MULTIPLE CORRESPONDANCE ANALYSIS - Assignment 1

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The data

In this exercise set we use **Finnish** sample from ISSP 2012 survey "Family and Changing Gender Roles IV". Original data involve 1171 observations of 8 variables (4 substantive and 4 demographic). All variables are categorical.

The 4 **substantive** variables, which values are masured in 1-5 scale, are:

A: Married people are generally happier than unmarried people. **B**: People who want children ought to get married. **C**: It is all right for a couple to live together without intending to get married. **D**: Divorce is usually the best solution when a couple can't seem to work out their marriage problems.

The **demographic** variables are: **g**: gender (1=male, 2=female) **a**: age group (1=16-25, 2=26-35, 3=36-45, 4=46-55, 5=56-65, 6= 66+) **e**: education (1=Primary, 2=Comprehensive, primary and lower secondary, 3= Post-comprehensive, vocational school or course, 4=General upper secondary education or certificate, 5= Vocational post-secondary non-tertiary education, 6=Polytechnics , 7= University, lower academic degree, BA, 8=University, higher academic degree, MA **p**: Living in steady partnership (1=Yes, have partner; live in same household, 2=Yes, have partner; don't live in same household, 3=No partner)

The data wrangling includes the following changes:

1. The missing data ae removed (this has already been provided). The number of observations without missing data is N=924.
2. For Task 2 a combined variable **ga**, describing the intraction of gender and age categories, is formed as: **ga = 6*(g-1) + a**

Graphical overview of the data and summaries of the variables

The preliminary treated data look as:

```
Finland <- read.table("Finland.txt")
head(Finland)
```

```
##   A B C D g a e p
## 1 3 3 1 2 1 2 4 3
## 2 3 2 3 2 1 4 2 3
## 3 3 3 1 3 1 3 8 1
## 4 3 2 2 3 2 2 6 1
## 5 2 2 2 3 2 4 5 1
## 6 3 3 1 3 1 3 7 3
```

```
dim(Finland)
```

```
## [1] 924   8
```

```
str(Finland)
```

```
## 'data.frame':    924 obs. of  8 variables:
##  $ A: int  3 3 3 3 2 3 3 2 2 3 ...
##  $ B: int  3 2 3 2 2 3 3 3 3 3 ...
##  $ C: int  1 3 1 2 2 1 1 1 2 3 ...
##  $ D: int  2 2 3 3 3 3 2 2 3 3 ...
```

```
## $ g: int 1 1 1 2 2 1 2 2 2 2 ...
## $ a: int 2 4 3 2 4 3 1 2 5 4 ...
## $ e: int 4 2 8 6 5 7 4 8 7 5 ...
## $ p: int 3 3 1 1 1 3 3 3 1 3 ...
```

Task 1: Cross-tabulations and correspondance analysis (CA).

The goal is to discover interesting relations among the measurements on A, . . . , D, g, . . . , p. Correspondence analysis is a descriptive/exploratory technique designed to analyse simple two-way and multi-way tables containing some measure of correspondence between the rows and columns. CA may also be defined as a special case of Principal Components Analysis of the rows and columns of a table, especially applicable to a cross-tabulation. Principal components analysis is used for tables consisting of continuous measurement, whereas CA is applied to contingency tables (i.e. cross-tabulations). The goal of CA is to transform a table of numerical information into a graphical display, in which each row and each column is depicted as a point. Thus CA can be used for data visualization or data pre-processing before applying methods for supervised learning. We can apply this technique for dimensionality reduction.

First we provide a cross-tabulation between the age variable **a** and answer **A**: Married people are generally happier than unmarried people. The variable **a** consists of 6 age groups, corresponding to rownames of crosstable: **a1** = 16-25, **a2**=26-35, **a3** = 36-45, **a4** = 46-55, **a5** = 56-65, **a6** = 66+ The categorical variable **A** assume 5 possible values, which correspond to column names of cross-table: **SA** - strongly agree; **A** - agree, **NN** - neither agree or disagree, **D** - disagree, **SD** - strongly disagree.

```
tab <-table(Finland[,"a"], Finland[,"A"])

rownames(tab) <- c("a1","a2","a3","a4","a5","a6")
colnames(tab) <- c("SA","A","NN","D","SD")

tab
```

```
##
##      SA  A NN  D SD
##   a1  5 18 42 29 29
##   a2  6 25 44 34 34
##   a3  6 30 55 30 17
##   a4  9 37 60 46 34
##   a5 15 67 58 49 15
##   a6  5 37 56 26  6
```

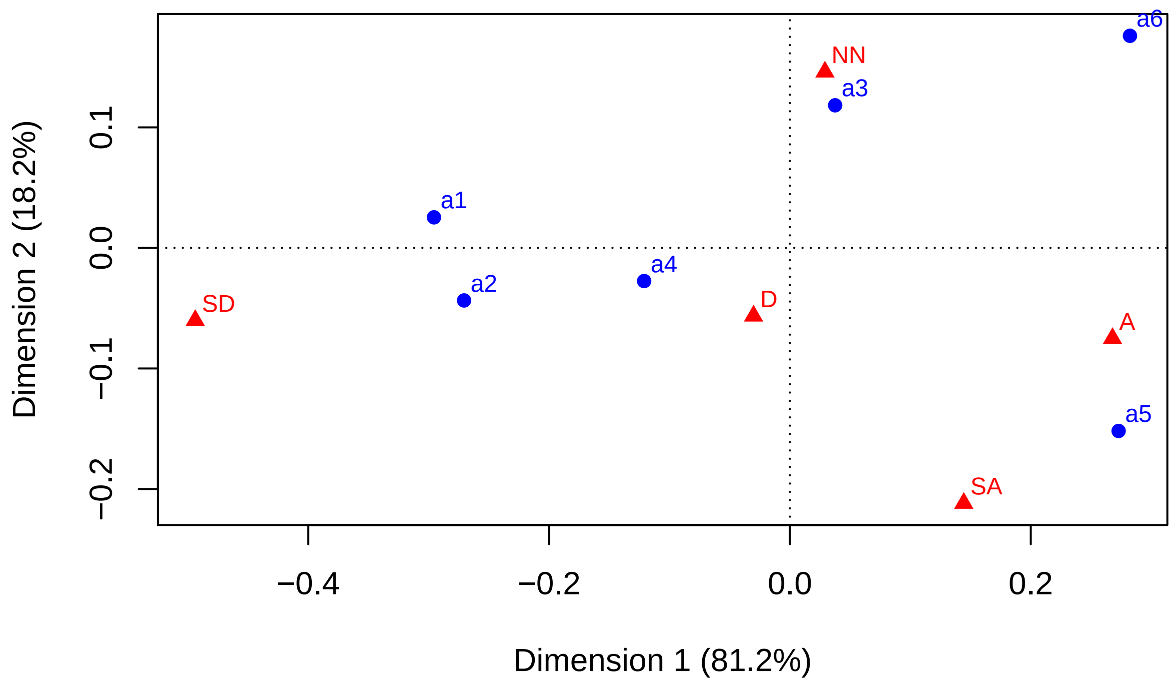
From this table we see that the biggest part of age group **a5** agree with question **A**. Since these values are absolute, we need more precise information wich could be obtained row and column profiles, calculated from this cross-table. We need also to calculate the chi-square statistic as it has been shown durint the second exercise session.

An alternative means of extracting the nature of the dependency between the rows and columns of the contingency table is to represent the row or column profiles graphically.

```
require(ca)
```

```
## Loading required package: ca
```

```
plot(ca(tab))
```



This plot is assymetric because the row profiles are plotted simultaneously with apexes representing the columns. **a3** is strongly "associated" with answer NN, which is clearly the case from the profile presented in the cross-table. Likewise we observe some proximity of **a5** to the apex representing **A** answer to question A. The fact that **a1**, **a2** and **a4** are positioned relatively far away from answers **SA** and **A** means that these younger people rather disagree then agree with the statement A. The age group **a6** is far from all answers of **A**, which probably means that people from this age group do not care so much about statement **A**.

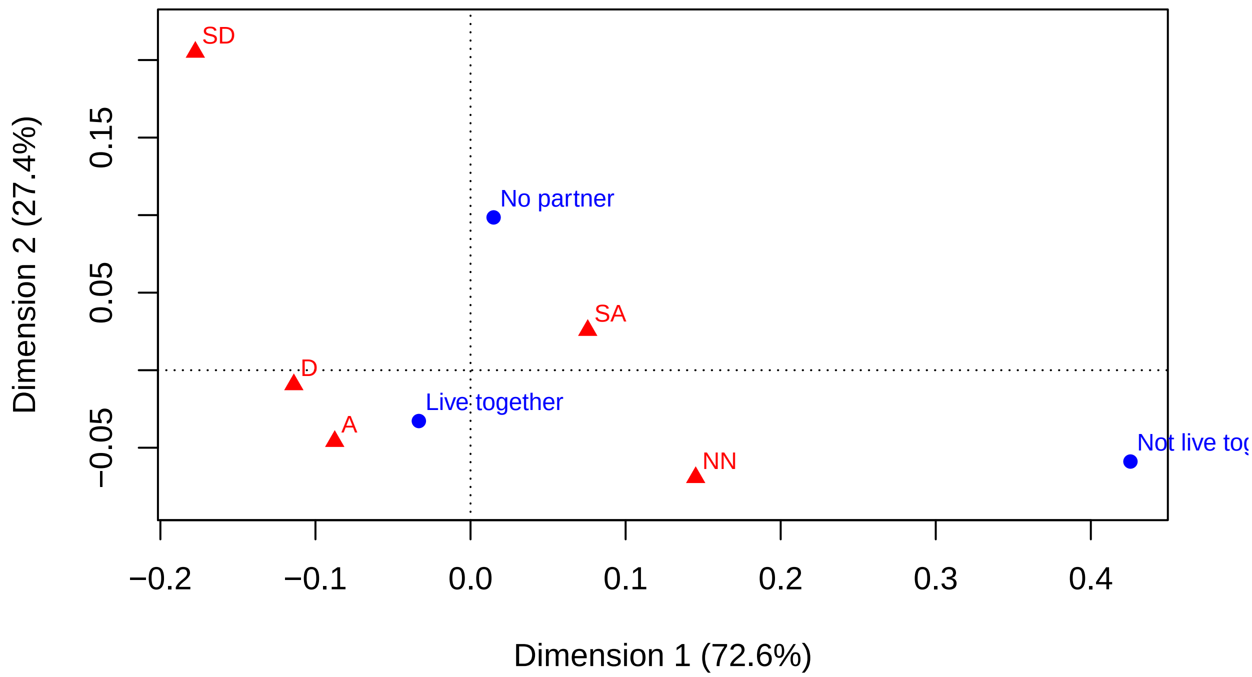
As a second example we conduct cross-tabulation between the variable **p** (if there is a partner and they live together) and answer **C*: C: It is all right for a couple to live together without intending to get married. The obtained cross-table and resulting plot from CA are:

```
#Cross-tabulations
#between g and D
tab2 <-table(Finland[,"p"], Finland[,"C"])

rownames(tab2) <- c("Live together","Not live together", "No partner")
colnames(tab2) <- c("SA","A","NN","D","SD")
tab2
```

```
##
##                     SA   A  NN   D  SD
##   Live together     291 247  44  32  29
##   Not live together  26  10   5   1   0
##   No partner        119  79  15  11  15
```

```
require(ca)
plot(ca(tab2))
```



