p-adic $\mathcal{D}'$

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1 p-adic Numbers

1.1 $\mathbb{Q}_p$

For $n \in \mathbb{Z}$, $n \neq 0$, and p prime, let $\mathrm{ord}_p(n)$ denote the exponent of p in the prime factorization of $|n|$ and let $\mathrm{ord}_p(0) = \infty$. For $a/b \in \mathbb{Q}$, we set $\mathrm{ord}_p(a/b) = \mathrm{ord}_p(a) - \mathrm{ord}_p(b)$. This definition is independent of the choice of representatives. We then define $|\cdot|_p : \mathbb{Q} \to \mathbb{R}$ by $|x|_p = p^{-\mathrm{ord}_p(x)}$. This is an absolute value on $\mathbb{Q}$ which satisfies a stronger form of the triangle inequality — the *ultrametric* inequality:

$$|x+y|_p \leqslant \max\Big(|x|_p, |y|_p\Big) \tag{UM}$$

It follows from $\mathrm{ord}_p(a/b + c/d) = \mathrm{ord}_p((ad+bc)/bd) = \mathrm{ord}_p(ad+bc) - \mathrm{ord}_p(bd) \geqslant \min(\mathrm{ord}_p(a) + \mathrm{ord}_p(d), \mathrm{ord}_p(b) + \mathrm{ord}_p(c)) - \mathrm{ord}_p(b) - \mathrm{ord}_p(d) = \min(\mathrm{ord}_p(a/b), \mathrm{ord}_p(c/d))$. We define $\mathbb{Q}_p$ to be the completion of $\mathbb{Q}$ with respect to the metric $d(x,y) = |x-y|_p$. This is again a field with the usual operations and the usual expansion of $|\cdot|_p$ to $\mathbb{Q}_p$ is again an absolute value. By the continuity of $\max : \mathbb{R}^2 \to \mathbb{R}$, we get the validity of (UM) for $x, y \in \mathbb{Q}_p$. It is $\mathbb{Q}_p \neq \mathbb{Q}_q$ for $p \neq q$, because $(p^n) \in 0$ in $\mathbb{Q}_p$, but $|p^n|_q = 1$ for all n. Interesting is the following

Lemma. *For all $x \in \mathbb{Q}_p \setminus \{0\}$ it is $|x|_p \in \{p^k \mid k \in \mathbb{Z}\}$.*

Proof. Let (x_n) be a Cauchy sequence in $\mathbb{Q}$ with respect to $|\cdot|_p$ that does not converge to zero, i. e., 0 is no accumulation point. Then there exist $\varepsilon > 0$ and $N \in \mathbb{N}$ such that for all $n \geqslant N$

$$|x_n|_p \geqslant \varepsilon \text{ and } |x_n - x_N|_p < \varepsilon.$$

With (UM), we get

$$|x_n|_p \leqslant \max(|x_n - x_N|_p, |x_N|_p) = |x_N|_p$$

and with the same argument

$$|x_N|_p \leqslant |x_n|_p$$

hence $|x_n|_p = |x_N|_p$ for all $n \geqslant N$. This shows that the sequence $(|x_n|_p)$ is eventually constant. $\square$

The elements of $\mathbb{Q}_p$ can be represented as follows.

Proposition ($\mathbb{Q}_p \leftrightarrow \mathbb{Z}/p\mathbb{Z}((X))$)**.** *Let $x \in \mathbb{Q}_p$. Then there exist uniquely determined $0 \leqslant a_j < p$ ($j \in \mathbb{Z}$) such that the sequence*

$$x_{n+N} = \sum_{j=-N}^{n} a_j p^j$$

is in x where $N = \max\{j \mid a_{-j} \neq 0\} \cup \{0\}$. In particular, the maximum exists.

