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Answer 1

a . $A \cap B \subseteq (A \cup \bar{B}) \cap (\bar{A} \cup B)$
Let's assume that $x \in (A \cap B)$. This means $x \in A$ and $x \in B$ by definition of intersetion. Then $x \in (A \cup \bar{B})$ and $x \in (\bar{A} \cup B)$ by definition of union. Using definition of intersection we conclude that $x \in (A \cup \bar{B}) \cap (\bar{A} \cup B)$. Then we conclude that $A \cap B \subseteq (A \cup \bar{B}) \cap (\bar{A} \cup B)$ by definition of subset.

b . $\bar{A} \cap \bar{B} \subseteq (A \cup \bar{B}) \cap (\bar{A} \cup B)$
Let's assume that $x \in (\bar{A} \cap \bar{B})$. This means $x \in \bar{A}$ and $x \in \bar{B}$ by definition of intersetion. Then $x \in (A \cup \bar{B})$ and $x \in (\bar{A} \cup B)$ by definition of union. Using definition of intersection we conclude that $x \in (A \cup \bar{B}) \cap (\bar{A} \cup B)$. Then we conclude that $A \cap B \subseteq (A \cup \bar{B}) \cap (\bar{A} \cup B)$ by definition of subset.

Answer 2

$$f^{-1}((A \cap B) \times C) = f^{-1}(A \times C) \cap f^{-1}(B \times C)$$

$$\begin{aligned}
(A \cap B) \times C &= \{(x, y) : x \in A \cap B, y \in C\} \\
&= \{(x, y) : x \in A, y \in C \ and\ x \in B, y \in C\} \\
&= \{(x, y) : (x, y) \in A \times C \ and\ (x, y) \in B \times C\} \\
&= (A \times C) \cap (B \times C) \\
so\ that\ f^{-1}((A \cap B) \times C) &= f^{-1}((A \times C) \cap (B \times C))
\end{aligned} \tag{1}$$

We will prove $f^{-1}((A \times C) \cap (B \times C)) = f^{-1}(A \times C) \cap f^{-1}(B \times C)$ by showing that each side is a subset of the other side.
Let $x \in f^{-1}((A \times C) \cap (B \times C))$ then $f(x) \in (A \times B) \cap (A \times C)$. So $f(x) \in (A \times B)$ and $f(x) \in (A \times C)$. Then $x \in f^{-1}(A \times B)$ and $x \in f^{-1}(B \times C)$. And finally $x \in f^{-1}(A \times B) \cap f^{-1}(B \times C)$. So we conclude that $f^{-1}((A \times C) \cap (B \times C)) \subseteq f^{-1}(A \times C) \cap f^{-1}(B \times C)$
Now let $x \in f^{-1}(A \times C) \cap f^{-1}(B \times C)$ then $x \in f^{-1}(A \times C)$ and $x \in f^{-1}(B \times C)$. So $f(x) \in (A \times B)$ and $f(x) \in (A \times C)$.Then $f(x) \in (A \times B) \cap (A \times C)$. Finally $x \in f^{-1}((A \times B) \cap (B \times C))$. We conclude that $f^{-1}(A \times C) \cap f^{-1}(B \times C) \subseteq f^{-1}((A \times C) \cap (B \times C))$.This completes the proof of identity.

Answer 3

a . $f(x) = ln(x^2 + 5)$
For this function to be one-to-one $\forall a \forall b(f(a) = f(b) \rightarrow a = b)$ but if we put $x = 1$ then $f(x) = ln6$, also if we put $x = -1$ then $f(x) = ln6$. This example doesn't satisfy the definition so this function is not one-to-one.
This function is onto because for every $y \in R$, there is $x \in R$ such that $f(x) = y$

b . $f(x) = e^{e^{x^7}}$
This function is strictly increasing $\forall x \forall y(x < y \rightarrow f(x) < f(y))$ so that $\forall a \forall b(f(a) = f(b) \rightarrow a = b)$. We conclude that this function is one-to-one by definition of one-to-one.
This function is onto because for every $y \in R$, there is $x \in R$ such that $f(x) = y$

Answer 4

a. If A and B are countable their elements can be arranged in a finite or infinite list;
$A = \{a_0, a_1, a_2, ...\},\ B = \{b_0, b_1, b_2, ...\}$
The elements of $A \times B$ can be arranged in an array:

$$
\begin{array}{cccccccccccc}
(a_0, & b_0)(a_0, & b_1)(a_0, & b_2)(a_0, & b_3)(a_0, & b_4) & . & . \\
(a_1, & b_0)(a_1, & b_1)(a_1, & b_2)(a_1, & b_3)(a_1, & b_4) & . & . \\
(a_2, & b_0)(a_2, & b_1)(a_2, & b_2)(a_2, & b_3)(a_2, & b_4) & . & . \\
(a_3, & b_0)(a_3, & b_1)(a_3, & b_2)(a_3, & b_3)(a_3, & b_4) & . & . \\
(a_4, & b_0)(a_4, & b_1)(a_4, & b_2)(a_4, & b_3)(a_4, & b_4) & . & . \\
 & . & . & . & . & . & . & \\
 & . & . & . & . & . & & .
\end{array}
$$

and subsequently, following the diagonals, listed:
$A \times B = \{(a_0, b_0), (a_0, b_1), (a_1, b_0), (a_0, b_2), (a_1, b_1), (a_2, b_0), (a_0, b_3), (a_1, b_2)....\}$
$A \times B$ elements can be arranged in a finite or infinite list so it is countable.

b. If A is uncountable and $A \subseteq B$, B is also uncountable.

c. If B is countable and $A \subseteq B$, A is also countable.

Answer 5

a. If $f_1(x)$ is $O(f_2(x))$, $|f_1(x)| \leq C_1|f_2(x)|$ *whenever* $x > k_1$. As both of them are increasing functions, $|ln|f_1(x)|| \leq C_2|ln|f_2(x)||$ is true *whenever* $x > k_2$ for every function $f_1(x)$ and $f_2(x)$ so $ln|f_1(x)|$ is $O(ln|f_2(x)|)$

b. If $f_1(x)$ is $O(f_2(x))$, $|f_1(x)| \leq C_1|f_2(x)|$ *whenever* $x > k_1$. As both of them are increasing functions, $|3^{f_1(x)}| \leq C_2|3^{f_2(x)}|$ is true *whenever* $x > k_2$ for every function $f_1(x)$ and $f_2(x)$ so $3^{f_1(x)}$ is $O(3^{f_2(x)})$

Answer 6

a.

b.

$$\begin{aligned} gcd(123, 277) &= gcd(123, 31) \\ &= gcd(31, 30) \\ &= gcd(30, 1) \\ &= 1 \end{aligned} \tag{2}$$