

# **A High-Performance Implementation of GNN-Based Trajectory Reconstruction on FPGA**

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# Outline

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- **Background**
  - *Challenges*
- **Related Work**
  - *Limitations*
- **Contributions**
- **Experiment**
- **Conclusion**
- **Reference**

# Background

- **Collaborative Program with CERN**

*European Organization for Nuclear Research*

- *Research Emphasis on High-Energy Physics – (HEP)*
- *Partnerships with Hundreds of International Universities*
  - University of Illinois Urbana-Champaign; University of Washington

- **Large Hadron Collider (LHC) Infrastructure Overview<sup>[1]</sup>**

- Collision-Event Analysis for Exploration of Novel Physical Phenomena

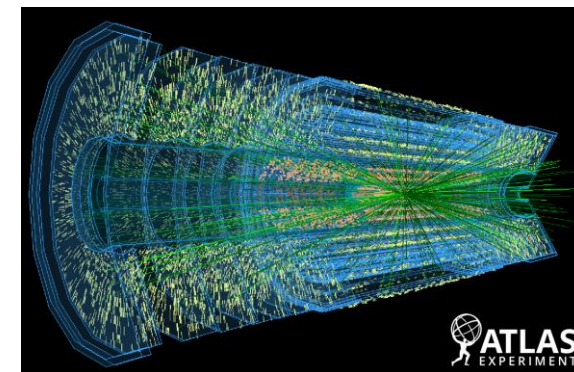
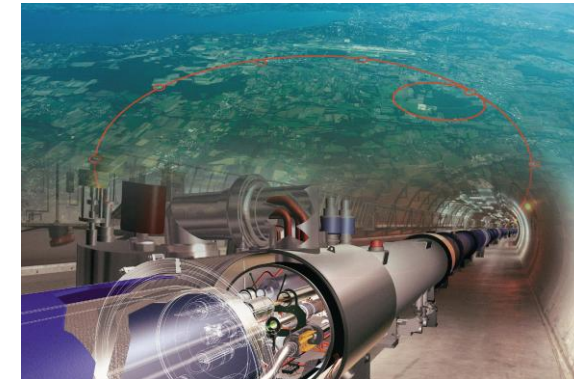
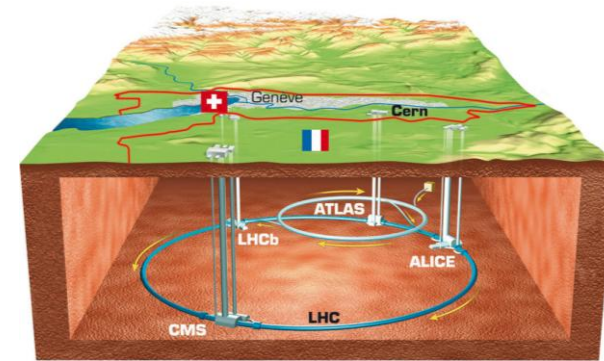
- **Methodology Framework for Collision Analysis**

- *Dual Proton-Beam Acceleration*
  - Near-Light Speed for High-Energy Collisions
  - High-Frequency Occurrence of Collision Events at 40 MHz
- *Generated Particles Pass through Detectors Yielding Hits*
- *Trajectory Reconstruction-Based Analysis of Hits*

**Detector**

**Hit**

**Trajectory Segment**



[1] L. Evans, "The large hadron collider," *New Journal of Physics*, vol. 9, no. 9, p. 335, 2007.

# Challenge

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- **Archiving High-Volume Collision Hits**

*Offline Analysis Capacity Considerations*

- ***Real-Time Data Reduction via Level-1 Trigger (L1T)<sup>[2]</sup> System***

- Selective Acquisition of Critical Data for Offline Processing
    - Trajectory Reconstruction in Trigger Decision Making

- **Stringent Latency and Throughput Constraints in L1T System**

- ***HL-LHC Upgrades<sup>[3]</sup> and CMS<sup>[4]</sup> Detector Enhancement Strategies***

*HL-LHC – High-Luminosity Large Hadron Collider*

- Latency Budget Allocation of 4  $\mu$ s for Data Selection
    - Event-Rate Processing of 2.22 MHz through Time-Multiplexing

*Time-Multiplexed Distribution of 40 MHz Collisions to 18 FPGAs*

- **Insufficiency of Current LHC Tracking Algorithm<sup>[5]</sup>**

- Inadequate Performance under Stringent Constraints

[2] “The Phase-2 Upgrade of the CMS Level-1 Trigger,” CERN, Geneva, Tech. Rep., 2020, final version.

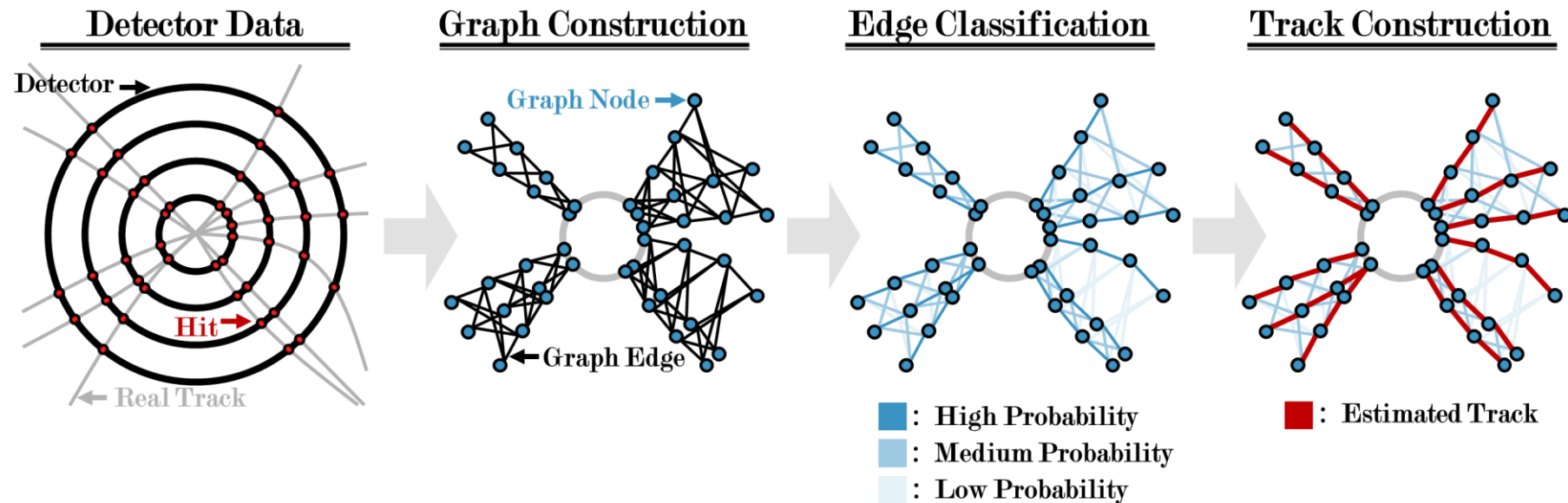
[3] O. Aberle, C. Adorisio, A. Adraktas, M. Ady, J. Albertone, L. Alberty, M. Alcaide Leon, A. Alekou, D. Alesini, B. Almeida Ferreira et al., “High-luminosity large hadron collider (hl-lhc): Technical design report,” 2020.

