### Assignment2\_Description

Neli Noykova June 11, 2017

### MULTIPLE CORRESPONDANCE ANALYSIS - Assignment 2

### The data

As in the previous Assignment 1, here we again use **Finnish** sample from ISSP 2012 survey "Family and Changing Gender Roles IV". Original data involve 1171 observations of 8 variables (4 substantive and 4 demographic). All variables are categorical.

The 4 **substantive** variables, which values are measured in 1-5 scale, are:

**A**: Married people are generally happier than unmarried people. **B**: People who want children ought to get married. **C**: It is all right for a couple to live together without intending to get married. **D**: Divorce is usually the best solution when a couple can't seem to work out their marriage problems.

The **demographic** variables are: **g**: gender (1=male, 2=female) **a**: age group (1=16-25, 2=26-35, 3=36-45, 4=46-55, 5=56-65, 6= 66+) **e**: education (1=Primary, 2=Comprehensive, primary and lower secondary, 3= Post-comprehensive, vocational school or course, 4=General upper secondary education or certificate, 5= Vocational post-secondary non-tertiary education, 6=Polytechnics, 7= University, lower academic degree, BA, 8=University, higher academic degree, MA **p**: Living in steady partnership (1=Yes, have partner; live in same household, 2=Yes, have partner; don't live in same household, 3=No partner)

Here we do not provide preliminary data treatment. We include all original variables, and also all missing data, denoted with number 9. During the data analyses we simply assign a new category, 9, to the missing data.

### Graphical overview of the data and summaries of the variables

The original data, including missing values denoted by number 9, look as:

```
Finland2 <- read.table("FinlandWithMissing.txt")</pre>
head(Finland2)
     ABCDgaep
## 1 3 3 1 2 1 2 4 3
## 2 3 2 3 2 1 4 2 3
## 3 3 3 1 3 1 3 8 1
## 4 9 4 1 1 2 2 4 2
## 5 3 2 2 3 2 2 6 1
## 6 9 4 2 1 2 5 6 3
dim(Finland2)
## [1] 1171
               8
str(Finland2)
                    1171 obs. of 8 variables:
## 'data.frame':
## $ A: int 3 3 3 9 3 9 2 1 3 3 ...
## $ B: int 3 2 3 4 2 4 2 1 3 3 ...
```

```
## $ C: int 1 3 1 1 2 2 2 1 1 1 ...

## $ D: int 2 2 3 1 3 1 3 9 3 2 ...

## $ g: int 1 1 1 2 2 2 2 2 1 2 ...

## $ a: int 2 4 3 2 2 5 4 2 3 1 ...

## $ e: int 4 2 8 4 6 6 5 5 7 4 ...

## $ p: int 3 3 1 2 1 3 1 3 3 3 ...
```

# Task 1: Multiple correspondence analysis (MCA) on the four substantive questions. Comparing the results with the case of excluded missing data.

As it was noted during the lectures, since we analyze the data at the nominal level, a missing value is treated as an additional category (in the example here category 9), which is included during the subset analysis.

We show the results for both data sets (without and with missing data) using the default symmetric plot, where both coordinates are principle. We recall the main formulas from MCA (from lecture slides) in order to show the mathematical expression of these coordinates.

If we denote the data table (Finland.txt or FinlandWithMissing.txt) as N, then the correspondence matrix P is obtained as:

$$\mathbf{P} = \left(\frac{1}{n}\right) \mathbf{N}$$

If we denote the row and column marginal totals (masses) of  $\bf P$  as  $\bf r$  and  $\bf c$  respectively, and

 $\mathbf{D}_{r}$ 

and

 $D_{c}$ 

are the diagonal matrices of these masses, then after applying singular value decomposition (SVD) we obtain:

$$\mathbf{S} = \mathbf{D_r^{(-1/2)}} \big( \mathbf{P} - \mathbf{r} \mathbf{c^T} \big) \mathbf{D_c^{(-1/2)}}$$

which is equivalent to

$$\mathbf{S} = \mathbf{D_r^{(1/2)}} \Big( \mathbf{D_r^{(-1)}PD_c^{(-1)}} - \mathbf{11^T} \Big) \mathbf{D_c^{(1/2)}}$$

According SVD

$$S = UD_{\alpha}V^{T}$$

Then the principal coordinates are presented as: - for rows:

$$\mathbf{F} = \mathbf{D_r^{(-1/2)}UD_{lpha}}$$

- for columns:

$$\mathbf{G} = \mathbf{D_c^{(-1/2)}VD_\alpha}$$

The total variance (inertia) is the sum of squares of the elements of

$$\mathbf{S} = \mathbf{trace}\left(\mathbf{S}\mathbf{S^T}\right) = \chi^{\mathbf{2}}/\mathbf{n}$$

The Chi-square distance

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(fo_{ij} - fe_{ij})^2}{fe_{ij}}$$

In this expression the observed and expected frequencies of the cell in row i and column j are denoted by

 $fo_{ij}$ 

and

 $fe_{ij}$ 

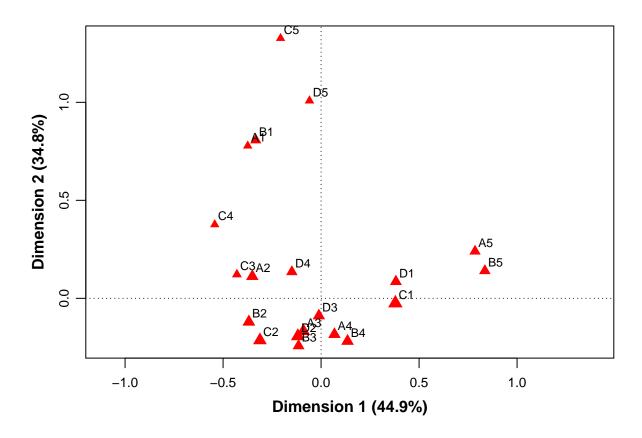
respectively.

We first provide MCA on data Finland.txt, where missed data are excluded and draw the default symmetric plot.

```
require(ca)
```

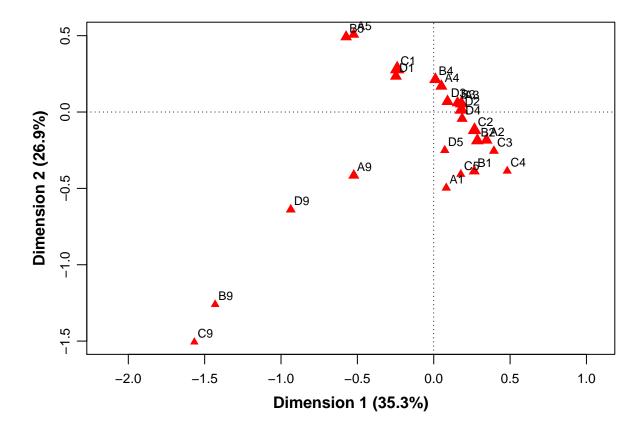
```
## Loading required package: ca
```

```
Finland <- read.table("Finland.txt")
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), cex.axis=0.8, font.lab=2, mfrow=c(1,1))
plot(mjca(Finland[,1:4], ps=""), mass=c(F,T))</pre>
```



Next perform the same MCA analysis and plot, but using also missed data as separate category = 9:

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), cex.axis=0.8, font.lab=2, mfrow=c(1,1))
plot(mjca(Finland2[,1:4], ps=""), mass=c(F,T))
```



We observe that the missing category 9 for all 4 substantive questions is situated on the right down part of the biplot, where both principal coordinates take negative values.

The symmetric plot represents the row and column profiles simultaneously in a common space. According Mike Bendixen (Marketing Bulletin, 2003, 14) this plot may lead to misinterpretation if examined in isolation or only visually because principal coordinates are presented for both rows and columns. These coordinates represent the row and column profiles and not the apexes for which the standard coordinates are required. Therefore in such cases asymmetric plots are recommended.

## Task 2: Burt matrix. Performing CA using a subset of non-missing rows and columns. Confidence regions of the demographic groups.

The part of Burt matrix on first 4 substantive questions can be obtained as:

