

Let n be a positive integer, and let f be a continuous real-valued function on $[0, 1]$ with the property that $\int_0^1 x^k f(x) dx = 1$ for $0 \leq k \leq n-1$. Prove that $\int_0^1 (f(x))^2 dx \geq n^2$.