[4] “The Phase-2 Upgrade of the CMS Tracker,” CERN, Geneva, Tech. Rep., 2017.

[5] R. Frühwirth, “Application of kalman filtering to track and vertex fitting,” *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 262, no. 2-3, pp. 444–450, 1987.

# Related Work

- **Interaction Network (IN)<sup>[6]</sup> Framework**
  - Specialized Graph Neural Network for Object-Object Interaction Modeling
- **GNN-Based Trajectory Reconstruction Framework**
  - **Graph Construction Stage** – Mapping Hits and Segment Candidates to Nodes and Directed Edges
  - **Edge Classification Stage** – Probabilistic Assessment of Edge Validity
  - **Track Construction Stage** – Integration of Edge Probabilities in Trajectory Reconstruction



[6] P. Battaglia, R. Pascanu, M. Lai, D. Jimenez Rezende et al., "Interaction networks for learning about objects, relations and physics," *Advances in neural information processing systems*, vol. 29, 2016.

# Related Work – Limitations

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- **Software-Driven Framework Leveraging CPUs or GPUs**
  - *Accuracy-Centric Methodology with Execution-Speed Agnosticism*<sup>[7,8]</sup>
    - CPU-Based Constraints Impeding Task-Specific Optimization Potential
    - GPU-Based Inefficiency in Single-Event and Latency-Critical Scenarios
    - Substantial Millisecond-Scale Deviation from L1T Microsecond Requirements
- **FPGA-Accelerated Framework**
  - *Throughput-Driven Processing Limited to Minor Graph Subregions*<sup>[11]</sup>
    - Impact of Small Subgraph on Accuracy Degradation
  - *Scope Constrained to GNN Edge-Classification Stage*<sup>[9,10]</sup>
    - Imposition of Host-to-Device Data Transfers Resulting in FPGA Underutilization

[7] G. DeZoort, S. Thais, J. Duarte, V. Razavimaleki, M. Atkinson, I. Ojalvo, M. Neubauer, and P. Elmer, “Charged particle tracking via edge-classifying interaction networks,” *Comput. Softw. Big Sci.*, vol. 5, no. 1, pp. 1–13, 2021.

[8] X. Ju, D. Murnane, P. Calafiura, N. Choma, S. Conlon, S. Farrell, Y. Xu, M. Spiropulu, J.-R. Vlimant, A. Aurisano et al., “Performance of a geometric deep learning pipeline for hl-lhc particle tracking,” *The European Physical Journal C*, vol. 81, pp. 1–14, 2021.

[9] A. Elabd, V. Razavimaleki, S.-Y. Huang, J. Duarte, M. Atkinson, G. DeZoort, P. Elmer, S. Hauck, J.-X. Hu, S.-C. Hsu et al., “Graph neural networks for charged particle tracking on fpgas,” *Frontiers in big Data*, vol. 5, p. 828666, 2022.

[10] S. Huang, Y. Yang, Y. Su, B. Lai, J. Duarte, S. Hauck, S. Hsu, J. Hu, and M. S. Neubauer, “Low latency edge classification gnn for particle trajectory tracking on fpgas,” in *2023 33rd International Conference on Field-Programmable Logic and Applications (FPL)*. Los Alamitos, CA, USA: IEEE Computer Society, sep 2023, pp. 294–298.

[11] Aneesh Heintz, Vesal Razavimaleki, Javier Duarte, Gage DeZoort, Isobel Ojalvo, Savannah Thais, Markus Atkinson, Mark Neubauer, Lindsey Gray, Sergio Jindari-ani, et al. 2020. Accelerated charged particle tracking with graph neural networkson FPGAs. *arXiv preprint arXiv:2012.01563* (2020).