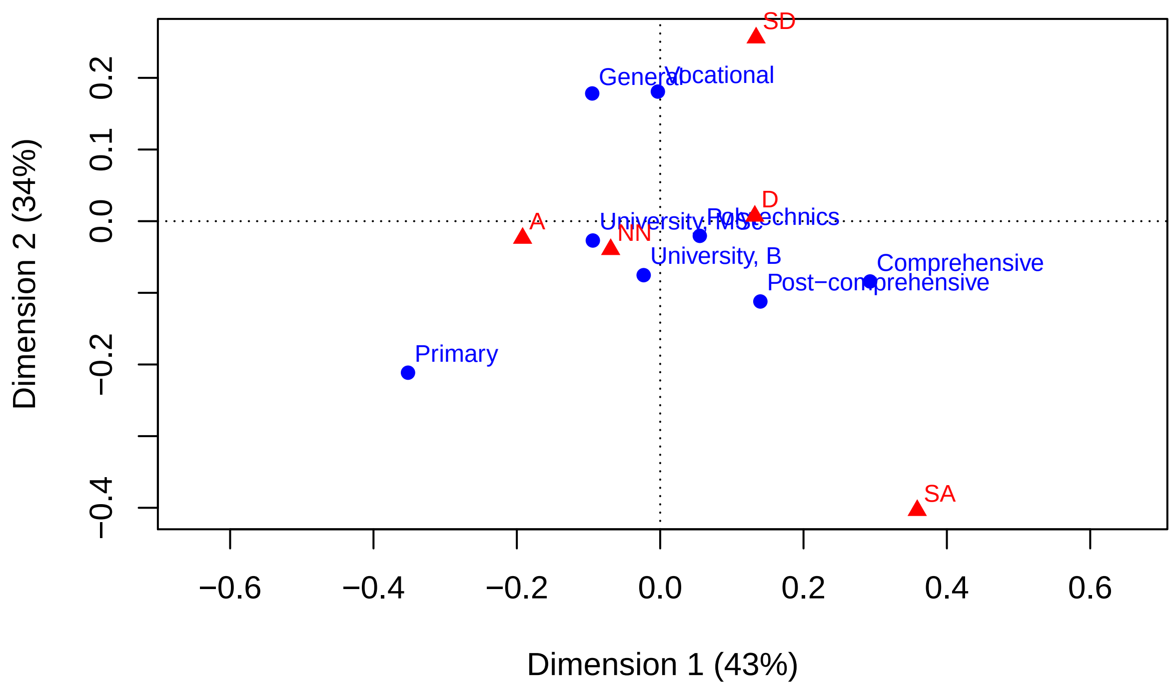
From the cross-tabulation and CA plot becomes clear that people, who live together, tend to give positive answer to the statement **C**.

As a third example we investigate relationship between education **e** and statement **A**:

```
tab1 <-table(Finland[,"e"], Finland[,"A"])
rownames(tab1) <- c("Primary","Comprehensive", "Post-comprehensive", "General", "Vocational", "Polytechn
colnames(tab1) <- c("SA","A","NN","D","SD")
tab1
```

```
##
##                      SA  A NN  D SD
##   Primary             3 28 34 14  3
##   Comprehensive       6  9 18 16 10
##   Post-comprehensive 16 37 71 54 26
##   General             1 20 39 18 18
##   Vocational          4 49 63 53 39
##   Polytechnics        7 25 28 22 16
##   University, B       2 11 23 16  5
##   University, MSc     7 35 39 21 18
```

```
require(ca)
plot(ca(tab1))
```



From this plot becomes clear that no one of the educational gruops strongly afrees with the statement **A**. The highly educated people with university degrees tend to answer using middle alternative.

Task 2: Cross-tabulation of ga demographic variable against one of the substantive variables. Performing CA.

The variable **ga**, which is the interaction of gender and age categories, is created as:

```
Finland$ga <- 6*(Finland$g-1) + Finland$a
head(Finland)
```

```
##   A B C D g a e p ga
## 1 3 3 1 2 1 2 4 3  2
## 2 3 2 3 2 1 4 2 3  4
## 3 3 3 1 3 1 3 8 1  3
## 4 3 2 2 3 2 2 6 1  8
## 5 2 2 2 3 2 4 5 1 10
## 6 3 3 1 3 1 3 7 3  3
```

```
str(Finland)
```

```
## 'data.frame':    924 obs. of  9 variables:
##  $ A : int  3 3 3 3 2 3 3 2 2 3 ...
##  $ B : int  3 2 3 2 2 3 3 3 3 3 ...
##  $ C : int  1 3 1 2 2 1 1 1 2 3 ...
##  $ D : int  2 2 3 3 3 3 2 2 3 3 ...
##  $ g : int  1 1 1 2 2 1 2 2 2 2 ...
```

```
##  $ a : int  2 4 3 2 4 3 1 2 5 4 ...
##  $ e : int  4 2 8 6 5 7 4 8 7 5 ...
##  $ p : int  3 3 1 1 1 3 3 3 1 3 ...
##  $ ga: num  2 4 3 8 10 3 7 8 11 10 ...
```

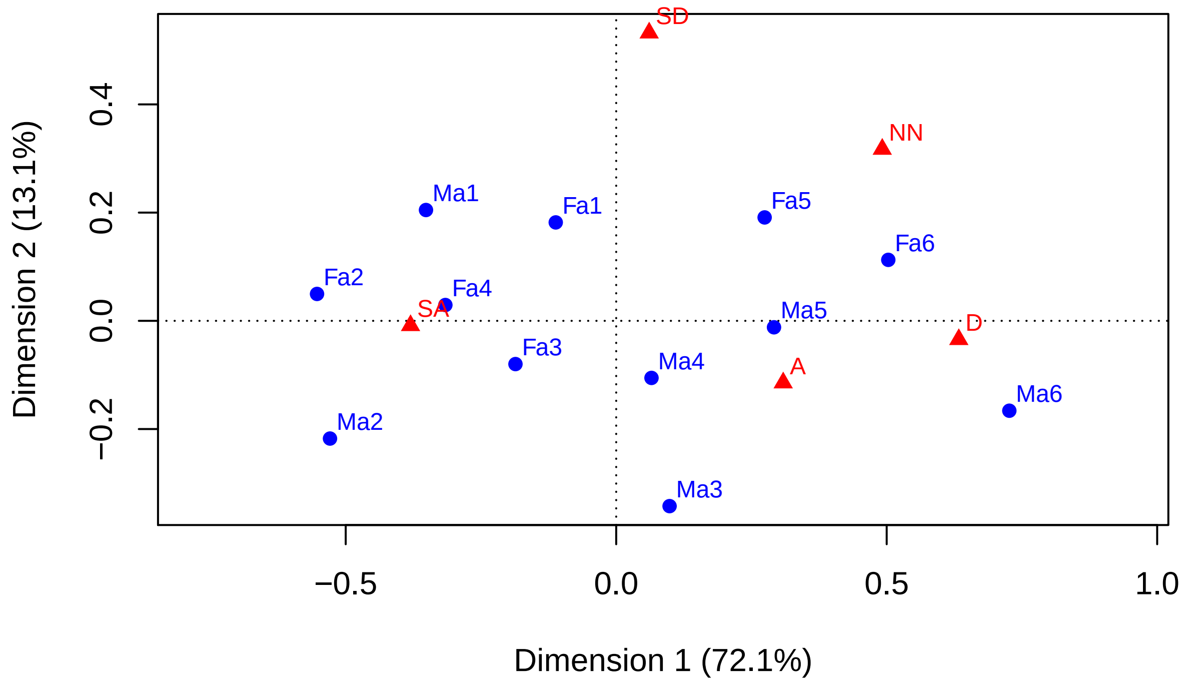
The cross-tabulation between **ga** and **A** is computed as:

```
tab3 <-table(Finland[,"ga"], Finland[,"C"])
rownames(tab3) <- c("Ma1","Ma2", "Ma3", "Ma4", "Ma5", "Ma6", "Fa1","Fa2", "Fa3", "Fa4", "Fa5", "Fa6")
colnames(tab3) <- c("SA","A","NN","D","SD")
tab3
```

```
##
##       SA  A NN  D SD
##   Ma1 30 11  3  0  4
##   Ma2 46 15  0  1  0
##   Ma3 26 31  0  4  1
##   Ma4 38 38  7  2  2
##   Ma5 29 36  9  7  3
##   Ma6  8 37  6  8  2
##   Fa1 38 21  1  6  9
##   Fa2 59 16  1  0  5
##   Fa3 43 25  4  2  2
##   Fa4 63 24  7  2  3
##   Fa5 42 45 18  8  7
##   Fa6 14 37  8  4  6
```

The CA between **ga** and **A**:

```
require(ca)
plot(ca(tab3))
```



From this plot becomes clear that female of age group 4 most often strongly agree with the statement **A**, while males in age group 5 most often agree with **A**.

Task 3:Computation of the Burt matrix on the questions and demographics together.

The most classical and standard approach to MCA is to apply a simple CA to the indicatormatrix Z. The indicator matrix Z = {zij} corresponds to a binary coding of the factors - instead of using a factor with Jq levels one uses Jq columns containing binary values, also called dummy variables. The Burt matrix C is obtained directly from the indicator matrix Z: C = ZTZ. Burt matrix concatenates all two-way cross-tabulations between pairs of variables

