Student Information

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Answer 1

Table 1: truth table for $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

p	**q**	$\neg$**q**	**p** $\rightarrow$ **q**	$\neg$**q** $\wedge$ **(p** $\rightarrow$ **q)**	$\neg$**p**	**(**$\neg$**q** $\wedge$ **(p** $\rightarrow$ **q))** $\rightarrow$ $\neg$**p**
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Table 2: truth table for $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

p	**q**	**r**	$\neg$**p**	**p**$\vee$**q**	$\neg$**p**$\vee$**r**	**q**$\vee$**r**	**(p**$\vee$**q)**$\wedge$**(**$\neg$**p**$\vee$**r)**	**((p**$\vee$**q)**$\wedge$**(**$\neg$**p**$\vee$**r))**$\rightarrow$**(q**$\vee$**r)**
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T

Answer 2

$$
\begin{aligned}
(p \rightarrow q) \vee (p \rightarrow r) &\equiv p \rightarrow (q \vee r) && \textit{Table 7} && (1)\\
&\equiv \neg(q \vee r) \rightarrow \neg p && \textit{Table 7} && (2)\\
&\equiv (\neg q \wedge \neg r) \rightarrow \neg p && De\ Morgan's\ 2.\ Law && (3)
\end{aligned}
$$

Answer 3

0.1

a) All cats are friends with some dogs.
b) Some cats are friends with all dogs.

0.2

a)$\forall x \forall y((Eats(x, y) \land Meal(y)) \rightarrow Customer(x))$
b)$\exists x \forall y(Chef(x) \land Meal(y) \land \neg Cooks(x, y))$
c)$\exists x \forall y \exists z((Customer(x) \land Eats(x, y) \land Meal(y)) \rightarrow (Cooks(z, y) \land Chef(z)))$
d)$\forall x \exists y \exists z(Chef(x) \land \neg Cooks(x, z) \land Meal(z) \land Knows(x, y) \land Chef(y) \land Cook(y, z))$

Answer 4

For this to be a deduction rule in a sound deductive system, if p $\rightarrow$ q and $\neg$p are true, $\neg$q must also be true.
However if we pick p false and q true, p $\rightarrow$ q and $\neg$p would be true but $\neg$q would be false so this example wouldn't satisfy the rule.

Answer 5

Table 3: $p \rightarrow q, q \rightarrow r, r \rightarrow p \vdash (p \leftrightarrow q) \land (p \leftrightarrow r)$

1	$p \rightarrow q$	*premise*
2	$q \rightarrow r$	*premise*
3	$r \rightarrow p$	*premise*
4	q	*assumption*
5	r	$\rightarrow$e $2, 4$
6	p	$\rightarrow$e $3, 5$
7	$q \rightarrow p$	$\rightarrow i\ 4 - 6$
8	p	*assumption*
9	q	$\rightarrow$e $1, 8$
10	r	$\rightarrow$e $2, 9$
11	$q \rightarrow r$	$\rightarrow i\ 8 - 10$
12	$p \leftrightarrow q$	$\leftrightarrow i\ 1, 7$
13	$p \leftrightarrow r$	$\leftrightarrow i\ 3, 11$
14	$(p \leftrightarrow q) \land (p \leftrightarrow r)$	$\land i\ 12, 13$

Answer 6

Table 4: $\forall x(Q(x) \to R(x)), \exists x(P(x) \to Q(x)), \forall x P(x) \vdash \exists x(P(x) \land R(x))$

1	$\forall x(Q(x) \to R(x))$	*premise*
2	$\exists x(P(x) \to Q(x))$	*premise*
3	$\forall x P(x)$	*premise*
4	$P(x_0) \to Q(x_0)$	*assumption*
5	$P(x_0)$	$\forall x$e 3
6	$Q(x_0)$	$\to$e $4, 5$
7	$Q(x_0) \to R(x_0)$	$\forall x$e 3
8	$R(x_0)$	$\to$e $6, 7$
9	$P(x_0) \land R(x_0)$	$\land$e $5, 8$
10	$\exists x(P(x_0) \land R(x_0))$	$\exists x$ i 9
11	$\exists x(P(x_0) \land R(x_0))$	$\exists x$e 2, $4 - 10$