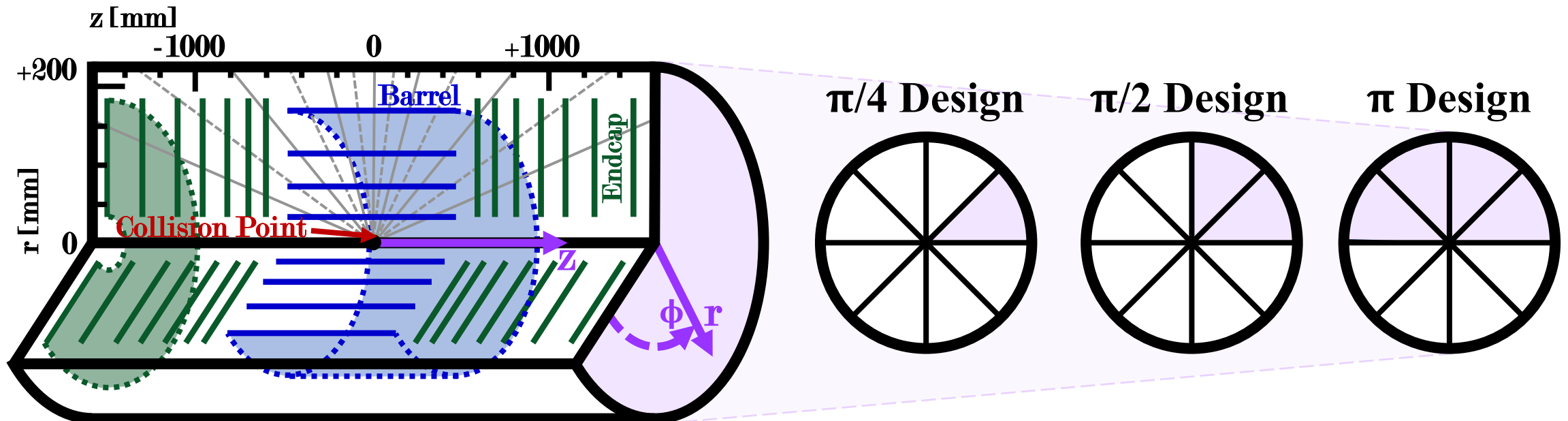
# Contribution

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- **End-to-End GNN-Based FPGA Accelerator for Trajectory Reconstruction**
  - *Algorithmic Optimizations Derived from Reference<sup>[7]</sup>*
    - Geometry-Aware Edge Pruning on Graph Construction
      - Computational Load Minimization
    - Probability-Based Sequential Building on Track Construction
      - FPGA Deployability Enhancement
  - *Data Streaming–Oriented High-Throughput Optimization Inspired by Reference<sup>[10]</sup>*
    - Batch Processing of Edge-Classification into Data-Streaming Paradigm
      - Achieved Latency Reduction of 52.3%
  - *Consolidated Three-Stage Pipeline within High-Performance FPGA Accelerator*
    - **65024× Acceleration** over Software-Based Approach with **Enhanced Accuracy Metrics<sup>[7,9]</sup>**
    - First End-to-End GNN-Based FPGA Accelerator for Trajectory Reconstruction
    - Event-Throughput Rate of 2.35 MHz with 2.36  $\mu$ s Latency Meeting L1T Criteria

# Configuration

- **Cross-Sectional View of Cylindrical Collider Detector Architecture**
  - Geometrical Configuration of 4× Cylindrical Barrels and 14× Planar Endcaps
- **Hit Distribution across Detector-Segment via Spatially Multiplexed FPGA**
  - Longitudinal Segmentation along the Z Axis
  - Azimuthal Segmentation along the  $\Phi$ -Axis into 2/4/8 Sectors for Scalability
- **Specifically Adapted to Support Three Distinct Design Variants**





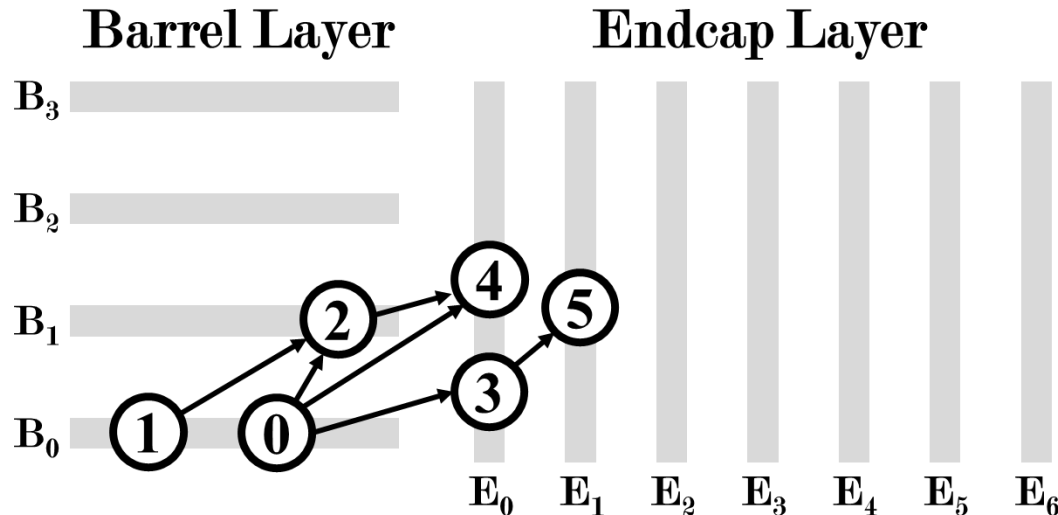
# Graph Construction Algorithm – Baseline

## • Graph Construction from High-Energy Collision Data<sup>[7]</sup>

- Node **Exhaustive Enumeration** in Adjacent Layers for Edge Feature Extraction
- Selection of Track-Segment Candidates via Feature Thresholding
- Mapping of Hits and Track-Segment Candidates onto Nodes with Directed Edges
- Conversion to Sparse Coordinate List (COO) Format for Input of GNN Stage
  - Specification of Source and Target Node Indices with Edge Feature Vectors

*Four-Dimensional Edge Feature:  $(\Delta r_{ij}, \Delta \Phi_{ij}, \Delta z_{ij}, \Delta R_{ij})$*

$$\Delta R_{ij} = \sqrt{(\Delta \Phi_{ij})^2 + \left( \ln \left( \frac{\tan \left( \frac{1}{2} \text{atan2}(r_i, z_i) \right)}{\tan \left( \frac{1}{2} \text{atan2}(r_j, z_j) \right)} \right) \right)^2}$$



	Source Node Index					
	0	1	2	3	4	5
0						
1						
2	$e_{0,2}$	$e_{1,2}$				
3	$e_{0,3}$					
4	$e_{0,4}$		$e_{2,4}$			
5				$e_{3,5}$		

