

MULTIPLE CORRESPONDANCE ANALYSIS - Assignment 1

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The data

In this exercise set we use **Finnish** sample from ISSP 2012 survey “Family and Changing Gender Roles IV”. Original data involve 1171 observations of 8 variables (4 substantive and 4 demographic). All variables are categorical.

The 4 **substantive** variables, which values are masured in 1-5 scale, are:

A: Married people are generally happier than unmarried people. **B:** People who want children ought to get married. **C:** It is all right for a couple to live together without intending to get married. **D:** Divorce is usually the best solution when a couple can't seem to work out their marriage problems.

The **demographic** variables are: **g:** gender (1=male, 2=female) **a:** age group (1=16-25, 2=26-35, 3=36-45, 4=46-55, 5=56-65, 6= 66+) **e:** education (1=Primary, 2=Comprehensive, primary and lower secondary, 3= Post-comprehensive, vocational school or course, 4=General upper secondary education or certificate, 5= Vocational post-secondary non-tertiary education, 6=Polytechnics , 7= University, lower academic degree, BA, 8=University, higher academic degree, MA **p:** Living in steady partnership (1=Yes, have partner; live in same household, 2=Yes, have partner; don't live in same household, 3=No partner)

The data wrangling includes the following changes:

1. The missing data ae removed (this has already been provided). The number of observations without missing data is N=924.
2. For Task 2 a combined variable **ga**, describing the intraction of gender and age categories, is formed as:
$$**ga = 6*(g-1) + a**$$

Graphical overview of the data and summaries of the variables

The preliminary treated data look as:

```
Finland <- read.table("Finland.txt")
head(Finland)

##   A B C D g a e p
## 1 3 3 1 2 1 2 4 3
## 2 3 2 3 2 1 4 2 3
## 3 3 3 1 3 1 3 8 1
## 4 3 2 2 3 2 2 6 1
## 5 2 2 2 3 2 4 5 1
## 6 3 3 1 3 1 3 7 3

dim(Finland)

## [1] 924   8

str(Finland)

## 'data.frame':   924 obs. of  8 variables:
##  $ A: int   3 3 3 3 2 3 3 2 2 3 ...
##  $ B: int   3 2 3 2 2 3 3 3 3 3 ...
##  $ C: int   1 3 1 2 2 1 1 1 2 3 ...
##  $ D: int   2 2 3 3 3 3 2 2 3 3 ...
```

```
## $ g: int 1 1 1 2 2 1 2 2 2 2 ...
## $ a: int 2 4 3 2 4 3 1 2 5 4 ...
## $ e: int 4 2 8 6 5 7 4 8 7 5 ...
## $ p: int 3 3 1 1 1 3 3 3 1 3 ...
```

Task 1: Cross-tabulations and correspondance analysis (CA).

The goal is to discover interesting relations among the measurements on A, \dots, D, g, \dots, p . Correspondence analysis is a descriptive/exploratory technique designed to analyse simple two-way and multi-way tables containing some measure of correspondence between the rows and columns. CA may also be defined as a special case of Principal Components Analysis of the rows and columns of a table, especially applicable to a cross-tabulation. Principal components analysis is used for tables consisting of continuous measurement, whereas CA is applied to contingency tables (i.e. cross-tabulations). The goal of CA is to transform a table of numerical information into a graphical display, in which each row and each column is depicted as a point. Thus CA can be used for data visualization or data pre-processing before applying methods for supervised learning. We can apply this technique for dimensionality reduction.

First we provide a cross-tabulation between the age variable **a** and answer **A**: Married people are generally happier than unmarried people. The variable **a** consists of 6 age groups, corresponding to rownames of crosstable: **a1** = 16-25, **a2**=26-35, **a3** = 36-45, **a4** = 46-55, **a5** = 56-65, **a6** = 66+ The categorical variable **A** assume 5 possible values, which correspond to column names of cross-table: **SA** - strongly agree; **A** - agree, **NN** - neither agree or disagree, **D** - disagree, **SD** - strongly disagree.

```
tab <-table(Finland[, "a"], Finland[, "A"])

rownames(tab) <- c("a1", "a2", "a3", "a4", "a5", "a6")
colnames(tab) <- c("SA", "A", "NN", "D", "SD")
```

```
tab
```

```
##
##      SA  A NN  D SD
## a1   5 18 42 29 29
## a2   6 25 44 34 34
## a3   6 30 55 30 17
## a4   9 37 60 46 34
## a5  15 67 58 49 15
## a6   5 37 56 26  6
```

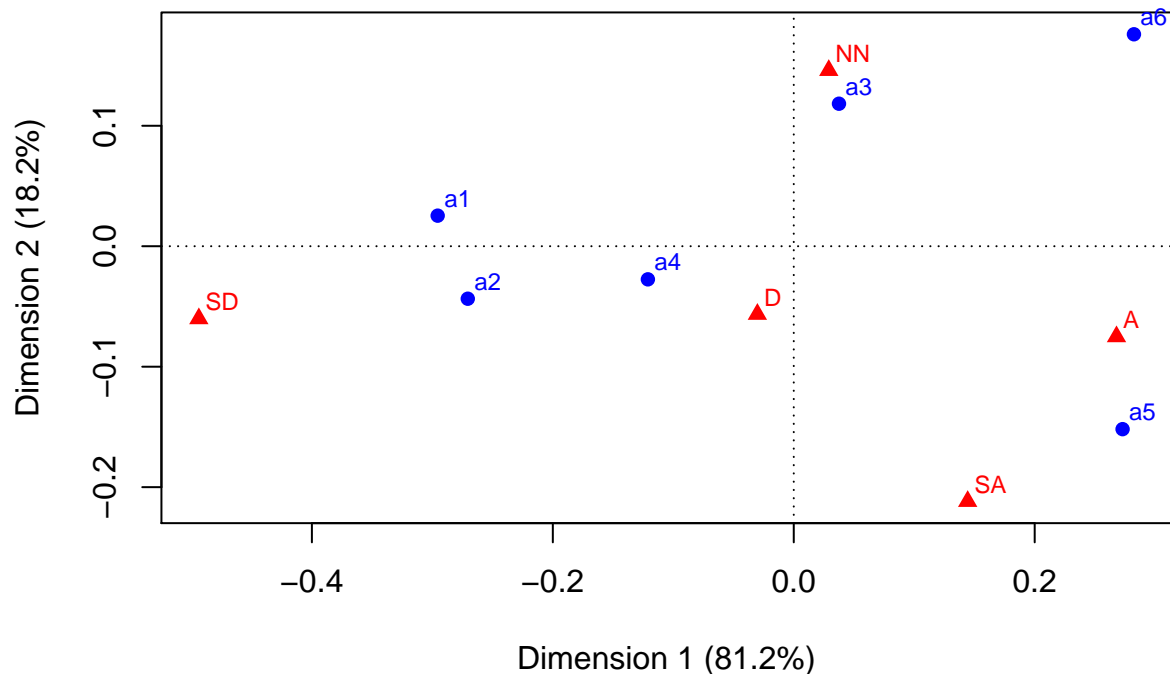
From this table we see that the biggest part of age group **a5** agree with question **A**. Since these values are absolute, we need more precise information which could be obtained row and column profiles, calculated from this cross-table. We need also to calculate the chi-square statistic as it has been shown during the second exercise session.

An alternative means of extracting the nature of the dependency between the rows and columns of the contingency table is to represent the row or column profiles graphically.

```
require(ca)
```

```
## Loading required package: ca
```

```
plot(ca(tab))
```



This plot is assymmetric because the row profiles are plotted simultaneously with apexes representing the columns. **a3** is strongly “associated” with answer **NN**, which is clearly the case from the profile presented in the cross-table. Likewise we observe some proximity of **a5** to the apex representing **A** answer to question **A**. The fact that **a1**, **a2** and **a4** are positioned relatively far away from answers **SA** and **A** means that these younger people rather disagree then agree with the statement **A**. The age group **a6** is far from all answers of **A**, which probably means that people from this age group do not care so much about statement **A**.

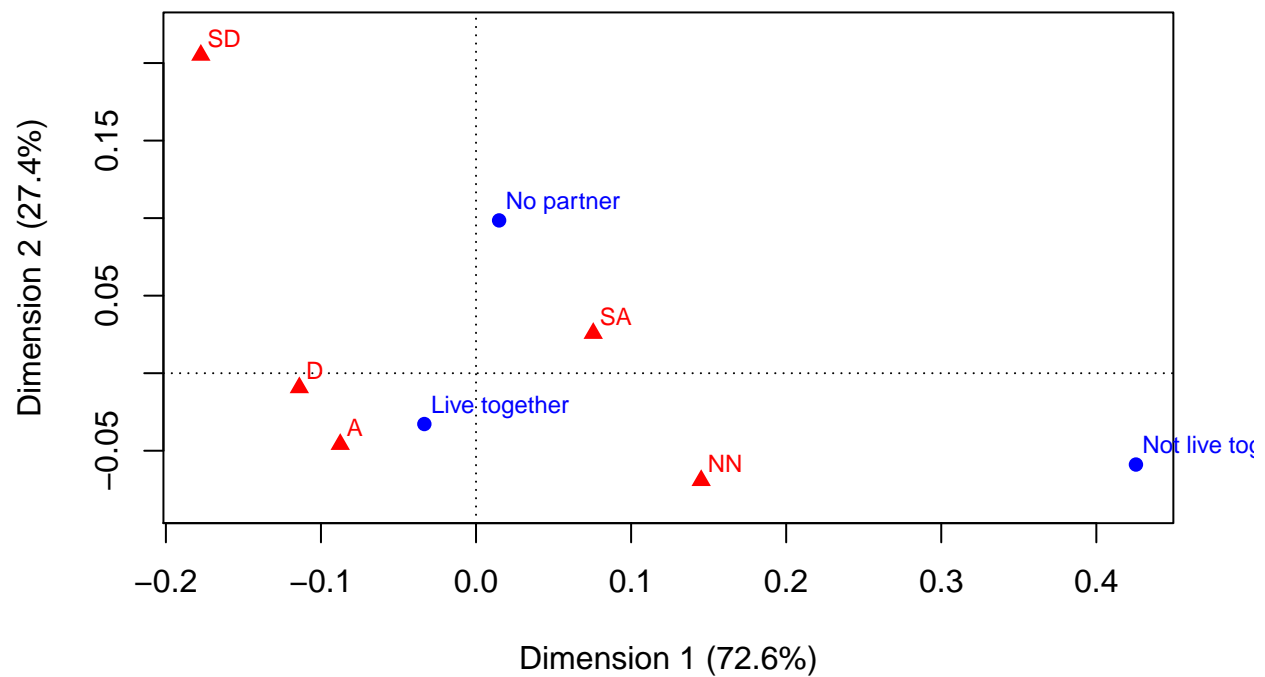
As a second example we conduct cross-tabulation between the variable **p** (if there is a partner and they live together) and answer ****C***: **C**: It is all right for a couple to live together without intending to get married. The obtained cross-table and resulting plot from **CA** are:

```
#Cross-tabulations
#between g and D
tab2 <-table(Finland[, "p"], Finland[, "C"])

rownames(tab2) <- c("Live together", "Not live together", "No partner")
colnames(tab2) <- c("SA", "A", "NN", "D", "SD")
tab2

##
##           SA    A  NN   D  SD
## Live together 291 247  44  32  29
## Not live together  26  10   5   1   0
## No partner    119  79  15  11  15

require(ca)
plot(ca(tab2))
```