```
require(ca)
Finland2.B <- mjca(Finland2[,1:4])$Burt</pre>
```

Since the missing values are involved in this matrix as separate category, we need to take the following part of Burt matrix:

```
Finland2.ABCD = Finland2.B[c(1:24), c(1:24)]
Finland2.ABCD

## A:1 A:2 A:3 A:4 A:5 A:9 B:1 B:2 B:3 B:4 B:5 B:9 C:1 C:2 C:3 C:4 C:5
```

```
## A:3
              0 344
                        0
                            0
                                 0
                                    28 100 105 87
                                                      22
                                                            2 153 134
                                                                         32
                                                                              14
## A:4
          0
              0
                   0 231
                            0
                                 0
                                    16
                                         47
                                             30 102
                                                       33
                                                            3 112
                                                                    93
                                                                         11
                                                                              10
                                                                                   3
## A:5
                        0 152
                                 0
                                     9
                                         12
                                               7
                                                  27
                                                       96
                                                            1 121
                                                                     17
                                                                          1
                                                                               0
                                                                                   12
## A:9
          0
                   0
                        0
                            0 152
                                    12
                                         41
                                             23
                                                  23
                                                       28
                                                           25
                                                                69
                                                                     49
                                                                          8
                                                                               3
                                                                                   4
              0
## B:1
         32
             53
                  28
                      16
                            9
                                12
                                   150
                                          0
                                               0
                                                   0
                                                        0
                                                            0
                                                                29
                                                                     41
                                                                         21
                                                                              18
                                                                                   38
## B:2
             99 100
                           12
                                41
                                     0 313
                                               0
                                                   0
                                                            0
                                                                66 175
                                                                              29
        14
                      47
                                                        0
                                                                         34
                                                                                    4
## B:3
                            7
                                     0
                                          0 208
                                                   0
                                                            0
                                                                98
          5
             38 105
                      30
                                23
                                                        0
                                                                     88
                                                                         19
                                                                               1
                                                                                    1
             27
                           27
## B:4
          4
                  87 102
                                23
                                     0
                                          0
                                               0 270
                                                        0
                                                            0 161
                                                                     99
                                                                          4
                                                                               3
                                                                                    1
## B:5
          1
             12
                  22
                      33
                           96
                                28
                                     0
                                          0
                                               0
                                                   0 192
                                                            0 178
                                                                      6
                                                                          0
                                                                               0
                                                                                    6
                   2
                                25
                                                   0
                                                                                    0
## B:9
          3
              4
                        3
                            1
                                     0
                                          0
                                               0
                                                        0
                                                           38
                                                               13
                                                                      8
                                                                          1
                                                                               0
## C:1
         21
             69 153 112 121
                                69
                                    29
                                         66
                                             98 161 178
                                                           13 545
                                                                      0
                                                                          0
                                                                                    0
## C:2
         15 109 134
                      93
                                49
                                    41 175
                                                  99
                                                                 0 417
                                                                          0
                                                                               0
                                                                                    0
                           17
                                             88
                                                        6
                                                            8
                                                                         79
## C:3
          7
             20
                  32
                      11
                            1
                                 8
                                    21
                                         34
                                             19
                                                   4
                                                        0
                                                            1
                                                                 0
                                                                      0
                                                                               0
                                                                                   0
## C:4
          7
                            0
                                 3
                                                   3
                                                                 0
                                                                      0
             17
                  14
                      10
                                    18
                                         29
                                               1
                                                        0
                                                            0
                                                                          0
                                                                              51
                                                                                    0
## C:5
             18
                   7
                        3
                           12
                                 4
                                    38
                                          4
                                                        6
                                                            0
                                                                 0
                                                                      0
                                                                          0
                                                                               0
                                                                                   50
          6
                                               1
                                                   1
## C:9
          3
              0
                   4
                        2
                            1
                                19
                                     3
                                          5
                                               1
                                                   2
                                                        2
                                                           16
                                                                 0
                                                                      0
                                                                          0
                                                                               0
                                                                                   0
## D:1
             31
                 58
                                26
                                    30
                                         39
                                             34
                                                  46
                                                       58
                                                            9 154
                                                                          8
                                                                               4
                                                                                    4
         15
                      45
                           41
                                                                     43
## D:2
         17
             97 121
                      86
                           44
                                47
                                    32 146
                                             82 104
                                                       45
                                                            3 158 208
                                                                         27
                                                                              13
                                                                                    3
## D:3
             49
                  96
                           27
                                                            4 110
                                                                                   7
          6
                      41
                                25
                                    24
                                         61
                                             55
                                                  59
                                                       41
                                                                     83
                                                                         28
                                                                              13
## D:4
          7
             37
                  43
                      44
                           12
                                16
                                    33
                                         39
                                             26
                                                  41
                                                       19
                                                                65
                                                                     55
                                                                          9
                                                                              13
                                                                                  16
## D:5
          7
             13
                  14
                        5
                           13
                                 6
                                    25
                                         11
                                               3
                                                   9
                                                        9
                                                            1
                                                                23
                                                                      4
                                                                          6
                                                                               7
                                                                                   17
## D:9
                  12
                      10
                           15
                               32
                                      6
                                         17
                                               8
                                                  11
                                                      20
                                                           20
                                                                35
                                                                     24
                                                                                    3
              6
                                                                          1
##
       C:9 D:1 D:2 D:3 D:4 D:5 D:9
## A:1
          3
                  17
                        6
                            7
                                 7
                                     7
             15
                                      6
## A:2
          0
             31
                 97
                      49
                           37
                                13
## A:3
          4
             58 121
                      96
                           43
                                14
                                    12
## A:4
          2
             45
                  86
                      41
                           44
                                 5
                                    10
                  44
## A:5
          1
             41
                      27
                           12
                                13
                                    15
## A:9
         19
             26
                  47
                      25
                           16
                                 6
                                    32
## B:1
          3
             30
                 32
                      24
                           33
                                25
                                     6
## B:2
          5
             39 146
                      61
                           39
                                11
                                    17
## B:3
          1
             34
                 82
                      55
                           26
                                 3
                                     8
## B:4
          2
             46 104
                      59
                           41
                                 9
                                    11
## B:5
             58
                 45
                                    20
          2
                      41
                           19
                                 9
## B:9
         16
              9
                   3
                        4
                            1
                                 1
                                    20
                           65
## C:1
          0 154 158 110
                                23
                                    35
## C:2
          0
             43 208
                      83
                           55
                                    24
## C:3
          0
              8
                  27
                      28
                            9
                                 6
                                     1
## C:4
          0
              4
                  13
                      13
                           13
                                 7
                                     1
## C:5
          0
              4
                   3
                        7
                           16
                                17
                                     3
## C:9
         29
              3
                   3
                                    18
                        3
                            1
                                 1
## D:1
          3 216
                   0
                        0
                            0
                                 0
                                     0
                            0
## D:2
          3
              0 412
                        0
                                 0
                                     0
                            0
                                 0
                                      0
## D:3
          3
              0
                   0 244
                        0 159
                                 0
                                      0
## D:4
          1
              0
                   0
## D:5
                   0
                                     0
          1
              0
                        0
                            0
                                58
## D:9 18
              0
                   0
                        0
                            0
                                 0
                                    82
```

### summary(Finland2.ABCD)

