## Markov\_model\_realworld

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#### 2023-04-12

# In this document, I provided line-by-line explanation of the code, 'Markov\_model\_realworld.' by Briggs et al

Overall structure of the code < Base-case analysis> 1. Define variables and parameters for the model 2. Prepare cost, qaly, and transitional cost matrix 3. Prepare transition probability matrix (this will change in simulation given age-specific mortality) 4. Prepare population matrix that keeps track of health state transition by cycle 5. Prepare a matrix ('trans') to keep track of number of people transition from one state to another state (this will be used to calculate transitional cost) 6. Prepare empty matrices to record QALYs and Cost outcomes by cycle and by strategy 7. Define a function to calculate time-dependent transition probability based on age-specific mortality 8. Simulate a cohort for 'no drug' and 'drug' scenario 9. Calculate cost and qaly of each scenario 10. Calculate ICER 11. Plot

1. Make the base-case analysis code as a function so that we can repeat the simulation by the sampled parameter set 2. Random sampling of parameters 3. Define cost and qaly matrices to record outcomes by parameter sample 4. Run PSA 5. Plot

#### summary(cars)

```
##
        speed
                         dist
##
           : 4.0
                           : 2.00
   Min.
                   Min.
   1st Qu.:12.0
                    1st Qu.: 26.00
   Median:15.0
                   Median : 36.00
##
##
    Mean
           :15.4
                   Mean
                           : 42.98
                    3rd Qu.: 56.00
##
    3rd Qu.:19.0
##
   Max.
           :25.0
                    Max.
                           :120.00
```

- 1. Define variables and parameters t names: strategy/scenario label n treatment: number of strategies
- s\_names: vector of health states n\_states: number of health states
- n\_cohort: cohort size cycle: cycle length n\_cylces: total number of cycles Initial\_age: age at the beginning of simulation effect: drug efficacy in decreasing the risk of progressing from asymptomatic to progressive disease

cAsymp: cost of having asymptomatic disease (per-cycle cost) cDeath: cost of death (per-cycle cost) cDrug: cost of drug (one-time cost) cProg: cost of having progressive disease (per-cycle cost) uAsymp: utility of having asymptomatic disease uProg: utility of having progressive disease oDr: discount rate for qaly cDr: discount rate for cost tpDcm: excess mortality with progressive disease tpProg: transition probability from asymptomatic to progressive disease tpDn: baseline mortality <- mortality 0.0379 # over 65 year old

```
t_names <- c("without_drug", "with_drug")
n_treatments <- length(t_names)</pre>
```

```
s_names <- c("Asymptomatic_disease", "Progressive_disease", "Dead")</pre>
n_states <- length(s_names)</pre>
n_cohort <- 1000
cycle <- 1
n_cycles <- 46
Initial age <- 55
effect <- 0.5
cAsymp <- 500
cDeath <- 1000
cDrug <- 1000
cProg <- 3000
uAsymp \leftarrow 0.95
uProg <- 0.75
oDr <- 0.06
cDr <- 0.06
tpDcm <- 0.15
tpProg <- 0.01
tpDn <- 0.0379 # over 65 year old
effect <- 0.5
```

2. Prepare cost, qaly, and transitional cost matrix cost and qaly matrix has the following structure row: strategies column: health states value: cost, qaly, or transitional cost Transitional cost matrix has the following structure row: health states (departing state) column: health states(arriving state) value: transitional cost(toll)

```
## Asymptomatic_disease Progressive_disease Dead
## without_drug 500 3000 0
## with_drug 1500 3000 0
```

3. Prepare transition probability matrix p\_matrix is time-homogeneous transition probability, meaning the elements of matrix would not change depending on cycle # Later in the code, this will be replaced by a time-dependent transition probability matrix when incorporating age-specific mortality

