General Differential Equation

$$\frac{\partial}{\partial t}(\rho\Phi) + div(\rho\, u\, \Phi) = div(\gamma\, grad\, \Phi) + S$$

Where

$\frac{\partial}{\partial t}(\rho\Phi)$ ➔Unsteady term

$div(\rho\, u\, \Phi)$ ➔Convective term

$div(\gamma\, grad\, \Phi)$ ➔Diffusion term

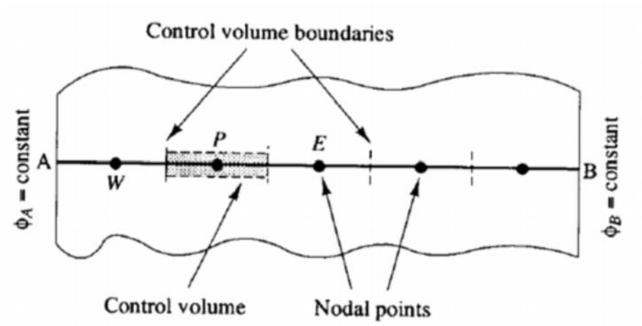
S ➔Source term

Φ ➔Dependent variable (example mass, pressure, temperature, etc.)

Finite volume method for 1D steady state Diffusion

The process of steady state 1D - diffusion of a property φ is governed by

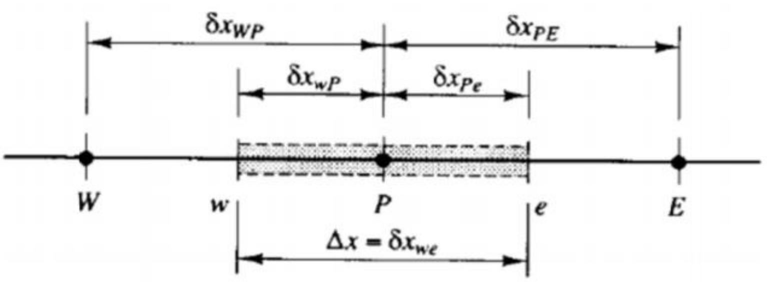
$$\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right) + \mathrm{S} = 0$$



Let us consider 1D heat conduction

Grid generation

The first step in the finite volume method is to divide the domain into discrete control volumes. The schematic of grid generation is shown in following Fig



A general nodal point is identified by P and its neighbours in a one- dimensional geometry, the nodes to the west and east, are identified by W and E respectively.

Discretization

The key step of the FVM is the integration of the governing equation(s) over a control volume to yield a discretized equation at its nodal point P. For the control volume defined above this gives

$$\int_{\delta V} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dV + \int_{\delta V} S dV = 0$$

as for 1D $\delta V = \delta x \times 1 \times 1$ $\int_{\delta x} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dx + \int_{\delta x} S dx = 0$

$$\int_{w}^{e} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dx + \int_{w}^{e} S dx = 0$$

$$\left(\Gamma A \frac{d\phi}{dx}\right)_e - \left(\Gamma A \frac{d\phi}{dx}\right)_w + \bar{S} dx = 0$$

In order to derive useful forms of the discretized equations, the interface diffusion coefficient Γ and the gradient dϕ/ dx at east face (e) and west face (w) are required. To calculate gradients (and hence fluxes) at the control volume faces, an approximate distribution of properties between nodal points is used.

Using Central differencing scheme. In a uniform grid linearly interpolated values for Γ_w and Γ_e are given by:

$\Gamma_w = \frac{\Gamma_W + \Gamma_P}{2}$ and $\Gamma_e = \frac{\Gamma_P + \Gamma_E}{2}$

and the diffusive flux terms are evaluated as:

$$\left(\Gamma A \frac{d\phi}{dx}\right)_e = \Gamma_e A_e \left(\frac{\phi_E - \phi_P}{\delta x_{PE}}\right)$$

$$\left(\Gamma A \frac{d\phi}{dx}\right)_w = \Gamma_w A_w \left(\frac{\phi_P - \phi_W}{\delta x_{WP}}\right)$$

The source term S may also depend on the dependent variable. In such cases the finite volume method approximates the source term by means of a linear form. Linearisation of source term can be expressed as:

$\bar{S}dV = S_c + S_p\phi_P$

Where, Sc is constant part of source term and Sp is a function of dependent variable. hence we get,

$\Gamma_e A_e\left(\frac{\phi_E - \phi_P}{\delta x_{PE}}\right) - \Gamma_w A_w\left(\frac{\phi_P - \phi_W}{\delta x_{WP}}\right) + \left(S_c + S_p\phi_P\right) = 0$

On arranging the terms we get:

$$\left(\frac{\Gamma_e}{\delta x_{PE}}A_e + \frac{\Gamma_w}{\delta x_{WP}}A_w - S_P\right)\phi_P = \left(\frac{\Gamma_w}{\delta x_{WP}}A_w\right)\phi_W + \left(\frac{\Gamma_e}{\delta x_{PE}}A_e\right)\phi_E + S_c$$

Which can also be written as:

$a_p\phi_p = a_W\phi_W + a_E\phi_E + S_c$ where:

a_w	a_E	a_P
$\frac{\Gamma_w}{\delta x_{WP}}A_w$	$\frac{\Gamma_e}{\delta x_{PE}}A_e$	$a_W + a_E - S_P$

Discretized equations for each of the control volume are formulated and suitable modifications are incorporated for the boundary control volumes. The resulting set of linear equations is solved using suitable linear solver (e.g. TDMA would be the best solver for the present example).

Rules for grid generation:

1) Locations of the CV faces are defined first.

2) Then nodal points are placed at the centers of the CV's.

3) Numbering starts from the boundary node at left.

4) All CV's have a volume of δx.A

5) Inter-nodal distances are equal to δx,(δxW P = δxP E = δx)

6) Near west boundary (node 2), δxW P = δx/2

7) Near east boundary (node 6), δxP E = δx/2