

Student Information

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Answer 1

Answer 2

a.

3 different tiles.

b.

$$a_0 = 0$$

$$a_2 = 2$$

$$a_4 = 5$$

$$a_6 = 14$$

$$a_n = 3 \times (a_{n-2}) - 1$$

c.

$$a_n \times x^n = 3a_{n-2} \times x^n - x^n$$

$$G(x) = \sum_{n=0}^{\infty} a_n \times x^n \quad \text{we denote } a_0 = 1$$

$$G(x) - 1 = \sum_{n=2}^{\infty} a_n \times x^n = \sum_{n=2}^{\infty} (3a_{n-2} \times x^n - x^n)$$

$$= 3 \times \sum_{n=2}^{\infty} (a_{n-2} \times x^n) - \sum_{n=2}^{\infty} x^n \tag{1}$$

$$= 3x^2 \times \sum_{n=2}^{\infty} (a_{n-2} \times x^{n-2}) - x \sum_{n=2}^{\infty} x^{n-2}$$

$$= 3x^2 \times \sum_{n=0}^{\infty} (a_n \times x^n) - x \sum_{n=0}^{\infty} x^n$$

$$G(x) - 1 = 3x^2 \times G(x) - \frac{x(1 - x^n)}{1 - x}$$

Answer 3

a.

The \subseteq relation on any set of sets is a partial order. Because \subseteq relation is reflexive on any set. (assume A is a set then $A \subseteq A$.) It is also antisymmetric because for any set A and B, $A \subseteq B$ and $B \subseteq A$ imply that $A=B$. Finally \subseteq is transitive, because for any set A, B and C, $A \subseteq B$ and $B \subseteq C$ imply that $A \subseteq C$.

b.

The relation $|$ of divisibility on the set of integers is a partial order. Because for any integer a , $a | a$ so it is reflexive. It is also antisymmetric because, if a and b are positive integers with $a | b$ and $b | a$, then $a = b$. Finally, it is transitive, Suppose that a divides b and b divides c . Then there are positive integers k and l such that $b = ak$ and $c = bl$. Hence, $c = a(kl)$, so a divides c .

c.

Relation R defined as “ aRb if there is a positive integer r such that $b = a^r$ ” on Z is a partial order. Because for any integer a , if we select $r = 1$, $a = a^1$ so it is reflexive. It is also antisymmetric because, if a and b are integers with $b = a^r$ and $a = b^r$, then $a = b$ and r must be 1. Finally, it is transitive, Suppose that $b = a^{r_1}$ and $c = b^{r_2}$, then $c = a^{r_1^{r_2}}$, there exists $r_3 = r_1 \times r_2$ so $c = a^{r_3}$.

Answer 4

a.

$$5 = 5$$

$$5 = 4 + 1$$

$$5 = 3 + 1 + 1$$

$$5 = 3 + 2$$

$$5 = 2 + 2 + 1$$

$$5 = 2 + 1 + 1 + 1$$

$$5 = 1 + 1 + 1 + 1 + 1$$

b.