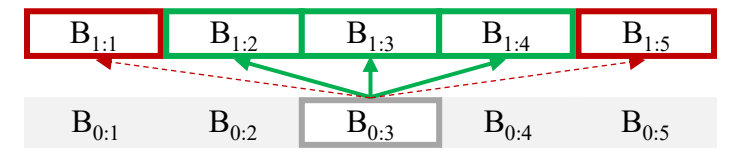
Source Node Index					
0	1	0	0	2	3

→ In order

Target Node Index					
2	2	3	4	4	5

Edge Features					
$e_{0,2}$	$e_{1,2}$	$e_{0,3}$	$e_{0,4}$	$e_{2,4}$	$e_{3,5}$

# Graph Construction Algorithm – Proposed



- **Exhaustive Connectivity Enumeration: Computational Infeasibility**

- Edge Candidates Formation Confined to  $\Delta\Phi$ -Span  $\ll \Phi$ -Range

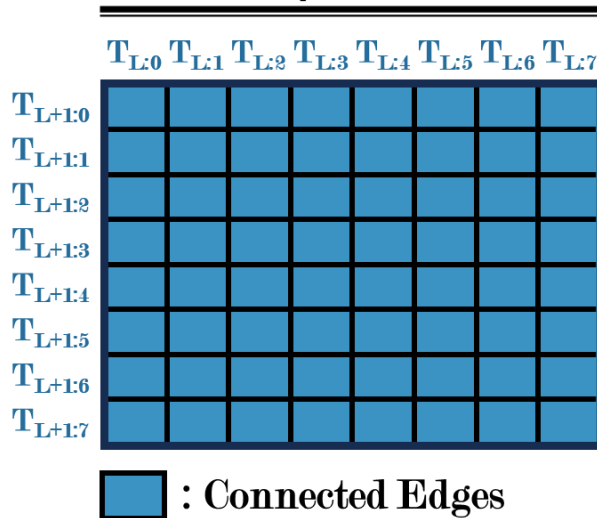
$\Phi$ -Range	$\pi/4$	$\pi/2$	$\pi$
$\Phi$ -Range/ $\Delta\Phi$ -Span	9.13	18.23	36.53

※ Ratios of Full  $\Phi$ -Range to Maximum  $\Delta\Phi$  for Potential Can-didate Edges Across Three Azimuthal Segmentation

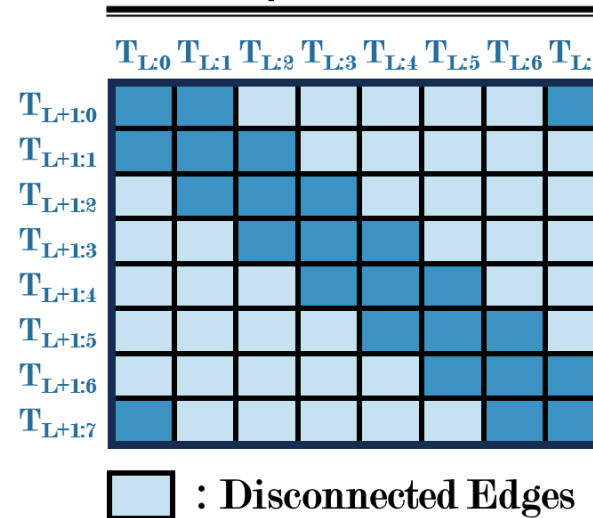
- **Neighborhood-Constrained Optimization Strategy**

- Adjacency-Based Candidate Edge Restriction
- **Computational Load Reduction and Scalability Enhancement**