```
##
        A:1
                        A:2
                                        A:3
                                                         A:4
##
   Min. : 0.000
                   Min. : 0.00
                                    Min. : 0.00
                                                    Min. : 0.00
   1st Qu.: 2.500
##
                   1st Qu.: 3.00
                                    1st Qu.: 3.50
                                                    1st Qu.: 2.75
                                                    Median : 13.50
##
   Median : 6.500
                   Median : 19.00
                                    Median : 25.00
   Mean : 9.833
                   Mean : 38.83
                                   Mean : 57.33
                                                    Mean : 38.50
```

```
3rd Qu.:14.250
                      3rd Qu.: 50.00
                                        3rd Qu.: 97.00
                                                          3rd Qu.: 45.50
##
                             :233.00
##
    Max.
           :59.000
                                        Max.
                                               :344.00
                                                          Max.
                                                                 :231.00
                      Max.
##
         A:5
                           A:9
                                             B:1
                                                               B:2
                                                                  : 0.00
##
    Min.
           :
              0.00
                      \mathtt{Min}.
                                0.00
                                        Min.
                                               :
                                                  0.00
                                                          Min.
##
    1st Qu.: 0.75
                      1st Qu.: 3.75
                                        1st Qu.: 5.25
                                                          1st Qu.: 4.75
##
    Median : 12.00
                      Median : 21.00
                                        Median : 22.50
                                                          Median: 31.50
##
    Mean : 25.33
                      Mean : 25.33
                                        Mean : 25.00
                                                          Mean : 52.17
                      3rd Qu.: 29.00
                                        3rd Qu.: 32.00
##
    3rd Qu.: 27.00
                                                          3rd Qu.: 62.25
##
    Max.
           :152.00
                      Max.
                             :152.00
                                        Max.
                                               :150.00
                                                          Max.
                                                                 :313.00
##
         B:3
                           B:4
                                             B:5
                                                              B:9
##
    Min.
           : 0.00
                             : 0.00
                                                                 : 0.000
                      Min.
                                        Min.
                                               : 0.0
                                                         Min.
    1st Qu.: 1.00
                      1st Qu.: 1.75
                                                         1st Qu.: 0.000
##
                                        1st Qu.:
                                                  0.0
##
    Median : 13.50
                      Median: 17.00
                                        Median: 10.5
                                                         Median : 2.500
                                        Mean
##
    Mean
          : 34.67
                      Mean
                             : 45.00
                                               : 32.0
                                                         Mean
                                                               : 6.333
    3rd Qu.: 42.25
                      3rd Qu.: 66.00
                                                         3rd Qu.: 8.250
##
                                        3rd Qu.: 35.0
##
    Max.
           :208.00
                            :270.00
                                        Max.
                                               :192.0
                                                         Max.
                                                                 :38.000
                      Max.
         C\!:\!1
                           C:2
                                                             C:4
##
                                            C:3
           : 0.00
                                0.0
                                              : 0.00
##
    Min.
                      Min.
                                       Min.
                                                        Min.
                                                                : 0.0
##
    1st Qu.: 19.00
                      1st Qu.: 5.5
                                       1st Qu.: 0.75
                                                        1st Qu.: 0.0
##
    Median: 67.50
                      Median: 42.0
                                       Median : 7.50
                                                        Median: 3.5
##
    Mean
          : 90.83
                      Mean
                            : 69.5
                                       Mean
                                              :13.17
                                                        Mean
                                                               : 8.5
##
    3rd Qu.:129.00
                      3rd Qu.: 94.5
                                       3rd Qu.:20.25
                                                        3rd Qu.:13.0
##
    Max.
           :545.00
                             :417.0
                                              :79.00
                                                        Max.
                                                               :51.0
                      Max.
                                       Max.
         C:5
                           C:9
##
                                             D:1
                                                               D:2
##
    Min.
           : 0.000
                      Min.
                             : 0.000
                                        Min.
                                                : 0.00
                                                          Min.
                                                                  : 0.00
##
    1st Qu.: 0.750
                      1st Qu.: 0.750
                                        1st Qu.: 3.75
                                                          1st Qu.: 3.00
    Median : 4.000
##
                      Median : 2.000
                                        Median : 28.00
                                                          Median : 38.00
           : 8.333
                                               : 36.00
##
    Mean
                             : 4.833
                                                                 : 68.67
                      Mean
                                        Mean
                                                          Mean
##
    3rd Qu.: 8.250
                      3rd Qu.: 3.250
                                        3rd Qu.: 43.50
                                                          3rd Qu.: 98.75
##
           :50.000
                             :29.000
                                               :216.00
                                                                  :412.00
    Max.
                      Max.
                                        Max.
                                                          Max.
##
         D:3
                           D:4
                                            D:5
                                                              D:9
##
    Min.
           : 0.00
                             : 0.0
                                              : 0.000
                                                                : 0.00
                      Min.
                                       Min.
                                                         Min.
    1st Qu.: 3.75
                      1st Qu.: 1.0
                                       1st Qu.: 1.000
                                                         1st Qu.: 1.00
                      Median: 16.0
##
    Median : 26.00
                                       Median : 6.500
                                                         Median: 9.00
           : 40.67
                             : 26.5
##
    Mean
                      Mean
                                       Mean
                                              : 9.667
                                                         Mean
                                                               :13.67
##
    3rd Qu.: 56.00
                      3rd Qu.: 39.5
                                       3rd Qu.:13.000
                                                         3rd Qu.:18.50
    Max.
           :244.00
                      Max.
                             :159.0
                                       Max.
                                              :58.000
                                                         Max.
                                                                 :82.00
```

For our work we need also to calculate the indicator matrix, obtained also via the MCA function mjca:

```
Finland2.Z <- mjca(Finland2[,1:4], ps="", reti=T)$indmat
head(Finland2.Z)</pre>
```

```
##
     A1 A2 A3 A4 A5 A9 B1 B2 B3 B4 B5 B9 C1 C2 C3 C4 C5 C9 D1 D2 D3 D4 D5 D9
                 0
                    0
                            0
                               0
                                   1
                                      0
                                          0
                                             0
                                                 1
                                                           0
                                                                         1
                                                                             0
      0
          0
              1
                 0
                    0
                        0
                                   0
                                      0
                                         0
                                             0
                                                 0
                                                    0
                                                               0
                                                                  0
                                                                      0
                                                                                0
                                                                                    0
                                                                                       0
                            0
                               1
                                                        1
                                                           0
                                                                         1
                                                                             0
      0
          0
             1
                 0
                    0
                        0
                            0
                               0
                                   1
                                      0
                                         0
                                             0
                                                 1
                                                    0
                                                        0
                                                           0
                                                               0
                                                                  0
                                                                      0
                                                                         0
                                                                             1
                                                                                0
                                                                                    0
                                                                                       0
      0
          0
             0
                 0
                    0
                        1
                            0
                               0
                                   0
                                      1
                                         0
                                             0
                                                 1
                                                    0
                                                        0
                                                           0
                                                               0
                                                                  0
                                                                      1
                                                                         0
                                                                             0
                                                                                0
                                                                                       0
      0
          0
              1
                 0
                    0
                        0
                            0
                               1
                                   0
                                      0
                                         0
                                             0
                                                 0
                                                    1
                                                        0
                                                           0
                                                               0
                                                                  0
                                                                      0
                                                                         0
                                                                             1
                                                                                0
                                                                                    0
                                                                                       0
      0
         0
             0
                 0
                    0
                        1
                           0 0 0 1
                                         0 0
                                                0
                                                    1
                                                       0
                                                           0
                                                              0
                                                                  0
                                                                      1
                                                                         0
                                                                             0
```

dim(Finland2.Z)

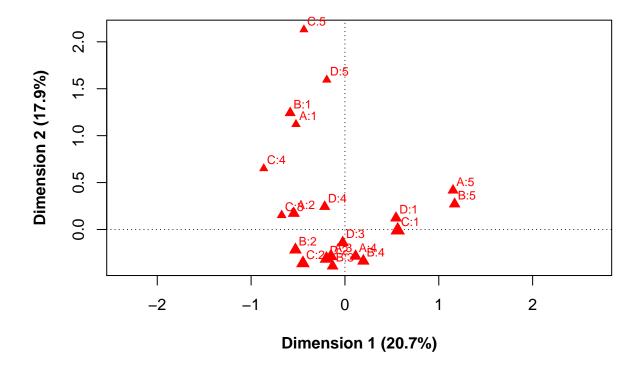
## [1] 1171 24

The relationship between both matrices is:

 $\mathbf{B} = \mathbf{Z}^{\mathbf{T}}\mathbf{Z}$ 

The non-missing categories are obtained by excluding the missing ones. Further analysis is provided using subset CA:

```
Finland2.nonmissing <- -c(6,12,18,24)
Finland2.mca1 <- ca(Finland2.B, subsetrow=c(Finland2.nonmissing), subsetcol=c(Finland2.nonmissing))
plot(Finland2.mca1, what=c("none", "all"), mass=c(F,T), font.lab=2)
```



### summary(Finland2.mca1)

```
##
## Principal inertias (eigenvalues):
##
##
    dim
           value
                        %
                            cum%
                                    scree plot
           0.243700
                      20.7
                             20.7
##
    1
##
    2
           0.210498
                      17.9
                             38.6
##
    3
           0.103194
                       8.8
                             47.4
           0.095390
##
    4
                       8.1
                             55.5
           0.075342
                             61.9
##
    5
           0.070659
##
    6
                       6.0
                             67.9
##
    7
           0.062970
                             73.3
##
    8
           0.057778
                        4.9
                             78.2
    9
           0.049824
                        4.2
                             82.4
##
##
    10
           0.047380
                        4.0
                             86.5
##
    11
           0.043140
                        3.7
                             90.1
           0.036612
##
    12
                        3.1
                             93.2
##
    13
           0.029151
                        2.5
                             95.7
##
    14
           0.022491
                        1.9
                             97.6
```

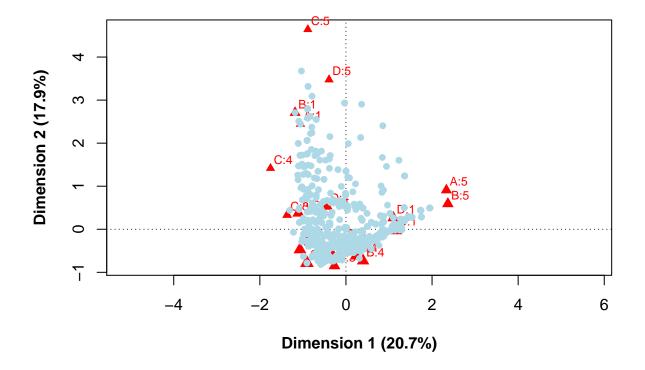
```
##
   15
          0.017302
                     1.5 99.1
##
          0.008924
                    0.8 99.9
   16
##
   17
          0.001268
                     0.1 100.0
##
   18
          0.000285
                     0.0 100.0
##
   19
          5.3e-050
                     0.0 100.0
##
          1.3e-050
                     0.0 100.0
   20
          -----
##
   Total: 1.175975 100.0
##
##
## Rows:
##
                              k=1 cor ctr
                                             k=2 cor ctr
       name
              mass qlt inr
## 1 |
         A1 |
                13
                    293
                          56 | -522 52
                                        14 | 1121 241
                                                       75 I
## 2 |
         A2 |
                   299
                          46 | -545 271 61 | 172 27
                50
                                                        7 |
## 3 |
         A3 |
                73
                    166
                          39 | -149 36
                                         7 | -285 130
                                                       28 I
## 4
     A4 |
                49
                    88
                          45 | 113 12
                                          3 | -284
                                                   76
                                                        19 |
## 5
    - 1
         A5 |
                    680
                          61 | 1152 602 177 | 417
                                                   79
                32
                                                        27 |
## 6
    B1 |
                32
                    805
                          64 | -584 145
                                        45 | 1243 660 235 |
## 7
                    397
                          47 | -528 340
                                        76 | -216 57
     B2 |
                67
                                                       15 l
## 8 |
         B3 |
                44
                    136
                          47 | -132 14
                                          3 | -390 122
                                                        32 I
## 9 |
         B4 |
                58
                    163
                          45 | 196 42
                                          9 | -336 122
                                                       31 l
## 10 |
         B5 |
                    782
                          64 | 1169 742 230 | 271 40
                41
                          39 | 563 796 151 | -8 0
## 11 |
         C1 |
                    797
               116
                                                        0 1
## 12 |
         C2 |
                    588
                          42 | -448 358
                                        73 | -360 231
                89
                                                        55 I
                                                        2 |
## 13 |
                                        31 | 152
         C3 |
                17
                    130
                          53 | -674 123
                                                    6
## 14 |
         C4 I
                11
                    197
                          55 | -865 126
                                        33 | 650 71
                                                       22 I
## 15 |
         C5 |
                    658
                          65 | -437
                                    27
                                         8 | 2130 631 230 |
                11
                          47 | 544 250
                                        56 | 122 13
## 16 |
         D1 |
                46
                    262
                                                        3 I
                          37 | -197
## 17 |
         D2 |
                88
                    277
                                    79
                                        14 | -312 198
                                                       41 l
## 18 |
         D3 |
                52
                     21
                          43 | -24
                                     1
                                         0 | -140 20
                                                        5 I
                                          6 | 243 36
## 19 |
         D4 |
                34
                     65
                          47 | -216
                                     28
                                                       10 |
## 20 |
         D5 |
                12
                   467
                          58 | -194
                                     7
                                          2 | 1595 461 150 |
##
## Columns:
## name
              mass qlt
                         inr k=1 cor ctr
                                            k=2 cor ctr
## 1 |
                13 293
                          56 | -522 52 14 | 1121 241
         A1 l
                                                      75 I
## 2 |
         A2 |
                50
                    299
                          46 | -545 271 61 | 172 27
## 3 |
         A3 |
                73
                    166
                          39 | -149 36
                                         7 | -285 130
                                                       28 I
                          45 | 113 12
## 4 |
         A4 |
                49
                    88
                                          3 | -284
                                                   76
                                                        19 I
## 5 |
         A5 |
                32
                    680
                          61 | 1152 602 177 | 417
                                                   79
                                                       27 I
                    805
                          64 | -584 145
                                        45 | 1243 660 235 |
## 6 |
         B1 |
                32
## 7 |
         B2 |
                67
                    397
                          47 | -528 340 76 | -216 57
                                                       15 l
                          47 | -132 14
                                          3 | -390 122
## 8 |
         B3 |
                44
                    136
                                                        32 I
## 9 |
                          45 | 196 42
                                          9 | -336 122
         B4 |
                58
                    163
                                                       31 l
                    782
                          64 | 1169 742 230 | 271 40
## 10 |
         B5 |
                41
## 11 |
         C1 |
                    797
                          39 | 563 796 151 | -8 0
               116
                                                        0 |
                                        73 | -360 231
## 12 |
         C2 |
                89
                    588
                          42 | -448 358
                                                        55 I
## 13 |
                    130
                          53 | -674 123
                                        31 | 152
                                                    6
                                                        2 |
         C3 |
                17
## 14 |
         C4 |
                11
                    197
                          55 | -865 126
                                         33 | 650 71
                                                       22 |
                          65 | -437
                                         8 | 2130 631 230 |
## 15 |
         C5 |
                11
                    658
                                    27
## 16 |
         D1 |
                    262
                          47 | 544 250
                                        56 | 122
                                                        3 I
                46
                                                  13
                          37 | -197 79
## 17 |
         D2 |
                88
                   277
                                        14 | -312 198
## 18 |
         D3 |
                52
                     21
                          43 I -24
                                     1
                                         0 | -140 20
                                                        5 I
## 19 |
                          47 | -216 28
                                          6 | 243 36 10 |
         D4 |
                34
                    65
```