Proof. We first consider the case $|x|_p \leqslant 1$, where we will have $N = 0$. Let (c_n) be a Cauchy sequence in x. For every $k \in \mathbb{N}$ let $N(k)$ denote a natural number such that for all $m, n \geqslant N(k)$, $|c_m - c_n|_p < p^{-k-1}$ and $N(k) < N(k+1)$. Now we set $x_{-1} = 0$ and choose $x_k \in \mathbb{Z} \cap [0, p^{k+1})$ such that

$$\left|x_k - c_{N(k)}\right|_p \leqslant p^{-k-1}$$

for $k \geqslant 0$. These exist, because if we write $c_{N(k)} = a/b$ with $(a,b) = 1$ we can find $\lambda, \mu \in \mathbb{Z}$ such that $\lambda b + \mu p^{k+1} = 1$, since $\mathrm{ord}_p(a/b) \geqslant 0$. We can set $x_k = \lambda a + \alpha p^{k+1}$ with a suitable $\alpha \in \mathbb{Z}$ and get

$$\left|\lambda a - \frac{a}{b} + \alpha p^{k+1}\right|_p \leqslant \max(\left|\frac{a}{b}\right|_p |\lambda b - 1|_p, \left|\alpha p^{k+1}\right|_p) \leqslant \max(\left|\mu p^{k+1}\right|_p, p^{-k-1}) = p^{-k-1}.$$

We are ready to define

$$a_j = \frac{x_j - x_{j-1}}{p^j} \quad (j \geqslant 0).$$

It is $a_j \in \mathbb{Z}$ because

$$|x_j - x_{j-1}|_p \leqslant \max(\left|x_j - c_{N(j)}\right|_p, \left|c_{N(j)} - c_{N(j-1)}\right|_p, \left|c_{N(j-1)} - x_{j-1}\right|_p) \leqslant p^{-j}$$

hence $p^j | (x_j - x_{j-1})$ and since

$$-p^j < x_j - x_{j-1} < p^{j+1}$$

we have $0 \leqslant a_j < p$. It is $(x_n) \sim (c_n)$, because for $n \geqslant N(k)$,

$$|x_n - c_n|_p \leqslant \max(|x_n - x_k|_p, \left|x_k - c_{N(k)}\right|_p, \left|c_{N(k)} - c_n\right|_p) \leqslant p^{-k-1}.$$

This proves the existence in our special case $|x|_p \leqslant 1$. Now the uniqueness. Let $\sum b_j p^j$ be another such sequence with $b_{j_0} \neq a_{j_0}$. Then

$$p^{j_0+1} \nmid \sum_{j=0}^{n} (a_j - b_j) p^j$$

for all $n \geqslant j_0$ and thus $(x_n) \not\sim \sum b_j p^j$.
The general case is handled by considering $p^m x$, whose p-adic absolute value is $\leqslant 1$ for sufficiently large m. □

We can thus identify a single element in $\mathbb{Q}_p$ with a formal Laurent series over the residue field $\mathbb{Z}/p\mathbb{Z}$ considering the mapping

$$x \mapsto \sum_{j \geqslant -N} \overline{a_j} X^j \tag{FLS}$$

which is a bijection, but *not* an isomorphism, since $\mathbb{Q}_p$ has characteristic 0, but $\mathbb{Z}/p\mathbb{Z}((X))$ has characteristic p. It is $|x|_p = p^{-k}$ where $k = \min\{j \mid a_j \neq 0\}$.

Definition. $\mathbb{Z}_p = \{x \in \mathbb{Q}_p \mid |x|_p \leqslant 1\}$.

This is a subring of $\mathbb{Q}_p$ with $\mathbb{Z}_p^\times = \{x \in \mathbb{Q}_p \mid |x|_p = 1\}$. Further, $\mathbb{Z}_p$ is integrally closed in $\mathbb{Q}_p$. It is $\mathbb{Q}_p$ the quotient field of $\mathbb{Z}_p$. Also, $\mathbb{Z} \subset \mathbb{Z}_p$.

1.2 Extensions of $\mathbb{Q}_p$

Definition. *Let v be an absolute value on the field K. We call v* non-Archimedean *if it satisfies* (UM)*, i. e., $v(x+y) \leqslant \max(v(x), v(y))$. We call v* discrete *if $v(K^\times)$ is discrete in $\mathbb{R}^+$. We also set*

$$A_K = \{x \in K \mid v(x) \leqslant 1\},$$
$$M_K = \{x \in K \mid v(x) < 1\}.$$

Proposition. *Let v be a non-Archimedean absolute value on K. Then, A_K and M_K are subrings of K and M_K is the unique maximal ideal of A_K. Thus, A_K is a* local ring.

