0.322)	(0.322)	(0.324)
Observations	8853	8853	8853	8853	8853	8853
Adjusted R^2	0.5603	0.6074	0.5629	0.6100	0.5635	0.6108
Country FE	Yes	No	Yes	No	Yes	No
Sector FE	Yes	No	Yes	No	Yes	No
Year FE	Yes	No	Yes	No	Yes	No
Country-Year FE	No	Yes	No	Yes	No	Yes
Sector-Year FE	No	Yes	No	Yes	No	Yes

Robust standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.001$

Notes: All columns have inventory-sales ratios measured in log as their dependent variables. Columns (2), (4), and (6) report the regression results for specification (1), (2), and (4), respectively. Their counterparts in columns (1), (3), and (5) differ from them by controlling for country fixed effects, sector fixed effects, and year fixed effects separately.

that a sector uses its upstream inputs in a more concentrated manner, or that its inputs consist of a smaller range, while a lower HHI means that a sector's production depends on a wider range of inputs. In the spirit of Kremer (1993) and Harrigan and Venables (2006), smooth production relies on the seamless coordination of intermediate inputs that complement each other. If one part is missing, it can render the the parts that are already available or finished on time unusable. Hence, a natural prediction is that sectors whose production relies on a wider range of inputs are more sensitive to the timeliness of their intermediate input deliveries and are more likely to hold inventories to buffer against risk.

$$\ln\left(IS_{jdt}\right) = \beta_0 + \beta_1 \mathbf{1}\left(complex\right)_j \times \ln\left(ImpDistance_{jdt}\right) + \beta_2 \ln\left(ImpDistance_{jdt}\right) + \beta_3 \ln\left(ImpShare_{jdt}\right) + \eta_{jt} + \mu_{dt} + \varepsilon_{jdt} \quad (4)$$

Shipping Delays and Geography. Next, I provide empirical evidence showing that delivery delays can indeed arise from geographical distance. Each unit of observation in the data is the schedule reliability of a shipping line connecting two subcontinents in a specific month. Each shipping line is operated by a container carrier, such as Evergreen or MCL. A single container carrier operates one shipping line between two subcontinents but can manage multiple shipping lines across different subcontinent pairs. For example, Evergreen operates both the West Coast North America to East Asia line and the line linking Europe to the East Coast of North America.

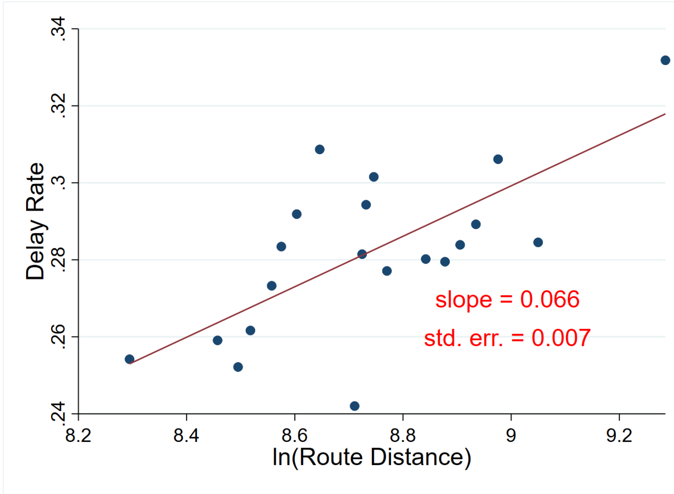


Figure 1: Delay Rate and Route Distance

Notes: The figure shows the binned scatterplot on the relationship between delay rates and the distance of shipping routes. Carrier-origin-month fixed effects and destination-month fixed effects are controlled. The data sample covers monthly records of schedule reliability of 69 container carriers operating on 34 trade lanes each linking two subcontinents in July 2017 to July 2018.

Figure 1 displays the correlation between route distance and delay probabilities. It clearly shows a significant positive relationship between shipping distance and shipping delay rates. The variation in the specification in the last column arises from within-carrier-destination and within-origin comparisons. For example, if Evergreen operates a line from the Middle East to East Asia, and another from the West Coast of North America to East Asia, the latter has a higher risk of delays due to its longer route distance. My specification employs carrier fixed effects to control for variations in shipping liner management or general productivity, as the ocean shipping and logistics literature documents vast heterogeneities at the shipping liner level. Additionally, it uses destination and origin fixed effects to account for conditions at the arrival and departure ports. See robustness check on other specifications and full regression results in Table A.2.

3 Model

In this section, I conceptualize supply chain disruption risks as delivery delay risks,[2] and incorporate them into a multi-country, multi-sector general equilibrium model of trade and inventory. The model features traders in many countries and many sectors who source bundles of intermediate inputs, facing idiosyncratic delivery delay risks and idiosyncratic demand uncertainties. Traders optimize their inventory holdings and orderings, trading off between revenue loss due to stockout and depreciation or obsolescence loss due to overstocking. The costs associated with buffering these risks add to conventional trade costs, which are passed through to producers, making high-risk countries less attractive for sourcing.

[2] Here I abstract from various sources where supply chain disruption risks can arise, and focus instead on the common outcome of these risks: delays in the optimal timing suggested by production or sales schemes.

Despite the high-dimensional state-space complexity of inventory management as a dynamic programming problem, the model yields tractable expressions for sector-level aggregate inventory holdings and orderings. It produces predictions consistent with the stylized facts and allows me to take it to aggregate sectoral data for quantification and counterfactual analysis.

3.1 Environment

There are multiple countries. Each country has final consumers, sectoral composite good producers, and traders. Final consumers consume goods produced by sectoral producers. For production, sectoral producers obtain intermediate inputs through traders. Sectors have different numbers of intermediate inputs. Traders serving for a sector adopt sourcing strategies that designate each input with a source country. Traders buy individual inputs from sectoral producers and assemble them into bundles.

Countries and sectors

There are N countries, each indexed by $n \in \mathcal{N}$. In each country, there are J sectors, each indexed by $j \in \mathcal{J}$. Each country n is inhabited by L_n individuals. They provide labor, consume goods, and earn dividends from traders.

Final consumers

In each country n in each period t, the representative final consumer derives utility from consuming the final consumption good. The final good $Y_{n,t}$ is a CES (Constant Elasticity of Substitution) aggregation over sectoral composite good produced in the home country,

$$Y_{n,t} = \left(\sum_{j \in \mathcal{J}} \left(\psi_n^j\right)^{\frac{1}{\xi}} \left(y_{n,t}^{c,j}\right)^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}},$$

where $y^{c,j}$ denotes final demand for sectoral composite good j, and ψ^j is the weight on sector j. Deliveries from sectoral composite good producers to the representative final consumer are not subject to delays.

Sectoral composite good producers

The sectoral composite good producer j in country n produces produce sectoral composite good $y_{n,t}^j$ using the following production technology:

$$y_{n,t}^j = A_{n,t}^j \left(L_{n,t}^j\right)^{\alpha_n^j} \underbrace{\left[\left(\left(\iota_n^j\right)^{\frac{1}{\zeta}} \left(Q_{n,t}^j\right)^{\frac{\zeta-1}{\zeta}} + \left(1-\iota_n^j\right)^{\frac{1}{\zeta}} \left(\tilde{Q}_{n,t}^j\right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \right]^{\left(1-\alpha_n^j\right)}}_{\equiv M_{n,t}^j}, \tag{5}$$

where $A_{n,t}^{j}$ is productivity, $L_{n,t}^{j}$ is labor, $M_{n,t}^{j}$ denotes the composite intermediate input, and α_n^j is the expenditure share on labor. The composite intermediate input $M_{n,t}^{j}$ is a CES aggregation of in-season and off-season market input bundles, $Q_{n,t}^{j}$ and $\tilde{Q}_{n,t}^{j}$, respectively, with weight ι_n^j for the in-season bundles.

The producer j in country n obtains all intermediate input bundles $Q_{n,t}^{j}$ and $\tilde{Q}_{n,t}^{j}$ from traders, and she sells her output $y_{n,t}^{j}$ to the final consumer and to traders that source the sectoral composite good j produced in country n as an input in their input bundles.

Sourcing strategies and traders

Traders in each sector j source input bundles by placing order x. Sectors differ in their number of upstream inputs. Let $\mathcal{U}(j) \subset \mathcal{J}$ denote the set of upstream sectors whose output is used as an input by sector j. Define a sourcing strategy for traders serving sector j as a vector

$$m(j) = \left(r_1, r_2, \cdots, r_{|\mathcal{U}(j)|}\right),$$

where $|\mathcal{U}(j)|$ is the number of upstream sectors for j, and the h-th coordinate represents a source country for the h-th input in an input bundle of j. Suppose input $k \in \mathcal{U}(j)$, is the h-th input in an input bundle of sector j, then sourcing strategy $m(j)$ assigns input k with a source country $m_k(j) = r_h$. For example, if an input bundle of the car producing sector is $(tire, engine)$, a sourcing strategy (US, CN) would mean sourcing tires from the US and engines from China, i.e., $m_{tire}(car) = US$, and $m_{engine}(car) = CN$.

Any country can be a source for any input. Denote the set containing all sourcing strategies of sector j by $\mathcal{M}(j)$, then the number of sourcing strategies for sector j is

$$|\mathcal{M}(j)| = N^{|\mathcal{U}(j)|}.$$

For each sourcing strategy $m(j)$, there is a unit mass of traders $i \in [0,1]$. Each trader uses only one sourcing strategy, i.e., for each input, he orders it from only one country. Traders sell input bundles q to the sectoral composite good producer. An input bundle sourced using strategy $m(j)$ by a trader of sector j in country n, $q_{m(j)n,t}^{j}$, aggregates each single input k using the Leontief technology:

$$q_{m(j)n,t}^{j}(i) = \min_{k \in \mathcal{U}(j)} \left\{ \chi_n^{kj} q_{m(j)n,t}^{kj}(i) \right\}, \tag{6}$$

where χ_n^{kj} denotes the fixed coefficient indicating how many units of input k are required in producing one unit of input bundle for sector j in country n, and $q_{m(j)n,t}^{kj}$ denotes the quantify of input k in the bundle. Traders can store unused input bundles and carry them forward as inventories z. Note that orders x, sales to the sectoral composite good producer q, and inventories z are all in the form of input bundles.

Traders sell input bundles to the sectoral composite good producer through two markets: the in-season and the off-season markets. The naming indicates how they value timeliness of sales differently, which is key for delivery delay risks to play a role. Since order processing is subject to a one-period lag, traders have to place their orders one period ahead. Moreover, the delivery time is stochastic: with a positive probability θ, an order will arrive later than the in-season market, but before the off-season market.[3]

In the in-season market, traders sell *differentiated* input bundles, and the market structure is monopolistic competition. The demand of the sectoral composite good producer for input bundles under strategy $m(j)$ is a CES aggregation of all varieties $q^j_{m(j)n,t}$ supplied by traders using $m(j)$,

$$Q^j_{m(j)n,t} = \left[\int_0^1 (\nu(i))^{\frac{1}{\sigma}} \left(q^j_{m(j)n,t}(i)\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where σ denotes the elasticity of substitution across varieties, and ν is an idiosyncratic demand shock. Each ν is drawn from a Pareto distribution with the cumulative distribution function $F(\nu) = 1 - \nu^{-\lambda}$ and support $[1, \infty)$. Shortages are punished in the in-season market. Having not enough available stock to meet a high demand shock ν would lead to revenue losses. In preparing available stock for the upcoming in-season market, between holding inventory and placing new orders, traders prefer inventories when new orders are subject to stochastic delays thus less reliable.

The composite in-season input bundle $Q^j_{m(j)n,t}$ is then an aggregation of input bundles from all sourcing strategies $m(j)$:

$$Q^j_{n,t} = \left(\sum_{m(j)\in\mathcal{M}(j)} \left(Q^j_{m(j)n,t}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{8}$$

where ρ is the Armington elasticity.

The off-season market is perfectly competitive. Input bundles are *homogenous* here. Traders face a common marginal value for their input bundles. This market is introduced to allow traders to even out their states and reduce dimensionality, hence maintain model tractability. The sectoral composite good producer simply adds up all homogenous input bundles from all traders under strategy $m(j)$, $\tilde{q}^j_{m(j)n,t}$:

$$\tilde{Q}^j_{m(j)n,t} = \int_0^1 \tilde{q}^j_{m(j)n,t}(i)\, di,$$

and aggregates input bundles from all sourcing strategies $m(j)$ to get the composite off-season input bundle $\tilde{Q}^j_{n,t}$,

$$\tilde{Q}^j_{n,t} = \left(\sum_{m(j)\in\mathcal{M}(j)} \left(\tilde{Q}^j_{m(j)n,t}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{9}$$

using the same Armington elasticity ρ as for the in-season bundles.

Figure 2 is a summary snapshot of a country. In the rest of the section, I first zoom in to a single trader's problem and focus on its ordering and inventory holding behaviors. I describe its

[3] Delays are one-period maximum.

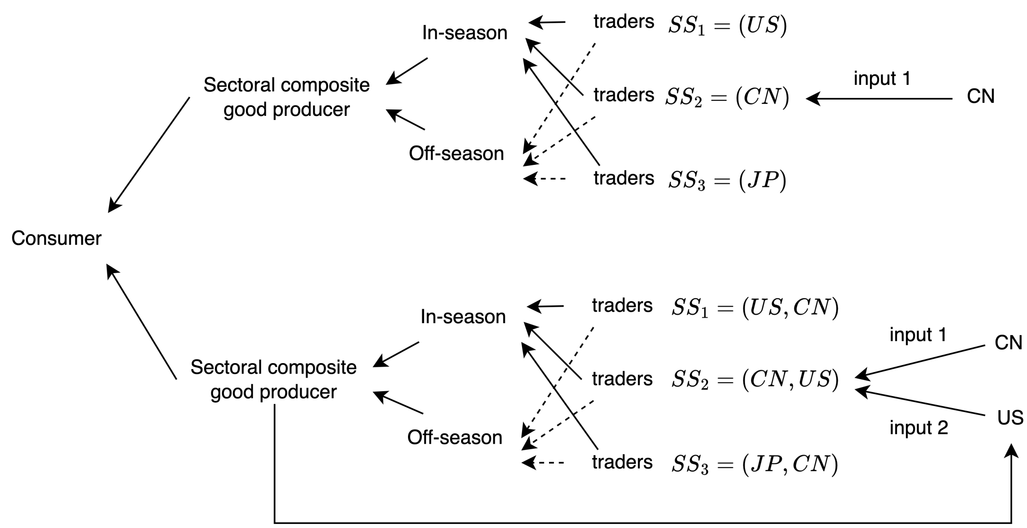


Figure 2: Setup within a country

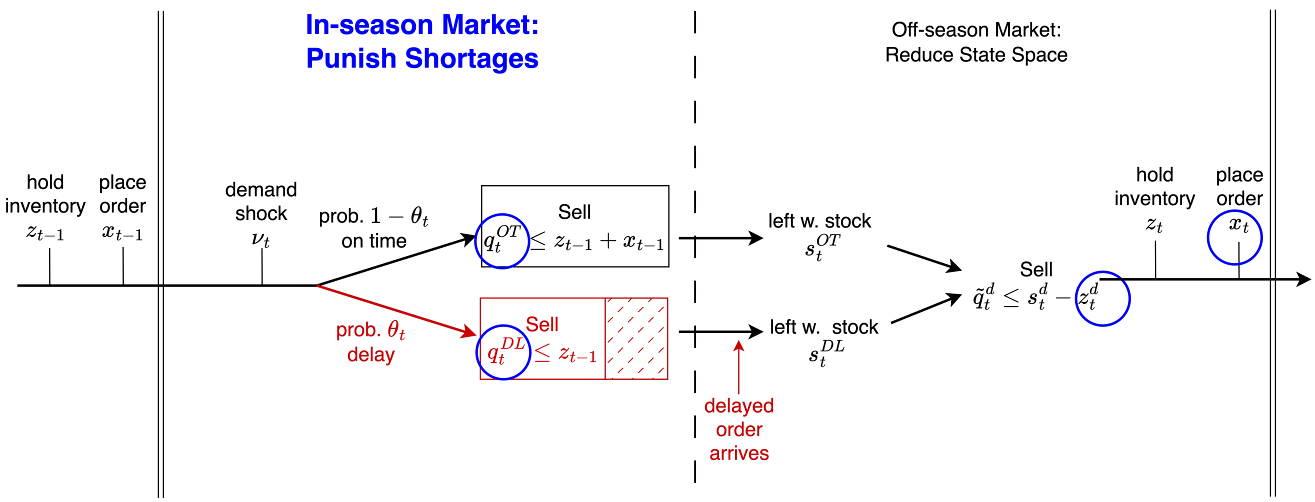


Figure 3: Timeline of a single trader

timeline and derive its policy functions. Then I aggregate single traders' optimal decisions, and define the general equilibrium.