```
## 20 | D5 | 12 467 58 | -194 7 2 | 1595 461 150 |
```

This plot is quite similar to the plot, obtained when missing data are removed from the data set.

Next we add responding points to the plot. For this purpose we first transform the corresponding frequencies in the table and draw a sequence of points at the specified coordinates:

```
Finland2.sum <- apply(Finland2.Z[,c(Finland2.nonmissing)], 1, sum)
Finland2.sum[Finland2.sum==0] <- 1
Finland2.rpc <- Finland2.Z[,c(Finland2.nonmissing)] %*% Finland2.mca1$colcoord / Finland2.sum
plot(Finland2.mca1, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc, pch=19, col="lightblue", cex=0.8)</pre>
```

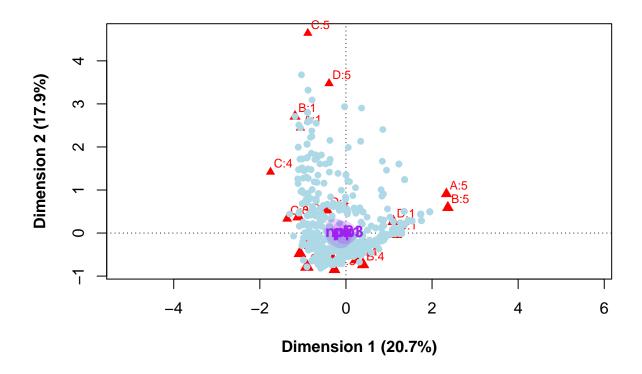


Next we compute confidence regions of demographic groups. For this purpose we have to add group ellipses and use Prof. Greenacre's program "confidence plots", which was given separately.

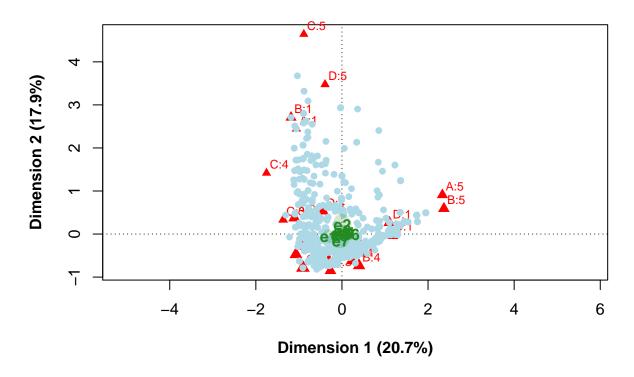
```
require(ellipse)

## Loading required package: ellipse
source("confidenceplots.R")
```

First we draw **confidence plots** for the demographic group **partnership** by generation asymmetric plot, adding points and finally draw confidence plots.

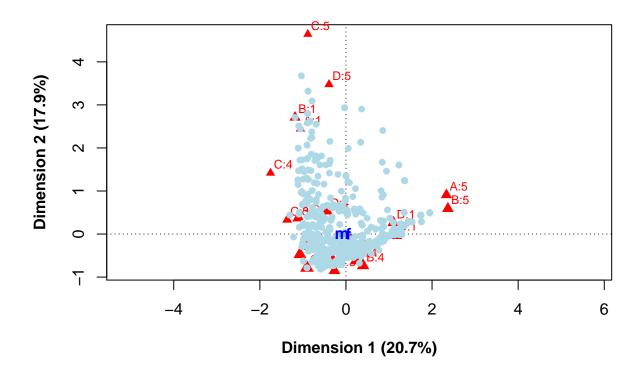


Next we draw confidence plot for the demographic group **education**.



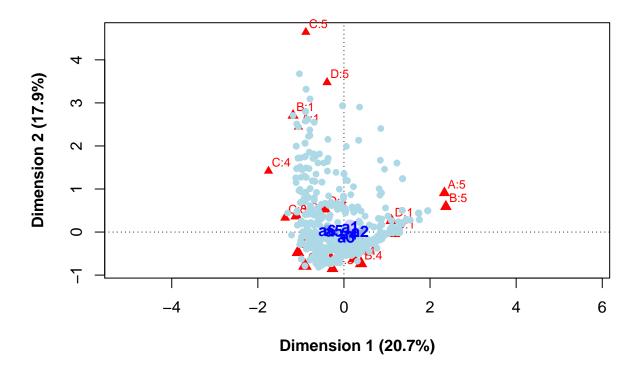
The confidence plot for the demographic group **gender** is:

```
plot(Finland2.mca1, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc, pch=19, col="lightblue", cex=0.8)
confidenceplots(Finland2.rpc[Finland2$g<3,1], Finland2.rpc[Finland2$g<3,2], group=Finland2$g[Finland2$g
groupnames=c("m","f"), shownames=T, add=T)</pre>
```



The confidence plot for the demographic group  $\mathbf{age}$  is:

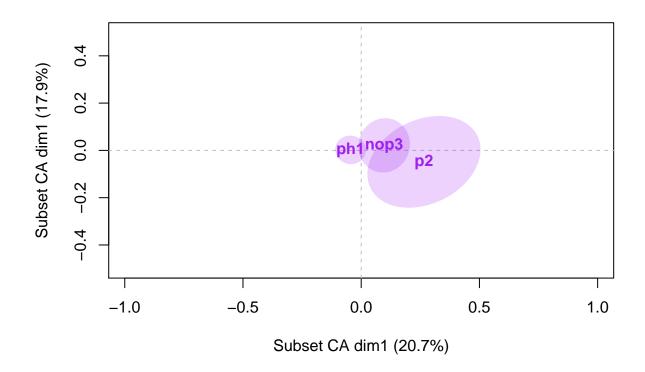
```
plot(Finland2.mca1, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc, pch=19, col="lightblue", cex=0.8)
confidenceplots(Finland2.rpc[Finland2$a<7,1], Finland2.rpc[Finland2$a<7,2], group=Finland2$a[Finland2$a
groupnames=c("a1","a2","a3","a4","a5","a6"), shownames=T, add=T)</pre>
```



If we plot **only the confidence plots (just the ellipses)** for every demographic group, the plots will look as follow:

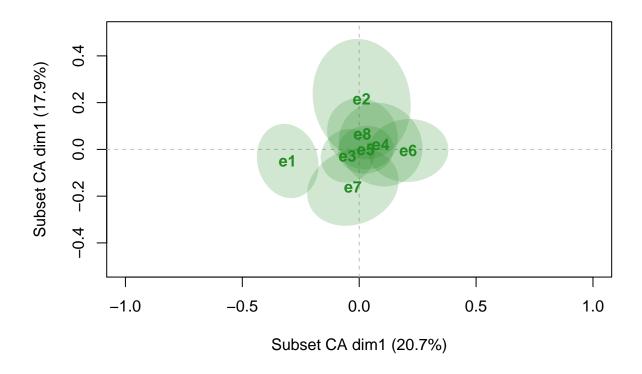
 $\bullet \;$  for the demographic group  ${\bf partnership} :$ 