Note: array is useful to create a multidimensional matrix. Here p\_matrix has 3-dimensional matrix with (health states x health states) x strategies

```
##
       = without drug
##
##
## from
                           Asymptomatic_disease Progressive_disease
     Asymptomatic_disease
                                          0.9521
                                                               0.0100 0.0379
##
                                          0.0000
##
     Progressive_disease
                                                               0.8121 0.1879
##
     Dead
                                          0.0000
                                                               0.0000 1.0000
##
##
       = with_drug
##
##
                          to
## from
                           Asymptomatic_disease Progressive_disease
##
     Asymptomatic_disease
                                          0.9571
                                                               0.0050 0.0379
##
     Progressive_disease
                                          0.0000
                                                               0.8121 0.1879
##
     Dead
                                          0.0000
                                                               0.0000 1.0000
```

4. Prepare population matrix that keeps track of health state transition by cycle pop matrix will record health state distribution in the population for each cycle by srategy (multidimensional matrix: n\_states x n\_cycles x n\_treatments)

In cycle = 1, everyone is in the asymptomatic disease state.

```
# Store population output for each cycle
# state populations
pop <- array(data = NA,</pre>
              dim = c(n_states, n_cycles, n_treatments),
              dimnames = list(state = s_names,
                                cycle = NULL,
                                treatment = t_names))
pop["Asymptomatic_disease", cycle = 1, ] <- n_cohort</pre>
pop["Progressive_disease", cycle = 1, ] <- 0</pre>
pop["Dead", cycle = 1, ] \leftarrow 0
head(pop)
   , , treatment = without_drug
##
##
                           cycle
                             [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
## state
##
     Asymptomatic_disease 1000
                                          NA
                                               NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                  NA
                                                                                        NA
                                    NA
                                                     NA
                                                          NA
##
     Progressive_disease
                                0
                                    NA
                                         NA
                                               NA
                                                          NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                  NA
                                                                                        NA
                                                     NΑ
                                0
                                    NA
                                         NA
                                               NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                        NA
##
     Dead
                                                     NA
                                                          NA
                                                                                  NA
                           cycle
##
## state
                                   [,13] [,14] [,15] [,16] [,17]
                                                                    [,18] [,19] [,20]
                            [,12]
##
     Asymptomatic_disease
                               NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NΑ
                                                                                     NΑ
##
     Progressive_disease
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
     Dead
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
                               NA
##
                           cycle
## state
                             [,21]
                                   [,22]
                                          [,23]
                                                [,24]
                                                       [,25]
                                                              [,26]
                                                                     [,27]
                                                                           [,28]
                                                                                  [,29]
##
     Asymptomatic_disease
                               NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
     Progressive_disease
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
     Dead
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
                           cycle
## state
                             [,30] [,31] [,32] [,33] [,34]
                                                              [,35] [,36] [,37] [,38]
##
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
     Asymptomatic_disease
                               NA
##
     Progressive_disease
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
     Dead
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
                                                                                     NA
##
                           cycle
## state
                            [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46]
##
     Asymptomatic_disease
                               NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
##
     Progressive_disease
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
##
     Dead
                                NA
                                      NA
                                             NA
                                                    NA
                                                          NA
                                                                 NA
                                                                        NA
                                                                              NA
##
##
   , , treatment = with_drug
##
##
                           cycle
##
                            [,1]
                                        [,3] [,4] [,5] [,6] [,7]
                                                                   [,8] [,9] [,10] [,11]
                                  [,2]
##
     Asymptomatic_disease 1000
                                    NΑ
                                         NA
                                               NA
                                                     NA
                                                          NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                  NA
                                                                                        MΔ
##
     Progressive_disease
                                0
                                    NA
                                          NA
                                               NA
                                                     NA
                                                          NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                  NA
                                                                                        NA
##
     Dead
                                0
                                    NA
                                         NA
                                               NA
                                                     NA
                                                          NA
                                                                NA
                                                                     NA
                                                                           NA
                                                                                  NA
                                                                                        NA
##
                           cycle
## state
                            [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20]
```

```
##
     Asymptomatic disease
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
                                                                                      NA
##
     Progressive_disease
                                NΑ
                                       NΑ
                                              NΑ
                                                    NΑ
                                                           NΑ
                                                                  NΑ
                                                                         NA
                                                                               NA
                                                                                      NΑ
     Dead
##
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
                                                                                      NA
##
                            cycle
## state
                             [,21]
                                    [,22]
                                          [,23] [,24] [,25]
                                                               [,26]
                                                                     [,27]
                                                                            [,28] [,29]
##
     Asymptomatic disease
                                NA
                                                                  NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                         NA
##
     Progressive disease
                                NA
                                                    NA
                                                                  NA
                                                                         NA
                                                                               NA
                                       NA
                                              NA
                                                           NA
                                                                                      NA
     Dead
                                NA
                                                                               NA
##
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                        NA
                                                                                      NA
##
                            cycle
                             [,30] [,31] [,32] [,33] [,34] [,35] [,36]
                                                                            [,37] [,38]
## state
##
     Asymptomatic_disease
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
##
     Progressive_disease
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
                                                                                      NA
##
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
                                                                                      NΑ
##
                            cycle
## state
                             [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46]
##
     Asymptomatic_disease
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
##
     Progressive_disease
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                         NA
                                                                               NA
                                NA
                                                                  NA
##
     Dead
                                NA
                                       NA
                                              NA
                                                    NA
                                                           NA
                                                                  NA
                                                                         NA
                                                                               NA
```

