CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a) **(i)**

$$\begin{aligned}
\bar{z} &= x - yj \\
3z + 4 &= 2j - \bar{z} \\
3x + 3yj + 4 &= 2j - x + yj \\
x = -1&, \quad y = 1 \\
z = -1 + j&, \quad \bar{z} = -1 - j \\
|z|^2 &= z.\bar{z} = (-1+j).(-1-j) = 2
\end{aligned} \tag{1}$$

(ii)

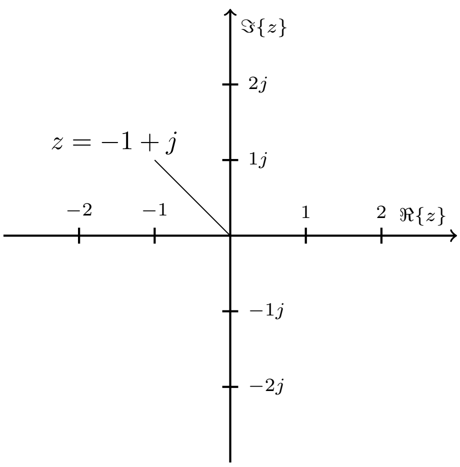

Figure 1:

(b)

$$
\begin{aligned}
z &= re^{j\theta}, \quad z^3 = 64j \\
z^3 &= 4^3.e^{j3\theta} \\
e^{j3\theta} &= j = cos3\theta + jsin3\theta \\
\theta &= \frac{\pi}{6} \\
z &= 4(cos\frac{\pi}{2} + jsin\frac{\pi}{2})
\end{aligned} \tag{2}
$$

(c)

$$
\begin{aligned}
z &= \frac{(1-j)(1+\sqrt{3}j)}{1+j} \\
&= \frac{(1-j)^2(1+\sqrt{3}j)}{2} \\
&= -j - \sqrt{3}j \\
\theta &= tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}, \quad |z| = 2
\end{aligned} \tag{3}
$$

(d)

$$
\begin{aligned}
z &= -je^{j\frac{\pi}{2}} \\
e^{j\frac{\pi}{2}} &= cos\frac{\pi}{2} + jsin\frac{\pi}{2} = j \\
z &= (-j).j = 1 \\
z &= cos2\pi + jsin2\pi
\end{aligned} \tag{4}
$$

2. Figure 2 is the graph of the given signal y(t)=x($\frac{1}{2}$t+1).

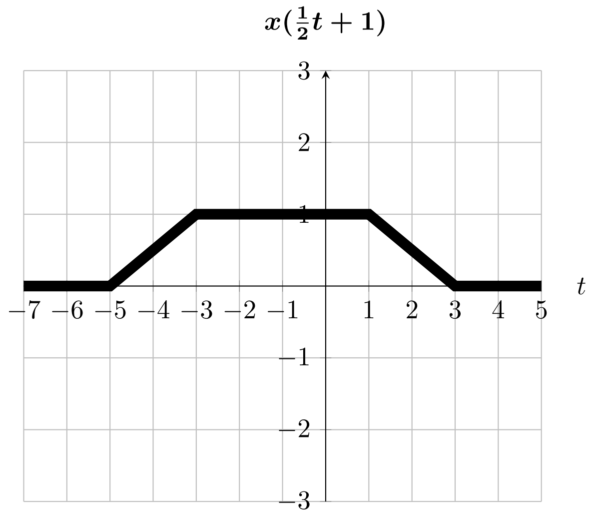

Figure 2: t vs. $x(\frac{1}{2}t+1)$.

3. (a) Figure 3 is the graph of the signal x[-n]+x[2n+1].

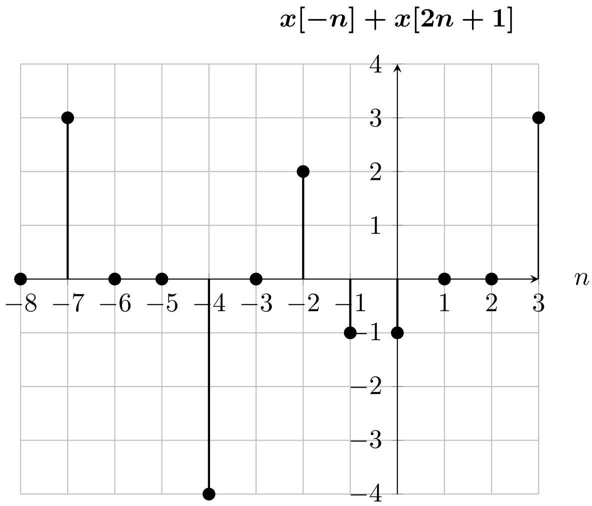

Figure 3: n vs. $x[-n] + x[2n+1]$.