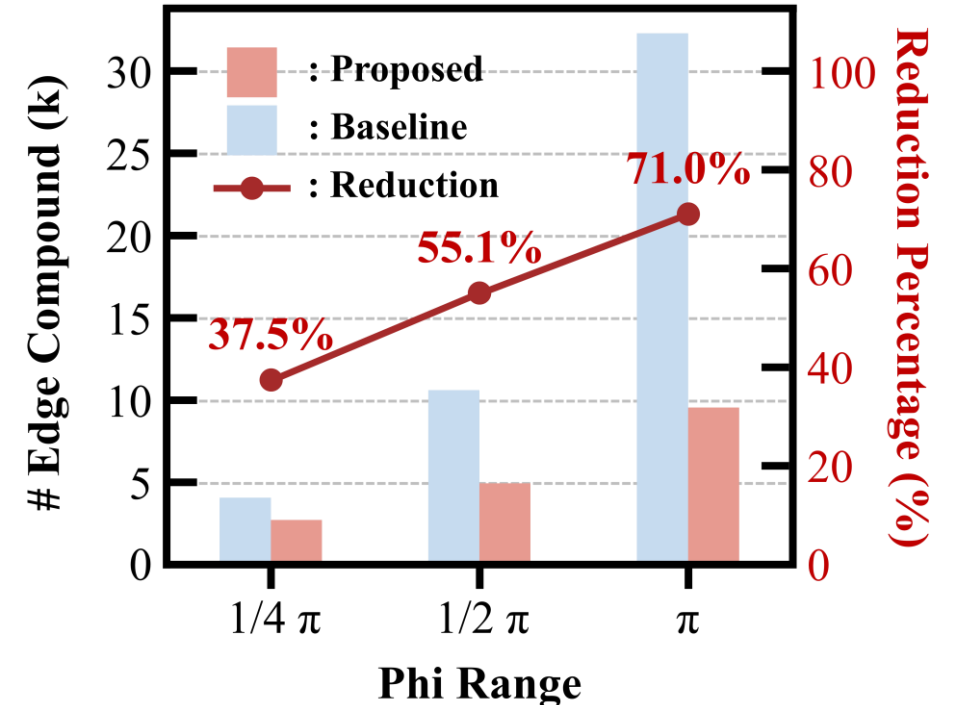
## Without $\phi$ -Subdivision



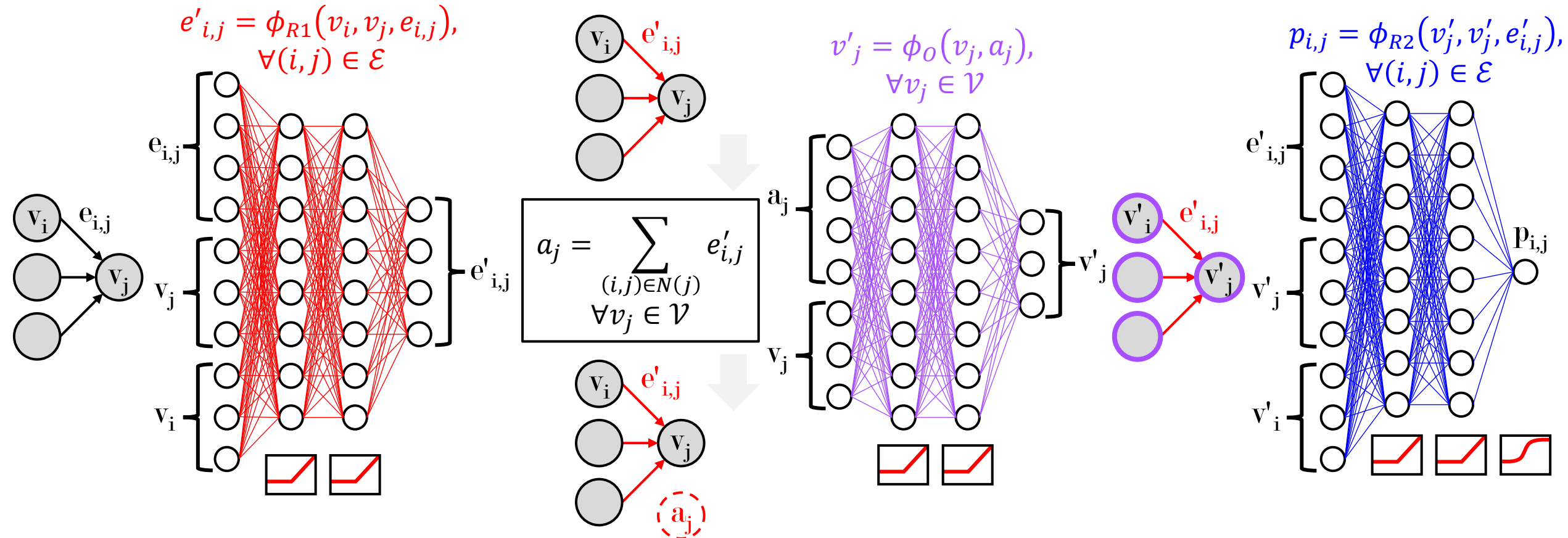
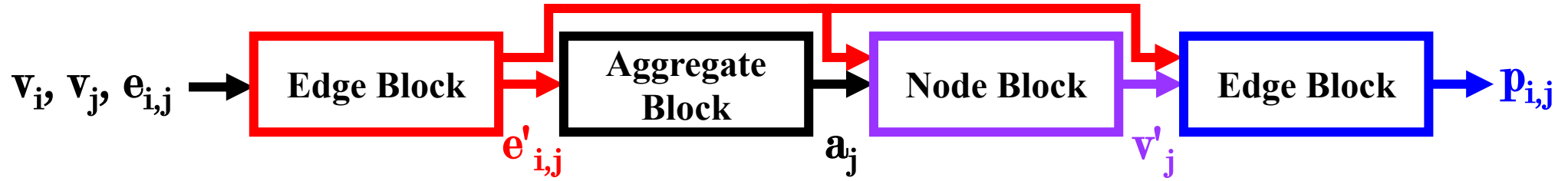
## With $\phi$ -Subdivision



$$L(T) \in \begin{cases} \{0, 1, 2, 3, 4, 5\}, & \text{if } T = E \\ \{0, 1, 2\}, & \text{if } T = B \end{cases}$$



# Edge Classification Algorithm



# Track Building Algorithm – Baseline

- **Retention of Edges Exceeding Probability Threshold**

*Distance Matrix  $\Delta R_{ij}$  Generation for Candidate Edges*

Cluster Indices	0	1
Node Indices	0, 2, 3	1, 4

- **Density-Based Spatial Clustering of Applications with Noise**

- Exhaustive Recursive Clustering for Localized Node Neighborhoods

*Number of Minimum Points = 2 in Neighborhood Range = 0.4*

- Treating Intra-Cluster Nodes as Individual Particle Paths

- **Inefficiency in Computation and Parallelism**

- Dense Matrix Interpretation for Sparse Graph

*Time-Complexity  $O(V^2)$  for Pair-Wise Distance Querying*

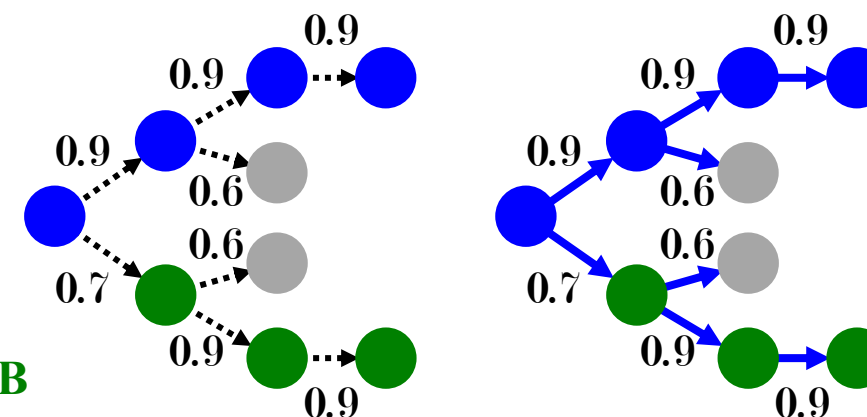
- Scheduling Requirements of Partitioned Intra-Cluster Node

- **Degradation of Accuracy**

- Complete Omission of Probability Information
- Suboptimal Differentiation through  $\Delta R$  Metrics
- Bifurcated Trajectory Paths