Lemma. *Let L be a finite field extension of K and v non-Archimedean absolute value on L. Then A_L/M_L is a field extension of A_K/M_K.*

Proof. Define $\iota : A_K/M_K \to A_L/M_L$, $\iota(x+M_K) = x+M_L$. This is well-defined and injective since $A_K \cap M_L = M_K$. □

Definition. *Let L be a finite field extension of K and v discrete non-Archimedean absolute value on L. Then we call the index of $v(K^\times)$ in $v(L^\times)$ the* degree of ramification of $L : K$. *We call*

$$[A_L/M_L : A_K/M_K]$$

the degree of the residue field extension of $L : K$. *Is the degree of ramification of $L : K$ equal to* 1*, then $L : K$ is* non-ramified*, is it equal to $[L : K]$, then $L : K$ is* totally ramified.

Theorem. *Let K be a with respect to the discrete non-Archimedean absolute value v complete field, L finite field extension K, e the degree of ramification and f the degree of the residue field extension of $L : K$. Then,*

$$[L : K] = e \cdot f.$$

Proof. Let $\varphi : A_L \to A_L/M_L$ the canonical epimorphism and $y_i \in A_L$ $(i \in I)$, such that $\{\varphi(y_i) \mid i \in I\}$ is a basis of A_L/M_L over A_K/M_K. Then it is $v(y_i) = 1$ for all i. Let further $\pi \in A_L$, such that $M_L = (\pi)$. The order of $v(\pi) \cdot v(K^\times)$ in the group $v(L^\times)/v(K^\times)$ is equal to e. We will show that

$$\left(y_i \pi^j\right)_{\substack{i \in I \\ 0 \leqslant j < e}}$$

is a basis of $L : K$. So, let

$$\sum_{\substack{i \in I \\ 0 \leqslant j < e}} \lambda_{ij} y_i \pi^j = \sum_{0 \leqslant j < e} \underbrace{\left(\sum_{i \in I} \lambda_{ij} y_i\right)}_{s_j} \pi^j = 0$$

with $\lambda_{ij} \in K$ not all 0. By division by the λ_{ij} with largest absolute value, we can suppose $\lambda_{ij} \in A_K$, where $\lambda_{ij} \notin M_K$ for a λ_{ij}. By application of φ, we see that at least one $s_j \neq 0$ and that for those s_j, $v(s_j) = v(\lambda_{ij}) \in v(K^\times)$. There have to be two $s_j \pi^j \neq 0$ with the same absolute value, $v(s_j \pi^j) = v(s_{j'} \pi^{j'})$ with $j < j'$. That is because $v(a) \neq v(b) \Rightarrow v(a+b) = \max(v(a), v(b))$. It follows that

$$v(\pi^{j'-j}) = \frac{v(\pi^{j'})}{v(\pi^j)} = \frac{v(s_j)}{v(s_{j'})} \in v(K^\times),$$

where $j' - j < e$. That is a contradiction to the choice of π.
We now define the A_K-modules

$$X = \sum_{\substack{1 \leqslant i \leqslant f \\ 0 \leqslant j < e}} A_K y_i \pi^j, \ Y = \sum_{1 \leqslant i \leqslant f} A_K y_i$$

and show $X = A_L$. Obviously,

$$X = \sum_{0 \leqslant j < e} \pi^j Y$$

and $A_L = Y + \pi A_L$. Repeated application of the last gives $A_L = \sum_{0 \leqslant j \leqslant e-1} \pi^j Y + \pi^e A_L$ and thus $A_L = X + M_K A_L$. Since $M_K X \subset X$ it follows that $A_L = X + M_K^\nu A_L$ for all $\nu \geqslant 1$ and since $(M_K^\nu A_L)_{\nu \in \mathbb{N}}$ is neighbourhood basis of 0 in A_L, it follows that $\overline{X} = A_L$. But it is X closed in A_L, because A_K is in K ist. This proves $X = A_L$. $\square$

Proposition. *Let K be a non-trivially valued locally compact field and V a finite dimensional vector space over K. Then there exists at most one Hausdorff topology on V such that V becomes a topological vector space.*

Proposition. *Let K be a finite field extension of $\mathbb{Q}_p$. Then there exists exactly one absolute value on K that extends $|\cdot|_p$.*

Theorem. *Let K be a finite field extension of $\mathbb{Q}_p$. Then there exists a uniquely determined intermediate field M of $K : \mathbb{Q}_p$ such that $M : \mathbb{Q}_p$ is unramified and $K : M$ is totally ramified.*