(b) $x[-n] + x[2n+1] = 3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n) + 3\delta(n-3)$

4. (a) We have the signal $x[n] = 3cos[\frac{13\pi}{10}n] + 5sin[\frac{7\pi}{3}n - \frac{2\pi}{3}]$ and we know that the period of cosine and sine functions is 2π. Therefore the period of the cosine part of this signal is $N_1 = \frac{20}{13}m_1$ and the period of the sine part is $N_2 = \frac{6}{7}m_2$. If we have $m_1 = 39$ and $m_2 = 70$ then the period of the whole signal is $N = 60$. Therefore the signal is periodic and the period of signal equals to $N = 60$.

(b) The given signal is $x[n] = 5sin[3n - \frac{\pi}{4}]$ and the period of this signal is $N = \frac{2\pi}{3}m$ where m is an integer. However period N must be an integer too because the signal is in discrete time. Therefore the signal is not periodic.

(c) The signal is $x(t) = 2cos(3\pi t - \frac{2\pi}{5})$. The period of the cosine function is 2π so the period of the given signal is $T = \frac{2\pi}{3\pi} = \frac{2}{3}$.

(d) The given signal is $x(t) = -je^{j5t}$ which equals to $-j(cos(5t) + jsin(5t)) = -jcos(5t) + sin(5t)$. The period of this signal is $T = \frac{2\pi}{5}$.

5. Figure 4 is the graph of the signal Even{x[n]}=$\frac{1}{2}\{x[n] + x[-n]\}$.

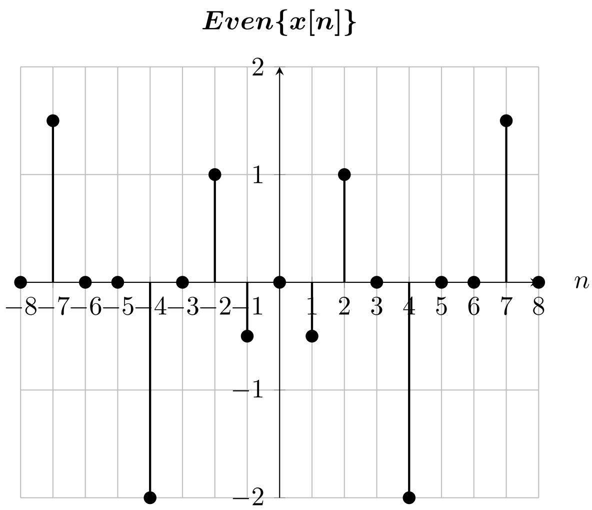

Figure 4: n vs. $Even\{x[n]\}$.

Figure 5 is the graph of the signal Odd{x[n]}=$\frac{1}{2}\{x[n]-x[-n]\}$.

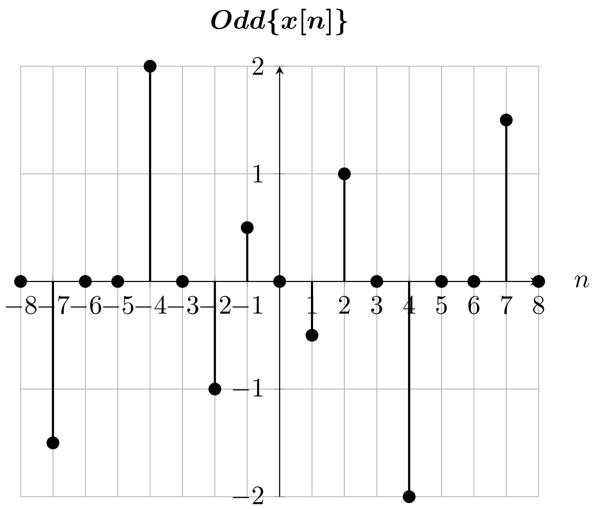

Figure 5: n vs. $Odd\{x[n]\}$.

6. (a) $y(t)=x(2t-3)$

memory: The system is NOT memoryless because for example when $t=2$, the output depends on past value of $x(t)$.

stability: The system is stable. If $|x(t)|\leq K$, then $|x(2t-3)|\leq K$ and $|y(t)|\leq K$ which shows that the system is bounded therefore stable.

causality: The system is NOT causal because for example when $t=4$, the output depends on future value of $x(t)$.

linearity: The system is linear because if

$$\begin{aligned} y_1(t) &= x_1(2t-3), \\ y_2(t) &= x_2(2t-3) \end{aligned} \tag{5}$$

and $x(t)=\alpha_1x_1(t)+\alpha_2x_2(t)$, then the output $y(t)$ corresponding to the input $x(t)$ is

$$\begin{aligned} y(t) &= (\alpha_1x_1+\alpha_2x_2)(2t-3) \\ &= \alpha_1x_1(2t-3)+\alpha_2x_2(2t-3) \\ &= \alpha_1y_1(t)+\alpha_2y_2(t) \end{aligned} \tag{6}$$

invertibility: The system is invertible and the inverse system is $w(t)=x(\frac{t+3}{2})$

time-invariance: The system is time varying.

