Middle East Technical University Department of Computer Engineering

Formal Languages and Abstract Machines
Take Home Exam 2

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1 Context-Free Grammars (10 pts)

a) Give the rules of the Context-Free Grammars to recognize strings in the given languages where $\Sigma = \{a, b\}$ and S is the start symbol.

$L(G) = \{w \mid\ w \in \Sigma^*;\ |w| \geq 3;$ (2/10 pts)
the first and the second from the last symbols of w are the same$\}$

$R = \{S \to aS \mid bS \mid aa \mid bb\}$

$L(G) = \{w \mid\ w \in \Sigma^*;$ the length of w is odd$\}$ (2/10 pts)

$R = \{S \to aA \mid bA,\ A \to aS \mid bS \mid e\}$

$L(G) = \{w \mid\ w \in \Sigma^*;\ n(w, a) = 2 \cdot n(w, b)\}$ where $n(w, x)$ is the number of x symbols in w (3/10 pts)

$R = \{S \to SaSaSbS \mid SaSbSbSaS \mid SbSaSaS \mid e\}$

b) Find the set of strings recognized by the CFG rules given below: (3/10 pts)

$S \to X \mid Y$
$X \to aXb \mid A \mid B$
$A \to aA \mid a$
$B \to Bb \mid b$

$Y \to CbaC$
$C \to CC \mid a \mid b \mid \varepsilon$

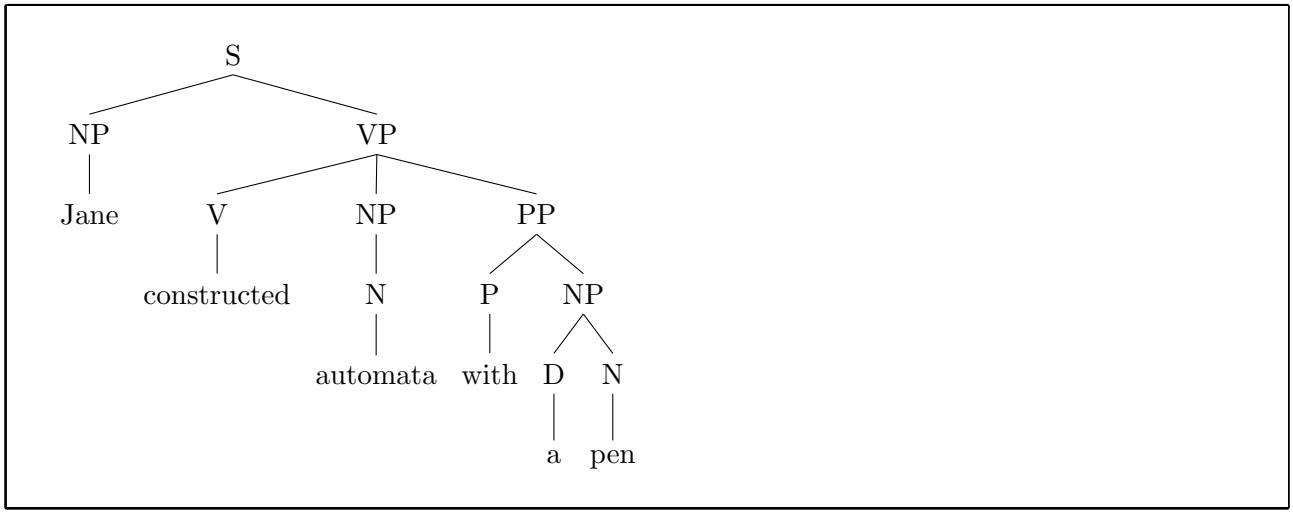
$L(G) = \{w \mid \ w \in \Sigma^*;\ w \ is \ not \ ab.\}$