The Burt matrix is computed as:

```
require(ca)
Finland.B <- mjca(Finland)$Burt
head(Finland.B)
```

```
##      A:1 A:2 A:3 A:4 A:5 B:1 B:2 B:3 B:4 B:5 C:1 C:2 C:3 C:4 C:5 D:1 D:2
## A:1   46   0   0   0   0  28  12   4   2   0  15  13   6   6   6  13  15
## A:2    0 214   0   0   0  50  92  38  25   9  64 101  17  15  17  28  91
## A:3    0   0 315   0   0  24  92  95  83  21 145 120  30  14   6  55 115
## A:4    0   0   0 214   0  15  42  30  97  30 104  88  10   9   3  41  84
## A:5    0   0   0   0 135   9  10   6  24  86 108  14   1   0  12  40  43
## B:1   28  50  24  15   9 126   0   0   0   0  26  34  17  15  34  29  24
##      D:3 D:4 D:5 g:1 g:2 a:1 a:2 a:3 a:4 a:5 a:6 e:1 e:2 e:3 e:4 e:5 e:6
```

```
## A:1   5   7   6  28  18   5   6   6   9  15   5   3   6  16   1   4   7
## A:2  46  36  13 113 101  18  25  30  37  67  37  28   9  37  20  49  25
## A:3  88  43  14 136 179  42  44  55  60  58  56  34  18  71  39  63  28
## A:4  41  43   5  85 129  29  34  30  46  49  26  14  16  54  18  53  22
## A:5  27  12  13  42  93  29  34  17  34  15   6   3  10  26  18  39  16
## B:1  20  29  24  59  67  15   8  16  16  41  30  15  10  27  12  32   7
##     e:7 e:8 p:1 p:2 p:3 ga:1 ga:2 ga:3 ga:4 ga:5 ga:6 ga:7 ga:8 ga:9 ga:10
## A:1   2   7  35   1  10    3    3    4    6   10    2    2    3    2     3
## A:2  11  35 171   8  35   10   12   16   22   29   24    8   13   14    15
## A:3  23  39 224  12  79   16   21   21   32   21   25   26   23   34    28
## A:4  16  21 141  14  59    8   14   13   20   20   10   21   20   17    26
## A:5   5  18  72   7  56   11   12    8    7    4    0   18   22    9    27
## B:1   4  19  97   2  27    7    3    8    8   19   14    8    5    8     8
##     ga:11 ga:12
## A:1     5     3
## A:2    38    13
## A:3    37    31
## A:4    29    16
## A:5    11     6
## B:1    22    16
```

```
summary(Finland.B)
```

```
##       A:1              A:2              A:3              A:4
##  Min.   : 0.000   Min.   :  0.00   Min.   :  0.00   Min.   :  0.00
##  1st Qu.: 3.000   1st Qu.: 13.00   1st Qu.: 21.00   1st Qu.: 14.00
##  Median : 6.000   Median : 25.00   Median : 34.00   Median : 22.00
##  Mean   : 8.118   Mean   : 37.76   Mean   : 55.59   Mean   : 37.76
##  3rd Qu.:10.000   3rd Qu.: 38.00   3rd Qu.: 67.00   3rd Qu.: 44.50
##  Max.   :46.000   Max.   :214.00   Max.   :315.00   Max.   :214.00
##       A:5              B:1              B:2              B:3
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.00   Min.   :  0.00
##  1st Qu.:  6.50   1st Qu.:  8.00   1st Qu.: 14.00   1st Qu.: 10.00
##  Median : 13.00   Median : 16.00   Median : 27.00   Median : 20.00
##  Mean   : 23.82   Mean   : 22.24   Mean   : 43.76   Mean   : 30.53
##  3rd Qu.: 28.00   3rd Qu.: 27.50   3rd Qu.: 52.00   3rd Qu.: 37.50
##  Max.   :135.00   Max.   :126.00   Max.   :248.00   Max.   :173.00
##       B:4              B:5              C:1              C:2
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.00   Min.   :  0.00
##  1st Qu.: 13.50   1st Qu.:  5.00   1st Qu.: 26.00   1st Qu.: 19.50
##  Median : 25.00   Median : 16.00   Median : 59.00   Median : 36.00
##  Mean   : 40.76   Mean   : 25.76   Mean   : 76.94   Mean   : 59.29
##  3rd Qu.: 50.50   3rd Qu.: 31.50   3rd Qu.:102.50   3rd Qu.: 77.00
##  Max.   :231.00   Max.   :146.00   Max.   :436.00   Max.   :336.00
##       C:3             C:4              C:5              D:1
##  Min.   : 0.00   Min.   : 0.000   Min.   : 0.000   Min.   :  0.00
##  1st Qu.: 3.00   1st Qu.: 2.000   1st Qu.: 2.000   1st Qu.: 12.00
##  Median : 7.00   Median : 5.000   Median : 5.000   Median : 22.00
##  Mean   :11.29   Mean   : 7.765   Mean   : 7.765   Mean   : 31.24
##  3rd Qu.:17.00   3rd Qu.:12.000   3rd Qu.: 9.500   3rd Qu.: 38.00
##  Max.   :64.00   Max.   :44.000   Max.   :44.000   Max.   :177.00
##       D:2              D:3              D:4              D:5
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.00   Min.   : 0.0
##  1st Qu.: 22.50   1st Qu.: 15.00   1st Qu.: 10.00   1st Qu.: 3.5
##  Median : 37.00   Median : 22.00   Median : 15.00   Median : 6.0
```

```
##  Mean   : 61.41   Mean   : 36.53   Mean   : 24.88   Mean   : 9.0
##  3rd Qu.: 85.00   3rd Qu.: 45.50   3rd Qu.: 30.50   3rd Qu.:11.0
##  Max.   :348.00   Max.   :207.00   Max.   :141.00   Max.   :51.0
##       g:1              g:2              a:1              a:2
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.00   Min.   :  0.00
##  1st Qu.: 26.50   1st Qu.: 33.50   1st Qu.:  0.00   1st Qu.:  0.00
##  Median : 61.00   Median : 76.00   Median : 13.00   Median : 17.00
##  Mean   : 71.29   Mean   : 91.76   Mean   : 21.71   Mean   : 25.24
##  3rd Qu.: 87.00   3rd Qu.:117.00   3rd Qu.: 30.00   3rd Qu.: 35.50
##  Max.   :404.00   Max.   :520.00   Max.   :123.00   Max.   :143.00
##       a:3              a:4              a:5             a:6
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.0   Min.   :  0.00
##  1st Qu.:  0.00   1st Qu.:  0.00   1st Qu.:  0.0   1st Qu.:  0.00
##  Median : 16.00   Median : 16.00   Median : 16.0   Median : 11.00
##  Mean   : 24.35   Mean   : 32.82   Mean   : 36.0   Mean   : 22.94
##  3rd Qu.: 33.00   3rd Qu.: 46.50   3rd Qu.: 53.5   3rd Qu.: 31.00
##  Max.   :138.00   Max.   :186.00   Max.   :204.0   Max.   :130.00
##       e:1             e:2             e:3              e:4
##  Min.   : 0.00   Min.   : 0.00   Min.   :  0.0   Min.   : 0.00
##  1st Qu.: 1.00   1st Qu.: 2.50   1st Qu.: 12.5   1st Qu.: 4.00
##  Median : 8.00   Median : 7.00   Median : 25.0   Median :11.00
##  Mean   :14.47   Mean   :10.41   Mean   : 36.0   Mean   :16.94
##  3rd Qu.:22.50   3rd Qu.:15.50   3rd Qu.: 48.0   3rd Qu.:22.00
##  Max.   :82.00   Max.   :59.00   Max.   :204.0   Max.   :96.00
##       e:5              e:6             e:7             e:8
##  Min.   :  0.00   Min.   : 0.00   Min.   : 0.00   Min.   :  0.00
##  1st Qu.:  9.00   1st Qu.: 3.50   1st Qu.: 2.00   1st Qu.:  5.00
##  Median : 24.00   Median :13.00   Median : 6.00   Median : 16.00
##  Mean   : 36.71   Mean   :17.29   Mean   :10.06   Mean   : 21.18
##  3rd Qu.: 51.00   3rd Qu.:22.50   3rd Qu.:13.00   3rd Qu.: 28.00
##  Max.   :208.00   Max.   :98.00   Max.   :57.00   Max.   :120.00
##       p:1             p:2              p:3              ga:1
##  Min.   :  0.0   Min.   : 0.000   Min.   :  0.00   Min.   : 0.000
##  1st Qu.: 46.0   1st Qu.: 3.000   1st Qu.: 16.00   1st Qu.: 0.000
##  Median : 88.0   Median : 5.000   Median : 34.00   Median : 3.000
##  Mean   :113.5   Mean   : 7.412   Mean   : 42.18   Mean   : 8.471
##  3rd Qu.:142.0   3rd Qu.: 9.500   3rd Qu.: 53.50   3rd Qu.:11.000
##  Max.   :643.0   Max.   :42.000   Max.   :239.00   Max.   :48.000
##       ga:2            ga:3            ga:4            ga:5
##  Min.   : 0.00   Min.   : 0.00   Min.   : 0.00   Min.   : 0.00
##  1st Qu.: 0.00   1st Qu.: 0.00   1st Qu.: 0.00   1st Qu.: 0.00
##  Median : 3.00   Median : 5.00   Median : 7.00   Median : 7.00
##  Mean   :10.94   Mean   :10.94   Mean   :15.35   Mean   :14.82
##  3rd Qu.:14.50   3rd Qu.:14.50   3rd Qu.:21.00   3rd Qu.:19.00
##  Max.   :62.00   Max.   :62.00   Max.   :87.00   Max.   :84.00
##       ga:6            ga:7            ga:8            ga:9
##  Min.   : 0.00   Min.   : 0.00   Min.   : 0.00   Min.   : 0.00
##  1st Qu.: 0.00   1st Qu.: 0.00   1st Qu.: 0.00   1st Qu.: 0.00
##  Median : 4.00   Median : 6.00   Median : 5.00   Median : 4.00
##  Mean   :10.76   Mean   :13.24   Mean   :14.29   Mean   :13.41
##  3rd Qu.:12.50   3rd Qu.:19.50   3rd Qu.:19.50   3rd Qu.:17.50
##  Max.   :61.00   Max.   :75.00   Max.   :81.00   Max.   :76.00
##      ga:10           ga:11            ga:12
##  Min.   : 0.00   Min.   :  0.00   Min.   : 0.00
```

```
##  1st Qu.: 0.00   1st Qu.:  0.00   1st Qu.: 0.00
##  Median : 6.00   Median : 10.00   Median : 5.00
##  Mean   :17.47   Mean   : 21.18   Mean   :12.18
##  3rd Qu.:24.00   3rd Qu.: 27.00   3rd Qu.:15.50
##  Max.   :99.00   Max.   :120.00   Max.   :69.00
```

We take the stacked tables of **A** and **D** substantive variables from the whole Burt matrix:

```r
Finland.AD = Finland.B[c(1:5,16:20), c(1:5,16:20)]
```

The principal intervals (eigenvalues) are:

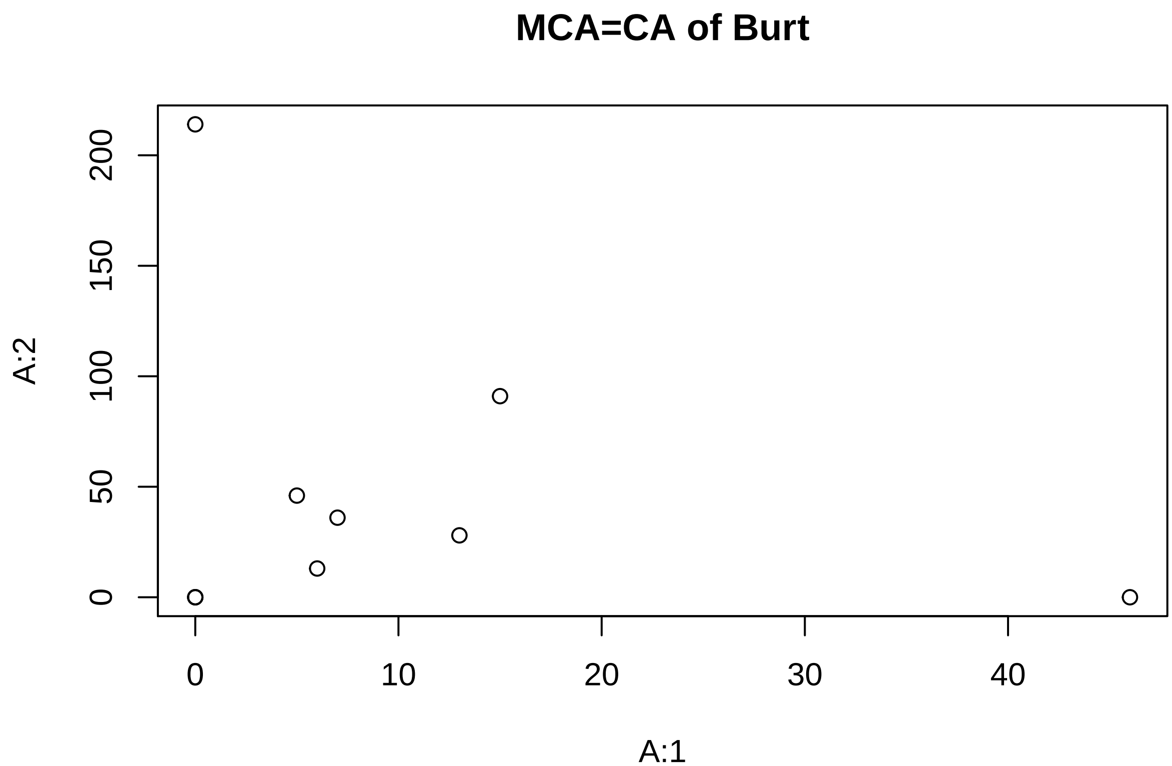
```r
summary(Finland.AD)
```

```
##       A:1            A:2             A:3              A:4
##  Min.   : 0.0   Min.   :  0.0   Min.   :  0.00   Min.   :  0.0
##  1st Qu.: 0.0   1st Qu.:  0.0   1st Qu.:  0.00   1st Qu.:  0.0
##  Median : 5.5   Median : 20.5   Median : 28.50   Median : 23.0
##  Mean   : 9.2   Mean   : 42.8   Mean   : 63.00   Mean   : 42.8
##  3rd Qu.:11.5   3rd Qu.: 43.5   3rd Qu.: 79.75   3rd Qu.: 42.5
##  Max.   :46.0   Max.   :214.0   Max.   :315.00   Max.   :214.0
##       A:5              D:1              D:2              D:3
##  Min.   :  0.00   Min.   :  0.00   Min.   :  0.00   Min.   :  0.00
##  1st Qu.:  0.00   1st Qu.:  0.00   1st Qu.:  0.00   1st Qu.:  0.00
##  Median : 12.50   Median : 20.50   Median : 29.00   Median : 16.00
##  Mean   : 27.00   Mean   : 35.40   Mean   : 69.60   Mean   : 41.40
##  3rd Qu.: 36.75   3rd Qu.: 40.75   3rd Qu.: 89.25   3rd Qu.: 44.75
##  Max.   :135.00   Max.   :177.00   Max.   :348.00   Max.   :207.00
##       D:4              D:5
##  Min.   :  0.00   Min.   : 0.0
##  1st Qu.:  0.00   1st Qu.: 0.0
##  Median :  9.50   Median : 5.5
##  Mean   : 28.20   Mean   :10.2
##  3rd Qu.: 41.25   3rd Qu.:13.0
##  Max.   :141.00   Max.   :51.0
```

