

Gaussian Mixture Model Distribution

$$p(s|x) = \sum_{k=1}^K \pi(a^k|x) \prod_{t=1}^T \phi(s_t|a^k, x)$$

$$\xrightarrow{\mathbf{y} = [s_1, s_2, \dots, s_T]}$$

$$p(s|x) = \sum_{k=1}^K \pi(a^k|x) \phi(s|a^k, x)$$

Negative Log-Likelihood Loss

Data: $\{(\mathbf{x}^m, \hat{\mathbf{s}}^m)\}_{m=1}^M$ $\hat{\mathbf{s}}^m = [s_1^m, s_2^m, \dots, s_T^m]$

$$l(\theta) = - \sum_{m=1}^M \sum_{k=1}^K \mathbf{1}(k = \hat{k}^m) \left[\log \pi(a^k | \mathbf{x}^m; \theta) + \sum_{t=1}^T \log \mathcal{N}(\mathbf{s}_t^m | a_t^k + \mu_t^k, \Sigma_t^k; \mathbf{x}^m; \theta) \right]$$

$$\downarrow$$

$$\boldsymbol{\mu}^k = [\mu_1^k, \mu_2^k, \dots, \mu_T^k]$$

$$\boldsymbol{\Sigma}^k = [\Sigma_1^k, \Sigma_2^k, \dots, \Sigma_T^k]$$

Data: $\{(\mathbf{x}^m, \mathbf{y}^m)\}_{m=1}^M$ $\mathbf{y}^m = [y_1^m, y_2^m, \dots, y_T^m]$

$$l(\theta) = - \sum_{m=1}^M \sum_{k=1}^K \mathbf{1}(k = \hat{k}^m) [\log \pi(a^k | \mathbf{x}^m; \theta) + \log \mathcal{N}(\mathbf{y}^m | a^k + \boldsymbol{\mu}^k, \boldsymbol{\Sigma}^k; \mathbf{x}^m; \theta)]$$

Data: $\{(\mathbf{x}^m, y^m)\}_{m=1}^M$

$$p(y|x) = \sum_{k=1}^K \pi(a^k|x) \phi(y|a^k, \mathbf{x})$$

$$l(\theta) = - \sum_{m=1}^M \sum_{k=1}^K \mathbf{1}(k = \hat{k}^m) [\log \pi(a^k|x; \theta) + \log \mathcal{N}(y^m|a^k + \boldsymbol{\mu}^k, \boldsymbol{\Sigma}^k; \mathbf{x}^m; \theta)]$$