```
plot(Finland2.rpc, type="n", asp=1, xlab="Subset CA dim1 (20.7%)", ylab="Subset CA dim1 (17.9%)", xlim=
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc[Finland2$p<4,1], Finland2.rpc[Finland2$p<4,2], group=Finland2$p[Finland2$p
groupnames=c("ph1","p2","nop3"), shownames=T, add=T)</pre>
```



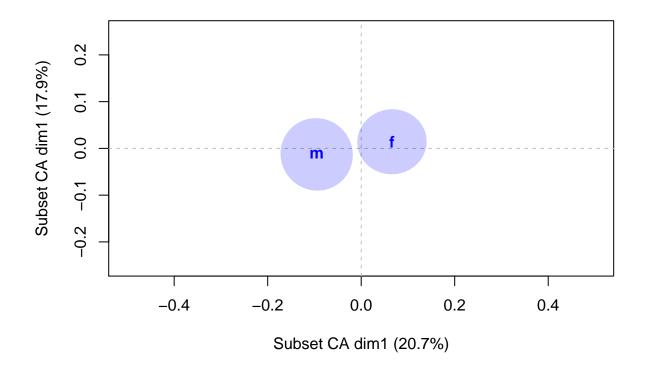
• for the demographic group **education**:

```
plot(Finland2.rpc, type="n", asp=1, xlab="Subset CA dim1 (20.7%)", ylab="Subset CA dim1 (17.9%)", xlim=
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc[Finland2$e<9,1], Finland2.rpc[Finland2$e<9,2], group=Finland2$e[Finland2$e
groupnames=c("e1","e2","e3","e4","e5","e6","e7","e8"), shownames=T, add=T)</pre>
```



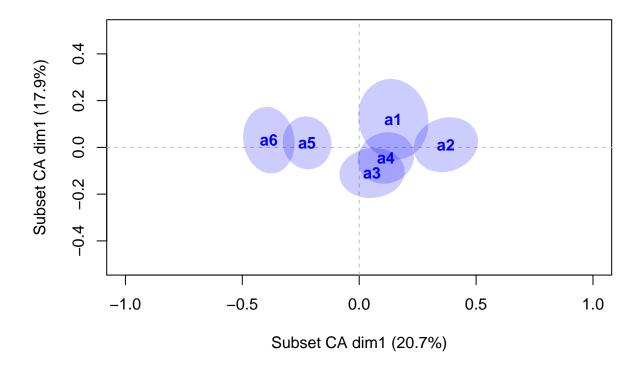
• for the demographic group **gender**:

```
plot(Finland2.rpc, type="n", asp=1, xlab="Subset CA dim1 (20.7%)", ylab="Subset CA dim1 (17.9%)", xlim=
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc[Finland2$g<3,1], Finland2.rpc[Finland2$g<3,2], group=Finland2$g[Finland2$g
groupnames=c("m","f"), shownames=T, add=T)</pre>
```



• for the demographic group **age**:

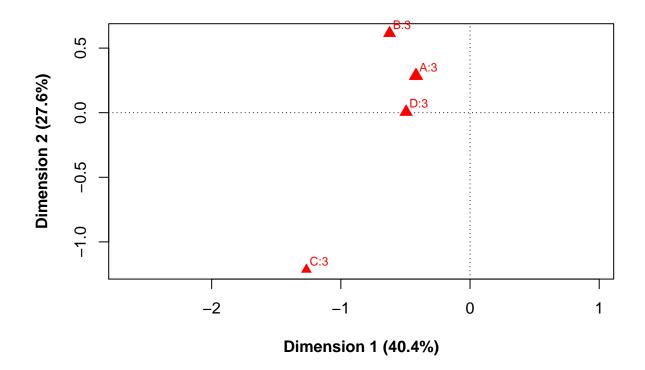
```
plot(Finland2.rpc, type="n", asp=1, xlab="Subset CA dim1 (20.7%)", ylab="Subset CA dim1 (17.9%)", xlim=
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc[Finland2$a<7,1], Finland2.rpc[Finland2$a<7,2], group=Finland2$a[Finland2$a
groupnames=c("a1","a2","a3","a4","a5","a6"), shownames=T, add=T)</pre>
```



Task 3: Analysis of the "middle" and "missing" values of the substantive variables. Confidence intervals of the demographic groups.

Analysis of "middle" category could be provided via CA of Burt matrix.

```
# investigation of "middle" category, via the Burt matrix
#seq() - generates a sequence of numbers
#seq(from=3, to=46, by=6): 3,9,15,21,27,33,39,45
Finland2.middle <- seq(3,22,6)
Finland2.mca2 <- ca(Finland2.B, subsetrow=Finland2.middle, subsetcol=seq(3,22,6))
#what - Vector of two character strings specifying the contents of the plot.
#First entry sets the rows and the second entry the columns. Allowed values # are "all" (all available
#"active" (only active points are displayed)
#"passive" (only supplementary points are displayed)
#"none" (no points are displayed)
#The status (active or supplementary) of columns is set in mjca using the option
#supcol.
# mass - area, first - rows, second - columns
#font.lab - labels for font?
plot(Finland2.mca2, what=c("none", "all"), mass=c(F,T), font.lab=2)</pre>
```



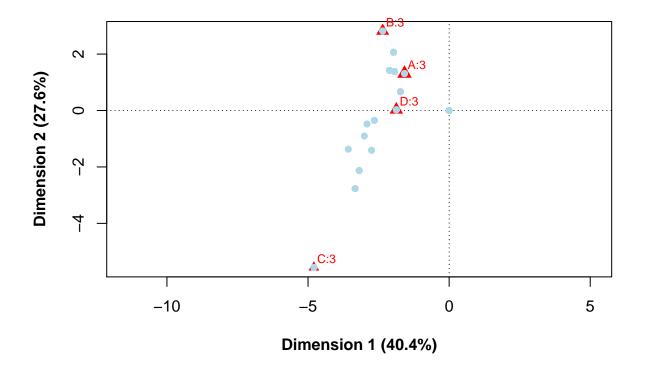
### summary(Finland2.mca2)

```
##
## Principal inertias (eigenvalues):
##
##
                       %
    dim
           value
                           cum%
                                   scree plot
##
    1
           0.069858
                      40.4 40.4
##
    2
           0.047719
                      27.6
                            68.0
                      19.8 87.8
##
           0.034199
           0.021173
                      12.2 100.0
##
##
    Total: 0.172949 100.0
##
##
##
## Rows:
                                                   k=2 cor ctr
##
              mass
                     qlt
                          inr
                                  k=1 cor ctr
       name
         A3 |
                73
                     563
                          194 | -418 384 184 |
## 2 |
         B3 |
                44
                     770
                          256 |
                                -623 389 246 |
                                                   616 381 353
## 3
         C3 |
                17
                     943
                          318 | -1267 491 387 | -1215 452 522
## 4 |
         D3 |
                     317
                          232 | -495 317 182 |
                                                      6
                52
                                                          0
                                                              0 |
##
## Columns:
##
       name
              mass
                     qlt
                          inr
                                  k=1 cor ctr
                                                   k=2 cor ctr
## 1 |
                73
                     563
                          194 |
                                -418 384 184 |
                                                   286 179 126
         A3 |
## 2 |
                     770
                          256 |
                                -623 389 246 |
                                                   616 381 353
         ВЗ
                 44
## 3 |
                          318 | -1267 491 387 | -1215 452 522 |
         C3 |
                17
                     943
```