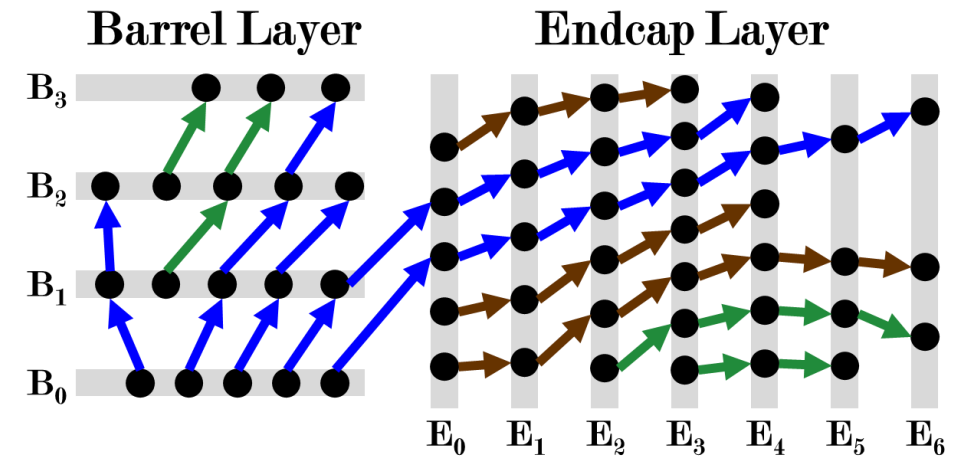
● : Particle A    ● : Particle B

	0	1	2	3	4
0	0	$\infty$	0.2	$\infty$	$\infty$
1	$\infty$	0	$\infty$	$\infty$	0.1
2	0.2	$\infty$	0	0.1	$\infty$
3	$\infty$	$\infty$	0.1	0	$\infty$
4	$\infty$	0.1	$\infty$	$\infty$	0



# Track Building Algorithm – Proposed

- **Establishment of Node Index Tables**
  - *Target / Untargeted Node Index Table*  
*Outgoing Edge with Highest Probability above Threshold*
  - *Layer-Level Parallelism*  
*Partial Data-Level Parallelism for Starting Node Collector*
  - *Sparse COO Format with  $O(E) \approx O(V)$  Complexity*
- **Probability-Based Sequential Track Building**
  - *LUT-Based Target-Node Mapping*  
*Initiated from B0 (Blue), E0 (Brown), Others (Green)*
  - *Fine-Grained Node-Level Parallelism*  
*Attaining Constant-Time Complexity  $O(1)$*



Edge Stream					
Source Indices	0	0	1	1	2
Target Indices	1	2	3	4	5
Probability	0.9	0.8	0.9	0.2	0.1

Graph Analysis				Cycle-Based Analysis			
Design	$\pi/4$	$\pi/2$	$\pi$	Design	$\pi/4$	$\pi/2$	$\pi$
# Nodes	113	201	378	Baseline	12,769	40,401	142,884
# Edges	196	334	596	Proposed	119	186	223
Ratio	1.73	1.65	1.57	<b>Speedup</b>	<b>107</b>	<b>217</b>	<b>641</b>

Target Node Index Table					
Source Indices	0	1	2		
Target Indices	1	3	N/A		...
Untargeted Node Index Table					
Node Indices	1	2	3	4	5
Value	F	T	F	T	T

# Overview – Architecture

## • Consecutive-Collision Processing

### • *AXI4-Stream Multi-Point Event Acquisition*

- Graph Construction Engine
- GNN Edge Classification Engine
- Track Building Engine

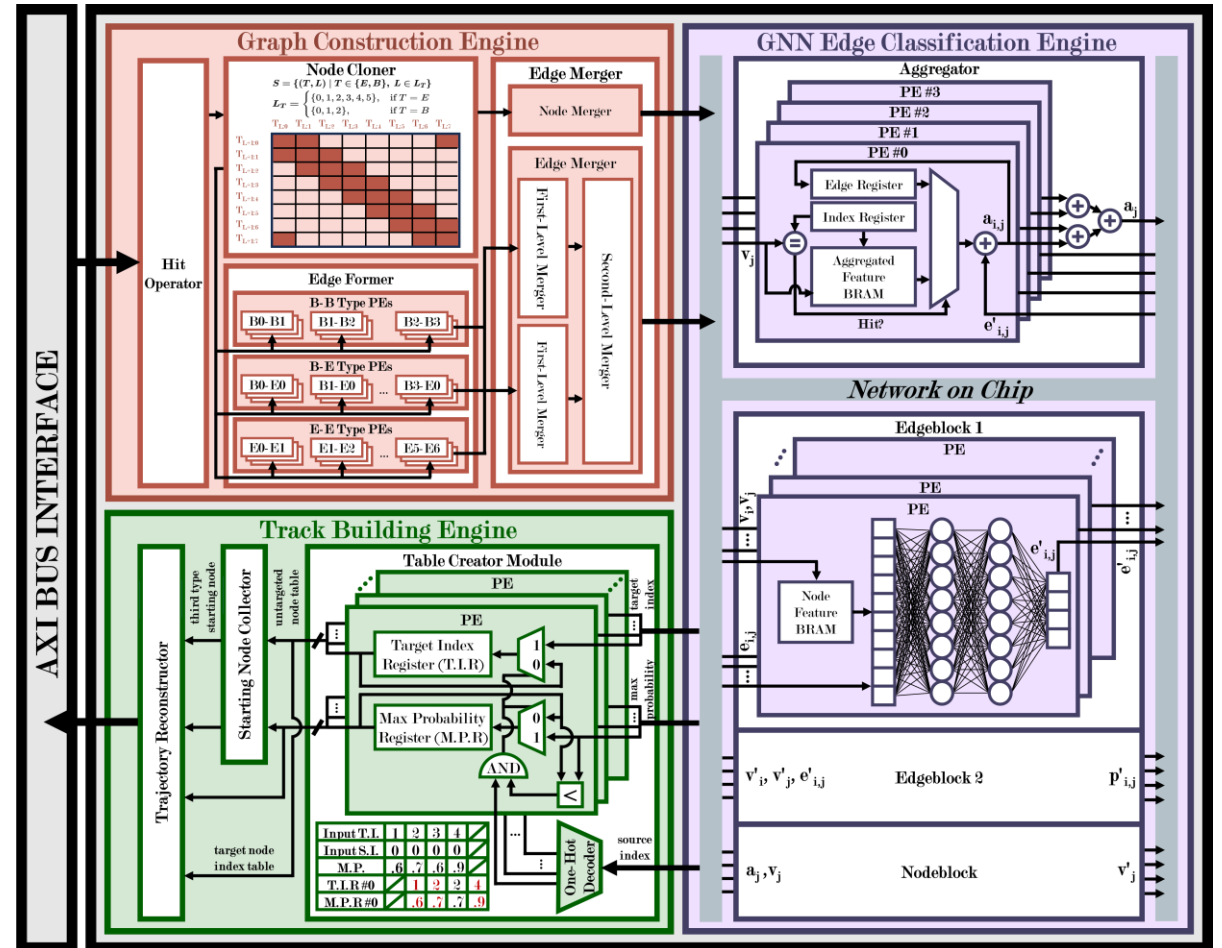
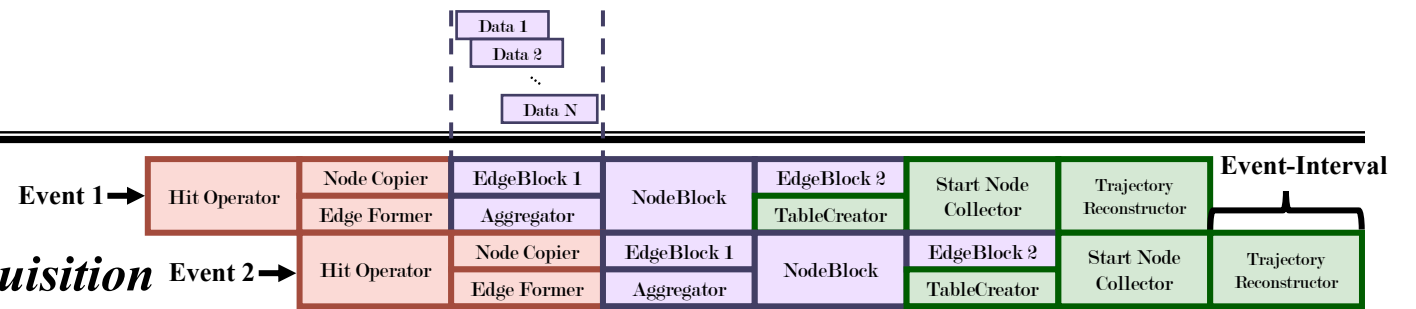
## • Throughput-Tuned Processing Pipeline