Theorem (Hensel lemma)**.** *Let K be finite extension of $\mathbb{Q}_p$, π prime in A_K. Further let $f \in K[X]$, $\alpha_0 \in A_K$ with $f(\alpha_0) \equiv 0(\pi)$ and $f'(\alpha_0) \not\equiv 0(\pi)$. Then there exists an $\alpha \in A_K$ with $f(\alpha) = 0$ and $\alpha \equiv \alpha_0(\pi)$.*

Theorem (Krasner lemma)**.** *Let $K : \mathbb{Q}_p$ be finite, $a, b \in \overline{\mathbb{Q}}_p$, $a = a_1, \ldots, a_n$ the conjugates of a. Further let*

$$|b - a|_p < |a_i - a|_p$$

for all $i \geqslant 2$. Then, $K(a) \subset K(b)$.

Proposition. *The algebraic closure $\overline{\mathbb{Q}}_p$ of $\mathbb{Q}_p$ is not complete.*

Proof. It is $[\overline{\mathbb{Q}}_p : \mathbb{Q}_p] = \aleph_0$. This means that $\overline{\mathbb{Q}}_p = \bigcup_{i \in \mathbb{N}} K_i$ with $K_i \subset K_{i+1}$ and K_i finite over $\mathbb{Q}_p$, i. e., K_i complete. This shows that all K_i are closed. Also, they have empty interior. By Baire's theorem, $\overline{\mathbb{Q}}_p$ is not complete. $\square$

Definition. *Let $\mathbb{C}_p$ be the completion of $\overline{\mathbb{Q}}_p$.*

Proposition. *It is $\mathbb{C}_p$ algebraically closed.*

2 p-adic Analysis

We will now restrict ourselves to $\mathbb{Q}_p$ again.

2.1 Basics

Theorem (Heine-Borel)**.** *Let $A \subset \mathbb{Q}_p$. It is A compact if and only if it is closed and bounded.*

Proof. We show that $\mathbb{Z}_p$ is compact. Let (x_n) be a sequence in $\mathbb{Z}_p$,

$$x_n = \sum_{k \geqslant 0} x_n^k p^k.$$

For all k, the integer sequences $(x_n^k)_n$ have an accumulation point a_k, because $x_n^k \in \{0, \ldots, p-1\}$. Then $a = \sum a_k p^k$ is an accumulation point of (x_n). □

Definition. *For $x \in \mathbb{Q}_p^n$, we set*

$$|x|_p = \max_{1 \leqslant i \leqslant n} |x_i|_p .$$

Let $a \in \mathbb{Q}_p^n$ and $\gamma \in \mathbb{Z}$. Then we set

$$\begin{aligned} B_\gamma(a) &= \{x \in \mathbb{Q}_p^n \mid |x-a|_p \leqslant p^\gamma\}, \\ S_\gamma(a) &= \{x \in \mathbb{Q}_p^n \mid |x-a|_p = p^\gamma\}, \\ \Delta_\gamma &= \chi_{B_\gamma(0)}. \end{aligned}$$

Also, let $B_\gamma = B_\gamma(0)$ and $S_\gamma = S_\gamma(0)$.

Definition. *Let $\gamma \in \mathbb{Z}$, $x_0 \in \mathbb{Q}_p$ such that*

$$f(x) = \sum_{n \geqslant 0} a_n (x - x_0)^n.$$

converges for all $x \in B_\gamma(x_0)$. Then f is an analytic function on $B_\gamma(x_0)$*. For $k \in \mathbb{N}$, we set*

$$f^{(k)}(x) = \sum_{n \geqslant 0} (n+k) \cdots (n+1) a_{n+k} x^n,$$

$$f^{(-k)}(x) = \sum_{n \geqslant k} \frac{1}{(n-k+1) \cdots n} a_{n-k} x^n.$$

Definition. *Let μ denote the unique positive regular translation-invariant Borel-measure on the locally compact group $(\mathbb{Q}_p, +)$ such that $\mu(B_0) = 1$. For μ-measurable $f : \mathbb{Q}_p^n \to \mathbb{C}$, we set*

$$\int f(x) dx = \int f(x) d\mu(x).$$

Theorem (Substitution rule)**.** *Let $A, B \subset \mathbb{Q}_p^n$ open and compact, $\Phi = (\varphi_1, \ldots, \varphi_n) : A \to B$ a homeomorphism such that all φ_i are analytic in A and*

$$\det \frac{\partial \Phi(x)}{\partial x} \neq 0$$

for all $x \in A$. Then there holds

$$\int_B f(x) dx = \int_A f(\Phi(x)) \left| \det \frac{\partial \Phi(x)}{\partial x} \right|_p dx$$

for all $f \in L^1(B)$.

