CENG 384 - Signals and Systems for Computer Engineers

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Written Assignment 2

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1. (a)

$$\begin{aligned} \frac{dy(t)}{dt} &= -4y(t) + x(t) \\ y(t) &= \frac{-\frac{dy(t)}{dt} + x(t)}{4} \end{aligned} \tag{1}$$

(b) $x(t) = (e^{-t} + e^{-2t})u(t)$

$$\begin{aligned} y(t) &= y_H(t) + y_p(t) \\ y_H(t) = Ke^{\alpha t} &\quad y_H^{'}(t) = K\alpha e^{\alpha t} \\ Ke^{\alpha t}(\alpha + 4) = 0 &\quad \alpha = -4 \\ y_H(t) &= Ke^{-4t} \quad \textit{zero response} \\ x(t) = (e^{-t} + e^{-2t})u(t) \; &then \; \lambda_1 = -1, \; \lambda_2 = -2 \\ H_1(\lambda) &= \frac{1}{4-1} = \frac{1}{3} \\ H_2(\lambda) &= \frac{1}{4-2} = \frac{1}{2} \\ y_P(t) &= \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t} \\ y(0) = 0, \; then \; K &+ \frac{1}{2} + \frac{1}{2} = 0, \; K = \frac{-5}{6} \\ y(t) &= \frac{-5}{6}e^{4t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t} \end{aligned} \tag{2}$$

2. (a)

$$\begin{aligned} y[n) &= x[n] * h[n] \\ &= \delta[n] + 2\delta[n-1] - 3\delta[n-2] - 3\delta[n-1] - 6\delta[n-2] + 9\delta[n-3] + \delta[n-2] + 2\delta[n-3] - 3\delta[n-4] \\ &= \delta[n] - \delta[n-1] - 8\delta[n-2] + 11\delta[n-3] - 3\delta[n-4] \end{aligned} \tag{3}$$

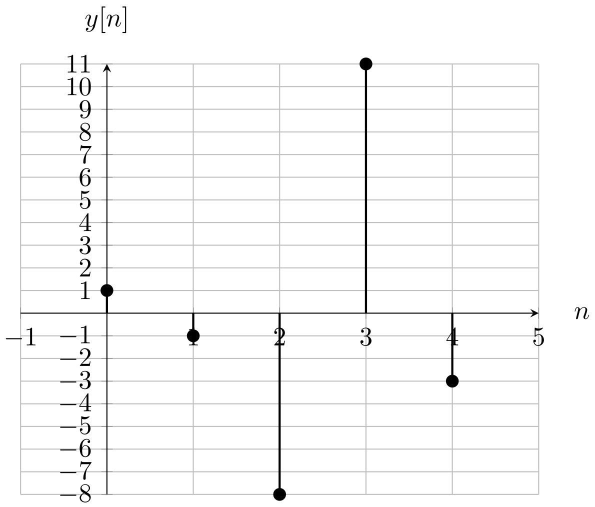

Figure 1: $y[n] = x[n] * h[n]$

(b)

$$
\begin{aligned}
y(t) &= \int \frac{dx(\tau)}{d\tau} h(t-\tau) d\tau \\
\frac{dx(t)}{dt} &= \frac{d}{dt}\{u(t) + u(t+1)\} \\
&= \delta(t) + \delta(t-1) \\
y(t) &= [\delta(t) + \delta t - 1] * h(t) \\
y(t) &= h(t) + h(t-1) = e^{2t} cos(t) u(t) + e^{-2(t-1)}.cos(t-1).u(t-1) \\
y(t) &= \begin{cases} 0, & \text{if } t < 0. \\ e^{-2t} cost, & \text{if } 0 \leq t < 1. \\ e^{-2t}(cost + e^{2} cos(t-1)), & \text{if } 1 \leq t < \text{inf.} \end{cases}
\end{aligned} \tag{4}
$$