3.2 A Single Trader's Timeline

Time is discrete. There are infinite periods. Superscripts indexing sector, and subscripts indexing the country and the sourcing strategy are suppressed for simplicity. Figure 3 visualizes the timeline of a single trader.

Beginning of period. Each trader i faces the end-of-period inventory of input bundles from last period, $z_{t-1}(i)$, and the order for input bundles he placed in the last period, $x_{t-1}(i)$.

In-season market. The in-season market is open. Traders sell input bundles to the sectoral composite good producer. The market structure is monopolistic competition, so each trader charges his own price. Each trader faces an idiosyncratic demand shock $\nu_t(i)$ from the sectoral composite good producer, and an idiosyncratic delivery delay shock of the placed order for input bundles. The order arrives on time with probability $1-\theta_t$, and is delayed with probability θ_t.

If the order arrives on time, the trader's available input bundles is the sum of his inventory and the newly delivered order. His sales quantity of input bundles q_t^{OT} needs to satisfy the availability constraint:

$$q_t^{OT}(i) \leq z_{t-1}(i) + x_{t-1}(i),$$

and after selling he is left with the off-season stock s_t^{OT}:

$$s_t^{OT}(i) = (1-\delta)\left[z_{t-1}(i) + x_{t-1}(i) - q_t^{OT}(i)\right],$$

where δ denotes the depreciation rate.

If the order is delayed, the trader's available input bundles is just his inventory. His sales quantity of input bundles q_t^{DL} needs to satisfy the availability constraint:

$$q_t^{DL}(i) \leq z_{t-1}(i).$$

The delayed order arrives after the in-season market is closed, and trader is left with the off-season stock s_t^{DL}:

$$s_t^{DL}(i) = (1-\delta)\left[z_{t-1}(i) + x_{t-1}(i) - q_t^{DL}(i)\right].$$

Off-season market. Then the off-season market is open. Traders sharing the same sourcing strategy can trade input bundles with other or sell extra input bundles to the sectoral composite good producer.

This market is perfectly competitive. The market clearing condition requires that the total supply of the off-season stock s_t, equals the total demand–the summation of inventory to hold z_t and the input bundles $\tilde{q}_t$ dumped to the sectoral composite good producer,

$$\int_0^1 s_t(i)\,di = \int_0^1 z_t(i)\,di + \int_0^1 \tilde{q}_t(i)\,di.$$

Trader i also places the new order $x_t(i)$ before period t ends.

Remarks: The off-season market plays two roles. First, it allows traders to pool their idiosyncratic risks by trading with each other and resetting their states, so there is no need to track individual traders over time. Second, it enables traders to sell any excess supplies to the sectoral composite good producer.

To understand this second role, consider the difference between two types of inventories: s_t and z_t. These inventories differ based on when goods held by traders are measured. s_t, or "off-season stock," is measured after the current in-season market and delayed orders arrive, but before the off-season market. It includes both goods that traders voluntarily retain from the in-season market and those they passively receive due to delays, making s_t influenced by both forward-looking and backward-looking factors. In contrast, z_t, held as "safety stock," is measured before the upcoming in-season market and is deliberately carried forward to buffer against future risks. If traders could only trade among themselves, off-season stock would automatically equal safety stock, making it difficult to distinguish whether traders hold inventories to buffer against future risks or simply because they are forced to stay with leftovers.

3.3 A Single Trader's Optimization Problem

Value function

Denote the price in the off-season market with ω_t, and the ordering cost for input bundles with c_t. The Bellman equation of a trader is,

$$
\begin{aligned}
V^d\left(z_{t-1}(i), x_{t-1}(i), \nu_t(i)\right) = \max_{p_t^d(i), q_t^d(i), z_t^d(i), x_t^d(i)} \; & \underbrace{p_t^d(i)\, q_t^d(i)}_{\text{in-season mkt. revenue}} \\
& \underbrace{-\omega_t \left[z_t^d(i) - (1-\delta)\left(z_{t-1}(i) + x_{t-1}(i) - q_t^d(i)\right)\right]}_{\text{off-season mkt. net revenue}} \\
& - \underbrace{c_t x_t^d(i)}_{\text{ordering cost}} + \beta \underbrace{\mathbb{E}V\left(z_t^d(i), x_t^d(i)\right)}_{\text{expected future value}}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
& s.t. \\
& q_t^d(i) = \min\left\{\underbrace{D_t\left(p_t^d(i), \nu_t(i)\right)}_{\text{in-season demand}}, \underbrace{a_t^d(i)}_{\text{avail. stock}}\right\}, \\
& a_t^d(i) = \begin{cases} z_{t-1}(i) + x_{t-1}(i), & d = OT \\ z_{t-1}(i), & d = DL \end{cases},
\end{aligned}
$$

where d denotes the delivery state, on-time OT, or delayed DL, β denotes the discount factor. The first term represents the in-season market revenue. $D_t\left(p_t^d(i), \nu_t(i)\right)$ is the function of in-season demand, and $a_t^d(i)$ denotes the available stock for the in-season market. The second term represents the net revenue from the off-season market, the third term is the ordering costs, and the fourth term is the expected future value, which can be expanded as

$$
\mathbb{E}V\left(z_t^d(i), x_t^d(i)\right) = (1-\theta_{t+1})\,\mathbb{E}_\nu V^{OT}\left(z_t^d(i), x_t^d(i), \nu\right) + \theta_{t+1}\mathbb{E}_\nu V^{DL}\left(z_t^d(i), x_t^d(i), \nu\right). \tag{11}
$$

Off-season market price ω_t, and ordering cost for input bundles c_t are exogenously given for each individual trader, and will be determined in equilibrium.

Having set up each single trader's optimization problem, I am ready to derive his policy functions. Consistent with the timeline, I will start by solving for the trader's optimal pricing p_t^d in the in-season market, which simultaneously determines his optimal sales quantity q_t^d and stock left for off-season market s_t^d. Then I solve for the optimal end-of-period inventory z_t^d, and new order x_t^d. In particular, I will focus on the trade-equilibrium where positive price ω_t exists to clear the off-season market, and show that traders' decisions on the end-of-period inventory z_t, and order x_t are symmetric in equilibrium and can be expressed in closed form.

Optimal in-season market pricing

In this subsection, I characterize the optimal behaviors in the in-season market, and make the point that shortages are punished in this market.

Following equation (7), an individual trader faces the following demand:

$$D_t\left(p_t^d(i), \nu_t(i)\right) = \nu_t(i)\left(\frac{p_t^d(i)}{P_t}\right)^{-\sigma} Q_t,$$

where Q_t is the sectoral composite good producer's aggregate demand for input bundles sourced via a specific sourcing strategy, and P_t is the corresponding price index. Then given the off-season market price ω_t, a trader follows a cutoff rule to decide the optimal price p_t^* to charge:

$$p_t^{d*}(i) = \begin{cases} \hat{p}_t \equiv \frac{\sigma}{\sigma-1}(1-\delta)\,\omega_t, & \nu_t(i) \leq \hat{\nu}_t^d(i) \\ \left(\frac{a_t^d(i)}{\nu_t(i)Q_t}\right)^{-\frac{1}{\sigma}} P_t, & \nu_t(i) > \hat{\nu}_t^d(i) \end{cases} \tag{12}$$

where $\hat{p}_t$ denotes the "cutoff price", a price such that the trader finds it indifferent between selling one more unit to the in-season market or to the off-season market, i.e., the marginal values in both markets are the same. $\hat{\nu}_t(i)$ denotes the "stockout cutoff". It is defined as the level of a demand shock at which the trader sells out his available stock at $\hat{p}_t$, and its expression can be obtained by equating both branches in (12),

$$\hat{\nu}_t^d(i) \equiv \frac{a_t^d(i)}{\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t}.$$

Clearly the expression agrees with the intuition that stockout is less likely to happen when the available stock level $a_t^d(i)$ is high.

Immediately following (12), one can rewrite the optimal price conditional on stockout as

$$p_t^{d*}(i) = \left(\frac{\hat{\nu}_t^d(i)}{\nu_t(i)}\right)^{-\frac{1}{\sigma}} \hat{p}_t, \tag{13}$$

which clearly shows that the optimal price is bounded from below by the "cutoff price" $\hat{p}_t$, and increases in the realization of the demand shock.

The pricing rule is intuitive. When a trader is hit by a low demand shock, he would rather sell to the off-season market than push all his stock to the in-season market and end up setting a low price. In this case, the selling price in the in-season market does not change to the value of ν_t. When a trader is hit by a high demand shock, however, he would sell out his available, and the selling price would increase in ν_t to let the demand just equal the available stock.

Having derived the optimal pricing in the in-season market, I can write out the maximized revenue

$$R_t\left(a_t^d\left(i\right), \nu_t\left(i\right)\right) = p_t^{d*}\left(i\right) D_t\left(p_t^{d*}\left(i\right), \nu_t\left(i\right)\right), \tag{14}$$

where the subscript t in notation $R_t\left(\cdot, \cdot\right)$ indicates that the revenue is also a function of aggregate variables that is exogenously given to individual trader i.

Note that conditional on stockout, plugging equation (13) into (14) yields

$$\begin{aligned} R_t\left(a_t^d\left(i\right), \nu_t\left(i\right)\right) &= a_t^d\left(i\right)\left(\frac{\hat{\nu}_t^d\left(i\right)}{\nu_t\left(i\right)}\right)^{-\frac{1}{\sigma}} \hat{p}_t \\ &= \left(a_t^d\left(i\right)\right)^{1-\frac{1}{\sigma}} \left(\nu_t\left(i\right)\right)^{\frac{1}{\sigma}} Q_t^{\frac{1}{\sigma}} P_t. \end{aligned} \tag{15}$$

Clearly, when the demand elasticity $\sigma > 1$, the stockout revenue increases in the available stock and the idiosyncratic demand shock. This way, shortages are punished.

Optimal inventory holding and ordering

In this subsection, I first show qualitatively why inventories are more appealing to traders than new orders with the presence of delivery delay risks. Then I show how my assumptions on the existence of the off-season market and the Pareto distribution of idiosyncratic shocks help deliver closed-form optimal inventory holdings that do not depend on individual traders' states. Those expressions serve for the tractability of the quantitative analysis.

Plugging in the optimal pricing into the Bellman equation, the optimization problem is simplified to be

$$\begin{aligned} V^d\left(z_{t-1}\left(i\right), x_{t-1}\left(i\right), \nu_t\left(i\right)\right) = \max_{z_t^d(i), x_t^d(i)} & \underbrace{R_t\left(a_t^d\left(i\right), \nu_t\left(i\right)\right)}_{\text{in-season mkt. revenue}} \\ & \underbrace{-\omega_t\left[z_t^d\left(i\right) - \left(1-\delta\right)\left(z_{t-1}\left(i\right) + x_{t-1}\left(i\right) - q_t^{d*}\left(i\right)\right)\right]}_{\text{off-season mkt. net revenue}} \\ & - \underbrace{c_t x_t^d\left(i\right)}_{\text{ordering cost}} + \beta \underbrace{\mathbb{E}V\left(z_t^d\left(i\right), x_t^d\left(i\right)\right)}_{\text{expected future value}}, \end{aligned} \tag{16}$$

$$s.t.$$

$$a_t^d(i) = \begin{cases} z_{t-1}(i) + x_{t-1}(i), & d = OT \\ z_{t-1}(i), & d = DL \end{cases}.$$

where $q_t^{d*}(i) = D_t\left(p_t^{d*}(i), \nu_t(i)\right)$, and it is a function of $a_t^d(i)$ and $\nu_t(i)$.

Applying the envelope theorem to (16) and iterate the results forward for one period yields the following:

1. conditional on that order x_t^d will be delivered on time in period $t+1$, the marginal value of inventory $z_t^d(i)$ is

$$V_z^{OT}\left(z_t(i), x_t^d(i), \nu_{t+1}(i)\right) = \begin{cases} \frac{\partial R_t\left(z_t^d(i)+x_t^d(i), \nu_{t+1}(i)\right)}{\partial z_t^d(i)}, & \nu_{t+1}(i) > \hat{\nu}_{t+1}^{OT}\left(z_t^d(i) + x_t^d(i)\right) \\ (1-\delta)\,\omega_{t+1}, & \nu_{t+1}(i) \leq \hat{\nu}_{t+1}^{OT}\left(z_t^d(i) + x_t^d(i)\right) \end{cases}, \quad (17)$$

and the marginal value of order $x_t^d(i)$ is

$$V_x^{OT}\left(z_t^d(i), x_t^d(i), \nu_{t+1}(i)\right) = \begin{cases} \frac{\partial R_t\left(z_t^d(i)+x_t^d(i), \nu_{t+1}(i)\right)}{\partial x_t^d(i)}, & \nu_{t+1}(i) > \hat{\nu}_{t+1}^{OT}\left(z_t^d(i) + x_t^d(i)\right) \\ (1-\delta)\,\omega_{t+1}, & \nu_{t+1}(i) \leq \hat{\nu}_{t+1}^{OT}\left(z_t^d(i) + x_t^d(i)\right) \end{cases}. \quad (18)$$

2. conditional on that order x_t^d will be delayed in period $t+1$, the marginal value of inventory $z_t^d(i)$ is

$$V_z^{DL}\left(z_t^d(i), x_t^d(i), \nu_{t+1}(i)\right) = \begin{cases} \frac{\partial R_t\left(z_t^d(i), \nu_{t+1}(i)\right)}{\partial z_t^d}, & \nu_{t+1}(i) > \hat{\nu}_{t+1}^{DL}\left(z_t^d(i)\right) \\ (1-\delta)\,\omega_{t+1}, & \nu_{t+1}(i) \leq \hat{\nu}_{t+1}^{DL}\left(z_t^d(i)\right) \end{cases}, \quad (19)$$

and the marginal value of order $x_t^d(i)$ is

$$V_x^{DL}\left(z_t^d(i), x_t^d(i), \nu_{t+1}(i)\right) = (1-\delta)\,\omega_{t+1}. \quad (20)$$

Note that (17) and (18) are followed by:

$$\mathbb{E}_\nu V_z^{OT}\left(z_t^d(i), x_t^d(i), \nu\right) = \mathbb{E}_\nu V_x^{OT}\left(z_t^d(i), x_t^d(i), \nu\right), \quad (21)$$

which implies that conditional on that the order arrives on time, the marginal values of inventory and order are the same.

How would inventories be more preferred to new orders then? Taking derivatives with respect to $z_t^d(i)$ and $x_t^d(i)$ on both sides of the expanded expected future value (11) yields:

$$\mathbb{E}V_z\left(z_t^d(i), x_t^d(i)\right) = (1-\theta_{t+1})\,\mathbb{E}_\nu V_z^{OT}\left(z_t^d(i), x_t^d(i), \nu\right) + \theta_{t+1}\mathbb{E}_\nu V_z^{DL}\left(z_t^d(i), x_t^d(i), \nu\right), \quad (22)$$

$$\mathbb{E}V_x\left(z_t^d(i), x_t^d(i)\right) = (1-\theta_{t+1})\,\mathbb{E}_\nu V_x^{OT}\left(z_t^d(i), x_t^d(i), \nu\right) + \theta_{t+1}\mathbb{E}_\nu V_x^{DL}\left(z_t^d(i), x_t^d(i), \nu\right). \quad (23)$$

Subtracting (23) from (22) yields

$$\mathbb{E}V_z\left(z_t^d(i), x_t^d(i)\right) - \mathbb{E}V_x\left(z_t^d(i), x_t^d(i)\right) = \theta_{t+1}\left[\mathbb{E}_\nu V_z^{OT}\left(z_t^d(i), x_t^d(i), \nu\right) - \mathbb{E}_\nu V_x^{DL}\left(z_t^d(i), x_t^d(i), \nu\right)\right], \quad (24)$$

which immediately follows (21). Equation (24) reveals that the difference of the expected marginal values of inventory and new order arises from the delay case. Equations (15), (19), and (20) together implies that this difference is greater than zero, and the gap increases in the delay risk θ_{t+1}. Intuitively, safety inventories and new orders are two options to reach the ideal level of available stock for the upcoming in-season market. Inventory holding is more appealing because they are immediately available and thus reliable while new orders can be subject to delivery delay risks; with higher the delay risks, inventories are more preferred.

