Student Information

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Answer 1

a.

Rational numbers are countable, they can be arrange (zigzag method) such a way that
$\frac{0}{1}, \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}, \frac{3}{1}, \frac{3}{2}, \frac{-3}{1}, \frac{-3}{2}, \frac{1}{4} \cdots$
Thus it is possible to set up a bijection between each rational number and it's position in the list, which is an element of N. Since the set of rational numbers inside the open interval (-1,0) is subset of the rational numbers it is also countable and it is infinite. (If S is countable and S' $\subseteq$ S, then S is also countable. Since S is countable, there is a bijection f : S $\rightarrow$ N. But then f(S')=N' is a subset of N, and f is a bijection between S' and N'.)

b.

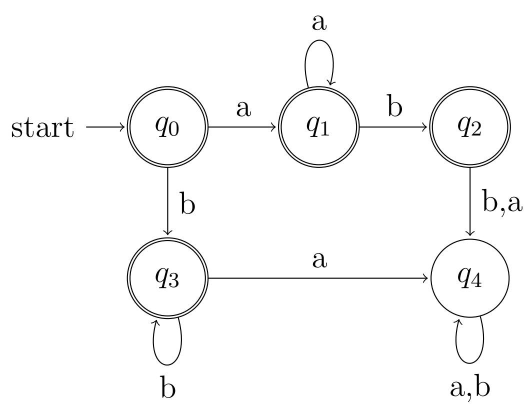
The set $D = \{L^+$: L is a finite language over the unary alphabet $\sum = \{a\}$ and L^+ is not regular.$\}$ is infinitely countable. Since only element of the alphabet is a, the members of L^+ can be enumerated by such a way that for each $k \geq 0$, all strings of length k are enumerated before before all strings of length $k + 1$.

c.

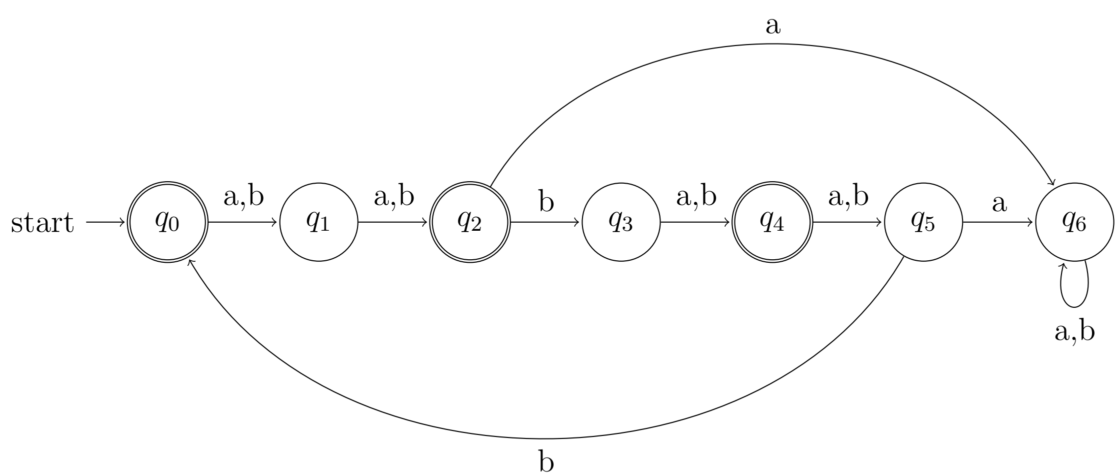
The set of all languages on the binary alphabet $\sum = \{a, b\}$ which cannot be recognized by any Finite Automaton are infinitly uncountable.