### *Support Multi-Level Granularity*

- *Module-Level Execution Pipeline*
  - Event-Interval Balancing
- *Data-Level Processing Pipeline*
  - Cycle-Wise Intra-Module Ingestion
- *Maximized Hardware-Resource Utilization*

## • Latency-Tuned Data Streaming

- *Compact FIFO with Interface Alignment*
  - Reduced Inter-Module Footprint
  - Overlap Execution across Modules





# Edge-Classification Architecture – Proposed

## • Data Streaming–Oriented High-Throughput Paradigm

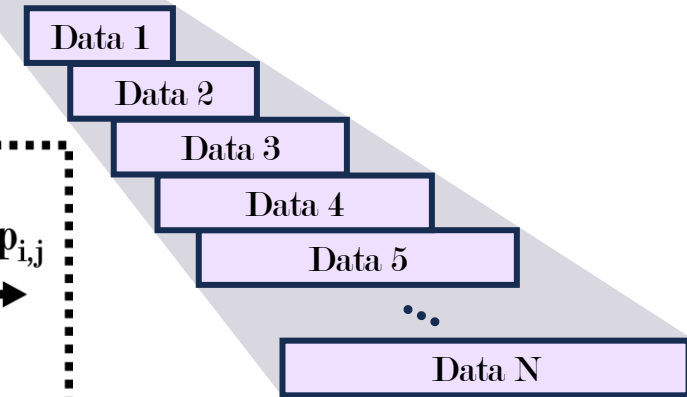
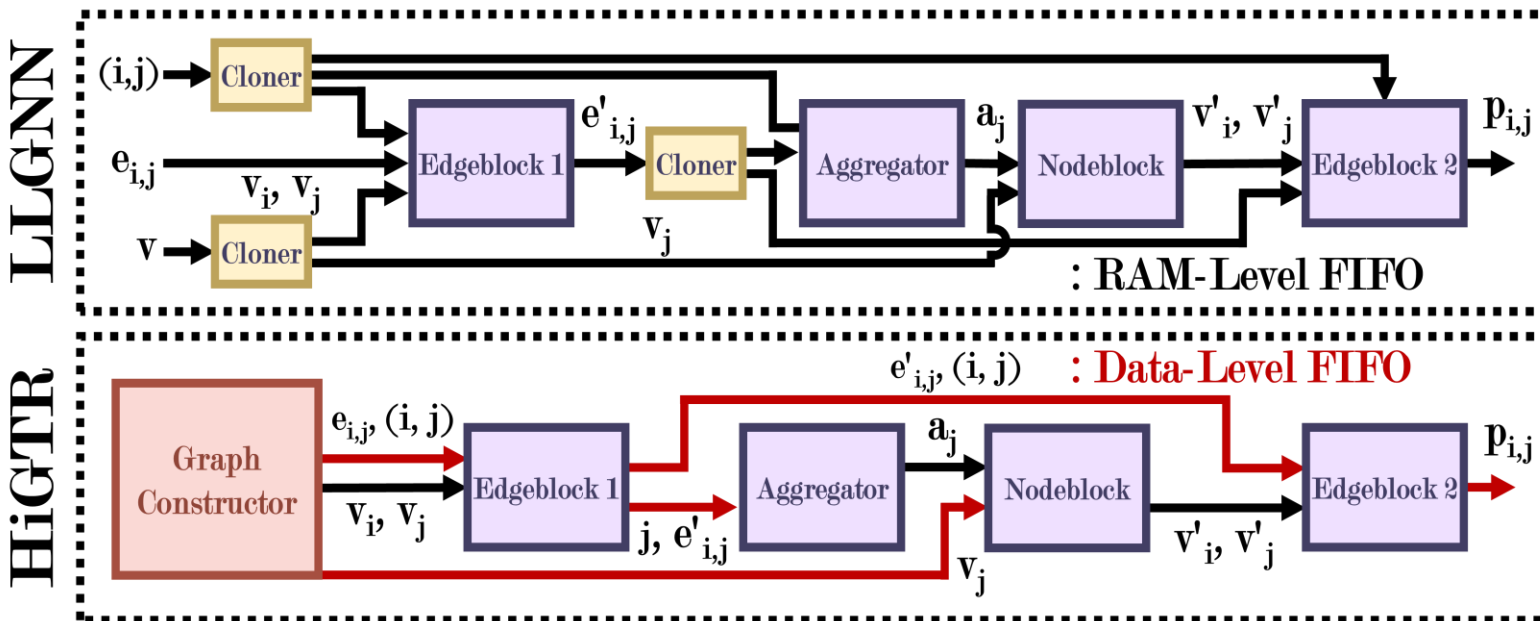
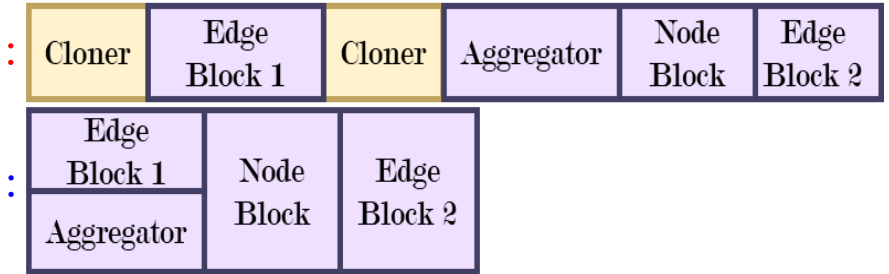
### • *Limitations of Batch Processing*