Knowing qualitatively how delivery delay risks incentivize inventory holdings, next I move to show how to derive closed-form optimal inventory levels to serve for tractable quantitative analysis.

Proposition 1. (Optimal Inventory Holding) *Given off-season market prices in the current period ω_t and the future period ω_{t+1}, the current unit ordering cost of an input bundle c_t, the aggregate demand for input bundles sourced via a sourcing strategy in the future period, Q_{t+1}, and its price index P_{t+1}, the optimal end-of-period inventory holding of a single trader using that sourcing strategy is,*

$$z_t^{d*}(i) = \underbrace{\theta_{t+1}^{\frac{1}{\lambda}}}_{\text{delay risk}} \underbrace{\left(\frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}}\right)^{\frac{1}{\lambda}}}_{\text{stockout avoidance}} \underbrace{\left(\frac{\omega_t - c_t}{\beta(1-\delta)\,\omega_{t+1}}\right)^{-\frac{1}{\lambda}}}_{\text{relative inventory holding cost}} \underbrace{\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}}_{\text{future demand}}, \quad (25)$$

and it increases in the delivery delay risk in the future period.

Proof. See Appendix B.1.

Equation (25) shows that, in addition to delivery delay risk, shown in the first term, the optimal inventory level is also determined by other factors. The second term captures the stockout-avoidance motive of inventory holding. Intuitively, a lower demand shock shape parameter λ implies a higher chance to draw a high demand shock; a higher demand elasticity σ, however, increases competition and lowers the markup that a trader can draw from the in-season market, as shown in both (15) and (12). Either way, traders' incentive to secure revenue in the in-season market would be affected. The third term captures the relative cost of holding inventories. The numerator is the difference between the direct cost of holding inventories in the current period ω_t, and the direct cost of its alternative–placing new orders c_t. The denominator is the lower bound of future marginal value–how much at least the inventory holding today will be worth in the future period. Intuitively, optimal inventory holding increases in the future marginal value, and decreases in the current marginal cost. The last term captures the future aggregate demand. It too incentivizes inventory holding.

Note that the optimal inventory holding of a single trader $z_t^{d*}(i)$ is a function of solely variables that are exogenously given to him. Since traders face the same exogenously given prices and demand, they choose the same optimal end-of-period inventory.

Proposition 2. (Optimal Ordering) *Given the current and future prices in the off-season market ω_t, ω_{t+1}, the current unit cost of an input bundle c_t, the aggregate demand for input bundles sourced via a sourcing strategy in the future period, Q_{t+1}, and its price index P_{t+1}, the optimal ordering amount of a single trader using that sourcing strategy is,*

$$x_t^*(i) = \left(\frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}}\right)^{\frac{1}{\lambda}} \left[\left(\frac{c_t - \beta(1-\delta)\omega_{t+1}}{\beta(1-\delta)(1-\theta_{t+1})\omega_{t+1}}\right)^{-\frac{1}{\lambda}} - \left(\frac{\omega_t - c_t}{\beta(1-\delta)\theta_{t+1}\omega_{t+1}}\right)^{-\frac{1}{\lambda}}\right] \left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}. \tag{26}$$

Proof. See Appendix B.2.

Note that in the partial equilibrium, the optimal new order $x_t^*(i)$ decreases in the delay risk. Intuitively, a higher delivery delay risk pushes more new orders to miss the profitable in-season market, which is less preferable. Note also that $x_t^*(i)$ depends on only variables exogenous to an individual trader, meaning that every trader ends up ordering the same.

3.4 Input Bundles and Delivery Delay Risks

In this subsection, I emphasize how bundling–the complementarity between inputs–works in affecting traders' exposure to delivery delay risks.

Following the Leontief bundling technology in (6), the ordering cost of an input bundle is the summation of the ordering costs for all inputs:

$$c^j_{m(j)n,t} = \sum_{k \in \mathcal{U}(j)} \chi_n^{kj} \tau_{m_k(j)n} p^k_{m_k(j),t},$$

where $p^k_{m_k(j)}$ denotes the price of sectoral composite good k produced in country $m_k(j)$, and $\tau_{m_k(j)n}$ denotes the iceberg cost incurred moving from country $m_k(j)$ to country n.

Assume that delivery shocks for each bilateral shipping routes are i.i.d draws. Then the delivery delay risk of an input bundle sourced via a sourcing strategy $m(j)$ is

$$\theta_{m(j)n} = 1 - \prod_{r \in \{m(j)\}} (1 - \mu_{rn}),$$

where $\{m(j)\}$ is a set that collects all coordinates in vector $m(j)$, i.e., all countries covered by a sourcing strategy $m(j)$, and μ_{rn} denotes the delay risk of a single shipping route connects country r and country n.

The focus on the timely delivery of an input bundle instead of a single input is in the spirit of the O-ring theory as in Kremer (1993) and Harrigan and Venables (2006), which highlights the complementarity between inputs: one missing input will fail the whole input bundle.

Proposition 3. (Delay Risks and Input Complexity) *Define the unweighted average delay risk over all sourcing strategies of sector j in country n as*

$$\bar{\theta}_n(j) = \frac{1}{|\mathcal{M}(j)|} \sum_{m(j)\in\mathcal{M}(j)} \left[1 - \prod_{r\in\{m(j)\}} (1-\mu_{rn})\right].$$

Then for any two sectors j_1 and j_2, if $|U(j_1)| < |U(j_2)|$, then the unweighted average delay risk over all sourcing strategies of sector j_1 is lower or equal to that of sector j_2, i.e., $\bar{\theta}_n(j_1) \leq \bar{\theta}_n(j_2)$.

Proof. See Appendix B.3.

Note that Proposition 3 does not necessarily imply higher *trade-weighted* average delay risks for sectors using a wider range of inputs. To reduce their delay risk exposure, they can reduce the import share from sourcing strategies that entail high delay risks.

3.5 Aggregation

I aggregate single traders' optimal solutions to the level of sourcing strategy. I add back the superscripts and subscripts indexing the country and the sector which a trader serves, and the sourcing strategy he uses. Henceforth, a notation like $u^j_{m(j)n,t}$ denotes the variable u in period t at the level of traders serving for sector j in country n and using sourcing strategy $m(j)$.

In-season market. Following (7), the corresponding price index to the aggregation of input bundles of traders using strategy $m(j)$ is

$$\begin{aligned}
P^j_{m(j)n,t} &= \left[\int_0^1 \nu(i)\left(p^j_{m(j)n,t}(i)\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
&= \left[\left(1-\theta_{m(j)n,t}\right)\int_0^1 \nu(i)\left(p^{j,OT}_{m(j)n,t}(i)\right)^{1-\sigma} di + \theta_{m(j)n,t}\int_0^1 \nu(i)\left(p^{j,DL}_{m(j)n,t}(i)\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
&= \hat{p}^j_{m(j)n,t}\left[\frac{\lambda^j_n}{\lambda^j_n-1} - \left(\frac{\lambda^j_n}{\lambda^j_n-1} - \frac{\lambda^j_n}{\lambda^j_n-\frac{1}{\sigma}}\right)\left(\left(1-\theta_{m(j)n,t}\right)\left(\hat{v}^{j,OT}_{m(j)n,t}\right)^{1-\lambda^j_n} + \theta_{m(j)n,t}\left(\hat{v}^{j,DL}_{m(j)n,t}\right)^{1-\lambda^j_n}\right)\right]^{\frac{1}{1-\sigma}}.
\end{aligned}$$

The second term within the square bracket is a linear combination of functions of stockout cutoffs of both on-time traders and delayed traders, $\hat{v}^{OT}_t$ and $\hat{v}^{DL}_t$, and the weight for each is the corresponding probability. Delayed traders face tighter constraints thus lower stockout cutoffs and on average charge higher prices, reflected by $\left(\hat{v}^{OT}_t\right)^{1-\lambda}$. The weight θ_t, the fraction of delayed traders, then determines the overall tightness of available constraints among all traders. Note that this channel solely does not necessarily imply that the price index increases in delay risk θ_t in equilibrium, because traders are also raising their inventory stock to buffer against risks, which can lower $\left(\hat{v}_t^{DL}\right)^{1-\lambda}$.

The point here is that the aggregate price charged by traders under a sourcing strategy is an outcome of traders' tradeoff between stockout and overstock. Traders minimize the cost they bear, and then pass it through to the sectoral composite good producer.

The total periodical profit earned by this unit mass of traders is

$$\Pi^j_{m(j)n,t} = \int_0^1 \pi^j_{m(j)n,t}(i)\, di.$$

See B.5 for the derivation and the closed-form expression of total profits of traders in using $m(j)$.

Off-season market. The off-season market provides a place for traders to even their states after the idiosyncratic shocks they have encountered in the in-season market, and dump the redundant inventories to the sectoral composite good producer.

The total supply to the off-season market is the sum of off-season stock held by all traders

$$S^j_{m(j)n,t} \equiv \int_0^1 s^j_{m(j)n,t}(i)\, di = (1-\delta)\int_0^1 \left[z^j_{m(j)n,t-1}(i) + n^j_{m(j)n,t-1}(i) - q^j_{m(j)n,t}(i)\right] di, \tag{27}$$

See B.4 for the derivation and the closed-form expression of the total off-season stock.

The clearing condition for the off-season market requires that

$$S^j_{m(j)n,t} = \tilde{Q}^j_{m(j)n,t} + \int_0^1 z^j_{m(j)n,t}(i)\, di, \tag{28}$$

where on the right-hand side, $\tilde{Q}^j_{m(j)n,t}$ is the demand from sectoral composite good producer via the off-season market, and $\int_0^1 z^j_{m(j)n,t}(i)\, di$ is traders' demand for safety inventory holdings to carry forward to the future period.

New orders. The aggregation of the new order for the input bundle is

$$\begin{aligned} X^j_{m(j)n,t} &= \int_0^1 x^j_{m(j)n,t}(i)\, di \\ &= \left(\frac{\frac{1}{\sigma}}{\lambda^j_n - \frac{1}{\sigma}}\right)^{\frac{1}{\lambda^j_n}} \left(\frac{\hat{p}^j_{m(j)n,t+1}}{P^j_{m(j)n,t+1}}\right)^{-\sigma} Q^j_{m(j)n,t+1} \\ &\quad \times \left[\left(\frac{c^j_{m(j)n,t} - \beta(1-\delta)\,\omega^j_{m(j)n,t+1}}{\beta(1-\delta)\left(1-\theta_{m(j)n,t+1}\right)\omega^j_{m(j)n,t+1}}\right)^{-\frac{1}{\lambda^j_n}} - \left(\frac{\omega^j_{m(j)n,t} - c^j_{m(j)n,t}}{\beta(1-\delta)\,\theta_{m(j)n,t+1}\omega^j_{m(j)n,t+1}}\right)^{-\frac{1}{\lambda^j_n}}\right], \end{aligned}$$

which immediately follows equation (26). And the order for a single input k in this bundle is

$$X^k_{m_k(j)n,t} = \frac{X^j_{m(j)n,t}}{\chi^k_{jn}}.$$

3.6 General Equilibrium

After deriving the optimal decisions for traders using each sourcing strategy, I am ready to put them into the general equilibrium. I specify the problems of other agents—the final consumer and the sectoral composite good producers, lay out the market clearing conditions, and then define the general equilibrium.

Final consumer. The maximization problem of the representative final consumer is

$$\begin{aligned}
&\max_{C_{n,t}} \sum_{t=1}^{\infty} \beta^{t-1} \ln\left(C_{n,t}\right) \\
&s.t. \\
&P_{n,t}C_{n,t} = \sum_j w_{n,t}L^j_{n,t} + \sum_j \sum_{m(j,n)} \Pi_{m(j,n),t} + DX_{n,t},
\end{aligned} \tag{29}$$

where $DX_{n,t} \equiv \sum_j DX^j_{n,t}$ is the summation of sectoral trade deficits. The trade deficit of sector j in country n is the difference between the total value of exported good j and the total value of imported good j:

$$DX^j_{n,t} \equiv p^j_{n,t} \sum_{r \neq n} \tau^j_{nr} X^j_{nr,t} - \sum_{m \neq n} \tau^j_{mn,t} p^j_{m,t} E^j_{mn,t},$$

where

$$E^j_{nr,t} = \sum_k \sum_{m_j(k)=n} X^j_{m_j(k)r,t},$$

and it means that country r's total imports for good j from country n is the summation of orders for input j placed by all traders from all sectors whose sourcing strategies guide them to source input j from country n.

Sectoral composite good producer. Let $P^j_{n,t}$ be the price index for $Q^j_{n,t}$, $\tilde{P}^j_{n,t}$ be the price index for $\tilde{Q}^j_{n,t}$, and $P^{M,j}_{n,t}$ be the price index for $M^j_{n,t}$. From equation (5), the producer's optimal demand for labor is

$$L^j_{n,t} = \frac{\alpha^j_n p^j_{n,t} y^j_{n,t}}{w_{n,t}}. \tag{30}$$

From equation (8), the optimal in-season demand for input bundles sourced via strategy $m(j)$ is

$$Q^j_{m(j)n,t} = \iota^j_n \left(\frac{P^j_{m(j)n,t}}{P^j_{n,t}}\right)^{-\rho} \left(\frac{P^j_{n,t}}{P^{M,j}_{n,t}}\right)^{-\zeta} M^j_{n,t}, \tag{31}$$

and from equation (9), the optimal off-season demand for input bundles sourced via strategy $m(j)$ is

$$\tilde{Q}^{j}_{m(j)n,t} = \left(1-\iota^{j}_{n}\right)\left(\frac{\omega^{j}_{m(j)n,t}}{\tilde{P}^{j}_{n,t}}\right)^{-\rho}\left(\frac{\tilde{P}^{j}_{n,t}}{P^{M,j}_{n,t}}\right)^{-\zeta} M^{j}_{n,t}. \tag{32}$$

Market clearing. Here I list the clearing conditions for the final good market, the sectoral composite good market, and the labor market in each country.

