

GCD

1. $\text{gcd}(24, 36)$,

- $\text{gcd}(36, (24 \text{ rem } 36))$... by (gcd 2)
- $\text{gcd}(36, 24)$... by (remainder)
- $\text{gcd}(24, (36 \text{ rem } 24))$... by (gcd 2)
- $\text{gcd}(24, 12)$... by (rem)
- $\text{gcd}(12, (24 \text{ rem } 12))$... by (gcd 2)
- $\text{gcd}(12, 0)$... by (rem)
- 12 ... by (gcd 1)

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$$\begin{cases} \text{eg } \text{gcd}(x, 0) = x, \dots \text{gcd} 1 \\ \text{eg } \text{gcd}(x, N_2 Y) = \text{gcd}(N_2 Y, x \text{ rem } N_2 Y), \dots \text{gcd} 2 \end{cases}$$

*) $m \text{ rem } n = (m - n \cdot \lfloor m/n \rfloor) = \text{Remainder}$

FACT

1. $\text{fact}(5)$,

- $5 \cdot \text{fact}(p 5)$... by (f2)
 - $5 \cdot \text{fact}(4)$... by (p)
 - $5 \cdot (4 \cdot \text{fact}(p 4))$... by (f2)
 - $5 \cdot (4 \cdot \text{fact}(3))$... by (p)
 - $5 \cdot (4 \cdot (3 \cdot \text{fact}(p 3)))$... by (f2)
 - $5 \cdot (4 \cdot (3 \cdot \text{fact}(2)))$... by (p)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot \text{fact}(p 2))))$... by (f2)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot \text{fact}(1))))$... by (p)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot (1 \cdot \text{fact}(p 1)))))$... by (f2)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot (1 \cdot \text{fact}(0)))))$... by (p)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot (1 \cdot 1))))$... by (f1)
 - $5 \cdot (4 \cdot (3 \cdot (2 \cdot 1)))$ → 120
 - $5 \cdot (4 \cdot (3 \cdot 2))$ → 120
 - $5 \cdot (4 \cdot 6)$
 - $5 \cdot 24$
- all by (x)

eg $\text{fact}(0) = 1, \dots f1$

eg $\text{fact}(N_2 X) = N_2 X \cdot \text{fact}(p N_2 X), \dots f2$

*) $p \text{ NN} = (-n - 1) = \text{Previous}$

OEDC-FACT

1. oedc-fact(5),

→ $g(5, 1)$... by(2)

→ $\text{Cond}(5 > 1, (g(5, 2 \cdot 1) \cdot g(\text{sd}(5, 1)), 2 \cdot 1), 5)$
... by(g)

→ $\text{Cond}(\text{true}, (g(5, 2 \cdot 1) \cdot g(\text{sd}(5, 1), 2 \cdot 1)), 5)$
... by(>)

→ $(g(5, 2) \cdot g(\text{sd}(5, 1), 2 \cdot 1))$... by(Cond 1)

→ $\text{Cond}(5 > 2, (g(5, 2 \cdot 2) \cdot g(\text{sd}(5, 1)), 2 \cdot 2), 5)$
... by(g)

→ $\text{Cond}(\text{true}, (g(5, 2 \cdot 2) \cdot g(\text{sd}(5, 2)), 2 \cdot 2), 5)$... by(>)

→ $(g(5, 4) \cdot g(\text{sd}(5, 2), 2 \cdot 2))$... by(Cond 1)

→ $\text{Cond}(5 > 4, (g(5, 2 \cdot 4) \cdot g(\text{sd}(5, 4)), 2 \cdot 4), 5)$... by(g)

→ $\text{Cond}(\text{true}, (g(5, 2 \cdot 4) \cdot g(\text{sd}(5, 4), 2 \cdot 4)), 5)$... by(>)

→ $(g(5, 8) \cdot g(\text{sd}(5, 4), 2 \cdot 4))$... by(Cond 1)

→ $\text{Cond}(5 > 8, (g(5, 2 \cdot 8) \cdot g(\text{sd}(5, 8), 2 \cdot 8)), 5)$... by(g)

