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Answer 1

$$(\sum_{k=1}^{n} k)^2 \geq \sum_{k=1}^{n} k^2$$

$$BASIS\ STEP\ P(1) \quad (\sum_{k=1}^{1} k)^2 \geq \sum_{k=1}^{1} k^2 \quad 1 \geq 1$$

$$INDUCTIVE\ STEP\ P(k) \quad (\sum_{n=1}^{k} n)^2 \geq \sum_{n=1}^{k} n^2 \quad (assume)$$

$$(\frac{n(n+1)}{2})^2 \geq \frac{n(n+1)(2n+1)}{6} \quad dividing\ both\ sides\ n(n+1)$$

$$\frac{n(n+1)}{4} \geq \frac{(2n+1)}{6}$$

$$P(k+1) \quad (\sum_{n=1}^{k+1} n)^2 \geq \sum_{n=1}^{k+1} n^2$$

$$(\frac{(n+1)(n+2)}{2})^2 \geq \frac{(n+1)(n+2)(2n+3)}{6} \quad dividing\ both\ sides\ (n+1)(n+2)$$

$$\frac{(n+1)(n+2)}{4} \geq \frac{(2n+3)}{6} \tag{1}$$

$$\frac{2n+3}{6} = \frac{2n+1}{6} + \frac{2}{6} \leq \frac{n(n+1)}{4} + \frac{2n+2}{4}\ (inductive\ hypothesis.Also,\ we\ know\ that\ \frac{2n+2}{4} \geq \frac{2}{6})$$

$$\frac{(2n+3)}{6} \leq \frac{n^2+2n+2)}{4} = \frac{(n+1)(n+2)}{4} multiplying\ both\ sides\ with\ (n+1)(n+2)$$

$$\frac{(n+1)(n+2)(2n+3)}{6} \leq \frac{(n+1)(n+2)}{2})^2\ which\ equals\ to$$

$$(\sum_{n=1}^{k+1} n)^2 \geq \sum_{n=1}^{k+1} n^2\ and\ this\ completes\ the\ prof \tag{2}$$

Answer 2

1.

2.

$a + b + c = 5$

C(3+5-1,5) = C(7,2) = $\frac{7!}{2!(7-2)!} = 21$

3.

As x_1, x_2, x_3 are positive integers, they are at least 1.

C(3+2-1,2)=C(4,2)=$\frac{4!}{2!(4-2)!} = 6$

Answer 3

$$\begin{array}{ll}
a_0(1-x)^{3n} & k=0 \\
a_1.x(1-x)^{3n-2} & k=1 \\
a_2.x^2(1-x)^{3n-4} & k=2 \\
\vdots & \\
a_n.x^n(1-x)^n & k=n
\end{array}$$

summing this we get:

$$\begin{aligned}(1-x)^n(a_0(1-x)^{2n} + a_1.x(1-x)^{2n-2} + a_2.x^2(1-x)^{3n-4} + \ldots + a_n.x^n) \\ = (1-x)^n(b(1-x)^2 + c.x)^n \ (\textit{by binomial theorem})\end{aligned} \tag{3}$$

$$(1-x^3)^n = (1-x)^n(1+x+x^2)^n \tag{2}$$

$$\begin{aligned}
(1-x)^n(b(1-x)^2 + c.x)^n &= (1-x)^n(1+x+x^2)^n \ \textit{dividing bthsides with } (1-x)^n \\
(b(1-x)^2 + c.x)^n &= (1+x+x^2)^n \\
b(1-x)^2 + c.x &= 1 + x + x^2 \qquad b = 1 \ c = 3 \\
a_r &= \binom{n}{r} b^{n-r} c^r \\
a_r &= \binom{n}{r} 3^r
\end{aligned}$$

Answer 4