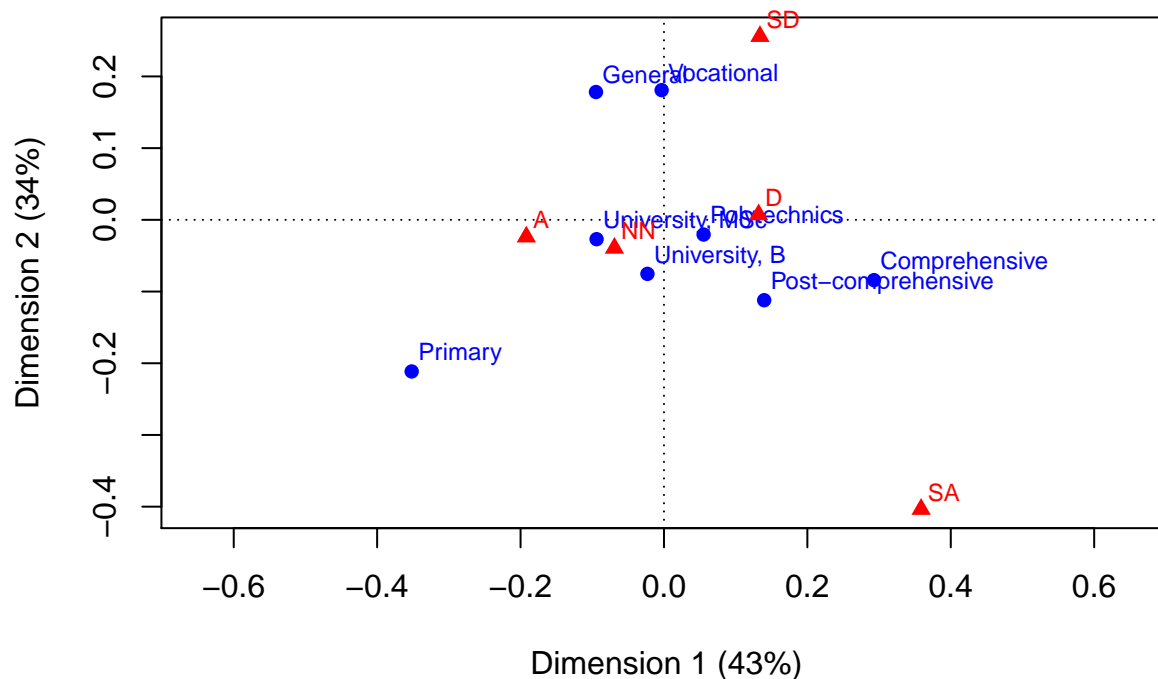
From the cross-tabulation and CA plot becomes clear that people, who live together, tend to give positive answer to the statement **C**.

As a third example we investigate relationship between education **e** and statement **A**:

```
tab1 <- table(Finland[, "e"], Finland[, "A"])
rownames(tab1) <- c("Primary", "Comprehensive", "Post-comprehensive", "General", "Vocational", "Polytechnics", "University, B", "University, MSc")
colnames(tab1) <- c("SA", "A", "NN", "D", "SD")
tab1
```

```
##
##           SA  A  NN  D  SD
## Primary      3 28 34 14  3
## Comprehensive  6  9 18 16 10
## Post-comprehensive 16 37 71 54 26
## General       1 20 39 18 18
## Vocational     4 49 63 53 39
## Polytechnics   7 25 28 22 16
## University, B  2 11 23 16  5
## University, MSc 7 35 39 21 18
```

```
require(ca)
plot(ca(tab1))
```



From this plot becomes clear that no one of the educational groups strongly agrees with the statement **A**. The highly educated people with university degrees tend to answer using middle alternative.

Task 2: Cross-tabulation of ga demographic variable against one of the substantive variables. Performing CA.

The variable **ga**, which is the interaction of gender and age categories, is created as:

```
Finland$ga <- 6*(Finland$g-1) + Finland$a
head(Finland)
```

```
##   A B C D g a e p ga
## 1 3 3 1 2 1 2 4 3  2
## 2 3 2 3 2 1 4 2 3  4
## 3 3 3 1 3 1 3 8 1  3
## 4 3 2 2 3 2 2 6 1  8
## 5 2 2 2 3 2 4 5 1 10
## 6 3 3 1 3 1 3 7 3  3
```

```
str(Finland)
```

```
## 'data.frame':   924 obs. of  9 variables:
##  $ A : int  3 3 3 3 2 3 3 2 2 3 ...
##  $ B : int  3 2 3 2 2 3 3 3 3 3 ...
##  $ C : int  1 3 1 2 2 1 1 1 2 3 ...
##  $ D : int  2 2 3 3 3 3 2 2 3 3 ...
##  $ g : int  1 1 1 2 2 1 2 2 2 2 ...
```

```
## $ a : int  2 4 3 2 4 3 1 2 5 4 ...
## $ e : int  4 2 8 6 5 7 4 8 7 5 ...
## $ p : int  3 3 1 1 1 3 3 3 1 3 ...
## $ ga: num  2 4 3 8 10 3 7 8 11 10 ...
```

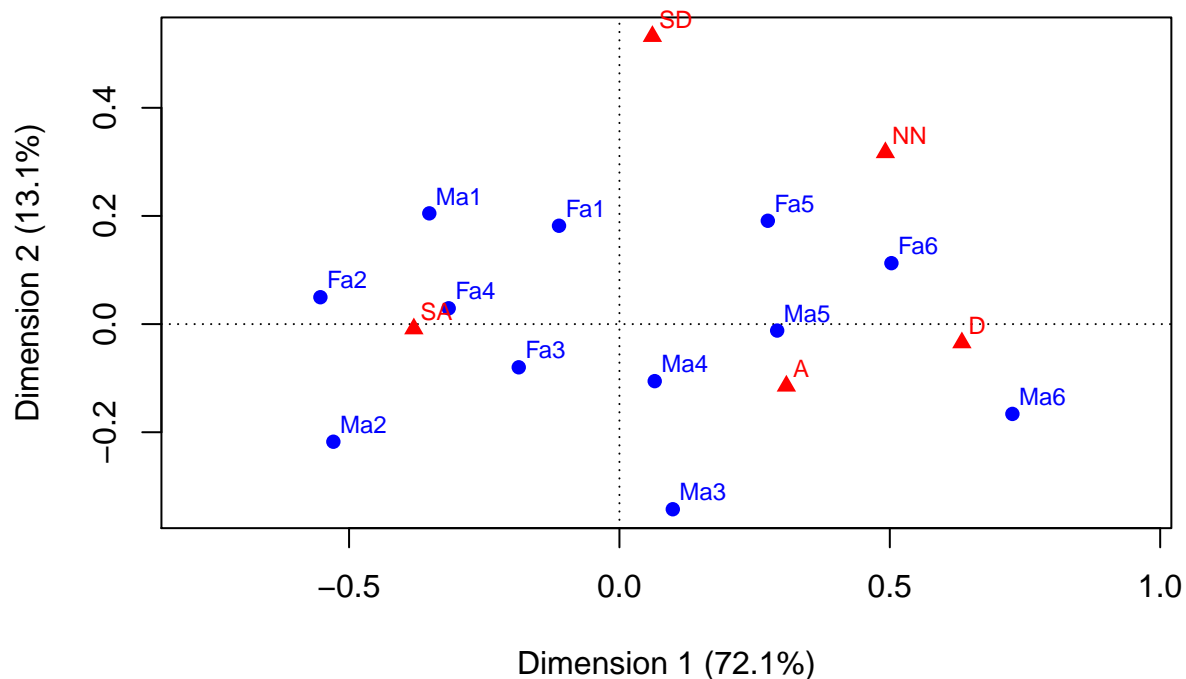
The cross-tabulation between **ga** and **A** is computed as:

```
tab3 <-table(Finland[,"ga"], Finland[,"C"])
rownames(tab3) <- c("Ma1","Ma2", "Ma3", "Ma4", "Ma5", "Ma6", "Fa1","Fa2", "Fa3", "Fa4", "Fa5", "Fa6")
colnames(tab3) <- c("SA","A","NN","D","SD")
tab3
```

```
##
##      SA  A NN  D SD
##  Ma1 30 11  3  0  4
##  Ma2 46 15  0  1  0
##  Ma3 26 31  0  4  1
##  Ma4 38 38  7  2  2
##  Ma5 29 36  9  7  3
##  Ma6  8 37  6  8  2
##  Fa1 38 21  1  6  9
##  Fa2 59 16  1  0  5
##  Fa3 43 25  4  2  2
##  Fa4 63 24  7  2  3
##  Fa5 42 45 18  8  7
##  Fa6 14 37  8  4  6
```

The CA between **ga** and **A**:

```
require(ca)
plot(ca(tab3))
```



From this plot becomes clear that female of age group 4 most often strongly agree with the statement **A**, while males in age group 5 most often agree with **A**.

Task 3: Computation of the Burt matrix on the questions and demographics together.

The most classical and standard approach to MCA is to apply a simple CA to the indicator matrix Z . The indicator matrix $Z = \{z_{ij}\}$ corresponds to a binary coding of the factors - instead of using a factor with J_q levels one uses J_q columns containing binary values, also called dummy variables. The Burt matrix C is obtained directly from the indicator matrix Z : $C = ZTZ$. Burt matrix concatenates all two-way cross-tabulations between pairs of variables

The Burt matrix is computed as:

```
require(ca)
Finland.B <- mjca(Finland)$Burt
head(Finland.B)
```

```
##      A:1 A:2 A:3 A:4 A:5 B:1 B:2 B:3 B:4 B:5 C:1 C:2 C:3 C:4 C:5 D:1 D:2
## A:1  46   0   0   0   0  28  12   4   2   0  15  13   6   6   6  13  15
## A:2   0 214   0   0   0  50  92  38  25   9  64 101  17  15  17  28  91
## A:3   0   0 315   0   0  24  92  95  83  21 145 120  30  14   6  55 115
## A:4   0   0   0 214   0  15  42  30  97  30 104  88  10   9   3  41  84
## A:5   0   0   0   0 135   9  10   6  24  86 108  14   1   0  12  40  43
## B:1  28  50  24  15   9 126   0   0   0   0  26  34  17  15  34  29  24
##      D:3 D:4 D:5 g:1 g:2 a:1 a:2 a:3 a:4 a:5 a:6 e:1 e:2 e:3 e:4 e:5 e:6
```

```

## A:1  5  7  6 28 18  5  6  6  9 15  5  3  6 16  1  4  7
## A:2 46 36 13 113 101 18 25 30 37 67 37 28 9 37 20 49 25
## A:3 88 43 14 136 179 42 44 55 60 58 56 34 18 71 39 63 28
## A:4 41 43  5  85 129 29 34 30 46 49 26 14 16 54 18 53 22
## A:5 27 12 13 42 93 29 34 17 34 15  6  3 10 26 18 39 16
## B:1 20 29 24 59 67 15  8 16 16 41 30 15 10 27 12 32  7
##      e:7 e:8 p:1 p:2 p:3 ga:1 ga:2 ga:3 ga:4 ga:5 ga:6 ga:7 ga:8 ga:9 ga:10
## A:1  2  7 35  1 10  3  3  4  6 10  2  2  3  2  3
## A:2 11 35 171  8 35 10 12 16 22 29 24  8 13 14 15
## A:3 23 39 224 12 79 16 21 21 32 21 25 26 23 34 28
## A:4 16 21 141 14 59  8 14 13 20 20 10 21 20 17 26
## A:5  5 18 72  7 56 11 12  8  7  4  0 18 22  9 27
## B:1  4 19 97  2 27  7  3  8  8 19 14  8  5  8  8
##      ga:11 ga:12
## A:1  5  3
## A:2 38 13
## A:3 37 31
## A:4 29 16
## A:5 11  6
## B:1 22 16

