## **Student Information**

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## Answer 1

If all vertices of a graph having degree at least 4, then there will be at least 4n/2=2n edges in the graph where n is the number of vertices by the handshaking theorem. This means that if there are 23 edges that n must be strictly less than 12 (because with 12 vertices there must be at least 24 edges). We can have a graph with 11 vertices and 23 edges and so this must be the maximum number of vertices possible.

#### Answer 2

By definition of Hamilton path and graph, the simple circuit  $x_0, x_1, ..., x_{n-1}, x_n, x_0$  is a Hamilton circuit if  $x_0, x_1, ..., x_{n-1}, x_n$  is a Hamilton path in the graph G=(V,E). Which means if we remove the edge which is connected to  $x_0$  and  $x_n$ , the graph G would still have the Hamilton path. Basically, if we remove any edge from the graph G with G wi

For every graph A, which has n>2 vertices and cointains a Hamilton path, by Dirac's theorem A's every vertex has a degree at least  $\frac{n}{2}$ . Now lets build an another graph B by removing an edge from A. As we proved before graph B must cointain a Hamilton path and B's every vertex degree is at least  $\frac{n}{2}-1$  as we remove an edge from the graph, at least 2 vertex's degree will be one less. Therefore for every graph G with n vertices, each of which has degree  $\frac{n-1}{2}$  (as  $\frac{n-1}{2}>\frac{n}{2}-1$ )contains a Hamilton path.

### Answer 3

If A is the adjacency matrix of a bipartite graph, it's diagonal entries must be 0 because bipartite graphs cannot contain a loop, as a loop connects the same vertex from the same set and adjancency matrix's diagonal entries are only not zero when the graph has loops. So that A's every odd power's diagonal entries is O.

# Answer 4

Choice	Edge	Weight
1	$_{\mathrm{e,f}}$	1
2	$_{ m d,a}$	2
3	$_{\mathrm{e,h}}$	2
4	$_{\mathrm{g,h}}$	2
5	$_{\mathrm{c,f}}$	3
6	$_{ m d,h}$	3
7	$_{ m g,d}$	3
8	$_{ m h,i}$	4
		total:20

a.

Choice	Edge	Weight
1	$_{\mathrm{e,f}}$	1
2	$_{\mathrm{e,h}}$	2
3	$_{ m h,g}$	2
4	$_{ m g,d}$	3
5	$_{ m d,a}$	2
6	$_{ m d,b}$	3
7	$_{\rm f,c}$	3
8	$_{ m h,i}$	4
		total:20

b.

Figure 1: Delete the related edges to display the acquired MST