Answer 2

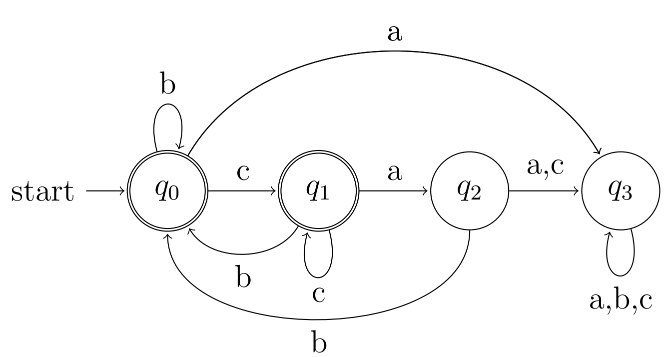
a.


start
q_0
q_1
q_2
q_3
q_4
a
a
b
b
b,a
a
b
a,b

b.


a
start
q_0
q_1
q_2
q_3
q_4
q_5
q_6
a,b
a,b
b
a,b
a,b
a
a,b
b

c.


a
b
start
q_0
q_1
q_2
q_3
c
a
a,c
b
c
a,b,c
b

Answer 3

a.

$$\begin{aligned}(q_0, abbb) &\vdash_M (q_1, bbb)\\ &\vdash_M (q_3, bbb)\\ &\vdash_M (q_3, bb)\\ &\vdash_M (q_3, b)\\ &\vdash_M (q_3, e)\end{aligned} \tag{1}$$

$w_1 =$ abbb is not in L(N) as it cannot reach to the final state.

b.

$$\begin{aligned}(q_0, ababa) &\vdash_M (q_1, baba)\\ &\vdash_M (q_3, baba)\\ &\vdash_M (q_3, aba)\\ &\vdash_M (q_5, ba)\\ &\vdash_M (q_1, ba)\\ &\vdash_M (q_3, ba)\\ &\vdash_M (q_4, a)\\ &\vdash_M (q_5, e)\end{aligned} \tag{2}$$

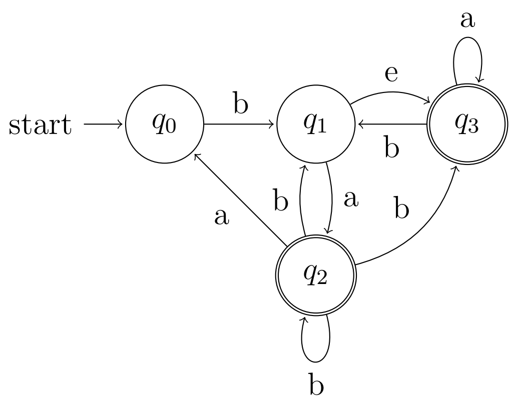
$w_2 =$ ababa is in L(N) as $(q_0, ababa) \vdash_M^* (q_5, e)$.

Answer 4

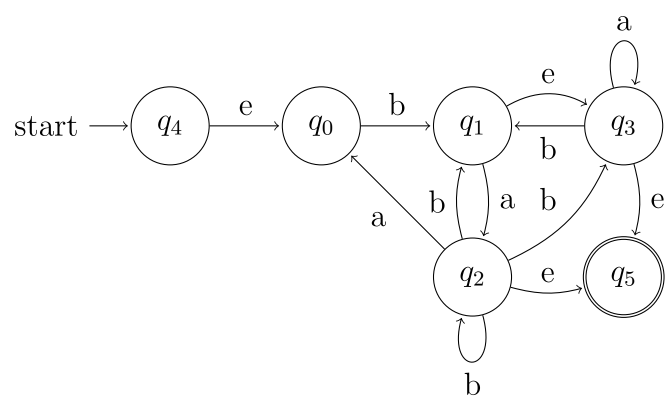
a.

$(ba(b \cup (b \cup ab)a)^*) \cup (b \cup ba(b \cup (b \cup ab)a)^* b \cup (b \cup ab)(a \cup b \cup ba(b \cup (b \cup ab)a)^* b \cup (b \cup ab))^* ba(b \cup (b \cup ab)a)^*)$

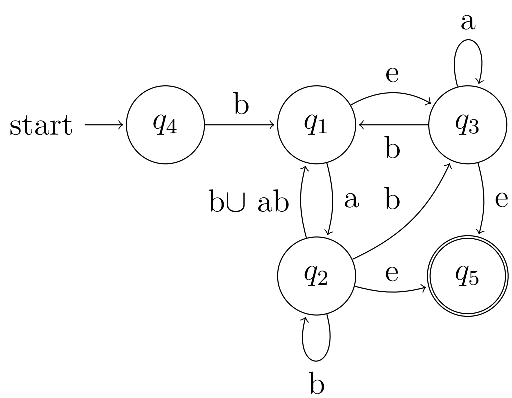
b.