```

summary(Finland.B)

```

##      A:1      A:2      A:3      A:4
## Min.   : 0.000  Min.   : 0.00  Min.   : 0.00  Min.   : 0.00
## 1st Qu.: 3.000  1st Qu.: 13.00  1st Qu.: 21.00  1st Qu.: 14.00
## Median : 6.000  Median : 25.00  Median : 34.00  Median : 22.00
## Mean   : 8.118  Mean   : 37.76  Mean   : 55.59  Mean   : 37.76
## 3rd Qu.:10.000  3rd Qu.: 38.00  3rd Qu.: 67.00  3rd Qu.: 44.50
## Max.   :46.000  Max.   :214.00  Max.   :315.00  Max.   :214.00
##      A:5      B:1      B:2      B:3
## Min.   : 0.00  Min.   : 0.00  Min.   : 0.00  Min.   : 0.00
## 1st Qu.: 6.50  1st Qu.: 8.00  1st Qu.: 14.00  1st Qu.: 10.00
## Median : 13.00  Median : 16.00  Median : 27.00  Median : 20.00
## Mean   : 23.82  Mean   : 22.24  Mean   : 43.76  Mean   : 30.53
## 3rd Qu.: 28.00  3rd Qu.: 27.50  3rd Qu.: 52.00  3rd Qu.: 37.50
## Max.   :135.00  Max.   :126.00  Max.   :248.00  Max.   :173.00
##      B:4      B:5      C:1      C:2
## Min.   : 0.00  Min.   : 0.00  Min.   : 0.00  Min.   : 0.00
## 1st Qu.: 13.50  1st Qu.: 5.00  1st Qu.: 26.00  1st Qu.: 19.50
## Median : 25.00  Median : 16.00  Median : 59.00  Median : 36.00
## Mean   : 40.76  Mean   : 25.76  Mean   : 76.94  Mean   : 59.29
## 3rd Qu.: 50.50  3rd Qu.: 31.50  3rd Qu.:102.50  3rd Qu.: 77.00
## Max.   :231.00  Max.   :146.00  Max.   :436.00  Max.   :336.00
##      C:3      C:4      C:5      D:1
## Min.   : 0.00  Min.   : 0.000  Min.   : 0.000  Min.   : 0.00
## 1st Qu.: 3.00  1st Qu.: 2.000  1st Qu.: 2.000  1st Qu.: 12.00
## Median : 7.00  Median : 5.000  Median : 5.000  Median : 22.00
## Mean   :11.29  Mean   : 7.765  Mean   : 7.765  Mean   : 31.24
## 3rd Qu.:17.00  3rd Qu.:12.000  3rd Qu.: 9.500  3rd Qu.: 38.00
## Max.   :64.00  Max.   :44.000  Max.   :44.000  Max.   :177.00
##      D:2      D:3      D:4      D:5
## Min.   : 0.00  Min.   : 0.00  Min.   : 0.00  Min.   : 0.0
## 1st Qu.: 22.50  1st Qu.: 15.00  1st Qu.: 10.00  1st Qu.: 3.5
## Median : 37.00  Median : 22.00  Median : 15.00  Median : 6.0

```


## Mean : 61.41	Mean : 36.53	Mean : 24.88	Mean : 9.0
## 3rd Qu.: 85.00	3rd Qu.: 45.50	3rd Qu.: 30.50	3rd Qu.:11.0
## Max. :348.00	Max. :207.00	Max. :141.00	Max. :51.0
## g:1	g:2	a:1	a:2
## Min. : 0.00	Min. : 0.00	Min. : 0.00	Min. : 0.00
## 1st Qu.: 26.50	1st Qu.: 33.50	1st Qu.: 0.00	1st Qu.: 0.00
## Median : 61.00	Median : 76.00	Median : 13.00	Median : 17.00
## Mean : 71.29	Mean : 91.76	Mean : 21.71	Mean : 25.24
## 3rd Qu.: 87.00	3rd Qu.:117.00	3rd Qu.: 30.00	3rd Qu.: 35.50
## Max. :404.00	Max. :520.00	Max. :123.00	Max. :143.00
## a:3	a:4	a:5	a:6
## Min. : 0.00	Min. : 0.00	Min. : 0.0	Min. : 0.00
## 1st Qu.: 0.00	1st Qu.: 0.00	1st Qu.: 0.0	1st Qu.: 0.00
## Median : 16.00	Median : 16.00	Median : 16.0	Median : 11.00
## Mean : 24.35	Mean : 32.82	Mean : 36.0	Mean : 22.94
## 3rd Qu.: 33.00	3rd Qu.: 46.50	3rd Qu.: 53.5	3rd Qu.: 31.00
## Max. :138.00	Max. :186.00	Max. :204.0	Max. :130.00
## e:1	e:2	e:3	e:4
## Min. : 0.00	Min. : 0.00	Min. : 0.0	Min. : 0.00
## 1st Qu.: 1.00	1st Qu.: 2.50	1st Qu.: 12.5	1st Qu.: 4.00
## Median : 8.00	Median : 7.00	Median : 25.0	Median :11.00
## Mean :14.47	Mean :10.41	Mean : 36.0	Mean :16.94
## 3rd Qu.:22.50	3rd Qu.:15.50	3rd Qu.: 48.0	3rd Qu.:22.00
## Max. :82.00	Max. :59.00	Max. :204.0	Max. :96.00
## e:5	e:6	e:7	e:8
## Min. : 0.00	Min. : 0.00	Min. : 0.00	Min. : 0.00
## 1st Qu.: 9.00	1st Qu.: 3.50	1st Qu.: 2.00	1st Qu.: 5.00
## Median : 24.00	Median :13.00	Median : 6.00	Median : 16.00
## Mean : 36.71	Mean :17.29	Mean :10.06	Mean : 21.18
## 3rd Qu.: 51.00	3rd Qu.:22.50	3rd Qu.:13.00	3rd Qu.: 28.00
## Max. :208.00	Max. :98.00	Max. :57.00	Max. :120.00
## p:1	p:2	p:3	ga:1
## Min. : 0.0	Min. : 0.000	Min. : 0.00	Min. : 0.000
## 1st Qu.: 46.0	1st Qu.: 3.000	1st Qu.: 16.00	1st Qu.: 0.000
## Median : 88.0	Median : 5.000	Median : 34.00	Median : 3.000
## Mean :113.5	Mean : 7.412	Mean : 42.18	Mean : 8.471
## 3rd Qu.:142.0	3rd Qu.: 9.500	3rd Qu.: 53.50	3rd Qu.:11.000
## Max. :643.0	Max. :42.000	Max. :239.00	Max. :48.000
## ga:2	ga:3	ga:4	ga:5
## Min. : 0.00	Min. : 0.00	Min. : 0.00	Min. : 0.00
## 1st Qu.: 0.00	1st Qu.: 0.00	1st Qu.: 0.00	1st Qu.: 0.00
## Median : 3.00	Median : 5.00	Median : 7.00	Median : 7.00
## Mean :10.94	Mean :10.94	Mean :15.35	Mean :14.82
## 3rd Qu.:14.50	3rd Qu.:14.50	3rd Qu.:21.00	3rd Qu.:19.00
## Max. :62.00	Max. :62.00	Max. :87.00	Max. :84.00
## ga:6	ga:7	ga:8	ga:9
## Min. : 0.00	Min. : 0.00	Min. : 0.00	Min. : 0.00
## 1st Qu.: 0.00	1st Qu.: 0.00	1st Qu.: 0.00	1st Qu.: 0.00
## Median : 4.00	Median : 6.00	Median : 5.00	Median : 4.00
## Mean :10.76	Mean :13.24	Mean :14.29	Mean :13.41
## 3rd Qu.:12.50	3rd Qu.:19.50	3rd Qu.:19.50	3rd Qu.:17.50
## Max. :61.00	Max. :75.00	Max. :81.00	Max. :76.00
## ga:10	ga:11	ga:12	
## Min. : 0.00	Min. : 0.00	Min. : 0.00	

```
## 1st Qu.: 0.00 1st Qu.: 0.00 1st Qu.: 0.00
## Median : 6.00 Median : 10.00 Median : 5.00
## Mean :17.47 Mean : 21.18 Mean :12.18
## 3rd Qu.:24.00 3rd Qu.: 27.00 3rd Qu.:15.50
## Max. :99.00 Max. :120.00 Max. :69.00
```

We take the stacked tables of **A** and **D** substantive variables from the whole Burt matrix:

```
Finland.AD = Finland.B[c(1:5,16:20), c(1:5,16:20)]
```

The principal intervals (eigenvalues) are:

```
summary(Finland.AD)
```

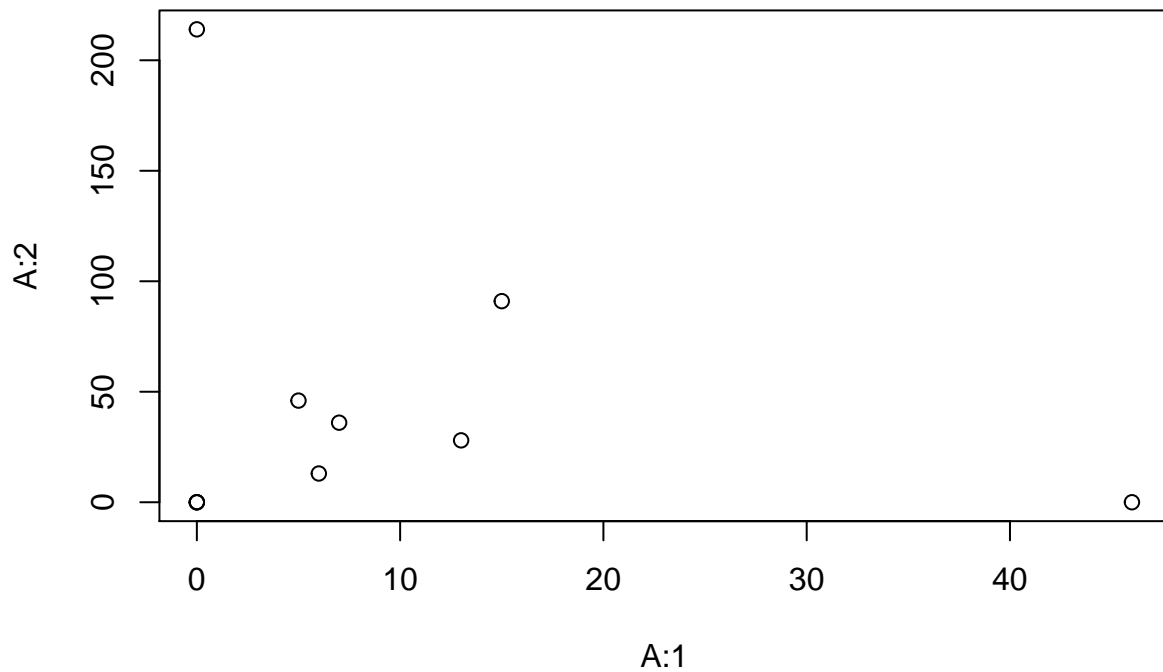
```
##      A:1      A:2      A:3      A:4
## Min.   : 0.0   Min.   : 0.0   Min.   : 0.00  Min.   : 0.0
## 1st Qu.: 0.0   1st Qu.: 0.0   1st Qu.: 0.00  1st Qu.: 0.0
## Median : 5.5   Median : 20.5  Median : 28.50  Median : 23.0
## Mean   : 9.2   Mean   : 42.8  Mean   : 63.00  Mean   : 42.8
## 3rd Qu.:11.5   3rd Qu.: 43.5  3rd Qu.: 79.75  3rd Qu.: 42.5
## Max.   :46.0   Max.   :214.0  Max.   :315.00  Max.   :214.0
##      A:5      D:1      D:2      D:3
## Min.   : 0.00  Min.   : 0.00  Min.   : 0.00  Min.   : 0.00
## 1st Qu.: 0.00  1st Qu.: 0.00  1st Qu.: 0.00  1st Qu.: 0.00
## Median :12.50  Median : 20.50  Median : 29.00  Median : 16.00
## Mean   :27.00  Mean   : 35.40  Mean   : 69.60  Mean   : 41.40
## 3rd Qu.:36.75  3rd Qu.: 40.75  3rd Qu.: 89.25  3rd Qu.: 44.75
## Max.   :135.00  Max.   :177.00  Max.   :348.00  Max.   :207.00
##      D:4      D:5
## Min.   : 0.00  Min.   : 0.0
## 1st Qu.: 0.00  1st Qu.: 0.0
## Median : 9.50  Median : 5.5
## Mean   :28.20  Mean   :10.2
## 3rd Qu.:41.25  3rd Qu.:13.0
## Max.   :141.00  Max.   :51.0
```

