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Answer 1

a.

This has a negative binomial disrubition which has the pmf,

$$
\begin{aligned}
P(x) &= \binom{x-1}{k-1}(1-p)^{x-k}p^k \\
P(X) &= \prod_{i=1}^{n} p(X_i) = \prod_{i=1}^{n} \binom{x_i-1}{k-1}(1-p)^{x_i-k}p^k \\
since \quad ln\prod_{i=1}^{n} P(X_i) &= \sum_{i=1}^{n} lnP(X_i) = lnP(X) \\
lnP(X) &= \sum_{i=1}^{n}[log\binom{x_i-1}{k-1} + klogp + (x_i-k)log(1-p)]
\end{aligned} \tag{1}
$$

We need to maximize $lnP(X)$. To do this we need to take derivative with respect to p and equating its derivative to 0.

$$
\frac{d(lnP(X))}{dp} = \sum_{i=1}^{n} \frac{k}{p} - \frac{x_i-k}{1-p} = 0 \tag{2}
$$

which has only one solution,

$$
\frac{nk}{p} = \frac{(\sum_{i=1}^{n} x_i) - nk}{1-p} \tag{3}
$$

since $n = N$ and $\hat{p} = \hat{\theta}$

$$
\hat{\theta} = \frac{nk}{\sum_{i=1}^{n} x_i} \tag{4}
$$

b.

This has binomial disrubition which has the pmf,

$$
\begin{aligned}
P(x) &= \binom{n}{x} p^x (1-p)^{n-x} \\
P(X) &= \prod_{i=1}^{n} p(X_i) = (\prod_{i=1}^{n} \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}) \\
&= (\prod_{i=1}^{n} \binom{n}{x_i} p^{x_i})(1-p)^{n-\sum_{i=1}^{n} x_i} \\
since \quad ln \prod_{i=1}^{n} P(X_i) &= \sum_{i=1}^{n} lnP(X_i) = lnP(X) \\
lnP(X) &= \sum_{i=1}^{n} x_i lnp + (n - \sum_{i=1}^{n} x_i) ln(1-p)
\end{aligned} \tag{5}
$$

We need to maximize $lnP(X)$. To do this we need to take derivative with respect to p and equating its derivative to 0.

$$
\frac{d(lnP(X))}{dp} = \frac{\sum_{i=1}^{n} x_i}{p} - (n - \sum_{i=1}^{n} x_i)\frac{1}{1-p} = 0 \tag{6}
$$

since $\hat{p} = \hat{\theta}$

$$
\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{k}{n} \tag{7}
$$

Answer 2

a.

$$
\begin{aligned}
f(x) &= \frac{1}{\sqrt{2\pi 0.8}} exp[-\frac{(x-\mu)^2}{1.6}] \\
f(X) &= \prod_{i=1}^{n} (\frac{1}{\sqrt{2\pi 0.8}} exp[-\frac{(x_i-\mu)^2}{1.6}]) \\
&= (\frac{1}{\sqrt{2\pi 0.8}})^n exp[-\frac{-\sum_{i=1}^{n}(x_i-\mu)^2}{1.6}]
\end{aligned} \tag{8}
$$

so the log-likehood of a sample can be written as

$$
lnf(X) = -\frac{n}{2} ln(2\pi 0.8) - \frac{-\sum_{i=1}^{n}(x_i-\mu)^2}{1.6} \tag{9}
$$

Taking its derivative w.r.t. the unknown parameter μ, equating it to 0, and solving for μ, we get,

$$\begin{aligned}\frac{d(lnf(X))}{\mu} &= \frac{1}{0.8}\sum_{i=1}^{n}(x_i-\mu)=0 \\ \mu &= \hat{X}\end{aligned} \tag{10}$$

b.

Confidence interval for the the mean ($\mu = \hat{X}$)

$$\hat{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{11}$$

where

$$\begin{aligned}\sigma &= \sqrt{\sigma^2} = \sqrt{0.8} = 0.89 \\ \hat{X} &= \frac{6.5, 8.8, 9.2, 9.9, 12.4}{6} = 9.05 \\ 1-\alpha &= 0.95 \\ \alpha = 0.05 &\quad \alpha/2 = 0.025 \\ q_{0.025} = -z_{0.025} &\quad q_{0.975} = z_{0.025}\end{aligned} \tag{12}$$

from dist. table (A4 in the book), we find that $q_{0.975} = 1.960$

$$\hat{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 9.05 \pm (1.960)\frac{0.89}{\sqrt{6}} \tag{13}$$

c.

In order to attain a margin ($z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$) of error Δ for estimating a population mean$(1-\alpha)$, a sample size $n \geq (\frac{z_{\alpha/2}.\sigma}{\Delta})^2$ is required.

$$\begin{aligned}1-\alpha &= 0.99 \\ \alpha = 0.01 &\quad \alpha/2 = 0.005 \\ q_{0.995} &= z_{0.005} = 2.58 \\ \Delta = 0.5 &\quad \sigma = 0.89 \\ n &\geq (\frac{2.58 \times 0.89}{0.5})^2 \\ n &\geq (4.6)^2 = 21.16\end{aligned} \tag{14}$$

Answer 3

$$n \geq 0.25(\frac{z_{\alpha/2}}{\Delta})^2 \tag{15}$$

$$\begin{aligned}
1-\alpha &= 0.8 \\
\alpha = 0.2 \quad \alpha/2 &= 0.1 \\
q_{0.9} = z_{0.1} &= 1.29 \\
\Delta &= 0.02 \\
n &\geq 0.25(\frac{1.29}{0.02})^2
\end{aligned} \tag{16}$$

a.

$$\begin{aligned}
n &\geq 0.25(\frac{1.29}{0.06})^2 \\
n &\geq 116
\end{aligned} \tag{17}$$

b.

half margin error would be 0.03

$$\begin{aligned}
n &\geq 0.25(\frac{1.29}{0.03})^2 \\
n &\geq 463 \\
463 - 116 &= 347
\end{aligned} \tag{18}$$

Answer 4

Start with computing Z-statistic,

$$Z = \frac{\hat{X}-\mu}{\sigma/\sqrt{n}} = \frac{123.3-120}{10/\sqrt{16}} = \frac{3.3}{2.5} = 1.32 \tag{19}$$

Since this is a two-sided test we compute P-value from table A4,

$$\begin{aligned}
P\{|Z| \geq |Z_obs|\} &= 2(1-\Phi(|Z_obs|)) \\
&= 2(1-\Phi(1.32)) = 2(1-0.906) = 0.1812
\end{aligned} \tag{20}$$

For testing H_0 with a P-value, we reject H_0 if $P < 0.01$, and accept H_0 if $P > 0.1$.
Since our P-value is $P > 0.1$ we accept it, we would have reject it if it were $P < 0.01$.