2.2 Distributions

Definition. *Let $f : \mathbb{Q}_p^n \to \mathbb{C}$. It is called f* locally constant *if for every $x \in \mathbb{Q}_p^n$ there exists an $l(x) \in \mathbb{Z}$ such that $f(x+\xi) = f(x)$ for all $\xi \in B_{l(x)}$. The set of all locally constant functions on $\mathbb{Q}_p^n$ is denoted by $\mathcal{E} = \mathcal{E}(\mathbb{Q}_p^n)$.*
If f is locally constant with compact support, then $l = l(f) = \sup l(x)$ is called the degree of constancy of f. *The set of all compact supported locally constant functions on $\mathbb{Q}_p^n$ is denoted by $\mathcal{D} = \mathcal{D}(\mathbb{Q}_p^n)$. Its elements are called* test functions.
We further set

$$\mathcal{D}_N^l = \mathcal{D}_N^l(\mathbb{Q}_p^n) = \{f \in \mathcal{D} \mid l(f) \geqslant l, \operatorname{supp} f \subset B_N\}.$$

It is $\mathcal{D}_N^l$ finite dimensional over $\mathbb{Q}_p$, hence complete, and $\mathcal{D}_M^k \subset \mathcal{D}_N^l$ for $M \leqslant N$ and $k \leqslant l$. We define the following notion of convergence in $\mathcal{D}$:
It is $\varphi_k \to 0$ iff there exist l and N such that $\varphi_k \in \mathcal{D}_N^l$ for all k and $\varphi_k(x) \to 0$ uniformly for all x. It is this the inductive limit of the spaces $\mathcal{D}_N^l$ with the uniform convergence topology. Hence, $\mathcal{D}$ is locally convex with this topology ([3], II, §4.4, ex. 2).

Proposition. *For all compact K, it is $\mathcal{D}$ dense in $C(K)$.*

Lemma. *It is $\varphi_k \to 0$ if and only if $\varphi_k(x) \to 0$ uniformly and for all $\psi \in \mathcal{D}$, $\langle \varphi_k, \psi \rangle \to 0$.*

Definition. *We define $\mathcal{D}' = \mathcal{D}'(\mathbb{Q}_p^n)$ to be the set of linear forms on $\mathcal{D}(\mathbb{Q}_p^n)$. These are continuous. Convergence in $\mathcal{D}'$ is weak convergence.*

Proposition. *It is $\mathcal{D}'$ complete.*

Proof. Let (f_k) be Cauchy in $\mathcal{D}'$. Then, $(\langle f_k, \varphi \rangle)$ is Cauchy in $\mathbb{C}$ for all φ. Thus, there exists a $C(\varphi) \in \mathbb{C}$ with $\langle f_k, \varphi \rangle \to C(\varphi)$. By the linearity of $\langle f_k, \cdot \rangle$, we get the linearity of C and thus $C \in \mathcal{D}'$. It is $f_k \to C$. □

Definition. *Let $f \in \mathcal{D}'(\mathbb{Q}_p^n), g \in \mathcal{D}'(\mathbb{Q}_p^m)$. Then we define the distribution $f \times g$ by*

$$\langle f(x) \times g(y), \varphi(x,y) \rangle = \langle f(x), g(y)\varphi(x,y) \rangle.$$

It is called the direct product of f and g.
For $f, g \in \mathcal{D}'(\mathbb{Q}_p^n)$, we define the convolution of f and g *by*

$$\langle f * g, \varphi \rangle = \lim_{k \to \infty} \langle f(x) \times g(y), \Delta_k(x)\varphi(x+y) \rangle.$$

Definition. *We set*

$$\delta_k = p^{nk} \Delta_{-k}.$$

Let $f \in \mathcal{D}'$. We call the sequence

$$f_k = \Delta_k(f * \delta_k)$$

the regularization of f.