The computation of MCA is the application of the (simple) CA algorithm to the Burt matrix C.

The following CA is conducted:

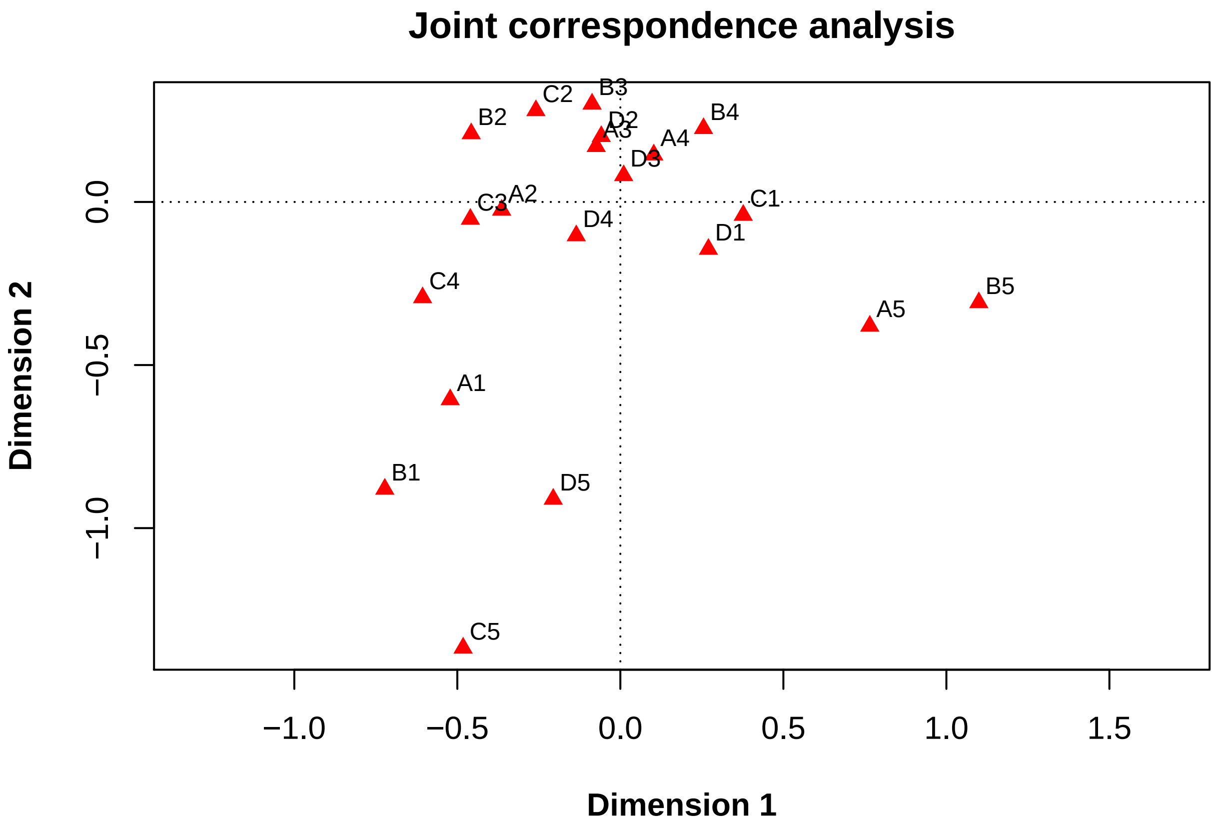
```r
plot(Finland.AD, main="MCA=CA of Burt")
```



Task 4: Multiple correspondence analysis (MCA) on the four substantive questions.

Here I use the script from Exercise set 2 about joint correspondance analysis and its fit to two-way tables:

```
Finland.mca <- mjca(Finland[,1:4], ps="", lambda="JCA")
par(mar=c(4.2,4,3,1), mfrow=c(1,1), font.lab=2)
plot(Finland.mca, main="Joint correspondence analysis")
```



Task 5: MCA on the four substantive questions: computing the case points, and adding confidence ellipses for some demographic groups.

The example code from Exercise set 3 is used, only data are different.

5.1 MCA on the four substantive questions

MCA is the CA of indicator matrix:

```
require(ca)
Finland.mca1 <- mjca(Finland[,1:4], lambda="indicator", ps="", reti=T)
sum(Finland.mca1$sv^2)
```

```
## [1] 4
```

```
summary(Finland.mca1)
```

```
##
## Principal inertias (eigenvalues):
##
##  dim    value      %   cum%   scree plot
##  1      0.506305  12.7  12.7  ***
##  2      0.475751  11.9  24.6  ***
##  3      0.322936   8.1  32.6  **
##  4      0.309594   7.7  40.4  **
```

```
##  5      0.274095   6.9  47.2  **
##  6      0.268731   6.7  53.9  **
##  7      0.250166   6.3  60.2  **
##  8      0.236639   5.9  66.1  *
##  9      0.225387   5.6  71.7  *
##  10     0.217098   5.4  77.2  *
##  11     0.209046   5.2  82.4  *
##  12     0.186252   4.7  87.0  *
##  13     0.162414   4.1  91.1  *
##  14     0.142888   3.6  94.7  *
##  15     0.125436   3.1  97.8  *
##  16     0.087262   2.2 100.0  *
##         -------- -----
##  Total: 4.000000 100.0
##
##
## Columns:
##      name   mass  qlt  inr     k=1 cor ctr    k=2 cor ctr
## 1  |   A1 |   12  198   57 |  -779  32  15 | 1783 167  83 |
## 2  |   A2 |   58  181   46 |  -730 161  61 |  257  20   8 |
## 3  |   A3 |   85   95   38 |  -187  18   6 | -384  76  26 |
## 4  |   A4 |   58   60   44 |   142   6   2 | -421  54  22 |
## 5  |   A5 |   37  509   63 |  1635 458 193 |  550  52  23 |
## 6  |   B1 |   34  616   65 |  -694  76  32 | 1850 540 245 |
## 7  |   B2 |   67  244   47 |  -767 216  78 | -276  28  11 |
## 8  |   B3 |   47   84   46 |  -239  13   5 | -555  71  30 |
## 9  |   B4 |   62  110   45 |   281  26  10 | -501  84  33 |
## 10 |   B5 |   40  588   66 |  1740 568 236 |  322  19   9 |
## 11 |   C1 |  118  559   39 |   789 556 145 |  -57   3   1 |
## 12 |   C2 |   91  379   42 |  -650 241  76 | -491 138  46 |
## 13 |   C3 |   17   65   52 |  -894  59  27 |  279   6   3 |
## 14 |   C4 |   12  101   54 | -1130  64  30 |  863  37  19 |
## 15 |   C5 |   12  472   66 |  -430   9   4 | 3041 462 231 |
## 16 |   D1 |   48  159   47 |   795 150  60 |  196   9   4 |
## 17 |   D2 |   94  155   36 |  -247  37  11 | -443 118  39 |
## 18 |   D3 |   56   12   42 |   -23   0   0 | -203  12   5 |
## 19 |   D4 |   38   35   46 |  -310  17   7 |  310  17   8 |
## 20 |   D5 |   14  313   59 |  -123   1   0 | 2310 312 155 |
```

We calculate: - the number of categories:

```r
J <- 4*5 -1
```

The number of substantive questions is Q=4. Then:

```r
Q <- 4
(J-Q)/Q
```

```
## [1] 3.75
```