2 Parse Trees and Derivations (20 pts)

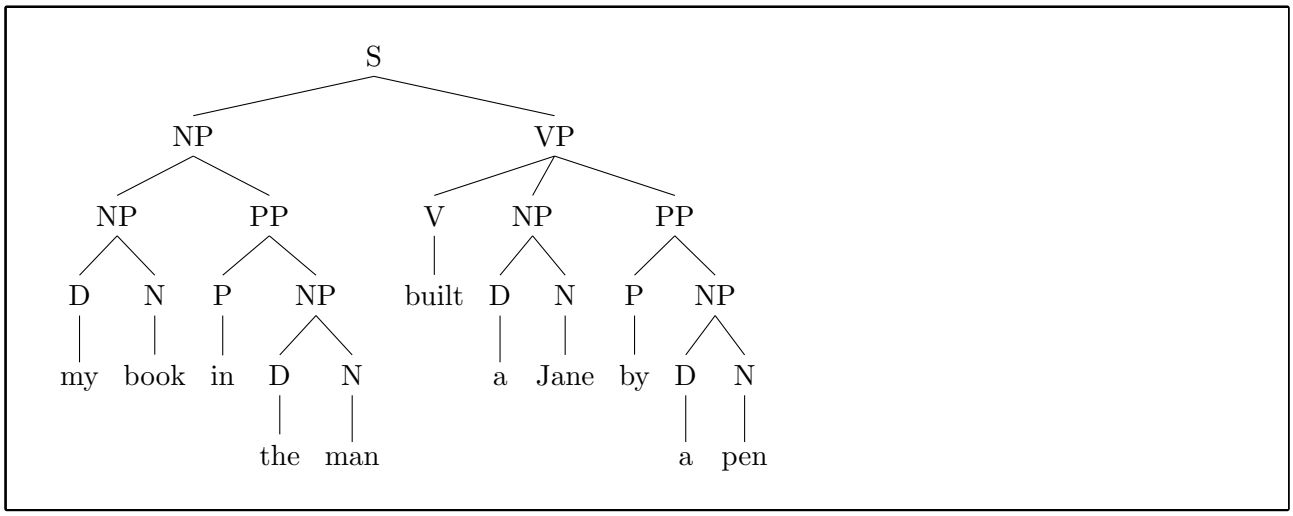
Given the CFG below, provide parse trees for given sentences in **a** and **b**.

```
S   → NP VP
VP  → V NP | V NP PP
PP  → P NP
NP  → N | D N | NP PP
V   → wrote | built | constructed
D   → a | an | the | my
N   → John | Mary | Jane | man | book | automata | pen | class
P   → in | on | by | with
```

a) Jane constructed automata with a pen (4/20 pts)



b) my book in the man built a Jane by a pen (4/20 pts)

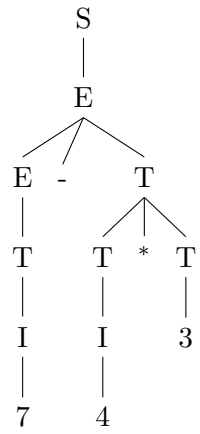


Given the CFG below, answer **c**, **d** and **e**

```
S  → E
E  → E + T | E - T | T
T  → T * I | T / I | I
I  → 0 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9
```

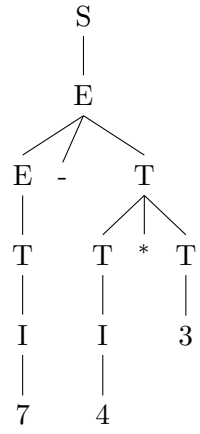
c) Provide the left-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \Rightarrow E \Rightarrow E-T \Rightarrow T-T \Rightarrow I-T \Rightarrow 7-T \Rightarrow 7-T^*I \Rightarrow 7-I*I \Rightarrow 7-4^*I \Rightarrow 7-4^*3$



d) Provide the right-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \Rightarrow E \Rightarrow E-T \Rightarrow E-T^*I \Rightarrow E-T^*3 \Rightarrow E-I^*3 \Rightarrow E-4^*3 \Rightarrow T-4^*3 \Rightarrow I-4^*3 \Rightarrow 7-4^*3$