It is $f_k \to f$.

Proposition. *For $f \in C(\mathbb{Q}_p^n)$, it is $f_k(x) = f * \delta_k(x) \to f(x)$ for all x.*

Proof. Let $\varepsilon > 0$. Since $\int \delta_k = 1$, we have

$$\begin{aligned} f_k(x) - f(x) &= \int (f(x-y) - f(x)) p^{nk} \chi_{B_{-k}}(y) dy \\ &= p^{nk} \int_{B_{-k}} f(x-y) - f(x) dy. \end{aligned}$$

Since f is continuous, there exists an $\gamma \in \mathbb{Z}$ such that $|f(x-y) - f(x)| < \varepsilon$ for $y \in B_\gamma$. It follows that

$$|f_k(x) - f(x)| \leqslant \varepsilon p^{nk} \int_{B_{-k}} dy = \varepsilon$$

for $k \geqslant \gamma$. □

2.3 Colombeau-type algebra of generalized functions

Definition. $\mathcal{P} = \mathcal{D}^{\mathbb{N}}$. *We define addition and multiplication on $\mathcal{P}$ componentwise (and pointwise). This forms an algebra over $\mathbb{C}$.*

$$\mathcal{N} = \{(f_k) \in \mathcal{P} \mid \textit{for all compact } K \textit{ there exists an } N \textit{ such that } f_k|K = 0 \textit{ for } k \geqslant N\}.$$

It is this an ideal of $\mathcal{P}$.

Definition. *We call $\mathcal{G}_p = \mathcal{P}/\mathcal{N}$ the p-*adic Colombeau-Egorov algebra*. Its elements are called* generalized functions.

Theorem. *The mapping $\Phi : \mathcal{D}' \to \mathcal{G}_p$, $\Phi(f) = (f_k) + \mathcal{N}$ is a vector space monomorphism, where (f_k) denotes the regularization of f. The mapping $\Psi : \mathcal{D} \to \mathcal{G}_p$, $\Psi(f) = (f) + \mathcal{N}$ is an algebra monomorphism.*

Proof. It is obvious that Φ is linear. We will now prove that $(f_k) \in \mathcal{N}$ implies $f = 0$. For $\varphi \in \mathcal{D}$, it is $\langle f_k, \varphi \rangle = \langle f, \delta_k * \varphi \rangle$. For all $k \geqslant -l(\varphi)$, we have $\delta_k * \varphi(x) = p^{nk} \int_{B_{-k}} \varphi(x-y) dy = \varphi(x)$ for all x. It follows that $\langle f, \varphi \rangle = \langle f_k, \varphi \rangle = 0$ for $k \geqslant \max(-l(\varphi), N(K))$, where $K = \operatorname{supp} \varphi$, i. e., $f = 0$. □

Definition. *We define $\mathcal{A} = \mathbb{C}^{\mathbb{N}}$ with componentwise operations, $\mathcal{I} = \{(c_k) \in \mathcal{A} \mid c_k \neq 0$ for at most finitely many $k\}$. It is $\mathcal{I}$ an ideal in $\mathcal{A}$. Let $\mathcal{C} = \mathcal{A}/\mathcal{I}$ be the set of* Colombeau-Egorov generalized numbers.
Further, we set $\mathcal{B}$ to be the set of bounded sequences in $\mathbb{Q}_p^n$ and $\mathcal{J} = \{(d_k) \in \mathcal{B} \mid d_k \neq 0$ for at most finitely many $k\}$. Again, $\mathcal{J}$ is an ideal of $\mathcal{B}$ and we set $\mathcal{M} = \mathcal{B}/\mathcal{J}$. Its elements are called generalized points.

Definition. *For $f = (f_k) + \mathcal{N} \in \mathcal{G}_p$ and $x = (x_k) + \mathcal{J} \in \mathcal{M}$, we set $(f_k(x_k)) + \mathcal{I} \in \mathcal{C}$ the* point value of f at x, *denoted by $f(x)$.*

Lemma. *The point value is well-defined.*

Theorem. *Let $f, g \in \mathcal{G}_p$. Suppose that for all $x \in \mathcal{M}$ we have $f(x) = g(x)$. Then, $f = g$.*

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