start
q_0
q_1
q_2
q_3
a
b
e
b
b
a
a
b
b

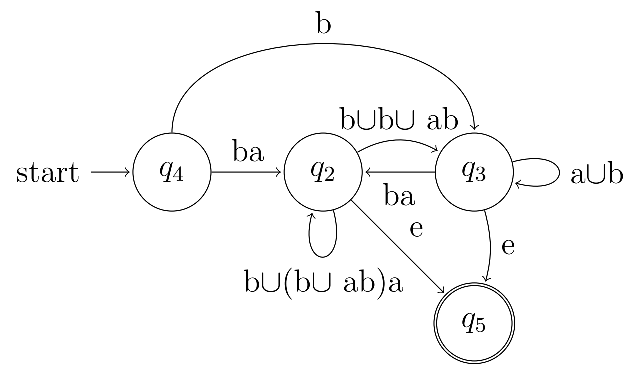
0.


start
q_4
q_0
q_1
q_2
q_3
q_5
a
e
b
e
b
a
b
a
b
e
e
b

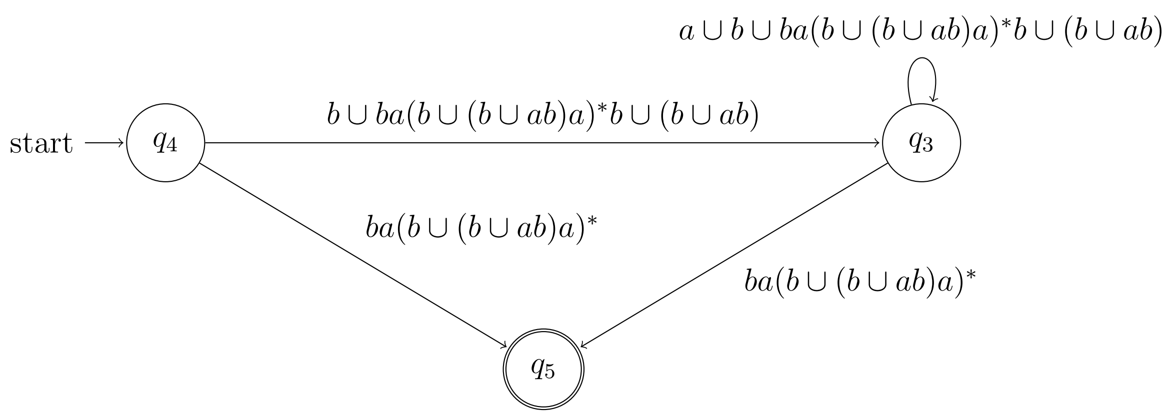
1.


start
q_4
q_1
q_2
q_3
q_5
a
b
e
b
b∪ ab
a
b
e
e
b

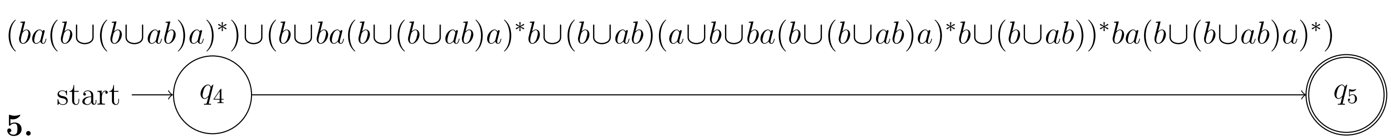
2.


b
b∪b∪ ab
start
q_4
q_2
q_3
q_5
ba
a∪b
ba
e
e
b∪(b∪ ab)a

3.


$a \cup b \cup ba(b \cup (b \cup ab)a)^{*} b \cup (b \cup ab)$
start
q_4
q_3
q_5
$b \cup ba(b \cup (b \cup ab)a)^{*} b \cup (b \cup ab)$
$ba(b \cup (b \cup ab)a)^{*}$
$ba(b \cup (b \cup ab)a)^{*}$

4.