The computation of MCA is the application of the (simple) CA algorithm to the Burt matrix C.

The following CA is conducted:

```
plot(Finland.AD, main="MCA=CA of Burt")
```

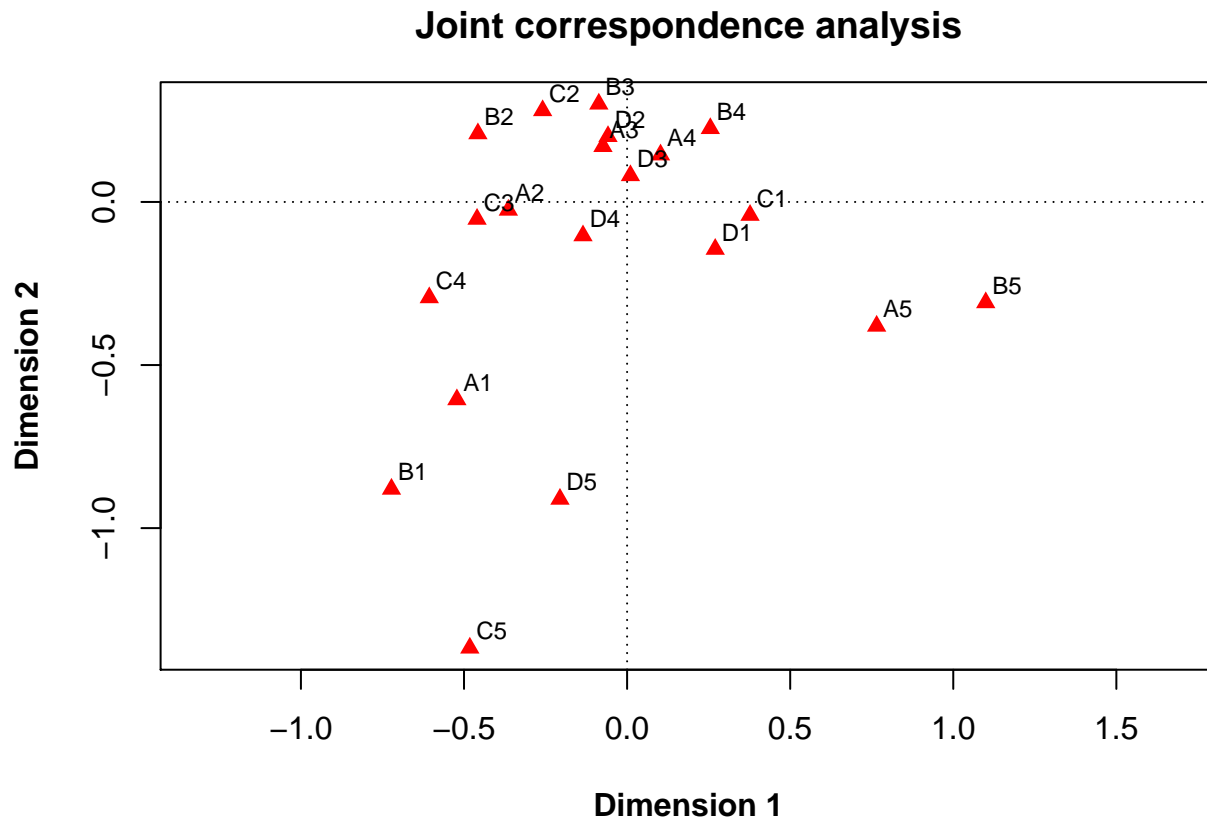
MCA=CA of Burt



Task 4: Multiple correspondence analysis (MCA) on the four substantive questions.

Here I use the script from Exercise set 2 about joint correspondence analysis and its fit to two-way tables:

```
Finland.mca <- mjca(Finland[,1:4], ps="", lambda="JCA")
par(mar=c(4.2,4,3,1), mfrow=c(1,1), font.lab=2)
plot(Finland.mca, main="Joint correspondence analysis")
```



Task 5: MCA on the four substantive questions: computing the case points, and adding confidence ellipses for some demographic groups.

The example code from Exercise set 3 is used, only data are different.

5.1 MCA on the four substantive questions

MCA is the CA of indicator matrix:

```
require(ca)
Finland.mca1 <- mjca(Finland[,1:4], lambda="indicator", ps="", reti=T)
sum(Finland.mca1$sv^2)
```

```
## [1] 4
```

```
summary(Finland.mca1)
```

```
##
## Principal inertias (eigenvalues):
##
## dim    value    %    cum%    scree plot
## 1      0.506305 12.7 12.7    ***
## 2      0.475751 11.9 24.6    ***
## 3      0.322936  8.1 32.6    **
## 4      0.309594  7.7 40.4    **
```

```
## 5      0.274095    6.9  47.2  **
## 6      0.268731    6.7  53.9  **
## 7      0.250166    6.3  60.2  **
## 8      0.236639    5.9  66.1   *
## 9      0.225387    5.6  71.7   *
## 10     0.217098    5.4  77.2   *
## 11     0.209046    5.2  82.4   *
## 12     0.186252    4.7  87.0   *
## 13     0.162414    4.1  91.1   *
## 14     0.142888    3.6  94.7   *
## 15     0.125436    3.1  97.8   *
## 16     0.087262    2.2 100.0   *
##      -----
## Total: 4.000000 100.0
##
##
## Columns:
##      name  mass  qlt  inr      k=1 cor ctr      k=2 cor ctr
## 1 |  A1 |  12 198  57 | -779 32 15 | 1783 167 83 |
## 2 |  A2 |  58 181  46 | -730 161 61 |  257  20  8 |
## 3 |  A3 |  85  95  38 | -187  18  6 | -384  76 26 |
## 4 |  A4 |  58  60  44 |  142  6  2 | -421  54 22 |
## 5 |  A5 |  37 509  63 | 1635 458 193 |  550  52 23 |
## 6 |  B1 |  34 616  65 | -694  76 32 | 1850 540 245 |
## 7 |  B2 |  67 244  47 | -767 216 78 | -276  28 11 |
## 8 |  B3 |  47  84  46 | -239  13  5 | -555  71 30 |
## 9 |  B4 |  62 110  45 |  281  26 10 | -501  84 33 |
## 10 | B5 |  40 588  66 | 1740 568 236 |  322  19  9 |
## 11 | C1 | 118 559  39 |  789 556 145 |  -57  3  1 |
## 12 | C2 |  91 379  42 | -650 241 76 | -491 138 46 |
## 13 | C3 |  17  65  52 | -894  59 27 |  279  6  3 |
## 14 | C4 |  12 101  54 | -1130 64 30 |  863  37 19 |
## 15 | C5 |  12 472  66 | -430  9  4 | 3041 462 231 |
## 16 | D1 |  48 159  47 |  795 150 60 |  196  9  4 |
## 17 | D2 |  94 155  36 | -247  37 11 | -443 118 39 |
## 18 | D3 |  56  12  42 |  -23  0  0 | -203  12  5 |
## 19 | D4 |  38  35  46 | -310 17  7 |  310  17  8 |
## 20 | D5 |  14 313  59 | -123  1  0 | 2310 312 155 |
```

We calculate: - the number of categories:

```
J <- 4*5 -1
```

The number of substantive questions is $Q=4$. Then:

```
Q <- 4
(J-Q)/Q
```

```
## [1] 3.75
```

