Student Information

Full Name : Zeynep Erdoğan
Id Number : 2171577

Answer 1

Size of a Monte Carlo study is calculated by

$$N \geq 0.25(\frac{z_{\alpha/2}}{\varepsilon})^2 \tag{1}$$

Since it is given that α = 1-0.95 = 0.05, $\varepsilon = 0.005$ and from table A4, $z_{\alpha/2} = z_{0.025} = -1.96$

$$N \geq 0.25(\frac{-1.96}{0.005})^2 = 38416 \tag{2}$$

Since we are asked about the number of creatures caught in 7 hours, and the given Poisson random variable is $\lambda = 5$ for an hour. We have a new poisson distrubiton X, with $\lambda = 7 * 5 = 35$

We can find the probablity $P\{W \geq 2S\}$, $w > 0$, $s > 0$ by

$$\begin{aligned} P\{W \geq 2S\} &= \int_0^\infty \int_{2s}^\infty wse^{-w-s}dw \\ &= \int_0^\infty se^{-s}[-we^{-w} - e^{-w}|_{2s}^\infty]ds \\ &= \int_0^\infty se^{-3s}(2s+1)ds \\ &= -\frac{(18s^2+21s+7)e^{-3s}}{27}|_0^\infty \\ &= 0.2593 \end{aligned} \tag{3}$$

Creatures caught in 7 hours having the relationship $W \geq 2S$ have a Binomial Distrubiton Y, with $n = X$ and $p = 0.2593$

Finally, we take proportion of runs with creatures caught in 7 hours having the relationship $W \geq 2S$ is bigger than 8 which is $\cong 0.5540$

Answer 2

We now have a Poisson Distrubiton X, with $\lambda = 50$

$f_W(w)$ can be find from the given joint pdf

$$\begin{aligned} f_W(w) &= \int_0^\infty wse^{-w-s}ds \\ &= we^{-w} \end{aligned} \tag{4}$$

Expected value of a random variable equals

$$\mu = E(X) = \int xf(x)dx \tag{5}$$

Then expected weight of the creature equals

$$\begin{aligned} \int wf_W(w)dw &= \int_0^\infty w^2e^{-w}ds \\ &= -(w^2 + 2w + 2)e^{-w} = 2 \end{aligned} \tag{6}$$

Since expected weight of one creature is 2, we need to multiply estimated total number of creatures caught in 10 hours by 2.

Answer 3

For this question, I have used Matlab's built-in functions for generating exponantial distrubition(A) and normal distrubition(B).
Then using A and B random variables, I calculated $E = \dfrac{2A + 3B}{3 + |2B|}$ in each loop.
After N loops finish, divinding sum of each monte carlo turn by N, I have found estimate of $E = \dfrac{2A + 3B}{3 + |2B|}$, which is $\cong 0.2289$