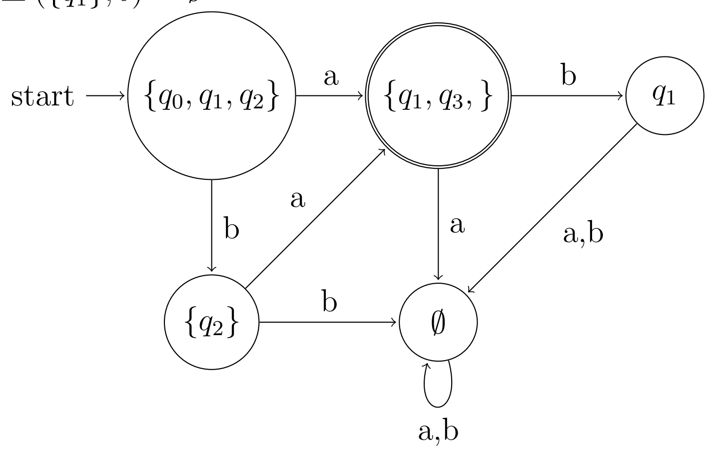

$(ba(b\cup(b\cup ab)a)^{*})\cup(b\cup ba(b\cup(b\cup ab)a)^{*}b\cup(b\cup ab)(a\cup b\cup ba(b\cup(b\cup ab)a)^{*}b\cup(b\cup ab))^{*}ba(b\cup(b\cup ab)a)^{*})$
start
q_4
q_5

5.

Answer 5

a.

$s' = E(q_0) = \{q_0, q_1, q_2\}$
$E(q_1) = \{q_1\} \quad E(q_2) = \{q_2\} \quad E(q_3) = \{q_1, q_3\}$
$\Delta'(\{q_0, q_1, q_2\}, a) = E(q_1) \cup E(q_3) = \{q_1, q_3\}$
$\Delta'(\{q_1, q_3\}, a) = \emptyset$
$\Delta'(\{q_1, q_3\}, b) = E(q_1) = \{q_1\}$
$\Delta'(\{q_2\}, a) = E(q_3) = \{q_1, q_3\}$
$\Delta'(\{q_2\}, b) = \emptyset$
$\Delta'(\{q_1\}, a) = \emptyset$
$\Delta'(\{q_1\}, b) = \emptyset$



b.

$\bar{L} = e \cup a(b \cup a \cup b(a \cup b)) \cup b(e \cup a(b \cup a(b \cup a \cup b(a \cup b)) \cup b)(a \cup b)^*$

Answer 6

Since $L_1 - L_2 = L_1 \cap \bar{L_2}$, we are going to construct $L_1 \cap \bar{L_2}$. Let $L_1 = L(M_1)$ *where* $M_1 = (K_1, \sum, \Delta_2, s_1, F_1)$ and let $L_2 = L(M_2)$ *where* $M_2 = (K_2, \sum, \Delta_2, s_2, F_2)$ and both M_1, M_2 are nondeterministic finite automata. To be able take complement of L_2, we have make M_2 deterministic finite automata. Then the complementary language $\bar{L_2} = \sum^* - L(M)$ is accepted by the deterministic finite automaton $\bar{M_2} = (K_2, \sum, \Delta_2, s_2, K_2 - F_2)$. Now the idea is to run M_1 and M_2 in parallel on the same input and accept if both M_1 and M_2 accept.
Consider $M = (K, \sum', \Delta, s, F)$ which is an automaton whose set of states is the Cartesian product of the sets of states of the M_1 and M_2 defined as
$K = K_1 \times K_2$
$\sum' = \sum$
$s = \langle s_1, s_2 \rangle$
$F = F_1 \times (K_2 - F_2)$

$\Delta(\langle p_1, p_2\rangle, a) = \Delta_1(p_1, a) \times \Delta_2(p_2, a)$.
This M accepts $L_1 \cap L_2$.

Answer 7

a.

L is not regular. Let's assume it were, Pumping Lemma theorem would apply for some integer n. Consider then the string $w = b^n a^{n^2}$, $w \in L$, $|w| \geq n$. Now let's divide w into xyz. For some integer $g > 0$, $x = b^{n-g}$, $y = (ba)^g$, $z = a^{n^2-g}$ $(a^{g+n^2-g} = a^{n^2})$. According to theorem xy^iz must be in L but it is impossible since every $i * g + n^2 - g$ cannot be equal to the square of a natural number for each $i \geq 0$ like the definition of L states.