We check the contents of these components of the mjca object (eigenvalues (squared singular values),standard coordinates of columns, standard coordinates of first 10 rows, variable names, category names, first 10 rows of 924 x 20 indicator matrix, first 20 rows and columns of 20 x 20 Burt matrix) :

```r
names(Finland.mca1)
```

```
##  [1] "sv"         "lambda"     "inertia.e"  "inertia.t"  "inertia.et"
##  [6] "levelnames" "factors"    "levels.n"   "nd"         "nd.max"
```

```
## [11] "rownames"   "rowmass"    "rowdist"    "rowinertia" "rowcoord"
## [16] "rowpcoord"  "rowctr"     "rowcor"     "colnames"   "colmass"
## [21] "coldist"    "colinertia" "colcoord"   "colpcoord"  "colctr"
## [26] "colcor"     "colsup"     "subsetcol"  "Burt"       "Burt.upd"
## [31] "subinertia" "JCA.iter"   "indmat"     "call"
```

```
Finland.mca1$sv^2                 # eigenvalues (squared singular values) on all 31 dimensions
```

```
##  [1] 0.50630492 0.47575147 0.32293568 0.30959403 0.27409529 0.26873127
##  [7] 0.25016609 0.23663902 0.22538662 0.21709794 0.20904570 0.18625197
## [13] 0.16241364 0.14288849 0.12543559 0.08726229
```

```
Finland.mca1$colcoord[,1:4]       # standard coordinates of columns
```

```
##              [,1]        [,2]        [,3]       [,4]
##  [1,] -1.09485075  2.58466228  0.38117469 -1.0611142
##  [2,] -1.02632480  0.37232989 -0.47157445  1.4201562
##  [3,] -0.26345610 -0.55714283 -0.80286444 -1.3121928
##  [4,]  0.19957060 -0.61095108  2.14671089 -0.1271033
##  [5,]  2.29834966  0.79755898 -0.91193285  1.3736196
##  [6,] -0.97470923  2.68220803  0.48209843 -0.6463525
##  [7,] -1.07792715 -0.39988664 -0.66328834  1.5112705
##  [8,] -0.33602699 -0.80484225 -1.22794762 -1.6797485
##  [9,]  0.39518935 -0.72610435  1.92930121 -0.7637834
## [10,]  2.44509059  0.46699649 -0.88686667  1.1895602
## [11,]  1.10913902 -0.08237802  0.03901019 -0.2765203
## [12,] -0.91287622 -0.71121198  0.25165618  0.6290356
## [13,] -1.25648834  0.40439859 -2.40903951 -1.9962482
## [14,] -1.58745288  1.25079576  0.30637839  1.2613963
## [15,] -0.60443232  4.40835272  0.88938548 -0.4212425
## [16,]  1.11702020  0.28392642 -0.07957831 -0.3599156
## [17,] -0.34677975 -0.64190277 -0.07750072  0.8506859
## [18,] -0.03261598 -0.29495380 -0.87687783 -0.9136392
## [19,] -0.43594396  0.44926861  1.62187492 -0.1096170
## [20,] -0.17281598  3.34972059 -0.11990279 -0.5442025
```

```
Finland.mca1$rowcoord[1:10,1:4]   # standard coordinates of first 10 rows
```

```
##              [,1]       [,2]       [,3]        [,4]
##  [1,]  0.05722575 -0.7561703 -0.9103450 -1.08632489
##  [2,] -1.03458890 -0.4329605 -1.7389019 -0.42526271
##  [3,]  0.16760566 -0.6304181 -1.2620137 -1.87904947
##  [4,] -0.80348254 -0.7115631 -0.9200550 -0.03842743
##  [5,] -1.07151274 -0.3746743 -0.7743111  1.18923762
##  [6,]  0.16760566 -0.6304181 -1.2620137 -1.87904947
##  [7,]  0.05722575 -0.7561703 -0.9103450 -1.08632489
##  [8,] -0.21080445 -0.4192814 -0.7646011  0.14134015
##  [9,] -0.81084974 -0.5214511 -1.0227208 -0.24451133
## [10,] -0.66354598 -0.4539851 -2.3389802 -2.65173623
```

```
Finland.mca1$colnames             # variable names
```

```
## [1] "A" "B" "C" "D"
```

```
Finland.mca1$levelnames           # category names
```

```
##  [1] "A1" "A2" "A3" "A4" "A5" "B1" "B2" "B3" "B4" "B5" "C1" "C2" "C3" "C4"
## [15] "C5" "D1" "D2" "D3" "D4" "D5"
```

```r
Finland.mca1$indmat[1:10,]          # first 10 rows of 924 x 20 indicator matrix
```

```
##    A1 A2 A3 A4 A5 B1 B2 B3 B4 B5 C1 C2 C3 C4 C5 D1 D2 D3 D4 D5
## 1   0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 2   0  0  1  0  0  0  1  0  0  0  0  0  1  0  0  0  1  0  0  0
## 3   0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  0  1  0  0
## 4   0  0  1  0  0  0  1  0  0  0  0  1  0  0  0  0  0  1  0  0
## 5   0  1  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  1  0  0
## 6   0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  0  1  0  0
## 7   0  0  1  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 8   0  1  0  0  0  0  0  1  0  0  1  0  0  0  0  0  1  0  0  0
## 9   0  1  0  0  0  0  0  1  0  0  0  1  0  0  0  0  0  1  0  0
## 10  0  0  1  0  0  0  0  1  0  0  0  0  1  0  0  0  0  1  0  0
```

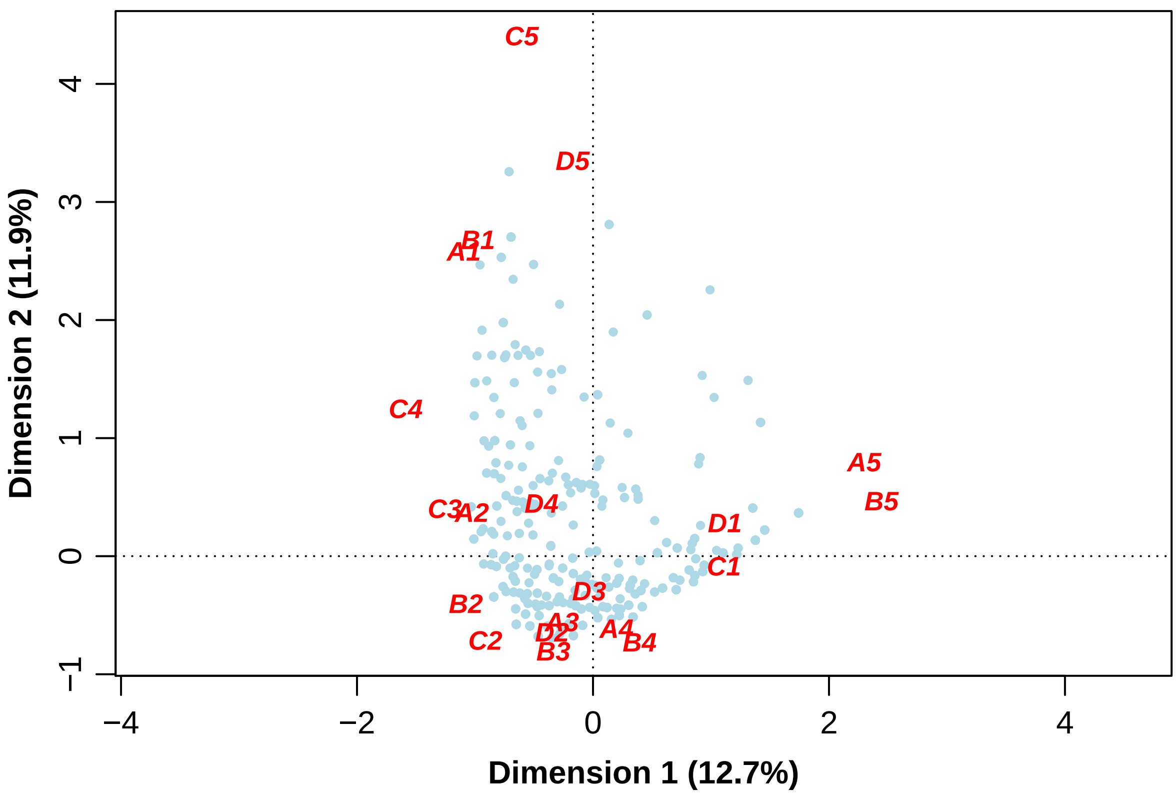
```r
Finland.mca1$Burt[1:20, 1:20]       # first 20 rows and columns of 20 x 20 Burt matrix
```

```
##    A1  A2  A3  A4  A5  B1  B2  B3  B4  B5  C1  C2 C3 C4 C5  D1  D2  D3  D4
## A1 46   0   0   0   0  28  12   4   2   0  15  13  6  6  6  13  15   5   7
## A2  0 214   0   0   0  50  92  38  25   9  64 101 17 15 17  28  91  46  36
## A3  0   0 315   0   0  24  92  95  83  21 145 120 30 14  6  55 115  88  43
## A4  0   0   0 214   0  15  42  30  97  30 104  88 10  9  3  41  84  41  43
## A5  0   0   0   0 135   9  10   6  24  86 108  14  1  0 12  40  43  27  12
## B1 28  50  24  15   9 126   0   0   0   0  26  34 17 15 34  29  24  20  29
## B2 12  92  92  42  10   0 248   0   0   0  52 141 27 25  3  30 119  53  37
## B3  4  38  95  30   6   0   0 173   0   0  79  75 17  1  1  28  75  45  23
## B4  2  25  83  97  24   0   0   0 231   0 142  82  3  3  1  41  90  55  37
## B5  0   9  21  30  86   0   0   0   0 146 137   4  0  0  5  49  40  34  15
## C1 15  64 145 104 108  26  52  79 142 137 436   0  0  0  0 128 137  93  57
## C2 13 101 120  88  14  34 141  75  82   4   0 336  0  0  0  35 175  73  51
## C3  6  17  30  10   1  17  27  17   3   0   0   0 64  0  0   7  22  22   7
## C4  6  15  14   9   0  15  25   1   3   0   0   0  0 44  0   3  12  12  11
## C5  6  17   6   3  12  34   3   1   1   5   0   0  0  0 44   4   2   7  15
## D1 13  28  55  41  40  29  30  28  41  49 128  35  7  3  4 177   0   0   0
## D2 15  91 115  84  43  24 119  75  90  40 137 175 22 12  2   0 348   0   0
## D3  5  46  88  41  27  20  53  45  55  34  93  73 22 12  7   0   0 207   0
## D4  7  36  43  43  12  29  37  23  37  15  57  51  7 11 15   0   0   0 141
## D5  6  13  14   5  13  24   9   2   8   8  21   2  6  6 16   0   0   0   0
##    D5
## A1  6
## A2 13
## A3 14
## A4  5
## A5 13
## B1 24
## B2  9
## B3  2
## B4  8
## B5  8
## C1 21
## C2  2
## C3  6
## C4  6
## C5 16
## D1  0
```

```
## D2  0
## D3  0
## D4  0
## D5 51
```

Next we compute positions of cases and plot them.

```
Finland.rpc <- Finland.mca1$indmat %*% Finland.mca1$colcoord[,1:2] / 4
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca1, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
```



The MCA is calculated as a CA of Burt matrix:

```
Finland.mca2 <- mjca(Finland[,1:4], lambda="Burt", ps="")
sum(Finland.mca2$sv^2)
```

```
## [1] 1.195211
```