```
## 4 | D3 | 52 317 232 | -495 317 182 | 6 0 0 |
```

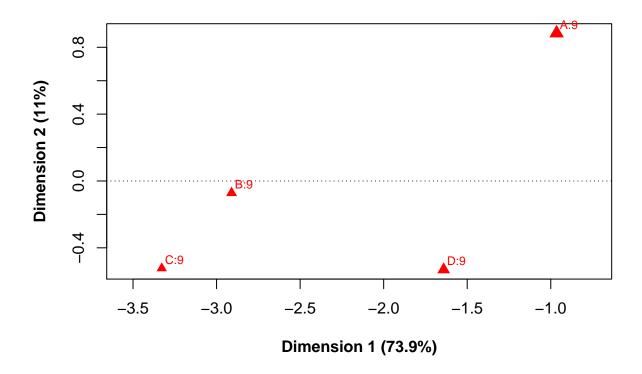
Adding respondent points only for middle categories:

```
#Finland2.Z - indicator matrix
# apply(variable, margin, function). - Returns a vector or array or list of values obtained by applying
#a function to margins of an array or matrix
#margin specifies if you want to apply by row (margin = 1),
#by column (margin = 2), or for each element (margin = 1:2).
Finland2.sum4 <- apply(Finland2.Z[,c(Finland2.middle)], 1, sum)
Finland2.sum4[Finland2.sum4==0] <- 1
Finland2.rpc5 <- Finland2.Z[,c(Finland2.middle)] %*% Finland2.mca2$colcoord / Finland2.sum4
plot(Finland2.mca2, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc5, pch=19, col="lightblue", cex=0.8)</pre>
```



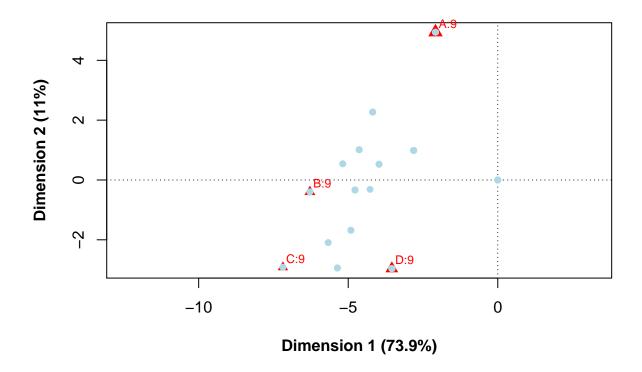
Only **missing** categories are obtained as:

```
Finland2.missing <- c(6,12,18,24)
Finland2.mca4 <- ca(Finland2.B, subsetrow=c(Finland2.missing), subsetcol=c(Finland2.missing))
plot(Finland2.mca4, what=c("none", "all"), mass=c(F,T), font.lab=2)
```



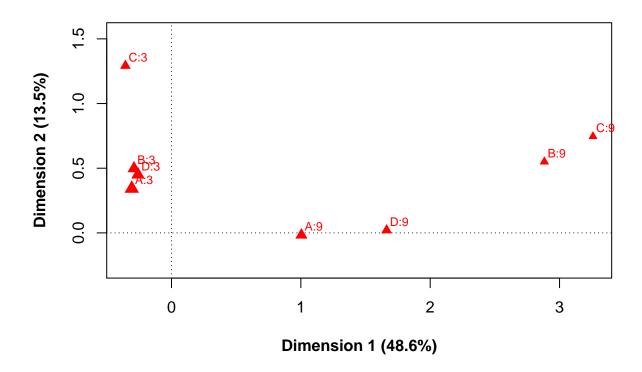
Adding respondent points only for **missing** categories:

```
Finland2.sum2 <- apply(Finland2.Z[,c(Finland2.missing)], 1, sum)
Finland2.sum2[Finland2.sum2==0] <- 1
Finland2.rpc3 <- Finland2.Z[,c(Finland2.missing)] %*% Finland2.mca4$colcoord / Finland2.sum2
plot(Finland2.mca4, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc3 , pch=19, col="lightblue", cex=0.8)</pre>
```



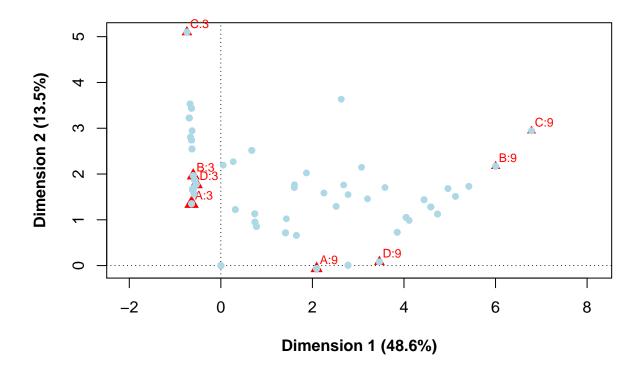
Both "middle" and missing categories are investigated again using CA of Burt matrix.

```
Finland2.missing <- c(6,12,18,24)
Finland2.mca3 <- ca(Finland2.B, subsetrow=c(Finland2.middle,Finland2.missing), subsetcol=c(Finland2.middle,Finland2.mca3, what=c("none", "all"), mass=c(F,T), font.lab=2)
```



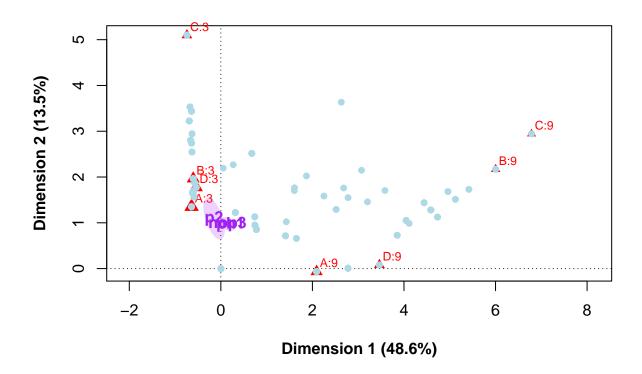
Adding respondent points only for both "middle" and missing categories:

```
Finland2.sum1 <- apply(Finland2.Z[,c(Finland2.middle,Finland2.missing)], 1, sum)
Finland2.sum1[Finland2.sum1==0] <- 1
Finland2.rpc2 <- Finland2.Z[,c(Finland2.middle,Finland2.missing)] %*% Finland2.mca3$colcoord / Finland2
plot(Finland2.mca3, what=c("none", "all"), mass=c(F,T), font.lab=2, map="rowprincipal")
points(Finland2.rpc2 , pch=19, col="lightblue", cex=0.8)</pre>
```

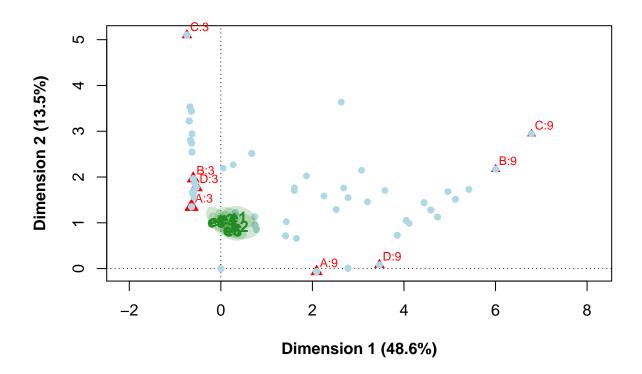


Next we compute confidence regions of demographic groups for both missing and "middle" groups.

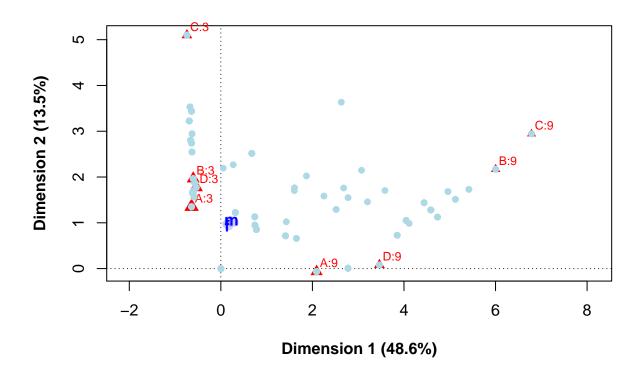
- confidence plot for the demographic group  ${\bf partnership}$ 



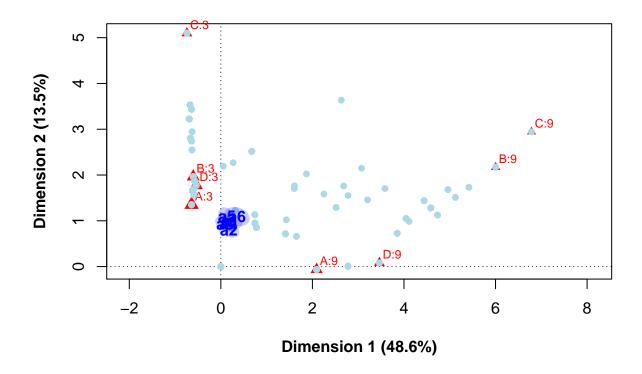
• confidence plot for the demographic group **education** 



 $\bullet\,$  confidence plot for the demographic group  ${\bf gender}$ 



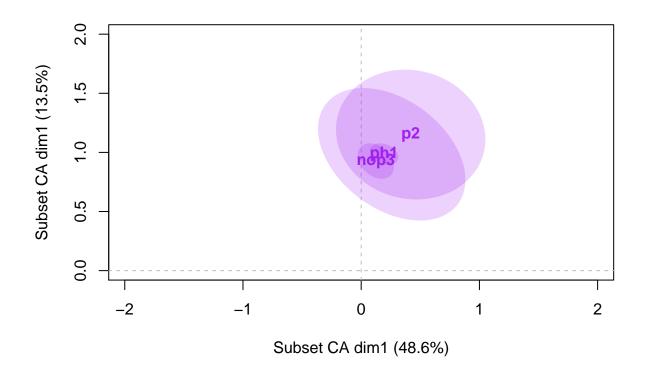
 $\bullet\,$  confidence plot for the demographic group  ${\bf age}$ 



Since from these plots the confidence regions are not so clear, similarly as in Task 2, we next plot "just the ellipses".

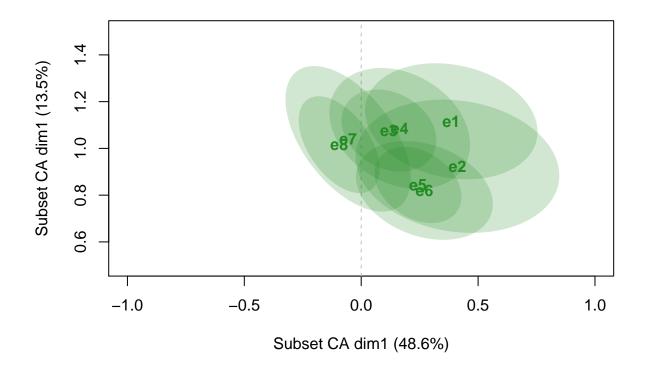
 $\bullet \;$  for the demographic group  ${\bf partnership} :$ 

```
plot(Finland2.rpc2, type="n", asp=1, xlab="Subset CA dim1 (48.6%)", ylab="Subset CA dim1 (13.5%)", xlim
abline(v=0, h=0, lty=2, col="grey")
#draw confidence plots
confidenceplots(Finland2.rpc2[Finland2$p<4,1], Finland2.rpc2[Finland2$p<4,2], group=Finland2$p[Finland2
groupnames=c("ph1","p2","nop3"), shownames=T, add=T)</pre>
```



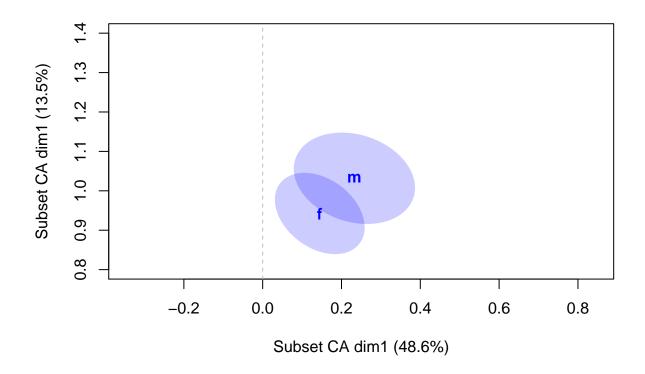
• for the demographic group **education**:

```
plot(Finland2.rpc2, type="n", asp=1, xlab="Subset CA dim1 (48.6%)", ylab="Subset CA dim1 (13.5%)", xlim
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc2[Finland2$e<9,1], Finland2.rpc2[Finland2$e<9,2], group=Finland2$e[Finland2
groupnames=c("e1","e2","e3","e4","e5","e6","e7","e8"), shownames=T, add=T)</pre>
```



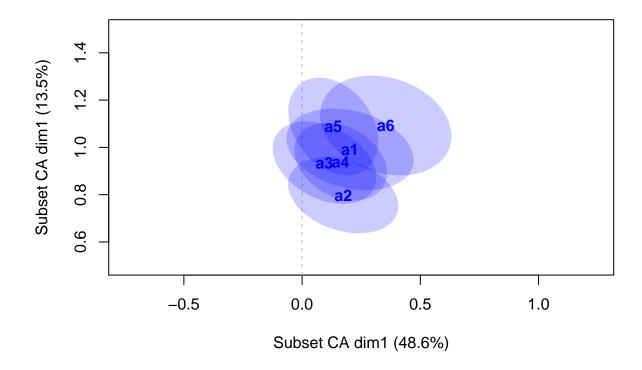
• for the demographic group **gender**:

```
plot(Finland2.rpc2, type="n", asp=1, xlab="Subset CA dim1 (48.6%)", ylab="Subset CA dim1 (13.5%)", xlim
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc2[Finland2$g<3,1], Finland2.rpc2[Finland2$g<3,2], group=Finland2$g[Finland2
groupnames=c("m","f"), shownames=T, add=T)</pre>
```



• for the demographic group **age**:

```
plot(Finland2.rpc2, type="n", asp=1, xlab="Subset CA dim1 (48.6%)", ylab="Subset CA dim1 (13.5%)", xlim
abline(v=0, h=0, lty=2, col="grey")
confidenceplots(Finland2.rpc2[Finland2$a<7,1], Finland2.rpc2[Finland2$a<7,2], group=Finland2$a[Finland2
groupnames=c("a1","a2","a3","a4","a5","a6"), shownames=T, add=T)</pre>
```



From all analyses, provided in Task 3, we can conclude that MCA is a powerful tool for investigating missing and "middle" categories. Both categories involve quite big amount of uncertainty, because of which most of the other methods just ignore these cases. Here we have seen how the information from both categories could be utilized applying MCA approach.

### Used and useful links

Package 'ca'

Oleg Nenadic and Michael Greenacre, Computation of Multiple Correspondence Analysis, with code in R Michael Greenacre, Biplots in practise

Multiple Correspondence Analysis Essentials: Interpretation and application to investigate the associations between categories of multiple qualitative variables - R software and data mining

Mike Bendixen, A Practical Guide to the Use of Correspondence Analysis in Marketing Research, Marketing Bulletin, 2003, 14, Technical Note 2.

 $[An\ Example\ R\ Markdown]\ (http://www.statpower.net/Content/310/R\%20Stuff/SampleMarkdown.html)\\ Writing\ Mathematic\ Fomulars\ in\ Markdown$