We check the contents of these components of the `mjca` object (eigenvalues (squared singular values), standard coordinates of columns, standard coordinates of first 10 rows, variable names, category names, first 10 rows of 924 x 20 indicator matrix, first 20 rows and columns of 20 x 20 Burt matrix) :

```
names(Finland.mca1)
```

```
## [1] "sv"          "lambda"      "inertia.e"   "inertia.t"   "inertia.et"
## [6] "levelnames" "factors"     "levels.n"    "nd"           "nd.max"
```

```
## [11] "rownames"      "rowmass"      "rowdist"      "rowinertia"   "rowcoord"
## [16] "rowpcoord"     "rowctr"       "rowcor"       "colnames"     "colmass"
## [21] "coldist"       "colinertia"   "colcoord"     "colpcoord"    "colctr"
## [26] "colcor"        "colsup"       "subsetcol"    "Burt"         "Burt.upd"
## [31] "subinertia"    "JCA.iter"     "indmat"       "call"

Finland.mca1$sv^2                # eigenvalues (squared singular values) on all 31 dimensions

## [1] 0.50630492 0.47575147 0.32293568 0.30959403 0.27409529 0.26873127
## [7] 0.25016609 0.23663902 0.22538662 0.21709794 0.20904570 0.18625197
## [13] 0.16241364 0.14288849 0.12543559 0.08726229

Finland.mca1$colcoord[,1:4]      # standard coordinates of columns

##           [,1]      [,2]      [,3]      [,4]
## [1,] -1.09485075  2.58466228  0.38117469 -1.0611142
## [2,] -1.02632480  0.37232989 -0.47157445  1.4201562
## [3,] -0.26345610 -0.55714283 -0.80286444 -1.3121928
## [4,]  0.19957060 -0.61095108  2.14671089 -0.1271033
## [5,]  2.29834966  0.79755898 -0.91193285  1.3736196
## [6,] -0.97470923  2.68220803  0.48209843 -0.6463525
## [7,] -1.07792715 -0.39988664 -0.66328834  1.5112705
## [8,] -0.33602699 -0.80484225 -1.22794762 -1.6797485
## [9,]  0.39518935 -0.72610435  1.92930121 -0.7637834
## [10,] 2.44509059  0.46699649 -0.88686667  1.1895602
## [11,] 1.10913902 -0.08237802  0.03901019 -0.2765203
## [12,] -0.91287622 -0.71121198  0.25165618  0.6290356
## [13,] -1.25648834  0.40439859 -2.40903951 -1.9962482
## [14,] -1.58745288  1.25079576  0.30637839  1.2613963
## [15,] -0.60443232  4.40835272  0.88938548 -0.4212425
## [16,]  1.11702020  0.28392642 -0.07957831 -0.3599156
## [17,] -0.34677975 -0.64190277 -0.07750072  0.8506859
## [18,] -0.03261598 -0.29495380 -0.87687783 -0.9136392
## [19,] -0.43594396  0.44926861  1.62187492 -0.1096170
## [20,] -0.17281598  3.34972059 -0.11990279 -0.5442025

Finland.mca1$rowcoord[1:10,1:4]  # standard coordinates of first 10 rows

##           [,1]      [,2]      [,3]      [,4]
## [1,]  0.05722575 -0.7561703 -0.9103450 -1.08632489
## [2,] -1.03458890 -0.4329605 -1.7389019 -0.42526271
## [3,]  0.16760566 -0.6304181 -1.2620137 -1.87904947
## [4,] -0.80348254 -0.7115631 -0.9200550 -0.03842743
## [5,] -1.07151274 -0.3746743 -0.7743111  1.18923762
## [6,]  0.16760566 -0.6304181 -1.2620137 -1.87904947
## [7,]  0.05722575 -0.7561703 -0.9103450 -1.08632489
## [8,] -0.21080445 -0.4192814 -0.7646011  0.14134015
## [9,] -0.81084974 -0.5214511 -1.0227208 -0.24451133
## [10,] -0.66354598 -0.4539851 -2.3389802 -2.65173623

Finland.mca1$colnames            # variable names

## [1] "A" "B" "C" "D"

Finland.mca1$levelnames          # category names

## [1] "A1" "A2" "A3" "A4" "A5" "B1" "B2" "B3" "B4" "B5" "C1" "C2" "C3" "C4"
## [15] "C5" "D1" "D2" "D3" "D4" "D5"
```

```
Finland.mca1$indmat[1:10,]
```

```
# first 10 rows of 924 x 20 indicator matrix
```

```
##      A1 A2 A3 A4 A5 B1 B2 B3 B4 B5 C1 C2 C3 C4 C5 D1 D2 D3 D4 D5
## 1    0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 2    0  0  1  0  0  0  1  0  0  0  0  0  1  0  0  0  1  0  0  0
## 3    0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  0  1  0  0
## 4    0  0  1  0  0  0  1  0  0  0  0  1  0  0  0  0  0  1  0  0
## 5    0  1  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  1  0  0
## 6    0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  0  1  0  0
## 7    0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 8    0  1  0  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 9    0  1  0  0  0  0  0  1  0  0  0  1  0  0  0  0  0  1  0  0
## 10   0  0  1  0  0  0  0  1  0  0  0  0  1  0  0  0  0  1  0  0
```

```
Finland.mca1$Burt[1:20, 1:20]
```

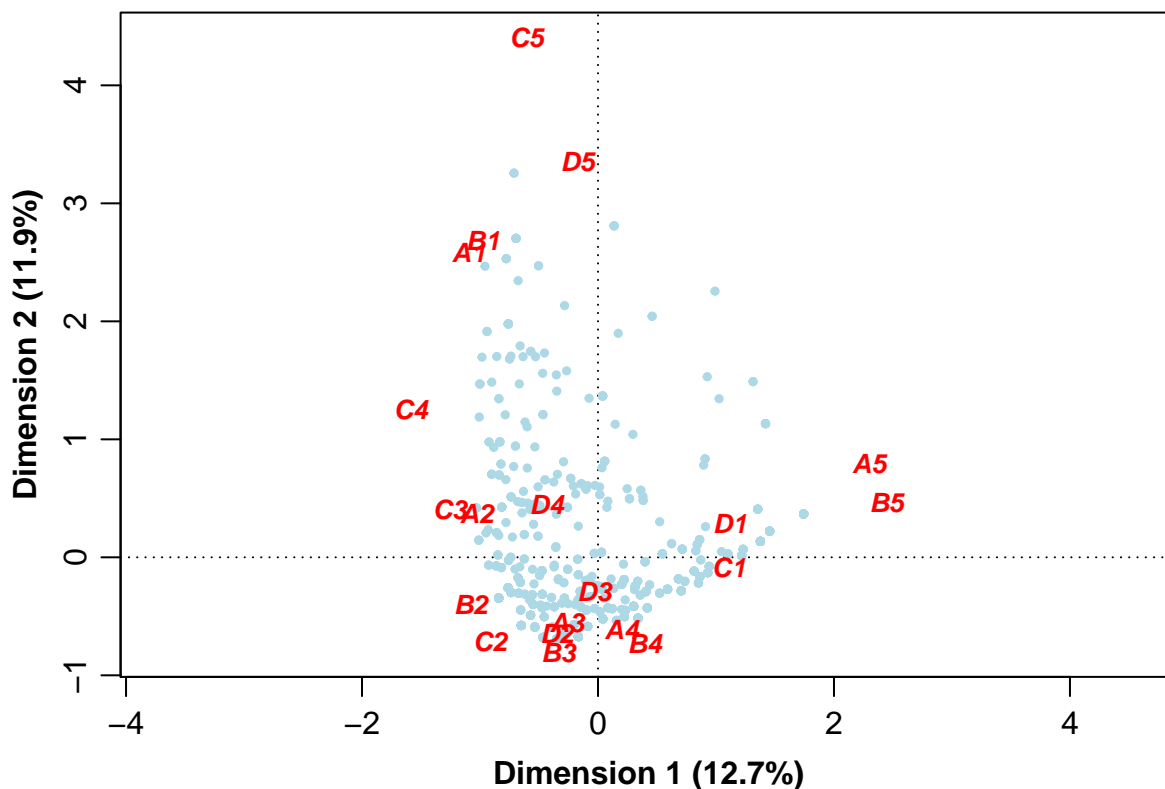
```
# first 20 rows and columns of 20 x 20 Burt matrix
```

```
##      A1  A2  A3  A4  A5  B1  B2  B3  B4  B5  C1  C2  C3  C4  C5  D1  D2  D3  D4
## A1 46    0    0    0    0 28   12   4    2    0 15   13   6   6   6  13  15   5   7
## A2 0 214    0    0    0 50   92  38   25    9 64  101  17  15  17  28  91  46  36
## A3 0    0 315    0    0 24   92  95   83   21 145  120  30  14   6  55 115  88  43
## A4 0    0    0 214    0 15   42  30   97   30 104   88  10   9   3  41  84  41  43
## A5 0    0    0    0 135    9  10    6  24   86 108   14   1   0  12  40  43  27  12
## B1 28  50  24  15    9 126    0    0    0    0 26   34  17  15  34  29  24  20  29
## B2 12  92  92  42  10    0 248    0    0    0 52  141  27  25   3  30 119  53  37
## B3 4   38  95  30    6    0    0 173    0    0 79   75  17   1   1  28  75  45  23
## B4 2   25  83  97  24    0    0    0 231    0 142   82   3   3   1  41  90  55  37
## B5 0    9  21  30  86    0    0    0    0 146  137    4   0   0   5  49  40  34  15
## C1 15   64 145 104 108  26   52  79 142  137 436    0   0   0   0 128 137  93  57
## C2 13  101 120  88  14  34 141  75  82    4    0 336    0   0   0  35 175  73  51
## C3 6   17  30  10    1  17  27  17    3    0    0   64   0   0   7  22  22   7
## C4 6   15  14    9    0  15  25    1    3    0    0   0  44   0   3  12  12  11
## C5 6   17    6    3  12  34    3    1    1    5    0    0   0  44   4    2    7  15
## D1 13  28  55  41  40  29  30  28  41  49 128  35   7   3   4 177    0    0    0
## D2 15  91 115  84  43  24 119  75  90  40 137 175  22  12   2    0 348    0    0
## D3 5   46  88  41  27  20  53  45  55  34  93  73  22  12   7    0    0 207    0
## D4 7   36  43  43  12  29  37  23  37  15  57  51   7  11  15    0    0    0 141
## D5 6   13  14    5  13  24    9    2    8    8  21    2   6   6  16    0    0    0    0
##      D5
## A1 6
## A2 13
## A3 14
## A4 5
## A5 13
## B1 24
## B2 9
## B3 2
## B4 8
## B5 8
## C1 21
## C2 2
## C3 6
## C4 6
## C5 16
## D1 0
```

```
## D2 0
## D3 0
## D4 0
## D5 51
```

Next we compute positions of cases and plot them.

```
Finland.rpc <- Finland.mca1$indmat %*% Finland.mca1$colcoord[,1:2] / 4
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca1, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
```



The MCA is calculated as a CA of Burt matrix:

```
Finland.mca2 <- mja(Finland[,1:4], lambda="Burt", ps="")
sum(Finland.mca2$sv^2)
```

```
## [1] 1.195211
```

```
summary(Finland.mca2)
```

```
##
## Principal inertias (eigenvalues):
##
## dim    value      %   cum%   scree plot
## 1     0.256345  21.4  21.4   *****
## 2     0.226339  18.9  40.4   *****
## 3     0.104287   8.7  49.1    **
```



