1. Consider the following assumption we might impose on $(\succ_x)_{x\in X}$:

$$y \succ_x x \Longrightarrow y \succ_y x$$

   (a) Interpret this assumption in words. Explain why (or in what context) this is, or is not, a reasonable assumption.

   (b) Note that this assumption implies that our example of an empty choice is ruled out. (Our example had $X = \{x, y\}$, $y \succ_x x$ and $x \succ_y y$.) True or false: This assumption implies choice in the RD model is non-empty. If true, prove; if false, find a counterexample.

2. Recall that we defined IIA as follows:

$$\forall x \in A \subset B \subset X, x = c(B) \Longrightarrow x = c(A).$$

   We can't quite apply this IIA to the RD model because now choice from a set is a subset, so we need to change the notation. But the same idea is captured with the following condition:

$$\forall x \in A \subset B \subset X, x \in c(B) \Longrightarrow x \in c(A).$$

   Prove that the RD model satisfies this version of IIA.

3. Consider the RD model. Assume $X = \{a, b, c\}$. True or false (prove if true, provide a counter-example if false): If $c_{\succ_x}(X) = X$ then $c_{\succ_x}(A) = A$ for all $A \subseteq X$. Hint: If $a \in c(X)$ then what can you say about whether $a \succ_a b$? What about $a \succ_a c$? Then, if $a \in A$ what can you say about whether or not $a \in c_{\succ_x}(A)$?

4. Expansion, as studied in the SR model, said: if $c(A) = c(B) = x$ then $c(A \cup B) = x$. This notion applies only when the choice is a singleton, denoted here by x. In the RD model choice need not be a singleton. Describe a change in the expansion condition which would apply also when choice is not a singleton.

5. Are the following choices consistent with the SR-model when $X = \{x, y, z\}$? If your answer is yes, find asymmetric binary relations that lead to these choices when used by a decision maker in the SR model. Make sure that the two asymmetric relations lead to a single choice from each possible subset of X. If your answer is no, say whether EXPANSION or SR-IIA is violated.

   (a) $c(\{x, y\}) = x$, $c(\{y, z\}) = y$, $c(\{z, x\}) = z$.

   (b) Now consider $X = \{x, y, z, w\}$, $c(\{x, z\}) = x$, $c(\{y, z\}) = y$, $c(\{x, y, z, w\}) = x$, $c(\{x, y, z\}) = z$.

6. Suppose you observe the choices from all sets in $2^X \setminus \emptyset$. The choices you observe satisfy SR-IIA and EXPANSION. Consider the following relations $\gg_1^R$ and $\gg_2^R$:

   - $x \gg_1^R y \Leftrightarrow \nexists S \in 2^X \setminus \emptyset$ such that $y = c(S)$ and $x \in S$
   - $x \gg_2^R y \Leftrightarrow x = c(\{x, y\})$

   This exercise asks you to show that when you observe behavior that satisfies SR-IIA and EXPANSION then a decision maker that behaves according to the SR model and uses $\gg_1^R$

in the first stage of elimination and $\gg_2^R$ in the second stage makes exactly the choices you observe. This implies that for any choices consistent with SR-IIA and EXPANSION there is an SR-model that explains them.

Remember that observed choices are singletons. Also remember that x is eliminated from A in the round if and only if $\exists y \in A$ with $y \gg_1 x$. x is **not** eliminated in the second round if and only if $\nexists y \in A$ that survives the first round and for which $y \gg_2 x$. The element that is not eliminated in either round is chosen.

(a) Show that x survives the first round of elimination if the decision maker behaves according to the SR model and uses $\gg_1^R$ in the first round of elimination.

(b) Suppose that there is some $y \in A$ with $y = c(\{x, y\})$. Show that if $\forall z \in A \setminus \{y\}$ there is a set B_{zy} with $y = c(B_{zy})$ and $z \in B_{zy}$, then we must have that $x \neq c(A)$. (To do this you might first want to apply EXPANSION to all B_{zy} and then SR-IIA using to union of all B_{zy}.)

(c) Show that x survives the second round of elimination if the decision maker behaves according to the SR model, uses $\gg_2^R$ in the second round of elimination, and $\gg_1^R$ in the first round. (You might want to do this by showing that all elements that are better than x according to $\gg_2^R$ are eliminated in the first round using the result from the previous part.)

7. Consider the SR model with $\gg_1$ and $\gg_2$ that leads to the choice c_{12} as defined in class. Define $c_{21}(A) = c_1(c_2(A))$. Find an example to show that it is possible that $c_{12}(A) \neq c_{21}(A)$.

8. Prove directly that IIA implies Expansion. That is, show that if choice c satisfies IIA then it satisfies Expansion. Do not use the result that choice satisfying IIA corresponds to choice from a complete-strict-preference model.

9. Read and understand the following indirect proof the of the preceding claim that does use the result you were told not to use: Since the SR-IIA model includes the complete-strict-preference model as a special case, the complete-strict-preference model must satisfy SR-IIA and Expansion. Since we know that any choice satysfying IIA comes from a complete-strict-preference model, we know that any choice satysfyinf IIA models satifies SR-IIA and Expansion.