5. Prepare a matrix ('trans') to record total transitional cost per cycle by state 'pop' matrix records total number of people in each health state, whereas trans records the number of people who 'enter' the state

dimension:  $n_{states} \times n_{cycles} \times n_{treatments}$  (here  $n_{states}$  indicate 'arrived state') So given treatment strategy and in a specific cycle, trans["asympomatic"] indicates the total "toll" or "transitional costs" to arrive the asymptomatic state by the end of the cycle

6. Prepare empty matrices to record QALYs and Cost outcomes by cycle and by strategy by cycle: cycle\_costs, cycle\_QALYs, cycle\_QALE, LE, LYs (dimension: n\_treatments, cycles) total: total\_costs, total\_QALYs (1xn\_treatments)

```
total_costs <- setNames(c(NA, NA), t_names)
total_QALYs <- setNames(c(NA, NA), t_names)</pre>
```

7. Define a function to calculate time-dependent transition probability based on age-specific mortality Because non-mortality transition probabilities are dependent on mortality, if mortality is age-dependent (or time-dependent), then other probabilities will be time-dependent as well

```
# Time-dependent probability matrix ----
p_matrix_cycle <- function(p_matrix, age, cycle,</pre>
                            tpProg = 0.01,
                            tpDcm = 0.15,
                            effect = 0.5) {
  tpDn_lookup <-
    c("(34,44]" = 0.0017,
      "(44,54]" = 0.0044,
      "(54,64]" = 0.0138,
      "(64,74]" = 0.0379,
      "(74,84]" = 0.0912,
      "(84,100]" = 0.1958)
  age\_grp \leftarrow cut(age, breaks = c(34,44,54,64,74,84,100)) # find the age group that this age falls into
  tpDn <- tpDn_lookup[age_grp]</pre>
  # Matrix containing transition probabilities for without_drug
  p_matrix["Asymptomatic_disease", "Progressive_disease", "without_drug"] <- tpProg*cycle</pre>
  p_matrix["Asymptomatic_disease", "Dead", "without_drug"] <- tpDn</pre>
  p_matrix["Asymptomatic_disease", "Asymptomatic_disease", "without_drug"] <- 1 - tpProg*cycle - tpDn
  p_matrix["Progressive_disease", "Dead", "without_drug"] <- tpDcm + tpDn</pre>
  p_matrix["Progressive_disease", "Progressive_disease", "without_drug"] <- 1 - tpDcm - tpDn
  p_matrix["Dead", "Dead", "without_drug"] <- 1</pre>
  # Matrix containing transition probabilities for with drug
  p_matrix["Asymptomatic_disease", "Progressive_disease", "with_drug"] <- tpProg*(1 - effect)*cycle
  p_matrix["Asymptomatic_disease", "Dead", "with_drug"] <- tpDn</pre>
  p_matrix["Asymptomatic_disease", "Asymptomatic_disease", "with_drug"] <-</pre>
    1 - tpProg*(1 - effect)*cycle - tpDn
  p_matrix["Progressive_disease", "Dead", "with_drug"] <- tpDcm + tpDn</pre>
  p_matrix["Progressive_disease", "Progressive_disease", "with_drug"] <- 1 - tpDcm - tpDn</pre>
  p_matrix["Dead", "Dead", "with_drug"] <- 1</pre>
  return(p_matrix)
}
```