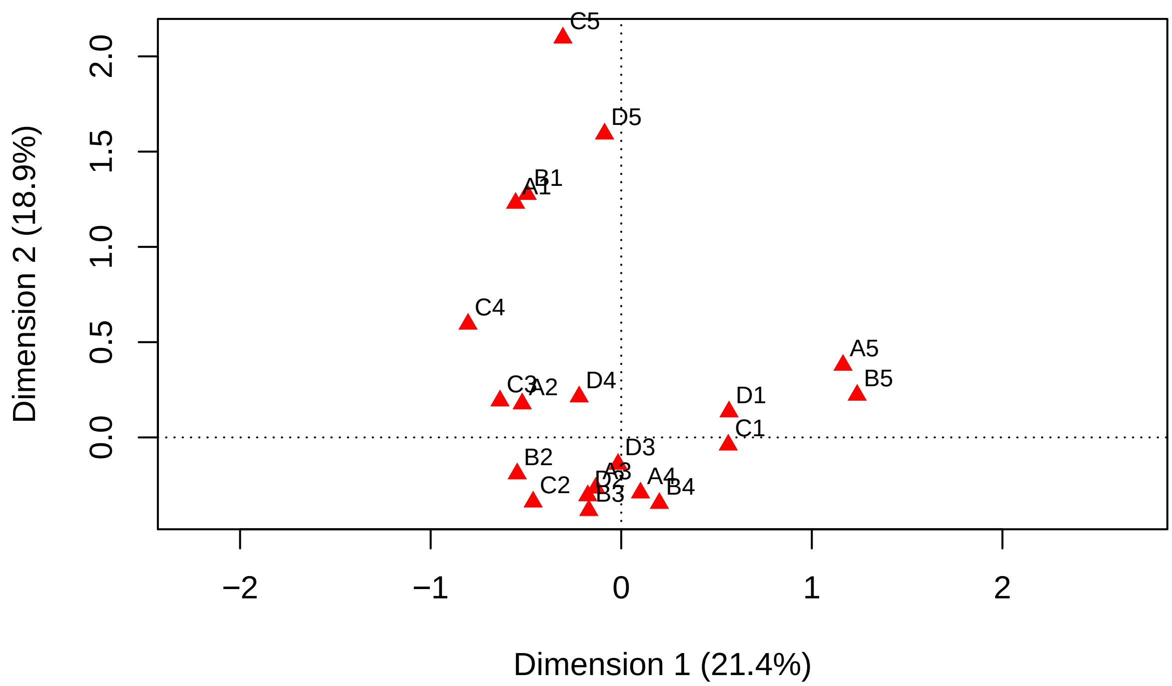
```
summary(Finland.mca2)
```

```
##
## Principal inertias (eigenvalues):
##
##  dim    value      %   cum%   scree plot
##  1      0.256345  21.4  21.4  *****
##  2      0.226339  18.9  40.4  *****
##  3      0.104287   8.7  49.1  **
```

```
##  4      0.095848   8.0  57.1  **
##  5      0.075128   6.3  63.4  **
##  6      0.072216   6.0  69.5  **
##  7      0.062583   5.2  74.7  *
##  8      0.055998   4.7  79.4  *
##  9      0.050799   4.3  83.6  *
##  10     0.047132   3.9  87.6  *
##  11     0.043700   3.7  91.2  *
##  12     0.034690   2.9  94.1  *
##  13     0.026378   2.2  96.3  *
##  14     0.020417   1.7  98.0
##  15     0.015734   1.3  99.4
##  16     0.007615   0.6 100.0
##         -------- -----
##  Total: 1.195211 100.0
##
##
## Columns:
##      name   mass  qlt  inr    k=1 cor ctr    k=2 cor ctr
## 1  |   A1 |   12  335   57 | -554  57  15 | 1230 278  83 |
## 2  |   A2 |   58  317   46 | -520 284  61 |  177  33   8 |
## 3  |   A3 |   85  165   38 | -133  33   6 | -265 132  26 |
## 4  |   A4 |   58  104   44 |  101  11   2 | -291  93  22 |
## 5  |   A5 |   37  724   63 | 1164 654 193 |  379  70  23 |
## 6  |   B1 |   34  825   65 | -494 107  32 | 1276 718 245 |
## 7  |   B2 |   67  400   47 | -546 357  78 | -190  43  11 |
## 8  |   B3 |   47  148   46 | -170  24   5 | -383 124  30 |
## 9  |   B4 |   62  186   45 |  200  47  10 | -345 139  33 |
## 10 |   B5 |   40  790   66 | 1238 765 236 |  222  25   9 |
## 11 |   C1 |  118  793   39 |  562 789 145 |  -39   4   1 |
## 12 |   C2 |   91  589   42 | -462 384  76 | -338 206  46 |
## 13 |   C3 |   17  123   52 | -636 113  27 |  192  10   3 |
## 14 |   C4 |   12  184   54 | -804 119  30 |  595  65  19 |
## 15 |   C5 |   12  681   66 | -306  14   4 | 2097 666 231 |
## 16 |   D1 |   48  288   47 |  566 273  60 |  135  16   4 |
## 17 |   D2 |   94  270   36 | -176  67  11 | -305 203  39 |
## 18 |   D3 |   56   22   42 |  -17   0   0 | -140  22   5 |
## 19 |   D4 |   38   65   46 | -221  34   7 |  214  32   8 |
## 20 |   D5 |   14  502   59 |  -87   2   0 | 1594 500 155 |
```

The default plot shows principal coordinates:

```
plot(Finland.mca2)
```



The adjusted MCA is the default of mjca function:

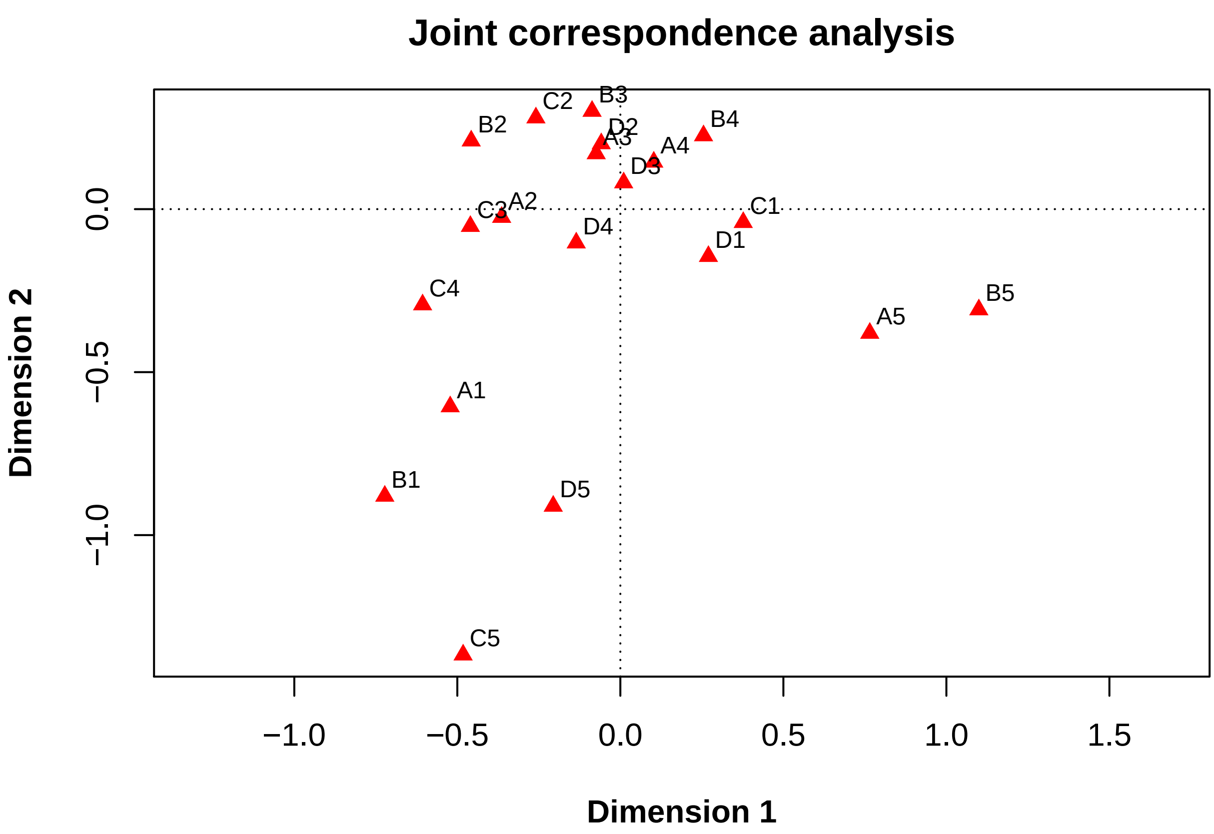
```
Finland.mca3 <- mjca(Finland[,1:4], ps="")
summary(Finland.mca3)
```

```
##
## Principal inertias (eigenvalues):
##
##  dim    value      %   cum%   scree plot
##  1      0.116786  44.9  44.9  *************
##  2      0.090602  34.8  79.7  **********
##  3      0.009457   3.6  83.3  *
##  4      0.006314   2.4  85.7  *
##  5      0.001032   0.4  86.1
##  6      0.000624   0.2  86.4
##  7      00000000   0.0  86.4
##         -------- -----
##  Total: 0.260281
##
##
## Columns:
##       name   mass  qlt  inr    k=1 cor ctr    k=2 cor ctr
## 1  |    A1 |   12  848   57 | -374 159  15 |  778 689  83 |
## 2  |    A2 |   58  847   46 | -351 769  61 |  112  78   8 |
## 3  |    A3 |   85  545   38 |  -90 122   6 | -168 423  26 |
## 4  |    A4 |   58  375   44 |   68  45   2 | -184 329  22 |
## 5  |    A5 |   37  830   63 |  785 759 193 |  240  71  23 |
```

```
## 6  |    B1 |   34  834   65 | -333 121   32 |  807 713 245 |
## 7  |    B2 |   67  738   47 | -368 666   78 | -120  71  11 |
## 8  |    B3 |   47  543   46 | -115 100    5 | -242 443  30 |
## 9  |    B4 |   62  460   45 |  135 127   10 | -219 333  33 |
## 10 |    B5 |   40  804   66 |  836 782  236 |  141  22   9 |
## 11 |    C1 |  118  902   39 |  379 899  145 |  -25   4   1 |
## 12 |    C2 |   91  898   42 | -312 611   76 | -214 288  46 |
## 13 |    C3 |   17  636   52 | -429 588   27 |  122  47   3 |
## 14 |    C4 |   12  756   54 | -542 510   30 |  376 246  19 |
## 15 |    C5 |   12  845   66 | -207  20    4 | 1327 825 231 |
## 16 |    D1 |   48  970   47 |  382 924   60 |   85  46   4 |
## 17 |    D2 |   94  829   36 | -119 227   11 | -193 602  39 |
## 18 |    D3 |   56  277   42 |  -11   4    0 |  -89 273   5 |
## 19 |    D4 |   38  557   46 | -149 305    7 |  135 252   8 |
## 20 |    D5 |   14  960   59 |  -59   3    0 | 1008 956 155 |
```

A joint correspondence analysis is used for optimal fit to two-way tables:

```
Finland.mca4 <- mjca(Finland[,1:4], ps="", lambda="JCA")
par(mar=c(4.2,4,3,1), mfrow=c(1,1), font.lab=2)
plot(Finland.mca4, main="Joint correspondence analysis")
```




```
summary(Finland.mca4)
```

```
##
## Principal inertias (eigenvalues):
##
##  dim    value
```

```
##  1      0.156300
##  2      0.101794
##  3      0.006669
##  4      0.005138
##  5      0.003304
##  6      0.000545
##  7      0.000228
##          --------
##  Total: 0.285989
##
##  Diagonal inertia discounted from eigenvalues: 0.090779
##  Percentage explained by JCA in 2 dimensions: 85.7%
##  (Eigenvalues are not nested)
##  [Iterations in JCA: 50 , epsilon = 0.0008615]
##
##
## Columns:
##      name   mass  inr    k=1   k=2    cor ctr
## 1  |   A1 |   12   57 | -522  -606 | 856  42 |
## 2  |   A2 |   58   46 | -364   -25 | 879  37 |
## 3  |   A3 |   85   38 |  -74   170 | 564  14 |
## 4  |   A4 |   58   44 |  103   145 | 362  10 |
## 5  |   A5 |   37   63 |  765  -381 | 912 121 |
## 6  |   B1 |   34   65 | -722  -880 | 970 136 |
## 7  |   B2 |   67   47 | -457   209 | 848  52 |
## 8  |   B3 |   47   46 |  -87   300 | 600  17 |
## 9  |   B4 |   62   45 |  255   225 | 528  21 |
## 10 |   B5 |   40   66 | 1100  -309 | 954 151 |
## 11 |   C1 |  118   39 |  377   -41 | 943  80 |
## 12 |   C2 |   91   42 | -259   280 | 918  60 |
## 13 |   C3 |   17   52 | -460   -53 | 711  17 |
## 14 |   C4 |   12   54 | -606  -293 | 793  24 |
## 15 |   C5 |   12   66 | -482 -1367 | 917 104 |
## 16 |   D1 |   48   47 |  270  -145 | 745  25 |
## 17 |   D2 |   94   36 |  -59   201 | 795  21 |
## 18 |   D3 |   56   42 |   10    81 | 257   2 |
## 19 |   D4 |   38   46 | -135  -103 | 487   6 |
## 20 |   D5 |   14   59 | -206  -911 | 917  60 |
```

5.2 Demographics is superimposed on an MCA

Call the used code about confidence plots, package ellipse, and calculated individual points:

```
source("confidenceplots.R")
require(ellipse)
```

```
## Loading required package: ellipse
```