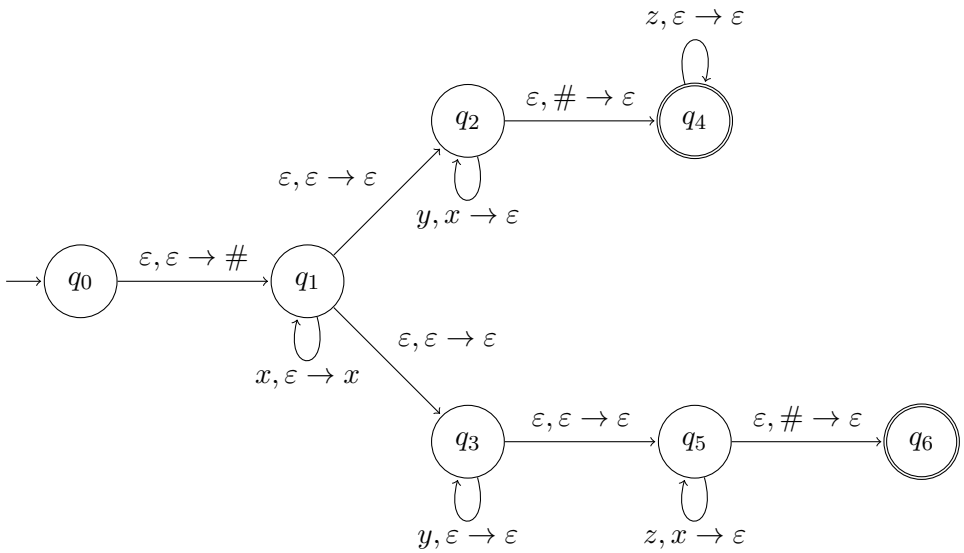


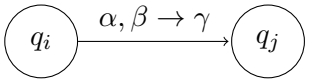
e) Are the derivations in **c** and **d** in the same similarity class? (4/20 pts)

Yes, they are in the same similarity class because they represent applications of the same rules at the same positions in the strings only differing in the relative order of these applications.

3 Pushdown Automata (30 pts)

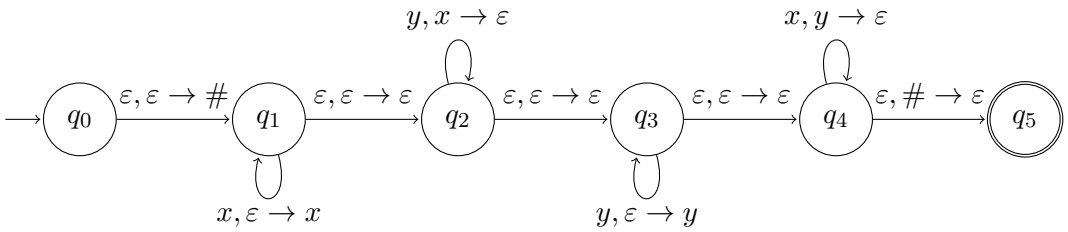
a) Find the language recognized by the PDA given below (5/30 pts)



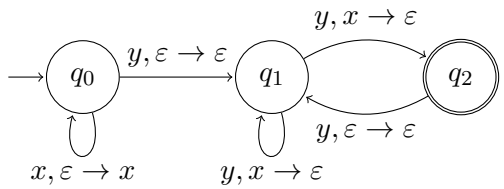
where the transition $((q_i, \alpha, \beta), (q_j, \gamma))$ is represented as: 

The language $\{x^n y^n z^m \text{ or } x^n y^m z^n \ : m, n \geq 0\}$

b) Design a PDA to recognize language $L = \{x^n y^{m+n} x^m \mid \ n, m \geq 0;\ n, m \in \mathbb{N}\}$ (5/30 pts)



c) Design a PDA to recognize language $L = \{x^n y^m \mid n < m \leq 2n;\ n, m \in \mathbb{N}^+\}$ (10/30 pts)
Do not use multi-symbol push/pop operations in your transitions.
Simulate the PDA on strings *xxy* (with only one rejecting derivation) and *xxyyyy* (accepting derivation) with transition tables.



$M = (K, \sum, \Gamma, \Delta, s, F)$, where $K = \{q_0, q_1, q_2\}$, $\sum = \{x, y\}$, $F = \{q_2\}$ and Δ is the following set of transitions;
(1) $((q_0, x, e), (q_0, x))$
(2) $((q_0, x, e), (q_1, e))$
(3) $((q_1, y, x), (q_1, e))$
(4) $((q_1, y, x), (q_2, e))$
(5) $((q_2, y, e), (q_1, e))$

Table 1: transition table for xxy

State	**Unread Input**	**Stack**	**Transition Used**
q_0	xxy	ε	-
q_0	xy	x	1
q_0	y	xx	1
q_1	ε	xx	2

Table 2: transition table for xxyyyy

State	**Unread Input**	**Stack**	**Transition Used**
q_0	xxyyyy	ε	-
q_0	xyyyy	x	1
q_0	yyyy	xx	1
q_1	yyy	xx	2
q_2	yy	x	4
q_1	yy	x	5
q_2	ε	ε	4

d) Given two languages L' and L as $L' = \{w \mid w \in L;\ |w| = 4n + 2\ for\ n \in \mathbb{N}\}$ (10/30 pts)
If L is a CFL, show that L' is also a CFL by constructing an automaton for L' in terms of another automaton that recognizes L.

answer here ...