- Latency Amplification by Downstream Batch Stalls
- High RAM Demand in Batch Data Transfers

### • *Advantages of Data Streaming*

- Early Downstream Processing in Overlapping Pipelines
- Minimal FIFO for Continuous Streams

LLGNN<sup>[10]</sup> :



	LLGNN	HiGTR	Gain
#LUT	161,308	145220	9.8%
#Flip-Flop	128826	125506	2.9%
#BRAM	24	18	25%
Latency	2.86 $\mu$ s	1.365 $\mu$ s	52.3%

# Experiment

- **FPGA Platform: AMD-Xilinx Virtex UltraScale+ VU9P**

*Designated FPGA Platform for L1T Subsystem in HL-LHC*

- **Development Toolkit** – Vitis HLS 2023.2
- **Resource Utilization** – Vivado 2023.2 Post-Place-And-Route Metric
- **Operating Frequency** – 200 MHz
- **Evaluation Dataset** – 1,000 Collision Events from TrackML<sup>[12]</sup>

Final Performance Metrics		
<i>Design</i>	<i>Throughput</i>	<i>Latency</i>
$\pi/4$	2.35 MHz	2.36 $\mu$ s
$\pi/2$	2.24 MHz	2.90 $\mu$ s
$\pi$	1.53 MHz	3.80 $\mu$ s
Target	2.22 MHz	4.00 $\mu$ s

※ Resource overutilization for  $\pi$  design

Final Accuracy Metrics				Comparison Table for $\pi/2$ Design Latency				
<i>Design</i>	<i>Baseline</i> <sup>[7,9]</sup> (Software)	<i>Proposed</i> (Software)	<i>Proposed</i> (Hardware)		<i>Graph Construction</i>	<i>Edge Classification</i>	<i>Track Building</i>	<i>Entire Flow</i>
$\pi/4$	87.31% / 93.04%	90.35% / 97.22%	88.38% / 95.89%	Software <sup>[7,9]</sup>	187 ms	0.58 ms	0.99 ms	188.57 ms
$\pi/2$	86.04% / 91.25%	91.64% / 97.63%	89.57% / 96.92%	Proposed	1.47 $\mu$ s	1.36 $\mu$ s	0.93 $\mu$ s	2.90 $\mu$ s
$\pi$	84.75% / 89.76%	92.34% / 97.94%	90.81% / 96.12%	Speedup	130,769	426	1,065	65,024

※ Perfect match efficiency / double-majority efficiency

[12] M. Kiehn, S. Amrouche, P. Calafiura, V. Estrade, S. Farrell, C. Germain, V. Gligorov, T. Golling, H. Gray, I. Guyon et al., “The trackml high-energy physics tracking challenge on kaggle,” in EPJ Web of Conferences, vol. 214. EDP Sciences, 2019, p. 06037



# Conclusion

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- **First End-to-End GNN-Driven FPGA Accelerator for Trajectory Reconstruction**
  - *Geometry-Aware Edge Pruning in Graph Construction*
    - Edge Count Reduction Achieving **37.5%–71.0%** Pruning [7]
  - *Linear-Time Probability-Driven Sequential Track Building*
    - Latency Reduction Ranging from **107x to 641x** [7]
    - Enhanced Tracking Accuracy Metrics
  - *Data-Streaming-Oriented High-Throughput Paradigm*
    - Latency Reduction Attaining **52.3%** [10]
  - *Consolidated Three-Stage Pipeline within High-Performance FPGA Accelerator*
    - **65024x Acceleration** over Conventional Software-Based Approach [7,9]
- **Performance Alignment Coupled with Accuracy Enhancements**
  - Substantial Potential for Practical Deployment within L1T System
  - Especially Significant for HL-LHC Upgrades

# Reference

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- [1] L. Evans, “The large hadron collider,” *New Journal of Physics*, vol. 9, no. 9, p. 335, 2007.
- [2] “The Phase-2 Upgrade of the CMS Level-1 Trigger,” CERN, Geneva, Tech. Rep., 2020, final version.
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- [12] M. Kiehn, S. Amrouche, P. Calafiura, V. Estrade, S. Farrell, C. Germain, V. Gligorov, T. Golling, H. Gray, I. Guyon et al., “The trackml high-energy physics tracking challenge on kaggle,” in *EPJ Web of Conferences*, vol. 214. EDP Sciences, 2019, p. 06037

# Question and Answer

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