Final good market. The final good market clears when the supply equals the representative consumer's demand:

$$Y_{n,t} = C_{n,t}. \tag{33}$$

Sectoral composite good market. The intermediate good market clears when the production equals the total demand from the final good producer in country n and importing groups from all countries:

$$y^{j}_{n,t} = y^{c,j}_{n,t} + \sum_{r} \tau^{j}_{nr,t} E^{j}_{nr,t}. \tag{34}$$

Labor market. The labor market clears when the labor endowment equals the total demand from all sectors in country n:

$$L_{n,t} = \sum_{j} L^{j}_{n,t}. \tag{35}$$

Definition 1. (Equilibrium) *Given the labor endowment in each country $\{L_{n,t}\}_{n=1,t=1}^{N,\infty}$, the productivities in each sector in each country $\left\{A^{j}_{n,t}\right\}_{j=1,n=1,t=1}^{J,N,\infty}$, the bilateral iceberg costs, $\left\{\tau^{j}_{rn,t}\right\}_{j=1,r=1,n=1,t=1}^{J,N,N,\infty}$, the bilateral delay risk $\{\mu_{rn,t}\}_{r=1,n=1,t=1}^{N,N,\infty}$, the initial inventory holdings $\left\{Z^{j}_{m(j)n,0}\right\}_{j=1,m(j)=1,n}^{J,|\mathcal{M}(j)|,N}$, and the initial orders for input bundles on the way, $\left\{X^{j}_{m(j)n,0}\right\}_{j=1,m(j)=1}^{J,|\mathcal{M}(j)|}$, a sequential competitive equilibrium is a sequence of prices in each country $\{w_{n,t}, P_{n,t}\}_{n=1,t=1}^{N,\infty}$, in each sector $\left\{p^{j}_{n,t}, P^{M,j}_{n,t}, P^{j}_{n,t}, \tilde{P}^{j}_{n,t}\right\}_{j=1,n=1,t=1}^{J,N,\infty}$ and for each sourcing strategy $\left\{P^{j}_{m(j)n,t}, \omega^{j}_{m(j)n,t}\right\}_{j=1,m(j)=1,n=1,t=1}^{J,|\mathcal{M}(j)|,N,\infty}$, allocations in each country $\{C_{n,t}, Y_{n,t}, DX_{n,t}\}_{n=1,t=1}^{N,\infty}$, in each sector $\left\{y^{j}_{n,t}, y^{c,j}_{n,t}, L^{j}_{n,t}, M^{j}_{n,t}, Q^{j}_{n,t}, \tilde{Q}^{j}_{n,t}\right\}_{j=1,n=1,t=1}^{J,N,\infty}$, for each sourcing strategy $\left\{Q^{j}_{m(j)n,t}, \tilde{Q}^{j}_{m(j)n,t}, \Pi^{j}_{m(j)n,t}, S^{j}_{m(j)n,t}, Z^{j}_{m(j)n,t}, X^{j}_{m(j)n,t}\right\}_{j=1,m(j)=1,n=1,t=1}^{J,|\mathcal{M}(j)|,N,\infty}$, and trade flows of single inputs $\left\{X^{k}_{m_k(j)n,t}\right\}_{j=1,k=1,m_k(j)=1,n=1,t=1}^{J,|\mathcal{U}(j)|,N,N,\infty}$ and $\left\{E^{j}_{rn,t}\right\}_{j=1,r=1,t=1}^{J,N,\infty}$ that*

1. *solves the optimization problems of representative consumers, representative producers, and traders, as characterized in equations (29), (30)-(32), (12), (25), and (26);*

2. *clears final good markets, sectoral composite good markets, and labor markets in each country as in equations* (33), (34), *and* (35)*;*

3. *clears in-season markets and off-season markets as in equations* (7) *and* (28).

The steady state is defined as an equilibrium where all endogenous variables in all countries and sectors are time-invariant.

Definition 2. (Steady State Equilibrium) *A steady state is an equilibrium where exogenous fundamentals* $\left\{\bar{L}_n, \bar{A}^j_n, \bar{\tau}^j_{rn}, \bar{\mu}_{rn}\right\}$*, as well as endogenous prices* $\left\{\bar{w}_n, \bar{P}_n, \bar{p}^j_n, \bar{P}^{M,j}_n, \bar{P}^j_n, \bar{\tilde{P}}^j_n, \bar{P}^j_{m(j)n}, \bar{\omega}^j_{m(j)n}\right\}$*, and endogenous allocations at the country or sectoral level,* $\left\{\bar{C}_n, \bar{Y}_n, \bar{DX}_n, \bar{y}^j_n, \bar{y}^{c,j}_n, \bar{L}^j_n, \bar{M}^j_n, \bar{Q}^j_n, \bar{\tilde{Q}}^j_n\right\}$*, at the sourcing-strategy level,* $\left\{\bar{Q}^j_{m(j)n}, \bar{\tilde{Q}}^j_{m(j)n}, \bar{\Pi}^j_{m(j)n}, \bar{S}^j_{m(j)n}, \bar{Z}^j_{m(j)n}, \bar{X}^j_{m(j)n}\right\}$*, and trade flows* $\left\{\bar{X}^k_{m_k(j)n}, \bar{E}^j_{rn}\right\}$*, are constant over time.*

3.7 Delay Risks and Icebergs

In this section, I characterize the key features of the general equilibrium defined in the previous section. Having the general equilibrium set up, I am ready to analyze how delivery delay risks impede trade as icebergs, and how they differ in their impacts on inventory.

Delay risks as a trade barrier

I use an example of a single sector multi-country setup to highlight how delay risks behave as iceberg trade costs in impeding trade.[4] In this setup, an input bundle is a single input, and a sourcing strategy is degenerated to a single source country, denoted by m.

A country n's total ordering cost for input from country m is

$$\begin{aligned} c_{mn,t} X_{mn,t} &= c_{mn,t} \left(\frac{\frac{1}{\sigma}}{\lambda_n - \frac{1}{\sigma}} \right)^{\frac{1}{\lambda_n}} \Psi_{mn,t} \left(\frac{\hat{p}_{mn,t+1}}{P_{mn,t+1}} \right)^{-\sigma} Q_{mn,t+1} \\ &= c_{mn,t} \left(\frac{\frac{1}{\sigma}}{\lambda_n - \frac{1}{\sigma}} \right)^{\frac{1}{\lambda_n}} \Psi_{mn,t} \left(\frac{\hat{p}_{mn,t+1}}{P_{mn,t+1}} \right)^{-\sigma} \left(\frac{P_{mn,t+1}}{P_{n,t+1}} \right)^{-\rho} Q_{n,t+1}, \end{aligned}$$

where

$$\Psi_{mn,t} \equiv \left(\frac{c_{mn,t} - \beta (1-\delta) \omega_{mn,t+1}}{\beta (1-\delta) (1-\theta_{mn,t+1}) \omega_{mn,t+1}} \right)^{-\frac{1}{\lambda_n}} - \left(\frac{\omega_{mn,t} - c_{mn,t}}{\beta (1-\delta) \theta_{mn,t+1} \omega_{mn,t+1}} \right)^{-\frac{1}{\lambda_n}}.$$

[4] In a multi-sector framework, a gravity-type equation can still be obtained but less clear for comparison with a standard Armington model.

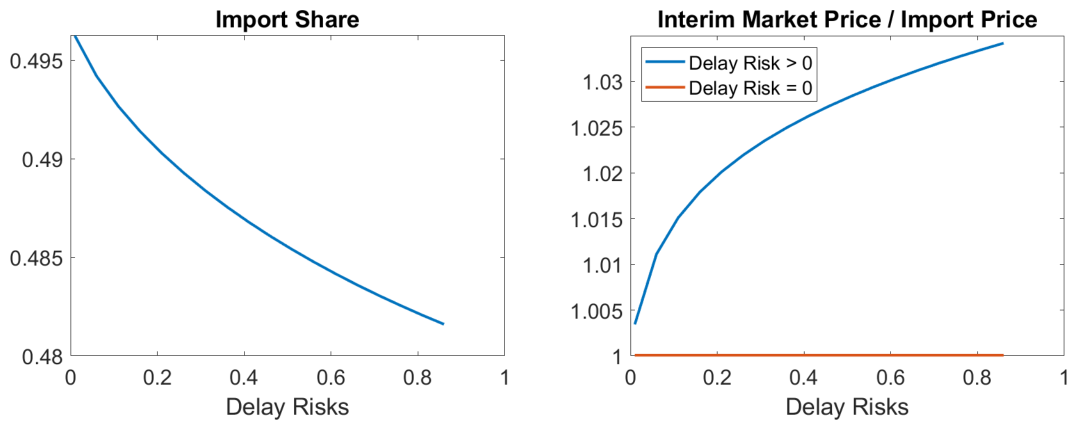


Figure 4: Delay Risks Impede Trade

Then the import share of this sector is

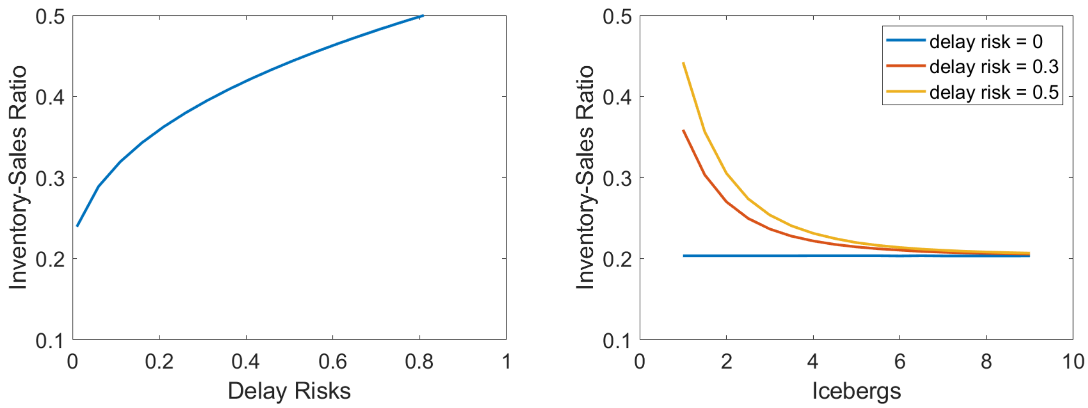
$$\begin{aligned}\frac{c_{mn,t}X_{mn,t}}{\sum_r c_{rn,t}X_{rn,t}} &= \frac{\Phi_n\left(\omega_{mn,t},\omega_{mn,t+1},c_{mn,t},P_{mn,t+1},\theta_{mn,t+1}\right)c_{mn,t}^{1-\rho}}{\sum_r \Phi_n\left(\omega_{rn,t}\omega_{rn,t+1},c_{rn,t},P_{rn,t+1},\theta_{rn,t+1}\right)c_{rn,t}^{1-\rho}} \\ &= \frac{\Phi_n\left(\omega_{mn,t},\omega_{mn,t+1},c_{mn,t},P_{mn,t+1},\theta_{mn,t+1}\right)\left(\tau_{mn,t}p_{mn,t}\right)^{1-\rho}}{\sum_r \Phi_n\left(\omega_{rn,t}\omega_{rn,t+1},c_{rn,t},P_{rn,t+1},\theta_{rn,t+1}\right)c_{rn,t}^{1-\rho}\left(\tau_{rn,t}p_{rn,t}\right)^{1-\rho}},\end{aligned} \tag{36}$$

where

$$\Phi_n\left(\omega_{mn,t},\omega_{mn,t+1},c_{mn,t},P_{mn,t+1},\theta_{mn,t+1}\right) \equiv \left(\frac{\hat{p}_{mn,t+1}}{P_{mn,t+1}}\right)^{-\sigma}\left(\frac{P_{mn,t+1}}{c_{mn,t}}\right)^{-\rho}\Psi_{mn,t},$$

and subscript n in notation $\Phi_n(\cdot)$ indicates that parameter λ_n can be different across source countries.

Note that differing from the gravity equation derived in a standard Armington framework, there is an extra wedge Φ in equation (36) determining the import shares. The wedge is a function of the delay risks in the future period, as well as prices within the sourcing group. In the partial equilibrium, given prices $\omega_{mn,t},\omega_{mn,t+1},c_{mn,t},P_{mn,t+1}$, wedge $\Phi_n(\cdot)$ is a decreasing function in delay risks. To verify that in the general equilibrium, I plot results of two comparative statics. The left panel in Figure 4 shows that in general equilibrium delivery delay risks indeed impede trade, and the right panel shows that the in-season market price faced by the sectoral composite good producer relative to traders' ordering cost. Indeed, when there is no delay risk, the two prices are the same; while there is a positive delay risk, there is a wedge between two prices. Intuitively, traders mitigate risks by adjusting inventories which are also costly, then they pass through the extra cost after buffering to the sectoral composite good producer.



(a) Delay Risk and Inventory-Sales Ratio

(b) Iceberg and Inventory-Sales Ratio

Figure 5: Trade Barriers and Inventory

Inventory in disentangling delay risks from icebergs

Knowing that delay risks can impede trade as iceberg costs, the next question is how to disentangle them from generic trade barriers? The answer is using inventory.

Figure 5 depicts the opposing directions of how two types of trade barriers impact inventory intensity. The left panel verifies the prediction discussed before in general equilibrium: delay risks raise inventory holdings. The right panel, by contrast, shows how iceberg costs have no impact on inventory intensity when delay risks do not exist and reduce inventory intensity when delay risks are positive. Intuitively, the overall inventory intensity can be decomposed to two factors: 1) the reliance on a specific source country, which is determined by the unit production cost plus the overall trade costs arise from that country; 2) the delivery delay risks associated with that source country. The pattern in the right panel in Figure 5 is driven by that iceberg costs reduce the reliance on the corresponding source country.

3.8 Modeling Choices and Connection to Existing Literature

I discuss how my inventory model is connected to those in the literature, and my other modeling choices.

My modeling on traders' behaviors builds on Alessandria et al. (2013). While in their setup delivery time is a deterministic one-period lag, my model allows for stochastic delivery delays, and allows the delivery delay risks to vary across bilateral country pairs.

Carreras-Valle (2021) also models continuous delivery delay frictions in a two-country setup. My model differs from hers in that I apply the law of large numbers to a unit mass of traders and allow for an off-season market to even out traders' idiosyncratic demand shocks and delivery delay shocks, which grants me the tractability to extend my model to a multi-country and multi-sector setup. Note that similar assumptions are made by Alessandria et al. (2013) to smooth out idiosyncratic demand shocks. They assume that buyers can return the unsold orders to sellers after one period of depreciation, which plays the same role as that in my setup traders can trade in the off-season market. In the literature of monetary search (e.g., Shi, 1997; Lagos and Wright, 2005; Rocheteau and Wright, 2005), to guarantee tractability, similar assumptions are used to reset agents' states after idiosyncratic shock exposure in matching processes.

In addition to inventory management, diversification at the individual firm level is an interesting angle to be considered as a risk mitigation strategy. However, Antràs, Fort, and Tintelnot (2017) document using the U.S. import data that the median number of countries from which a firm imports a particular product is one, even for the 95th percentile of firms. This suggests that in practice, at any given moment, firm-level sourcing diversification across countries is not common, hence my model abstracts from this decision.

4 Quantification

In this section, I describe how I take the model to data. I first describe the data sources, the parameters to be calibrated, and the identification strategy. Then I show the model fit, and calibration results.

4.1 Data

Following Levchenko (2007), I define the input complexity of a sector based on the Herfindahl-Hirschman Index of the share of input-use as defined in (3). A higher HHI means that a sector uses its upstream inputs in a more concentrated manner. I determine that a sector with a HHI above the medium to be an "single-input" sector (Sector 1), and that with a HHI below the medium to be an "multi-input" sector (Sector 2), and aggregate all sectors into these two sectors. I cover 17 economies, i.e., G-20 countries with European countries aggregated as a single economy. All data are taken for the year 2016.

4.2 Parameters Calibrated Independently

The elasticity of substitution across sourcing strategies ρ is set to be 4, which is standard in an Armington framework. Elasticity of substitution across varieties of individual traders using the same sourcing strategy σ is set to be 3, which follows Alessandria et al. (2013). The depreciation rate of inventory holdings δ is set to be 0.015 following Wen (2011). I calibrate the off-season market share using the ratio of the value of the secondary markets to GDP in the U.S.. I get the market size of the secondary markets from Rogers, Rogers, and Chen (2022), which is 644 billion dollars in 2022, approximately 3% to the GDP. So I take the in-season market share $\iota = 0.97$.

Labor endowment data L_n are from the PWT (version 10.01) and trade deficits DX_n are computed from the OECD input-output tables. Sectoral price indexes are taken from the US KLEMS datasets. Expenditure shares on labor α_n and final expenditure shares on sectoral composite good

Table 2: Parameters Uniform across Countries and Sectors

Parameter	Description	Value
ρ	elasticities of substitution across sourcing strategies	4
ζ	elasticities of substitution between in-season and off-season goods	0.5
σ	elasticities of substitution across individual traders	3
ξ	elasticities of substitution across sectoral goods	1.5
β	discount factor	0.99
δ	inventory depreciation rate	0.015
ι	share of in-season goods in intermediate inputs	0.97

ψ_n^j are computed from the input-output tables. Leontief coefficients χ_n^{kj} are obtained by combining the sectoral price indices with the expenditure shares from the input-output tables.