- 8. Simulate a cohort for 'no drug' and 'drug' scenario two for loops are implemented in this block
- (1) given strategy
- (2) given cycle #
- calculate age-specific transition probability matrix
- update pop matrix in the current cycle given pop matrix in last cycle and transition probability matrix
- calculate trans matrix to count the number of people move from one to another state within a cycle

A \* B: element-wise multiplication A %\*% B: matrix multiplication (take the row of A, column of B and sum the product )

To calculate the transitional cost per cycle to arrive in each state, For example, the transitional cost of arriving in "progressive disease" is N\_1C\_12P\_12 + N\_2C\_22P\_22+N+3C\_32P\_32 = c(N\_1, N\_2, N\_3) %%  $t(c(C_12P_12, C_22P_22, C_32P_32)) = c(N_1, N_2, N_3)$  %%  $t(c(C_12P_12, C_22, C_32))$  c(P\_12,P\_22,P\_32))

trans\_c\_matrix \* p\_matrix -> transition probability-weighted transitional cost c(1,1,1) %\*% column vector() -> sum of elements in the column vector

```
## Run model ----
for (i in 1:n treatments) {
  age <- Initial age
  for (j in 2:n_cycles) {
    p_matrix <- p_matrix_cycle(p_matrix, age, j - 1)</pre>
    pop[, cycle = j, treatment = i] <-</pre>
      pop[, cycle = j - 1, treatment = i] %*% p_matrix[, , treatment = i]
    trans[, cycle = j, treatment = i] <-</pre>
      pop[, cycle = j - 1, treatment = i] %*% (trans_c_matrix * p_matrix[, , treatment = i])
    age <- age + 1
  cycle_state_costs[i, ] <-</pre>
    (state_c_matrix[treatment = i, ] %*% pop[, , treatment = i]) * 1/(1 + cDr)^(1:n_cycles - 1)
  # discounting at _previous_ cycle
  cycle trans costs[i, ] <-</pre>
    (c(1,1,1) \% *\% trans[, , treatment = i]) * 1/(1 + cDr)^(1:n_cycles - 2)
  cycle_costs[i, ] <- cycle_state_costs[i, ] + cycle_trans_costs[i, ]</pre>
  # life expectancy
  LE[i, ] \leftarrow c(1,1,0) \% \% pop[, , treatment = i]
  # life-years
  LYs[i, ] \leftarrow LE[i, ] * 1/(1 + oDr)^(1:n_cycles - 1)
  # quality-adjusted life expectancy
  cycle_QALE[i, ] <-</pre>
    state_q_matrix[treatment = i, ] %*% pop[, , treatment = i]
  # quality-adjusted life-years
  cycle_QALYs[i, ] <- cycle_QALE[i, ] * 1/(1 + oDr)^(1:n_cycles - 1)</pre>
  total_costs[i] <- sum(cycle_costs[treatment = i, -1])</pre>
  total QALYs[i] <- sum(cycle QALYs[treatment = i, -1])
}
```