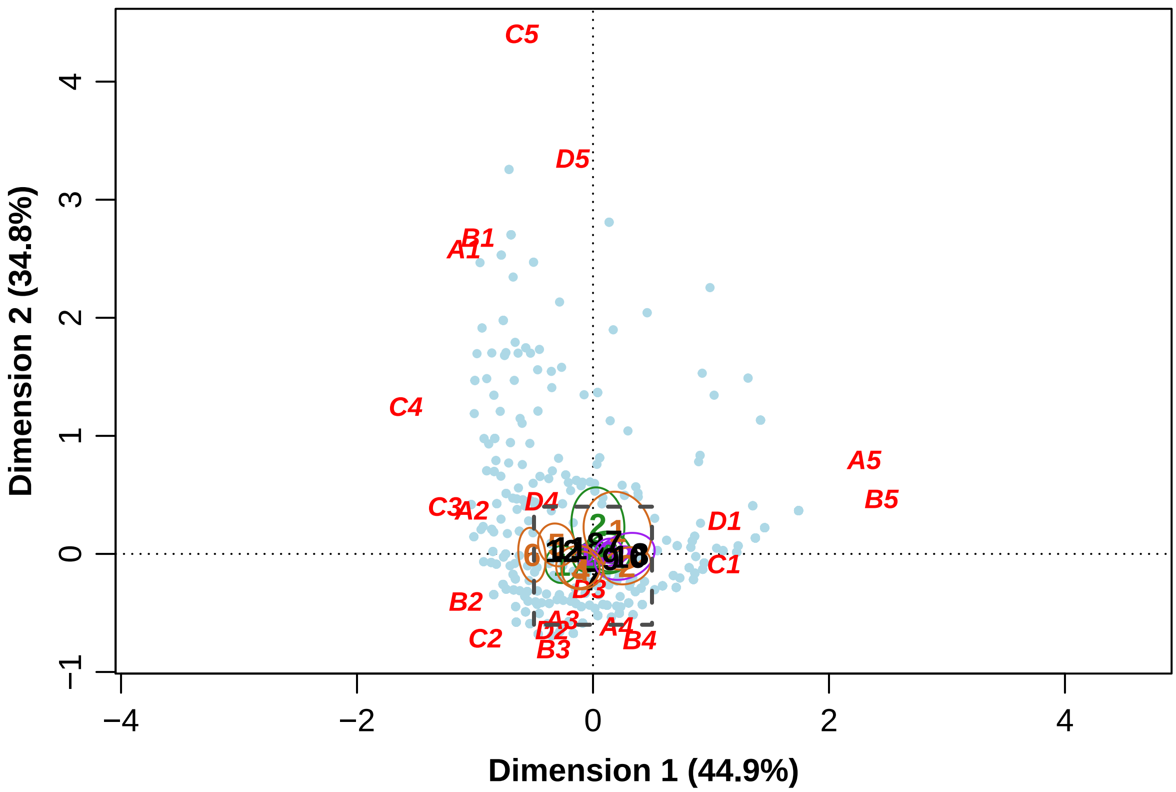
```
Finland.rpc <- Finland.mca1$indmat %*% Finland.mca1$colcoord[,1:2] / 4
```

The demographic groups are: education e, partnership p and ga groups.Their confidence plots are :

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
```

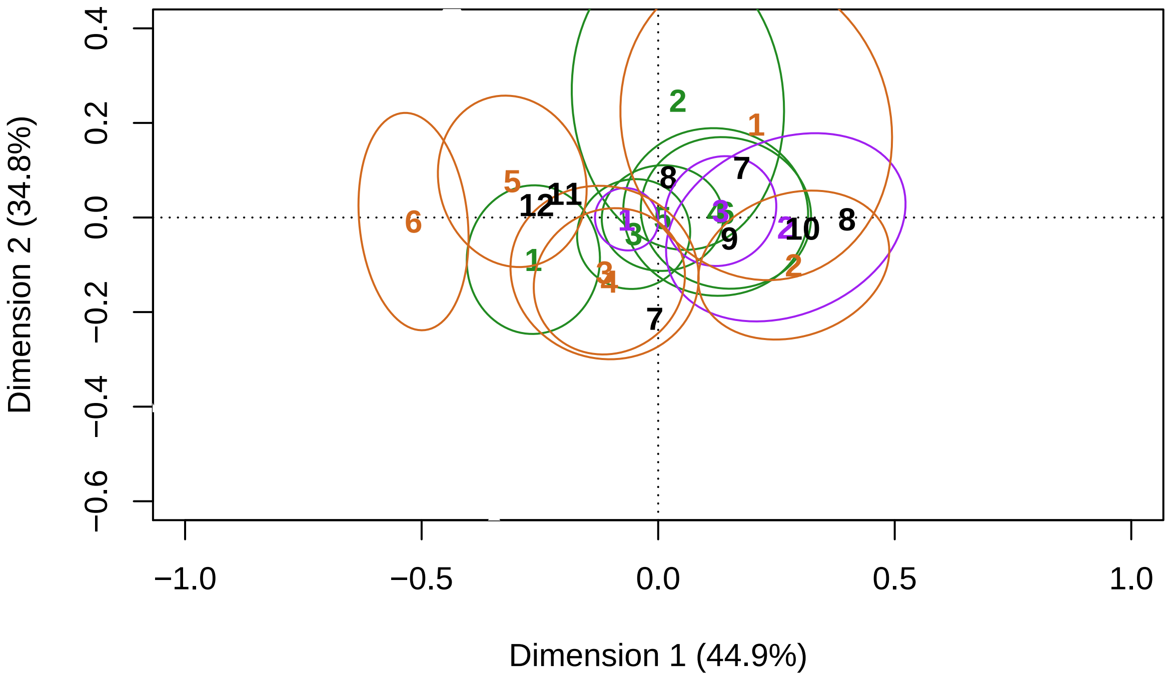
```
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"e"]), groupcols=rep("forestgre
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"p"]), groupcols=rep("purple",6)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=rep("chocolat

lines(matrix(c(-0.5,-0.6, -0.5,0.4, 0.5,0.4, 0.5,-0.6, -0.5,-0.6), ncol=2, byrow=T), lty=2, col="gray30
```

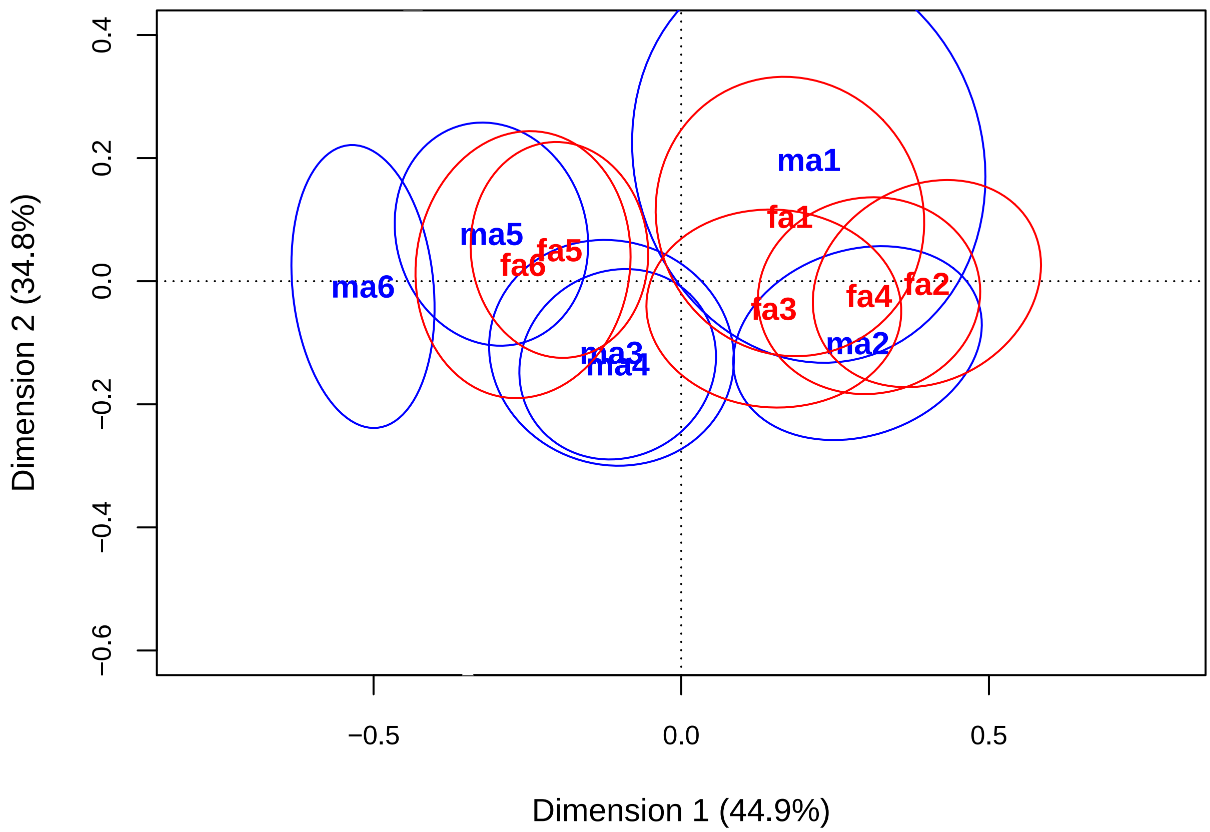


We zoom in to the group means and confidence ellipses:

```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"e"]), groupcols=rep("forestgre
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"p"]), groupcols=rep("purple",6)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=rep("chocolat
```

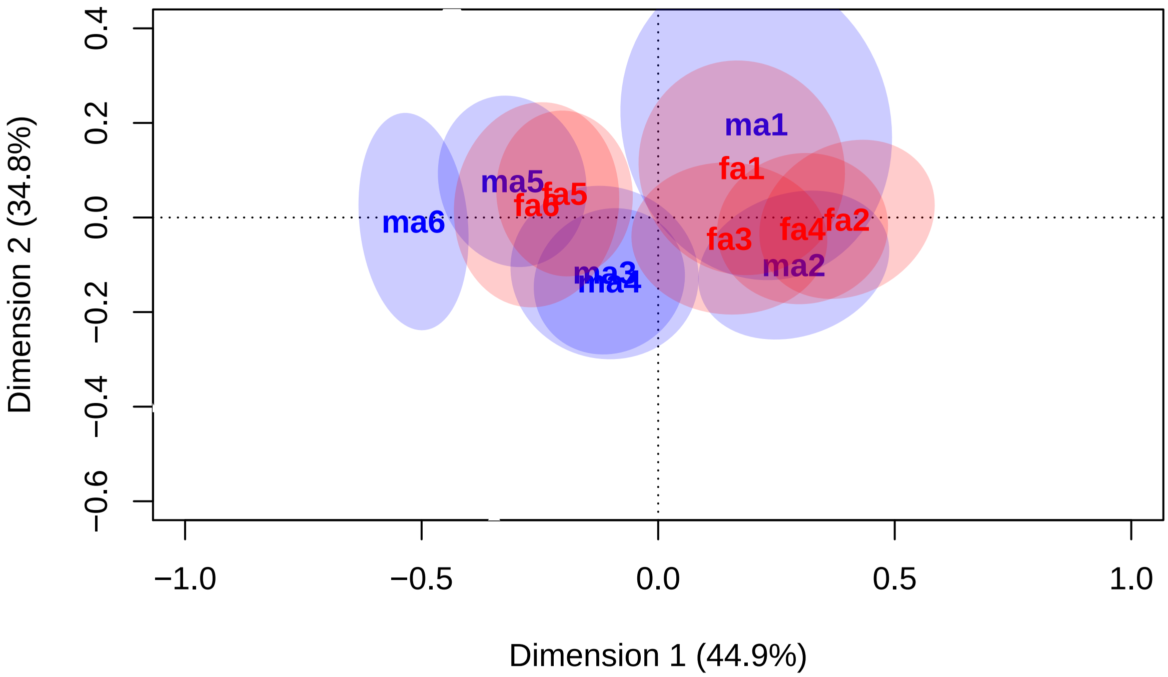