3. (a)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_0^t e^{-\tau} e^{-3(t-\tau)} d\tau \\
&= \int_0^t e^{-3\tau} e^{2\tau} d\tau \\
&= e^{-3t}(e^{2t} - 1) \\
&= (e^{-t} - e^{-3t}) u(t) \quad for \quad t \geq 0
\end{aligned} \tag{5}
$$

(b)

$$
\begin{aligned}
y_1(t) &= \int_1^t x(\tau)h(t-\tau)d\tau \\
&= |_1^t - e^{t-2} = e^{t-1} - 1 \\
y_2(t) &= \int_1^2 e^{t-\tau} d\tau \\
&= |_1^2 - e^(t-\tau) = e^{t-1} - e^{t-2} \\
y(t) &= \begin{cases} 0, & \text{if } t < 1. \\ e^{t-1} - 1, & \text{if } 1 \leq t < 2. \\ e^{t-1} - e^{t-2}, & \text{if } 2 \leq t < \text{inf}. \end{cases}
\end{aligned} \tag{6}
$$

4. (a)

$$
\begin{aligned}
\lambda^2 - 15\lambda + 26 = 0 \\
\lambda_1 = 13, \lambda_2 = 2 \\
y[n] = c_1 13^n + c_2 2^n \\
y[0] = c_1 + c_2 = 10 \\
y[1] = 13c_1 + 2c_2 = 42 \\
c_1 = 2, c_2 = 8 \\
y[n] = 2.13^n + 8.2^n
\end{aligned} \tag{7}
$$

(b)

$$
\begin{aligned}
&\lambda^2 - 3\lambda + 1 = 0 \\
&\lambda_1 = \frac{3+\sqrt{5}}{2}, \quad \lambda_2 = \frac{3-\sqrt{5}}{2} \\
&y[n] = c_1 \frac{3+\sqrt{5}}{2}^n + c_2 \frac{3-\sqrt{5}}{2}^n \\
&y[0] = c_1 + c_2 = 1 \\
&y[1] = \frac{3+\sqrt{5}}{2} c_1 + \frac{3-\sqrt{5}}{2} c_2 = 2 \\
&c_1 = \frac{1}{2} - \frac{\sqrt{5}}{5}, \quad c_2 = \frac{1}{2} + \frac{\sqrt{5}}{5} \\
&y[n] = (\frac{1}{2} - \frac{\sqrt{5}}{5})(\frac{3-\sqrt{5}}{2})^n + (\frac{1}{2} + \frac{\sqrt{5}}{5})(\frac{3+\sqrt{5}}{2})^n
\end{aligned} \tag{8}
$$

5. (a)

$$\begin{aligned}
h_H(t) &= Ke^{\alpha t} \\
h_H^{'}(t) &= K\alpha e^{\alpha t} \\
h_H^{''}(t) &= K\alpha^2 e^{\alpha t} \\
Ke^{\alpha t}(\ alpha^2 + 6\alpha + 8) &= 0 \quad \alpha_1 = -4 \quad \alpha_2 = -2 \\
h_H(t) &= K_1 e^{-2t} + K_2 e^{-4t} \\
\int_{0^-}^{0^+} h^{''}(t)dt + \int_{0^-}^{0^+} 6h'(t)dt + \int_{0^-}^{0^+} 8h(t)dt &= \int_{0^-}^{0^+} 2\delta(t)dt \\
[h^{''}(0^+) - h^{''}(0^-)] + 6[h^{'}(0^+) - h^{'}(0^{'})] + 0 &= 2 \\
h'(0^+) + 6h(0^+) &= 2 \\
h_H(0^+) &= K_1 + K_2 \\
h_H^{'}(0^+) &= K_1 - 2 - 4K_2 \\
4K_1 + 2K_2 &= 2 \\
K_1 = \frac{5}{7},\ K_2 &= \frac{-3}{7} \\
h_H(t) &= (\frac{5}{7}e^{-2t} + \frac{-3}{7}e^{-4t})u(t)
\end{aligned} \tag{9}$$

(b) **causality:** The system is causal

memory: The system is not memoryless

stability: The system is unstable

invertibility: The system is non-invertible