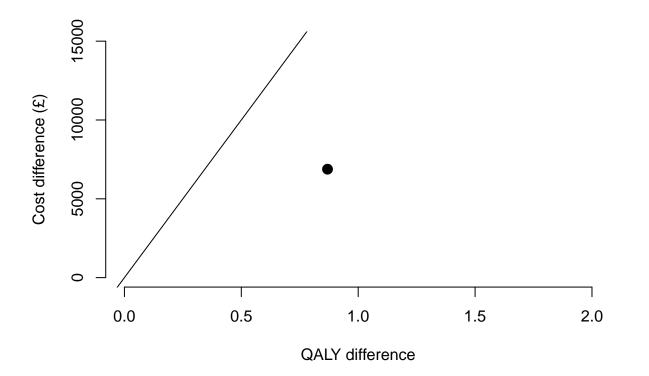
9. Calculate cost and galy of each scenario

```
## Plot results ----
# Incremental costs and QALYs of with_drug vs to without_drug
c_incr <- total_costs["with_drug"] - total_costs["without_drug"]
q_incr <- total_QALYs["with_drug"] - total_QALYs["without_drug"]</pre>
```

10. Calculate ICER

```
# Incremental cost-effectiveness ratio
ICER <- c_incr/q_incr</pre>
```

11. Plot



PSA code 1. Turn the base-case analysis code into a function so that we can repeat the simulation by the sampled parameter set

##

```
cycle_trans_costs[i, ] <-
    (c(1,1,1) %*% trans[, , treatment = i]) * 1/(1 + cDr)^(1:n_cycles - 2)

cycle_costs[i, ] <- cycle_state_costs[i, ] + cycle_trans_costs[i, ]

# life expectancy
LE[i, ] <- c(1,1,0) %*% pop[, , treatment = i]

# life-years
LYs[i, ] <- LE[i, ] * 1/(1 + oDr)^(1:n_cycles - 1)

# quality-adjusted life expectancy
cycle_QALE[i, ] <-
    state_q_matrix[treatment = i, ] %*% pop[, , treatment = i]

# quality-adjusted life-years
cycle_QALYs[i, ] <- cycle_QALE[i, ] * 1/(1 + oDr)^(1:n_cycles - 1)

total_costs[i] <- sum(cycle_costs[treatment = i, -1])

total_QALYs[i] <- sum(cycle_QALYs[treatment = i, -1])</pre>
```

```
# Probability Sensitivity Analysis (PSA)
ce_markov <- function(start_pop,</pre>
                     p_matrix,
                     state_c_matrix,
                     trans_c_matrix,
                     state_q_matrix,
                     n_{cycles} = 46,
                     init_age = 55,
                     s_names = NULL,
                     t_names = NULL) {
 n_states <- length(start_pop)</pre>
 n_treat <- dim(p_matrix)[3]</pre>
 pop <- array(data = NA,
              dim = c(n_states, n_cycles, n_treat),
              dimnames = list(state = s_names,
                              cycle = NULL,
                              treatment = t_names))
 trans <- array(data = NA,
                dim = c(n_states, n_cycles, n_treat),
                dimnames = list(state = s_names,
                                cycle = NULL,
                                treatment = t_names))
 for (i in 1:n_states) {
   pop[i, cycle = 1, ] <- start_pop[i]</pre>
```

```
}
cycle_empty_array <-</pre>
  array(NA,
        dim = c(n_treat, n_cycles),
        dimnames = list(treatment = t_names,
                          cycle = NULL))
cycle_state_costs <- cycle_trans_costs <- cycle_empty_array</pre>
cycle_costs <- cycle_QALYs <- cycle_empty_array</pre>
LE <- LYs <- cycle_empty_array # life expectancy; life-years</pre>
cycle_QALE <- cycle_empty_array # quality-adjusted life expectancy</pre>
total_costs <- setNames(rep(NA, n_treat), t_names)</pre>
total_QALYs <- setNames(rep(NA, n_treat), t_names)</pre>
for (i in 1:n_treat) {
  age <- init_age
  for (j in 2:n_cycles) {
    # difference from point estimate case
    # pass in functions for random sample
    # rather than fixed values
    p_matrix <- p_matrix_cycle(p_matrix, age, j - 1,</pre>
                                 tpProg = tpProg(),
                                  tpDcm = tpDcm(),
                                 effect = effect())
    # Matrix multiplication
    pop[, cycle = j, treatment = i] <-</pre>
      pop[, cycle = j - 1, treatment = i] %*% p_matrix[, , treatment = i]
    trans[, cycle = j, treatment = i] <-</pre>
      pop[, cycle = j - 1, treatment = i] %*% (trans_c_matrix * p_matrix[, , treatment = i])
    age <- age + 1
  }
  cycle_state_costs[i, ] <-</pre>
    (state_c_matrix[treatment = i, ] %*% pop[, , treatment = i]) * 1/(1 + cDr)^(1:n_cycles - 1)
  cycle_trans_costs[i, ] <-</pre>
    (c(1,1,1) \% *\% trans[, , treatment = i]) * 1/(1 + cDr)^(1:n_cycles - 2)
  cycle_costs[i, ] <- cycle_state_costs[i, ] + cycle_trans_costs[i, ]</pre>
  LE[i, ] \leftarrow c(1,1,0) \% *\% pop[, , treatment = i]
  LYs[i, ] \leftarrow LE[i, ] * 1/(1 + oDr)^(1:n_cycles - 1)
  cycle_QALE[i, ] <-</pre>
```