4.3 Parameters Calibrated in the Equilibrium

The primary objective is to calibrate parameters that determine bilateral delivery delay risks and iceberg trade costs to achieve two key insights: first, to assess the scale of delivery delay risks and their variations across countries; and second, to evaluate the impact of delivery delay risks as a trade barrier in comparison to standard iceberg trade costs. To this end, I begin by parameterizing the bilateral delivery delay risks. Then I parameterize the parameters that govern other fundamentals.

Delivery Delay Risks. Literature in maritime shipping (Vernimmen et al., 2007; Chung and Chiang, 2011) documents that there are several factors that affect the shipping schedule reliabilities: departure or arrival port conditions such as congestions and communication technologies, en-route shipping delay due to weather conditions, the port or passages along the shipping route. Guided by this, I assume that the bilateral delivery delay probability is a function of shipping distance, the exporter-country-specific factor, and the conditions of gateways that the shipping route crosses, and it is expressed as

$$\mu_{rn} = 1 - exp\left(- \underbrace{\phi_g^T v_{rn}}_{gateways} - \underbrace{\zeta_r}_{exp.\ FE} - \underbrace{\gamma \cdot seadist_{rn}}_{shipping\ distance} \right), \tag{37}$$

where v_{rn} is a vector of dummies each indicating if a shipping route crosses a certain gateway, and five major gateways in maritime shipping are included:[5]

$$v_{rn}^T = (\mathbf{1}\,(\text{Suez})\,, \mathbf{1}\,(\text{Panama})\,, \mathbf{1}\,(\text{Malacca})\,, \mathbf{1}\,(\text{Good Hope})\,, \mathbf{1}\,(\text{Magellan}))\,.$$

[5]For the importance of these gateways among all, see Meza et al. (2022) and `https://porteconomicsmanagement.org/pemp/contents/part1/interoceanic-passages/main-maritime-shipping-routes/`.

The idea of the this specification is that: to arrive at the final destination on time, each part has to be completed on time. The sea distance term captures the cumulative hazard probability during the shipping process. γ can be regarded as the constant hazard rate of each shipping interval. ζ_r captures the exporter fixed effects such as the exporter port conditions, or other domestic logistics conditions in the exporter country. ϕ_g is a vector summarizing factors that can slow down the movement of shipments through an ocean passage or a gateway, e.g., the extent of congestion in Suez Canal.

Dispersion of Demand Shocks. Disruption chain disruption risks are not the only driving force of inventory holdings. Regression results in Section 2 also suggests that country fixed effects and sector fixed effects account for a considerable part of the country-sector inventory intensity. For calibration, I use the shape parameter of idiosyncratic demand shocks, λ, to absorb these average effects that cannot be explained by delivery delay risks. Specifically, parameter λ_n^1 determines the overall demand dispersion at the in-season market in country n. A lower λ implies a fatter upper tail of the distribution of idiosyncratic demand shocks, meaning that it is more likely to draw a high demand shock. I also include a gap $\bar{\lambda}^j$ between sector j and the base sector, which is uniform across countries. This captures the sector-specific fixed effects in the distribution of idiosyncratic demand shocks,[6] and leads the shape parameter of sector j in country n to be $\lambda_n^j = \lambda_n^1 + \bar{\lambda}^j$.[7]

Remarks: Note that except the shipping distance which is origin-destination-specific, no other parameter in the specification (37) of the bilateral delivery delay risk μ_{rn} is destination-specific. It would be ideal to include destination-specific factors that capture conditions in importing ports etc. However, since inventory intensities are observed at the country-sector level, one cannot tell the composition of inventories based on their source countries. Importer fixed effects are therefore not included into the specification because there would be no way to distinguish if a country holds a lot of inventories due to high overall demand uncertainty, or due to congestion in the importing ports.[8]

Icebergs. I assume that $\tau_{nn}^j = 1$, normalizing the domestic iceberg trade cost to one. I then specify the sectoral iceberg trade costs to be a product of importer-sector-specific parameter and a

[6] Here the demand uncertainty is broad, and can be interpreted as systematic operating differences between sectors which can increase the difficulty in predicting future demand.

[7] The interpretation for this that, in any country,

$$\frac{P\left(\nu_1 > \nu\right)}{P\left(\nu_2 > \nu\right)} = \frac{1 - F_1\left(\nu\right)}{1 - F_2\left(\nu\right)} = \exp\left(\bar{\lambda}_2 \nu\right)$$

i.e., two sectors' ratio of probabilities of drawing a demand shock greater than a given level ν is a constant, and does not vary across countries.

[8] If inventory could be broken down to foreign and domestic parts, one might be able to differ the effect of overall demand uncertainty from that of importing ports, because the latter would cause the foreign inventory share to increase while the overall demand uncertainty would not.

Table 3: Impacts of Friction Parameters on Trade Share and Inventory Intensity

Parameters	IS_n	Ω_{nn}	Ω_{rn}	$\Omega_{hn} \forall h \neq r, n$
$\tau_{rn}\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
$\phi_g, \zeta_r, \gamma\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$\lambda_n\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$

bilateral mean trade costs following Johnson and Moxnes (2023),

$$\tau_{rn}^j = \tau_n^j \bar{\tau}_{rn},$$

where $\bar{\tau}_{rn}$ is the assumed to be symmetric between country r and country n, i.e., $\bar{\tau}_{rn} = \bar{\tau}_{nr}$, and not sector-specific. Note that this specification still allows for asymmetric trade costs between two countries because of τ_n^j.

Denote the share of values of good j produced in country r and imported by country n by Ω_{rn}^j:

$$\Omega_{rn}^j \equiv \frac{c_{rn,t}^j X_{rn,t}^j}{\sum_m c_{mn,t}^j X_{mn,t}^j}.$$

Table 3 illustrates how in a multi-country setup, bilateral trade shares together with inventory intensities can be used to distinguish the roles of iceberg trade costs, delivery delay risks, and demand dispersion. The iceberg trade cost differs from delivery delay risks and demand uncertainty in its impact on inventory intensity, as discussed in Section 3.7. Demand dispersion differs from delivery delay risks in that, the former would reduce the overall share of imports, while a delivery delay risk–arising from a specific source route–would induce the substitution effects across foreign countries.

Productivity. I calibrate the country-sectoral productivities to match the GDP per capita data relying on the following equation,

$$p_{n,t}^j = \frac{w_{n,t}^{\alpha} \left(P_{n,t}^{M,j}\right)^{1-\alpha}}{A_{n,t}^j}.$$

I normalize the wage rate and the TFP of sector 1 in the U.S. to 1. I match the relative price of Sector 2 to Sector 1 to its counterpart in the data of the U.S., and wage rates to the GDP per capita relative to the U.S. There are N parameters, $\left\{A_{US}^2, A_{-US}^1\right\}$, and N moments, $\left\{\frac{P_{US}^2}{P_{US}^1}, w_{-US}\right\}$.

Table 4: Parameters Calibrated in Equilibrium

Parameter	Description	Dimension	Targeted Moments
Icebergs: $\tau^j_{rn} = \tau^j_n \bar{\tau}_{rn}$			
$\bar{\tau}_{rn}$	average bilateral iceberg trade costs	$\frac{N(N-1)}{2}$	Head-Ries Index $\Omega^j_{rn}\Omega^j_{nr}$
τ^j_n	importer-sector-specific iceberg multipliers	$J \times N$	
Supply Chain Frictions: delivery delay risks + demand dispersion			
ϕ_g	gateway fixed effects	$G \times 1$	Head-Ries Index $\Omega^j_{rn}\Omega^j_{nr}$, inventory intensity IS^j_n
ζ_r	exporter fixed effects	$N \times 1$	
γ	en-route hazard rate	1	
λ^1_n	Pareto shape parameter of demand shocks for the base sector	$N \times 1$	
$\bar{\lambda}^j$	difference in Pareto shape parameters between sector j and the base sector	$(J-1) \times 1$	
Productivity: $A_{2,US}, A_{1,-US}$			
$A_{2,US}, A_{1,-US}$	country-sector TFP	$N \times 1$	GDP per capita, sectoral price ratios $\frac{P^2_{US}}{P^1_{US}}$

Objective Function and Algorithm

To summarize, the set of parameters to be calibrated in equilibrium is

$$\Theta = \left\{ \underbrace{\bar{\tau}_{rn}, \tau^j_n}_{\text{iceberg}}, \underbrace{\phi_c, \zeta_r, \gamma}_{\text{delay risk}}, \underbrace{\lambda^1_n, \bar{\lambda}^j}_{\text{demand dispersion}}, \underbrace{A^1_n, A^j_{US}}_{\text{productivity}} \right\},$$

and Table 4 shows the description, dimension, and targeted moments of these parameters. I jointly calibrate these parameters. There are $\frac{1}{2}N(N-1) + (J+3)N + G + J$ parameters and $N(N-1) + (J+1)N$ moments; hence when $J = 2$ and $N = 17$, there is an over-identification.

The calibration is to find the values for those parameters through the following optimization problem:

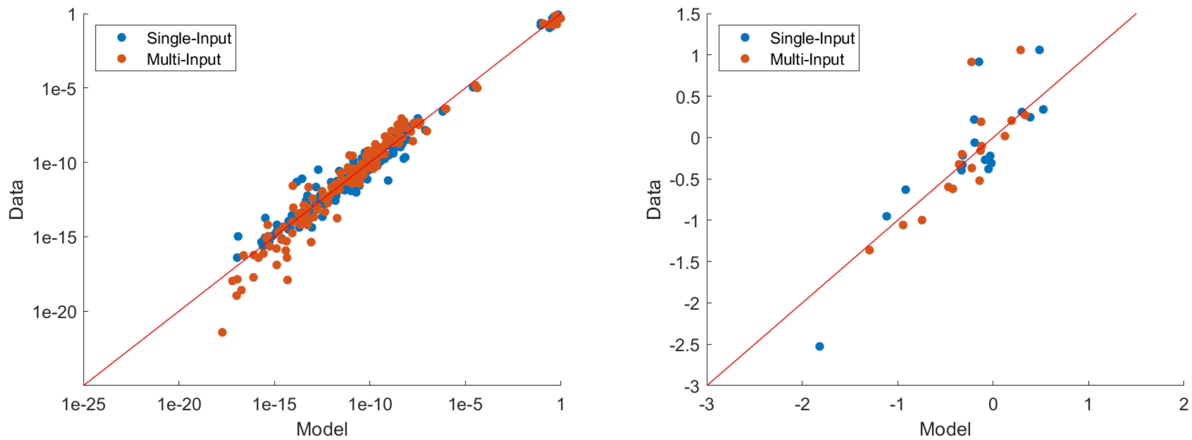
$$\hat{\Theta} = \min_{\Theta} \left(\log m_{model}(\Theta) - \log m_{data}\right)^T \left(\log m_{model}(\Theta) - \log m_{data}\right),$$

where $m_{model}(\Theta)$ denotes the vector containing all the model moments and m_{data} contains all data moments.

The algorithm for the calibration consists of two layers.

1. Inner layer.

   Given any set of icebergs, delay risk parameters, and demand shape parameters, I adjust the TFP parameters until the equilibrium wage rates across countries are consistent with



(a) Product of Bilateral Trade Shares (Head-Ries Index)

(b) Inventory Intensity (log)

Figure 6: Model Fit

Notes: The left panel shows the model fit of the Head-Ries indices. The right panel shows the model fit of inventory intensities.

the relative wage in data. When the calibrated TFP is obtained, the model also delivers the equilibrium inventories, output, and trade flows.

2. Outer layer.

   I then jointly adjust icebergs, delay risk parameters, and demand shape parameters to match the equilibrium inventory intensity and bilateral trade shares generated from Step 1 to their data moments.

4.4 Calibrated Results

Model Fit

Figure 6 compares the model-generated moments against the data moments on trade shares, which is captured by the Head-Ries indices, and inventory intensities at the country-sectoral level. It demonstrates that the calibrated model fits well the data moments: the model captures 96% of variation of the Head-Ries indices and 71% of variation of inventory intensities.

Sources of Delivery Delay Risks

Figure 7 depicts how each factor in specification (37) contributes to delay risks. On average, shipping distance takes account for the majority of the source of delivery delay risks. Exporter fixed effects come the second. Ocean gateways are important for specific countries including Argentina, Saudi Arabia, and South Africa.

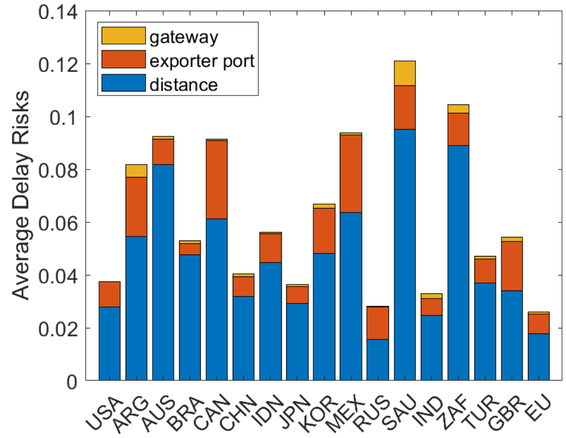


Figure 7: Sources of Delivery Delay Risks

Determinants of Inventory Intensity

Figure C.2 shows the change of inventory intensity after shutting down the cross-border delivery delay risks. It shows these risks on average can explain 7% of the country-level overall inventory intensities. Figure C.3 shows the tight correlation between inventory intensities and the values of demand dispersion parameters across countries and sectors. The comparison suggests that, the vast country-specific heterogeneity is the main determinant of overall inventory intensities across countries.

Iceberg Costs

Figure C.1 shows the histogram of bilateral iceberg trade costs in both sectors. As is typical of estimated iceberg trade costs from gravity equations, the levels of the trade costs are very large. But since in the model I do not pre-assume higher weight for home goods, iceberg trade costs may also reflect a home-bias effect.

5 Counterfactuals

In this section, I evaluate how sectors' inventory holdings, and sourcing decisions respond to changes in the delivery delay risks, and the implications of these responses for sectors' trade patterns and consumer prices, and also the difference in the complexity of sectors' input structure.

I conduct two counterfactual exercises the calibrated results obtained from the previous section. First, I remove all the cross-border delivery risks to get a sense of the magnitude of the overall delivery delay risks as a trade barrier. Second, to leverage that the multi-country multi-sector structure and the rich geographical component of my quantitative model, I raise the delay risks for shipping routes that cross three major gateways respectively to investigate the role of individual maritime infrastructures on trade and their heterogenous effects on countries.

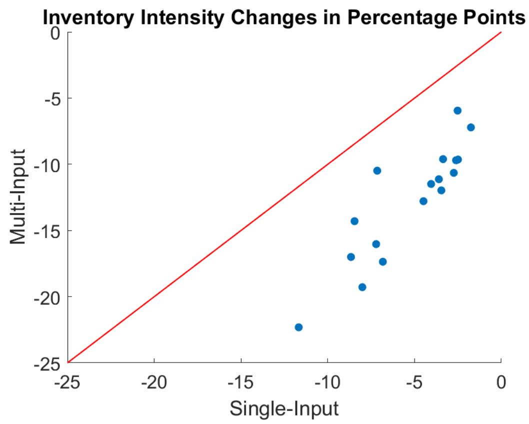


Figure 8: Changes in Inventory Intensity

5.1 Impact of Cross-border Delivery Delay Risks

To get a sense of the magnitude of the delay risks which arise from the foreign sources, I shut down the cross-border bilateral delivery delay risks by setting them to be the same as the domestic level.[9]

Reducing Inventories

Figure 8 shows how the elimination of cross-border delay risks affect inventory intensities across countries and sectors. Two patterns emerge. First, both sectors experience a decline on their inventory intensities. The drops range from 2% to 12% for single-input sectors, and 5% to 23% for multi-input sectors. Intuitively, reducing delivery delay risks reduces the incentive of inventory holdings for risk buffering.

Second, inventory intensities of multi-input sectors decline more aggressively than single-input sectors. This is because, facing the same shipping routes, sectors requiring more complex inputs are more restricted to delivery delay risks because things can get wrong more easily. Hence, when the overall delay risks get lower, the restriction to multi-input sectors gets loosened more than that to single-sectors.