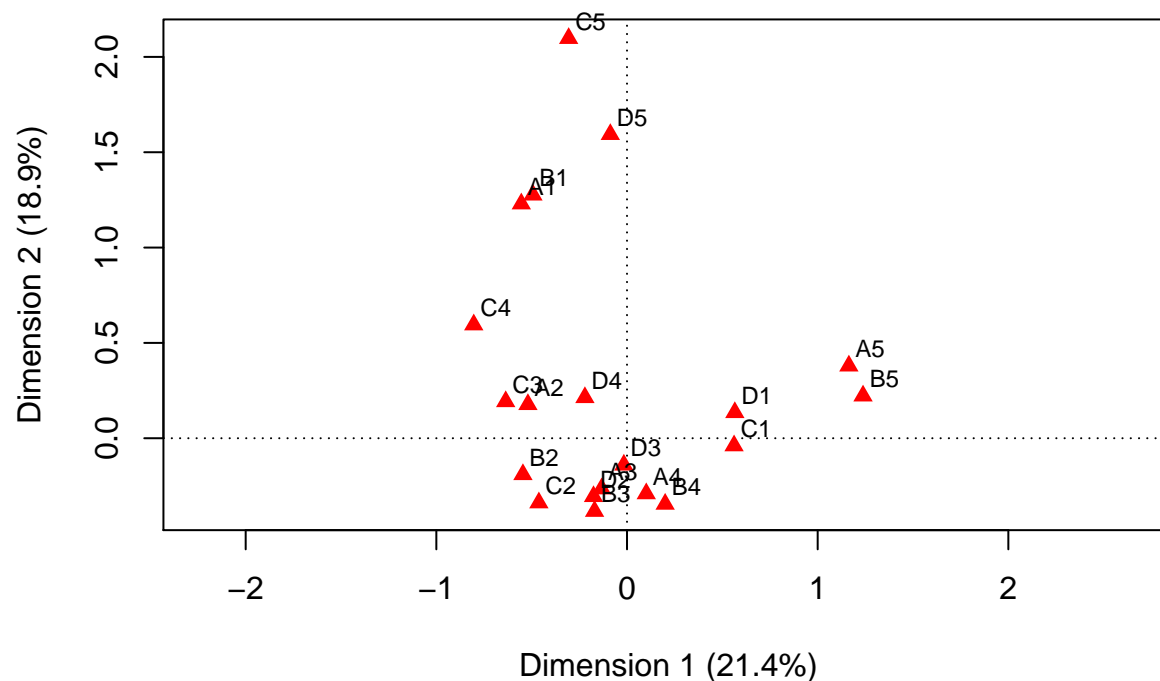
```

## 4      0.095848   8.0  57.1  **
## 5      0.075128   6.3  63.4  **
## 6      0.072216   6.0  69.5  **
## 7      0.062583   5.2  74.7   *
## 8      0.055998   4.7  79.4   *
## 9      0.050799   4.3  83.6   *
## 10     0.047132   3.9  87.6   *
## 11     0.043700   3.7  91.2   *
## 12     0.034690   2.9  94.1   *
## 13     0.026378   2.2  96.3   *
## 14     0.020417   1.7  98.0
## 15     0.015734   1.3  99.4
## 16     0.007615   0.6 100.0
##      -----
## Total: 1.195211 100.0
##
##
## Columns:
##      name  mass  qlt  inr   k=1 cor ctr   k=2 cor ctr
## 1 |  A1 |  12  335  57 | -554  57  15 | 1230 278  83 |
## 2 |  A2 |  58  317  46 | -520 284  61 |  177  33   8 |
## 3 |  A3 |  85  165  38 | -133  33   6 | -265 132  26 |
## 4 |  A4 |  58  104  44 |  101  11   2 | -291  93  22 |
## 5 |  A5 |  37  724  63 | 1164 654 193 |  379  70  23 |
## 6 |  B1 |  34  825  65 | -494 107  32 | 1276 718 245 |
## 7 |  B2 |  67  400  47 | -546 357  78 | -190  43  11 |
## 8 |  B3 |  47  148  46 | -170  24   5 | -383 124  30 |
## 9 |  B4 |  62  186  45 |  200  47  10 | -345 139  33 |
## 10 | B5 |  40  790  66 | 1238 765 236 |  222  25   9 |
## 11 | C1 | 118  793  39 |  562 789 145 |  -39   4   1 |
## 12 | C2 |  91  589  42 | -462 384  76 | -338 206  46 |
## 13 | C3 |  17  123  52 | -636 113  27 |  192  10   3 |
## 14 | C4 |  12  184  54 | -804 119  30 |  595  65  19 |
## 15 | C5 |  12  681  66 | -306  14   4 | 2097 666 231 |
## 16 | D1 |  48  288  47 |  566 273  60 |  135  16   4 |
## 17 | D2 |  94  270  36 | -176  67  11 | -305 203  39 |
## 18 | D3 |  56   22  42 |  -17   0   0 | -140  22   5 |
## 19 | D4 |  38   65  46 | -221  34   7 |  214  32   8 |
## 20 | D5 |  14  502  59 |  -87   2   0 | 1594 500 155 |

```

The default plot shows principal coordinates:

```
plot(Finland.mca2)
```



The adjusted MCA is the default of `mjca` function:

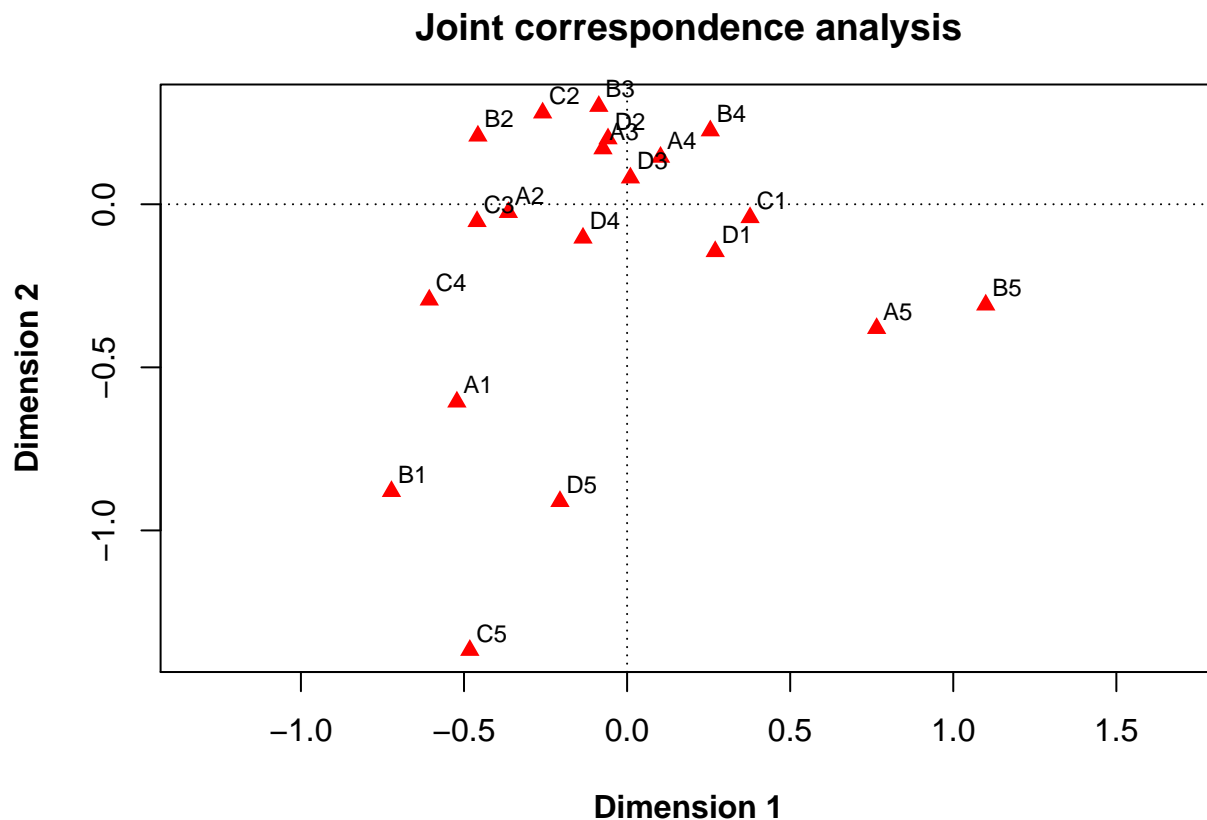
```
Finland.mca3 <- mjca(Finland[,1:4], ps="")
summary(Finland.mca3)
```

```
##
## Principal inertias (eigenvalues):
##
## dim    value      %   cum%   scree plot
## 1      0.116786  44.9  44.9   *****
## 2      0.090602  34.8  79.7   *****
## 3      0.009457   3.6  83.3    *
## 4      0.006314   2.4  85.7    *
## 5      0.001032   0.4  86.1
## 6      0.000624   0.2  86.4
## 7      0.000000   0.0  86.4
## -----
## Total: 0.260281
##
##
## Columns:
##   name  mass  qlt  inr   k=1 cor ctr   k=2 cor ctr
## 1 |  A1 |  12 848  57 | -374 159 15 |  778 689 83 |
## 2 |  A2 |  58 847  46 | -351 769 61 |  112  78  8 |
## 3 |  A3 |  85 545  38 |  -90 122  6 | -168 423 26 |
## 4 |  A4 |  58 375  44 |   68  45  2 | -184 329 22 |
## 5 |  A5 |  37 830  63 |  785 759 193 |  240  71 23 |
```

```
## 6 | B1 | 34 834 65 | -333 121 32 | 807 713 245 |
## 7 | B2 | 67 738 47 | -368 666 78 | -120 71 11 |
## 8 | B3 | 47 543 46 | -115 100 5 | -242 443 30 |
## 9 | B4 | 62 460 45 | 135 127 10 | -219 333 33 |
## 10 | B5 | 40 804 66 | 836 782 236 | 141 22 9 |
## 11 | C1 | 118 902 39 | 379 899 145 | -25 4 1 |
## 12 | C2 | 91 898 42 | -312 611 76 | -214 288 46 |
## 13 | C3 | 17 636 52 | -429 588 27 | 122 47 3 |
## 14 | C4 | 12 756 54 | -542 510 30 | 376 246 19 |
## 15 | C5 | 12 845 66 | -207 20 4 | 1327 825 231 |
## 16 | D1 | 48 970 47 | 382 924 60 | 85 46 4 |
## 17 | D2 | 94 829 36 | -119 227 11 | -193 602 39 |
## 18 | D3 | 56 277 42 | -11 4 0 | -89 273 5 |
## 19 | D4 | 38 557 46 | -149 305 7 | 135 252 8 |
## 20 | D5 | 14 960 59 | -59 3 0 | 1008 956 155 |
```

A joint correspondence analysis is used for optimal fit to two-way tables:

```
Finland.mca4 <- mjca(Finland[,1:4], ps="", lambda="JCA")
par(mar=c(4.2,4,3,1), mfrow=c(1,1), font.lab=2)
plot(Finland.mca4, main="Joint correspondence analysis")
```



```
summary(Finland.mca4)
```

```
##
## Principal inertias (eigenvalues):
##
## dim    value
```

```

## 1      0.156300
## 2      0.101794
## 3      0.006669
## 4      0.005138
## 5      0.003304
## 6      0.000545
## 7      0.000228
## -----
## Total: 0.285989
##
## Diagonal inertia discounted from eigenvalues: 0.090779
## Percentage explained by JCA in 2 dimensions: 85.7%
## (Eigenvalues are not nested)
## [Iterations in JCA: 50 , epsilon = 0.0008615]
##
##
## Columns:
##      name  mass  inr   k=1   k=2   cor ctr
## 1 |   A1 |   12   57 | -522  -606 | 856  42 |
## 2 |   A2 |   58   46 | -364   -25 | 879  37 |
## 3 |   A3 |   85   38 |  -74   170 | 564  14 |
## 4 |   A4 |   58   44 |  103   145 | 362  10 |
## 5 |   A5 |   37   63 |  765 -381 | 912 121 |
## 6 |   B1 |   34   65 | -722 -880 | 970 136 |
## 7 |   B2 |   67   47 | -457   209 | 848  52 |
## 8 |   B3 |   47   46 |  -87   300 | 600  17 |
## 9 |   B4 |   62   45 |  255   225 | 528  21 |
## 10 |  B5 |   40   66 | 1100 -309 | 954 151 |
## 11 |  C1 |  118   39 |  377   -41 | 943  80 |
## 12 |  C2 |   91   42 | -259   280 | 918  60 |
## 13 |  C3 |   17   52 | -460   -53 | 711  17 |
## 14 |  C4 |   12   54 | -606 -293 | 793  24 |
## 15 |  C5 |   12   66 | -482 -1367 | 917 104 |
## 16 |  D1 |   48   47 |  270 -145 | 745  25 |
## 17 |  D2 |   94   36 |  -59   201 | 795  21 |
## 18 |  D3 |   56   42 |   10    81 | 257   2 |
## 19 |  D4 |   38   46 | -135 -103 | 487   6 |
## 20 |  D5 |   14   59 | -206 -911 | 917  60 |

```

5.2 Demographics is superimposed on an MCA

Call the used code about confidence plots, package ellipse, and calculated individual points:

```

source("confidenceplots.R")
require(ellipse)

```

```
## Loading required package: ellipse
```

```
Finland.rpc <- Finland.mca1$indmat %*% Finland.mca1$colcoord[,1:2] / 4
```

