Practice for rev1(rev1(L))

Proof by structural induction on L

WTS = What to show

Fresh constants

e = Elt. E

l = List

Base Case

WTS : rev1(rev1(nil)) = nil

rev1(rev1(nil)) → rev1(nil) by rev1-1 (eq rev1(nil) = nil)

→ nil by rev1-1

Induction Case

WTS: rev1(rev1(e|l)) = e|l

IH: rev1(rev1(l)) = l

rev1(rev1(e|l)) → rev1(rev1(l)@(e|nil)) by rev-2

↑ 이 이상 나아갈 수 없으므로, Lemma를 준비한다.

rev1(rev1(L)@(E|nil)) = E|L

[rev1(rev1(L)@L2) = rev1(L2)@L] ... Lemma ← rev(L1@L2) = rev1(L2)@rev1(L1)

rev1(l)@(e|nil) → rev1(e|nil)@l by lemma

→ (rev1(nil)@(e|nil))@l by rev1-2

→ (nil@(e|nil))@l by rev1-1

→ (e|nil)@l by @1

→ e|(nil@l) by @2

→ e|l by @1

End of Proof

이어서 lemma 의 증명

rev1 (rev1(L) @ L2) = rev1(L2) @ L

Proof By Structual Induction on L

Fresh constants

e = Elt, E

l, l2 = List

nil이 사라진다는 lemma 필요

Base case

W2S: rev1 (rev1(nil) @ l2) = rev1(l2) @ nil

rev1 (rev1(nil) @ l2)

→ rev1(nil @ l2) by rev1-1

→ rev1 (l2) by @1

[L @ nil = L] ··· lemma 2

rev1 (l2) @ nil → rev1(l2)

Induction Case

W2S : rev1(rev1(e|l) @ l2) = rev1 (l2) @ (e|l)

IH : rev1(rev1(l) @ L2) = rev1(L2) @ l

rev1 (rev1 (e|l) @ l2)

→ rev1 ((rev(l) @ (e|nil)) @ l2) by rev1-2

→ rev1 (rev(l) @ ((e|nil) @ l2)) by assoc@

→ rev1 ((e|nil) @ l2) @ l by IH

→ rev1 (e|(nil @ l2)) @ l by @2

→ rev1 (e|l2) @ l by @1

→ (rev1(l2) @ (e|nil)) @ l by rev1-2

→ rev1 (l2) @ ((e|nil) @ l) by assoc@

→ rev1 (l2) @ (e|(nil @ l)) by @2

→ rev1 (l2) @ (e|l) by @1 End of Proof

Lemma 2의 증명

$L @ nil = L$

Proof By structural Induction on L

Fresh Constants

$e = Elt,\ E$

$l = List$

Base Case

$W2S = nil @ nil = nil$

$nil @ nil \rightarrow nil$ by @1

Induction Case

$W2S = (e \mid l) @ nil = e \mid l$

$IH = l @ nil = l$

$(e \mid l) @ nil$

$\rightarrow e \mid (l @ nil)$ by @2

$\rightarrow e \mid l$ by IH

End of Proof