PROJECT — NOOR HAKAM

DUE: 5 DECEMBER 2022

$$\Upsilon = \underbrace{M(q)\ddot{q}}_{\text{INERTIA}} + \underbrace{V_m(q,\dot{q})\dot{q}}_{\text{CENTRIPETAL CORIOLIS}} + \underbrace{f_d\dot{q}}_{\text{DAMPING}} + \underbrace{T_d}_{\text{DISTURBANCE}}$$

Υ → TORQUE CONTROLLER

*DO NOT INCLUDE T_d IN CONTROLLER

q JOINT ANGLES → q_d DESIRED TRAJECTORY

$\dot{q}$ ANGULAR VELOCITY

$\ddot{q}$ ANGULAR ACCELERATION

CONTROL OBJECTIVE: $e = q_d - q$, AS $t \to \infty$, $e \to 0$

AUXILLARY SIGNAL: $r = \dot{e} + \alpha e$

$$\dot{r} = \ddot{e} + \alpha\dot{e}$$
$$\dot{r} = \ddot{q}_d - \ddot{q} + \alpha\dot{e}$$
$$M\dot{r} = M\ddot{q}_d - M\ddot{q} + M\alpha\dot{e}$$
$$M\dot{r} = M\ddot{q}_d - \left[\Upsilon - V_m\dot{q} - f_d\dot{q} - \cancel{T_d}\right] + M\alpha\dot{e}$$
$$M\dot{r} = M\ddot{q}_d - \Upsilon + V_m\dot{q} + f_d\dot{q} + \cancel{T_d} + M\alpha\dot{e}$$

LYAPUNOV FUNCTION: $V = \frac{1}{2}r^TMr + \frac{1}{2}e^Te$ → V IS P.D, DECRESENT, AND R.U

$$\dot{V} = r^TM\dot{r} + \frac{1}{2}r^T\dot{M}r + e^T\dot{e}$$
$$= r^T\left[M\ddot{q}_d - \Upsilon + V_m\dot{q} + f_d\dot{q} + \cancel{T_d} + M_\alpha\dot{e}\right] + \frac{1}{2}r^T\dot{M}r + e^T\dot{e}$$

↳ $r = \dot{e} + \alpha e$ $\quad$ $r = \dot{q}_d - \dot{q} + \alpha e$

$\dot{e} = r - \alpha e$ $\quad$ $\dot{q} = \dot{q}_d + \alpha e - r$

$$= r^T\left[M\ddot{q}_d + V_m\dot{q}_d + V_m\alpha e - V_m r + f_d\dot{q} + \cancel{T_d} + M\alpha\dot{e} - \Upsilon\right] + \frac{1}{2}r^T\dot{M}r + e^Tr - e^T\alpha e$$
$$= r^T\left[\frac{1}{2}\dot{M} - V_m\right]r + r^T\left[M\ddot{q}_d + V_m\dot{q}_d + V_m\alpha e + f_d\dot{q} + \cancel{T_d} + M\alpha\dot{e} + e - \Upsilon\right] - e^T\alpha e$$

↳ $r^T\left[\frac{1}{2}\dot{M} - V_m\right]r = 0$ BY SKEW SYMMETRY

$$\Upsilon = M\ddot{q}_d + V_m\dot{q}_d + V_m\alpha e + f_d\dot{q} + \cancel{T_d} + M\alpha\dot{e} + e + kr$$
$$= r^T\left[\cancel{M\ddot{q}_d} + \cancel{V_m\dot{q}_d} + \cancel{V_m\alpha e} + \cancel{f_d\dot{q}} + \cancel{M\alpha\dot{e}} + \cancel{e} - \left(\cancel{M\ddot{q}_d} + \cancel{V_m\dot{q}_d} + \cancel{V_m\alpha e} + \cancel{f_d\dot{q}} + \cancel{M\alpha\dot{e}} + \cancel{e} + kr\right)\right] - e^T\alpha e$$
$$= -r^Tkr - e^T\alpha e$$

→ $\dot{V}$ IS N.D

EXPONENTIAL STABILITY: $V = \frac{1}{2}r^TMr + \frac{1}{2}e^Te$

$$\frac{1}{2}\cdot\min\left[\lambda_{MIN}(M), 1\right]\left(\|r\|^2 + \|e\|^2\right) \le V \le \frac{1}{2}\cdot\max\left[\lambda_{max}(M), 1\right]\left(\|r\|^2 + \|e\|^2\right)$$

↳ $P_1 = \min\left[\lambda_{min}(M), 1\right]$

$P_2 = \max\left[\lambda_{max}(M), 1\right]$

$z = \left[e^T \quad r^T\right]^T$

$$\frac{1}{2}P_1\|z\|^2 \le V \le \frac{1}{2}P_2\|z\|^2$$
$$V \le \frac{1}{2}P_2\|z\|^2$$
$$\frac{2}{P_2}V \le \|z\|^2$$
$$\dot{V} = -r^Tkr - e^T\alpha e$$
$$\dot{V} \le -\min\left[k, \alpha\right]\left[\|r\|^2 + \|e\|^2\right]$$

↳ $\nu = \min\left[k, \alpha\right]$

$$\dot{V} \le -\nu\left[\|r\|^2 + \|e\|^2\right]$$
$$\dot{V} \le -\nu\|z\|^2$$
$$\dot{V} \le -\frac{2\nu}{P_2}V$$
$$V(t) \le V_0 e^{-2\nu/P_2 t}$$

→ G.E.S

↳ $\nu = \min\left[k, \alpha\right]$

$P_2 = \max\left[\lambda_{max}(M), 1\right]$