2. Random sampling of parameters

```
# replace point values with functions to random sample

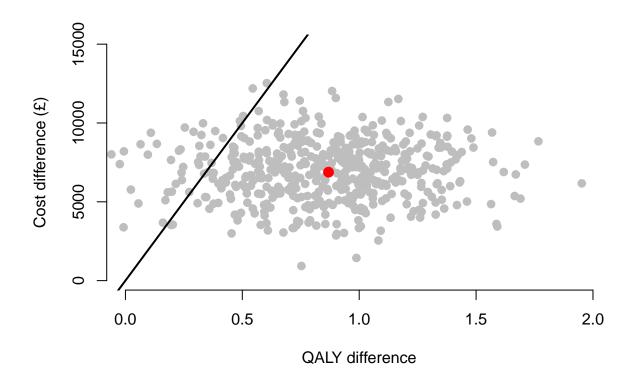
cAsymp <- function() rnorm(1, 500, 127.55)
cDeath <- function() rnorm(1, 1000, 255.11)
cDrug <- function() rnorm(1, 1000, 102.04)
cProg <- function() rnorm(1, 3000, 510.21)
effect <- function() rnorm(1, 0.5, 0.051)
tpDcm <- function() rbeta(1, 29, 167)
tpProg <- function() rbeta(1, 15, 1506)
uAsymp <- function() rbeta(1, 69, 4)
uProg <- function() rbeta(1, 24, 8)</pre>
```

3. Define cost and qaly matrices to record outcomes by parameter sample

```
# Define cost and QALYs as functions
state c matrix <- function() {</pre>
 matrix(c(cAsymp(), cProg(), 0,
                                          # without drug
           cAsymp() + cDrug(), cProg(), 0), # with drug
           byrow = TRUE,
           nrow = n_treatments,
           dimnames = list(t_names,
                            s names))
}
state_q_matrix <- function() {</pre>
 matrix(c(uAsymp(), uProg(), 0, # without drug
           uAsymp(), uProg(), 0), # with drug
         byrow = TRUE,
         nrow = n_treatments,
         dimnames = list(t_names,
                          s names))
}
trans_c_matrix <- function() {</pre>
```

4. Run PSA

5. Plot



## pdf ## 2