The demographic groups are: education e, partnership p and ga groups. Their confidence plots are :

```

par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")

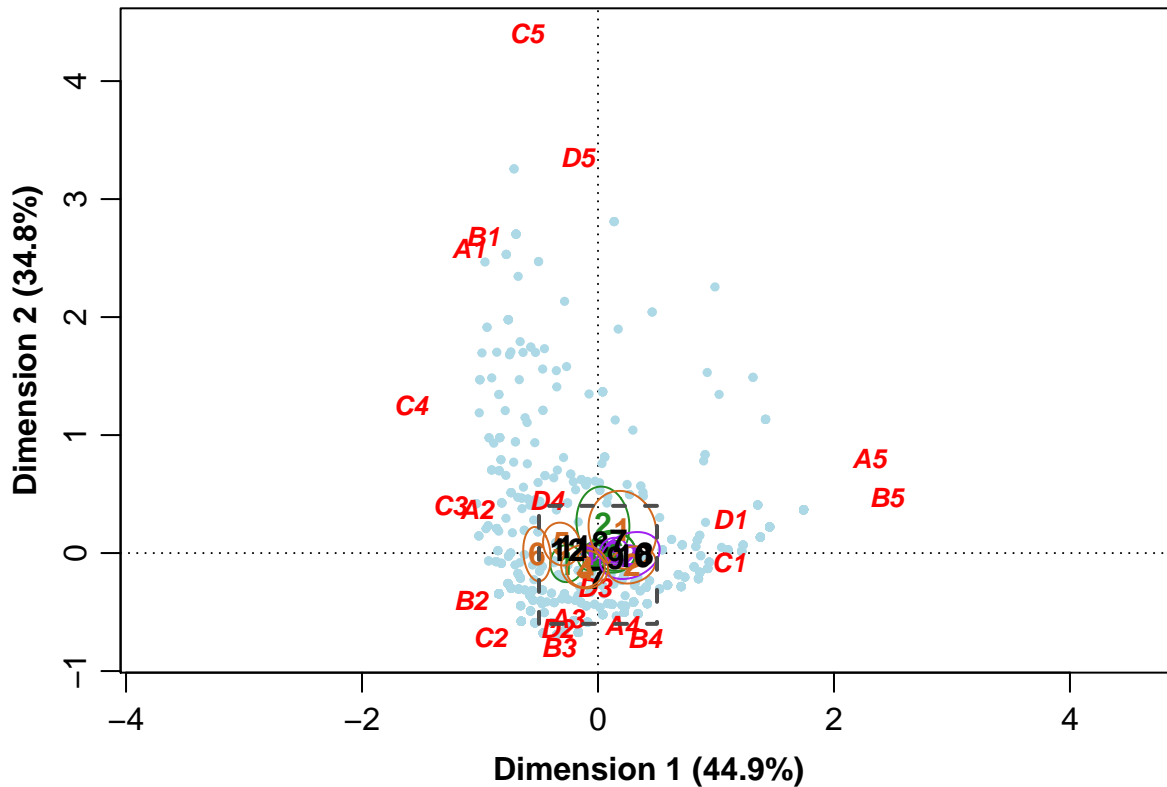
```

```

text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "e"]), groupcols=rep("forestgreen", 6))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "p"]), groupcols=rep("purple", 6))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=rep("chocolate", 6))

lines(matrix(c(-0.5,-0.6, -0.5,0.4, 0.5,0.4, 0.5,-0.6, -0.5,-0.6), ncol=2, byrow=T), lty=2, col="gray30")

```

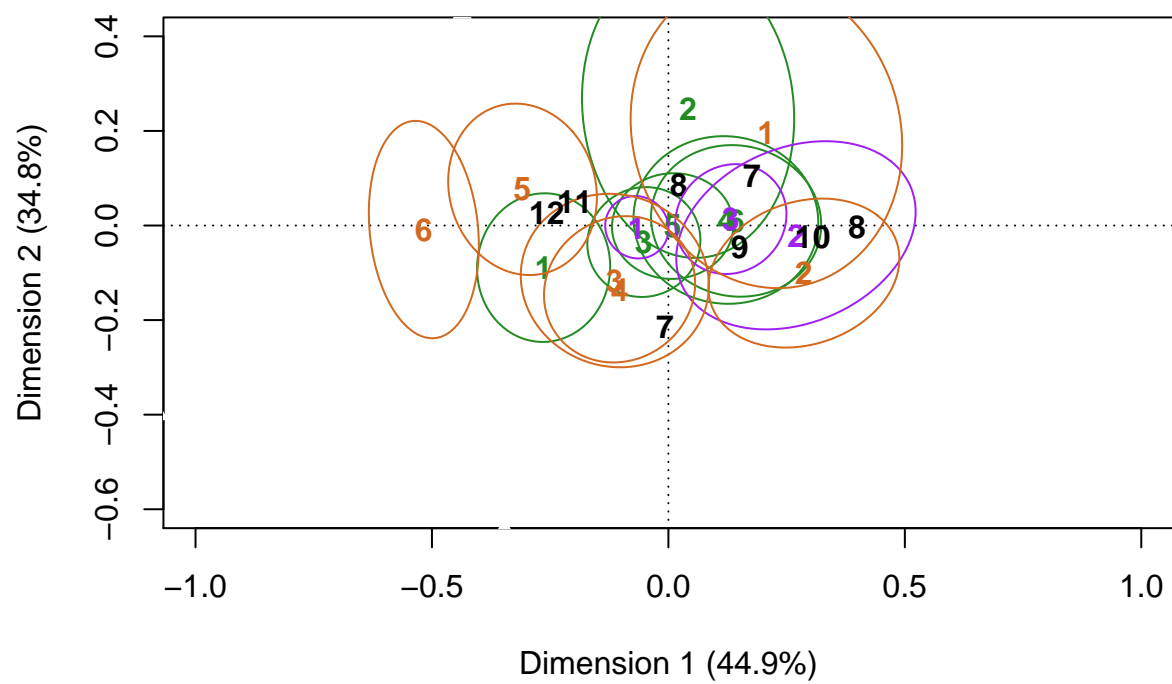


We zoom in to the group means and confidence ellipses:

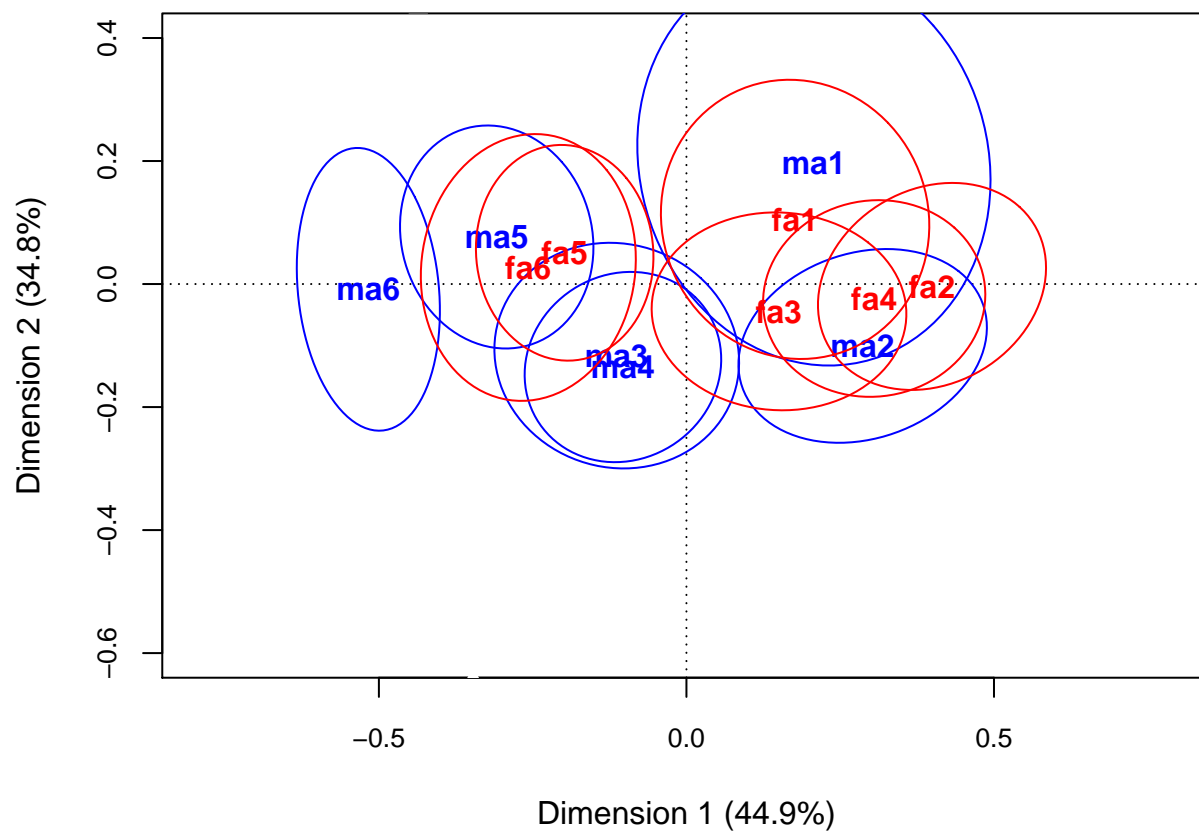
```

plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(-0.5,0.5))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "e"]), groupcols=rep("forestgreen", 6))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "p"]), groupcols=rep("purple", 6))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=rep("chocolate", 6))

```

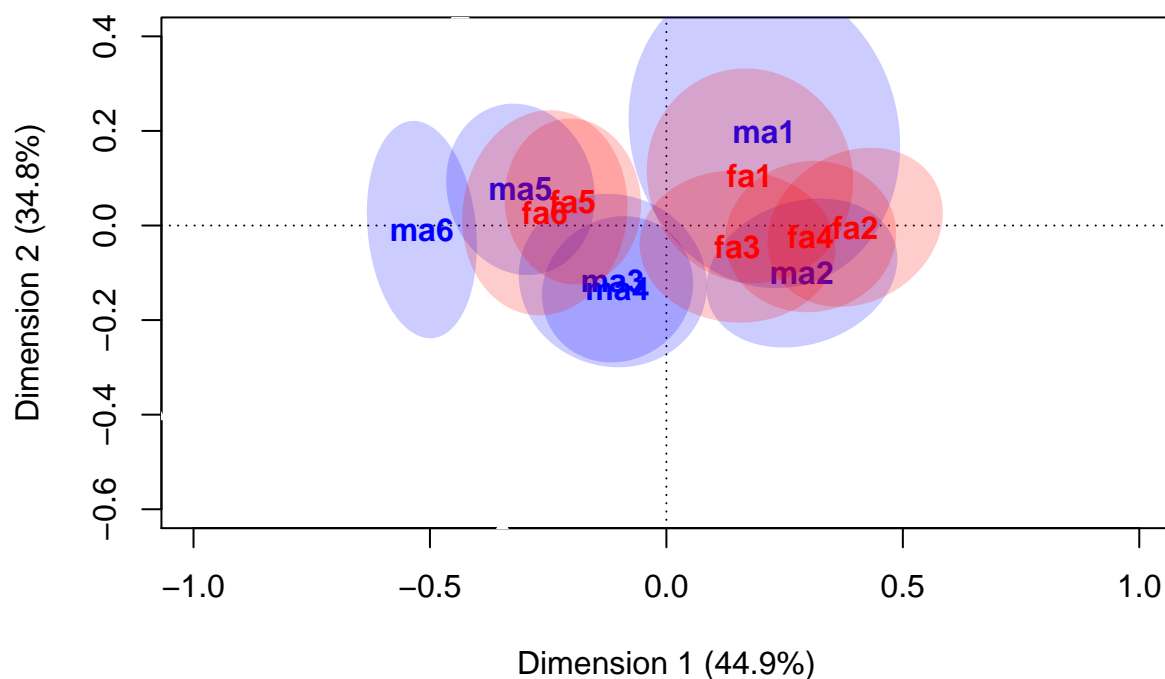


```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(-0.5,0.5),
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=c(rep("blue", 6), rep("red", 6)),
groupnames=c("ma1", "ma2", "ma3", "ma4", "ma5", "ma6", "fa1", "fa2", "fa3", "fa4", "fa5", "fa6"), sl=c(0.05, 0.05))
```