→ $\text{Cond}(\text{false}, (g(5, 2 \cdot 8) \cdot g(\text{sd}(5, 8), 2 \cdot 8)), 5)$... by(>)

→ $(5 \cdot g(\text{sd}(5, 4), 2 \cdot 4))$... by(Cond 2)

→ $g(1, 8)$... by(sd + (x)op)

→ $\text{Cond}(1 > 8, (g(1, 2 \cdot 8) \cdot g(\text{sd}(1, 8), 2 \cdot 8)), 1)$... by(g)

→ $\text{Cond}(\text{false}, (g(1, 2 \cdot 8) \cdot g(\text{sd}(1, 8), 2 \cdot 8)), 1)$... by(>)

→ $(5 \cdot 1)$... by(Cond 2)

→ $(5 \cdot g(\text{sd}(5, 2), 2 \cdot 2))$... by((x)op)

→ $g(3, 4)$... by(sd + (x)op)

→ $\text{Cond}(3 > 4, (g(3, 2 \cdot 4) \cdot g(\text{sd}(3, 4), 2 \cdot 4)), 3)$... by(g)

→ $\text{Cond}(\text{false}, (g(3, 2 \cdot 4) \cdot g(\text{sd}(3, 4), 2 \cdot 4)), 3)$... by(>)

$$\text{eq } \text{oedc-fact}(0) = 1, \dots \text{of } 1$$

$$\text{eq } \text{oedc-fact}(N_2 X) = g(N_2 X, 1), \dots \text{of } 2$$

$$\text{eq } g(X, Y) = \text{Cond}(X > Y, g(X, 2 \cdot Y) \cdot g(\text{sd}(X, Y), 2 \cdot Y), X), \dots \text{in } g$$

$$\text{eq } \text{Cond}(\text{true}, X, Y) = X, \dots \text{Cond } 1$$

$$\text{eq } \text{Cond}(\text{false}, X, Y) = Y, \dots \text{Cond } 2$$

$\rightarrow (5, 3) \dots \text{by}(\text{cond } 2)$
 $\rightarrow (15, g(\text{sd}(5, 1), 2, 1)) \dots \text{by}(\text{op})$
 $\rightarrow g(4, 2) \dots \text{by}(\text{sd} + \text{op})$
 $\rightarrow \text{cond}(4 > 2, (g(4, 2, 2) \cdot g(\text{sd}(4, 2), 2, 2)), 4) \dots \text{by}(g)$
 $\rightarrow \text{cond}(\text{true}, (g(4, 2, 2) \cdot g(\text{sd}(4, 2), 2, 2)), 4) \dots \text{by}(>)$
 $\rightarrow (g(4, 4) \cdot g(\text{sd}(4, 2), 2, 2)) \dots \text{by}(\text{cond } 1) + (\text{op})$
 $\rightarrow \text{cond}(4 > 4, (g(4, 2, 4) \cdot g(\text{sd}(4, 4)), 2, 4), 4) \dots \text{by}(g)$
 $\rightarrow \text{cond}(\text{false}, (g(4, 2, 4) \cdot g(\text{sd}(4, 4)), 2, 4), 4) \dots \text{by}(>)$
 $\rightarrow (4 \cdot g(\text{sd}(4, 2), 2, 2)) \dots \text{by}(\text{cond } 2)$
 $\rightarrow g(2, 4) \dots \text{by}(\text{sd} + \text{op})$
 $\rightarrow \text{cond}(2 > 4, (g(2, 2, 4) \cdot g(\text{sd}(2, 4), 2, 4)), 2) \dots \text{by}(g)$
 $\rightarrow \text{cond}(\text{false}, (g(2, 2, 4) \cdot g(\text{sd}(2, 4), 2, 4)), 2) \dots \text{by}(>)$
 $\rightarrow (4, 2) \dots \text{by}(\text{cond } 2)$
 $\rightarrow (15, 8) \dots \text{by}(\text{op})$
 $\rightarrow 120$