$$\begin{aligned} x(t-t_0)\rightarrow system \rightarrow x(2t-t_0-3) &\neq y(t-t_0) \\ y(t-t_0) &= x(2t-2t_0-3) \end{aligned} \tag{7}$$

(b) $y(t)=tx(t)$

memory: The system is memoryless because y(t) depends only on the present value of x(t).

stability: The system is NOT stable. Let's select $x(t)$ as $u(t)$ which is a bounded since it's output is either 1 or 0.

$$\begin{aligned} x(t) \to & System \to tx(t) \\ u(t) \to & System \to tu(t) \end{aligned} \tag{8}$$

where $tu(t)$ is the unit ramp signal which goes to infinity so it is not bounded therefore not stable.

causality: The system is memoryless hence causal.

linearity: The system is linear because if

$$\begin{aligned} y_1(t) &= tx_1(t), \\ y_2(t) &= tx_2(t) \end{aligned} \tag{9}$$

and $x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$, then the output $y(t)$ corresponding to the input $x(t)$ is

$$\begin{aligned} y(t) &= t(\alpha_1 x_1 + \alpha_2 x_2)(t) \\ &= t\alpha_1 x_1(t) + t\alpha_2 x_2(t) \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned} \tag{10}$$

invertibility: The system is invertible

time-invariance: The system is time varying.

$$\begin{aligned} x(t - t_0) \to system \to tx(t - t_0) &\neq y(t - t_0) \\ y(t - t_0) &= (t - t_o)x(t - t_0) \end{aligned} \tag{11}$$

(c) $y[n] = x[2n - 3]$

memory: The system is NOT memoryless because for example when $n = 2$, the output depends on the input at the past time.

stability: The system is stable. If $|x[n]| \leq K$, then $|x[2n - 3]| \leq K$ and $|y[n]| \leq K$ which shows that the system is bounded therefore stable.

causality: The system is NOT causal because for example when $n = 4$, the output depends on future value of $x[n]$.

linearity: The system is linear because if

$$\begin{aligned} y_1[n] &= x_1[2n - 3], \\ y_2[n] &= x_2[2n - 3] \end{aligned} \tag{12}$$

and $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$, then the output $y[n]$ corresponding to the input $x[n]$ is

$$\begin{aligned} y(n) &= (\alpha_1 x_1 + \alpha_2 x_2)[2n - 3] \\ &= \alpha_1 x_1[2n - 3] + \alpha_2 x_2[2n - 3] \\ &= \alpha_1 y_1[n] + \alpha_2 y_2[n] \end{aligned} \tag{13}$$

invertibility: The system is invertible and the inverse system is $w[u] = x[\frac{u+3}{2}]$

time-invariance: The system is time varying.

$$\begin{aligned} x[n-n_0] \to system \to x[2n-n_0-3] \neq y[n-n_0] \\ y[n-n_0] = x[2n-2n_0-3] \end{aligned} \tag{14}$$

(d) $y[n] = \sum_{k=1}^{\infty} x[n-k]$

memory: The system is NOT memoryless because the output depends on input values at past times.

stability: The system is unstable. For example, if the input to the accumulator is $u[-n]$, the system becomes

$$y[n] = \sum_{k=1}^{\infty} u[-n+k] \tag{15}$$

Then $y[1] \to \infty$, $y[1] \to \infty$... which means there is no bound for the system therefore unstable.

causality: The system is causal since each output of the system depends only past and present values of $x[n]$.

linearity: The system is linear because if

$$\begin{aligned} y_1[n] &= \sum_{k=1}^{\infty} x_1[n-k], \\ y_2[n] &= \sum_{k=1}^{\infty} x_2[n-k] \end{aligned} \tag{16}$$

and $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$, then the output $y[n]$ corresponding to the input $x[n]$ is

$$\begin{aligned} y[n] &= \sum_{k=1}^{\infty}(\alpha_1 x_1[n-k] + \alpha_2 x_2[n-k]) \\ &= \sum_{k=1}^{\infty} \alpha_1 x_1[n-k] + \sum_{k=1}^{\infty} \alpha_2 x_2[n-k] \\ &= \alpha_1 y_1[n] + \alpha_2 y_2[n] \end{aligned} \tag{17}$$

invertibility: The system is NOT invertible

time-invariance: The system is time invariant.

$$x[n-n_0] \to system \to \sum_{k=1}^{\infty} x[n-n_0-k] = y[n-n_0] \tag{18}$$