4 Closure Properties (20 pts)

Let L_1 and L_2 be context-free languages which are not regular, and let L_3 be a regular language. Determine whether the following languages are necessarily CFLs or not. If they need to be context-free, explain your reasoning. If not, give one example where the language is a CFL and a counter example where the language is not a CFL.

a) $L_4 = L_1 \cap (L_2 \setminus L_3)$ (10/20 pts)

b) $L_5 = (L_1 \cap L_3)^*$ (10/20 pts)

5 Pumping Theorem (20 pts)

a) Show that $L = \{a^n m^n t^i \mid n \leq i \leq 2n\}$ is not a Context Free Language using Pumping Theorem for CFLs. (10/20 pts)

answer here ...

b) Show that $L = \{a^n b^{2n} a^n \mid n \in \mathbb{N}+\}$ is not a Context Free Language using Pumping Theorem for CFLs. (10/20 pts)

answer here ...

6 CNF and CYK (not graded)

a) Convert the given context-free grammar to Chomsky Normal Form.

$S \to XSX \mid xY$
$X \to Y \mid S$
$Y \to z \mid \varepsilon$

answer here ...

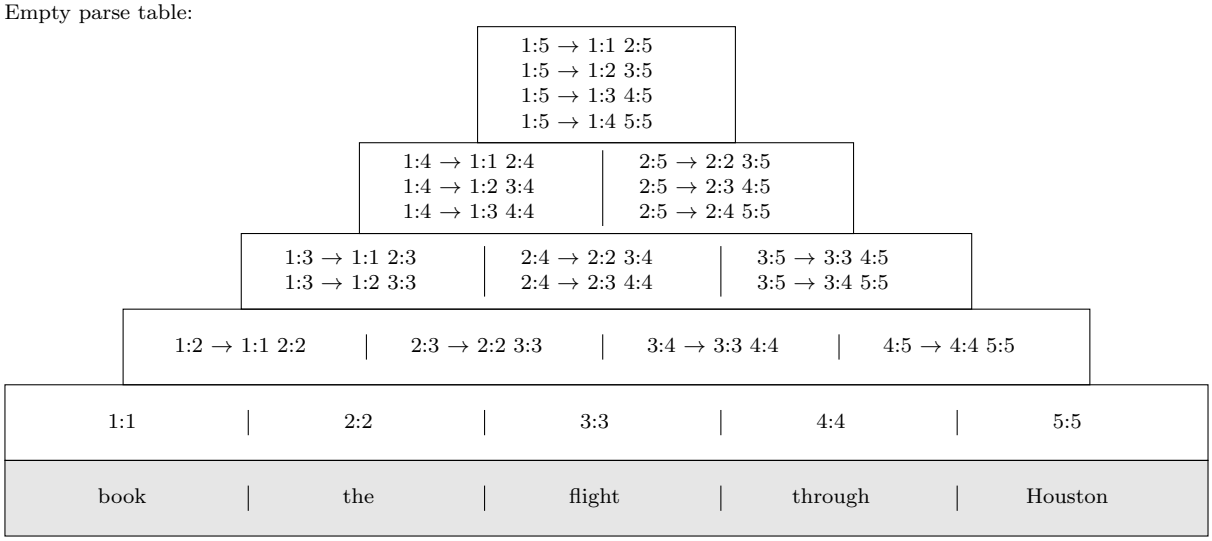
b) Use the grammar below to parse the given sentence using Cocke–Younger–Kasami algorithm. Plot the parse trees.

S → NP VP
S → X1 VP
X1 → Aux NP
S → book | include | prefer
S → Verb NP
S → X2 PP
S → Verb PP
S → VP PP
NP → I | she | me | Houston
NP → Det Nom
Nom → book | flight | meal | money
Nom → Nom Noun
Nom → Nom PP

VP → book | include | prefer
VP → Verb NP
VP → X2 PP
X2 → Verb NP
VP → Verb PP
VP → VP PP
PP → Prep NP
Det → that | this | the | a
Noun → book | flight | meal | money
Verb → book | include | prefer
Aux → does
Prep → from | to | on | near | through

book the flight through Houston

Empty parse table:



rest of the answer here ...

7 Deterministic Pushdown Automata (not graded)

Provide a DPDA to recognize the given languages, the DPDA must read its entire input and finish with an empty stack.

a) $a^*bc \cup a^n b^n c$

answer here ...

b) $(aa)^*c \cup a^n b^n c$

answer here ...