Lowering Prices Faced by Producers

Figure 9 shows that the gap between the ordering cost faced by traders and the price faced by producers narrows relative to the baseline case. The gap drops by up to 7% among all bilateral pairs. This implies that when delay risks are reduced, traders are less likely to face shortages that arise from international supply chains, and accordingly, the costs that they have to pay to buffer against risks also decrease. This way, they pass through lower extra costs to producers.

[9]Because the baseline inventories rely on the interaction between demand uncertainty and domestic uncertainty, in my model, allowing for zero delay risk would lead to no inventory at the end of a period. It is not realistic in the sense that, evidence shows that firms hold inventories purchased from home country also. Intuitively, within a border, transaction is not perfectly smooth. I choose to normalize the domestic within-border delay risk to be 0.0001. Domestic variations in inventory intensity are absorbed by the overall demand uncertainty.

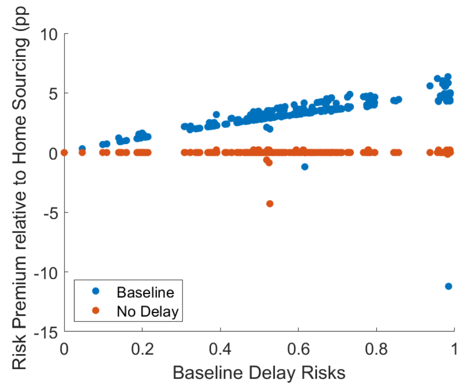


Figure 9: Gap between ordering cost and using price

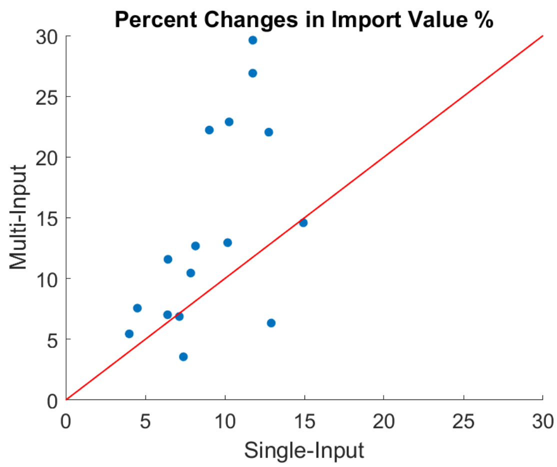


Figure 10: Changes in Cross-border Trade

Increasing Trade

Global cross-border trade value increase by 8.2%. Country-sector import changes are plotted in Figure 10. Most sectors increase their import shares because they need to bear lower delivery delay risks, which means they need to incur lower inventory holding costs. Among them, multi-input sectors increase their imports more sharply than single-input sectors, because they are more loosened from the the removal of those risks.

To better understand the magnitude of cross-sectional delay risks, I calculate their tariff-equivalent. Departing from the baseline scenario, I uniformly reduce bilateral iceberg trade costs by a specific percentage to achieve the same increase in the value of global cross-border trade. The analysis reveals that a 3.2% reduction in bilateral iceberg trade costs is necessary to produce an equivalent increase. This finding implies that cross-border delay risks are, on average, equivalent to imposing a 25.8% tariff.

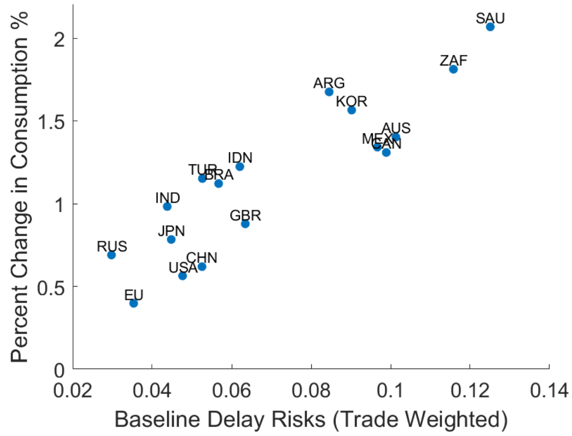


Figure 11: Baseline Delay Risks and Gains in Consumption

Table 5: Delivery Delay Risks and Welfare Changes (%)

Economies	Change in Welfare	Economies	Change in Welfare
US	0.56	Mexico	1.34
Argentina	1.68	Russia	0.69
Australia	1.40	Saudi Arabia	2.07
Brazil	1.12	India	0.98
Canada	1.31	South Africa	1.81
China	0.62	Turkey	1.15
Indonesia	1.22	UK	0.88
Japan	0.78	EU	0.40
South Korea	1.57		

Increasing Welfare

Consumption-equivalent welfare gains range from 0.4-2.1 percent, varying across countries. See Table 5 for details. Figure 11 plots the positive correlation between welfare gains and the baseline delivery delay risk exposures across countries, meaning that those countries that have suffered more from delay risks would gain more.

5.2 Role of Maritime Infrastructure

To utilize the rich geographic component in the quantification exercise, I conduct a second experiment to showcase how this framework can be applied to study the impact of existing maritime infrastructure on trade. Specifically, I adjust the parameter governing the gateway fixed effects for three major gateways—Suez, Panama, and Malacca—until the unweighted mean percentage change in delay rates for all shipping routes crossing each respective gateway reaches 5%. Note that the current framework does not allow for rerouting; in other words, country pairs continue using the same shipping routes as in the baseline scenario. This suggests that, in response to increased delay rates from a specific gateway, importers adjust their inventory holdings and divert

Table 6: Impact of Suez, Panama, and Malacca on World Exports

	Change in Exports (%)	Max (%)	Min (%)	Most Affected	Change in Welfare (%)
Suez	-0.0198	0.1813	-0.2006	Saudi Arabia	-0.0219
Panama	-0.0322	0.0009	-0.2574	Brazil	-0.0302
Malacca	-0.0116	0.1436	-0.1394	South Korea	-0.0117

Notes: Column (1) presents the percentage change in the value of global cross-border exports. Columns (2) and (3) present the maximum and minimum of total export value changes across economies. Column (5) presents the changes in welfare for the most affected economy in the corresponding scenario.

to other sources that do not rely on the affected gateway.

Table 6 presents the results. Inefficiency in any of the three gateways would reduce global trade flows. For example, increasing the average delay rates at the Suez Canal by only 5% would decrease the global trade value by approximately 0.0198%, equivalent to about 3.06 billion US dollars based on the 2016 global merchandise trade value of 15.46 trillion US dollars. Different economies depend on these gateways to varying extents. Among the covered economies, Saudi Arabia would suffer the most from the inefficiency of the Suez Canal, experiencing a 0.022% drop in welfare. Inefficiency in the Panama Canal would impact Brazil the most, reducing its welfare by 0.03%. Finally, the Malacca Strait, which is important for East Asian countries to connect with India, the Middle East, and Europe, would hurt South Korea the most by reducing its welfare by 0.012%.

6 Conclusion

This paper introduces a tractable multi-country, multi-sector general equilibrium trade model that explicitly incorporates supply chain disruption risks and strategic inventory holdings. The model not only explains empirical patterns in the relationship between inventory and trade, but also quantifies the significant impact of these risks on global trade flows and welfare.

Through calibration using data from 17 economies, the model reveals that removing cross-border delivery delay risks could increase global trade by 8.2% and improve consumption-equivalent welfare by up to 2.1% in some countries. These results underscore the substantial burden that supply chain disruptions impose on international trade, especially for sectors that rely on complex input structures, which are particularly vulnerable to delays. The counterfactual exercises highlight the sensitivity of trade flows to geopolitical risks, demonstrating the value of this model as a tool for policymakers. For instance, a hypothetical increase in delivery delay risks through the Suez Canal underscores how regional instability can ripple across global supply chains, affecting trade and inventory patterns in multiple sectors and countries.

Overall, this research bridges a gap in the literature by offering a quantitative framework to assess the interplay between supply chain risks, inventory management, and international trade, and apply it to reveal and measure unobservable risk exposures by countries and sectors, which is crucial to understand and mitigate the vulnerabilities in global trade networks.

References

ALESSANDRIA, G., J. KABOSKI, AND V. MIDRIGAN (2013): "Trade Wedges, Inventories, and International Business Cycles," *Journal of Monetary Economics*, 60, 1–20.

ALESSANDRIA, G., J. P. KABOSKI, AND V. MIDRIGAN (2010): "Inventories, Lumpy Trade, and Large Devaluations," *American Economic Review*, 100, 2304–2339.

ALESSANDRIA, G., S. Y. KHAN, A. KHEDERLARIAN, C. MIX, AND K. J. RUHL (2022): "The Aggregate Effects of Global and Local Supply Chain Disruptions: 2020–2022," 43.

ANDERSON, J. E. AND E. VAN WINCOOP (2003): "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, 93, 170–192.

——— (2004): "Trade Costs," *Journal of Economic Literature*, 42, 691–751.

ANTRÀS, P. (2023): "An 'Austrian' Model of Global Value Chains," *working paper*.

ANTRÀS, P., T. C. FORT, AND F. TINTELNOT (2017): "The Margins of Global Sourcing: Theory and Evidence from US Firms," *American Economic Review*, 107, 2514–2564.

ANTRÀS, P. AND A. GORTARI (2020): "On the Geography of Global Value Chains," *Econometrica*, 88, 1553–1598.

CALIENDO, L. AND F. PARRO (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

CARRERAS-VALLE, M.-J. (2021): "Increasing Inventories: The Role of Delivery Times," *UMN Job Market Paper*.

CHUNG, C.-C. AND C.-H. CHIANG (2011): "The Critical Factors: An Evaluation of Schedule Reliability in Liner Shipping," .

DJANKOV, S., C. FREUND, AND C. S. PHAM (2010): "Trading on Time," *The Review of Economics and Statistics*, 92, 166–173.

EVANS, C. L. AND J. HARRIGAN (2005): "Distance, Time, and Specialization: Lean Retailing in General Equilibrium," *The American Economic Review*, 95, 292–313.

FEYRER, J. (2021): "Distance, Trade, and Income — The 1967 to 1975 Closing of the Suez Canal as a Natural Experiment," *Journal of Development Economics*, 153, 102708.

GROSSMAN, G. M., E. HELPMAN, AND H. LHUILLIER (2023): "Supply Chain Resilience: Should Policy Promote International Diversification or Reshoring?" *Journal of Political Economy*, 131, 3462–3496.

Grossman, G. M., E. Helpman, and A. Sabal (2024): "Optimal Resilience in Multitier Supply Chains," *working paper*, qjae024.

Guasch, J. L. and J. Kogan (2001): *Inventories in Developing Countries: Levels and Determinants : A Red Flag for Competitiveness and Growth*, World Bank Publications.

Harrigan, J. and A. J. Venables (2006): "Timeliness and Agglomeration," *Journal of Urban Economics*, 59, 300–316.

Head, K. and J. Ries (2001): "Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade," *American Economic Review*, 91, 858–876.

Hummels, D. (2007): "Transportation Costs and International Trade in the Second Era of Globalization," *Journal of Economic Perspectives*, 21, 131–154.

Hummels, D. L. and G. Schaur (2010): "Hedging Price Volatility Using Fast Transport," *Journal of International Economics*, 82, 15–25.

——— (2013): "Time as a Trade Barrier," *American Economic Review*, 103, 2935–2959.

Johnson, R. C. and A. Moxnes (2023): "GVCs and Trade Elasticities with Multistage Production," *Journal of International Economics*, 145, 103796.

Kremer, M. (1993): "The O-Ring Theory of Economic Development*," *The Quarterly Journal of Economics*, 108, 551–575.

Lagos, R. and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

Leibovici, F. and T. Adamopoulos (2024): "Trade Risk and Food Security," Tech. rep.

Levchenko, A. A. (2007): "Institutional Quality and International Trade," *The Review of Economic Studies*, 74, 791–819.

Limão, N. and A. J. Venables (2001): "Infrastructure, Geographical Disadvantage, Transport Costs, and Trade," *The World Bank Economic Review*, 15, 451–479.

Liu, E. and A. Tsyvinski (2024): "A Dynamic Model of Input–Output Networks," *The Review of Economic Studies*, rdae012.

Meza, A., I. Ari, M. A. Sada, and M. Koç (2022): "Disruption of Maritime Trade Chokepoints and the Global LNG Trade: An Agent-Based Modeling Approach," *Maritime Transport Research*, 3, 100071.

Olken, B. A. and P. Barron (2009): "The Simple Economics of Extortion: Evidence from Trucking in Aceh," *Journal of Political Economy*, 117, 417–452.

ROCHETEAU, G. AND R. WRIGHT (2005): "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73, 175–202.

ROGERS, Z. S., D. S. ROGERS, AND H. CHEN (2022): "The Importance of Secondary Markets in the Changing Retail Landscape: A Longitudinal Study in the United States and China," *Transportation Journal*, 61, 18–59.

SEQUEIRA, S. (2016): "Corruption, Trade Costs, and Gains from Tariff Liberalization: Evidence from Southern Africa," *American Economic Review*, 106, 3029–3063.

SHI, S. (1997): "A Divisible Search Model of Fiat Money," *Econometrica*, 65, 75–102.

STEINWENDER, C. (2018): "Real Effects of Information Frictions: When the States and the Kingdom Became United," *American Economic Review*, 108, 657–696.

VERNIMMEN, B., W. DULLAERT, AND S. ENGELEN (2007): "Schedule Unreliability in Liner Shipping: Origins and Consequences for the Hinterland Supply Chain," *Maritime Economics & Logistics*, 9, 193–213.

WEN, Y. (2011): "Input and Output Inventory Dynamics," *American Economic Journal: Macroeconomics*, 3, 181–212.

YI, K.-M. (2010): "Can Multistage Production Explain the Home Bias in Trade?" *American Economic Review*, 100, 364–393.

A Quantification

A.1 Recover Inventory Stocks

A.1.1 Initial Inventory Stocks

Since the OECD input-output tables provide only changes in inventories, I need to first obtain the initial stocks to recover the stock levels during the period of interest. I digitize and assemble the values of inventory stocks and output data for two-digit ISIC during 1987 and 1990 for over 25 countries from the Industrial Statistics Yearbooks released by the United Nations. I then compute the inventory-sales ratios in 1990 for each country-sector,

$$IS_{1990} = \frac{\text{value of inventory stock}_{1990}}{\text{value of output}_{1990}},$$

and then use IS_{1990} to compute the inventory stocks in 1995, assuming that the annual inventory-sales ratios were stable during 1990 and 1995,

$$\text{value of inventory stock}_{1995} = \text{value of output}_{1995} \times IS_{1990},$$

and use them as the initial inventory stocks for the period 1995-2018.[10]

A.1.2 Changes in Inventories

1993 SNA requires the valuation of inventories in national accounts to follow the perpetual inventory method (PIM). Following that, the BEA defines the change in private inventories as the value of the change in the physical volume of inventories owned by private business over a specified period, valued in the average prices of that period, i.e.,

$$\begin{aligned} \Delta\text{inventories} &\equiv \bar{P}_t \Delta S_t \\ &\equiv \bar{P}_t \left(S_t - S_{t-1}\right), \\ &\equiv \bar{P}_t \left(\left(1-\delta\right) S_{t-1} - X_t + I_t - S_{t-1}\right), \end{aligned}$$

where S_t denotes the volume or quantity of inventories at the end of period t, X_t denotes the withdrawal from the inventory stock during period t, I_t denotes the addition to the inventory stock during period t, and δ is the depreciation rate.

Since data in the OECD input-output tables are recorded in current price US dollars, I first use time series of exchanges rates to translate them into their current price local currencies. I then deflate changes in inventories with the period-mean GDP deflators, having the 1995 prices of each

[10]The Industrial Statistics Yearbooks have stopped releasing data of inventory stocks since 1991, making 1990 the last year when the inventory stock data of an ideal number of countries are available.

country as their baselines,

$$\Delta\hat{S}_t = \frac{\Delta\text{inventories}_t}{1+\bar{\pi}_t},$$

where $\bar{\pi}_{1995} = 0$.