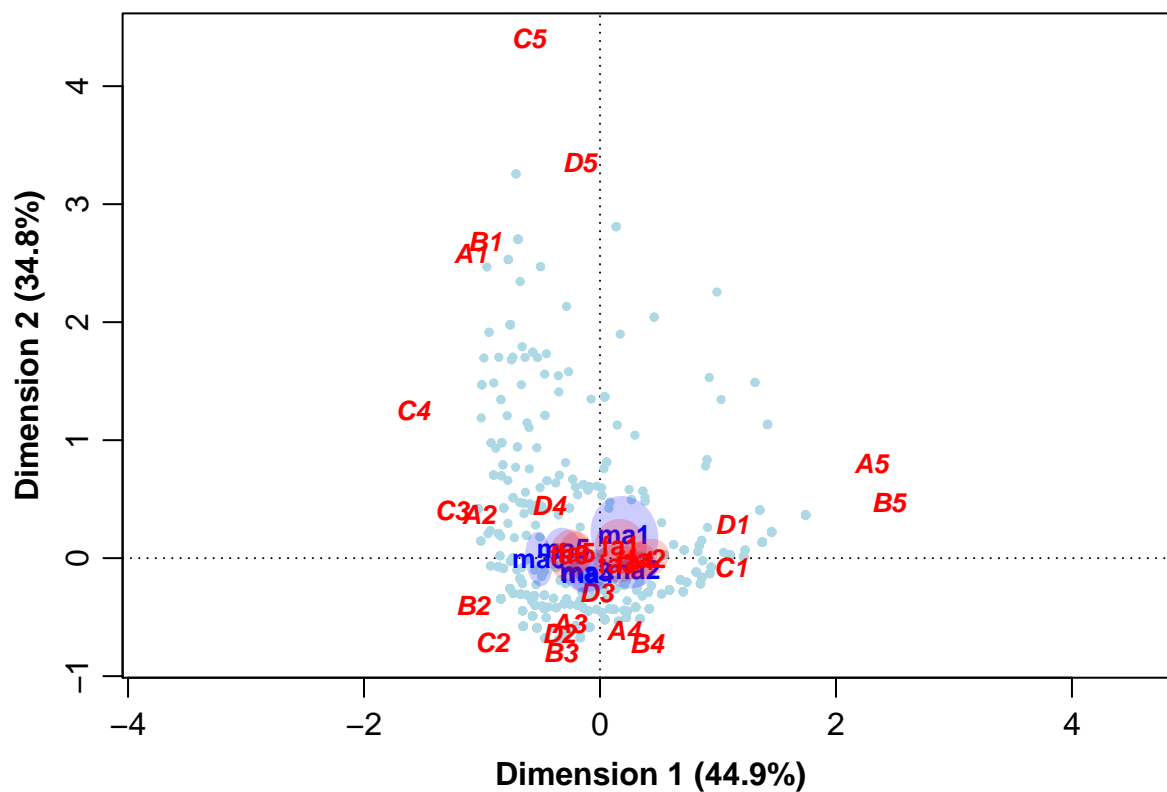
And add shading:

```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(-0.6,0.4),
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=c(rep("blue", 6), rep("red", 6)),
groupnames=c("ma1", "ma2", "ma3", "ma4", "ma5", "ma6", "fa1", "fa2", "fa3", "fa4", "fa5", "fa6"), s
```



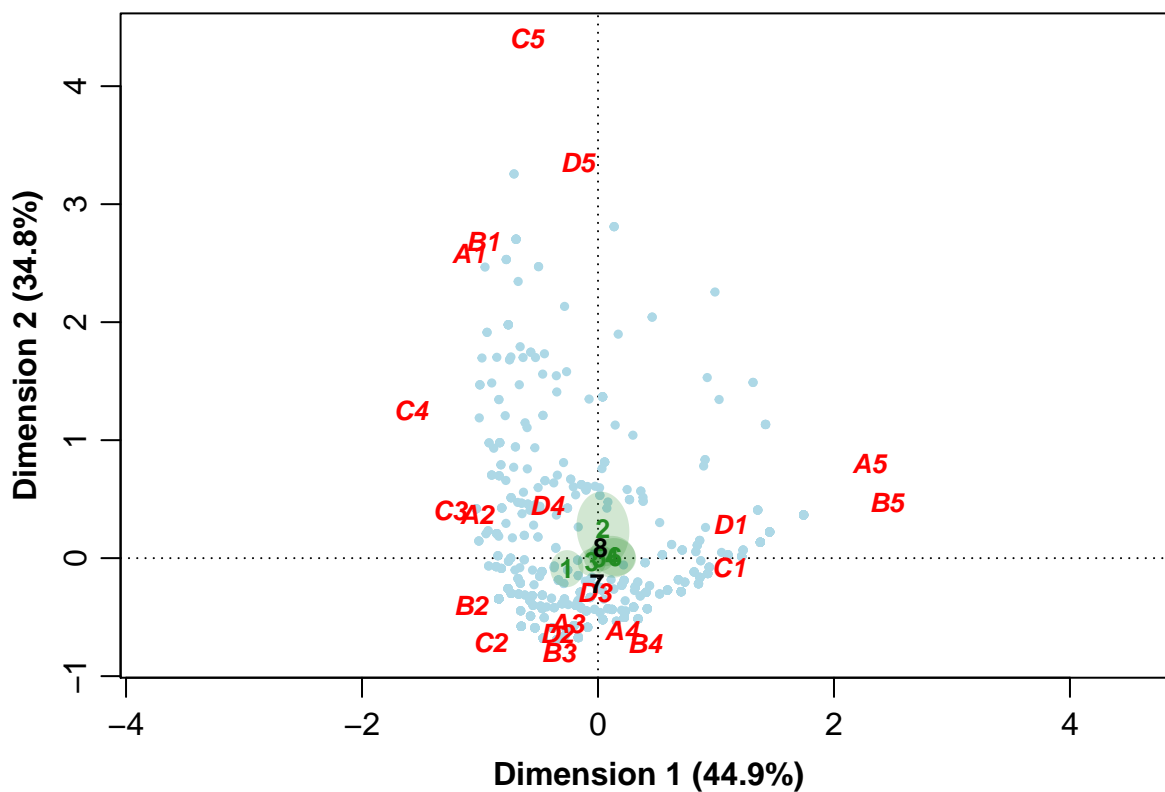
Next we go back to original plot, with category labels, gender-age labels slightly smaller (cex=0.8)

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=c(rep("blue", 6), rep("red", 6)),
               groupnames=c("ma1", "ma2", "ma3", "ma4", "ma5", "ma6", "fa1", "fa2", "fa3", "fa4", "fa5", "fa6"), sl
```

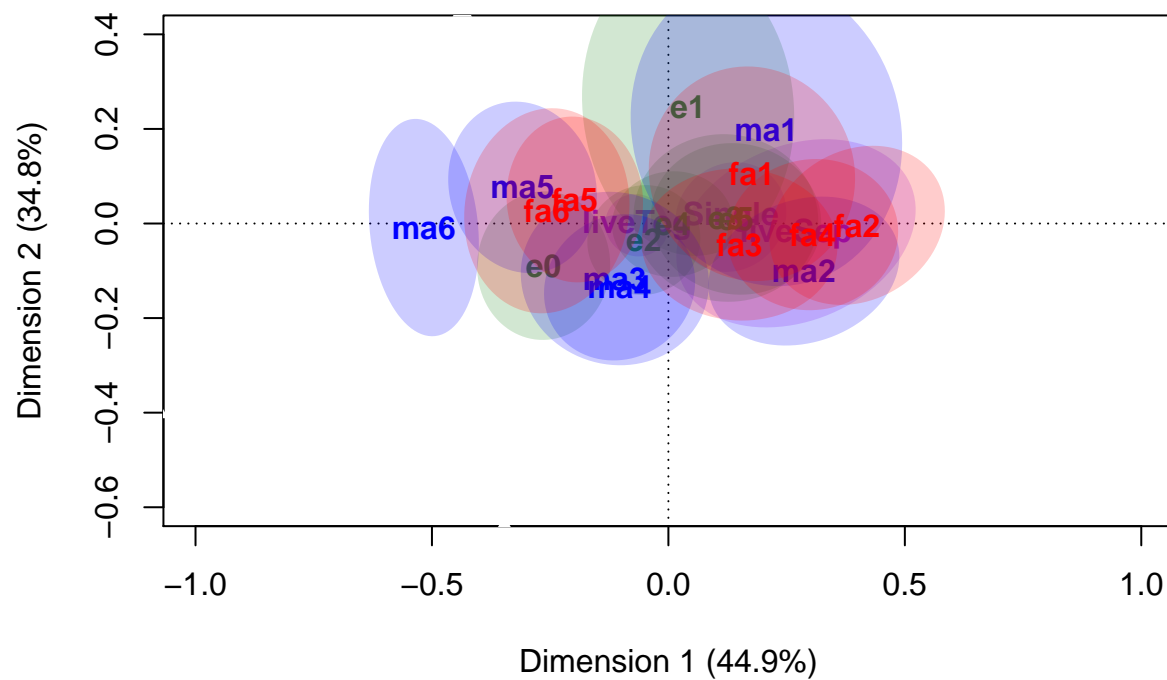
Back to other demographic variable, education:

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "e"]), groupcols=rep("forestgreen", 2),
  shownames=T, add=T, cex=0.8)
```



Plot confidence ellipses for all demographic groups together in a zoom:

```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(-0.5,0.5))
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "p"]), groupcols=rep("purple", 6),
  groupnames=c("liveTog", "liveSep", "Single"), shownames=T, add=T)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "e"]), groupcols=rep("forestgreen", 6),
  groupnames=c("e0", "e1", "e2", "e3", "e4", "e5"), shownames=T, add=T)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[, "ga"]), groupcols=c(rep("blue", 6), rep("red", 6)),
  groupnames=c("ma1", "ma2", "ma3", "ma4", "ma5", "ma6", "fa1", "fa2", "fa3", "fa4", "fa5", "fa6"), shownames=T, add=T)
```



Used and useful links

Mike Bendixen, A Practical Guide to the Use of Correspondence Analysis in Marketing Research

Oleg Nenadic and Michael Greenacre, Computation of Multiple Correspondence Analysis, with code in R

Biplots in practise

Multiple Correspondence Analysis Essentials: Interpretation and application to investigate the associations between categories of multiple qualitative variables - R software and data mining