CENG 384 - Signals and Systems for Computer Engineers
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Written Assignment 4

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1. (a)

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n] \tag{1}$$

(b) We take Fourier Transform of the equation above,

$$Y(e^{jw}) - \frac{3}{4}e^{-jw}Y(e^{jw}) + \frac{1}{8}e^{-2jw}Y(e^{jw}) = 2X(e^{jw}) \tag{2}$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{-2}{1-\frac{1}{4}e^{-jw}} + \frac{4}{1-\frac{1}{2}e^{-jw}} \tag{3}$$

(c) The Fourier Transform of $a^n u(n)$ is $\frac{1}{1-ae^{-jw}}$. By using this equation we can find the impulse response of the system.

$$h[n] = (-2(\frac{1}{4})^n + 4(\frac{1}{2})^n)u[n] \tag{4}$$

(d) The Fourier Transform of $x[n] = (\frac{1}{4})^n u[n]$ is

$$X(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}} \tag{5}$$

By using $Y(e^{jw}) = X(e^{jw})H(e^{jw})$ we find the Fourier Transform of y[n].

$$Y(e^{jw}) = \frac{-2}{(1-\frac{1}{4}e^{-jw})^2} + \frac{-4}{1-\frac{1}{4}e^{-jw}} + \frac{8}{1-\frac{1}{2}e^{-jw}} \tag{6}$$

By using Fourier Transform pairs we obtain

$$y[n] = (\frac{1}{4})^n u[n](-2n-6) + 8(\frac{1}{2})^n u[n] \tag{7}$$

2. The overall frequency response is $H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$.

$$h_1(n) = (\frac{1}{3})^n u(n) \Longleftrightarrow H_1(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}} \tag{8}$$

$$H_1(e^{jw}) + H_2(e^{jw}) = \frac{5e^{-jw} - 12}{e^{-2jw} - 7e^{-jw} + 12} = \frac{-2}{1-\frac{1}{4}e^{-jw}} + \frac{1}{1-\frac{1}{3}e^{-jw}} \tag{9}$$

$$H_2(e^{jw}) = \frac{-2}{1-\frac{1}{4}e^{-jw}} \tag{10}$$

$$h_2(n) = -2(\frac{1}{4})^n u(n) \tag{11}$$

3. (a)

$$\begin{aligned} x_1(t) &= \frac{sin2\pi t}{\pi t} \\ x_2(t) &= cos3\pi t \\ X_1(jw) &= 1 \quad when \quad |w| < 2\pi \\ &= 0 \quad when \quad |w| > 2\pi \\ X_2(jw) &= \pi[\delta(w-3\pi) + \delta(w+3\pi)] \end{aligned}$$

$$X(jw) = \begin{cases} 1, & -2\pi < w < 2\pi \\ \pi & w = 3\pi \\ \pi & w = -3\pi \\ 0, & \text{otherwise} \end{cases}$$

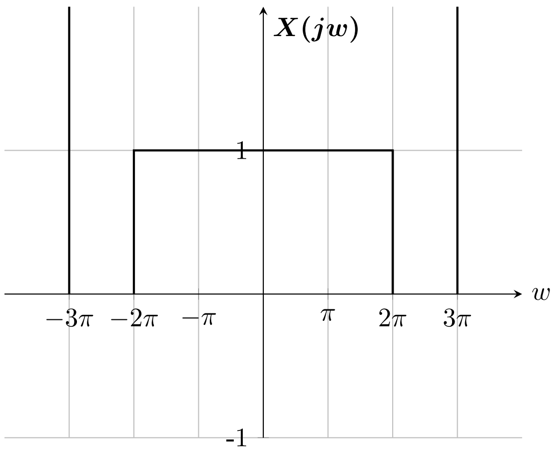

Figure 1: $X(jw)$ vs. w.

(b)

$$2w_m = 6\pi \quad NyquistRate$$
$$w_s > 2w_m$$
$$T_s < \frac{1}{3}$$

(c)

$$X_p(jw) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(j(w-6k\pi))$$
$$X_p(jw) = 3\sum_{n=-\infty}^{+\infty} X(j(w-6k\pi))$$

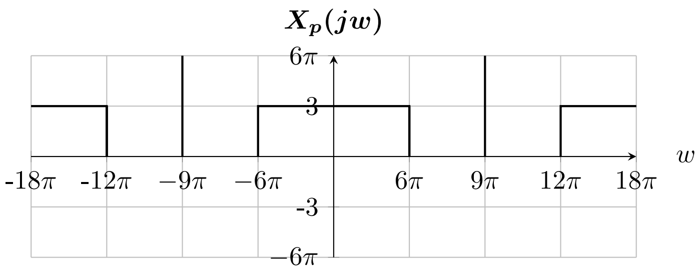

Figure 2: $X_p(jw)$ vs. w.

4. (a)

$$X_d(e^{jw}) = X_p(j\frac{w}{T})$$
$$X_p(jw) = \frac{1}{T}\sum_{k=-\infty}^{\infty} x_c(j(w-kw_s)) \quad where \quad T = 2$$

$$X_p(jw) = \begin{cases} \frac{1}{2}\sum_{k=-\infty}^{\infty} \frac{4}{\pi}(w-k\pi) & |w| \le \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases}$$

$$X_d(e^{jw}) = X_p(j\frac{w}{T}) = \begin{cases} \frac{1}{2}\sum_{k=-\infty}^{\infty} \frac{4}{\pi}(\frac{w}{2}-k\pi) & |w| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

(b) The fourier transform $cos n\pi$

$$H(e^{jw}) = \pi(\delta(w-\pi) + \delta(w+\pi))$$

(c)

$$Y_d(e^{jw}) = \frac{1}{\pi} X_d(e^{jw}) * H(e^{jw})$$

$$Y_d(e^{jw}) = \begin{cases} \frac{1}{4\pi} \sum_{k=-\infty}^{\infty} 4(w - 2k\pi) & |w| \leq \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$