```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=c(rep("blue",
                groupnames=c("ma1","ma2","ma3","ma4","ma5","ma6","fa1","fa2","fa3","fa4","fa5","fa6"),sh
```



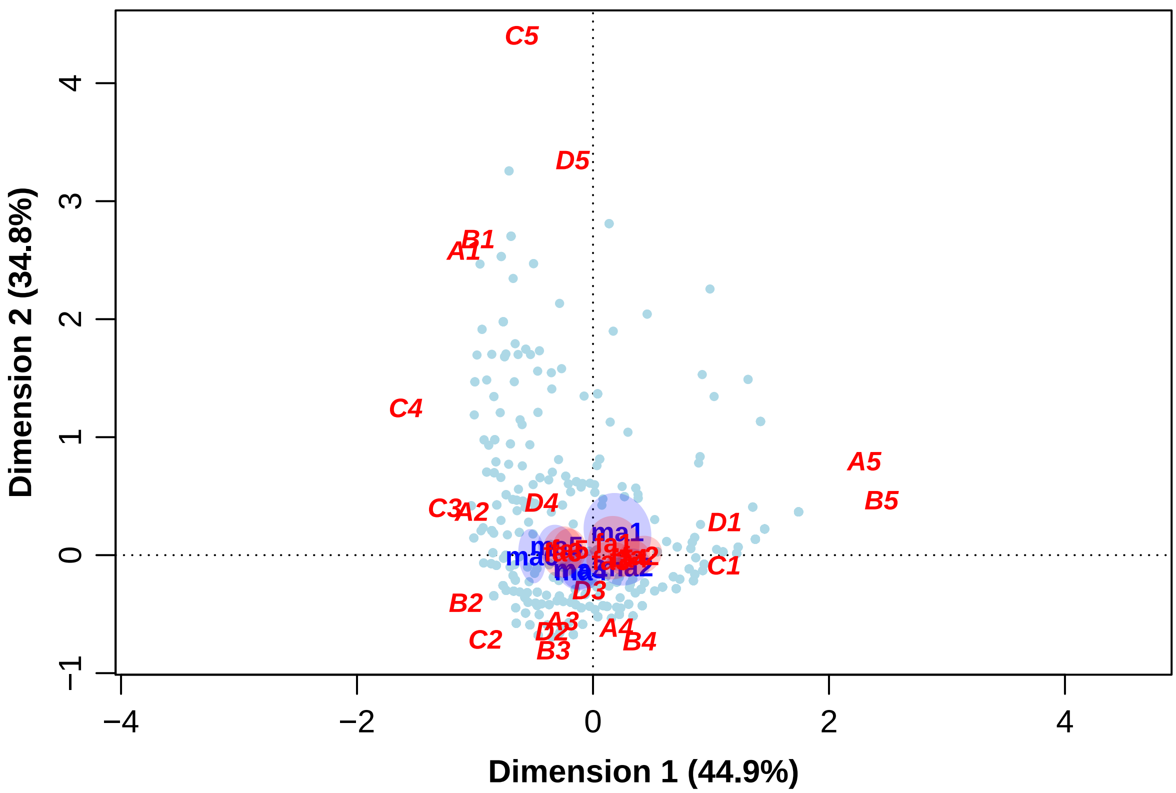
And add shading:

```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(-
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=c(rep("blue",6
                groupnames=c("ma1","ma2","ma3","ma4","ma5","ma6","fa1","fa2","fa3","fa4","fa5","fa6"),sh
```



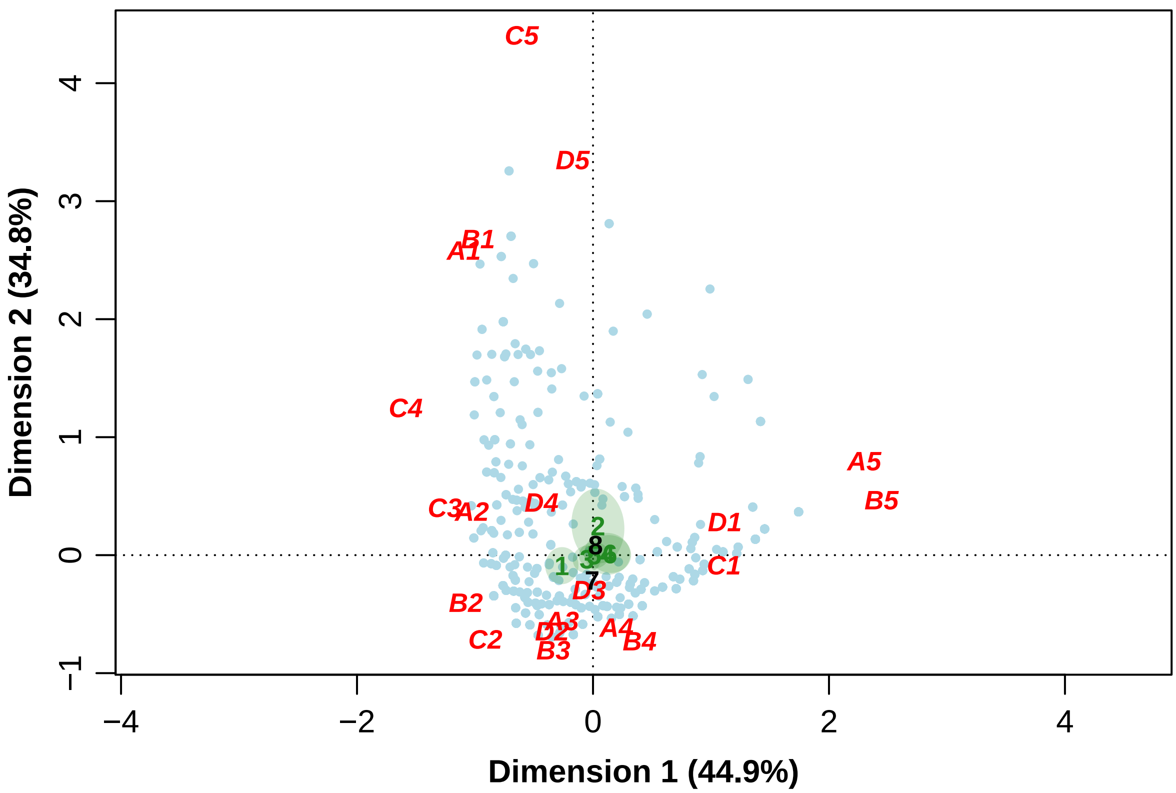
Next we go back to original plot, with category labels, gender-age labels slightly smaller (cex=0.8)

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=c(rep("blue",6
                groupnames=c("ma1","ma2","ma3","ma4","ma5","ma6","fa1","fa2","fa3","fa4","fa5","fa6"),sh
```



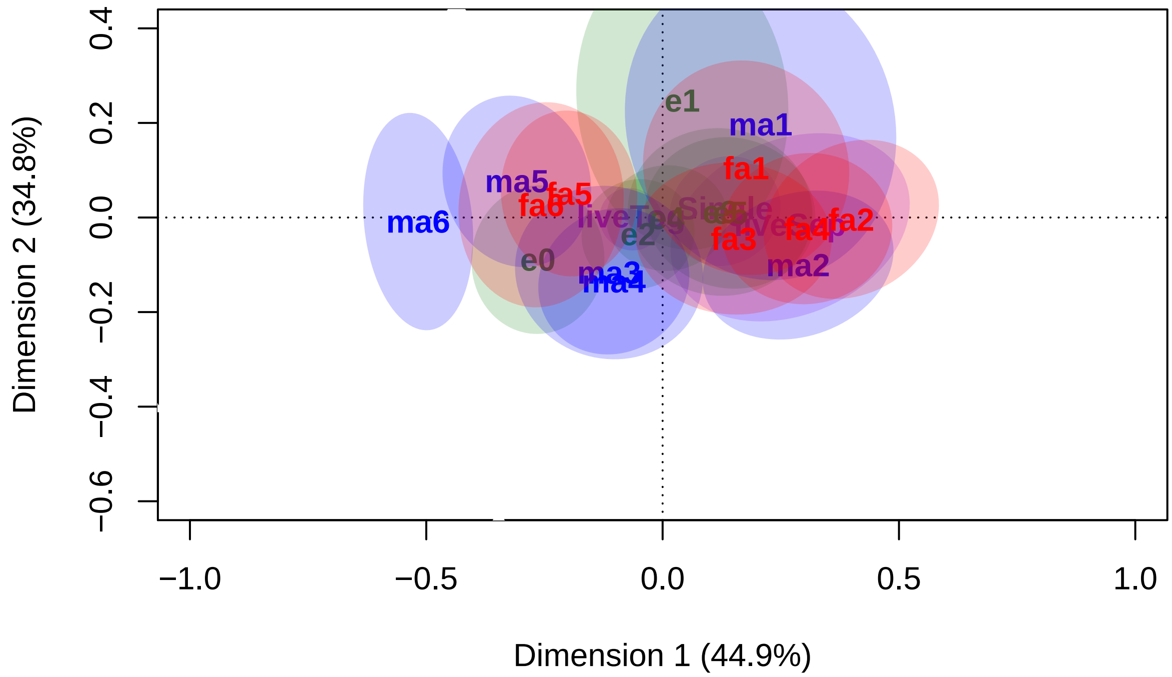
Back to other demographic variable, education:

```
par(mar=c(4.2,4,1,1), mgp=c(2,0.7,0), mfrow=c(1,1), font.lab=2)
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"))
points(Finland.rpc, pch=19, cex=0.5, col="lightblue")
text(Finland.mca1$colcoord, labels=Finland.mca1$levelnames, col="red", font=4, cex=0.8)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"e"]), groupcols=rep("forestgre
                shownames=T, add=T, cex=0.8)
```



Plot confidence ellipses for all demographic groups together in a zoom:

```
plot(Finland.mca3, labels=c(0,0), map="rowprincipal", col=c("black","white"), xlim=c(-0.5,0.5), ylim=c(
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"p"]), groupcols=rep("purple",6
                groupnames=c("liveTog","liveSep","Single"), shownames=T, add=T)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"e"]), groupcols=rep("forestgre
                groupnames=c("e0","e1","e2","e3","e4","e5"), shownames=T, add=T)
confidenceplots(Finland.rpc[,1], Finland.rpc[,2], group=factor(Finland[,"ga"]), groupcols=c(rep("blue",
                groupnames=c("ma1","ma2","ma3","ma4","ma5","ma6","fa1","fa2","fa3","fa4","fa5","fa6"),sh
```



Used and useful links

Mike Bendixen, A Practical Guide to the Use of Correspondence Analysis in Marketing Research

Oleg Nenadic and Michael Greenacre, Computation of Multiple Correspondence Analysis, with code in R

Biplots in practise

Multiple Correspondence Analysis Essentials: Interpretation and application to investigate the associations between categories of multiple qualitative variables - R software and data mining