A.1.3 Time Series of Inventory Stocks

Once I obtain the initial inventory stocks and the approximate changes in inventory quantities for each country-sector, I use forward iterations to recover the whole sequence of inventory stocks,

$$S_t = S_{1995} + \sum_{\tau=1996}^{t} \Delta\hat{S}_\tau,$$

where S_t denotes the end-of-period inventory stock of period t, δ denotes the depreciation rate, and ΔS denotes the net increase in inventories, or the changes in inventories.

A.1.4 Time Series of Inventory-Sales Ratios

Then I compute the inventory-sales ratios,

$$IS_t = \frac{S_t}{y_t},$$

where y_t is the real output obtained by deflating values of output relative to the 1995 baseline.

I validate the recovered inventory-sales ratios for countries that regularly release their inventory stocks over the period of interest, including the U.S., Canada, China, and Japan.

A.2 Robustness Check: Inventory Intensity and Importing Distance

Table A.1: Importing Distance and Inventory Intensity

	(1)	(2)
ln(import distance)	0.1144***	0.1297***
	(0.0200)	(0.0351)
ln(import share)	0.0464***	0.0818***
	(0.0066)	(0.0165)
Constant	-3.4653***	-3.8625***
	(0.1994)	(0.3801)
Observations	6856	5954
Adjusted R^2	0.7787	0.8855
Fixed Effects	*n,j,t*	*nj,nt,jt*

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

A.3 Robustness Check: Schedule Reliability and Route Distance

Table A.2: Delay Rate and Route Distance

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ln(sea distance)	0.012**	-0.031***	0.015**	0.012*	0.050***	0.017*	0.066***
	(0.005)	(0.008)	(0.007)	(0.007)	(0.006)	(0.010)	(0.007)
Constant	0.173***	0.550***	0.150**	0.177***	-0.155***	0.137*	-0.308***
	(0.048)	(0.066)	(0.060)	(0.058)	(0.052)	(0.083)	(0.064)
Observations	7522	7522	7522	7522	7522	7522	5217
Adjusted R^2	0.3327	0.0719	0.1827	0.3526	0.4541	0.2025	0.5680
Fixed Effects	*t,c*	*t,o*	*t,d*	*t,o,c*	*t,d,c*	*t,o,d*	*cot,dt*

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B Model

B.1 Symmetry of End-of-Period Inventories and New Orders

Proposition 1 Proof.

Proof. For simplicity, I suppress index i in the following derivation.

Given ω_t, and c_t, the first order conditions for z_t^d, and x_t^d are

$$\omega_t = \beta \mathbb{E} V_z \left(z_t^d, x_t^d \right) \tag{B.1}$$

$$c_t = \beta \mathbb{E} V_n \left(z_t^d, x_t^d \right) \tag{B.2}$$

where $V_z \left(z_t, x_t \right) \equiv \frac{\partial V(z_t, x_t)}{\partial z_t}$, and $V_x \left(z_t, x_t \right) \equiv \frac{\partial V(z_t, x_t)}{\partial x_t}$.

Applying the envelope theorem to (16) in period $t+1$ yields the following:

1) if order x_t^d is delivered on time in period $t+1$

$$V_z^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \begin{cases} \frac{\partial R_t \left(z_t^d + x_t^d, \nu_{t+1} \right)}{\partial z_t^d}, & \nu_{t+1} > \hat{\nu}_{t+1}^{OT} \left(z_t^d + x_t^d \right) \\ \left(1 - \delta \right) \omega_{t+1}, & \nu_{t+1} \leq \hat{\nu}_{t+1}^{OT} \left(z_t^d + x_t^d \right) \end{cases}, \tag{B.3}$$

and

$$V_x^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \begin{cases} \frac{\partial R_t \left(z_t^d + x_t^d, \nu_{t+1} \right)}{\partial n_t^d}, & \nu_{t+1} > \hat{\nu}_{t+1}^{OT} \left(z_t^d + x_t^d \right) \\ \left(1 - \delta \right) \omega_{t+1}, & \nu_{t+1} \leq \hat{\nu}_{t+1}^{OT} \left(z_t^d + x_t^d \right) \end{cases}. \tag{B.4}$$

2) if order x_t^d is delayed in period $t+1$

$$V_z^{DL} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \begin{cases} \frac{\partial R_t \left(z_t^d, \nu_{t+1} \right)}{\partial z_t^d}, & \nu_{t+1} > \hat{\nu}_{t+1}^{DL} \left(z_t^d \right) \\ \left(1 - \delta \right) \omega_{t+1}, & \nu_{t+1} \leq \hat{\nu}_{t+1}^{DL} \left(z_t^d \right) \end{cases},$$

and

$$V_x^{DL} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \left(1 - \delta \right) \omega_{t+1}.$$

Notice that following (B.3) and (B.4), we have

$$\mathbb{E}_\nu V_z^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \mathbb{E}_\nu V_x^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right). \tag{B.5}$$

Then expanding the expected marginal values of z_t and n_t yields:

$$\mathbb{E} V_z \left(z_t^d, x_t^d \right) = \left(1 - \theta_{t+1} \right) \mathbb{E}_\nu V_z^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) + \theta_{t+1} \mathbb{E}_\nu V_z^{DL} \left(z_t^d, x_t^d, \nu_{t+1} \right), \tag{B.6}$$

$$\mathbb{E} V_x \left(z_t^d, x_t^d \right) = \left(1 - \theta_{t+1} \right) \mathbb{E}_\nu V_x^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) + \theta_{t+1} \mathbb{E}_\nu V_x^{DL} \left(z_t^d, x_t^d, \nu_{t+1} \right). \tag{B.7}$$

Subtracting both sides of equation (B.7) from both sides of equation (B.6) yields

$$\mathbb{E}V_z\left(z_t^d, n_t^d\right) - \mathbb{E}V_x\left(z_t^d, x_t^d\right) = \theta_{t+1}\left[\mathbb{E}_\nu V_z^{OT}\left(z_t^d, x_t^d, \nu_{t+1}\right) - \mathbb{E}_\nu V_x^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right)\right], \tag{B.8}$$

following (B.5), i.e., conditional on the order arrives on time, the marginal values of inventory and order are the same.

Differencing (B.1) and (B.2) yields

$$\omega_t - c_t = \beta\left[\mathbb{E}V_z\left(z_t^d, x_t^d\right) - \mathbb{E}V_x\left(z_t^d, x_t^d\right)\right]. \tag{B.9}$$

Combining (B.8) and (B.9), and organizing terms yields

$$\mathbb{E}_\nu V_z^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right) = \frac{1}{\beta}\left(\frac{\omega_t - c_t}{\theta_{t+1}}\right) + (1-\delta)\,\omega_{t+1}. \tag{B.10}$$

Then we expand $\mathbb{E}_\nu V_z^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right)$. We start by expanding $\mathbb{E}_\nu V^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right)$:

$$\begin{aligned}
\mathbb{E}_\nu V^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right) =& \underbrace{\int_1^{\hat{\nu}} \hat{p}_{t+1}\nu\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}\,dF(\nu) + \int_{\hat{\nu}}^{\infty} z_t^d\left(\frac{z_t^d}{\nu Q_{t+1}}\right)^{-\frac{1}{\sigma}} P_{t+1}\,dF(\nu)}_{\text{expected in-season market revenue}} \\
& \underbrace{+ (1-\delta)\,\omega_{t+1}\left\{\int_1^{\hat{\nu}}\left[\left(z_t^d + x_t^d\right) - \nu\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}\right] dF(\nu) + \int_{\hat{\nu}}^{\infty} x_t^d\,dF(\nu)\right\}}_{\text{expected market value of the interim stock}} \qquad \text{(B.11)} \\
& - \omega_{t+1} z_{t+1}^d - c_{t+1} x_{t+1}^d + \beta\mathbb{E}V\left(z_{t+1}^d, x_{t+1}^d\right) \\
=& \frac{1}{\sigma-1}\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}\left(1-\hat{\nu}^{1-\lambda}\right) + \left(1-\hat{\nu}^{-\lambda}\right)(1-\delta)\,\omega_{t+1} z_t^d + \hat{p}_{t+1}\frac{\lambda}{\lambda-\frac{1}{\sigma}}\hat{\nu}^{-\lambda} z_t^d \\
& + (1-\delta)\,\omega_{t+1}\int_{\hat{\nu}}^{\infty} x_t^d\,dF(\nu) - \omega_{t+1} z_{t+1}^d - c_{t+1} x_{t+1}^d + \beta\mathbb{E}V\left(z_{t+1}^d, x_{t+1}^d\right),
\end{aligned}$$

where

$$\hat{\nu} = \hat{\nu}_{t+1}^{DL}\left(z_t^d\right) \equiv \frac{z_t^d}{\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}}.$$

Applying the envelope theorem to (B.11) yields

$$\frac{\partial\mathbb{E}_\nu V^{DL}\left(z_t^d, x_t^d, \nu_{t+1}\right)}{\partial z_t^d} = (1-\delta)\,\omega_{t+1} + \frac{\frac{1}{\sigma}}{\lambda-\frac{1}{\sigma}}(1-\delta)\,\omega_{t+1}\left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\lambda\sigma} Q_{t+1}^{\lambda}\left(z_t^d\right)^{-\lambda}. \tag{B.12}$$

Replacing the LHS in (B.10) with the expression in (B.12) yields

$$z_t^{d*} = \theta_{t+1}^{\frac{1}{\lambda}} \left(\frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}} \right)^{\frac{1}{\lambda}} \left(\frac{\omega_t - c_t}{\beta (1-\delta) \omega_{t+1}} \right)^{-\frac{1}{\lambda}} \left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\sigma} Q_{t+1}. \tag{B.13}$$

Q.E.D.

B.2 Proposition 2 Proof

Proof. Plugging (B.10) into (B.6) yields

$$\mathbb{E}V_z \left(z_t^d, x_t^d \right) = (1-\theta_{t+1}) \mathbb{E}_\nu V_z^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) + \frac{1}{\beta} (\omega_t - c_t) + (1-\delta) \theta_{t+1} \omega_{t+1}.$$

Plugging in (B.1) and organizing terms yields

$$\mathbb{E}_\nu V_z^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) = \frac{c_t - \beta (1-\delta) \theta_{t+1} \omega_{t+1}}{\beta (1-\theta_{t+1})}. \tag{B.14}$$

Expanding the $\mathbb{E}_\nu V^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right)$ yields

$$\begin{aligned}
\mathbb{E}_\nu V^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right) = & \underbrace{\int_1^{\hat{\nu}} \hat{p}_{t+1} \nu \left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\sigma} Q_{t+1} dF(\nu) + \int_{\hat{\nu}}^{\infty} \left(z_t^d + x_t^d \right) \left(\frac{z_t^d + x_t^d}{\nu Q_{t+1}} \right)^{-\frac{1}{\sigma}} P_{t+1} dF(\nu)}_{\text{expected in-season market revenue}} \\
& \underbrace{+ (1-\delta) \omega_{t+1} \left\{ \int_1^{\hat{\nu}} \left[\left(z_t^d + x_t^d \right) - \nu \left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\sigma} Q_{t+1} \right] dF(\nu) \right\}}_{\text{expected market value of the interim stock}} \qquad \text{(B.15)} \\
& - \omega_{t+1} z_{t+1}^d - c_{t+1} x_{t+1}^d + \beta \mathbb{E}V \left(z_{t+1}^d, x_{t+1}^d \right) \\
= & \frac{1}{\sigma - 1} \left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\sigma} Q_{t+1} \left(1 - \hat{\nu}^{1-\lambda} \right) + \left(1 - \hat{\nu}^{-\lambda} \right) (1-\delta) \omega_{t+1} \left(z_t^d + x_t^d \right) \\
& + \hat{p}_{t+1} \frac{\lambda}{\lambda - \frac{1}{\sigma}} \hat{\nu}^{-\lambda} \left(z_t^d + x_t^d \right) - \omega_{t+1} z_{t+1}^d - c_{t+1} x_{t+1}^d + \beta \mathbb{E}V \left(z_{t+1}^d, x_{t+1}^d \right),
\end{aligned}$$

where

$$\hat{\nu} = \hat{\nu}_{t+1}^{OT} \left(z_t^d + x_t^d \right) \equiv \frac{z_t^d + x_t^d}{\left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\sigma} Q_{t+1}}.$$

Applying the envelope theorem to (B.15) yields

$$\frac{\partial \mathbb{E}_\nu V^{OT} \left(z_t^d, x_t^d, \nu_{t+1} \right)}{\partial z_t^d} = (1-\delta) \omega_{t+1} + \frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}} (1-\delta) \omega_{t+1} \left(\frac{\hat{p}_{t+1}}{P_{t+1}} \right)^{-\lambda\sigma} Q_{t+1}^{\lambda} \left(z_t^d + x_t^d \right)^{-\lambda}. \tag{B.16}$$

Replacing the LHS in (B.14) with the expression in (B.16) yields

$$z_t^{d*} + x_t^{d*} = (1-\theta_{t+1})^{\frac{1}{\lambda}} \left(\frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}}\right)^{\frac{1}{\lambda}} \left(\frac{c_t}{\beta(1-\delta)\,\omega_{t+1}} - 1\right)^{-\frac{1}{\lambda}} \left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}. \tag{B.17}$$

Then plugging in the expression of optimal inventory holding z_t in equation (B.13) into equation (B.17) yields

$$x_t^{d*} = \left(\frac{\frac{1}{\sigma}}{\lambda - \frac{1}{\sigma}}\right)^{\frac{1}{\lambda}} \left[\left(\frac{c_t - \beta(1-\delta)\,\omega_{t+1}}{\beta(1-\delta)(1-\theta_{t+1})\,\omega_{t+1}}\right)^{-\frac{1}{\lambda}} - \left(\frac{\omega_t - c_t}{\beta(1-\delta)\,\theta_{t+1}\omega_{t+1}}\right)^{-\frac{1}{\lambda}}\right] \left(\frac{\hat{p}_{t+1}}{P_{t+1}}\right)^{-\sigma} Q_{t+1}.$$

Q.E.D.

B.3 Proposition 3 Proof

Proof. Since $K(j_1) < K(j_2)$,

$$|\mathcal{M}(j_1)| = N^{K(j_1)} < N^{K(j_2)} = |\mathcal{M}(j_2)|$$

and

$$|\mathcal{M}(j_2)| = |\mathcal{M}(j_1)|\, N^{K(j_1)-K(j_2)}. \tag{B.18}$$

For any $m(j_1)$, one can find $N^{(K(j_1)-K(j_2))}$source strategies $\tilde{m}(j_2)$ such that their sub-vectors containing their first $K(j_1)$ coordinates equal $m(j_1)$, hence

$$\prod_{r\in\{m(j_1)\}} (1-\mu_{rn}) \geq \prod_{r\in\{\tilde{m}(j_2)\}} (1-\mu_{rn}),$$

and

$$\prod_{r\in\{m(j_1)\}} (1-\mu_{rn}) \geq \frac{1}{N^{(K(j_1)-K(j_2))}} \sum_{\tilde{m}(j_2)} \left[\prod_{r\in\{\tilde{m}(j_2)\}} (1-\mu_{rn})\right],$$

and

$$1 - \prod_{r\in\{m(j_1)\}} (1-\mu_{rn}) \leq \frac{1}{N^{(K(j_1)-K(j_2))}} \sum_{\tilde{m}(j_2)} \left[1 - \prod_{r\in\{\tilde{m}(j_2)\}} (1-\mu_{rn})\right]$$

and

$$\frac{1}{\mathcal{M}(j_1)} \sum_{m(j_1)\in\mathcal{M}(j_1)} \left[1 - \prod_{r\in\{m(j_1)\}} (1-\mu_{rn})\right] \leq \frac{1}{\mathcal{M}(j_1)} \frac{1}{N^{(K(j_1)-K(j_2))}} \sum_{m(j_1)\in\mathcal{M}(j_1)} \left\{\sum_{\tilde{m}(j_2)} \left[1 - \prod_{r\in\{\tilde{m}(j_2)\}} (1-\mu_{rn})\right]\right\}. \tag{B.19}$$

Applying (B.18) to (B.19) yields

$$\bar{\theta}_n(j_1) \leq \bar{\theta}_n(j_2).$$

Q.E.D.

B.4 Closed-Form $S^{j}_{m(j)n,t}$

The stock for the off-season market consists of two parts: 1) the stock left from the in-season market and the active holdings and 2) the delayed orders,

$$S_t = (1-\delta)\left[\underbrace{(1-\theta_t)\int_1^{\hat{\nu}^{OT}} s_t^{OT}(\nu)\,dF(\nu) + \theta_t \int_1^{\hat{\nu}^{DL}} s_t^{DL}(\nu)\,dF(\nu)}_{\text{inventory left from the in-season market}} + \underbrace{\theta_t x_{t-1}}_{\text{delayed orders}}\right]. \tag{B.20}$$

The stock kept by traders with their orders delivered on time is

$$\begin{aligned} s_t^{OT}(\nu) &= (1-\delta)\left[z_{t-1} + x_{t-1} - q_t(\nu)\right] \\ &= (1-\delta)\left[z_{t-1} + x_{t-1} - \nu_t \left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t\right]. \end{aligned}$$

Integrating it over all traders who have their orders derived on time and do not have stockout yields

$$\begin{aligned} \int_1^{\hat{\nu}^{OT}} s_t^{OT}(\nu)\,dF(\nu) &= (1-\delta)\int_1^{\hat{\nu}^{OT}} \left[z_{t-1} + x_{t-1} - \nu_t \left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t\right] dF(\nu) \\ &= (1-\delta)\left[\left(1-\left(\hat{\nu}^{OT}\right)^{-\lambda}\right)(z_{t-1}+x_{t-1}) - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t \left(1-\left(\hat{\nu}^{OT}\right)^{1-\lambda}\right)\right] \\ &= (1-\delta)\left[(z_{t-1}+x_{t-1}) - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t - (z_{t-1}+x_{t-1})^{1-\lambda}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\right. \\ &\quad \left. + \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}(z_{t-1}+x_{t-1})^{1-\lambda}\right] \\ &= (1-\delta)\left[(z_{t-1}+x_{t-1}) - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \frac{1}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}(z_{t-1}+x_{t-1})^{1-\lambda}\right], \end{aligned}$$

and similarly for those traders whose orders are delayed, the stock they carry from the in-season market is

$$\int_1^{\hat{\nu}^{DL}} s_t^{DL}(\nu)\,dF(\nu) = (1-\delta)\left[z_{t-1} - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \frac{1}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}(z_{t-1})^{1-\lambda}\right].$$

Hence equation (B.20) can be expanded as

$$\begin{aligned}\frac{S_t}{1-\delta} =& (1-\theta_t)\left[(z_{t-1}+x_{t-1}) - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \frac{1}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}+x_{t-1}\right)^{1-\lambda}\right] \\ &+\theta_t\left[z_{t-1} - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \frac{1}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}\right)^{1-\lambda}\right] + \theta_t x_{t-1} \\ =& z_{t-1}+x_{t-1} - \frac{\lambda}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \frac{1}{\lambda-1}\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left[\left(1-\theta_t\right)\left(z_{t-1}+x_{t-1}\right)^{1-\lambda} + \theta_t\left(z_{t-1}\right)^{1-\lambda}\right].\end{aligned}$$

Hence, adding back the superscripts and subscripts and applying equations (25) and (26) yield the complete formula for the stock for the off-season market

$$\begin{aligned}S^j_{m(j)n,t} =& (1-\delta)\left\{Z^j_{m(j)n,t-1} + X^j_{m(j)n,t-1} - \frac{\lambda^j_n Q^j_{m(j)n,t}}{\lambda^j_n - 1}\left(\frac{\hat{p}^j_{m(j)n,t}}{P^j_{m(j)n,t}}\right)^{-\sigma}\right. \\ &\left.+\frac{\left(Q^j_{m(j)n,t}\right)^{\lambda^j_n}}{\lambda^j_n-1}\left(\frac{\hat{p}^j_{m(j)n,t}}{P^j_{m(j)n,t}}\right)^{-\sigma\lambda^j_n}\left[\left(1-\theta_{m(j)n,t}\right)\left(Z^j_{m(j)n,t-1}+X^j_{m(j)n,t-1}\right)^{1-\lambda^j_n} + \theta_{m(j)n,t}\left(X^j_{m(j)n,t-1}\right)^{1-\lambda^j_n}\right]\right\}.\end{aligned}$$

B.5 Closed-Form $\Pi_{m(j,n),t}$

The profits of selling to the in-season market also consists of two parts: those whose orders arrive on time, and those whose orders do not:

$$\pi_t = \left(1-\theta_t\right)\pi_t^{OT} + \theta_t \pi_t^{DL}$$

where

$$\pi_t = \underbrace{\left(1-\theta_t\right)R_t^{OT} + \theta_t R_t^{DL}}_{\text{in-season market revenue}} + \underbrace{\left(S_t - z_t\right)\omega_t}_{\text{off-season market net revenue}} - \underbrace{c_t x_t}_{\text{ordering cost}}.$$

The in-season-market revenue of firms whose orders arrive on time is

$$\begin{aligned}R_t^{OT} =& \int_1^{\hat{\nu}^{OT}} \hat{p}_t \nu \left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t dF(\nu) + \int_{\hat{\nu}^{OT}}^{\infty} \hat{p}_t\left(\frac{\nu}{\hat{\nu}^{OT}}\right)^{\frac{1}{\sigma}}\left(z_{t-1}+x_{t-1}\right) dF(\nu) \\ =& \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t\left(1-\left(\hat{\nu}^{OT}\right)^{1-\lambda}\right) + \frac{\lambda}{\lambda-\frac{1}{\sigma}}\left(\hat{\nu}^{OT}\right)^{-\frac{1}{\sigma}}\hat{p}_t\left(z_{t-1}+x_{t-1}\right)\left(\hat{\nu}^{OT}\right)^{\frac{1}{\sigma}-\lambda} \\ =& \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t\left(1-\left(\hat{\nu}^{OT}\right)^{1-\lambda}\right) + \frac{\lambda}{\lambda-\frac{1}{\sigma}}\hat{p}_t\left(z_{t-1}+x_{t-1}\right)\left(\hat{\nu}^{OT}\right)^{-\lambda} \\ =& \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t - \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}+x_{t-1}\right)^{1-\lambda} + \frac{\lambda}{\lambda-\frac{1}{\sigma}}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}+x_{t-1}\right)^{1-\lambda} \\ =& \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \left(\frac{\lambda}{\lambda-\frac{1}{\sigma}} - \frac{\lambda}{\lambda-1}\right)\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}+x_{t-1}\right)^{1-\lambda}\end{aligned}$$

and the in-season market revenue of firms whose orders are delayed is

$$R_t^{DL} = \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t + \left(\frac{\lambda}{\lambda-\frac{1}{\sigma}} - \frac{\lambda}{\lambda-1}\right)\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left(z_{t-1}\right)^{1-\lambda}$$

Hence the overall profits are

$$\begin{aligned}\pi_t =& \frac{\lambda}{\lambda-1}\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma} Q_t \\ &+ \left(\frac{\lambda}{\lambda-\frac{1}{\sigma}} - \frac{\lambda}{\lambda-1}\right)\hat{p}_t\left(\frac{\hat{p}_t}{P_t}\right)^{-\sigma\lambda} Q_t^{\lambda}\left[\left(1-\theta_t\right)\left(z_{t-1}+x_{t-1}\right)^{1-\lambda} + \theta_t\left(z_{t-1}\right)^{1-\lambda}\right] \\ &+ \left(S_t - z_t\right)\omega_t - c_t x_t.\end{aligned}$$

Hence, adding back the superscripts and subscripts yields the complete formula for the periodical profits of a trader:

$$\begin{aligned}\Pi^j_{m(j)n,t} &= \int_0^1 \pi^j_{m(j)n,t}(i)\,di \\ &= \left(\frac{\lambda_n^j}{\lambda_n^j-\frac{1}{\sigma}} - \frac{\lambda_n^j}{\lambda_n^j-1}\right)\hat{p}^j_{m(j)n,t}\left(\frac{\hat{p}^j_{m(j)n,t}}{P^j_{m(j)n,t}}\right)^{-\sigma\lambda_n^j}\left(Q^j_{m(j)n,t}\right)^{\lambda_n^j} \\ &\quad\times\left[\left(1-\theta_{m(j)n,t}\right)\left(Z^j_{m(j)n,t-1}+X^j_{m(j)n,t-1}\right)^{1-\lambda_n^j} + \theta_{m(j)n,t}\left(Z^j_{m(j)n,t-1}\right)^{1-\lambda_n^j}\right] \\ &\quad+ \frac{\lambda_n^j}{\lambda_n^j-1}\hat{p}^j_{m(j)n,t}\left(\frac{\hat{p}^j_{m(j)n,t}}{P^j_{m(j)n,t}}\right)^{-\sigma} Q^j_{m(j)n,t} + \left(S^j_{m(j)n,t} - Z^j_{m(j)n,t}\right)\omega^j_{m(j)n,t} - c^j_{m(j)n,t}X^j_{m(j)n,t}.\end{aligned}$$

B.6 Algorithm to speed up the solution for ω_t

Take the sector using two inputs for example.

1. Start with the guesses for ω_t, the dimension is $1 \times M \times M \times N$. Vectorize it together with all the exogenous variables.

2. Create a dummy variable, $I_a^{(i)}$, to keep track of the *absolute* position and another, $I_r^{(i)}$, for the *relative* position of a wholesaler group, the agent. For both dummy variables, value one (true) indicates that ω_t of the agent has not converged.

3. Pick out agents whose *relative* dummy is true, and evaluate them by running the program 'solve_Z_2'. The input vectors include $\omega_t^{(i)}$ and other vectors in their $I_r^{(i)}$-tailored version. The output vectors S, z, and $\tilde{Q}$. They share the same dimension, $D^{(i)} = \sum I_r^{(i)} = \sum I_a^{(i)}$.

   (a) all exogenous input vectors can tailored using $X^{(i)} = X^{(0)}\left(I_a^{(i)}\right)$.

4. Store ω_t to the vector with the fixed dimension $N^3 \times 1$ using $I_a^{(i)}$.

5. Evaluate $d_\omega^{(i)} = |S - z - \tilde{Q}|$. Check if $d_\omega^{(i)}$ is larger than the tolerance. Get the logical vector in which false value indicates convergence. Entries with true values are agents to be taken to the next iteration. Update the *relative* dummy with the logical vector, and get $I_r^{(i+1)}$.

6. Update $I_a^{(i)}$ with $I_r^{(i+1)}$and get $I_a^{(i+1)}$: $I_a^{(i+1)}\left(I_a^{(i)} > 0\right) = I_r^{(i+1)}$.

7. Update guesses $\omega_t^{(i+1)}$ for $I_r^{(i+1)} = 1$. Vector with the same superscript i shares the same dimension. Ideally, $D^{(i+1)} \leq D^{(i)}$.

   (a) update the dratio matrix, $H^{(i)} = \left(d_\omega^{(i)} ./ d_{\omega,old}^{(i)} \geq 1\right)$, and obtain $H^{(i+1)} = H^{(i)}\left(I_r^{(i+1)}\right)$.

   (b) update $\omega^{(i+1)}$

      i. $\omega_1^{(i)} = f\left(speed^{(i)}, S - z - \tilde{Q}, H^{(i)}, \omega_{lb}, \omega_{up}\right)$

      ii. $\omega_2^{(i)} = f\left(speed^{(i)}, S - z - \tilde{Q}, H^{(i)}, \omega_{lb}, \omega_{up}, \omega_s\right)$ where ω_s is obtained by updating the demand side

      iii. $\omega_{new}^{(i)}$ is the combination between $\omega_1^{(i)}$ and $\omega_2^{(i)}$

      iv. $\omega^{(i+1)}$=$\omega_{new}^{(i)}\left(I_r^{(i+1)}\right)$.

   (c) $\omega_{old}^{(i+1)}$ is updated with the holding matrix, the $\omega_{old}^{(i+1)} = H^{(i+1)} \cdot \omega_{old}^{(i)}\left(I_r^{(i+1)}\right) + \left(1 - H^{(i+1)}\right) \cdot \omega^{(i)}\left(I_r^{(i+1)}\right)$.

   (d) dimension of $d_{\omega,old}^{(i+1)}$ equals the $D^{(i+1)}$. $d_{\omega,old}^{(i+1)} = d_\omega^{(i)}\left(I_r^{(i+1)}\right)$.

8. Go back to Step 3 and iterate until $D^{(i+1)} = 0$.

C Quantification

C.1 Calibrated Iceberg Trade Costs

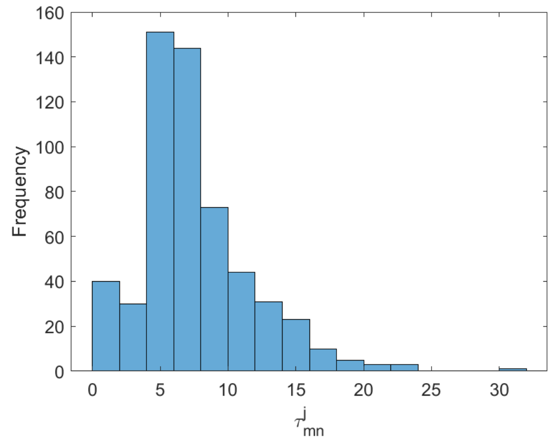
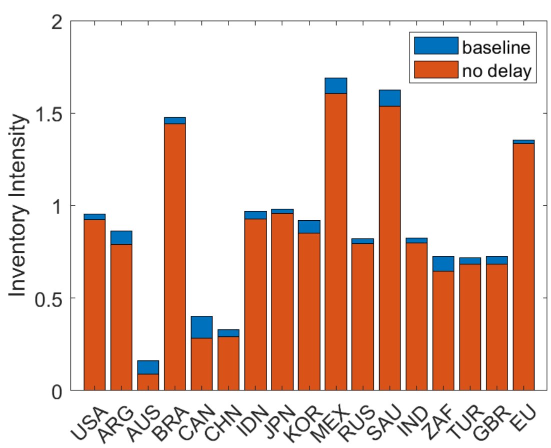


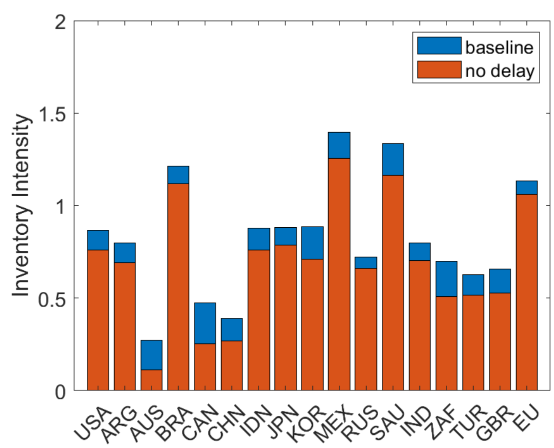
Figure C.1: Distribution of the Calibrated Iceberg Trade Costs

Note: Histogram displays the distribution of calibrated values for bilateral iceberg trade costs τ^j_{mn}.

C.2 Determinants to Inventory Intensity



(a) Single-Input Sector (Sector 1)



(b) Multi-Input Sector (Sector 2)

Figure C.2: Delivery Delay Risks and Inventory Intensity

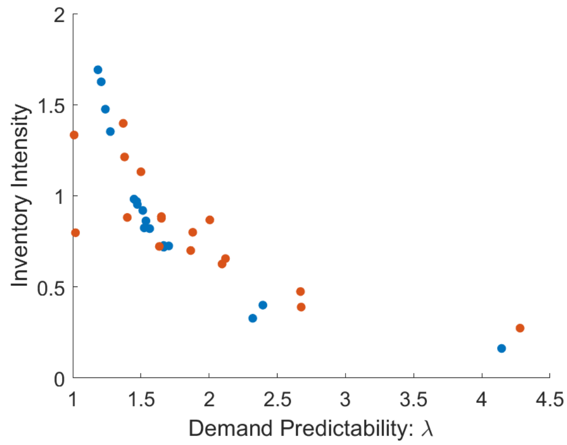


Figure C.3: Demand Uncertainty and Inventory Intensity